A Trade-off Theory of Ownership and Capital Structure*

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Abstract

This paper investigates the ownership connections between two units, when a tax-bankruptcy trade-off determines their cost of debt. We show that a hierarchical organization, with a parent that fully owns a subsidiary, is optimal when the parent tax rate is high enough to induce positive debt. Full ownership enables the parent to use all subsidiary dividends for the service of debt. Value is instead insensitive to subsidiary ownership when debt is optimally concentrated in the subsidiary, while the zero leverage parent helps it contain bankruptcy costs. Horizontal or pyramidal groups may become value maximizing when an enriched trade-off considers intercorporate dividend taxes and limits to interest deductions, beyond corporate income taxes. These ownership mutations may even neutralize such additional tax provisions.

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1. Introduction

Shareholders in control of multiple activities may directly own equity in each of them. This gives rise to a horizontal group, an organization that often characterizes family firms (Masulis et al., 2011). Groups are instead hierarchical when the controlling investors indirectly own the shares of one unit through another unit. Outside investors may buy minority stakes in affiliates of a hierarchical group, giving rise to pyramidal groups (La Porta et al., 1999), but this is not the rule. For instance, parent companies in U.S. multinational groups often fully own their subsidiaries (Lewellen and Robinson, 2013). One common trait of connected units is their reliance on debt financing (Huizinga et al., 2008; Kolasinski, 2009; Masulis, 2011). Connected units generate a total value added of 28 US$ trillion in over 200 countries (Altomonte and Rungi, 2013), yet finance scholars mostly focus on the case of one unit in isolation.

This paper sets out to explain the ownership structure of units that share a common controlling entity. We call them "units", instead of firms or banks, because in our model, as in Leland (2007), there is no explicit production or intermediation activity and hence no real synergy. The controlling entity initially owns the stochastic cash flows from the two activities. It maximizes their value, net of corporate taxes, choosing whether to own them directly, in a horizontal group, or indirectly, in a hierarchical group. When there is positive intercorporate ownership, a dividend flows to the parent, in proportion to its equity ownership in the subsidiary. The dividend flow also depends on the level and the cost of debt, both of which we determine together with ownership. As in trade-off theory, debt provides a tax shield because interest is deductible from the corporate income tax. At the same time, higher debt increases the expected default costs.

Our first result is that the value maximizing ownership of units is hierarchical, with a levered parent company that owns 100% of its subsidiary shares. This result holds for any combination of corporate income tax rates and proportional default costs. This hierarchical arrangement dominates both horizontal and pyramidal groups because the entire subsidiary dividend flows to the parent before eventually reaching ultimate shareholders. Dividends may thus help the parent repay its own debt, allowing it to better exploit the tax shield.

This ownership result refers however to a special case, that overlooks the presence of formal guarantees and informal bailout of affiliates in both industrial and financial groups (see Bodie and Merton, 1992; and Herring and Carmassi, 2009, respectively). Our general analysis therefore allows the parent company to support its profitable but insolvent affiliate as long as both of them survive (as in Boot et al., 1993; and Luciano and Nicodano, 2014). In this enlarged model, debts in the two units become substitutes due to the interplay of ownership and bailout connections. Higher parent debt reduces the level of its cash flows that remains available for the bailout of its affiliate, leading to lower optimal subsidiary debt and tax shield. Viceversa, higher subsidiary debt reduces the dividend transfer to the parent, and, in turn, optimal parent leverage and tax shield.

Our main result is that full ownership of subsidiary shares remains value maximizing if the income tax rate of the parent company is high enough to induce the parent company to reduce its tax burden through leverage. Value is instead insensitive to subsidiary ownership when the parent company tax rate is small enough so that zero leverage is optimal. Such zero-leverage unit may equally be a proper parent of a fully owned subsidiary, a proper parent in a pyramidal group or a unit in a horizontal group (that we will anyway call the "parent"). The parent gives up its tax savings to provide maximal support to the levered connected unit. This ownership irrelevance result is hard to reconcile with the typical trade-off intuition, that leads to interior optimal leverage instead of the concentration of all group debt in the subsidiary. This derives from endogenous debt pricing and costly default. As debt increases, lenders' losses upon default grow inducing a higher tax-deductible interest spread, while the deadweight cost of default is mitigated by bailout transfers. Such positive feedback from higher loss upon default to higher tax savings explains the possibility of total debt shifting onto the subsidiary.

So far the model only considers income taxes, for consistency with prior capital structure literature. However, regulatory authorities restrict group activities because of governance (Kandel et al., 2015), or financial stability (Herring and Carmassi, 2009) or tax motives (OECD, 2016). Our structural model allows the analysis of other provisions applying to groups. We therefore enrich the classical tax-bankruptcy trade-off by considering Intercorporate Dividend Taxes (IDT), that weigh on internal (or intercorporate) dividends, on top of the taxes on distributions to ultimate shareholders[1]. The imposition of IDT and, more generally, of any cost associated to subsidiary ownership, is able to break ownership irrelevance, triggering a mutation into a horizontal group, that leaves tax savings unchanged. Ownership transformations may occur as well in response to "Thin Capitalization" (TC) rules, that aim to limit interest deductions in groups. With a low parent tax rate, TC rules are unable to alter the tax shield when they apply to proper subsidiaries of hierarchical groups only, as it often happens. The proper subsidiary transforms into a directly owned firm, so as to preserve its value. When the parent tax rate is high, the introduction of TC does not alter full optimal ownership, but increases optimal parent debt so as to counterbalance the sub-optimal exploitation of the tax shield in the subsidiary, due to the binding cap on interest deductions. The obligation to pay IDT, beyond TC, makes partial subsidiary ownership optimal. Thus, the parent company owns subsidiary shares along with the controlling entity, minority financiers, or both.

Finally, we analyze the interaction of group-specific provisions. To this end, consider the prohibition to support a subsidiary Special Purpose Vehicle (SPV) imposed on banks by the Volcker Rule. A combination of IDT and the Volcker Rule may increase default costs, despite lowering the level of optimal debt, because IDT damages the only remaining internal support channel - the one based on internal dividends. On the contrary, a combination of IDT and TC rules may prevent excessive leverage while containing default costs by leaving the bailout channel at work. This juxtaposition reveals both the relevance of the internal capital market as well as the sensitivity of financial stability outcomes to tax design.

In the light of our analysis, internal support and the tax-bankruptcy trade-off help explain why group affiliates are often fully owned by their parent companies (Lewellen and Robinson, 2013). They may also explain some contrasting features of groups in the European Union (E.U.) and in the U.S.. E.U. parent units display a higher leverage than their subsidiaries and often own 100% of their affiliates (Bloch and Kremp, 1999; Bianco and Nicodano, 2006; Faccio and Lang, 2002). Moreover, the association between larger internal dividend payments and parent debt financing is visible in France (De Jong et al., 2012). These observations

[1]See Morck and Yeung (2005) for a historical perspective on intercorporate dividend taxes in the US and a discussion of their rationale.

are consistent with the fact that E.U. tax authorities do not tax internal dividends. In the U.S., on the contrary, intercorporate dividends are subject to taxes unless parent ownership exceeds a high threshold. Accordingly, evidence on family ownership (Villalonga and Amit, 2009; Masulis et al., 2011) shows that direct control via a horizontal structure is more common.

Other model implications remind aspects of both the private equity and securitization industries. Like the parent company in our ownership irrelevance proposition, the private equity fund (which may enjoy a low corporate income tax due to incorporation in tax havens) issues no debt and owns 100% of its leveraged portfolio firms, at times contributing to their debt restructurings. Tax savings in the Leveraged Buyout (LBO) deals contribute to value creation (Acharya et al., 2013; Kaplan, 1989; Renneboog et al., 2007), as in our results. In a financial conduit, there is a bailout mechanism in place because both the sponsoring unit (the parent) and the investors informally agree upon the state contingent support of the vehicle (Gorton and Souleles, 2006). Selling the cash flow rights of the supported vehicle (subsidiary) to outsiders generates an orphan subsidiary that avoids tax provisions applying to proper subsidiaries, while enjoying interest deductions. According to Han et al. (2015), securitization increases with the corporate tax rate, i.e. with incentives to exploit the tax shield.

The rest of the paper is organized as follows. Section 2 refers to the related literature, Section 3 presents the model. Section 4 characterizes the optimal internal ownership and leverage choices. Section 5 generalizes the trade off to IDT, highlighting its interaction with TC and no bailout provisions. Section 6 summarizes some relevant stylized facts. All proofs are in the Appendix.

2. Related Literature

This paper contributes to the theory of corporate ownership. Grossman and Hart (1986) determine the allocation of control rights in two productive units that minimize distortions in asset specific investment, assuming no separation between ownership and control. Subsequent theories highlight the choice of ownership by a controlling entity, that extracts private benefits from control, thereby determining the ownership share of minority shareholders and the control wedge. In Zingales (1995), the controlling entity separates ownership from control, through a pyramid or an alternative mechanism, to strengthen its bargaining power in future negotiations. In Almeida and Wolfenzon (2006) and Almeida et al. (2011), the choice between hierarchical and horizontal groups relies on the possibility to use minority shareholders' funds. The acquisition of a subsidiary through a parent permits the controlling family to use the retained earnings in the parent, that partly belong to minority shareholders, for funding projects with lower net present value in the subsidiary. The direct purchase of the subsidiary by the family does not instead permit diversion of retained earnings from the parent. We also study the choice of ownership by a controlling party. In our setting, without private benefits from control, value is insensitive to the control wedge because the controlling party and minority shareholders earn the same return, as in Demsetz and Lehn (1985). What matters is the parent ownership of subsidiary equity, because the transfer of subsidiary profits affects the tax-bankruptcy trade-off. Our shift of focus towards debt stems from the observation that groups very often receive all their outside funding from lenders rather than minority shareholders. An intermediate focus is present in Chemmanur and John (1996), who study the joint design of debt and ownership of two projects. Their model accommodates a wholly owned group when the entrepreneur has limited wealth: both separate incorporation and debt financing of the subsidiary protect private benefits from control.

Our work contributes to the taxes and corporate finance literature in two dimensions. Several papers analyze the effect of personal dividend taxes on the dividend payout, investment and equity issues (see Chetty and Saez, 2010, and references therein), ignoring the internal capital market and leverage. We instead fix payout, investment and equity issues and analyze how an enriched trade-off shapes leverage and internal support. As far as IDT are concerned, Morck (2005) observes that their introduction improves on corporate governance by making pyramidal groups more expensive, thereby discouraging the separation of ownership from control. Our model indicates that IDT may transform a wholly owned subsidiary into a partially owned one. Thus, IDT may coexist with pyramids, unless the statutory IDT tax rate decreases in the ownership share of the parent company.[2] Our model confirms that IDT dismantles pyramidal groups when ownership irrelevance prevails, prior to the introduction of IDT. In this case, however, such transformation affects neither the corporate leverage nor expected default costs.

Finally, our analysis advances our understanding of capital structure. First, we clarify the properties of the stand-alone firm developed in Leland (2007), showing that it has positive debt, due to the tax savings - cost of default feedback, even with a zero risk-free rate. The possibility of costly default generates, in fact, a positive interest spread and a tax shield. As for connected units, Huizinga, Laeven and Nicodeme (2008) study a trade-off model of group capital structure, showing that a change in the tax treatment of any affiliate prompts a change in leverage of all affiliates. This implication carries over to our model, that differs in three respects. First, we study the optimal internal capital market arrangements, that enhance the tax-bankruptcy trade-off. Second, we endogenously determine both default and the interest spread through a structural model of credit risk. This implies that higher debt financing increases tax savings not only directly, but also indirectly, through a rise in the tax-deductible interest spread. Both channels explain why the group may concentrate all leverage in one affiliate. Third, we endogenously determine subsidiary ownership.

Previous studies, that also price liabilities in connected units, explain that a merger allows for both a higher debt and a higher tax shield, with respect to independent units, because of support between its diversified segments. However, the contagion costs may offset the gains stemming from the tax shield when cash flow correlation is moderate (Leland, 2007). This is why a zero-leverage parent that supports its subsidiary exploits both the tax shield and diversification, while avoiding contagion thanks to limited liability (Luciano and Nicodano, 2014). Such a group has exogenous ownership and zero subsidiary payout. Our paper characterizes optimal group leverage when the payout ratio is positive and subsidiary ownership is endogenous. Thus, it provides a general trade-off theory of ownership and capital structure.

[2]This observation provides a rationale for the statutory IDT tax rate in the US tax code, which is decreasing in the ownership share.

3. The Model

This section describes our set-up, that follows Leland (2007) in modeling endogenous leverage and bankruptcy costs. The following section provides details on the internal linkages. At time 0, a controlling entity owns two units, $i = P, S$.[3] Each unit has a random operating cash flow X_i which is realized at time T. We denote with $G(\cdot)$, the cumulative distribution function and with $f(\cdot)$ the density of X_i, identical for the two units; $g(\cdot, \cdot)$ is the joint distribution of X_P and X_S. At time 0, the controlling entity selects the face value F_i of the zero-coupon risky debt to issue, so as to maximize the total arbitrage free value (ν_{PS}) of equity, E_i, and debt, D_i:

$$\nu_{PS} = \max_{F_P, F_S} \sum_{i=P,S} (E_i + D_i) \,. \tag{1}$$

At time T, realized cash flows are distributed to financiers. Equity is a residual claim: shareholders receive operational cash flow net of corporate income taxes and the face value of debt paid back to lenders. A unit is declared insolvent when it cannot meet its debt obligations. Its income, net of the deadweight loss due to default costs, is distributed first to the tax authority and then to the lenders.

The unit pays a flat proportional income tax at an effective rate $0 < \tau_i < 1$ and suffers proportional dissipative costs $0 < \alpha_i < 1$, in the case of default. Interest on debts are deductible from taxable income.[4] The presence of a tax advantage for debt generates a trade-off for the unit: on the one hand, increased leverage results in tax benefits, while on the other, it leads to higher expected default costs since – everything else being equal – a highly levered unit is more likely to default. Maximizing the value of debt and equity is equivalent to minimizing the cash flows the controlling entity expects to lose in the form of taxes (T_i) or of default costs (C_i):

$$\nu_{PS} = \min_{F_P, F_S} \sum_{i=P,S} T_i + C_i. \tag{2}$$

The expected tax burden of each unit is proportional to the expected taxable income, that

[3] The subsidiary, S, can be thought of as the consolidation of all other affiliates.
[4] No tax credits or carry-forwards are permitted.

is to the operational cash flow X_i, net of the tax shield X_i^Z. In turn, the tax shield coincides with interest deductions, which are equal to the difference between the nominal value of debt F_i, and its market value D_i: $X_i^Z = F_i - D_i$. The tax shield is a convex function of F_i.

The expected tax burden in each unit separately – each taken as a stand-alone (SA) unit – is equal to (Leland, 2007):

$$T_{SA}^i(F_i) = \tau_i \phi \mathbb{E}[(X_i - X_i^Z)^+], \tag{3}$$

where the expectation is computed under the risk neutral probability and ϕ is the discount factor. The superscripts and subscripts i indicate whether the stand alone unit is endowed with the parent ($i = P$) or subsidiary ($i = S$) parameters. Increasing the nominal value of debt increases the tax shield, thereby reducing the tax burden because the market value of debt, D_i, increases with F_i at a decreasing rate (reflecting a higher risk).

Similarly, default costs are proportional to cash flows when a default takes place (i.e., when net cash flow is insufficient to reimburse lenders). A default occurs when the level of realized cash flow is lower than the default threshold, $X_i^d = F_i + \frac{\tau_i}{1-\tau_i} D_i$:

$$C_{SA}^i(F_i) = \alpha_i \phi \mathbb{E}\left[X_i 1_{\{0 < X_i < X_i^d\}}\right]. \tag{4}$$

Default costs represent a deadweight loss to the economy. They increase in the default cost parameter, α_i, as well as in (positive) realized cash flows when the unit goes bankrupt. A rise in the nominal value of debt, F_i, increases the default threshold, X_i^d, thereby increasing the expected default costs. It can be shown that the default threshold X_i^d of a stand-alone unit is concave in the face value of debt F_i. Luciano and Nicodano (2014) show that the stand alone company has positive optimal debt if the sum of tax burden and default costs is convex in the face value of debt. This stand alone company raises positive debt even if the riskfree rate is zero, differently from a stand-alone with a riskless cash flow. Moreover, it can be shown that it raises more debt than a "traditional" stand alone company. Indeed, while the latter trades off constant marginal tax savings with increasing marginal default costs, the cost of debt and tax savings both increase at an increasing rate in the former.

The following section introduces transfers from connected units, that reduce the expected

Ownership Structures

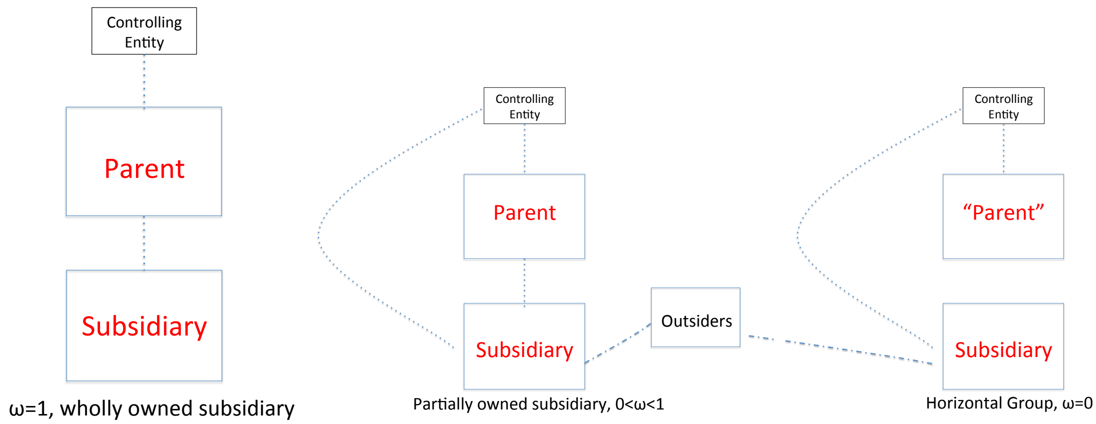

Figure 1: This figure depicts the three organizations that our model can reproduce: a parent with its fully-owned subsidiary (left panel), a parent that partially owns its subsidiary(center panel) or a horizontal group (right panel).

default costs generated by a given level of debt. It will become clear that bailout transfers also reduce lenders' recovery upon default, thereby shifting the cost of debt to its tax-deductible component.

3.1. Internal Bailouts and Ownership

This section provides details on linkages between units. We first model internal ownership and bailout transfers that characterize complex organizations. Next, we assess how the two links impact on both the tax burden and the default costs of the group, given exogenous debt levels.

The parent owns a fraction, ω, of its subsidiary's equity. When ω=1, the controlling entity owns the subsidiary only indirectly, a case that appears in the left-hand side of Figure 1. When ω=0, there is no parent ownership. The controlling entity will either own the subsidiary directly or will sell it to outside shareholders or both, as in the right-hand side of Figure 1. The intermediate case is a mix of parent, controlling and/or outside shareholding.[5]

The subsidiary distributes its profit after paying the tax authority and lenders, $(X_S^n - F_S)^+$, where X_S^n are its cash flows, net of corporate income taxes. Assuming a unit payout

[5]The controlling entity in this model has no role but the design of ownership and capital structure. There are several ways a party may keep control without ownership, see the case of SPVs, private equity and families (Villalonga and Amit, 2009).

ratio, the parent receives a share ω of the subsidiary profits at time T. Let the effective (i.e., gross of any tax credit) tax rate on the intercorporate dividend be equal to $0 \leq \tau_D < 1$.[6] Thus, the IDT is equal to a fraction $\omega\tau_D$ of the subsidiary cash flows. Consequently, the expected present value of the after tax internal dividend is equal to:

$$ID = \phi\omega\mathbb{E}\left[(1-\tau_D)(X_S^n - F_S)^+\right]. \tag{5}$$

The cash flow available to the parent, after receiving the internal dividend, increases to:

$$X_P^{n,\omega} = X_P^n + (1-\tau_D)\omega(X_S^n - F_S)^+. \tag{6}$$

Equation (6) indicates that internal dividends provide the parent with an extra buffer of cash that can help it remain solvent in adverse contingencies in which it would default as a stand alone company. It follows that the dividend transfer generates an internal rescue mechanism, whose size increases in the parent ownership, ω, and falls in the dividend tax rate, τ_D, given the capital structure. In other words, positive subsidiary ownership by the parent contributes to the creation of an internal capital market. Equation (6) highlights that internal flows are useful for avoiding the parent company's default, instead of serving for financing investment projects, as in Stein (1997).

We do not analyze personal dividend and capital gains taxation levied on shareholders (other than the parent). Therefore, we assume that the positive personal dividend (and capital gains) tax rate is already included in τ_i, which is indeed an effective tax rate. We also assume that the personal tax rate on distributions is equal across the parent and the subsidiary, so as to rule out straightforward tax arbitrage between the two.

Another characteristic of the internal capital market is the internal bailout, that we model following Luciano and Nicodano (2014). The parent transfers cash flows to the other unit if the latter, which is insolvent but profitable ($0 < X_S^n < F_S$), becomes solvent thanks to the transfer $F_S - X_S^n$, while the parent remains solvent ($X_P^n - F_P \geq F_S - X_S^n$).[7]

[6] Note that we can interpret IDT as any negative synergy, related to internal ownership, that depletes dividend distribution to the parent.

[7] In a static model, the ex-post enforcement of bailouts must rely on courts. In practice, enforcement

Note that neither dividends nor the bailouts affect the solvency of the provider. They thus differ from both internal loans and unconditional guarantees, that help the recipient unit service its debt impairing the service of the giver's debt. Bailouts, moreover, are contingent on positive subsidiary cash flows, while dividends, loans and guarantees are not. This conditionality implies that the subsidiary is less likely to go bankrupt when it has positive cash flows, a fact that reduces lenders' recovery upon default. Anticipating this, lenders will then require a higher spread, which is tax deductible. This feature will turn out to affect the trade-off between parent and subsidiary debt. Clearly, it is in principle possible to model a zero dividend payout contingent on the parent default, making dividends exactly symmetric to bailouts. However, we stick to an unconditional payout, that captures the ability for parent's lenders to recover subsidiary profits through a revocatory action.

We can now show how dividends and the bailout promise affect default costs and the tax burden of the group.

3.2. The Tax Bankruptcy Trade-Off in Complex Organizations

This section amends the tax-bankruptcy trade-off for the presence of both the bailout (following Luciano and Nicodano, 2014) and internal dividends. Recall that Equations (3) and (4) define the expected tax burden, $T^i_{SA}(F_i)$, and default costs $C^i_{SA}(F_i)$ for a stand alone unit. Below we analyze their change given the debt levels, F_P and F_S, that will become endogenous later on. As in previous studies of groups, we assume the limited liability of the parent unit with respect to the debt of its subsidiary, even in the limiting case when the parent is the sole owner of its subsidiary. However, we also assume that the assets of the subsidiary are subject to the claims of the parent lenders, should the parent default, if internal ownership is positive.[8]

Default costs in the subsidiary, C_S, fall when there is a bailout transfer from the parent, while the tax burden of the group is unaffected. The reduction in expected default costs (Γ)

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mechanisms vary from reputation (Boot et al., 1993) to the purchase of the junior tranche by the sponsoring parent (De Marzo and Duffie, 1999).

[8]This *de facto* asymmetry of claims with equal priority appears in group restructurings (see, for instance, Penati and Zingales (1997)).

is equal to:

$$\Gamma = C_{SA}^{S}(F_S) - C_S = \alpha_S \phi \mathbb{E}\left[X_S 1_{\{0<X_S<X_S^d, X_P \geq h(X_S)\}}\right] \geq 0. \tag{7}$$

Subsidiary default costs are lower, the greater the ability of the parent to rescue its subsidiary. The indicator function $1_{\{\cdot\}}$ defines the set of states of the world in which the rescue occurs, when the subsidiary defaults without transfers (first term) and the parent funds are sufficient for rescue (second term). The function h, which is defined in the Appendix, implies that the rescue by the parent is likelier the smaller the parent debt, F_P.

Dividends leave the subsidiary trade off unchanged, affecting however both the default costs and the tax burden of the parent as follows. The cum-dividend cash flow in the parent – defined in equation (6) – increases with internal ownership, ω. Such additional cash flow raises the chances that the parent is solvent. It also increases the lenders' recovery rate in insolvency, should the parent go bankrupt anyway. This last feature differentiates internal dividend transfers from the bailout transfers described earlier. It captures the asymmetry of limited liability in groups, that protects the parent from but subjects the subsidiary to the claims of the other unit lenders. Expected default costs saved by the parent, ΔC, are then equal to:

$$\Delta C = C_{SA}^{P}(F_P) - C_P = \alpha_P \phi \mathbb{E}\left[X_P \left(1_{\{0<X_P^n<F_P\}} - 1_{\{0<X_P^{n,\omega}<F_P\}}\right)^+\right] \geq 0. \tag{8}$$

The first (second) term in square brackets measures the amount of parent cash flows that is lost in the default, without (with) the dividend transfer. It is easy to prove that the parent default costs fall in the dividend receipts, net of taxes. These, in turn, increase in $\omega(1-\tau_D)$ and decrease in subsidiary debt.

Finally, when there are taxes on intercorporate dividends, the group tax burden increases, relative to the case of two stand alone units. We denote this change as ΔT, defined as:

$$\Delta T = T_S + T_P - T_{SA}^{P}(F_P) - T_{SA}^{S}(F_S) = \phi \omega \tau_D \mathbb{E}[(X_S^n - F_S)^+] \geq 0. \tag{9}$$

This is positive and increasing in the subsidiary dividend. In turn, the dividend increases in the profits after the service of debt, $(X_S^n - F_S)^+$, and in the internal ownership ω.

To conclude this section, the following lemma summarizes the response of additional default costs and tax burden with respect to the debt levels and internal ownership.

Lemma 1. *The default costs saved by the subsidiary, Γ, are non-increasing in parent debt. The default costs saved by the parent, ΔC, are non-decreasing in internal ownership. Both internal dividends and the additional tax burden, ΔT, are non-increasing in the subsidiary debt, insensitive to parent debt, and non-decreasing in internal ownership.*

This lemma highlights a trade-off between increasing debt in the parent and debt in the subsidiary, due to internal ownership. The higher the subsidiary debt, the lower the subsidiary dividends, given its exogenous cash flow and the subsidiary ownership, ω. This dividend reduction results in higher default costs in the parent, at each level of debt. IDT enhances this trade-off.

4. Ownership and Capital Structure

This section determines the capital structure (F_P and F_S) and the internal ownership share ω that maximize the joint value (or minimize total default costs and tax burdens) of the two units, solving:

$$\min_{F_S \geq 0, F_P \geq 0, 0 \leq \omega \leq 1} T_S + T_P + C_S + C_P. \tag{10}$$

Throughout the paper, we maintain the standard technical assumption of convexity of the objective function, with respect to the face values of debt.[9] We report the Kuhn-Tucker conditions associated with the minimum program at the beginning of Appendix B. We denote the optimal debt and ownership, solving (10), as F_S^*, F_P^*, ω^*. The value-maximizing organization may result in a hierarchical, $\omega^* > 0$, or horizontal, $\omega^* = 0$, group.

Section 4.1 presents and discusses the main theorem of this paper, assuming no IDT. The model allows to compute several endogenous magnitudes, including taxes, the dividend and bailout transfers characterizing the internal capital market, the cost and market value of debt, the market value of equity, default probability and losses upon default - beyond the

[9]Extensive numerical analysis shows that convexity in the relevant solution range holds true, unless the effective tax rate is much higher than the proportional default cost rate. Also, for technical reasons, we require the joint density g(x,y) to be increasing in its second argument for every $x \in [0, 2F_S]$ and $y \in [0, \frac{F_S}{1-\tau_S}]$.

book value of debts and the ownership share. A presentation of two numerical exercises, based on Leland (2007) calibration, allows to appreciate the heterogeneity of optimal capital structure for units with the same tax-bankruptcy parameters.

4.1. Optimal Hierarchical Groups

Let us first consider the optimal choice of ownership without bailouts, for simplicity. In this no bailout case, it is easy to show that full subsidiary ownership is optimal. We in fact know that stand-alone firms face default costs because they always have positive optimal leverage, as shown in Luciano and Nicodano (2014). The ownership link helps improving the tax bankruptcy trade-off by decreasing default costs in the parent, at the stand-alone level of debt, without worsening the tax-bankruptcy trade-off in the subsidiary.

Lemma 2. ***No bailout.*** *Let there be no bailout. Then, the optimal subsidiary ownership is full ($\omega^* = 100\%$) and the parent has positive leverage ($F_P^* > 0$). Moreover, optimal debt in the parent exceeds the one in a stand-alone unit ($F_P^* > F_{SA}^{P*}$), if the condition* (B.9)*, stated in the Appendix, holds.*

This lemma highlights that optimal subsidiary ownership by the parent is full because internal dividends are value increasing. Under a mild sufficient condition, dividends induce the parent company to raise additional debt relative to such a comparable stand alone unit. This condition requires that not all the dividend is lost in the parent default, when the realized cash flow of the parent is equal to the default threshold. It is worth stressing that this is only a sufficient condition, the result holding for a much wider set of cases. However, the condition suggests that it is unlikely that the parent will raise all group debt, even if it presented a very favourable tax-bankruptcy trade-off. The reason is that the chances of losing all the dividend in the parent default increase together with parent debt.

In the general case in which both mechanisms interact in the internal capital market, there is a tension between the payment of internal dividends and the bailout transfer. The effect described in Lemma 2 is counterbalanced by an opposite incentive towards larger subsidiary debt, and smaller parent debt, to exploit the value of the bailout. The following theorem characterizes the optimal internal ownership and capital structure, maintaining the absence of dividend taxes.

Theorem 1. ***Bailout and dividends.***

(i) Assume $\tau_P \leq z$. Then, internal ownership (ω^*) is indefinite. Moreover, the parent has zero leverage ($F_P^* = 0$). Finally, group debt exceeds the debt of two comparable stand-alone units, if and only if the ratio of default costs and tax savings of the subsidiary is lower than a constant Q;

(ii) If $\tau_P > z'$, then the parent fully owns its subsidiary ($\omega^* = 1$). Moreover, it has positive leverage ($F_P^* > 0$).

Part (i) of the theorem is an ownership-irrelevance proposition. It implies that horizontal groups, along with hierarchical and pyramidal groups, deliver the same value to their shareholders, if the parent tax rate, τ_P, is sufficiently low. On the contrary, Part (ii) states that only hierarchical groups with full internal ownership maximize value when the tax rate of the parent is high enough.

The explanation for these ownership patterns is possible thanks to their joint characterization with optimal capital structure. Under the conditions of Part (i), the parent raises no debt. Minimal parent debt maximizes the magnitude of the set of states in which the parent is able to provide a bailout transfer, allowing for both large subsidiary debt and tax gains. It also maximizes the tax burden in the parent company, whose value is however limited for a sufficiently low parent tax rate. As shown by Luciano and Nicodano (2014), in this case, the total debt raised by the group is greater than that of two stand-alone units, if the ratio of default cost to tax rates in the subsidiary is not excessive.

When the conditions of Part (ii) apply, the parent does not specialize in providing support, because it gains more from increasing its own tax shield than from lowering the cost of debt financing in the subsidiary via larger bailout transfers. Subsidiary dividends, in turn, help service parent debt, thereby allowing it to increase its own tax shield. Thus, Part (ii) indicates that the result of Lemma 2 may carry over to the general case, with full indirect ownership allowing for maximal dividend support.

The parent company in the hierarchical group has positive optimal debt. This implication radically departs from the one in Luciano and Nicodano (2014), for two reasons. The main difference rests in endogenous ownership, that is an additional equation (condition (viii)) in the system (B.1) of Kuhn Tucker conditions. This change matters because we allow for a positive dividend payout, which is set to zero in Luciano and Nicodano (2014), implying that internal dividends complement internal bailouts in shaping the internal capital market. In sum, Luciano and Nicodano (2014) highlight a striking special case of capital structure, that reveals the power of bailouts when the cost of debt and default are endogenous. Our Theorem 1 is a general trade-off theory of capital and ownership structure.

The cut-off parent tax rate levels, z and z', are increasing in τ_S and inversely u-shaped in α_S. A higher corporate tax rate in the subsidiary increases the likelihood that the parent has zero leverage, as marginal tax savings obtained from additional subsidiary debt are higher. In turn, a higher default cost rate in the subsidiary triggers two effects. On the one hand it makes the parent support more valuable, in the set of states when rescue succeeds. On the other hand, it increases the cost of subsidiary debt and the default threshold, making it more likely that the subsidiary will default. For low α_S, the first effect prevails and z increases together with the chances of zero parent leverage, while the second effect overcomes for high α_S. Finally, z' is decreasing in α_P indicating that positive parent debt becomes less likely, while z is independent of α_P being computed at $F_P = 0$.

4.1.1. Understanding Heterogeneous Ownership and Leverage

This section illustrates, by means of a numerical example, how the tax bankruptcy trade-off shapes groups, their leverage and their internal capital market. The tables below show the optimal characterization of the group for different parametric combinations, when cash flows are jointly normally distributed. Table 1 will help understand the heterogeneity in firm capital structure and valuation, the complementarity between debt and non-debt tax shields and the asymmetric consequences of dividends and bailouts. Table 2 will reveal that optimal debt is lower in more diversified groups.

Table 1 reports changes, in response to variation in both the parent tax rate and cash flow volatility, in a well diversified group with affiliate cash flow correlation equal to -0.8. When the parent tax rate is high, as in the first two columns, the parent fully owns its subsidiary. Ownership irrelevance instead prevails in the last column, when the parent tax rate falls. The table displays, in each row, several endogenous variables including the ownership share in the subsidiary and the optimal face value of debt. In this parametrization, the expected value of cash flows is set to 100 in each unit, so as to standardize each endogenous balance sheet item highlighted in the rows.

The benchmark case is the parametrization of Leland (2007) for BBB-rated firms, in the third column of Table 1. In this column, all parameters are equal across units. With both corporate tax rates at 20%, subsidiary ownership turns out to be irrelevant for group value and the parent has zero leverage. The bailout transfer is large (47.74), supporting the high subsidiary debt (183) and containing its cost (resulting from the difference between the face and the market value of debt, that is 183-133.58). Such high debt implies high default costs (1.91), despite the bailout transfer, but allows for a low tax burden (12.45) compared to the one of the zero-leverage parent (20.01). The dividend, which is distributed to the parent only if the group is hierarchical, is negligible precisely because the service of subsidiary debt absorbs most of the resources.

A higher parent tax rate, 26% vs. 20%, leads to a hierarchical group with 100% subsidiary ownership (as in the first and second columns). In the second column, when all other parameters follow Leland (2007) choices, the large internal dividend (79.30) supports the highly-levered parent (158), that enjoys a relatively low tax burden (18.97) when considering the 26% tax rate, while the almost zero-leverage subsidiary pays higher taxes (19.98) despite its lower tax rate.

In both cases, group capital structure is extreme. The group may also display a more balanced capital structure, as in the first column, where the standard deviation of cash-flows is twice as large as in the previous case. Higher risk makes the tax shield more valuable, because the firm pays taxes when profitable but does not receive refunds when unprofitable. Increased volatility also implies greater default probability, and greater chances that dividends will be lost in the default of the parent. This induces a substitution effect towards subsidiary debt. The group exploits in this case both support channels. In all the three cases, the internal capital market increases group value above the one of unconnected, stand alone companies, reported in the last row of the table.

There are three additional insights deriving from this exercise. The first concerns the heterogeneity of optimal leverage in units with the same tax-bankruptcy parameters, first observed by Bernanke, Campbell and Whited (1990). The subsidiary in the third column

Table 1: Group Structure, Parent Tax Rates and Cash Flow Volatility			
	Full ownership		**Ownership irrelevance**
	$\tau_P = 26\%, \sigma = 44\%$	$\tau_P = 26\%, \sigma = 22\%$	$\tau_P = 20\%, \sigma = 22\%$
Value (ν)	165.50 (77.63; 87.87)	160.52 (159.74;0.78)	166.32 (32.74;133.58)
Ownership share (ω)	100%	100%	Any
Face Value of Debt (F)	182 (40;142)	159 (158;1)	183 (0;183)
Debt Value (D)	118.09 (30.22;87.87)	123.70 (122.92;0.78)	133.58 (0;133.58)
Cum-Dividend Equity Value (E)	62.30(47.42;14.88)	116.12(36.82;79.30)	33.17 (32.74;0.43)
Default Costs (C)	2.85 (0.24;2.61)	0.78 (0.78;0)	1.91 (0;1.91)
Tax Burden (T)	38.69 (25.14;13.55)	38.95 (18.97;19.98)	32.45 (20.01;12.45)
Dividend Transfer (ID)	14.88	79.30	0.43 $\times\omega$
Bailout Transfer (B)	15.35	0	47.74
Value of SA units	164.91 (80.07;84.84)	157.52 (76.05;81.47)	162.94 (81.47;81.47)

Table 1: This table displays the optimal group structure with different parameteric combinations, leading to full ownership or ownership irrelevance. Unspecified parameters follow the basecase in Leland (2007): $\alpha_P = \alpha_S = 23\%, \tau_S = 20\%$. Cash flows are jointly Gaussian, with correlation $\rho = -0.8$. $\sigma_S = \sigma_P = \sigma$ is the annualized percentage volatility of cash flows. Their discounted expected value is $X_0^S = X_0^P = 100$. Parent and Subsidiary figures are in brackets. The subsidiary Cum-Dividend Equity Value includes the dividend transferred to the parent. Value is the sum of the debt and equity (net of dividend) claims in each unit.

displays the same tax-bankruptcy parameters of the proper subsidiary in the second column, but has much higher debt (183 vs. 1). An intermediate case is the stand-alone unit, that has debt equal to 57. This debt heterogeneity is due to transfers to/from connected units. Both explain, in turn, the heterogeneous cum-dividend equity values, ranging from 0.43 in the first case, to 79.30 in the proper subsidiary and to 81.47 in the stand-alone unit.

The second takeaway regards the relationship between non-debt tax sheltering and the debt tax shield. In a stand-alone unit, they are substitutes (De Angelo and Masulis, 1980; Graham and Tucker, 2006) because the reduction in the effective corporate income tax weakens the incentive to raise debt. In connected units, they become complements. Consider in fact a non-debt tax shelter, such as the incorporation of the parent in a lower tax rate jurisdiction. The consequences of this choice are represented by the change from the second to the third column. Optimal debt increases (from 159 to 183), because the subsidiary has unchanged tax rate while the lower parent tax rate permits the provision of larger bailout funds.

The last insight concerns the asymmetry between dividend and bailouts. When the units display the same tax rates and default costs, as in the third column of Table 1, zero parent debt is optimal. This pattern indicates that the bailout guarantee is more valuable than the dividend transfer. The reason is subtle, and appears only because of endogenous default and

Table 2: Hierarchical Group Structure and Cash Flow Correlation					
	Correlation				
	-0.8	-0.5	0	0.5	0.8
Value (ν)	168.95 (128.26; 40.69)	168.56 (101.18; 67.38)	168.63 (102.27; 66.36)	169.71 (129.12; 40.59)	171.56 (128.08; 43.48)
Ownership share (ω)	100%	100%	100%	100%	100%
Face Value of Debt (F)	153 (94; 59)	174 (67;107)	198 (85; 113)	241 (176; 65)	258 (187; 71)
Debt Value (D)	113.05 (72.36; 40.69)	115.84 (48.46; 67.38)	121.08 (54.72; 66.36)	132.69 (92.10; 40.59)	135.27 (91.79; 43.48)
Cum-Dividend Equity Value (E)	106.56 (55.90; 50.66)	81.21 (52.72; 28.49)	74.06 (47.55; 26.51)	84.83 (37.00; 47.83)	81.08 (36.28; 44.80)
Default Costs (C)	1.68 (1.45; 0.23)	3.30 (1.54; 1.76)	5.46 (2.56; 2.90)	10.24 (8.92; 1.32)	10.74 (9.04; 1.70)
Tax Burden (T)	37.56 (23.03; 14.53)	35.86 (23.57; 12.29)	32.93 (21.35; 11.58)	27.10 (13.22; 13.88)	25.28 (11.74; 13.54)
Dividend Transfer (ID)	50.66	28.49	26.51	47.83	44.80
Bailout Transfer (B)	2.58	6.38	3.79	0.09	0.003

Table 2: Cash flows are jointly Gaussian with $X_0^S = X_0^P = 100, \sigma_S = \sigma_P = \sigma = 44\%$. The parameters are $\tau_P = 0.26, \tau_S = 0.16, \alpha_P = \alpha_S = 0.23, \tau_D = 0$. Parent and Subsidiary figures are in brackets. Cum-Dividend Equity Value is the value of equity in each unit, that, for the subsidiary, includes the dividend transferred to the parent. Value is the sum of the debt and equity claims in each unit, where subsidiary equity value is ex-dividend.

debt pricing. A share α_P of dividends gets lost when the parent defaults, while the bailout is conditional on the survival of the subsidiary.[10] The resulting lower recovery upon default for subsidiary lenders leads to a higher interest spread, which is tax deductible. In other words, a larger share of the subsidiary's cost of debt is tax deductible.

Table 2 displays the structure of a hierarchical group as cash flow correlation varies. We stick to the same parameters as in the first column of the previous table, but change to a lower subsidiary tax rate, that ensures full subsidiary ownership. As correlation increases, so do total debt, total default costs and total taxes. This pattern, which reverses the intuition "more diversification leads to higher debt capacity" (Leland, 2007), is persistent across parameter sets. The logic is as follows. Lenders to diversified groups charge relatively low spreads, anticipating support against insolvency through the internal capital market. This contains tax savings, debt and default costs. On the contrary, when there is high positive correlation, lenders anticipate that the unit with larger debt will likely default when it has non-positive cash-flows. They will therefore charge a high spread, that will lead to high tax savings, high debt and high default costs. The table reveals that the parent company does not always display higher debt than its subsidiary, despite the large tax rate differential. Accordingly, the ratio of dividend to bailout transfers does not behave monotonically with cash flow correlation.

In this section, we have considered no other duty beside corporate income taxes so as to

[10]This modeling captures the fact that a proper parent company has claims to subsidiary's assets, while the parent company enjoys limited liability.

nest prior results. The next section allows for a general tax-bankruptcy trade-off that also encompasses group-specific regulatory provisions.

5. A Generalized Tax Bankruptcy Trade Off

Complex organizations are of interest to regulators because of either governance, or tax receipts or financial stability. IDT have a governance rationale. According to Morck (2005), IDT help dismantle pyramids, that permit the expropriation of minority shareholders when the controlling entity enjoys private benefits from control. Sections 5.2 and 5.3 will in turn highlight TC rules and "no bailout" rules, that respectively stem from a tax receipt and a financial stability rationale, also in combination with IDT. Major OECD countries impose TC rules, and enforce them especially on domestic subsidiaries of foreign groups. Her Majesty Revenue and Customs (INTM541010) describes their rationale: "Thin capitalization commonly arises where a company is funded...by a third party...but with guarantees...provided to the lender by another group company... The effect of funding a .. company.. with excessive ... guaranteed debt is...excessive interest deductions. It is the possibility of excessive deductions for interest which the ..legislation on thin capitalization seeks to counteract." Finally, the Volcker Rule (i.e. Section 619 of the Dodd-Frank Act) amends the Bank Holding Company Act of 1956, prohibiting banking entities from entering into transactions with funds for which they serve as investment advisers and in particular to rescue them. In the UK, the Banking Reform of 2013 similarly rules out the voluntary support by depository institutions to other financial entities in distress within the same bank holding company (see Segura, 2014). Against this background, we generalize the tax-bankruptcy trade-off to allow for group-specific provisions, so as to identify the optimal transformations in connected units. Our positive analysis will highlight optimal changes in ownership, in the tax burden and in default costs.

Section 5.1 focuses on IDT. Sections 5.2 and 5.3 will in turn highlight the interaction of IDT with TC rules and "No Bailout" rules. Section 5.4 hints at an additional group-specific provision, namely tax consolidation.

5.1. IDT and optimal ownership mutations

The following theorem characterizes optimal internal ownership and capital structure in the presence of IDT.

Theorem 2. ***IDT.***

Let the tax rate on intercorporate dividend be positive ($0 < \tau_D < 1$). Then:

(i) if $\tau_P \leq z$, the optimal internal ownership is zero ($\omega^* = 0$);

(ii) if $\tau_P \geq z'$, then optimal internal ownership is less than full ($0 \leq \omega^* < 1$) if $\tau_D > \underline{\tau}_D$.

Theorem 2 proves that IDT discourages full subsidiary ownership. Under the conditions of Theorem 2(i), as soon as the tax rate τ_D is non-null, the optimal internal ownership drops to zero, so as to avoid the double taxation of dividends. The associated capital structure remains optimal after this spin-off of the subsidiary. Indeed, the bailout still ensures the optimal exploitation of the tax bankruptcy trade-off. Theorem 2 (ii) states that partial or zero internal ownership may be optimal even when the parent has positive leverage, provided that the dividend tax rate is high enough.

Extensive numerical analysis shows that the cut-off level $\underline{\tau}_D$, defined in the Appendix, is increasing in τ_S and τ_P, increasing in α_P and insensitive to changes in α_S, for fixed capital structure. This means that the higher the marginal benefits from leverage and the marginal default costs in the parent saved through dividends, the higher the IDT rate needs to be to discourage full ownership. However, when we consider the effects that a change in the parameter has on optimal capital structure as τ_D changes, we find that $\underline{\tau}_D$ is increasing in τ_P and α_S, and decreasing in τ_S and α_P. Indeed, $\underline{\tau}_D$ increases when it is more profitable to increase debt in the parent (higher tax savings, lower default costs) or less profitable to increase debt in the subsidiary (lower tax savings, higher default costs).

Corollary 1 summarizes the effects of IDT.

Corollary 1. *The introduction of a tax on intercorporate dividend transforms a hierarchical group into a pyramid ($0 < \omega^* < 1$), if $\tau_D > \underline{\tau}_D$ or a horizontal group ($\omega^* = 0$), if $\tau_D > \bar{\tau}_D$ where $\underline{\tau}_D \leq \bar{\tau}_D$. However, such transformations affect neither the value nor the leverage if $\tau_P < z$.*

In line with Morck (2005), Corollary 1 highlights the ability of IDT to dismantle the hierarchical groups. In our setting, Corollary 1 points out that dismantling the hierarchical structure may affect neither tax revenues nor dissipative default costs when only the guaranteed unit levers up. The cut-off level $\bar{\tau}_D$ responds to changes in the parameters exactly like $\underline{\tau}_D$, as discussed above.

A few remarks are useful. First, the dismantling result holds, as long as the payout ratio is positive and inflexible.[11] If the subsidiary payout ratio is set to zero, the hierarchical group survives following the introduction of IDT provided that the parent lenders can claim their share of profits in the subsidiary. Second, recall that we collapsed the personal dividend tax into the effective corporate income tax to avoid cumbersome notation. Theorem 1, and thus, the previous corollary, hold as long as the personal tax rate on dividends from the parent is the same as the one on dividends from its subsidiary. Otherwise, the shift from indirect to direct ownership may no longer be neutral, also with a zero-leverage supporting unit. Third, the reader may wonder whether the parent still bails out its subsidiary when it sells out its cash flows. We have allowed for an endogenous probability of bailout, set by the controlling entity. The ex-ante value maximizing probability is equal to one, indicating that subsidiary bailouts increase tax gains and value.[12] Fourth, so far, there are no costs associated with ownership transformations. These can be sizable when real synergies explain the group structure.[13] We discuss this case after considering the TC rules.

5.2. TC rules

Tax authorities observe that guaranteed subsidiaries display thin equity capital due to the exploitation of the tax shield. TC rules limit such behavior. These measures directly cap interest deductions in subsidiaries or indirectly restrict them by constraining debt/equity ratios below a certain level. Theorem 3 characterizes the optimal capital structure and

[11] Dividend payouts for corporate shareholders appear not to adjust to corporate tax clienteles (Barclay et al., 2009; Dahlquist et al., 2014).

[12] Absent court enforcement, the incentive depends on both the multiperiod structure of the game (as modeled, for instance, in Luciano and Nicodano (2014)) and on the size of private benefits from control due to the subsidiary).

[13] The benefits of ownership mutations may be large, as well. Cooper et al. (2015) illustrates that US business income migrates away from traditional corporations, with a 31.6% average tax rate, into pass-through partnerships, with a 15.9% rate.

ownership following the introduction of TC rules in combination with IDT. Then, Theorem 4 compares such connected units, subject to group specific taxes, with the stand alone organization.

Theorem 3. ***IDT and TC.*** *Let the leverage constraint in the guaranteed unit be binding ($F_S^{**} = K < F_S^*$) and let $0 < \underline{\tau}_D < 1$. If $0 \leq K \leq \bar{K}$, then:*
i) the parent has optimal positive leverage ($F_P^* > 0$);
ii) optimal subsidiary ownership is partial ($\omega^* < 1$) if $\tau_D > \underline{\tau}_D$;
iii) parent debt F_P^* exceeds the comparable stand-alone level of debt F_{SA}^{P*} when the ratio of parent to subsidiary default costs exceeds a constant C, defined in the Appendix, $\tau_D < \underline{\tau}_D$ and condition (B.9) *in the Appendix holds.*

The first part of the theorem shows that debt shifts to the parent, when debt in the subsidiary is constrained to be lower than a level, $\bar{K}$, even under the conditions of Theorem 1 part i). The forced reduction in subsidiary debt makes a zero-leverage parent suboptimal, even for a low level of τ_P. Indeed, increased tax savings in the parent substitute for the forced reduction in subsidiary tax savings. The second part of the theorem states that, for a sufficiently high dividend tax rate, the parent will no longer own all the shares in its subsidiary. Thus, partial subsidiary ownership does no longer depend, as in Theorem 2 (ii), on specific ranges of the parent corporate income tax rate. Finally, the third part builds on the tension between the bailout guarantee on one side and of the dividend transfer on the other. When parent debt increases, the parent saves additional default costs thanks to the dividend transfer but subsidiary default costs increase due to the reduced bailout transfer. Indeed, only when the default cost rate in the parent is high enough relative to that of the subsidiary ($\frac{\alpha_P}{\alpha_S} > C$) such parent debt increase generates value.

Note that Theorem 3 holds true only if the tax authority enforces TC rules in every supported unit, including affiliates of horizontal groups. If it limits enforcement to proper subsidiaries in hierarchical groups, as it is often the case in practice, the irrelevance Theorem 2 part i) characterizing IDT carries over to TC rules.

Table 3 exemplifies the results characterized in Theorem 3, following the calibration in Leland (2007).

Table 3: Group-Specific Taxes and Ownership Mutations				
	No Thin Cap	**Thin Cap, $F_S = F_{SA}^{S*}$**		
	$\tau_D = 0\%$	$\tau_D = 0\%$	$\tau_D = 1\%$	$\tau_D = 7\%$
Ownership (ω)	any	100%	87%	0%
Value (ν)	166.59	163.88	163.51	163.36
Face Value of Debt (F)	220 (0;220)	138 (81;57)	131 (74;57)	112 (55;57)
Default costs (C)	8.13 (0;8.13)	1.56 (1.12;0.44)	1.23 (0.85;0.38)	1.02 (0.78;0.24)
Tax Burden (T)	25.40 (20.01;5.39)	34.69 (16.85;17.84)	35.35 (17.18;18.17)	35.57 (17.81; 17.76)
Dividend Tax (IDT)	0	0	0.30	0

Table 3: This table reports the optimal ownership, value, debt, default costs, tax burden and dividend tax levied with different levels of dividend taxation, τ_D. Figures of the parent and subsidiary unit, respectively, are reported in brackets. Cash flows are jointly normally distributed with correlation 0.2. Parameters are set according to Leland (2007): $\alpha_S = \alpha_P = 23\%$, $\tau_S = \tau_P = 20\%$, $\phi = 0.78$, $r = 5\%$, $\sigma_S = \sigma_P = 0.22$, $X_0^S = X_0^P = 100$.

The table illustrates the levels of the IDT rate that, together with the TC cap on subsidiary debt, trigger the ownership changes described in Theorem 3. With equal corporate income tax rates in the parent and in the subsidiary, subsidiary ownership is irrelevant when both TC rules and IDT are absent (first column). Moreover, the subsidiary displays high debt (220) and default costs (8.13), along with a low tax burden (5.39), while the parent has zero leverage, zero default costs but a 20.01 tax burden. When TC rules constrain subsidiary debt to the stand alone level (as in the second to fourth columns), the parent company owns 100% of the equity in the supported unit when $\tau_D = 0$. Subsidiary ownership drops to 87% when $\tau_D = 1\%$ and to 0% when $\tau_D = 7\%$. Hence, the group may be organized as a fully integrated hierarchical group, as a pyramid (with the presence of outside ownership), or as a horizontal group, depending on the IDT rate. Both parent leverage and group value decrease with the dividend tax rate, because IDT impairs dividend support. Default costs are dramatically lower than in the case without TC (\$8.13 vs. values ranging from \$1.02 to \$1.56, depending on the IDT rate). They are also decreasing in the dividend tax rate.

Theorem 4 compares groups, subject to IDT and TC, to unconnected units:

Theorem 4. ***Group vs. Stand Alone Units.*** *A group shows lower leverage than the stand alone organization, when $F_S^{**} = F_{SA}^*$, and $\tau_D > \bar{\tau}_D$. However, its value remains higher.*

The theorem states that a proper combination of TC rules and IDT may generate groups with lower leverage than stand alone organizations. Despite lower debt, the bailout guarantee and dividend payouts allow the group to reach a higher value than the stand alone organization. This provides additional evidence that connected units are able to optimize the tax-bankruptcy trade-off. The intuition for this result is as follows. When the IDT tax rate is high, the parent does not own the subsidiary and therefore receives no dividends. The relevant link between the units remains the bail out of the subsidiary, which increases value by reducing the default probability. To increase this bailout transfer, the parent optimally maintains lower leverage than it would as a stand-alone firm. What needs an additional explanation is the reason why an increase in the tax burden does not wipe out value gains in the group, given the debt reduction. Indeed, the group tax burden exceeds only slightly the one in the stand-alone organization, because the parent decreases its debt just slightly. Default costs instead decrease sharply in the subsidiary, due to the optimal exploitation of the bailout guarantee.

Figure 2 provides a visual comparison between unconnected units and the connected units of the previous table, displaying value, tax burden, debt and internal dividend in the four quadrants. The value of connected units falls towards the level of unconnected ones as TC rules constrain subsidiary leverage (second pillar), and IDT complements TC with an increasing rate (third and fourth). The fourth combination generates a horizontal group with the highest tax burden, and yet, connecting the units via the bailout link creates value, relative to the case of unconnected units, thanks to lower default costs.

The analysis we have carried out so far implies that units display a rich variety of capital structures, despite equal tax rates, bankruptcy cost rates, cash flows distributions and no synergies. While connected units usually have, on average, higher debt and lower tax burdens than unconnected ones, the last proposition indicates that we might actually observe regulated cases displaying lower leverage and lower default costs.

5.3. The Volcker Rule and IDT

Prudential rules proposed by the Volcker Rule and the Vickers Committee limit the possibility for banking units to bail out their SPVs.[14] This section analyzes the interplay of such prudential rules with the generalized tax-bankruptcy trade-off.

Without the possibility of a bailout, it never pays to concentrate leverage in the sub-

[14] See the discussion in Segura (2014).

Group-Specific Taxes, Ownership Mutations vs. Unconnected Units

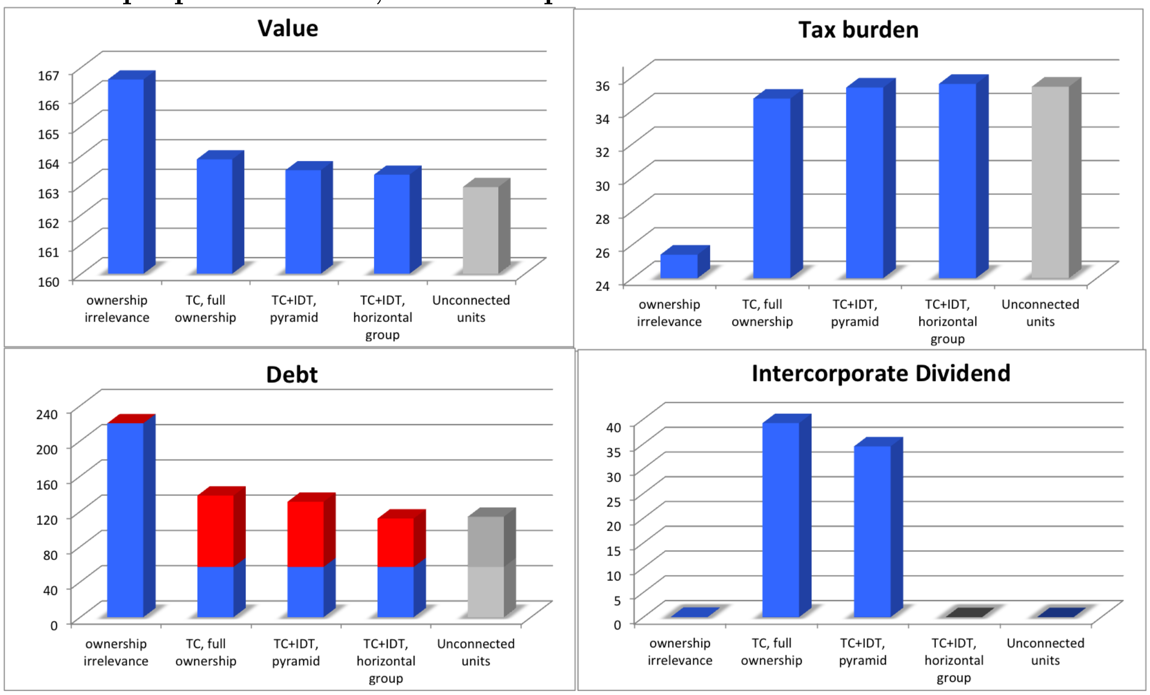

Figure 2: This figure shows value, tax burden, debt and dividend in connected units with: a) no group-specific tax provisions (ownership irrelevance); b) TC rules (TC, full ownership); c) TC rules and 1% IDT rate (TC+IDT, pyramid); d) TC rules and 7% IDT rate (TC+IDT, horizontal group) and e) unconnected units. Cash flows are jointly normally distributed with parameters that follow Leland (2007): $\alpha_S = \alpha_P = 23\%$, $\tau_S = \tau_P = 20\%$, $\phi = 0.78$, $r = 5\%$, $\rho = 0.2$, $\sigma_S = \sigma_P = 0.22$, $X_P^0 = X_S^0 = 100$. Unconnected units have $\omega = 0$, and no bailout transfer.

sidiary. As a consequence, the parent company has positive optimal debt and fully owns its subsidiary, because subsidiary dividends reduce its default probability, without affecting the set of states in which subsidiary defaults (see Lemma 2). Table 4 numerically illustrates the effects of a ban on bailouts, with different levels of the IDT rate and the same parametrization used in Table 3 of the previous section. With a zero IDT rate, a ban on bailouts causes a reduction in the subsidiary debt, from 220 in the unrestricted case (see the first column of Table 3) to 42. At the same time, there is debt shifting towards the parent unit, from zero up to 94. The reduction in group debt due to the Volcker Rule, from 220 to 136, reduces default costs from 8.13% to 1.67% of cash flows, provided the IDT tax rate is zero.

As the IDT tax rate increases, the optimal ownership structure changes. A higher dividend tax ($\tau_D = 1\%$) generates a pyramid, in which parent ownership of the subsidiary is 90%. In this case, total debt issued by the organization is slightly higher than in the absence

Table 4: Effects of IDT on ownership and leverage, no bailouts			
	$\tau_D = 0\%$	$\tau_D = 1\%$	$\tau_D = 7\%$
Ownership (ω)	100%	90%	0%
Value (ν)	163.67	163.20	162.94
Face Value of Debt (F_P)	136 (94;42)	137 (86;51)	114 (57;57)
Default Costs (C)	1.67 (1.36;0.31)	1.89 (1.29;0.60)	1.78 (0.89;0.89)
Tax Burden (T)	34.77 (16.33;18.44)	34.98 (16.62;18.36)	35.62 (17.81;17.81)
Dividend Tax (IDT)	0	0.35	0

Table 4: This table reports the optimal ownership, value,debt, default costs, tax burden and dividend tax levied with different levels of dividend taxation, τ_D, when bailouts are not allowed. Figures in brackets refer to the parent and the subsidiary unit, respectively. Parameters used in the analysis follow Leland (2007): $\alpha_S = \alpha_P = 23\%$, $\tau_S = \tau_P = 20\%$, $\phi = 0.78$, $r = 5\%$, $\rho = 0.2$, $\sigma = 22\%$, $X_0 = 100$.

of dividend taxes (137 vs. 136), but more balanced between the two units (86 in the parent, 51 in the subsidiary). However, value is lower (163.20 vs. 163.67) as both the tax burden (34.89 vs. 34.77) and default costs (1.89 vs. 1.67) are higher. If τ_D is high (7%), as in the third column, optimal ownership of the subsidiary by the parent falls to zero. The SPV is directly owned by the controlling party or sold to outside financiers. In this case, the two units never transfer funds internally and their optimal capital structure decisions coincide with those of stand alone firms. Surprisingly, default costs increase, relative to the zero IDT rate case (1.78 vs. 1.67), even though the overall leverage decreases (114 vs. 136).

This analysis shows that IDT interferes with prudential regulation, that by itself eliminates the incentive to create a highly levered subsidiary SPV, containing default costs. The presence of IDT may increase default costs, contrary to the case of combinations with TC, by damaging the only remaining internal support channel - the one based on internal dividends.

5.4. Hierarchical Group Synergies: Tax Consolidation

In the previous sections, group affiliates only exploit financial synergies. Internal ownership can generate other synergies, relating for instance to investment choices (Stein, 1997 and Matvos and Seru, 2014) or to product market competition and workers' incentives (Fulghieri and Sevilir, 2011). Real synergies can make leverage decisions less sensitive to interest deductions, if they make default costs positive also when cash flows are negative. At the same time, ownership decisions react less to the introduction of IDT if dismantling the group reduces real synergies.

Another important and widespread group-related synergy is tax consolidation, by which a profitable parent can use subsidiary losses to reduce its taxable income, and viceversa.

We discuss the consequences of consolidation in this section. Suppose that the group can exploit the consolidation option whenever its internal ownership exceeds a certain threshold, $\bar{\omega} > 0$.[15] Such option is valuable, because it implies that the tax burden of the group never exceeds the one of stand alone units, and is typically smaller. However, a trade-off, involving the choice of leverage, may emerge between the optimization of consolidation gains on the one hand and that of tax shield gains on the other. The controlling entity can avoid such trade-off by setting up separate vehicles, characterized by a low τ_P, that sell cash flow rights to outsiders and optimize the tax shield, while the rest of the group exploits consolidation. In this case, the analysis in the previous sections holds for these separate "tax arbitrage vehicles".

Without ad hoc vehicles, the ownership irrelevance result of Theorem 1i) holds for a sufficiently high cash flow correlation between the units, because the tax shield option is more valuable, relative to the consolidation option, the higher is correlation. For low correlation levels, the case of ownership irrelevance is less likely to hold and may disappear: the minimum optimal internal ownership is at least equal to the prescribed ownership threshold for consolidation, $\bar{\omega} > 0$, in order to trigger consolidation gains on top of tax shield optimization.

The introduction of IDT impacts on the consolidation vs. tax shield trade-off. Increasing ownership up to the prescribed threshold, $\bar{\omega}$, lowers the tax burden through consolidation on the one hand, but increases taxes paid on intercorporate dividends on the other. Given a high τ_P, the presence of consolidation synergies implies that internal ownership drops below 100% for a higher cutoff level of the IDT tax rate in Theorem 2 (ii). At the consolidation threshold, $\bar{\omega}$, it takes a discontinuous increase in the IDT tax rate to dismantle the hierarchical group.

Consolidation benefits, along with the absence of IDT and the presence of TC rules, provide an additional reason for the existence of wholly owned subsidiaries in EU non-financial groups, as well as larger debt raised by parent companies. In the US, the threshold for consolidation ($\bar{\omega} = 80\%$)[16] triggers a zero tax rate on intercorporate dividends. This tax design eliminates the above mentioned trade-off associated with internal ownership above

[15]Tax consolidation is an option at the Federal level in the US and in other EU jurisdictions, such as France, Italy and Spain, provided internal ownership exceeds some predetermined thresholds. It is forbidden in certain jurisdictions, such as the UK and some US states.

[16]A minority interest may however be sufficient for financial conduits.

the threshold.

6. Model Implications and Observations on Group Structure

This section scans through some empirically documented behavior of groups, following the logic of our analysis. Our logic highlights the (privately) efficient management of internal resources, in a tax-bankruptcy perspective, without touching upon real synergies and the allocation to low-q projects of minority shareholders' funds. It also shifts the emphasis away from pyramids to highlight an array of group shapes, with a prominent role for those where the controlling entity fully owns each affiliate unit.

The case of multinationals appears to fit this logic. Parent companies ultimately wholly-own the vast majority of their foreign subsidiaries in a sample of 634 major U.S. multinational firms. Almost 40 percent of firms are directly owned by their U.S. parent company. The remaining firms, comprising $33,051$ foreign subsidiaries, are arranged into more complex ownership structures. Out of this total, 37 percent of subsidiaries are part of hierarchical structures. Various tax considerations are an important factor in structuring affiliate ownership (Lewellen and Robinson, 2013). This evidence documents the existence of an even richer set of ownership structures responding to a richer array of foreign tax considerations. Hierarchical groups are not special to multinationals. The investigation of 270,374 groups in 129 countries reveals that two thirds of the 1,519,588 affiliates are held domestically. US groups represent an exception, in that affiliates abroad represent almost half of the total (Altomonte and Rungi, 2013).

Our theory highlights the use of internal resources to improve the tax-bankruptcy trade-off, with internal dividends helping support parent companies. This rationale may explain the reason why continental European groups have larger debt in parent companies than in subsidiaries (Bianco and Nicodano, 2006) and display positive correlation between subsidiary dividends and the parent firms' debt (De Jong et al., 2012). These hierarchical groups, that also rely on their deep pockets to shield affiliates from competitors (Boutin et al., 2013), are often not widely held (Bloch and Kremp, 1999; Faccio and Lang, 2002), as our results would suggest. The use of intra-group mutual support to contain bankruptcy risk and enhance borrowing capacity is present in family groups around the world (Masulis et al., 2011).

Group firms have higher mean dividend yield than non-group firms. The dividend difference with respect to non-group firms is higher for bottom than for apex firms. Group firms are also able to borrow more than their non-group peers. Within pyramidal groups, companies in the upper layers borrow more than companies at the bottom.

As far as valuation is concerned, Masulis et al. (2011) uncover lower equity value (Q) of group affiliates with respect to non-group peers in apex companies, where the separation of ownership from control is minimal, but not in bottom companies. This evidence is hard to reconcile with expropriation of minorities through pyramids. Our analysis may help explain this evidence, associating lower equity value of group affiliates relative to stand alone firms with both internal parent support and higher debt commitments. Our numerical example in Table 1 shows that parent companies have lower excess equity value when they borrow more than their subsidiaries, especially when they also provide bailout transfers to them.

Our model implies that the presence of group-specific tax provisions prompts ownership transformations, towards partial or zero subsidiary ownership. Masulis et. al (2011) document that the percentage of both family groups and family pyramids in a cross-section of countries are negatively related to an index of intra-group tax provisions, that measures the stringency of a countrys tax laws related to intra-group transactions, including Thin Capitalization rules. In the 1930s and 1940s, hierarchical pyramids were pervasive in the U.S.. Some were widely diversified, while many others were focused in a single industry. Many groups were controlled by business families, but about half by widely held companies. Kandel et al. (2015) show that groups flattened out in response to the introduction of IDT, although some kept their layers by exploiting the exemptions granted to ownership stakes in excess of a high threshold. Currently, direct control via a horizontal structure is a common form of family ownership, although 80 percent of the firms display some indirect ownership through trusts, foundations, corporations and limited partnership (Villalonga and Amit, 2009).

Our model indicates that intercorporate links allow the group to better exploit the trade-off between lower tax burden and higher default costs, when interest deductions distort debt incentives, as documented by Desai et al. (2004). Another implication of our model, that emerges in the sample of multinational companies studied by Brok (2016), is that a change in the tax rate of one affiliate has nonlinear effects on total group debt. As far as TC rules are concerned, our model implies debt shifting onto the parent company when TC rules are strictly enforced on subsidiaries. Blouin et al. (2014) find that foreign affiliates' leverage responds to the introduction of TC rules in U.S. multinationals. However, the ratio of debt to assets of the consolidated worldwide firm does not react to the introduction of TC rules. This observation suggests that the multinational firm engages in debt shifting towards other group affiliates.

The debt tax shield is relevant in the private equity industry, as well. The combination of a zero-leverage parent and a levered subsidiary, in our model, reminds of the private equity fund with its LBO firm. The private equity fund participates in the restructurings of the firm via a state-contingent support mechanism, i.e. when it is insolvent, but profitable. Leverage contributes to tax savings and value creation (Acharya et al., 2013; Kaplan, 1989; Renneboog et al., 2007) along with efficiency gains (see Axelson et al., 2009), that are absent in our set up. Certain characteristics of financial conduits align with the model implications. Conduits, that can be incorporated as orphan Special Purpose Vehicles (SPVs), are structured to be tax neutral, as they would otherwise be subject to double taxation (Gorton and Souleles, 2006). In our model, selling the cash flow rights of the supported subsidiary to outsiders avoids IDT and other provisions for proper subsidiaries, while enjoying interest deductions. Consistently with this observation, Han et al. (2015) show that securitization increases with the corporate tax rate (i.e., with incentives to exploit the tax shield). In conduits, there is a mechanism akin to a bailout transfer: the sponsoring unit and the investors agree upon the state contingent subsidization of the vehicle, beyond the formal obligations of the sponsor (Gorton and Souleles, 2006).

Last but not least, our theory may explain the heterogeneity in the capital structure choices of companies with the same tax-bankruptcy parameters, first observed in the U.S. by Bernanke et al. (1990). The capital structure in a given firm depends not only on its own tax-bankruptcy parameters, but on the transfers to and from (possibly unlisted) connected units as well.[17] For instance, a unit may have zero optimal leverage when it supports other units. This result provides a tax-bankruptcy rationale for the existence of zero-leverage

[17] This type of reasoning is the same underlying the study of Faulkender and Smith (2014), who are however interested in dividend repatriation provisions.

companies (see Strebulaev and Yang (2013)). Our model, differently from Luciano et al. (2014), does not require that zero-leverage firms belong to a hierarchical group. If the parent effective tax rate is relatively low, this zero-leverage unit may belong to a horizontal group.

7. Summary and Concluding Comments

This paper determines group structure, abandoning the textbook fiction of the stand-alone firm. The paper indicates how tax provisions, intertwined with endogenous default and leverage, generate the variety of ownership structures of connected units that we commonly observe. Hierarchical groups with fully owned subsidiaries maximize value, when the parent tax rate is sufficiently high with respect to the subsidiary tax rate. Optimal hierarchical groups usually exploit both bailouts and internal dividends as supporting mechanisms. Given tax rates, they are more likely to emerge when groups are diversified across industries and countries, so that cash flow correlation across affiliates is low. Any group ownership structure may instead emerge, when the tax rate of the supporting unit is small, leading to an unbalanced capital structure, with debt concentrated in the supported unit. A small tax rate reminds of tax arbitrage vehicles, that reduce their effective tax rate through either non-debt tax shields or incorporation in low tax jurisdictions. The endogenous interest spread plays a crucial role, together with bailouts, in generating ownership irrelevance. It is the positive feedback from higher default costs to higher tax savings that concentrates debt in the subsidiary.

Our model embeds group specific provisions in the trade-off analysis, extending it beyond income taxes. The presence of IDT discourages indirect ownership, and either pyramids or horizontal groups may become value maximizing. TC rules, applying to proper subsidiaries only, do not contain leverage in tax arbitrage vehicles, that can change their ownership structure to preserve tax savings. On the contrary, they are effective if they apply to all guaranteed affiliates. No bailout rules contain both tax savings and default costs. However, IDT may interfere with this prudential rule by limiting the only remaining internal support mechanism. In a nutshell, we highlight the tax-bankruptcy motives of complex structures, leaving aside real synergies, contractual incompleteness, agency problems and inter-temporal issues stressed in prior capital structure literature.

Our structural analysis implies that the observed heterogeneity in the capital structure choices of listed companies with the same tax/bankruptcy parameters depends on the transfers to and from (possibly unlisted) connected units. It therefore suggests an agenda for future empirical work, jointly considering capital and ownership structure.

Appendix A. Definition of the $h(\cdot)$ function

The function $h(X_S)$ defines the set of states of the world in which the parent company has enough funds to intervene in saving its affiliate from default while at the same time remaining solvent. The rescue happens if the cash flows of the parent X_P are enough to cover both the obligations of the parent and the remaining part of those of the subsidiary. The function $h(X_S)$, which defines the level of parent cash flows above which the rescue occurs, is defined in terms of the cash flows of the subsidiary as:

$$h(X_S) = \begin{cases} X_P^d + \frac{F_S}{1-\tau} - \frac{X_S}{1-\tau} & X_S < X_S^Z, \\ X_P^d + X_S^d - X_S & X_S \geq X_S^Z. \end{cases}$$

When $X_S < X_S^Z$ the cash flow X_S of the subsidiary does not give rise to any tax payment, as it is below the tax shield generated in that unit.

Appendix B. Proofs

Proof that the stand-alone has positive optimal debt also with a zero risk-free interest rate, $\phi = 1$.

First of all, let us suppress the SA subscript and the i superscript and subscript for notational simplicity. The derivative of debt D with respect to F for a stand-alone unit is

$$\frac{dD}{dF} = \phi \left[-\alpha \frac{dX^d}{dF} X^d f(X^d) + \tau \frac{dX^Z}{dF} \left(G(X^d) - G(X^Z) \right) + 1 - G(X^d) \right]. \tag{B.1}$$

When $F = 0$, it follows that

$$\frac{dD(0)}{dF} = \phi(1 - G(0)).$$

The derivative of the tax shield X^Z in F is $dX^Z/dF = 1 - \frac{dD}{dF}$, while that of the default threshold X^d is $dX^d/dF = 1 + \frac{\tau}{1-\tau}\frac{dD}{dF}$.

Default costs are non-decreasing in F, since $\frac{dC}{dF} = \alpha X^d f(X^d) \frac{dX^d}{dF} \phi$. This derivative is 0 in $F = 0$. The tax burden is decreasing in F:$\frac{dT}{dF} = -\tau \phi \frac{dX^Z}{dF} \left(1 - G(X^Z)\right)$.

It follows that the sum of taxes and default costs is always non-increasing in F at $F = 0$ when $r = 0$, i.e. $\phi = 1$:

$$\frac{dT(0)}{dF} + \frac{dC(0)}{dF} = -\tau G(0)\,(1 - G(0)) \leq 0, \tag{B.2}$$

and it strictly decreasing unless $G(0) = 1$ or $G(0) = 0$, i.e. the firm is never profitable or always profitable, respectively.When $0 < G(0) < 1$, then, the Kuhn-Tucker conditions of the minimum program for a stand-alone unit are violated at $F = 0$. Such conditions are indeed:

$$\begin{cases} \frac{dT(F)}{dF} + \frac{dC(F)}{dF} = \lambda & (i) \\ \lambda F = 0 & (ii) \\ \lambda \geq 0. & (iii) \end{cases}$$

When $F = 0$, $\lambda \geq 0$ and condition (i) is violated, since its l.h.s., when $\phi = 1$, is strictly negative, as equation (B.2) shows.

Kuhn-Tucker conditions of the minimum program

We provide the set of Kuhn-Tucker conditions of the minimization program (10). To keep the notation simple, here and in the following proofs, we only report the dependence of the functions on the parent and subsidiary debt and the evaluation of the conditions at

ω^* and π^*, when necessary.

$$\begin{cases} \frac{dT_{SA}^P(F_P^*)}{dF_P} + \frac{dC_{SA}^P(F_P^*)}{dF_P} - \frac{\partial\Gamma(F_P^*,F_S^*)}{\partial F_P} - \frac{\partial\Delta C(F_P^*,F_S^*)}{\partial F_P} + \frac{\partial\Delta T(F_P^*,F_S^*)}{\partial F_P} = \mu_1, & (i) \\ F_P^* \geq 0, & (ii) \\ \mu_1 F_P^* = 0, & (iii) \\ \frac{dT_{SA}^S(F_S^*)}{dF_S} + \frac{dC_{SA}^S(F_S^*)}{dF_S} - \frac{\partial\Gamma(F_P^*,F_S^*)}{\partial F_S} - \frac{\partial\Delta C(F_P^*,F_S^*)}{\partial F_S} + \frac{\partial\Delta T(F_P^*,F_S^*)}{\partial F_S} = \mu_2, & (iv) \\ F_S^* \geq 0, & (v) \\ \mu_2 F_S^* = 0, & (vi) \\ \mu_1 \geq 0, \mu_2 \geq 0 & (vii) \\ -\frac{\partial\Delta C(F_P^*,F_S^*)}{\partial\omega} + \frac{\partial\Delta T(F_P^*,F_S^*)}{\partial\omega} = \mu_3 + \mu_4 & (viii) \\ \omega^* - 1 \leq 0 & (ix) \\ \omega^* \geq 0 & (x) \\ \mu_3(\omega^* - 1) = 0 & (xi) \\ \mu_4(\omega^*) = 0 & (xii) \\ \mu_3 \leq 0, \mu_4 \geq 0 & (xiii) \end{cases} \tag{B.3}$$

Proof of Lemma 1

The integral expressions of Γ, ΔC and ΔT are:

$$\begin{aligned} \Gamma &= \alpha_S \int_0^{X_S^d} \int_{h(X_S)}^{+\infty} xg(x,y)dydx \\ \Delta C &= \alpha_P \phi \int_{X_S^d}^{+\infty} \int_{\left(X_P^d - \omega(1-\tau_D)\left[(1-\tau_S)y + \tau X_S^Z - F_S\right]\right)^+}^{X_P^d} xg(x,y)dxdy \\ &= \alpha_P \phi \int_{X_S^d}^{\frac{X_P^d}{\omega(1-\tau_D)(1-\tau_S)} + X_S^d} \int_{\left(X_P^d - \omega(1-\tau_D)\left[(1-\tau_S)y + \tau X_S^Z - F_S\right]\right)}^{X_P^d} xg(x,y)dxdy + \\ &+ \alpha_P \phi \int_{\frac{X_P^d}{\omega(1-\tau_D)(1-\tau_S)} + X_S^d}^{+\infty} \int_0^{X_P^d} xg(x,y)dxdy, \end{aligned}$$

$$\Delta T = \phi\omega\tau_D \int_{X_S^d}^{+\infty} [(1-\tau_S)x + \tau_S X_S^Z - F_S)] f(x)dx.$$

We now compute the first derivatives of ΔC and ΔT with respect to F_S and F_P, and we prove our statement:

$$\begin{aligned}
\frac{\partial \Gamma}{\partial F_P} &= -\alpha_S \phi \frac{\partial X_P^d}{\partial F_P} \left[\int_0^{X_S^Z} xg\left(x, X_P^d + \frac{F_S}{1-\tau_S} - \frac{x}{1-\tau_S}\right) dx + \int_{X_S^Z}^{X_S^d} xg\left(x, X_P^d + X_S^d - x\right) dx \right] \leq 0, \qquad \text{(B.4)} \\
\frac{\partial \Delta C}{\partial F_P} &= \alpha_P \phi \frac{\partial X_P^d}{\partial F_P} \int_{X_S^d}^{+\infty} X_P^d g(X_P^d, y) dy + \\
&- \alpha_P \phi \left[\frac{\partial X_P^d}{\partial F_P} - \omega(1-\tau_D)\tau_S \frac{\partial X_S^Z}{\partial F_P} \right] \int_{X_S^d}^{\frac{X_P^d}{\omega(1-\tau_D)(1-\tau_S)} + X_S^d} \left(X_P^d - \omega(1-\tau_D)\left[(1-\tau_S)y + \tau_S X_S^Z - F_S\right]\right) \times \\
&\times\ g\left(\left(X_P^d - \omega(1-\tau_D)\left[(1-\tau_S)y + \tau_S X_S^Z - F_S\right]\right), y\right) dy, \qquad \text{(B.5)}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Delta C}{\partial F_S} &= \alpha_P \phi \frac{\partial X_P^d}{\partial F_S} \int_{X_S^d}^{+\infty} X_P^d g(X_P^d, y) dy + \\
&- \alpha_P \phi \left[\frac{\partial X_P^d}{\partial F_S} - \omega(1-\tau_D) \left[\tau_S \frac{\partial X_S^Z}{\partial F_S} - 1 \right] \right] \times \\
&\times \int_{X_S^d}^{\frac{X_P^d}{\omega(1-\tau_D)(1-\tau_S)} + X_S^d} (X_P^d - \omega(1-\tau_D)\left[(1-\tau_S)y + \tau_S X_S^Z - F_S\right]) \times \\
&\times\ g\left(y, (X_P^d - \omega(1-\tau_D)\left[(1-\tau_S)y + \tau_S X_S^Z - F_S\right])\right) dy, \qquad \text{(B.6)}
\end{aligned}$$

$$\frac{\partial \Delta T}{\partial F_P} = \phi\omega\tau_D \frac{\partial X_S^Z}{\partial F_P} \int_{X_S^d}^{+\infty} \tau_S f(x) dx \geq 0,$$

$$\frac{\partial \Delta T}{\partial F_S} = \phi\omega\tau_D \left[\tau_S \frac{dX_S^Z}{dF_S} - 1 \right] (1 - G(X_S^d)) \leq 0.$$

The previous set of expressions results from the fact that $\frac{\partial X_S^Z}{\partial F_P} \geq 0$.

$$\begin{aligned}
\frac{\partial \Delta C}{\partial \omega} &= \alpha_P \phi \int_{X_S^d}^{\frac{X_P^d}{\omega(1-\tau_D)(1-\tau_S)} + X_S^d} (1-\tau_D)[(1-\tau_S)y + \tau_S X_S^Z - F_S] \times \\
&\times\ (X_P^d - \omega(1-\tau_D)[(1-\tau_S)y + \tau_S X_S^Z - F_S])] \times \qquad \text{(B.7)} \\
&\times\ g\left(X_P^d - \omega(1-\tau_D)\left[(1-\tau_S)y + \tau_S X_S^Z - F_S\right], y\right) dy \geq 0.
\end{aligned}$$

ΔC is non-decreasing in ω, as default costs saved in the parent through dividends are higher the higher the dividend transfer from the subsidiary. The change in the tax burden due to IDT is always non-decreasing in ω as well, as – ceteris paribus – higher dividend taxes are paid when the ownership share is higher:

$$\frac{\partial \Delta T}{\partial \omega} = \phi \tau_D \int_{X_S^d}^{+\infty} (x(1-\tau_S) + \tau_S X_S^Z - F_S) f(x) dx \geq 0. \tag{B.8}$$

This derivative has value zero when $\tau_D = 0$.

Proof of Lemma 2.

Let us analyze the Kuhn-Tucker condition (i) when the bailout is absent and thus $\frac{\partial \Gamma}{\partial F_P} = 0$. The condition is violated for every F_S, when evaluated at $F_P = 0$. Indeed, $\frac{\partial T_{SA}^P(0)}{\partial F_P} + \frac{\partial C_{SA}^P(0)}{\partial F_P} < 0$, because of convexity of the stand alone problem. The other three terms of the l.h.s. are zero when $F_P = 0$, as one can easily see from (B.4),(B.5),(B.6). As a consequence, condition (i) is never satisfied (the r.h.s. of the inequality being zero) and the parent optimally raises non-zero debt: $F_P^* > 0$.

We now study the choice of optimal ownership when $F_P^* > 0$. Let us first consider $\omega^* = 0$, i.e. $\mu_4 \geq 0$ and $\mu_3 = 0$. Condition (viii) is violated, since the l.h.s. is negative at $\omega = 0$ from (B.7). Hence, zero ownership is not optimal. The existence of an interior solution, $0 < \omega^* < 1$, requires both $\mu_3 = 0$ and $\mu_4 = 0$. Condition (viii) is satisfied only for $\omega^* \to \infty$, which violates condition (ix). Hence, no interior solution satisfies the Kuhn-Tucker conditions. Finally, let us analyze the corner solution $\omega^* = 1$, which requires $\mu_3 \leq 0, \mu_4 = 0$. Condition (viii) is satisfied for an appropriate μ_3: all other conditions can be satisfied at $F_S^*, F_P^*, \omega^* = 1$. It follows that $\omega^* = 1$ when $\tau_D = 0$. This proves the first part of the lemma.

We now turn to the proof that F_P^* exceeds the stand-alone level of debt F_{SA}^{P*}. It is sufficient, given our convexity assumption, to show that $\frac{\partial \Delta C(F_{SA}^{P*}, F_{SA}^{P*})}{\partial F_P} > 0$. This rules out the possibility of a solution with $F_P^* \leq F_{SA}^{P*}$, as the Kuhn-Tucker condition (i) in (B.3) is never satisfied.

Consider then expression (B.5). All terms are positive, since $\frac{\partial X_P^d}{dF_P} \geq 0, \frac{\partial X_S^Z}{dF_P} \leq 0$, but

$$\int_{X_S^d}^{\frac{X_P^d}{\omega(1-\tau_D)(1-\tau_S)} + X_S^d} X_P^d g\left(X_P^d - DIV(y; F_S), y\right) dy,$$

where $DIV(y; F) = \omega(1-\tau_D)\left[(1-\tau_S)y + \tau_S X_S^Z - F_S\right]$. One sufficient condition is

$$\int_{X_{SA}^{S,d*}}^{+\infty} g(X_{SA}^{P,d*}, y)dy - \int_{X_{SA}^{S,d*}}^{X_{SA}^{S,d*}+\frac{X_{SA}^{P,d*}}{\omega(1-\tau_S)}} g\left(X_{SA}^{P,d*} - DIV(y; F_{SA}^{S*}), y\right) dy \geq 0, \tag{B.9}$$

where $X_{SA}^{P,d*}$ is the default threshold of a stand-alone counterpart of the parent, evaluated at its optimal debt F_{SA}^{P*}. In words, as the debt of the parent increases, the parent cash flow potentially lost in bankruptcy increases as well, as its default threshold increases. Thus, internal dividends potentially save a larger parent cash flow. However, the chances that the dividend itself is insufficient to support the parent also increase in parent debt. This condition requires the first probability to exceed the second, when the two units have their respective stand alone optimal debts and $\tau_D = 0$. If condition (B.12) holds, $F_P^* \leq F_{SA}^{P*}$ violates the Kuhn Tucker condition (i). Hence, $F_P^* > F_{SA}^{P*}$.

Proof of Theorem 1

Consider the Kuhn-Tucker conditions (i) to (xiii) in (B.3). Under our convexity assumption, these conditions are necessary and sufficient. We first investigate the existence of a solution in which $F_P^* = 0$, holding $F_S > 0$. If such a solution does not exist, then convexity implies the existence of another solution with $F_P^* > 0$. After determining optimal capital structure, we turn to optimal ownership structure.

A solution for capital structure characterized by $F_P^* = 0$, $F_S^* > 0$ implies $\mu_1 \geq 0$ and $\mu_2 = 0$. We focus on condition (iv) first. We have to prove that the term $-\frac{\partial \Delta C(F_P^*=0, F_S^*)}{\partial F_S} + \frac{\partial \Delta T(F_P^*=0, F_S^*)}{\partial F_S}$ has a negative limit as subsidiary debt, F_S tends to zero, and a positive limit when F_S goes to infinity, since the rest of the l.h.s. does, under the technical assumptions that $xf(x)$ converges as $x \longrightarrow +\infty$ (see Luciano and Nicodano, 2014).
Note that $\frac{\partial \Delta C(0, F_S)}{\partial F_S} = 0$ for every F_S, because this derivative measures the change in parent default costs associated with subsidiary dividends, when the parent has zero debt. Moreover, $\frac{\partial \Delta T}{\partial F_S}$ is always lower than or equal to zero, and has a negative limit as F_S goes to zero, since $\lim_{F_S \longrightarrow 0} \frac{\partial X_S^Z}{\partial F_S} = 1 - \phi(1 - G(0)) > 0$. When F_S goes to infinity, $\frac{\partial \Delta T}{\partial F_S}$ goes to zero, as $G(X_S^d)$ tends to one. This proves that, when $F_P^* = 0$, there exists an $F_S^* > 0$, which solves the equation that equates the l.h.s. of condition (iv) to zero.
Let us now turn to condition (i). We recall that the derivative $\frac{\partial \Delta C}{\partial F_P}$ vanishes at $F_P^* = 0$, see (B.5). Hence, we look for conditions such that the l.h.s. of condition (i) is positive and set

it equal to μ_1 to fulfill it. The l.h.s. of (i) is positive is

$$\frac{\tau_P(1-\tau_P)G(0)(1-G(0))}{1-\tau_P G(0)} \leq \alpha_S \left[\int_0^{X_S^Z} xg\left(x, \frac{F_S}{1-\tau_S} - \frac{x}{1-\tau_S}\right) dx + \right.$$
$$\left. + \int_{X_S^Z}^{X_S^d} xg(x, X_S^d - x)dx \right] = A(F_S). \qquad \text{(B.10)}$$

Under our assumptions on the joint density function $g(\cdot,\cdot)$, $A(F_S)$ is increasing in F_S in the relevant range, $[0,\mathrm{F}_S^{max}], where \mathrm{F}_S^{max} = argmax_{A(F_S)}$. Hence, a sufficient condition for the existence of a zero-leverage parent is

$$\frac{\tau_P(1-\tau_P)G(0)(1-G(0))}{1-\tau_P G(0)} \leq \alpha_S \left[\int_0^{X_{SA}^{S,Z*}} xg\left(x, \frac{F_{SA}^{S*}}{1-\tau_S} - \frac{x}{1-\tau_S}\right) dx + \right.$$
$$\left. + \int_{X_{SA}^{Z*}{}_S}^{X_{SA}^{S,d*}} xg(x, X_{SA}^{S,d*} - x)dx \right] = A(F_{SA}^{S*}). \qquad \text{(B.11)}$$

where $G(0)$ is the cumulative probability of a (parent) cash-flow lower than or equal to zero, F_{SA}^{S*} is the optimal debt of the stand alone counterpart of the subsidiary (i.e. that receives no bailout transfer from its parent and transfers no dividend to its parent) and $X_{SA}^{S,Z*}, X_{SA}^{S,d*}$ represent its optimal tax shield and default threshold, respectively. Both sides of this inequality are non-negative. The l.h.s. is increasing in τ_P, while the r.h.s. is independent of τ_P. Hence, for any fixed value of the r.h.s., there exists a low enough value of τ_P satisfying this condition. We define $z(\rho, \alpha_S, \alpha_P, \tau_S)$ as the cutoff value of the parent tax rate. Then $\tau_P < z$ is a sufficient condition for the existence of a solution in which $F_P^* = 0$. Straightforward differentiation shows that $z(\rho, \alpha_S, \alpha_P, \tau_S)$ is independent of α_P, increasing in τ_S and inversely u-shaped in α_S.

The second result concerning capital structure contained in part (i) follows directly from Theorem 2 part 2) in Luciano and Nicodano (2014). The constant Q is equal to

$$Q = \frac{Pr(X_S > X_S^{Z**})\frac{\partial X_S^{Z**}}{\partial F_S}}{X_S^{d**}\frac{\partial X_S^{d**}}{\partial F_S}Pr(X_S = X_S^{d**}, X_P < 0) + \frac{\partial h}{\partial F_S}\int_0^{X_S^{d**}} xg(x, h(x))dx},$$

where the notation Pr(Y) is used to refer to the probability of event Y. We are now ready to turn to ownership, thereby completing the proof of part (i). When $\tau_P \leq z$, implying $F_P^* = 0$, condition (viii) is satisfied for any ω. The dividend from the subsidiary, ω, has no effect on boththe parent default costs ($\Delta C = 0$ because $F_P^* = 0$, implying $X_P^d = 0$) and the tax burden ($\Delta T = 0$ because $\tau_D = 0$ by assumption). Hence ω has no effect on the value of the parent. Similarly, the tax burden of the subsidiary and its value are independent of ω. It follows that ω^* is indefinite.

Let us now prove part (ii). Let us consider again the Kuhn-Tucker condition (i). This condition is not satisfied at $(F_P = 0, F_S = F_S^*)$ when

$$\frac{\tau_P(1-\tau_P)G(0)(1-G(0))}{1-\tau_P G(0)} > A(F_S^{max}), \tag{B.12}$$

where we recall that F_S^{max} is the $argmax_{F_S} A(F_S)$. We define the level of τ_P above which condition (B.12) is satisfied as z'. Notice that $z' > z$, since $F_S^{max} \geq F_S^* \geq F_{SA}^*$ when $F_P = 0$. Then, when $\tau_P > z'$, the parent optimally raises debt, as there exists no solution in which $F_P^* = 0$. We consider now the choice of optimal ownership when $F_P^* > 0$.

Let us first assume $\omega^* = 0$, $\mu_4 \geq 0$ and $\mu_3 = 0$. Condition (viii) is violated, since the l.h.s. is negative at $\omega = 0$ from (B.7). Hence, zero ownership is not optimal. The existence of an interior solution, $0 < \omega^* < 1$, requires both $\mu_3 = 0$ and $\mu_4 = 0$. Condition (viii) is satisfied only for $\omega^* \to \infty$, which violates condition (ix). Hence, no interior solution satisfies the Kuhn-Tucker conditions. Finally, let us analyze the corner solution $\omega^* = 1$, which requires $\mu_3 \leq 0, \mu_4 = 0$. Condition (viii) is satisfied for an appropriate μ_3: all other conditions can be satisfied at $F_S^*, F_P^*, \omega^* = 1$. It follows that $\omega^* = 1$ when $\tau_D = 0$; as such, part (ii) is proven.

Proof of Theorem 2

Let us start with part (i). Theorem 1 proves that $F_P^* = 0$ if $\tau_P \leq z$, absent IDT. When $\tau_D > 0$, $\omega^* = 0$ is the only value of ω which does not contradict condition (viii). In fact, $\frac{\partial \Delta C(0,F_S)}{\partial \omega} = 0$ for every F_S, while $\frac{\partial \Delta T}{\partial \omega}$ is strictly positive as soon as $\tau_D > 0$, leading to a contradiction unless $\omega^* = 0$, and hence, $\mu_3 = 0$. This proves part (i) of Theorem 2.We can also prove that there is no other change relative to the characterization of Theorem 1, part (i). Indeed, when $\omega = 0$ both ΔC and ΔT are equal to 0 for every (F_P, F_S) couple. We can repeat the discussion of the Kuhn-Tucker conditions performed in the proof of Theorem 1 part (i) to see that there exists a solution $F_P^* = 0, F_S^* > 0$, even if $\tau_D > 0$, because $\omega^* = 0$.

As a consequence, the presence of IDT is irrelevant at the optimum for value and capital structure choices. To prove part (ii), recall first that, according to the Proof of Theorem 1ii), $\tau_P \geq z'$ implies $F_P^* > 0$. We then check for the existence of solutions with $\omega^* = 1$. Condition (viii), evaluated at $\omega^* = 1$, is

$$
\begin{aligned}
- \quad & \alpha_P \phi \int_{X_S^{d*}}^{\frac{X_P^{d*}}{(1-\tau_D)(1-\tau_S)}+X_S^{d*}} (1-\tau_D)[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*] \times \\
\times \quad & (X_P^{d*} - (1-\tau_D)[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*])] \times \\
\times \quad & g\left(X_P^{d*} - (1-\tau_D)\left[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*\right], y\right) dy + \\
+ \quad & \phi \tau_D \int_{X_S^{d*}}^{+\infty} (x(1-\tau_S) + \tau_S X_S^{Z*} - F_S^*) f(x) dx = \mu_3,
\end{aligned}
$$

and $\mu_3 \leq 0$. When $\tau_D = 0$ the first term of the sum on the l.h.s. of the equation is negative and the second term disappears, whereas when $\tau_D = 1$ the first term disappears, while the second term is positive. Being the l.h.s. continuous and increasing in τ_D, there exists a level of τ_D, that we define as $\underline{\tau}_D$, above which there is no solution ath $\omega^* = 1$. This cutoff $\underline{\tau}_D$ solves:

$$
\begin{aligned}
- \quad & \alpha_P \phi \int_{X_S^{d*}}^{\frac{X_P^{d*}}{(1-\underline{\tau}_D)(1-\tau_S)}+X_S^{d*}} (1-\underline{\tau}_D)[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*] \times \\
\times \quad & (X_P^{d*} - (1-\underline{\tau}_D)[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*])] \times \\
\times \quad & g\left(X_P^{d*} - (1-\underline{\tau}_D)\left[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*\right], y\right) dy + \\
+ \quad & \phi \underline{\tau}_D \int_{X_S^{d*}}^{+\infty} (x(1-\tau_S) + \tau_S X_S^{Z*} - F_S^*) f(x) dx = 0, \qquad \text{(B.13)}
\end{aligned}
$$

Hence, when $\tau_D > \underline{\tau}_D$, $0 \leq \omega^* < 1$. Notice also that $\frac{\partial^2 \Delta C}{d\omega^2} \leq 0$, while $\frac{\partial \Delta T}{\partial \omega} = 0$. This implies convexity of the objective function w.r.t. ω and more importantly, that the l.h.s. of the equation (B.13) is increasing in ω. Hence, when $\tau_D < \underline{\tau}_D$, no solution with $0 \leq \omega^* < 1$ is feasible because this would contradict the Kuhn Tucker condition (viii) that requires $-\frac{\partial \Delta C}{\partial \omega}|_{\omega=\omega^*} + \frac{\partial \Delta T}{\partial \omega}|_{\omega=\omega^*} = \mu_4$, with $\mu_4 \geq 0$. This concludes our proof of part ii) of the theorem.

Proof of Corollary 1

The condition for the existence of a pyramid descends directly from part ii) of Theorem 2. A condition for the existence of horizontal groups is a condition on τ_D such that $\omega^* = 0$ is the only feasible solution. $\omega^* = 0$ implies $\mu_4 \geq 0$, $\mu_3 = 0$ in (B.3). Condition (viii) in (B.3) when $\omega^* \to 0$ reads

$$\begin{aligned} - & \quad \alpha_P \phi(1-\tau_D) \int_{X_S^{d*}}^{+\infty} \left[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*\right] X_P^{d*} g(X_P^{d*}, y) dy + \\ + & \quad \phi \tau_D \int_{X_S^{d*}}^{+\infty} (x(1-\tau_S) + \tau_S X_S^{Z*} - F_S^*) f(x) dx = \mu_4, \end{aligned} \tag{B.14}$$

where we considered that the upper limit of integration, $\frac{X_P^d}{\omega(1-\tau_D)(1-\tau_S)} + X_S^d$, tends to $+\infty$ when ω goes to 0 and we denoted with X_i^{Z*} and X_i^{d*} for $i = P, S$ the tax shield and default threshold, respectively, evaluated at the optimum. The l.h.s. of the above equation is non-positive for $\tau_D = 0$ and is increasing in τ_D, since its first derivative with respect to τ_D is strictly positive. It follows that a necessary condition for the existence of a solution where $\omega^* = 0$, for given F_S^* and F_P^*, is that τ_D is higher than a certain level, which we define as $\bar{\tau}_D$, solving

$$\begin{aligned} - & \quad \alpha_P \phi(1-\bar{\tau}_D) \int_{X_S^{d*}}^{+\infty} \left[(1-\tau_S)y + \tau_S X_S^{Z*} - F_S^*\right] X_P^{d*} g(X_P^{d*}, y) dy + \\ + & \quad \phi \bar{\tau}_D \int_{X_S^{d*}}^{+\infty} (x(1-\tau_S) + \tau_S X_S^{Z*} - F_S^*) f(x) dx = 0 \end{aligned} \tag{B.15}$$

Also, recall that the l.h.s. of the Kuhn Tucker condition (viii) is increasing in ω. Hence, for every $\tau_D > \bar{\tau}_D$, $\omega^* = 0$ is the only feasible solution, because $\omega^* > 0$ would contradict condition (viii) that requires the l.h.s. to be lower than or equal to zero. Also, notice that convexity of the objective function in ω implies also that $\underline{\tau}_D \leq \bar{\tau}_D$. Finally, value and default costs neutrality under the conditions of the corollary follow directly from Theorem 2 part i).

Proof of Theorem 3

We prove part (i) of the theorem first. The presence of a cap on subsidiary debt introduces a further constraint in the optimization program: $F_S^{**} \leq K$, where K is the imposed cap and $(F_P^{**}, F_S^{**}, \omega^{**})$ denotes the solution to such a constrained program. We thus consider the set of Kuhn-Tucker conditions in (B.3) and modify them appropriately:

$$
\begin{aligned}
(iv)': &\quad \frac{\partial T^S_{SA}(F^{**}_S)}{\partial F_S} + \frac{\partial C^S_{SA}(F^{**}_S)}{\partial F_S} - \frac{\partial \Gamma(F^{**}_P, F^{**}_S)}{\partial F_S} - \frac{\partial \Delta C(F^{**}_P, F^{**}_S)}{\partial F_S} + \frac{\partial \Delta T(F^{**}_P, F^{**}_S)}{\partial F_S} = \mu_2 - \mu_3, \\
(vii)': &\quad \mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0 \\
(xiv)': &\quad \mu_3(F^{**}_S - K) = 0.
\end{aligned}
$$

Let us consider the case in which the newly introduced constraint (xiv)' is binding, so that $F^{**}_S = K$. We look for the conditions under which the parent can be unlevered. Hence, $\mu_1 \geq 0, \mu_2 = 0, \mu_3 \geq 0$. We focus on condition (i), and we refer the reader to the proof of Theorem 1 for the discussion of other conditions, which is immediate. Condition (i), when $F^{**}_P = 0$ and $F^{**}_S = K$, becomes:

$$
\begin{aligned}
- &\quad \tau_P(1 - G(0)) \frac{\partial X^Z_P(0, K)}{\partial F_P} - \frac{\partial X^Z_S(0, K)}{\partial F_P} \int_{X^d_S(0,K)}^{+\infty} \tau_S f(x) dx + \\
+ &\quad \alpha_S \phi \frac{\partial X^d_P(0, K)}{\partial F_P} \left[\int_0^{X^Z_S(0,K)} x g(x, \frac{K}{1-\tau} - \frac{x}{1-\tau}) dx + \right. \\
+ &\quad \left. \int_{X^Z_S(0,K)}^{X^d_S(0,K)} x g(x, X^d_S(0, K) - x) dx \right] = \mu_1
\end{aligned}
\tag{B.16}
$$

The first term on the l.h.s. is negative, the second is negative as well and increasing in K (as X^Z_S is increasing and convex in F_P), while the third one is null when $K = 0$ and is increasing in K.

It follows that condition (i) can be satisfied only for a K high enough. We define $\bar{K}$ as the cap above which the parent is optimally unlevered. It solves the following equation:

$$
\begin{aligned}
& \alpha_S \phi \frac{\partial X_P^d(0,\bar{K})}{\partial F_P} \left[\int_0^{X_S^Z(0,\bar{K})} x g(x, \frac{\bar{K}}{1-\tau_S} - \frac{x}{1-\tau_S}) dx + \right. \\
+ & \left. \int_{X_S^Z(0,\bar{K})}^{X_S^d(0,\bar{K})} x g(x, X_S^d(0,\bar{K}) - x) dx \right] + \\
- & \frac{\partial X_S^Z(0,\bar{K})}{\partial F_P} \int_{X_S^d(0,\bar{K})}^{+\infty} \tau_S f(x) dx \\
= & \mu_1 + \tau_P (1 - G(0)) \frac{\partial X_P^Z(0,\bar{K})}{\partial F_P}
\end{aligned}
$$

Considerations similar to the unconstrained case apply to condition (iv)', which is met at $F_S^{**} = K$ by an appropriate choice of μ_3. Notice that the higher α_S, the lower the required cap level $\bar{K}$ that allows for the presence of an optimally unlevered parent company. This concludes our proof of part (i).

Part (ii) descends directly from Theorem 1 part ii), because it can be easily noticed that its proof relies on the parent being levered.

To prove part (iii), we discuss the first Kuhn-Tucker condition when $F_P = F_{SA}^{P*}, F_S = F_{SA}^{S*}$, considering the case in which $\tau_D < \underline{\tau}_D$. In this case, the only feasible solution implies $\omega^* = 1$. Then, given our convexity assumption, we can find a sufficient condition for $F_P^* > F_{SA}^{P*}$:

$$
\left(-\frac{\partial \Gamma}{\partial F_P} - \frac{\partial \Delta C}{\partial F_P} + \frac{\partial \Delta T}{\partial F_P} \right) |_{F_P = F_{SA}^{P*}, F_S = F_{SA}^{S*}, \omega^* = 1} < 0. \tag{B.17}
$$

This implies $\frac{\alpha_P}{\alpha_S} > C$, where C is equal to

$$
\frac{\frac{\partial X_P^d}{\partial F_P} \left[\int\limits_0^{X_{SA}^{Z*}} x g\left(x, X_{SA}^{P,d*} + \frac{F_{SA}^{S*}}{1-\tau_P} - \frac{x}{1-\tau_P}\right) dx + \int_{X_{SA}^{S,Z*}}^{X_{SA}^{S,d*}} x g\left(x, X_{SA}^{P,d*} + X_{SA}^{S,d*} - x) dx\right) \right] + \tau_D \int_{X_{SA}^{S,d*}}^{+\infty} \tau_S \frac{\partial X_S^Z}{\partial F_P} f(x) dx}{\frac{\partial X_P^d}{\partial F_P} \int\limits_{X_{SA}^{S,d*}}^{+\infty} X_{SA}^{P,d*} g\left(X_{SA}^{P,d*}, y\right) dy - \left[\frac{\partial X_P^d}{\partial F_P} - (1-\tau_D)\tau_S \frac{\partial X_S^Z}{\partial F_P} \right] \int\limits_{X_{SA}^{S,d*}}^{\frac{X_{SA}^{P,d*}}{(1-\tau_D)(1-\tau_S)} + X_{SA}^{S,d*}} (X_{SA}^{P,d*} - DIV(y)) g\left(X_{SA}^{P,d*} - DIV(y), y\right) dy}, \tag{B.18}
$$

and all derivatives are evaluated at $(F_{SA}^{P*}, F_{SA}^{S*})$.

Proof of Theorem 4

We know from Luciano and Nicodano (2014) that conditional guarantees are value increasing. As a consequence, the value of the parent/subsidiary structure when there is a bailout guarantee is $\nu_{PS}(F_P^{**}, F_{SA}^{S*}) \geq \nu_{SA}(F_{SA})$, where $\nu_{SA}(F_{SA}^{S*}, F_{SA}^{P*}) = \nu_{PS}(F_{SA}^{P*}, F_{SA}^{S*}, \Gamma = 0, \omega = 0)$. We know from the previous considerations that the f.o.c. for a solution to the PS problem when $F_P^{**} > 0$ include:

$$\frac{\partial T_{SA}^P(F_P^{**})}{\partial F_P} + \frac{\partial C_{SA}^P(F_P^{**})}{\partial F_P} - \frac{\partial \Gamma(F_P^{**}, F_{SA}^{P*})}{\partial F_P} - \frac{\partial \Delta C(F_P^{**}, F_{SA}^{P*})}{\partial F_P} + \frac{\partial \Delta T(F_P^{**}, F_{SA}^{P*})}{\partial F_P} = 0. \quad \text{(B.19)}$$

The equivalent equation in the stand-alone case is simply

$$\frac{\partial T_{SA}^P(F_{SA}^{P*})}{\partial F_{SA}} + \frac{\partial C_{SA}^P(F_{SA}^{P*})}{\partial F_{SA}} = 0.$$

We also know that $\frac{\partial \Gamma(F_P^{**}, F_{SA}^{P*})}{\partial F_P} \leq 0$, since the guarantee is non-zero and more valuable the lower F_P is. Also, when $\tau_D > \bar{\tau}_D$, $\Delta C = 0$ and $\Delta T = 0$ for all F_P and F_S since $\omega^* = 0$. Since by assumption $T_{SA}^P + C_{SA}^P$ is convex in the face value of debt, it follows that $F_P^{**} < F_{SA}^{P*}$.

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