NETTING*

Jason Roderick Donaldson[†]   Giorgia Piacentino[‡]

April 7, 2017

Abstract

We present a model to explain why banks hold off-setting debts without netting them out. We find that off-setting debts help a bank to raise liquidity with new debt from a third party, since diluting old debt subsidizes the new debt. Even though a diluted bank is worse off ex post, a network of gross debts is stable ex ante. This is because it provides banks with valuable liquidity co-insurance, since each bank exercises its option to dilute when it needs liquidity most. However, the network harbors systemic risk: since one bank's liabilities are other banks' assets, a liquidity shock can transmit through the network in a default cascade.

[†]Washington University in St. Louis; jasonrdonaldson@gmail.com.

[‡]Washington University in St. Louis and CEPR; piacentino@wustl.edu.

1 Introduction

Gross interbank positions are an order of magnitude larger than net interbank positions. For example, Barclay's gross repo position is 28 billion pounds, almost ten times its net position.[1] JP Morgan's gross derivatives position is 1.5 trillion dollars, almost twenty times its net position.[2] These banks are not atypical. In Germany, a quarter of the average bank's assets and liabilities comprise unnetted interbank exposures of various maturities, with an average maturity longer than a year (Bluhm, Georg, and Krahnen (2016)). These banks choose to hold these gross positions rather than net them out, even when they constitute off-setting debts with the same bank—i.e. a bank is likely to borrow from another bank and simultaneously lend to that bank.[3] Banks maintain these positions even though practitioners and policy makers alike champion the benefits of netting. Indeed, according to ISDA, "Support for netting is well-nigh universal in the financial industry as well as among policy markers" (Mengle (2010), p. 2). The literature suggests that cross-holdings of demand deposits may function as emergency credit lines (Allen and Gale (2000)). But why do banks choose not to net out fixed-term debts of longer maturities?

In this paper, we develop a model to address this question. We find that off-setting debts help a bank to raise liquidity with new debt from a third party, since diluting old debt subsidizes the new debt. Even though a diluted bank is worse off ex post, a network of gross debts is stable ex ante. This is because it provides banks with valuable liquidity co-insurance, since each bank exercises its option to dilute when it needs liquidity most. However, the network harbors systemic risk: since one bank's liabilities are other banks' assets, a liquidity shock can transmit through the network in a default cascade.

Model preview. In the model, there are N banks, $B_1, ..., B_N$, each of which suffers from a maturity mismatch—it has long-term assets in place, but may need short-term liquidity due to a "liquidity shock" before its assets mature. To meet this liquidity need, a bank can borrow in the market from an outside creditor C. However, its ability to do this is impeded by two contracting frictions. First, liquidity shocks are non-contractable

---

[1]Barclay's 2015 annual report (available online at <https://www.home.barclays/barclays-investor-relations/results-and-reports/annual-reports.html>) lists £28.187 billion in assets as "reverse repurchase agreements and other similar secured lending" and £25.035 billion in liabilities as "repurchase agreements and other similar secured borrowings."

[2]Corbi (2012) reports JP Morgan's net derivatives position as $80 billion and estimates its gross position as $1.485 trillion.

[3]For example, the bankruptcy proceedings of Lehman Brothers reveal that it held at least 500 million USD in off-setting claims with Bank of America and at least 145 million USD in off-setting claims with UBS (see, e.g., Cuillerier and Valdez (2012), Bank of America v. Lehman Brothers Holdings (2010), and Howard (2011)).

and, second, asset pledgeability is limited.

These frictions make it hard for a bank to avoid having to liquidate its assets to meet a liquidity shock—since liquidity shocks are non-contractable, a bank cannot pay a small upfront premium in exchange for a large payoff in the event of a shock; since asset pledgeability is limited, a bank cannot mortgage its assets to meet its liquidity need in the event of a shock. In contrast, without either friction, banks would insure completely and there would be no liquidity risk at all.

FIGURE 1: BALANCE SHEET BEFORE AND AFTER LIQUIDITY SHOCK

Gross Positions			Cash from Sale and New Debt	
assets	liabilities		assets	liabilities
long-term assets	risky short-term liabilities		long-term assets	risky short-term liabilities
debt from B$_j$	debt to B$_j$	$\longrightarrow$	cash from debt sale	debt to B$_j$
			cash from new debt	debt to C
	equity			equity

Results preview. Our first main result is that entering into off-setting debt positions allows banks to circumvent these frictions and thus to co-insure liquidity risk. To see why, suppose that you take on off-setting debts with bank B$_j$, i.e. you have a debt from B$_j$ on the assets side of your balance sheet and an off-setting debt to B$_j$ on the liabilities side of your balance sheet. Now, if you are hit by a liquidity shock, you have two ways to raise liquidity in the market from C. (i) You can sell the debt on the assets side of your balance sheet and (ii) you can take on new, *senior* debt on the liabilities side of your balance sheet (see Figure 1). Taking on this senior debt from C dilutes your debt to B$_j$: the cost of your default is transferred to B$_j$ and, hence, the new debt to C is subsidized. As a result, C is more willing to lend to you, which allows you to meet your liquidity shock.

Our second main result is that the network of off-setting debts is pairwise stable, i.e. you and B_j both want to enter into off-setting debts ex ante. Why does B_j want to enter into this position with you, even though it knows you will dilute its debt when you are hit by a liquidity shock? Because the off-setting debts allow B_j to raise liquidity by diluting your debt when it is shocked, just as they allow you to raise liquidity by diluting its debt when you are shocked. In other words, off-setting debts implement valuable liquidity co-insurance for you and B_j, since each of you exercises your option to dilute the other when you need liquidity most (and only when you need liquidity most). Hence, your zero-net debts do not have zero net present value.

This mechanism explains not only why a bank may borrow from one bank and simultaneously lend to that bank—i.e. the absence of bilateral netting—but also why a bank may borrow from one bank and simultaneously lend to another bank—i.e. the absence of multilateral netting, or so-called "novation." In particular, networks like the ring network (Subsection 5.1), in which banks intermediate between other banks, also implement liquidity insurance. There, banks accept the risk of being diluted by their borrowers in exchange for the option to dilute their creditors. Indeed, since banks rely on new debt to a third party to circumvent the limited-pledgeability friction, interbank debt is *optimally non-exclusive* in our model. Thus, our model suggests a rationale for why there is a lot of borrowing and lending within groups of banks generally and, as such, may cast light on the core-periphery structure of interbank debt networks.[4]

Our third main result is that, although the network of off-setting debts provides banks with liquidity insurance, it harbors systemic risk. Since your liability is B_j's asset, B_j may be unable to repay its creditors if you default. Specifically, if enough banks are hit by liquidity shocks, then all banks default, including those that are not shocked. In other words, whereas interconnectedness helps the financial system to sustain shocks in normal times, it can propagate shocks in the event of a liquidity crisis. This is in line with results in the networks literature (see below).

Policy. Some policy makers have suggested that "close-out netting," i.e. netting in the event of default,[5] may be enough to mitigate systemic risk (see, e.g., Mengle (2010)). The idea is that if you default on B_j, B_j can just net out its off-setting debt, so your default does not transmit to B_j's creditors. Our analysis suggests that such close-out netting may not be enough. This is because debts can be traded in the market before your default and, as a result, even if B_j has an off-setting position with you today, it may not tomorrow—B_j may owe money to a third party who bought its debt from you.

---

[4]To be specific, we view the banks $B_1, ..., B_N$ that hold unnetted gross positions as the core of the network and we view the liquidity shocks and long-term assets as the periphery of the network—liquidity shocks represent drawdowns from short-term creditors and long-term assets represent loans to long-term borrowers. Such core-periphery networks are pervasive in interbank markets, as documented in, e.g., Craig and von Peter (2010), in 't Veld and van Lelyveld (2014), and Peltonen, Scheicher, and Vuillemey (2013).

[5]For the legal details of close-out netting, see, e.g., Johnson (2015) and Paech (2014).

Thus, B_j may not have a debt to net out against you and your default may transmit to its creditors.

Empirical content. Our analysis is motivated by the empirical observation that banks often do not net out off-setting debts, but it may cast light on a number of other stylized facts as well. (i) In our model, banks with off-setting debts accept the costs of dilution without asking for increased repayments, since they gain reciprocal liquidity insurance. This is consistent with Afonso, Kovner, and Schoar's (2014) finding that "borrowers get...lower rates from lenders that they also lend to...consistent with a model where...borrowers and lenders...insure each other against liquidity shocks at favorable rates" (p. 3). (ii) In our model, banks use a mix of junior debt (to one another) and senior debt (to C), which we interpret as a mix of unsecured debt (e.g. bonds and loans) and secured debt (e.g. repos). This casts light on why banks' capital structure contains a mix of secured and unsecured debt and why banks may lend unsecured despite the risk of dilution. (iii) In our model, banks lend to one another in a network of zero-net positions to co-insure liquidity shocks from outside this core network. This may cast light on the core-periphery structure of banking systems, as touched on above (footnote 4).

The main frictions in our model seem to have been important in real-world liquidity crises. Consider the failures of LTCM, Bear Stearns, and Lehman Brothers. All involved liquidity shocks, suggesting liquidity shocks are hard to insure against via direct contracting. Also, even though LTCM and Lehman had valuable long-term assets, they could not raise enough liquidity against their collateral, suggesting limited pledgeability restricted their debt capacity. Furthermore, these banks all responded to liquidity shocks like the banks in our model: they raised liquidity by selling securities and taking on new secured debt (via repos).[6]

Related literature. We make four main contributions to the literature.

First, we uncover a mechanism by which off-setting non-contingent, non-exclusive debts implement state-contingent liquidity insurance, since they embed the option to dilute. This complements the idea that the option to default can implement valuable contractual contingencies when contracts are incomplete (see Allen and Gale (1998) and Zame (1993), as well as Hart and Moore (1998), in which the option to buy assets plays a similar role).

Second, we show that non-exclusive contracts can mitigate the inefficiencies arising from limited pledgeability, i.e. the ability to take on new debt to a third party can loosen borrowing constraints.[7] This is the counterpart of Donaldson, Gromb, and Piacentino's

---

[6]See, e.g., Jorion (2000) (pp. 282–284) on LTCM, Rose, Bergstresser, and Lane (2009) (pp. 11–13) on Bear, and Valukas (2010) (pp. 3 and 9–10) on Lehman.

[7]Dilutable debt also arises in Diamond's (1993) asymmetric-information model, since it makes it costly for bad borrowers to pool with good borrowers, and in Hart and Moore's (1995) agency model, since it loosens

(2016) finding that limited pledgeability can mitigate inefficiencies arising from non-exclusive contracts. Our result also gives a new, positive perspective on non-exclusivity, which the literature has generally considered as a friction.[8]

Third, we provide an explanation for why banks hold off-setting fixed-term debts. This complements Allen and Gale's (2000) finding that cross-holdings of demand deposits work as emergency credit lines when there is no market to borrow in.[9] Such cross-holdings of demand deposits do not help in our model, since they do not allow for the dilution which is at the heart of our mechanism. Rather, banks must hold off-setting fixed-term debts, as observed in practice.[10]

Fourth, we find that, despite providing liquidity insurance, the interbank network harbors systemic risk. This extends the "robust-yet-fragile" results from the networks literature to our environment, in which the network is endogenous and interbank debts are optimal.[11]

2   Model

In this section, we present the model.

---

constraints on management that are too tight if debts are strictly prioritized.

   [8]See, e.g., Acharya and Bisin (2014), Admati, DeMarzo, Hellwig, and Pfleiderer (2013), Attar, Casamatta, Chassagnon, and Décamps (2015), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Bizer and De-Marzo (1992), Brunnermeier and Oehmke (2013), Faure-Grimaud and Gromb (2004), Kahn and Mookherjee (1998), Leitner (2012), and Parlour and Rajan (2001).

   [9]If there is a debt market at the interim date in Allen and Gale's (2000) environment, then there is no role for cross-holdings of deposits, since banks can just borrow in the market to get liquidity. However, the possibility of borrowing at the interim date can also distort investment decisions at the initial date. Thus, in Allen and Gale's (2000) setup, a mechanism designer may wish to close debt markets (see their discussion on p. 26 as well as Allen and Gale (2004), Farhi, Golosov, and Tsyvinski (2009), and Jacklin (1987)). In our model, we take it as a primitive that debt markets exist and find that a mechanism designer does not want to close them, but rather relies on them.

   [10]Our finding that mutual gross debts help shocked banks to raise liquidity from a third party is related to a 2011 lecture by John Moore. He outlines a macro model based on the assumption that banks with larger gross interbank positions can borrow more from households. Our analysis suggests that Moore's reduced-from idea that mutual gross positions facilitate new borrowing is right. However, unlike in Moore's setup, access to new debt does not come for free in our model, since creditors bear the risk of dilution.

   [11]Papers that study systemic risk in interbank networks or credit chains include Acemoglu, Ozdaglar, and Thabaz-Salehi (2015), Allen, Babus, and Carletti (2012), Allen and Gale (2000), Babus (2016), Blumh, Faia, and Krahnen (2013), Brusco and Castiglionesi (2007), Cabrales, Gottardi, and Vega-Redondo (2013), Di Maggio and Tahbaz-Salehi (2015), Donaldson and Micheler (2016), Elliott, Golub, and Jackson (2014), Gale and Kariv (2007), Farboodi (2015), Freixas, Parigi, and Rochet (2000), Kiyotaki and Moore (2001), and Zawadowski (2013); however, the equilibrium network is endogenous in only a few of these papers.

2.1 Dates and Players

There are three dates, $t \in \{0, 1, 2\}$, and N symmetric banks, $B_1, ..., B_N$, as well as a creditor C, representing a competitive capital market. Everyone is risk-neutral, consumes only at Date 2, and discounts the future at rate zero. Banks have long-term assets but relatively short-term liabilities. Specifically, each bank has riskless assets y realized at Date 2, but the possibility of a "liquidity shock" at Date 1. If a bank is hit by a liquidity shock, it must generate liquidity $L < y$, otherwise it defaults and its assets are destroyed. We assume that exactly M banks are hit with liquidity shocks at Date 1, each with probability $\pi := M/N$. Since M is deterministic, there is no aggregate risk and thus there is the possibility of perfect insurance. C has enough wealth w to provide liquidity to all the shocked banks, but not to all the banks, i.e. $w \in [ML, NL)$.

2.2 Contracting Frictions

Two contracting frictions limit banks' ability to insure against liquidity shocks. First, liquidity shocks are non-contractable.[12] Second, asset pledgeability is limited, i.e. a bank can divert a fraction $1 - \theta$ of its assets y.[13]

These frictions are especially important for banks. Liquidity shocks are a major concern for banks since, almost by definition, they invest in long-term assets funded by short-term liabilities. Bank regulations such as Basel III's Liquidity Coverage Ratio expressly target these liquidity risks. This regulation reflects bank's inability to insure against liquidity shocks via direct insurance contracts—if banks could insure directly, they would not have to insure indirectly by holding securities as "liquidity coverage." Banks' complexity contributes to limited pledgeability, making it hard to value assets ex ante and costly to liquidate them ex post (Shleifer and Vishny (1992)).

2.3 Payoffs

If a bank is liquidated at Date 1, its assets are destroyed and it gets zero. Otherwise, it decides whether to default or to repay its debt. If it defaults, it gets the diversion value of its long-term assets $(1 - \theta)y$. If it repays, it gets y minus the *net* repayments from its financial assets, i.e. the repayments it makes to other banks less the repayments it

---

[12]In the baseline model, we assume for simplicity that liquidity shocks are observable but not verifiable following the incomplete-contracts paradigm. However, in Subsection 5.2, we show that our results also obtain if liquidity shocks are unobservable.

[13]Limited pledgeability is a catch-all way to generate financial constraints. It has micro-foundations in terms of numerous frictions, including capital diversion, non-contractable effort, and asymmetric information (see DeMarzo and Fishman (2007)).

gets from other banks and any cash it may hold:

$$\text{B}_i\text{'s payoff} = \begin{cases} 0 & \text{if liquidated at Date 1,} \\ (1-\theta)y & \text{if default at Date 2,} \\ y - \text{net repayments} & \text{if no default.} \end{cases} \quad (1)$$

It may be worth emphasizing that the bank cannot divert financial assets, but only the long-term assets y. In other words, cash and interbank debts are perfectly pledgeable, whereas only the fraction θ of long-term assets is pledgeable. This assumption may reflect the fact that (i) a real-world bank may be able to "divert" clients or employees in the event of default, but not to steal financial securities outright, or (ii) financial assets are easily redeployable and thus put creditors in a strong bargaining position in the event of default, which may prevent diversion as in Hart and Moore (1994, 1998), for example.[14]

2.4 Timeline

We now describe the sequence of moves. The capital market is competitive, so whenever we say that a bank borrows from or sells to C, it implies that the bank makes a take-it-or-leave-it offer.

Date 0   Banks contract bilaterally with C or among themselves.

Date 1 M random banks suffer liquidity shocks.

Banks sell assets to or borrow from C.

Each shocked bank either pays L or defaults.

Date 2   Asset payoffs realize, banks either divert and default or repay their debt.

2.5 Equilibrium

Below we consider various Date-0 contractual relationships. Given these contracts, we find the subgame perfect equilibrium. I.e. $\text{B}_1, ..., \text{B}_N$, and C act to maximize their expected utility given their beliefs about players' strategies and these beliefs are consistent with players' strategies.

---

[14]To be concrete, in models like Hart and Moore (1994, 1998), a borrower cannot commit not to renegotiate repayments. At renegotiation, creditors' threat point is to seize assets and redeploy them. Thus, redeployment values determine equilibrium repayments. As a result, financial assets are more pledgeable than other long-term assets, since creditors' redeployment value of financial securities is almost equal to borrowers' long-term value, whereas creditors' redeployment value of other long-term assets (e.g. relationship loans) is substantially discounted.

2.6 Assumption: Liquidity Risk

We assume that the mortgage value θy of a bank's assets is between its expected liquidity need $\pi L = ML/N$ and its realized liquidity need L given a shock,

$$\frac{M}{N}L < \theta y < L. \qquad (\star)$$

Intuitively, this assumption says that (i) there is enough liquidity to go around at Date 1, since the total mortgage value of banks' assets $N\theta y$ exceeds the total need for liquidity ML, but that (ii) no shocked bank has enough liquidity to self-insure, since the mortgage value of its own assets θy is less than its liquidity need L if it is shocked.

3 Benchmarks

In this section, we consider four benchmark allocations.

3.1 First Best

The first best is the allocation that maximizes the total surplus accrued to $B_1, ..., B_N$, and C. Since $L < y$, liquidation is inefficient. Thus, all liquidity shocks are met in the first best: a bank generates surplus $y - L$ if it is hit by a liquidity shock and y otherwise.

LEMMA 1. (FIRST BEST.) *The first-best surplus is* $N(y - \pi L)$.

3.2 Second Best

We define the second-best allocation as the allocation that maximizes banks' payoffs given limited pledgeability and individual rationality, but not the non-contractability of the liquidity shocks. In this case, a bank B_i can insure with C by paying a premium in every state for full insurance in the event of a shock. Specifically, at Date 2, B_i pays C the premium πL (which is less than θy by $(\star)$). And, at Date 1, C pays L to B_i whenever B_i suffers a liquidity shock (which occurs with probability π).

LEMMA 2. (SECOND BEST.) *The second-best surplus equals the first-best surplus* $N(y - \pi L)$ *from Lemma 1.*

3.3 Autarky

We now turn to the extreme case in which banks are in autarky, i.e. there are no transactions whatsoever among $B_1, ..., B_N$, and C. Due to maturity mismatch, a bank's

assets are destroyed whenever it is hit by a liquidity shock. Thus, a bank gets zero with probability π and y otherwise.

LEMMA 3. (AUTARKY.) *If banks are in autarky, the surplus is $N(1 - \pi)y$.*

3.4 No Interbank Market

We now turn to the case in which there is no interbank market, i.e. banks cannot contract with one another, but only with C (subject to the contracting frictions). In this setting, a bank cannot contract on the liquidity shock to insure fully, as in the second best (Subsection 3.2).

LEMMA 4. (NO INTERBANK MARKET.) *With no interbank market, full insurance is impossible. The surplus is strictly less than the first best (Lemma 1) and is given by*

$$N(1 - \pi)y + \pi \left\lfloor \frac{w}{L} \right\rfloor (y - L). \tag{2}$$

Notice this surplus is above the autarky surplus. This is because C can insure a bank B_i by transferring it L at Date 0 as a "liquidity buffer." This allows B_i to endure a liquidity shock at Date 1 and hence improves efficiency. However, since C has limited wealth, $w \in [ML, NL)$, it can insure at most $w/L < N$ banks this way, so inefficient liquidation is unavoidable and thus the surplus is less than the first best.

4 Mutual Gross Debts

In this section, we consider a network of mutual gross interbank debts. We show it implements the first-best surplus in equilibrium and is stable ex ante. However, it can harbor systemic risk.

4.1 Mutual Gross Debts Implement First Best

Consider the following network of contracts among the banks $B_1, ..., B_N$. Every pair of banks B_i and B_j enters an off-setting position: B_i and B_j exchange promises to repay F, as depicted in Figure 2. Suppose that these debts are paid proportionally (pari passu) but a bank B_i can take on new senior debt to C at Date 1. Thus, the interbank debts can be interpreted as unsecured, e.g. loans or bonds, and the debt to C as secured, e.g. repos.

PROPOSITION 1. (EQUILIBRIUM WITH MUTUAL GROSS DEBTS.) *The complete network of gross positions with $F = (L - \theta y)/(N - M)$ implements the first best.*
    The following constitutes an equilibrium outcome.
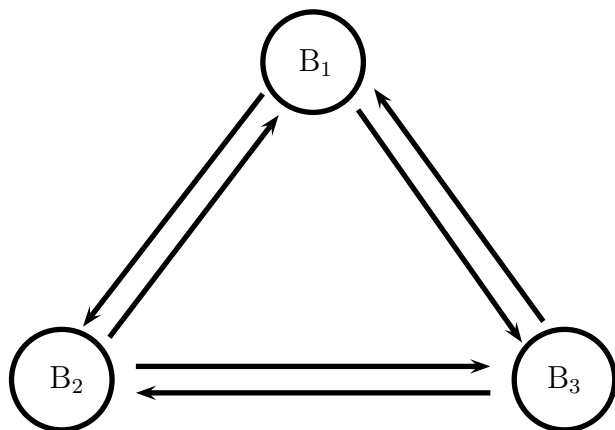
assets	liabilities
long-term assets y	risky short-term liabilities L
debt from B$_1$	debt to B$_1$
$\vdots$	$\vdots$
debt from B$_{i-1}$	debt to B$_{i-1}$
debt from B$_{i+1}$	debt to B$_{i+1}$
$\vdots$	$\vdots$
debt from B$_N$	debt to B$_N$
	equity

Date 1 *Shocked banks sell all the debt they hold and borrow θy from C via senior debt. They keep their long-term assets in place.*

Not-shocked banks keep their long-term assets in place.

Date 2 *Shocked banks default and get $(1 - \theta)y$.*

Not-shocked banks do not default and get $y - MF$.

This says that holding off-setting debts allows banks to insure liquidity shocks, so that shocked banks can avoid liquidation at Date 1 and keep their long-term assets in place until Date 2. The reason is that shocked banks can take on new debt, diluting not-shocked banks. This dilution implements a transfer from the not-shocked banks to shocked banks, which is exactly the insurance transfer that banks would commit to if they could contract on liquidity shocks (Lemma 2).

This mechanism works because, given the face value F, not-shocked banks' assets are valuable enough relative to their liabilities that they have incentive to repay their debt and avoid default. Given they never default, not-shocked banks have no way to benefit from dilution, since there can be no dilution without default. As a result, not-shocked banks' debt sells at par. Hence, a shocked bank can raise $(N - M)F$ from selling not-shocked banks' debt—it holds the debt of $N - 1$ other banks of which $M - 1$ are shocked. It can also raise θy from taking on new senior debt to C. Thus, in total a

shocked bank can raise

$$(N - M)F + \theta y = (N - M)\frac{L - \theta y}{N - M} + \theta y = L. \tag{3}$$

I.e. a shocked bank can raise exactly the liquidity L it needs to sustain its shock.

4.2 Mutual Gross Debts Are Stable

Now turn to Date 0. Given the network of mutual gross debts, would a pair of banks B_i and B_j like to alter their bilateral agreement? For example, would B_i and B_j like to change the face value of debt that they owe each other? In other words, is this network of mutual gross debts pairwise stable? Or are covenants or other contractual provisions necessary?

PROPOSITION 2. (STABILITY OF MUTUAL GROSS DEBTS.) *The network in which all banks have debts to and from all other banks with face value $F = (L - \theta y)/(N - M)$ is pairwise stable in the sense that no pair of banks can enter into another pair of bilateral (pari passu or prioritized) debts that makes one strictly better off without making the other strictly worse off.*

A pair of banks may try to increase its joint surplus at the expense of the other banks, i.e. alter their bilateral contracts to dilute other banks. With debt contracts, they can do this in two ways: (i) they can prioritize each others' debt, e.g. securing the assets θy to ensure seniority, or (ii) they can lower each others' face values, so they make lower repayments to each other relative to other banks. However, both (i) and (ii) are undesirable because they undermine the liquidity insurance role of off-setting debts—they lead a bank to default when it is hit by a liquidity shock. This is because both arrangements reduce the amount that a bank can raise in the market from C: prioritizing debt reduces the amount that a bank can borrow from C, since it reduces the value of the assets that a bank pledges to C as security; decreasing face values decreases the amount that a bank can raise from debt sales, since it reduces the market value of the other bank's debt.

In the N-bank environment, we establish network stability within the class of debt contracts. However, our analysis speaks to the optimality of debt as well, as we show formally for $N = 2$.

COROLLARY 1. (OPTIMALITY OF DEBT.) *Suppose $N = 2$ and $M = 1$. Off-setting non-prioritized debts with face value $F = L - \theta y$ are globally optimal bilateral contracts between B_1 and B_2.*

The fact that these off-setting debts implement the first best outcome between B_1 and B_2 (Proposition 1) implies that the two-bank network in which B_1 and B_2 take off-

setting debts is stable to deviation to any contract, not just debt contracts. Further, B_1 and B_2 optimally choose not to prioritize their debts. Each accepts the possibility of being diluted with new debt to C in exchange for the option to dilute the other with new debt to C.[15] This works because each bank exercises its option to dilute only when it is shocked, given that dilution is only valuable when default is possible. This mechanism relies on *non-exclusive* contracts—those that allow banks to enter into a new relationship with C—to help banks to co-insure, suggesting a positive side to non-exclusive contracting which contrasts with the literature. Further, banks use a mix of junior debt (to each other) and senior debt (to C) to implement efficiency. This casts light on the coexistence of unsecured and secured debt in real-world banks' capital structure.

4.3   Mutual Gross Debts Harbor Systemic Risk

Now consider a "large" liquidity need given the network of gross debts above. What happens if $M' > M$ banks need liquidity? Do mutual gross debts still serve as liquidity insurance?

PROPOSITION 3. (SYSTEMIC CRISES.) *Suppose that all banks have unsecured debt to and from each other with face value $F = (L - \theta y)/(N - M)$ and the number of shocked banks is M', where*

$$M' > \frac{(N - M)\theta y}{L - \theta y}. \tag{4}$$

There is a "systemic crisis" in which all banks default: the M' shocked banks fail to meet their liquidity shocks at Date 1 and the $N - M'$ not-shocked banks default at Date 2.

Even though mutual gross debts allocate liquidity efficiently whenever there is enough of it to go around, we now see that gross debts can also exacerbate liquidity shortages when there is not enough of it to go around. Indeed, if the number of shocked banks M' is large enough, liquidity shocks transmit through the network of gross debts leading *all* banks to default. The shock passes from one bank to the next in a *default cascade*: since each bank's liabilities are another bank's assets, a bank's failure to repay its debt harms not only its creditors but also its creditors' creditors. This finding echoes results in the networks literature that financial linkages can mitigate the effects of some shocks while amplifying the effects of others (see, e.g., Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) and Allen and Gale (2000)).

---

[15]This idea that financial networks form because banks voluntarily expose themselves to dilution complements Farboodi's (2015) idea that financial networks form because banks voluntarily expose themselves to counterparty risk.

assets	liabilities
long-term assets y	risky short-term liabilities L
debt from B_{i-1}	debt to B_{i+1}
	equity
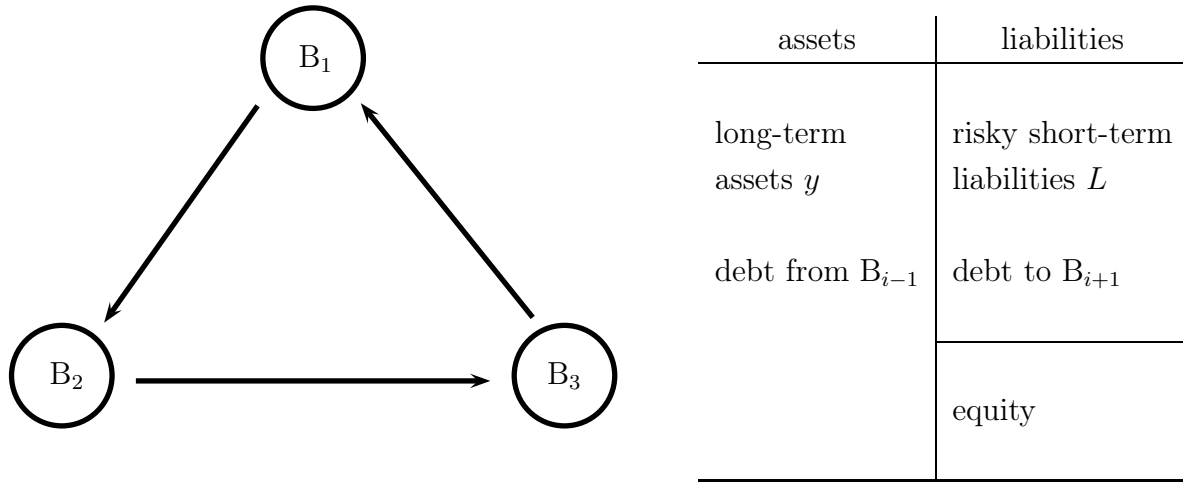
5 Extensions

We now consider extensions of the baseline setup.

5.1 Multilateral Netting and the Ring Network

So far we have focused on why banks hold mutual gross debts, i.e. on the absence of bi-lateral netting. However, our mechanism also casts light on why banks lend and borrow from many other banks, i.e. on the absence of multilateral netting ("novation"). For example, consider the *ring network* in which B_i has debt F to B_{i+1} for $i \in \{1, ..., N-1\}$ and B_N has debt to B_1, as depicted in Figure 3. This zero-net position among the banks $B_1, ..., B_N$ can also implement liquidity insurance, albeit under more restrictive conditions.

LEMMA 5. (RING NETWORK.) *Suppose $M = 1$ and $L \leq 2\theta y$. The ring network with $F = L - \theta y$ implements the first best.*

This result implies that gross exposures within networks provide liquidity insurance via the option to dilute generally; the mechanism is not specific to the complete network. Observe, however, that the ring network implements the first best only for $M = 1$. This is because it cannot provide insurance if two adjacent banks are shocked: if B_i and B_{i+1} are both shocked, then B_{i+1} cannot raise enough liquidity in the market from selling B_i's debt.

5.2 Unobservable Liquidity Shocks

So far, we have assumed that liquidity shocks were observable but non-contractable, following the incomplete-contracts paradigm. However, if liquidity shocks are unobservable, gross positions can still implement liquidity insurance. The core mechanism is unchanged: shocked banks liquidate debt in the market and mortgage their assets to meet their liquidity shocks. However, there is one twist on the equilibrium above: not only do shocked banks sell to avoid liquidation, but not-shocked banks also sell to pool with the shocked banks. This is because they sell at a subsidy, since their portfolios are worse on average than not-shocked banks' portfolios—not-shocked banks hold the debts of M shocked banks and $N - 1 - M$ not-shocked banks, whereas shocked banks hold the debts of $M - 1$ shocked banks and $N - M$ not-shocked banks.

We make one additional assumption to construct the equilibrium: there are two markets at Date 1, first, a resale market for debt in which banks sell debts to C and, second, a market for new loans, in which banks can borrow from C or from other banks. This assumption allows all banks to sell debts at a pooling price in the market as well as for banks to reallocate liquidity among themselves.

LEMMA 6. *Suppose the model is as specified in Section 2, except liquidity shocks are unobservable and there are Date-1 markets as described above.*

Let

$$F = \frac{L - \theta y}{(1 - \pi)(N - 1)}. \tag{5}$$

As a long $w > N(L - \theta y)$, the complete network of gross positions with face value F implements the first best.

The following constitutes an equilibrium outcome.

Date 1	*Both shocked banks and not-shocked banks sell each of the $N - 1$ debts they hold at price $(1 - \pi)F$.*
	Shocked banks borrow θy in the market via new senior debt.

Date 2	*Shocked banks default and get $(1 - \theta)y$. Not-shocked banks do not default and get $y - MF$.*

The out-of-equilibrium beliefs are that any bank that deviates is believed to be not-shocked.

6   Conclusion

Why do banks choose to hold off-setting gross positions rather than net them out? Because they embed the option to dilute. Since banks exercise this option when they

need liquidity most, unnetted gross positions implement mutually beneficial liquidity insurance. Indeed, as long as there is enough liquidity to go around, these mutual gross positions allocate liquidity efficiently. However, if there is not enough liquidity to go around, they can exacerbate liquidity shortages—with gross positions, a liquidity shortage can even trigger a systemic crisis in which all banks default.

A Proofs

A.1 Proof of Lemma 1

The result follows immediately from the argument in the text given that the probability that each of the N banks is shocked is π. $\qquad\square$

A.2 Proof of Lemma 2

The result follows almost immediately from the argument in the text. It remains only to check that C has sufficient wealth to transfer L to each shocked bank, i.e. that $N\pi L = ML \leq w$, which holds by assumption. $\qquad\square$

A.3 Proof of Lemma 3

The result follows immediately from the argument in the text. $\qquad\square$

A.4 Proof of Lemma 4

We first show that it is individually rational for C to insure any given bank B_i with a non-contingent contract: C transfers a "liquidity buffer" L to B_i at Date 0 in exchange for the promised repayment R at Date 2. This is bilaterally optimal and avoids inefficient liquidation of B_i's assets. However, C has limited wealth w and cannot make a non-contingent transfer of L to more than w/L banks. This leads to inefficient liquidation of some banks.

Bilateral insurance. Suppose C transfers L to B_i at Date 0 and B_i promises to repay R at Date 2. If B_i is shocked, it uses L to meet its liquidity need. At Date 2, it has assets y at and repays $\min\{R, \theta y\}$ to C. If B_i is not hit by a liquidity shock, it has assets $y + L$ at Date 2 and repays $\min\{R, \theta y + L\}$ to C. Thus, C's break-even condition is

$$L = \pi \min\{R, \theta y\} + (1 - \pi)\min\{R, \theta y + L\}, \tag{6}$$

which has the solution

$$R = \frac{L - \pi\theta y}{1 - \pi}. \tag{7}$$

This is between θy and $L + \theta y$ by $(\star)$, so if B_i is shocked C gets less than L from seizing B_i's assets and if B_i is not shocked C gets more than L from repayment. C breaks even and B_i gets $y - \pi L$, as in the first best.

Wealth constraint. To minimize the number of liquidations and maximize the surplus, C transfers L to as many banks as it can subject to its wealth constraint. Since C has wealth $w \in [ML, NL)$, it can make the non-contingent transfer L to $\lfloor w/L \rfloor$

random banks. Thus, a proportion $\lfloor w/L \rfloor /N$ of shocked banks avoid liquidation. In summary, all $N - M$ not-shocked banks get y, a fraction $\lfloor w/L \rfloor /N$ of the M shocked banks get $y - L$, and the remaining banks are liquidated and get zero. This gives the expected surplus

$$\text{expected surplus} = (N - M)y + \frac{M}{N} \left\lfloor \frac{w}{L} \right\rfloor (y - L). \tag{8}$$

This is less than the first-best surplus since $w < NL$. □

A.5 Proof of Proposition 1

We prove the result using the one-shot deviation principle as follows. Given the supposed equilibrium, we find the market prices of debt for shocked and not-shocked banks and show (i) that there are no profitable deviations at Date 2 and (ii) that there are no profitable deviations at Date 1.

Debt prices. In the proposed equilibrium, the shocked banks repay nothing on their interbank debts, since they must secure all their pledgeable assets θy to C in order to raise L to meet their liquidity shocks (see equation (3)). Thus, the Date-1 market price of shocked banks' debt is zero. In contrast, the not-shocked banks repay the face value of their interbank debts. Thus, the Date-1 market price of not-shocked banks debt is $F = (L - \theta y)/(N - M)$.

Date 2. A shocked bank has total debt $(N - 1)F + \theta y$, comprising the interbank debt it took on with the $N - 1$ other banks at Date 0 and the debt to C it took on at Date 1. It defaults since its payoff from diversion is greater than its payoff from repayment:

$$(1 - \theta)y \geq y - (N - 1)F - \theta y. \tag{9}$$

A not-shocked bank has total debt $(N - 1)F$, comprising the interbank debt it took on with the $N - 1$ other banks at Date 0. It has assets $y + (N - 1 - M)F$, comprising the value of its long-term assets y and the debt owed to it from the $(N - 1 - M)F$ other not-shocked banks. It does not default since its payoff from repayment is greater than its payoff from diversion:

$$y - \left((N - 1)F - (N - 1 - M)F \right) \geq (1 - \theta)y \tag{10}$$

or

$$\theta y \geq MF = M \frac{L - \theta y}{N - M}. \tag{11}$$

This can be re-written as $N\theta y \geq ML$, which is satisfied by ($\star$). The bank gets payoff

$$y - \left((N - 1)F - (N - 1 - M)F \right) = y - MF. \tag{12}$$

17

Date 1. Given the prices above, each shocked bank has interbank debt worth

$$\big((N-1)-(M-1)\big)F = L - \theta y, \tag{13}$$

since it holds the debt of all of the $N-1$ other banks, of which $M-1$ are shocked. Thus, the only way each shocked bank can raise L is to mortgage its assets entirely. It always prefers to do this, since if it does not raise L, it is liquidated and gets zero. Thus it has no incentive to deviate to any other strategy.

A not-shocked bank can sell assets or borrow from the market to raise cash from C. There are two cases: (i) the bank stays solvent or (ii) the bank defaults.[16]

Case (i): Not-shocked bank is solvent. Intuitively: two risk-neutral players—the not-shocked bank and C—cannot enter into a profitable trade, so there is no profitable deviation for the bank. More formally: if a not-shocked bank raises c by selling assets (i.e. selling debts from other banks) and stays solvent, the repayment it receives from other banks is decreased by c, since it raised c at the fair price. This is not a profitable deviation. If it raises c by borrowing and stays solvent, it must repay c to C, since it borrowed at the fair price. This is not a profitable deviation.

Case (ii): Not-shocked bank defaults. If a not-shocked becomes insolvent, it diverts and gets $(1-\theta)y$. This is not a profitable deviation because it is less than the payoff from staying solvent, as established above (equations (11) and (12)). $\qquad\square$

A.6 Proof of Proposition 2

The main idea behind the proof reflects the fact that if two banks, B_i and B_j, deviate to dilute other banks, it will make it impossible for either B_i or B_j to endure a liquidity shock, as discussed in the text (p. 11). In the formal proof, we must show that the benefits of dilution are always outweighed by the cost of liquidation for any deviation (F_i, F_j), where F_i denotes the face value of debt B_i owes B_j and F_j denotes the face value of debt B_j owes B_i.

There are three types of deviations to unsecured debt, either F_i and F_j both decrease, both increase, or one increases and one decreases. We rule each of these out in turn and then rule out deviations to prioritized (i.e. "secured") debt.

The rest of the proof proceeds as follows.

0. We summarize maintained assumptions and notation.

1. We show there is no profitable deviation in which one bank increases and the other decreases its face value, i.e. in which $F_i \leq F \leq F_j$.

---

[16]Note that there is no case in which a bank defaults with some probability $p \in (0,1)$, since all uncertainty is resolved at Date 1 and we have not allowed C to sell lotteries (but only to buy debt and lend).

2. We show that there is no profitable deviation in which both banks decrease their face values, i.e. in which both $F_i < F$ and $F_j < F$.

3. We show that there is no profitable deviation in which both banks increase their face values, i.e. in which both $F_i > F$ and $F_j > F$.

4. We show that there is no profitable deviation to prioritized debts.

Step 0: Summary of maintained assumptions and notation. To show the network is stable we need only to show that it is stable for a given equilibrium. Thus, we restrict attention to equilibria in which a shocked bank always mortgages its assets with C at Date 1. This is always incentive compatible, since it can never reduce a shocked banks' payoff—a shocked bank can always hold cash and repay C at Date 2.

Throughout, F refers to the equilibrium face value $F = (L - \theta y)/(N - M)$.

Finally, note that since B_i and B_j are symmetric, the results we prove for one bank hold for the other. We do not re-write the results. E.g., when we prove that it cannot be that $F_j \leq F < F_i$, it should be understood that thus it cannot be that $F_i \leq F < F_j$ either.

Step 1: No deviation in which one bank increases and the other deceases its face value. If either $F_i < F \leq F_j$ or $F_i \leq F < F_j$, then B_j is worse off: it repays at least as much in total and gets at most as much from from B_i. Thus, this cannot be a profitable deviation for B_j.

Step 2: No deviation in which both banks decrease face values. Before proceeding, we note the following lemma, which allows us to focus on states in which one bank is shocked and the other is not.

LEMMA 7. *For any deviation to be profitable, the joint surplus when one bank is shocked and the other is not must increase, i.e. the sum of B_i's payoff and B_j's payoff when B_i is shocked and B_j is not shocked must be greater given the deviation to F_i and F_j than given the equilibrium F.*

Proof. We first explain that the joint surplus must increase for a deviation to be profitable and then argue that we can restrict attention to states in which one bank is shocked and the other is not.

If the joint surplus is lower, then at least one bank must be worse off. Thus, the joint surplus must increase for both banks to be willing to deviate.

Now observe that the joint surplus cannot increase if either neither bank is shocked or both banks are shocked.

If neither bank is shocked, then there are two cases. Either (i) no bank defaults or (ii) a bank defaults. If (i), then the joint surplus is unchanged, since payments net out. If (ii), then the defaulting bank is worse off, since if it defaults when neither bank is shocked then it always defaults, and thus it gets less than it would without deviating.

If both banks are shocked, then they must mortgage all their assets at Date 1 to avoid liquidation. Thus, they both are at best not liquidated at Date 1 and default at Date 2. The joint surplus is at best unchanged (since both banks default if they do not deviate).[17] □

Given this lemma, we focus on how the deviation affects the joint surplus when B_i is shocked and B_j is not shocked. B_i sells other banks' debts and gets

$$\Big((N-2)-(M-1)\Big)F + F_j < (N-M)F = L - \theta y, \tag{14}$$

since $F_j < F$ by hypothesis. Thus, B_i cannot raise L even if it mortgages all its assets for θy. B_i must liquidate and gets zero. This reduces its payoff by $(1-\theta)y$, since it gets $(1-\theta)y$ in equilibrium if it meets its liquidity at Date 1 and defaults at Date 2.

Whereas B_i is worse off, B_j may be better off, since it has less debt to repay at Date 2 given $F_j < F$. Indeed, B_j's payoff is increased by $F - F_j$.

In order for the joint surplus to be increased, it must be that $F - F_j > (1-\theta)y$, i.e. the benefit to B_j outweighs the cost to B_i. Since $F_j \geq 0$, this implies that $F \geq (1-\theta)y$ or

$$\frac{L - \theta y}{N - M} \geq (1-\theta)y, \tag{15}$$

which can be re-written as

$$L \geq y + (N - M - 1)(1-\theta)y \geq y, \tag{16}$$

which is violated since $L < y$ by assumption. Thus, the joint surplus is decreased.

Step 3: No deviation in which both banks increase face values. Given Lemma 7 above, we can focus on how the deviation affects the joint surplus when B_i is shocked and B_j is not shocked. There are two cases: Case (i) in which B_i does not default and Case (ii) in which B_i defaults. Below, we show that in neither of these cases is the joint surplus increased. Intuitively, if B_i does not default, it does not benefit from dilution, so the joint surplus is lower. If B_i does default, B_i's payoff is unchanged and B_j's payoff cannot increase, so the joint surplus is also (weakly) lower.

We can restrict attention to cases in which B_j, the not-shocked bank, does not default, since if it defaults its payoff (and hence the joint surplus) is lowered further.

Case (i): B_i does not default. In this case, B_i's payoff is $y - $ net repayment $- L$.

---

[17]To see why both banks necessarily default when they are both shocked in a bit more detail, suppose that B_i were not to default. In this case, B_i would make a strictly lower net repayment to B_j. However, this would leave B_j unable to meet its liquidity shock. As a result, B_j would be liquidated and, thus, B_j would repay zero to B_i. But then the net repayment from B_i to B_j could not be less than zero. Since in equilibrium the net repayment is zero (both banks pay the other zero when shocked), the net repayment is not lowered, a contradiction.

Given that if there is no deviation, B_i defaults and gets $(1 - \theta)y$, the change in B_i's payoff is

$$y - \text{net repayment} - L - (1 - \theta)y \tag{17}$$

$$= y - \left(\big((N - 2)F + F_i\big) - \big((N - 2 - (M - 1))F + F_j\big) \right) - L - (1 - \theta)y \tag{18}$$

$$= \theta y - L + F_j - F_i - (M - 1)F. \tag{19}$$

Since B_j is not shocked and does not default, the change in B_j's payoff is at most $F_i - F_j$. Thus, the change in the joint surplus is at most

$$\theta y - L + F_j - F_i - (M - 1)F + F_i - F_j = \theta y - L - (M - 1)F \tag{20}$$

$$= \theta y - L - \frac{M - 1}{N - M}(L - \theta y) \tag{21}$$

$$= -\frac{M - 1}{N - M}(L - \theta y) < 0, \tag{22}$$

since $L > \theta y$ by $(\star)$. Thus, the joint surplus is decreased.

Case (ii): B_i defaults. In this case, B_i's payoff is $(1-\theta)y$, which is the same as when there is no deviation. In contrast, B_j does not default and must repay an additional $F_j - F$. However, it may also recover more from B_i given B_i's default. With no deviation, B_j recovered zero from B_i given default. Given the deviation, it may recover something positive. In particular, it may get the increase in B_i's total pledgeable assets (θy plus its financial assets), which is equal to

$$\theta y + \left(N - 2 - (M - 1) \right) F + F_j - L = (N - M)F + F_j - F + \theta y - L \tag{23}$$

$$= F_j - F, \tag{24}$$

having substituted for the equilibrium face value $F = (L - \theta y)/(N - M)$ in some places. Thus, the change in B_j's payoff is at most

$$-\left(F_j - F\right) + \left(F_j - F\right) = 0. \tag{25}$$

Thus, the joint surplus is not increased.[18]

Step 4: No deviation to prioritized debt. Finally, suppose B_i prioritizes B_j's debt. We first consider B_i's payoff when it is shocked and then when it is not shocked.

If B_i is shocked, then B_i cannot mortgage its assets to raise liquidity from C, since the equilibrium face value F is determined such that C must have the senior claim on

---

[18]It might be worth pointing out that the joint surplus will actually be decreased, since we have assumed that debt is pari passu and therefore B_j would not capture all the increase in B_i's total pledgeable assets as in equation (24).

all the pledgeable assets θy (equation (3)). Thus, B_i is liquidated. B_i gets zero and, hence, cannot repay B_j (even though B_j was prioritized).

If B_i is not shocked and it does not default, B_i's repayments are not affected by priority. It is no better off than in any deviation to unsecured debt, as ruled out above. If B_i is not shocked and it defaults, it is worse off than it would be if it did not deviate (since then it defaults only if it is shocked).

In summary, if a bank prioritizes its assets, it is worse when it is shocked and no better off when it is not shocked. $\qquad\square$

A.7 Proof of Corollary 1

The result follows immediately from Proposition 1. $\qquad\square$

A.8 Proof of Proposition 3

We first show that shocked banks are liquidated at Date 1 and then that not-shocked banks default at Date 2.

Shocked banks fail at Date 1. A shocked bank can raise at most $\big((N-1) - (M'-1)\big)F = (N-M')F$ from selling its debt. Thus, the maximum it can raise is

$$\theta y + (N-M')F = \theta y + (N-M')\frac{L-\theta y}{N-M} < L. \tag{26}$$

This holds since $M' > M$, following from the hypothesis that $M' > (N-M)\theta y/(L-\theta y)$ and $(\star)$.

Not-shocked banks default at Date 2. A not-shocked bank's payoff after repayment is lower than its payoff $(1-\theta)y$ from default:

$$y + (N-M'-1)F - (N-1)F = y - M'\frac{L-\theta y}{N-M} \tag{27}$$

$$< (1-\theta)y, \tag{28}$$

since $M' > (N-M)\theta y/(L-\theta y)$ by hypothesis. $\qquad\square$

A.9 Proof of Lemma 5

The equilibrium strategies are exactly as in Proposition 1 and almost the same proof applies. We must only check the corresponding conditions for the ring network, that (i) the single shocked bank can raise enough liquidity at Date 1 and (ii) the not-shocked banks do not default at Date 2.

Shocked bank. In the ring network, the shocked bank, say B_i, has the debt of one bank, B_{i-1}, to sell to raise liquidity, where we suppose $1 < i < N-1$ w.l.o.g. to keep

notation simple below. Thus, it raises $F = L - \theta y$ and it can meet its liquidity shock L by mortgaging its assets to raise θy form C. It defaults at Date 2 and the value of its debt is zero at Date 1.

Not-shocked banks. No not-shocked bank defaults at Date 2. To see this, consider B_{i+1}, that holds the debt of the shocked bank B_i as an asset. B_{i+1} does not default whenever its payoff from repaying F to B_{i+1} is greater than its payoff from diversion, or

$$y - F = y - (L - \theta y) \geq (1 - \theta)y. \tag{29}$$

This can be re-written as $2\theta y \geq L$, which holds by hypothesis.

B_{i+1} has the same liabilities as all other not-shocked banks but lower assets. Since B_{i+1} does not default, no other not-shocked bank defaults either. $\qquad \blacksquare$

A.10   Proof of Lemma 6

The proof is analogous to the proof of Proposition 1. We apply the one-shot deviation principle as follows. Given the supposed equilibrium, we find the market price of debt for shocked and not-shocked banks and show (i) that there are no profitable deviations at Date 2 and (ii) that there are no profitable deviations at Date 1.

Debt price. In the proposed equilibrium, the shocked banks repay nothing on their interbank debts (since they have secured all their pledgeable assets θy). Thus, the Date-2 value of shocked banks' debt is zero. In contrast, the not-shocked banks repay the face value of their interbank debts. Given we have supposed a pooling equilibrium, all debt trades at the average price at Date 1

$$\text{Date-1 debt price} \ = \ \frac{M}{N} \times 0 + \frac{N - M}{N} \times F = (1 - \pi)F. \tag{30}$$

Date 2. A shocked bank has total debt $(N - 1)F + \theta y$, comprising the interbank debt it took on with the $N - 1$ other banks at Date 0 and the debt it took on to C at Date 1. It defaults since its payoff from diversion exceeds its payoff from repayment:

$$(1 - \theta)y \geq y - \left((N - 1)F + \theta y\right). \tag{31}$$

Recall that, unlike in the equilibrium of the baseline model, in this equilibrium a not-shocked bank raises cash $(N - 1)(1 - \pi)F$ at Date 1 as well, which it either holds in cash or lends out senior at the risk-free rate (zero) in the debt market. Still, as in the equilibrium of the baseline model, it does not default since its payoff from repayment exceeds its payoff from diversion:

$$y - \left((N - 1)F - (N - 1)(1 - \pi)F\right) \geq (1 - \theta)y \tag{32}$$

or, substituting in for F,

$$(N-1)\pi\frac{L-\theta y}{(1-\pi)(N-1)} \le \theta y. \tag{33}$$

This can be re-written as $\pi L \le \theta y$, which is satisfied by ($\star$).

Date 1. Given the price above, each bank has interbank debt worth

$$(N-1)(1-\pi)F = L - \theta y. \tag{34}$$

Thus, the only way that a shocked bank can raise L is to mortgage its assets entirely. It always prefers to do this, since if it does not raise L, it is liquidated and gets zero.

A not-shocked bank can sell its assets or not. If it pools with the shocked bank, it sells other banks' debt at the pooling price in equation (30). Otherwise, it is believed to be not-shocked, given the out-of-equilibrium beliefs. At Date 1, it holds the debt of $N-1$ other banks, M of which are shocked (with debt worth zero) and $N-M$ of which are not-shocked (with debt worth F). Thus, a not-shocked bank holds other banks' debt with average value

$$\frac{M}{N-1} \times 0 + \frac{(N-1-M)}{N-1} \times F < \frac{N-M}{N}F = \text{ Date-1 debt price.}$$

I.e., if the not-shocked bank separates, its assets are worth less than their market price given the pooling equilibrium. Thus, a not-shocked bank strictly prefers to sell at the pooling price than to separate.

We conclude the proof by checking that C has enough liquidity to buy all banks' debt. In the complete network, each of the N banks holds the debt of each of the $N-1$ other banks. Thus, the total amount of liquidity that C needs to buy all banks' debt is

$$N \times (N-1) \times \left(\text{Date-1 debt price}\right) = N(N-1)(1-\pi)F \tag{35}$$

$$= N(N-1)(1-\pi)\frac{L-\theta y}{(1-\pi)(N-1)} \tag{36}$$

$$= N(L-\theta y) \tag{37}$$

$$< w, \tag{38}$$

by hypothesis.

Shocked banks meet their liquidity needs by borrowing via senior debt in the anonymous market from both C and not-shocked banks with excess cash, who are indifferent between lending and holding cash at rate zero. $\qquad\square$

B   Table of Notations

Dates and Players	
$t \in \{0,1,2\}$	time index
B_i	ith bank for $i \in \{1, ..., N\}$
C	outside creditor/"competitive capital market"

Assets, Liquidity Shocks, and Endowments	
y	banks' Date-2 asset value
θ	pledgeable fraction of assets
L	banks' liquidity need if hit by liquidity shock
M	number of banks hit by liquidity shocks
$\pi \equiv M/N$	probability a bank is hit by liquidity shock
w	C's wealth

Prices	
F	Face value of interbank debt

References

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2015, February). Systemic risk and stability in financial networks. *American Economic Review 105*(2), 564–608.

Acharya, V. and A. Bisin (2014). Counterparty risk externality: Centralized versus over-the-counter markets. *Journal of Economic Theory 149*(C), 153–182.

Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. Pfleiderer (2013). The Leverage Ratchet Effect. Working paper series of the Max Planck Institute for Research on Collective Goods, Max Planck Institute for Research on Collective Goods.

Afonso, G., A. Kovner, and A. Schoar (2014). Trading partners in the interbank lending market. Federal Reserve Bank of New York staff report no. 620.

Allen, F., A. Babus, and E. Carletti (2012). Asset commonality, debt maturity and systemic risk. *Journal of Financial Economics 104*(3), 519–534.

Allen, F. and D. Gale (1998). Optimal financial crises. *The Journal of Finance 53*, 1245–1284.

Allen, F. and D. Gale (2000). Financial Contagion. *Journal of Political Economy, University of Chicago Press 108*(1), 1–33.

Allen, F. and D. Gale (2004). Financial Intermediaries and Markets. *Econometrica 72*(4), 1023–1061.

Attar, A., C. Casamatta, A. Chassagnon, and J.-P. Décamps (2015). Multiple lenders, strategic default, and the role of debt covenants. Working paper.

Babus, A. (2016). The formation of financial networks. *The RAND Journal of Economics 47*(2), 239–272.

Bank of America v. Lehman Brothers Holdings (2010). (In re Lehman Brothers Holdings Inc.), Adv. Proc. No. 08-01753, 2010 Bankr. LEXIS 3867 (Bankr. S.D.N.Y. Nov. 16).

Bisin, A. and P. Gottardi (1999). Competitive equilibria with asymmetric information. *Journal of Economic Theory 87*, 1–48.

Bisin, A. and P. Gottardi (2003). Competitive markets for non exclusive contracts with adverse selection: The role of entry fees. *Review of Economic Theory 6*(2), 313–338.

Bisin, A. and A. A. Rampini (2005). Exclusive contracts and the institution of bankruptcy. *Economic Theory 27*(2), 277–304.

Bizer, D. S. and P. M. DeMarzo (1992). Sequential Banking. *Journal of Political Economy 100*(1), 41–61.

Bluhm, M., E. Faia, and J. Krahnen (2013). Endogenous banks' network, cascades and systemic risk. Working paper.

Bluhm, M., C.-P. Georg, and J.-P. Krahnen (2016). Interbank intermediation. Working paper, Deutsche Bundesbank, Research Centre.

Brunnermeier, M. and M. Oehmke (2013). The maturity rat race. *The Journal of Finance 68*(2), 483–521.

Brusco, S. and F. Castiglionesi (2007). Liquidity coinsurance, moral hazard, and financial contagion. *The Journal of Finance 62*(5), 2275–2302.

Cabrales, A., P. Gottardi, and F. Vega-Redondo (2013). Risk-sharing and contagion in networks. Working paper we1301, Universidad Carlos III.

Corbi, A. (2012). Netting and offsetting: Reporting derivatives under U.S. GAAP and under IFRS. Technical report, International Swaps and Derivatives Association.

Craig, B. and G. von Peter (2010). Interbank tiering and money center banks. BIS Working Papers 322, Bank for International Settlements.

Cuillerier, I. and Y. Valdez (2012). Lehman bankruptcy court denies contractual right to triangular setoff. *Futures & Derivatives Law Report 32*.

DeMarzo, P. M. and M. J. Fishman (2007). Agency and optimal investment dynamics. *Review of Financial Studies 20*(1), 151–188.

Di Maggio, M. and A. Tahbaz-Salehi (2015). Collateral shortages and intermediation networks. Working paper, Columbia University.

Diamond, D. W. (1993). Seniority and maturity of debt contracts. *Journal of Financial Economics 33*(3), 341–368.

Donaldson, J., D. Gromb, and G. Piacentino (2016). The Paradox of pledgeability. Working paper, Washington University and HEC.

Donaldson, J. R. and E. Micheler (2016). Resaleable debt and systemic risk. Working paper, Washington University.

Elliott, M., B. Golub, and M. Jackson (2014). Financial networks and contagion. *American Economic Review 104*(10), 3115–53.

Farboodi, M. (2015). Intermediation and voluntary exposure to counterparty risk. Working paper, Princeton University.

Farhi, E., M. Golosov, and A. Tsyvinski (2009). A theory of liquidity and regulation of financial intermediation. *Review of Economic Studies 76*(3), 973–992.

Faure-Grimaud, A. and D. Gromb (2004). Public trading and private incentives. *Review of Financial Studies 17*(4), 985–1014.

Freixas, X., B. M. Parigi, and J.-C. Rochet (2000). Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money, Credit and Banking 32*(3), 611–638.

Gale, D. and S. Kariv (2007). Financial networks. *The American Economic Review 97*(2), 99–103.

Hart, O. and J. Moore (1994). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics 109*(4), 841–79.

Hart, O. and J. Moore (1995). Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management. *American Economic Review 85*(3), 567–85.

Hart, O. and J. Moore (1998). Default and renegotiation: A dynamic model of debt. *The Quarterly Journal of Economics 113*(1), 1–41.

Howard, S. (2011). BofA fights to keep $500M Lehman setoff. https://www.law360.com/banking/articles/246742/bofa-fights-to-keep-500m-lehman-setoff.

in 't Veld, D. and I. van Lelyveld (2014). Finding the core: Network structure in interbank markets. *Journal of Banking & Finance 49*, 27 – 40.

Jacklin, C. (1987). Demand deposits, trading restrictions, and risk sharing. In E. Prescott and N. Wallace (Eds.), *Contractual Arrangements for Intertemporal Trade*, Chapter II, pp. 26–47. Minneapolis: University of Minnesota Press.

Johnson, V. (2015). International financial law: The case against close-out netting. *Boston University International Law Journal 33*, 101–125.

Jorion, P. (2000). Risk management lessons from Long-Term Capital Management. *European Financial Management 6*(3), 277–300.

Kahn, C. M. and D. Mookherjee (1998). Competition and incentives with nonexclusive contracts. *The RAND Journal of Economics 29*(3), 443–465.

Kiyotaki, N. and J. Moore (2001). Liquidity, business cycles, and monetary policy. Clarendon Lectures, Oxford.

Leitner, Y. (2012). Inducing agents to report hidden trades: A theory of an intermediary. *Review of Finance 16*, 1013–1042.

Mengle, D. (2010). The importance of close-out netting. Working paper, International Swaps and Derivatives Association.

Moore, J. (2011). Leverage stacks and the financial system. Slides available at `http://homepages.econ.ed.ac.uk/~jhmoore/papers/JohnMooreLeverageStacks16June2011.pdf`.

Paech, P. (2014). Close-out netting, insolvency law and conflict-of-laws. *Journal of Corporate Law Studies 14*(2), 419–452.

Parlour, C. A. and U. Rajan (2001). Competition in loan contracts. *American Economic Review 91*(5), 1311–1328.

Peltonen, T. A., M. Scheicher, and G. Vuillemey (2013). The network structure of the CDS market and its determinants. Working Paper Series 1583, European Central Bank.

Rose, C. S., D. B. Bergstresser, and D. Lane (2009). The tip of the iceberg: JP Morgan Chase and Bear Stearns (A).

Shleifer, A. and R. W. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance 47*(4), 1343–66.

Valukas, A. R. (2010). Lehman Brothers Holdings Inc. ch. 11 proceedings.

Zame, W. R. (1993). Efficiency and the Role of Default When Security Markets Are Incomplete. *American Economic Review 83*(5), 1142–1164.

Zawadowski, A. (2013). Entangled financial systems. *Review of Financial Studies 26*(5), 1291–1323.