The Joint Dynamics of U.S. and Euro-area Inflation Expectations with Time-varying Uncertainty*

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Abstract

We use several U.S. and euro-area surveys of professional forecasters to estimate a dynamic factor model of inflation with time-varying uncertainty. We obtain survey-consistent distributions of future inflation at any horizon, both in the United States and in the Euro area. Our methodology allows us to compute, in closed form, survey-consistent measures of inflation expectations, inflation uncertainty, inflation expectations anchoring, deflation probabilities, and inflation co-movements of the United States and the Euro area. Our results suggest strong commonalities in inflation dynamics in the two economies.

JEL codes: E31, E44, G15
Keywords: inflation, surveys of professional forecasters, dynamic factor model with stochastic volatility, term structure of inflation expectations and inflation uncertainty, anchoring of inflation expectations

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1 Introduction

The Federal Reserve System (the Fed) and the European Central Bank (the ECB) are two of many central banks that have adopted a mandate for price stability devised to foster economic activity and employment. To meet this objective, both the Fed and the ECB pay close attention to various measures of inflation expectations implied both by financial market data and by surveys of professional forecasters. Surveys, in particular, have received considerable attention from policymakers and academic researchers. This notably reflects their documented success in forecasting inflation (e.g., Ang, Bekaert, and Wei, 2007). Surveys are thus closely monitored and often mentioned in various monetary policy communications. For instance, the Federal Open Market Committee (FOMC) minutes for the 2016 June FOMC meeting state: *"The Michigan survey measure of longer-run inflation expectations fell to its lowest level on record in early June, but other measures of such expectations–including those from the Survey of Professional Forecasters and from the Desk's Survey of Primary Dealers and Survey of Market Participants–were generally little changed, on balance, in recent months."*[1] Similarly, the accounts of the October 2016 monetary policy meeting of the ECB Governing Council state: *"It was highlighted that survey-based measures of longer-term inflation expectations, such as the ECB's Survey of Professional Forecasters, were broadly unchanged for the period five years ahead, at around 1.8%. However, while the average of point forecasts was stable, the Survey of Professional Forecasters also provided a probability distribution around the point forecasts which remained tilted to the downside."*[2] Overall, regular references to survey-based measures of inflation expectations in central banking communications reflect their importance for monetary policy decision-making process.

In this paper we exploit rich information contained in several surveys of pro-

[1] See the June 14-15, 2016 FOMC minutes at https://www.federalreserve.gov/monetarypolicy/fomcminutes20160615.htm. In general, almost every release of the FOMC minutes refers to survey-based measures of inflation expectations, which clearly points out to the importance of these surveys.

[2] See the account of the ECB's Governing Council meeting on October 6, 2016, at https://www.ecb.europa.eu/press/accounts/2016/html/mg161006.en.html.

fessional forecasters with the goal of understanding the evolution of both inflation expectations and inflation uncertainty in the United States and in the euro area. Survey-based measures — unlike market-based measures of inflation expectations — are not affected by inflation risk premium, which may be considerable.[3]

We contribute to the literature of survey-based inflation expectations by constructing a model that takes survey-based inflation forecasts (for various horizons, at varying frequencies, and with different definitions) as inputs and produces survey-consistent distributions of inflation at any horizon. Overall, the goal of the paper is not to forecast inflation, but rather construct comparable measures of inflation expectations and inflation uncertainty, for each area, respectively, that would be consistent with surveys' forecasts. Our dynamic factor model has several noteworthy features. First, common latent factors are allowed to drive the dynamics of inflation rates in both economies, reflecting ever-increasing interconnectedness between developed economies (Monacelli and Sala, 2009; Ciccarelli and Mojon, 2010). Second, our model features stochastic volatility of inflation, hence allowing for time-varying inflation uncertainty.[4] Third, our model is highly tractable because it offers closed-form solutions for conditional first and second moments of future inflation rates at *any horizon*. This tractability is due to the fact that the factors in our econometric model follow so-called affine processes. The affine property of the factors implies that the model can be easily cast in the state-space form and then estimated by Kalman filtering techniques. These techniques easily handle missing data issues, which is particularly useful in our case, because different surveys are released at different points of time.

In our model estimation, we rely both on survey-based consensus inflation forecasts that correspond to an average scenario and on probability distributions of

[3] Hordahl and Tristani (2010) estimate a joint model of the inflation risk premium in the United States and the euro area and find a small positive inflation risk premium, which is increasing with the horizon.

[4] Engle (1982) was the first who emphasized time-varying inflation uncertainty in the context of the econometric model by specifying a new class of stochastic processes called autoregressive conditional heteroscedastic (ARCH) processes; Zarnowitz and Lambros (1987) were the first who emphasized time-varying inflation uncertainty in the context of the second moment of the survey-based inflation distribution, the concept that we use in our model to proxy for inflation uncertainty.

future inflation rates that provide information about uncertainty surrounding this scenario.[5] Point inflation forecasts at various horizons allow to estimate expected inflation at *any horizon*, while survey-based probability distributions of inflation allow to construct model-implied probability distributions at *any horizon*, from which inflation uncertainty measure (as a second moment of the probability distribution) can be drawn. Importantly, even though conditional moments reported in surveys are not based on the same inflation measure or do not coincide in terms of horizons, model-implied conditional distributions of inflation can be made perfectly comparable across the U.S. and euro-area economies.[6]

Our empirical results are as follows.

First, we find substantial commonalities in the dynamics of inflation rates of the United States and the Euro area. In particular, out of four common latent factors that drive the level of inflation rates in both economies in our model, the same factor turns out to be the most important one for the U.S. and euro-area economies, and, out of two common latent factors that drive conditional variances of inflation rates in both economies, the same factor appears to be the most important one for the variances of inflation rates. Our results support previous findings that inflation in industrialized countries is largely a global phenomenon. For example, Monacelli and Sala (2009) find that one international common factor explains between 15 and 30 percent of the variance in consumer prices in four OECD countries (Unites States, Germany, France, and United Kingdom), while Ciccarelli and Mojon (2010) find that inflations in 22 OECD countries have a common factor that accounts for nearly 70 percent of their variance. In addition, relatively high estimated conditional correlations at one- and five-year horizons (roughly ranging between 0.60 to 0.75 over our sample period of 1999-2016) in our model support the story of strong comovement of inflation rates between the two global economies. The literature has

[5]Surveys provide point forecasts and (often but not always) probability distributions at a forecaster level. However, in this paper we abstract from considering disaggregated data.

[6]Different surveys ask respondents to provide inflation forecasts according to different definitions, such as, for example, year-on-year growth rate of the price index, annualised growth rate over a given period, or average of year-on-year growth rates.

so far only focused on the analysis of first-order moments — or point estimates — of future inflation rates across different economies.[7] By contrast, our approach makes it possible to study their joint distribution and generates moments of inflation distributions that are adequately comparable across economies. For example, we find that conditional covariances of future inflation rates are related to the Economic Policy Uncertainty (EPU) indices (Baker, Bloom, and Davis, 2015), as well as to the European Commission and University of Michigan Economic Sentiment indices.

Second, model-implied moments of inflation distributions suggest two insights. First, our results are in line with other studies indicating that inflation expectations declined after 2010 in both economies, especially at short to intermediate horizons up to 5 years.[8] While it is straightforward to infer inflation expectations from the surveys, our measure is unique in a sense that it succinctly combines information from several relevant surveys. The second insight is about inflation uncertainty — the second moment of inflation probability distributions — drawn from the surveys. We find that, compared with the euro-area, inflation uncertainty has been higher in the United States over our entire sample period of 1999-2016 and considerably higher in the first half of our sample. Higher U.S. inflation uncertainty may possibly reflect the fact that the Federal Reserve adopted explicit inflation target only in January 2012.[9] However, inflation uncertainty substantially declined in the United States in our sample, at short and intermediate horizons, while the euro-area inflation uncertainty increased somewhat at all considered horizons (up to 10 years). Interestingly, we find that longer-term inflation uncertainty also increased in our sample, despite the Fed's adoption of the inflation target. This finding is consistent with Nagel (2015) who also finds higher inflation uncertainty at longer horizons in

[7] An exception includes the paper by Charemza, Díaz, and Makarova (2015), who use copula functions to estimate the uncertainty of Canadian inflation forecasts conditional on the United States forecast uncertainty. Note that uncertainty here is defined as the forecast error of a time series econometric model.

[8] Other measures that point out to declining inflation expectations are inflation compensation—the difference between the nominal and TIPS yields of comparable maturities — and inflation swaps. However, these measures are affected by the inflation risk premia and therefore, may not reflect inflation expectations accurately.

[9] See the press release at `http://www.federalreserve.gov/newsevents/press/monetary/20120125c.htm`.

the United States, based on market data.

Third, we contribute to a growing literature on inflation expectations' anchoring (see, e.g., Bernanke, 2007; Gurkaynak, Levin, Marder, and Swanson, 2007; Beechey, Johannsen, and Levin, 2011; De Pooter, Robitaille, Walker, and Zdinak, 2014; Mehrotra and Yetman, 2014; Kumar, Afrouzi, Coibion, and Gorodnichenko, 2015; Mertens, 2015; Nagel, 2015; Buono and Formai, 2016; Łyziak and Paloviita, 2016; Speck, 2016). While different measures are used to define "anchored" expectations, all of these measures mainly reflect stability of the conditional mean of inflation.[10] However, the conditional mean (first-order moment) can be stable even if conditional variance (second-order moment) is relatively high. We therefore propose a measure of "anchored" inflation expectations in terms of conditional distributions. Namely, we measure the extent to which inflation expectations are anchored in terms of probabilities of future inflation being in a certain range that is consistent with inflation targets in the United States or the Euro area. As with other model-implied quantities, these probabilities can be computed for future inflation rates at *any horizon*. We find that, overall in our sample period, longer-term inflation expectations in the Euro area are more anchored than in the United States given higher levels of these probabilities. However, the probability of five-year five years forward euro-area inflation being in the [1.5;2.5]-percent range trended down somewhat from about 90 percent to just under 80 percent, while the probability of the U.S. inflation being in the same range over the same horizon increased notably since the financial crisis, from about 40 to about 70 percent, possibly reflecting the adoption of the explicit inflation target by the Fed.

Fourth, we compute probabilities of deflation and low inflation outcomes for both economies. We find that (a) probabilities of deflation varied a lot during

[10] One popular measure of inflation expectation anchoring is the sensitivity of the interest rates and inflation compensation measures to incoming macroeconomic news (see, e.g., Gurkaynak, Levin, Marder, and Swanson, 2007; Beechey, Johannsen, and Levin, 2011; De Pooter, Robitaille, Walker, and Zdinak, 2014). Other measures include the closeness of average beliefs to the inflation targets of the central banks, the dispersion of these beliefs across agents, confidence (i.e., relatively low uncertainty) in beliefs of the agents in the long-run, revisions in forecasts, and a comovement between long-run and short-run inflation expectations (see, e.g., Kumar, Afrouzi, Coibion, and Gorodnichenko, 2015).

our sample period; (b) towards the end of our sample (mid-2016) probabilities of one-year ahead deflation were reasonably low in the Euro area and negligible in the United States; and (c) in mid-2016 probabilities of one-year ahead inflation falling below 1 percent were around 10 and 40 percent in the United States and in the Euro area, respectively. Other studies, such as Kitsul and Wright (2012) and Fleckenstein, Longstaff, and Lustig (2013) use inflation derivatives data to obtain these probabilities and find that market-based deflation probabilities at comparable near-term horizons in 2012-2013 are around 5 and 10 percent, respectively.

In the remainder of the paper Section 2 provides a literature review, Section 3 introduces surveys, Section 4 describes our model and estimation strategy, Section 5 presents empirical results, and Section 6 concludes. Appendix 7 gathers proofs, technical results, and additional data descriptions.

2 Literature review

Surveys became a popular tool in assessing expectations of inflation (and other macroeconomic variables). Growing empirical evidence suggests that surveys-based inflation forecasts outperform numerous statistical forecasting methods. For example, surveys appear (1) to outperform simple time-series benchmarks in forecasting inflation (Grant and Thomas, 1999; Thomas, 1999; Mehra, 2002); (2) to outperform other forecasting methods such as term structure models and the Philips curve (Ang, Bekaert, and Wei, 2007; Chun, 2012); (3) to beat other forecasts in real time (as opposed to ex-post revised data) (Faust and Wright, 2009; Croushore, 2010); and, finally, (4) to be consistent with inflation expectations embedded in Treasury yields (Chernov and Mueller, 2012). Faust and Wright (2013) provide a comprehensive overview of these inflation forecasting methods: they find that the Philadelphia Fed's Survey of Professional Forecasters, Blue Chip surveys, and the Fed staff's Greenbook forecasts outperform numerous other methods.

As a result of this favourable attention, many researchers started using surveys as measures of inflation expectations (Roberts, 1995; Brissimis and Magginas, 2008; Bekaert and Engstrom, 2010; Grishchenko and Huang, 2013; Chun, 2014; Grishchenko, Vanden, and Zhang, 2016) and as inputs to constructing the term structure of inflation expectations (Chernov and Mueller, 2012; Aruoba, 2014). Several studies also incorporated inflation surveys in models aimed at capturing the joint dynamics of nominal and real yield curves (Chernov and Mueller, 2012; Haubrich, Pennacchi, and Ritchken, 2012a; Piazzesi, Salomao, and Schneider, 2015; D'Amico, Kim, and Wei, 2016).[11]

Despite the fact that surveys were extensively used to extract consensus inflation forecasts, survey-based inflation uncertainty measures only recently attracted attention for two reasons: (1) inflation uncertainty appears to be time-varying (e.g., Zarnowitz and Lambros, 1987) and (2) it seems to be an important risk factor in determining the nominal bond risk premium (Buraschi and Jiltsov, 2005; Piazzesi and Shneider, 2007; Rudebusch and Swanson, 2008; Campbell, Sunderam, and Viceira, 2013). It is hence important to measure this risk accurately. Surveys appear to be a natural data set to measure inflation uncertainty but few surveys contain inflation probability distribution functions.[12] Therefore, disagreement over point inflation forecasts often serves as a proxy for uncertainty because two measures are known to be closely correlated (Zarnowitz and Lambros, 1987; Giordani and Soderlind, 2003; Wright, 2011). However, recent literature has also studied disagreement and uncertainty survey measures (Conflitti, 2011; Lahiri and Sheng, 2010; Rich, Song, and Tracy, 2012; Andrade and Le Bihan, 2013; Boero, Smith, and Wallis, 2014; D'Amico and Orphanides, 2014). Using the law of total variance, our uncertainty measure is the sum of disagreement and the average of forecasters' perceived uncertainty.

Therefore, a natural extension of the term structure models of inflation expectations would be a model where both first and second moments of inflation are

[11] Surveys are included in the estimation of the term structure mainly for two (somewhat connected) reasons: (1) to overcome the small-sample problems related to the highly persistent nature of interest rates and estimate more precisely parameters related to the physical drift of the state variables (see, e.g., Kim and Orphanides (2012) who use survey data on short-term interest rates to address this issue); (2) to better estimate the physical expectation of inflation that is reflected in surveys' forecasts.

[12] See sections 7.4.1 and 7.4.2 for details.

time-varying. Stock and Watson (2007) propose such a model without, however, investigating its term structure implications. By contrast, Haubrich, Pennacchi, and Ritchken (2012b) and Fleckenstein, Longstaff, and Lustig (2013) exploit stochastic volatility models of inflation to derive conditional moments of inflation. While Kitsul and Wright (2012) and Fleckenstein, Longstaff, and Lustig (2013) only use market data to estimate their model, Haubrich, Pennacchi, and Ritchken (2012b) use both market and survey data. In the latter paper, however, authors only use conditional first-order moments and do not involve second-order moments of survey inflation distributions.

3 Survey data

In our model estimation, we use several surveys of professional forecasters for the United States and for the Euro area. We obtain inflation forecast data from the following surveys: the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia (US-SPF), the Survey of Primary Dealers conducted by the Federal Reserve Bank of New York (SPD), Blue Chip Survey of Financial Forecasts and Economic Indicators (Blue Chip, or BCFF and BCEI hereafter), the Survey of Professional Forecasters conducted by the European Central Bank (ECB-SPF), and the Consensus Economics Survey (CES). Our sample period is from January 1999 (the onset of the European Union and the start date of the ECB SPF) to June 2016.

We construct a detailed database of inflation expectation surveys at various horizons for the above surveys. Several issues need to be accounted for when surveys are used in the estimation of our model. First, different surveys use different definitions of inflation.[13] Second, different surveys provide inflation forecasts for different horizons. Third, surveys differ in frequency of their availability (from quarterly to FOMC to monthly frequency). Finally, surveys typically provide point estimates

[13] Specifically, there are three different definitions of inflation in the various surveys used and we account for all three in our model estimation.

but may also provide information on the distribution of inflation as well as individual forecasters' estimates.[14] Table 1 summarizes the forecast variables extracted from different surveys and Sections 7.4.1 and 7.4.2 provide specific details about the surveys.

[Insert Table 1 about here.]

4 Model and estimation strategy

4.1 Inflation and its driving factors

Let us denote by $\pi^{(i)}_{t,t+h}$ the annualized inflation rate in economy i between dates t and dates $t+h$, defined as the log difference in the price index $P^{(i)}_t$:

$$\pi^{(i)}_{t,t+h} = \frac{12}{h}\log\left(\frac{P^{(i)}_{t+h}}{P^{(i)}_t}\right). \tag{1}$$

We assume that the annual inflation rate, $\pi^{(i)}_{t-12,t}$, is a linear combination of factors gathered in the $n \times 1$ vector $Y_t = (Y_{1,t}, \dots, Y_{n,t})'$. As specified below, the dynamics of Y_t is such that the marginal mean of Y_t is zero. Importantly, $Y_{j,t}$ factors, where $j \in \{1, \dots, n\}$, may be common to different economies. Specifically:

$$\pi^{(i)}_{t-12,t} = \bar{\pi}^{(i)} + \delta^{(i)\prime} Y_t. \tag{2}$$

We assume that the distribution of Y_t is Gaussian conditional on its past realization $\underline{Y_{t-1}} = \{Y_{t-1}, Y_{t-2}, \dots\}$ and another $q \times 1$ exogenous vector $z_t = (z_{1,t}, \dots, z_{q,t})'$ that affects the variance of Y_t. In particular, we assume the following functional form for Y_t:

$$Y_t = \Phi_Y Y_{t-1} + \Theta(z_t - \bar{z}) + diag(\sqrt{\Gamma_{Y,0} + \Gamma'_{Y,1} z_t})\varepsilon_{Y,t}, \quad \varepsilon_{Y,t} \sim \mathcal{N}(0, I), \tag{3}$$

[14] Our model assumes the existence of a representative forecaster. Hence, it does not account for the heterogeneity associated with the availability of individual estimates. Accordingly, in our estimation, we use the average of survey outputs (i.e. point estimates and/or distributions).

where $\bar{z}$ is the unconditional mean of z_t, $\Gamma_{Y,0}$ is an $n \times 1$ vector, and $\Gamma_{Y,1}$ is a $q \times n$ matrix. According to (3) z_t affects both the conditional expectation and variance of Y_t, so that the Y_t process features stochastic volatility. Similar modelling has been entertained in the literature, (see, e.g., Capistrán and Timmermann, 2009; Caporale, Paesani, and Onorante, 2010; Andrade, Fourel, Ghysels, and Idier, 2014). Vector z_t is essential for modelling time-varying inflation variances so we refer to z_t as the uncertainty vector (and to $z_{j,t}$ as the uncertainty factors) hereinafter.

The specification of the conditional variance in (3) implies that the entries of $\Gamma_{Y,0} + \Gamma'_{Y,1} z_t$ have to be non-negative for all t. To achieve this, we assume that all elements of Γ_Y vectors are non-negative and that z_t follows a multivariate autoregressive gamma process (Appendix 7.2). As shown in the Appendix, the dynamics of z_t admits the following weak VAR representation:

$$z_t = \mu_z + \Phi_z z_{t-1} + diag(\sqrt{\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}})\varepsilon_{z,t}, \tag{4}$$

where, conditional on $\underline{z_{t-1}}$, $\varepsilon_{z,t}$ has a zero mean and a unit diagonal covariance matrix, and where $\Gamma_{z,0}$ is a $q \times 1$ vector and $\Gamma_{z,1}$ is a $q \times q$ matrix.

Given the dynamics for Y_t and z_t, the VAR form of the dynamics followed by $X_t = (Y'_t, z'_t)'$ is:

$$X_t = \begin{bmatrix} Y_t \\ z_t \end{bmatrix} = \mu_X + \Phi_X \begin{bmatrix} Y_{t-1} \\ z_{t-1} \end{bmatrix} + \Sigma_X(z_{t-1})\varepsilon_{X,t}, \tag{5}$$

where $\varepsilon_{X,t}$ is a $(n+q)$-dimensional unit-variance martingale difference sequence and where:

$$\mu_X = \begin{bmatrix} -\Theta\Phi_z(I-\Phi_z)^{-1}\mu_z \\ \mu_z \end{bmatrix}, \quad \Phi_X = \begin{bmatrix} \Phi_Y & \Theta\Phi_z \\ 0 & \Phi_z \end{bmatrix},$$

$$\Sigma_X(z_{t-1})\Sigma_X(z_{t-1})' = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}' & \Sigma_{22} \end{bmatrix}$$

$$\text{with} \quad \begin{cases} \Sigma_{11} = \Theta \times diag(\Gamma_{z,0} + \Gamma_{z,1}' z_{t-1}) \times \Theta' + diag(\Gamma_{Y,0} + \Gamma_{Y,1}'(\mu_z + \Phi_z z_{t-1})), \\ \Sigma_{22} = diag(\Gamma_{z,0} + \Gamma_{z,1}' z_{t-1}), \\ \Sigma_{12} = \Theta\Sigma_{22}. \end{cases}$$

An important property of X_t is that this process is affine (see Appendix 7.1.1). In particular, this implies that, at any date t, the first and second conditional moments of any linear combination of future X_t are affine functions of X_t. Since the realized log annual growth rate of the price index $\pi_{t-12,t}^{(i)}$ is an affine transformation of X_t (see eq. (2)), its first and second moments can be written as an affine function of X_t factors as well:

$$\mathbb{E}_t(\pi_{t+h-12,t+h}^{(i)}) = \bar{\pi}^{(i)} + a_h^{(i)} + b_h^{(i)\prime} X_t \tag{6}$$

$$\mathbb{V}ar_t(\pi_{t+h-12,t+h}^{(i)}) = \alpha_h^{(i)} + \beta_h^{(i)\prime} X_t \tag{7}$$

where $\mathbb{E}_t(\bullet)$ and $\mathbb{V}ar_t(\bullet)$ respectively denote the expectations and variances conditional on $\underline{X_t}$. In our empirical analysis we consider inflation forecast rates over different horizons because of the nature of the surveys we fit. In particular, we calculate the annualized h-period ahead inflation rates $\pi_{t,t+h}^{(i)} := (12/h)\log(P_{t+h}^{(i)}/P_t^{(i)})$, which are also affine functions of X_t:

$$\pi_{t,t+h}^{(i)} = \frac{1}{k}\delta^{(i)\prime}(X_{t+12} + X_{t+24} + \cdots + X_{t+h}), \tag{8}$$

where $h = 12 \times k$. Therefore, their first and second moments can be also written as affine functions of X_t:

$$\mathbb{E}_t(\pi^{(i)}_{t,t+h}) = \bar{\pi}^{(i)} + \bar{a}^{(i)}_h + \bar{b}^{(i)'}_h X_t \tag{9}$$

$$\mathbb{V}ar_t(\pi^{(i)}_{t,t+h}) = \bar{\alpha}^{(i)}_h + \bar{\beta}^{(i)'}_h X_t. \tag{10}$$

Appendices 7.1.2 and 7.1.3 outline the recursive algorithms used to compute the parameters of the moments of the realized ($a^{(i)}_h$, $\alpha^{(i)}_h$, $b^{(i)}_h$, $\beta^{(i)}_h$) and forecast ($\bar{a}^{(i)}_h$, $\bar{\alpha}^{(i)}_h$, $\bar{b}^{(i)}_h$, $\bar{\beta}^{(i)}_h$) inflation rates.[15]

4.2 State-space model and Kalman-filter estimation

4.2.1 Objective and strategy

In addition to model parameters, we have to estimate the factors X_t that are not observed by the econometrician. We handle both estimations using Kalman filtering techniques. The affine property of the process X_t is key to the tractability of the estimation. Specifically, not only do we have closed-form formulae but the latter are also affine, allowing us to cast the model into the linear state-space form, which is the required form of the model for the Kalman filter algorithm. This is a fundamental difference between our approach and alternative inflation models exhibiting stochastic volatility (see, e.g., Stock and Watson, 2007; Mertens, 2015). Indeed, while the latter models entail closed-form expressions for the first two conditional moments of inflations, the second-order moments are non-linear in the unobserved factors, which substantially complicates the model estimation.

A state-space model consists of two types of equations: transition equations and measurement equations. Transition equations describe the dynamics of the latent factors, as in eq. (5). Measurement equations specify the relationship between the observed variables and the latent factors. A by-product of the Kalman filter algorithm is the likelihood function. Parameter estimates can therefore be obtained

[15] $a^{(i)}_h$, $\alpha^{(i)}_h$, $b^{(i)}_h$ and $\beta^{(i)}_h$ are obtained by setting $\gamma_1 = \cdots = \gamma_{h-1} = \mathbf{0}$ and $\gamma_h = [\delta^{(i)'}, \mathbf{0}]'$ and $\bar{a}^{(i)}_h$, $\bar{\alpha}^{(i)}_h$, $\bar{b}^{(i)}_h$ and $\bar{\beta}^{(i)}_h$ are obtained by setting $\gamma_1 = \cdots = \gamma_{11} = \gamma_{13} = \cdots = \gamma_{12k-1} = \mathbf{0}$ and $\gamma_{12} = \gamma_{24} = \cdots = \gamma_h = [\delta^{(i)'}, \mathbf{0}]'$ in the recursive equations (16) and (17).

by maximising this function.

4.2.2 Measurement equations

The state-space model involves three types of the measurement equations:

(a) The first set of equations states that, for each economy i, the realised inflation rate is equal to linear combination of factors Y_t, as stated by eq. (2), with area-specific loadings.

(b) The second set of equations states that, up to the measurement error, survey-based expectations of future inflation rates are equal to the model-implied ones, that is:

$$SPF_t = \bar{\boldsymbol{\pi}} + \mathbf{a} + \mathbf{b}' X_t + diag(\sigma^{avg}) \eta_t^{avg} \tag{11}$$

where η_t^{avg} is a vector of *iid* Gaussian measurement errors, SPF_t gathers all survey-based expected inflations available at date t, and the entries of the vector $\bar{\boldsymbol{\pi}}$, the vector $\mathbf{a}$ and the matrix $\mathbf{b}$ are naturally based on the appropriate $\pi^{(i)}$s, $a_h^{(i)}$s, $b_h^{(i)}$s, $\bar{a}_h^{(i)}$s and $\bar{b}_h^{(i)}$s (see eqs. (6) and (9)).

(c) The third set of equations states that, up to the measurement error, survey-based variances are equal to the model-implied ones, i.e.:

$$VSPF_t = \boldsymbol{\alpha} + \boldsymbol{\beta}' X_t + diag(\sigma^{var}) \eta_t^{var} \tag{12}$$

where η_t^{var} is a vector of *iid* Gaussian measurement errors, $VSPF_t$ gathers all survey-based conditional variances of inflation forecasts available at date t, and the entries of the vector $\boldsymbol{\alpha}$ and the matrix $\boldsymbol{\beta}$ are based on the appropriate $\alpha_h^{(i)}$s, $\beta_h^{(i)}$s, $\bar{\alpha}_h^{(i)}$s and $\bar{\beta}_h^{(i)}$s (see eqs. (7) and (10)).

Let us denote by S_t the vector of observations used in the state-space model. Since the latter is based on equations of types (a), (b), and (c), we have $S_t = [\pi_t^{(1)}, \pi_t^{(2)}, SPF_t', VSPF_t']'$. Using obvious notations, the measurement equations of the state-space model read:

$$S_t = A + B'X_t + diag(\sigma^S)\eta_t^S, \tag{13}$$

where $\mathbb{V}ar(\eta_t^S) = Id$.

4.2.3 Discussion of model estimation

At this stage, three remarks are in order. First, most survey forecasts are not released every month (with the exception of the Blue Chip surveys), so SPF_t and $VSPF_t$ variables are not available every month and thus these series contain missing observations when measured at a monthly frequency.[16] Fortunately, it is straightforward to adjust the Kalman filter in order to handle missing observations (for details see Harvey and Pierse, 1984; Harvey, 1989). For the months when no SPF_t and $VSPF_t$ variables are available, the filter is still able to produce estimates of all latent factors, though with lower precision.

The second remark is about the Kalman filter performance in our case. While the affine form of the transition and measurement equations facilitates the implementation of the filter, the filter we eventually run is not optimal. It would have been optimal if the conditional covariance matrix $\Sigma_X \Sigma_X'$ in eq. (5) were not dependent on X_{t-1}. However, it is not the case since some entries of $\Gamma_{Y,1}$ are non-null.[17] Therefore, we estimate our model using a quasi-maximum-likelihood (QML) approach (see, e.g., Duan and Simonato, 1999; de Jong, 2000).

The third remark is about the number of factors used in our subsequent empirical analysis. In our model two types of factors are used, namely, level and un-

[16] An alternative, but equivalent, view would be that the vectors and matrices $\bar{\boldsymbol{\pi}}$, $\mathbf{a}$, $\mathbf{b}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ have time-varying sizes.

[17] Our filter algorithm makes use of the standard forecasting and updating steps of the Kalman filter except that, at iteration t, we replace the unobserved covariance matrix of the X_t innovations $(\Sigma_X(z_{t-1})\Sigma_X(z_{t-1})')$ by $\Sigma_X(z_{t-1|t-1})\Sigma_X(z_{t-1|t-1})'$, where $z_{t-1|t-1}$ denotes our filtered estimate of z_{t-1} (using the information up to date $t-1$). Another adjustment we have to make to the filter pertains to the fact that factors z_t are non-negative. For this purpose, after each updating step of the algorithm, negative entries in the z_t estimate are replaced by 0. Monte Carlo analyses by Duan and Simonato (1999) and Zhou (2001) suggest that in the case of linear but heteroskedastic models, that kind of approximation may be of limited importance in practice (see also Duffee and Stanton (2012)).

certainty factors. We consider principal component analysis as a method to reduce the dimensionality of our data. The use of survey data comes with the difficulty of having to deal with many missing observations. To avoid such complications, we choose to proxy information on the level and uncertainty of inflation expectations using market-based data (i.e., inflation swaps and market-based variances), which are available at a higher frequency.[18] Let us first determine the number of level factors. To do so, we first regress euro-area inflation swaps on euro-area realised inflation and U.S. inflation swaps on U.S. realised inflation (in line with eq. (11)). The residuals of these bivariate regressions are collected and a principal component analysis is conducted on them. Panel A of Table 2 displays the results of this analysis, which indicate that the two first components explain about 93% of the cross-sectional variation observed. We now proceed in regressing every inflation swap on the realised inflation of the two economies and on the first two principal components (i.e., on four factors). Panel B of Table 2 displays the R^2 coefficients of these regressions. The average of all R^2 coefficients amounts to roughly 96%. This exercise suggests that four level factors is a reasonable choice.[19] In order to assess the number of uncertainty factors, we conduct a principal component analysis on market-based variances. Panel C of Table 2 displays the results of a principal component analysis on market-based variances. Results indicate that the first two principal components explain roughly 90% of the cross-sectional variation observed in market-based variances.

[Insert Table 2 about here.]

[18] In most studies that model inflation swaps (or breakeven inflation rates —nominal yields minus real yields —), both inflation expectations and swaps are linear combinations of latent factors. In this setting, the number of factors driving inflation expectations coincides with those driving inflation swaps (Chernov and Mueller, 2012; Haubrich, Pennacchi, and Ritchken, 2012a; Piazzesi, Salomao, and Schneider, 2015; D'Amico, Kim, and Wei, 2016). By extension, the same principle can be applied for variances. For further details on market-based data, refer to Appendix 7.6.

[19] Note that this exercise does not account for the effect of uncertainty factors on the level (see eqs. (3)). However, data on derivatives (floors) start in 2010, thus restricting capturing the appropriate number of level factors.

5 Results

5.1 Estimated model

Table 3 presents parameter estimates of the model described in section 4. We assume that there are four Y_t factors that explain inflation variations and two z_t volatility factors that explain inflation uncertainty (see section 4.2.3).[20] We observe the autoregressive parameters pertaining to the first and fourth factors, namely, $\Phi_Y[1,1]$ and $\Phi_Y[4,4]$ are close to 1, suggesting these factors appear to be very persistent in our estimation.

[Insert Table 3 about here.]

Figure 1 displays the factor loadings of the estimated model. The first factor appears to be the most important one both for the euro area (top left panel) and for the U.S. (top right panel). This factor has a similar loading for both economies. The second most important factor is the third one for the euro area and the fourth one for the U.S. economy. Two middle panels suggest that the first volatility factor is important for inflation expectations at shorter horizons (up to about five years) for both economies.

[Insert Figure 1 about here.]

The fit of the surveys is illustrated in Figure 2. Even though we are fitting the first two moments of many different types of inflation expectations across different economies, the fit is satisfactory.

[Insert Figure 2 about here.]

[20]The results we hereby disclose are those of a joint estimation for the two currency areas, including both first and second conditional moments of surveys. Individual estimations per country, as well as estimations excluding second conditional moments have been conducted and their results are available upon request.

5.2 Model-implied conditional distributions

Figure 3 compares the one-year ahead survey-based inflation histograms to the one-year ahead model-implied distributions of inflation. For the model-implied distributions, two-standard-deviation confidence intervals are reported. These standard deviations reflect uncertainty associated with the estimation of the latent factors X_t and are obtained by applying the delta method on the function relating X_t factors to the conditional cumulative distribution function (c.d.f.) of future inflation.[21] Note that raw survey data are not comparable across areas unless the inflation measure is the same, which is not the case here. By contrast, our model generates moments of inflation distributions that are adequately comparable across economies. Both economies' distributions have shifted noticeably to the left from 2005 to 2014, suggesting a decline in inflation expectations. The euro area's inflation distribution flattened, somewhat indicating an increase in the variance of inflation expectations, and, thus, greater inflation uncertainty, as suggested by Figure 2. Similar findings are reported in Rich, Song, and Tracy (2012), where they find that uncertainty measures stemming from the ECB's survey of professional forecasters display countercyclical behavior, and find evidence of increased inflation uncertainty since 2007.

[Insert Figure 3 about here.]

Figure 4 displays model-implied average annualized expected inflation rates (top charts) and one-year forward inflation rates (bottom charts) for two dates: (November 2004, April 2016) for the Euro area and (January 2005, January 2016) for the United States. The figure also displays the 5th and 95th quantiles associated with the conditional distributions.[22] The top charts demonstrate that survey-based inflation expectations declined noticeably on the short to intermediate horizons (up to roughly 5 years) in the past decade and also declined on the longer horizons (5 to 10 years), but to a lesser extent. As the top figures also demonstrate, the term

[21] The covariance matrix of the filtered values of X_t stems from the Kalman filter. Appendix 7.3 details the computation of the c.d.f. of future inflation rates.

[22] The quantiles are derived from closed-form formulas given in Appendix 7.3.

structure of the euro-area inflation expectations is flatter compared to those of the United States: Intermediate- and longer-horizon euro-area inflation expectations on average have been lower than those in the United States despite the fact that the very short (1- to 2-year) expected inflation rates have been similar. As the bottom two charts demonstrate, one-year forward expected inflation rates declined notably at the short- to intermediate- horizons, but little moved on the longer horizons, over the past decade.

[Insert Figure 4 about here.]

Figure 5 displays model-implied variances of the average annualized spot inflation rates (top charts) and the one-year forward inflation rates (bottom charts) for the same two dates as Figure 4 does. We interpret these variances as inflation uncertainty about expected inflation rates.[23] This figure demonstrates a few interesting points. First, according to the top charts, inflation uncertainty has been higher in the United States than in the Euro area at all horizons. Second, in the past decade, U.S. inflation uncertainty declined at the horizons up to 6 years and slightly rose at the longer horizons. At the same time, euro-area inflation uncertainty increased and more so at the short to intermediate horizons.[24] Third, the euro-area uncertainty associated with the one-year forward inflation rates increased about uniformly at all horizons, while the U.S. uncertainty about one-year forward rates declined at shorter horizons but increased at longer horizons.

[Insert Figure 5 about here.]

Figure 6 compares model-implied (physical) probabilities of negative and lower than 1 percent future inflation rates to their risk-neutral counterparts. These are provided for one- and three-year ahead horizons. The risk-neutral probabilities

[23] Appendix 7.5 provides details on the definition of the uncertainty measure.

[24] This point is also confirmed by the the 5 to 95 percentiles' bounds in Figure 4, uncertainty associated with the expected inflation rates has been lower on the long end of the term structure of expectations in the two economies.

are based on inflation derivatives, namely, zero-coupon inflation swaps and inflation floors.[25] The grey shaded areas are two-standard-deviation confidence intervals for the model-implied probabilities.[26] Unsurprisingly, low inflation probabilities are higher in the short run than in the long run. Both economies faced high low-inflation probabilities shortly after the Lehman Brothers collapse. Importantly, risk-neutral probabilities are higher than their physical counterparts and their difference is substantial. Similarly, Kitsul and Wright (2012) and Fleckenstein, Longstaff, and Lustig (2013)find that risk-neutral probabilities assign larger values to tail events (either deflation or high inflation) relative to their time series counterparts, supporting a U-shaped empirical pricing kernel. This is consistent with the fact that the deflation state (or high inflation state) is perceived by agents as a bad state of the world, characterized by a high rate of marginal utility.

[Insert Figure 6 about here.]

Top three panels in Figure 7 display conditional variances, covariances, and correlations of future inflation rates for the two areas. The bottom panel shows the times series of the joint probability of deflation, i.e. $\mathbb{P}(\pi_{t+h}^{(E.A.)} \leq 0, \pi_{t+h}^{(U.S.)} \leq 0 | \underline{S_t^a})$. We plot these series for one-year ($h = 12$ months) and five-year ($h = 60$ months) horizons. Our model reveals that there are differences in uncertainty measures.[27] Although we observe some convergence for the uncertainty measures over the next five years across the two areas, with the U.S. variances of inflation declining and the euro-area variances slightly increasing, the euro-area variances remain smaller. This may be explained by the average lower level of inflation in the Euro area, and also by the absence of the explicit inflation target in the U.S, that has been adopted

[25] The risk-neutral distributions –more precisely the forward-neutral distributions– are assumed to be of the generalised Beta type (see Appendix 7.6). These distributions are specified by four parameters. For each area, each date, and each maturity, these four parameters are chosen so as to minimise the weighted sum of squared pricing errors.

[26] These standard deviations are obtained by applying the delta method on the function relating X_t factors to the conditional cumulative distribution function (c.d.f.) of future inflation.

[27] Raw data stemming from surveys do not allow to carry out comparisons because surveys are different in nature and cannot be directly compared.

only in January 2012.[28] We also find that the variances of one-year ahead inflation converged toward the end of our sample, in the late 2015 to mid-2016. This apparently suggests that the uncertainty about the near-term inflation is similar in the two areas. Correlation between the euro-area and the U.S. inflation rates appears substantial and having increased in the post-crisis period, from about 60 percent to about 75 percent. This finding supports the idea of joint inflation movements due to the interdependent nature of economies nowadays and is in line with earlier findings (e.g., Monacelli and Sala, 2009; Ciccarelli and Mojon, 2010) who also find substantial commonalities in the inflation movements in the industrialized countries. The joint deflation probability in the United States and the Euro area varied a lot during our sample period, especially for the one-year ahead horizon. However, in the period surrounding the euro-area crisis period, 2012-2014, (indicated by the pink bar on the charts) the joint probability of the five-year ahead deflation was higher than the one for the one-year ahead horizon. At the height of the financial crisis in 2009 and in the beginning of 2016 the probability of the joint one-year ahead deflation was substantially higher than the joint probability of the five-year ahead deflation. Finally, at the end of our sample, in the mid-2016, the joint deflation probability does not exceed 2 percent at either horizon.

[Insert Figure 7 about here.]

Table 4 reports regression results that relate deflation probabilities, inflation covariances and inflation variances to various explanatory variables. We find that Economic Policy Uncertainty (EPU), inflation risk premia and sentiment indices have a higher explanatory power relative to the stock markets' volatility when it comes to co-movement indicators such as joint deflation probabilities and covariances as well as individual deflation probabilities and variances.[29] EPU indices are positively correlated and particularly interesting for the five-year (joint and area-specific) deflation probabilities, as well as for inflation covariances, euro-area inflation variances,

[28] See the January 25, 2012 Monetary Policy Press Release http://www.federalreserve.gov/newsevents/press/monetary/20120125c.htm.

[29] Note that Charemza, Díaz, and Makarova (2015) find a puzzling lack of correlation between inflationary forecast errors and the EPU index for Canada.

and U.S. five-year inflation variances. Both euro-area and U.S. economic sentiment indices have low explanatory power for joint and euro area deflation probabilities, but high explanatory power for one-year ahead inflation covariances and U.S. variances. Moreover, economy-specific inflation risk premia are relevant for deflation probabilities and variances of that particular economy. Interestingly, we also find that euro-area indicators (i.e. EPU and risk premia) seem to be more useful in explaining joint and U.S. deflation probabilities than their U.S. counterparts, suggesting interactions between the two economies.

[Insert Table 4 about here.]

Figure 8 reports time-series of conditional probabilities of future inflation rates being in a certain range,

$$\mathbb{P}(\pi^{(i)}_{t+h-m,t+h} \in I_j | \underline{X_t}), \quad j = 1,\, 2, \tag{14}$$

where h is the horizon measured in months, m - tenor of the future expected rate measured in months, $I_1 = [1.5\%, 2.5\%]$, and $I_2 = [1\%, 3\%]$. We suggest that these conditional probabilities of the future inflation rate (that can be computed for any requested horizon) capture the spirit of the anchoring of inflation expectations and uncertainty around inflation expectations is an important factor to gauging the anchoring of those expectations.

European Central Bank has adopted *the medium-term* inflation target in May 2003, formulated as follows: *"The primary objective of the ECB's monetary policy is to maintain price stability. The ECB aims at inflation rates of below, but close to, 2% over the medium term."*[30] The Federal Reserve has adopted *the longer-run* inflation target in January 2012, stated as follows: *"The inflation rate over the longer run is primarily determined by monetary policy, and hence the Committee has the ability to specify a longer-run goal for inflation. The Committee judges that inflation at the rate of 2 percent, as measured by the annual change in the*

[30] See https://www.ecb.europa.eu/mopo/html/index.en.html.

price index for personal consumption expenditures, is most consistent over the longer run with the Federal Reserve's statutory mandate."[31] Thus, the ECB and the Fed target inflation over slightly different periods. We interpret the "medium-term" as the average annual inflation four years head, and the "long-term" as the average annual inflation five-year five years ahead. We plot conditional probabilities (14) for one-year four years ahead inflation (black solid line), for five-year five years ahead inflation (dotted line), and for one-year nine years ahead inflation (grey solid line). The latter horizon was used by Beechey, Johannsen, and Levin (2011) to study the anchoring of inflation expectations as the sensitivity of the interest rates and inflation compensation to macroeconomic news releases.[32]

Overall, Figure 8 conveys a few interesting findings. First, the probabilities of inflation expectations have been higher throughout our sample in the Euro area than in the United States. Second, the I_1 and I_2-range euro-area probabilities have decreased by up to 10 percent at the considered horizons since the end of 2008. This finding appears to be in line with Łyziak and Paloviita (2016), where the authors find that euro-area long-term inflation forecasts are found to be more sensitive to the shorter-term forecasts and to the realized HICP inflation in the post-crisis period. Third, probabilities of the one-year ahead inflation either four years or nine years ahead declined slightly in the Euro area and increased somewhat in the United States. Nevertheless, such probabilities are notably higher in the Euro area (around 0.6) than in the Untied States (around 0.4), according to the I_1 range. The results are qualitatively similar for the I_2 range. Fourth, the U.S. I_1 probability of five-year five years ahead inflation has increased substantially after the introduction of the explicit inflation target by the Fed and exceeded its pre-crisis levels towards the end of our sample, in 2016. Fifth, the euro-area I_1 probability of the five-year five years ahead inflation is still found to be a touch higher compared to the United States towards the end of our sample (despite an increase in probabilities in the United

[31] See https://www.federalreserve.gov/newsevents/press/monetary/20120125c.htm.

[32] Our approach is fundamentally different as our measure is available only at monthly frequency, and therefore, cannot be used to study the responsiveness of inflation expectations to macro news.

States), suggesting that euro-area inflation expectations still remain somewhat better anchored. These results are in line with the findings of Beechey, Johannsen, and Levin (2011) and Buono and Formai (2016). Finally, a post-crisis increase in the anchoring probabilities of the U.S. future inflation rates may potentially reflect an explicit inflation target announced by the Federal Open Market Committee in January 2012.

[Insert Figure 8 about here.]

6 Conclusion

We build a dynamic latent factor model with stochastic volatility for the joint estimation of inflation expectations and inflation uncertainty in the United States and the euro area. The main contribution of our paper is that our model produces survey-consistent measures of inflation expectations and of inflation uncertainty in the two currency areas. We use different types of inflation projections from the surveys of professional forecasters to fit the first and the second moments of distribution of future inflation rates. We find strong commonalities in the movements of inflation expectations and inflation uncertainty in both economies over our sample period of 1999-2016. Surveys suggest that since 2010 short- and intermediate-term inflation expectations moved down noticeably in both economies, while long-term inflation expectations remained stable. U.S. inflation uncertainty declined substantially over our sample period but still remains higher than the euro-area inflation uncertainty, despite a noticeable increase in the latter one. Post-financial-crisis inflation expectations in the United States became more anchored, while euro-area inflation expectations — slightly less anchored. Probabilities of deflation varied substantially over our sample period, especially during the financial crisis, declining to low levels toward the end of our sample in 2016. Our findings should be interesting to market participants, policymakers, and anyone who regularly gauges inflation expectations and uncertainty as our measures can be regularly updated.

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7 Appendix

7.1 Conditional means and variances of X_t

In this appendix we compute conditional expectations and variances of linear combinations of future X_ts. Formally, we consider the first two moments of the random variable $\Sigma_{i=1}^{h}\gamma_i' X_{t+i}$ conditionally on the information available as of date t (i.e. $\underline{X_t}$).

Appendix 7.1.1 shows that X_t is an affine process. This property implies that the first two conditional moments of X_t are affine in X_t. That is, there exist functions a_h, b_h, α_h and β_h such that, for any set of γ_is:

$$
\begin{aligned}
\mathbb{E}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= a_h(\gamma_1,\dots,\gamma_h) + b_h(\gamma_1,\dots,\gamma_h)' X_t \\
\mathbb{V}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \alpha_h(\gamma_1,\dots,\gamma_h) + \beta_h(\gamma_1,\dots,\gamma_h)' X_t.
\end{aligned}
$$

Appendix 7.1.2 (Appendix 7.1.3) provide the recursive formulas that can be used to compute a_h and b_h (α_h and β_h).

7.1.1 Affine property of X_t

Showing that X_t has an affine dynamics amounts to showing that the Laplace transform of X_{t+1}, conditional on $\underline{X_t}$, is exponential affine in X_t.

Lemma 7.1 *The Laplace transform of X_{t+1}, conditional on $\underline{X_t}$, is given by:*

$$
\begin{aligned}
& \mathbb{E}(\exp(u' X_{t+1})|\underline{X_t}) \\
= \ & \exp(u_Y' \Phi_Y Y_t + b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2)' z_t + \\
& a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2) - u_Y' \Theta \bar{z} + 0.5\Gamma_{Y,0}' u_Y^2), \qquad (15)
\end{aligned}
$$

where $u = (u_Y', u_z')'$, $u_Y^2 = u_Y \odot u_Y$ (by abuse of notation), Γ_Y is a $q \times n$ matrix and where the functions a_z and b_z define the conditional Laplace transform of z_t (see Appendix 7.2, eq. (18) and (19).

Proof We have:

$$\begin{aligned}
& \mathbb{E}(\exp(u'X_{t+1})|\underline{X_t}) \\
= \;& \mathbb{E}(\exp(u_Y'Y_{t+1} + u_z'z_{t+1})|\underline{X_t}) \\
= \;& \mathbb{E}(\mathbb{E}[\exp(u_Y'Y_{t+1} + u_z'z_{t+1})|\underline{X_t}, z_{t+1}]|\underline{X_t}) \\
= \;& \exp(u_Y'\{\Phi_Y Y_t - \Theta\bar{z}\})\mathbb{E}\{\exp((u_z + \Theta' u_Y)' z_{t+1}) \times \\
& \mathbb{E}[\exp(u_Y' diag(\sqrt{\Gamma_{Y,0} + \Gamma_{Y,1}' z_{t+1}})\varepsilon_{Y,t+1})|\underline{X_t}, z_{t+1}]|\underline{X_t}\} \\
= \;& \exp(u_Y'\{\Phi_Y Y_t - \Theta\bar{z}\})\mathbb{E}(\exp((u_z + \Theta' u_Y)' z_{t+1} + 0.5 u_Y' diag(\Gamma_{Y,0} + \Gamma_{Y,1}' z_{t+1}) u_Y)|\underline{X_t}) \\
= \;& \exp(u_Y'\{\Phi_Y Y_t - \Theta\bar{z}\} + 0.5\Gamma_{Y,0}' u_Y^2)\mathbb{E}(\exp((u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2)' z_{t+1})|\underline{X_t}) \\
= \;& \exp(u_Y'\Phi_Y Y_t + b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2)' z_t + \\
& a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2) - u_Y'\Theta\bar{z} + 0.5\Gamma_{Y,0}' u_Y^2),
\end{aligned}$$

which leads to the result. ■

The fact that X_t follows an affine process implies the following result.

Lemma 7.2 *The multi-horizon Laplace transforms of X_t, conditional on $\underline{X_t}$, are exponential affine in X_t. Specifically, for any set of vectors u_i, $i \in [1, h]$, we have:*

$$\mathbb{E}(\exp(u_1'X_{t+1} + \cdots + u_h'X_{t+h})|\underline{X_t}) = \exp(A_h(u_1, \ldots, u_h) + B_h(u_1, \ldots, u_h)'X_t),$$

where the functions A_i and B_i are given by:

$$\left\{\begin{array}{l} A_h([u_Y', u_z']') = a_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2) - u_Y'\Theta\bar{z} + 0.5\Gamma_{Y,0}' u_Y^2 \\ B_h([u_Y', u_z']') = [u_Y'\Phi_Y, b_z(u_z + \Theta' u_Y + 0.5\Gamma_{Y,1} u_Y^2)']' \end{array}\right. \quad \text{if } h = 1,$$

and

$$\left\{\begin{array}{l} A_h(u_1, \ldots, u_h) = A_{h-1}(u_2, \ldots, u_h) + A_1(u_1 + B_{h-1}(u_2, \ldots, u_h)) \\ B_h(u_1, \ldots, u_h) = B_1(u_1 + B_{h-1}(u_2, \ldots, u_h)) \end{array}\right. \quad \text{otherwise.}$$

Proof eq. (15) proves that Lemma 7.2 is valid for $h = 1$. Assume Lemma 7.2 is valid for a given $h \geq 1$, we have:

$$\begin{aligned}
& \mathbb{E}(\exp(u_1'X_{t+1} + \cdots + u_{h+1}'X_{t+h+1})|\underline{X_t}) \\
= \;& \mathbb{E}\{\exp(u_1'X_{t+1})\mathbb{E}[\exp(u_2'X_{t+2} + \cdots + u_{h+1}'X_{t+h+1})|\underline{X_{t+1}}]|\underline{X_t}\} \\
= \;& \mathbb{E}\{\exp(u_1'X_{t+1})\exp(A_h(u_2, \ldots, u_{h+1}) + B_h(u_2, \ldots, u_{h+1})'X_{t+1})|\underline{X_t}\} \\
= \;& \exp(A_h(u_2, \ldots, u_{h+1}) + A_1(u_1 + B_h(u_2, \ldots, u_{h+1})) + B_1(u_1 + B_h(u_2, \ldots, u_{h+1})'X_t)),
\end{aligned}$$

which leads to the result. ■

7.1.2 Computation of a_h and b_h

We have:

$$\begin{aligned}
\mathbb{E}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \mathbb{E}_t\left(\mathbb{E}_{t+1}\sum_{i=1}^{h}\gamma_i' X_{t+i}\right)\\
&= \mathbb{E}_t\left(\gamma_1' X_{t+1} + a_{h-1}(\gamma_2,\dots,\gamma_h) + b_{h-1}(\gamma_2,\dots,\gamma_h)' X_{t+1}\right)\\
&= a_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h)) +\\
&\quad b_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h))' X_t,
\end{aligned}$$

which implies that:

$$\left\{\begin{array}{l}
a_h(\gamma_1,\dots,\gamma_h) = a_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h))\\
b_h(\gamma_1,\dots,\gamma_h) = b_1(\gamma_1 + b_{h-1}(\gamma_2,\dots,\gamma_h)),
\end{array}\right. \tag{16}$$

with $a_1(\gamma) := \gamma'\mu_X$ and $b_1(\gamma) := \Phi_X'\gamma$.

7.1.3 Computation of α_h and β_h

We have:

$$\begin{aligned}
\mathbb{V}_t\left(\sum_{i=1}^{h}\gamma_i' X_{t+i}\right) &= \mathbb{V}_t\left(\mathbb{E}_{t+1}\left[\sum_{i=1}^{h}\gamma_i' X_{t+i}\right]\right) + \mathbb{E}_t\left(\mathbb{V}_{t+1}\left[\sum_{i=1}^{h}\gamma_i' X_{t+i}\right]\right)\\
&= \mathbb{V}_t\left(\gamma_1' X_{t+1} + \mathbb{E}_{t+1}\left[\sum_{i=2}^{h}\gamma_i' X_{t+i}\right]\right) + \mathbb{E}_t\left(\mathbb{V}_{t+1}\left[\sum_{i=2}^{h}\gamma_i' X_{t+i}\right]\right)\\
&= \mathbb{V}_t\left(a_{h-1}(\gamma_2,\dots,\gamma_h) + (b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1)' X_{t+1}\right) +\\
&\quad \mathbb{E}_t\left(\alpha_{h-1}(\gamma_2,\dots,\gamma_h) + \beta_{h-1}(\gamma_2,\dots,\gamma_h)' X_{t+1}\right)\\
&= \alpha_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + \beta_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1)' X_t +\\
&\quad \alpha_{h-1}(\gamma_2,\dots,\gamma_h) + a_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h)) + b_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h))' X_t.
\end{aligned}$$

Therefore:

$$\left\{\begin{array}{l}
\alpha_h(\gamma_1,\dots,\gamma_h) = \alpha_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + \alpha_{h-1}(\gamma_2,\dots,\gamma_h) +\\
\quad a_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h))\\
\beta_h(\gamma_1,\dots,\gamma_h) = \beta_1(b_{h-1}(\gamma_2,\dots,\gamma_h) + \gamma_1) + b_1(\beta_{h-1}(\gamma_2,\dots,\gamma_h)),
\end{array}\right. \tag{17}$$

where, with $\quad S_p = \sum_{i=1}^{p} [e_i^{(p)} \otimes e_i^{(p)}] e_i^{(p)\prime}$:

$$\begin{cases} \alpha_1(\gamma) = (\gamma_Y \otimes \gamma_Y)'[(\Theta \otimes \Theta) S_q \Gamma_{z,0} + S_n \Gamma_{Y,0} + S_n \Gamma'_{Y,1} \mu_z] + (\gamma_z \otimes \gamma_z)' S_q \Gamma_{z,0} \\ \quad +2(\gamma_z \otimes \gamma_Y)'(I_q \otimes \Theta) S_q \Gamma_{z,0}, \\ \beta_1(\gamma)' = (\gamma_Y \otimes \gamma_Y)'[(\Theta \otimes \Theta) S_q \Gamma'_{z,1} + S_n \Gamma'_{Y,1} \Phi_z] + (\gamma_z \otimes \gamma_z)' S_q \Gamma'_{z,1} \\ \quad +2(\gamma_z \otimes \gamma_Y)'(I_q \otimes \Theta) S_q \Gamma'_{z,1}. \end{cases}$$

7.2 Auto-regressive Gamma processes

The vector z_t follows a multivariate $\text{ARG}_\nu(\varphi, \mu)$ process. This process, introduced by Gouriéroux and Jasiak (2006), is the time-discretized Cox, Ingersoll, and Ross (1985) process (see also Monfort, Pegoraro, Renne, and Roussellet (2015)).

Conditionally on $\underline{z_{t-1}} = \{z_{t-1}, z_{t-2}, \dots\}$, the different components of z_t, denoted by $z_{i,t}$, are independent and drawn from non-centered Gamma distributions, i.e.:

$$z_{i,t} | \underline{z_{t-1}} \sim \gamma_{\nu_i}(\varphi_i' z_{t-1}, \mu_i),$$

where ν, μ, φ_1, ..., φ_{q-1} and φ_q are q-dimensional vectors. (Recall that W is drawn from a non-centered Gamma distribution $\gamma_\nu(\varphi, \mu)$, iif there exists an exogenous $\mathcal{P}(\varphi)$-distributed variable Z such that $W|Z \sim \gamma(\nu + Z, \mu)$ where $\nu + Z$ and μ are, respectively, the shape and scale parameters of the gamma distribution.)

Importantly, it can be shown that this process is affine, in the sense that its conditional Laplace transform is exponential affine. Formally, the conditional log-Laplace transform of z_{t+1}, denoted by ψ_t, is given by:

$$\psi_t(w) := \log(E_t[\exp(w' z_{t+1})]) = a_z(w) + b_z(w)' z_t,$$

with

$$a_z(w) \quad = \quad -\nu' \log(1 - \mu \odot w) \tag{18}$$

$$b_z(w) \quad = \quad \varphi \left(\frac{w \odot \mu}{1 - w \odot \mu} \right), \tag{19}$$

where φ is the $q \times q$ matrix equal to $[\varphi_1, \dots, \varphi_q]$, where $\odot$ is the element-by-element (Hadamard) product and where, by abuse of notations, the log and division operator are applied element-by-element wise.

The weak vector auto-regressive form of process z_t is given by:

$$z_t = \mu_z + \Phi_z z_{t-1} + diag(\sqrt{\Gamma_{z,0} + \Gamma'_{z,1} z_{t-1}}) \varepsilon_{z,t},$$

where, conditionally on $\underline{z_{t-1}}$, $\varepsilon_{z,t}$ is of mean zero and has a covariance matrix equal to the identity matrix and where:

$$\mu_z = \mu \odot \nu, \quad \Phi_z = (\mu \mathbf{1}'_{q\times 1}) \odot (\varphi'), \quad \Gamma_{z,0} = \mu \odot \mu \odot \nu \quad \text{and} \quad \Gamma'_{z,1} = 2[(\mu \odot \mu)\mathbf{1}'_{q\times 1}] \odot (\varphi').$$

Assuming that the eigenvalues of Φ_z lie (strictly) within the unit circle, this last formula notably implies that the unconditional mean of z_t is equal to $(I_q - \Phi_z)^{-1}\mu_z$ whilst z_t's unconditional variance is equal to $(I_{q^2} - \Phi_z \otimes \Phi_z)^{-1} S_q(\Gamma_{z,0} + \Gamma'_{z,1}\bar{z})$.

7.3 Computation of model-implied conditional distributions

In the model, inflation rates of different areas are equal to the linear combinations of the affine process X_t. This implies the existence of closed-form formulas to derive the conditional distribution functions of future inflation rates for any maturity (see Duffie, Pan, and Singleton (2000)). Specifically, we have:

$$\mathbb{P}(\gamma_1' X_{t+1} + \cdots + \gamma_h' X_{t+h} < y | \underline{X_t}) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\Psi_h(iv\boldsymbol{\gamma}, X_t)]e^{-ivy}}{v} dv,$$

where $Im(c)$ denotes the imaginary part of $c \in \mathbb{C}$ and where Ψ_h is the multi-horizon Laplace transform of X_t, defined by:

$$\Psi_h(\mathbf{u}, X_t) = \mathbb{E}(\exp(u_1' X_{t+1} + \cdots + u_h' X_{t+h}) | \underline{X_t}), .$$

with $\mathbf{u} = [u_1, \ldots, u_h]$. A simple computation of the latter Laplace transform is provided by Lemma 7.2 in Appendix 7.1.1.

7.4 Survey data

7.4.1 U.S. surveys

U.S. surveys used in our study include the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (US-SPF), Blue Chip Financial Forecasts (BCFF) and Blue Chip Economic Indicators (BCEI) surveys, the Survey of Primary Dealers (SPD) published by the Federal Reserve Bank of New York, and the Consensus Economics Survey (CES). Below we provide a brief description of each of them. Panel A of Table 1 summarizes the data set described below.

The US-SPF is conducted quarterly and provides forecasts on a wide range of macroeconomic and financial variables.[33] For the purpose of this study, we use a few different inflation forecasts from the US-SPF.

[33] The US-SPF survey was formerly conducted by the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER), began in 1968:Q4 and was taken over by the Philadelphia Fed in 1990:Q2.

First, we use density forecasts for the price change in the GDP price deflator (survey variable PRPGDP) for the current and the following calendar year, Y0 and Y1, respectively.[34] US-SPF defines a price change as the annual-average over annual-average percent change in the level of the GDP price index that is available quarterly. The US-SPF inflation measure is thus consistent with the following inflation target:

$$\frac{1}{4}(\pi_{t+h-21,t+h-9} + \pi_{t+h-18,t+h-6} + \pi_{t+h-15,t+h-3} + \pi_{t+h-12,t+h}),$$

where h is the forecast horizon measured in months. The density functions are available both on an individual and aggregate basis but we use information from the aggregated (averaged) forecast density functions. We use these density projections to obtain a survey-based inflation forecast uncertainty measure using the variance of the average density function.[35] Since the forecast density functions are the *fixed event* forecasts (namely for the current and the following years), therefore, the forecast horizon changes relatively to the survey's timing, we are able to construct the first and the second moments of the density functions four, five, six, seven, and eight quarters out. Our sample for the density functions is from 1999:Q1 and to 2016:Q2.[36]

Second, we use the US-SPF five-year average headline CPI inflation consensus forecasts (survey variable CPI5YR) in order to identify more distant-horizon inflation forecasts. This projection is defined as the annual average inflation rate over the next five years. The "next five years" includes the year in which the survey is conducted and the following four years. This average inflation forecast corresponds to Y0-Y4 in our notations and is consistent with the following definition, at time t:

$$\frac{1}{5}(\pi_{t-1,t} + \pi_{t,t+1} + \pi_{t+1,t+2} + \pi_{t+2,t+3} + \pi_{t+3,t+4}).$$

Our sample for this variable is from 2005:Q3 (its starting point in the US-SPF) to 2016:Q2.

BCFF and BCEI surveys are published monthly. These surveys represent a rea-

[34] US-SPF started providing the density projections of the core Consumer Price Index (survey variable PRCCPI) and of the core Personal Consumer Expenditures Index (survey variable PRCPCE) only in 2007:Q1. Therefore we decided to concentrate on the density projections of the GDP price deflator (despite small level differences with the headline CPI index) in order to have information about the second moments of the future U.S. inflation rates starting from the beginning of our sample, 1999:Q1. US-SPF does not provide any density projections about the headline CPI inflation.

[35] In this study we abstract from heterogeneity issues implied by individual forecasters' inflation projections. Information about the structure of the survey and definitions of the variables can be obtained in the `spf-documentation.pdf` file in `https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters`. Since we obtain aggregate histograms, which are averages of the corresponding individual forecasters' histograms, these distributions ought to be smoothed, ideally. Appendix 7.6 provides details.

[36] The beginning of our sample is motivated by the onset of the euro-zone and availability of the euro-area surveys.

sonably stable panel of about 50 top professional analysts who forecast a range of financial (in the case of the BCFF) and macroeconomic (in the case of the BCEI) variables. The panels of the BCFF and BCEI analysts are different yet a lot of panel members participate in both surveys.[37] Both Blue Chip surveys provide individual point estimates of inflation forecasts, from which consensus and disagreement measures can be obtained. While Blue Chip inflation consensus forecasts have been used in the literature extensively (Chun, 2011; Chernov and Mueller, 2012; Grishchenko and Huang, 2013; D'Amico, Kim, and Wei, 2016; Grishchenko, Vanden, and Zhang, 2016), its inflation disagreement measures only recently became popular (Wright, 2011; Buraschi and Whelan, 2012; D'Amico and Orphanides, 2014; Shi, 2014). Monthly surveys provide inflation forecasts up to six quarters out. In addition to those, BCFF and BCEI surveys publish long-range forecasts twice a year. These long-range forecasts contain average annual forecasts usually five years out from the survey publication year, and the average five-year forecast of the next five years afterwards (Y6-Y10 in our notations). We use Y6-Y10 consensus inflation forecasts in our model estimation.

The Survey of Primary Dealers (SPD) has started only in 2004, and to the best of our knowledge, we are among the first to use this survey in the academic literature. A concurrent study of Crump, Eusepi, and Moench (2016) also uses SPD to study nominal term premia. Prior to each FOMC meeting, the survey asks primary dealers (currently 22) a number of questions including inflation density forecasts. The survey questions sometimes vary depending on the economic environment.[38] Nonetheless, certain questions such as the density forecasts for the headline CPI inflation are routinely asked so the sufficient time series can already be gathered. In particular, starting from the March 2007 FOMC meeting, survey participants are asked to provide a percent chance attached to the five-year average annual CPI inflation five years ahead falling below 1%, between 1.01% and 1.50%, between 1.51% and 2%, between 2.01% and 2.50%, between 2.51% and 3%, and above 3.01%.[39] Starting from the December 2014 FOMC meeting, primary dealers are also asked to provide the same inflation density forecasts over the next five years. Thus, the Y6-Y10 density measure is available from 3/2007 until 6/2016 and the Y0-Y4 measure is available from 12/2014 to 6/2016. The SPD forecasts are consistent with the inflation measure $\pi_{t,t+h}$ defined in eq. (8).[40] Thus, the SPD survey nicely complements information from the US-SPF that provides density inflation forecasts for the shorter horizons

[37] For example, out of 47 and 53 participating analysts in the December 2015 BCFF and November 2015 BCEI surveys, respectively, 35 analysts were participating in both surveys.

[38] See posted questions on the website of the Federal Reserve Bank of New York: `https://www.newyorkfed.org/markets/primarydealer_survey_questions.html`.

[39] The bins did not change over the time of the survey.

[40] For the five-year annual average CPI inflation five years ahead, this inflation measure is adjusted in order to get conditional moments of $\frac{1}{60}(\pi_{t+1,t+61} + \cdots + \pi_{t+60,t+120})$.

with those for the longer horizons.

The CES is an additional survey that provides inflation forecasts for a range of developed countries, on a monthly basis. CES survey participants provide point estimates for the average annual percent change of the headline CPI index relative to the previous calendar year. These projections are available for the current and the next calendar years, Y0 and Y1 in our notations.

7.4.2 Euro-area surveys

Euro-area surveys include the Survey of Professional Forecasters published by the European Central Bank and the Consensus Economics Survey. We briefly describe each of these two surveys below. Panel B of Table 1 summarizes these surveys.

The ECB-SPF survey was launched in the first quarter of 1999 and has received a considerable amount of attention by academics and practitioners in recent years (see for instance (see, e.g., Conflitti, 2011; Rich, Song, and Tracy, 2012; Andrade and Le Bihan, 2013)). The survey provides GDP forecasts, inflation expectations, and unemployment forecasts on a quarterly frequency. It also provides assumptions made by different forecasters. Up to date, the panel of forecasters includes 79 listed international and European institutions as well as a number of other participants who chose to remain anonymous. More than half of the survey participants involved in the first survey remain in the pool of participants today and the size of the survey panel remains about unchanged given that more than 20 new survey respondents were added throughout the years. The panel of veteran forecasters is thus relatively stable and the average number of panel members who answer all inflation-related questions across surveys remains stable and equals, on average, to 34. Participants are asked to provide point forecasts and probability distributions for rolling horizons (one and two years ahead year-on-year forecasts) and longer-term inflation expectations (five years ahead) implied by changes in the Harmonized Index of Consumer Prices (HICP). ECB-SPF inflation measures are defined as $\pi_{t+h-12,t+h}$, where h is the forecast horizon measured in months.[41]

CES survey participants provide point estimates for the average annual percent change of HICP relative to the previous calendar year. These projections are available for the current and the next calendar year, since January 1999 (in the case of the Euro area), on a monthly basis. There are roughly 20 institutions participating in the euro area survey; less than half of which coincide with disclosed ECB-SPF participants. Importantly, CE also publishes their long-term forecasts on a semi-annual

[41] The survey also provides fixed calendar year horizons (current year, and two more following years) but we do not include this information due to the fact that only point forecasts are supplied. Moreover, the nature of the fixed-horizon forecasts may not allow for consistent comparisons of uncertainty across time given that we ought to see a decrease in uncertainty at every survey round in which more information becomes available.

basis (i.e., in April and October), in which five-year five years ahead (Y6-Y10) inflation projections are available. We use these long-term forecasts to complement the ECB-SPF survey information.

7.5 Survey-based uncertainty

Measuring uncertainty has gained a lot of attention in recent years. Several proxies for uncertainty, such as stock market volatility, conditional volatility of series, cross-sectional dispersions, and keyword counts in news articles, have been used. However, survey-based measures of uncertainty, also known as subjective measures of uncertainty, have become increasingly popular due to their model-free nature.

The literature distinguishes between three survey-based uncertainty measures: i) disagreement among forecasters (*ex-ante* measure), ii) variance of the surveys' aggregate probability distribution (*ex-ante* measure), and iii) average individual forecast error variance (*ex-post* measure). *Ex-ante* measures of uncertainty have been found to be more adequate representations of uncertainty in real time so it is important to distinguish between them.

Due to its simplicity and availability, the first measure — a disagreement among forecasters — became one of the most commonly used survey-based uncertainty proxies in the literature. It is defined as follows:

$$d_{t,h} = \frac{1}{N}\sum_{i=1}^{N}(f_{i,t,h} - \bar{f}_{t,h})^2, \tag{20}$$

where N is the number of forecasters, $f_{i,t,h}$ is the forecast at time t for horizon h of a forecaster i and $\bar{f}_{t,h}$ is the mean forecast (i.e., consensus). This proxy of uncertainty, though easily computable, becomes irrelevant if heterogeneity among forecasters vanishes.

The second measure of uncertainty — variance of the aggregate probability distribution provided by the surveys — is defined as follows:

$$\sigma^2_{agg,t,h} = d_{t,h} + \frac{1}{N}\sum_{i=1}^{N}\sigma^2_{i,t,h}, \tag{21}$$

where $\sigma^2_{agg,t,h}$ and $\sigma^2_{i,t,h}$ are the conditional variance of the aggregate distribution and the variance associated with a forecaster i's distribution, respectively. Thus, this measure captures both heterogeneity among forecasters (via the cross-sectional variance of individual means (i.e., disagreement)) and aggregated uncertainty of individual forecasters. In this paper, we consider this particular survey-based uncertainty measure.

An important strand of the literature studies similarities and differences between survey-based measures of disagreement and uncertainty (Conflitti, 2011; Rich, Song, and Tracy, 2012; Andrade and Le Bihan, 2013; Boero, Smith, and Wallis, 2014; D'Amico and Orphanides, 2014). Notably, Giordani and Soderlind (2003) find that disagreement is a fairly good proxy for other measures of uncertainty that are more theoretically appealing, but less easily available.[42] Lahiri and Sheng (2010) decompose forecast errors into common and idiosyncratic shocks, and show that aggregate forecast uncertainty can be expressed as the sum of the disagreement among forecasters and the perceived variability of future aggregate shocks. This finding implies that the reliability of disagreement as a proxy for uncertainty depends primarily on the stability of the forecasting environment.

7.6 Smoothing survey-based and risk-neutral distributions

7.6.1 Overview

Our analysis makes use of the generalised beta distribution twice. First, we use it in order to convert the forecasters' views about the probabilities of future inflation outcomes into smoothed distributions. Second, the generalised beta distribution is used to convert inflation option prices into risk-neutral distributions. While the former distributions are essential in the estimation of our model, the latter are used after the estimation, when we study our model outputs.

In both cases, the spirit of the smoothing methodology, that broadly builds on Engelberg, Manski, and Williams (2009) (see also Boero, Smith, and Wallis, 2014; Clements, 2014), is the same. We consider the data associated with a specific inflation distribution, as defined by: (a) one area, (b) one measure of inflation (year-on-year growth rate of the price index, annualised growth rate over a given period or average of year-on-year growth rates), (c) one horizon and (d) a given type of probability measure (historical in the first case, risk-neutral in the second case). Then, we assume that these data are coherent with a generalised Beta distribution and look for the parametrisation of this distribution that provides the closest fit to the considered data (by minimising the sum of weighted squared deviations between the data and its theoretical counterpart).

In the first case, the data consists of survey-based probabilities of future inflation outcomes falling within given ranges (see Appendix 7.4). These survey-based data provide us with evaluations of the cumulative distribution function (c.d.f.) of the associated distribution at the bounds of the bins. Let us stress that these smoothed

[42] According to their paper, previous research on SPF data implies a weak correlation between disagreement and other measures of uncertainty possibly due to failure in using a long enough sample and the failure in fitting a normal distribution to each histogram for obtaining a robust measure of disagreement.

distributions are fundamentally different from those resulting from the approach developed in the present paper. Indeed, the latter are coherent across time and horizons, which is not the case of the former. Heuristically, the smoothing approach presented in this appendix constitutes a preliminary processing of the data before using them in the model estimation.

In the second case, the data consists of market quotes of inflation derivatives, namely inflation floors and swaps. As explained in Subsection 7.6.3, these market quotes closely relate to the forward-neutral distribution of inflation, which is a probability measure that is equivalent to the physical one. As soon as one observes a sufficiently large number of inflation derivatives' quotes, one can estimate the generalised Beta distribution that provides the closest set of "theoretical" quotes. For each considered horizon and date, we use six market quotes to estimate the forward-neutral distribution: five prices of inflation floors (with strikes of -2%, -1%, 0%, 1% and -2%) and the inflation swap rate.

7.6.2 Generalised Beta distribution

X is distributed as a generalised Beta distribution of parameters (a, b, c, d) if $(X - c)/(d - c)$ is distributed as $B(a, b)$. In that case, we use the following notation: $X \sim \mathcal{B}(a, b, c, d)$.

If $X \sim \mathcal{B}(a, b, c, d)$, we have $\mathbb{P}(X < x) = \mathbb{P}(Y < (x - c)/(d - c))$, where Y is distributed as $B(a, b)$. Therefore, the c.d.f. of X is:

$$F(x) = \frac{Beta((x - c)/(d - c); a, b)}{B(a, b)},$$

where $Beta(x; a, b)$ is the incomplete Beta function, defined by:

$$Beta(x; a, b) := \int_0^x t^{a-1}(1 - t)^{b-1} dt.$$

The distribution function of X then is:

$$f(x; a, b, c, d) := \mathbb{I}_{\{x \in [c,d]\}} \frac{1}{(d - c)B(a, b)} \left(\frac{x - c}{d - c}\right)^{a-1} \left(\frac{d - x}{d - c}\right)^{b-1}.$$

7.6.3 T-forward-neutral distribution of inflation

Denoting by f_t^{T-t} the T-forward-neutral distribution of the inflation rate $\pi_{t,T}$, the price of a zero-coupon inflation floor, as of date t, with an exercise rate of S and an expiry date $t+h$ is given by:

$$\begin{aligned} floor_{t,h}(S) &= e^{-hr_{t,h}} \int_{-\infty}^{S} \left\{(1+x)^h - (1+S)^h\right\} f_t^h(x)dx \\ &\approx he^{-hr_{t,h}} \int_{-\infty}^{S} (S-x) f_t^h(x;a,b,c,d)dx, \end{aligned} \tag{22}$$

where $r_{t,h}$ is the risk-free interest rate between dates t and $t+h$ (known at date t).

Let us assume that the T-forward neutral distribution of $\pi_{t,t+h}$ is $\mathcal{B}(a,b,c,d)$. In that case, the price of the previous floor is approximately equal to:

$$\begin{aligned} & e^{-hr_{t,h}} h \int_c^S (S-x) \frac{1}{(d-c)B(a,b)} \left(\frac{x-c}{d-c}\right)^{a-1} \left(\frac{d-x}{d-c}\right)^{b-1} dx \\ = \; & e^{-hr_{t,h}} h(d-c) \int_0^{\frac{S-c}{d-c}} \left(\frac{S-c}{d-c} - y\right) \frac{1}{B(a,b)} y^{a-1}(1-y)^{b-1} dy \\ = \; & e^{-hr_{t,h}} h(d-c) \frac{S-c}{d-c} \int_0^{\frac{S-c}{d-c}} \frac{1}{B(a,b)} y^{a-1}(1-y)^{b-1} dy - \\ & e^{-hr_{t,h}} h(d-c) \int_0^{\frac{S-c}{d-c}} \frac{1}{B(a,b)} y^{a}(1-y)^{b-1} dy \\ = \; & \frac{e^{-hr_{t,h}} h}{B(a,b)} \times \\ & \left\{(S-c) Beta\left(\frac{S-c}{d-c}; a, b\right) - (d-c) Beta\left(\frac{S-c}{d-c}; a+1, b\right)\right\} \end{aligned} \tag{23}$$

Moreover, in that context, the inflation swap rate of maturity h, denoted by $s_{t,t+h}$ is such that

$$\int_{-\infty}^{+\infty} \left\{(1+x)^h - (1+S)^h\right\} f_t^h(x)dx = 0,$$

which implies that:

$$s_{t,t+h} \approx \int_c^d x f_t^h(x)dx = \frac{bc+ad}{a+b}.$$

Table 1: Summary of the surveys data

Survey	Horizon	Description	Inflation Rate Def	Freq.	Sample
Panel A: U.S. surveys of inflation forecasts					
US SPF	Y0	Density	Annual-avg over annual-avg	Q	1999:Q1 - 2016:Q2
US SPF	Y1	Density	Annual-avg over annual-avg	Q	1999:Q1 - 2016:Q2
US SPF	Y0-Y4	P.E.	Q4 over Q4	Q	2005:Q3 - 2016:Q2
BCFF & BCEI	Y6-Y10	P.E.	Annual average	4/year	3/1999 - 6/2016
SPD	Y0-Y4	Density	Annual average	8/year	12/2014 - 6/2016
SPD	Y6-Y10	Density	Annual average	8/year	3/2007 - 6/2016
CES	Y0	P.E.	Annual average	M	1/1999 - 6/2016
CES	Y1	P.E.	Annual average	M	1/1999 - 6/2016
Panel B: Euro-area surveys of inflation forecasts					
ECB SPF	Y1	Density	Annual inflation	Q	1/1999 - 6/2016
ECB SPF	Y1-Y2	Density	Annual inflation	Q	1/1999 - 6/2016
ECB SPF	Y4-Y5	Density	Annual inflation	Q	1/1999 - 6/2016
CES	Y6-Y10	P.E.	Annual average	SA	4/2003 - 4/2016

This table summarizes inflation forecast variables from the U.S. and euro-area surveys used in the study. Column 1 specifies the surveys: US SPF (Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia), BCFF and BCEI (Blue Chip Financial Forecasts and Economic Indicators surveys), SPD (Survey of Primary Dealers conducted by the Federal Reserve Bank of New York), CES (Consensus Economics Survey), ECB SPF (Survey of Professional Forecasters conducted by the European Central Bank). Column 2 specifies the horizon of the inflation forecast: Y0 - current year, Y1 - next year, Y0-Y4 - 5-year horizon, and so on. Column 3 specifies whether point estimates (P.E.) of the forecasts or the first two moments of the reported density projections (density) are used in the model estimation. Column 4 provides a definition of the inflation rate forecast reported in surveys. Column 5 specifies the available frequency of the forecasts, and Column 6 - the sample period used in the estimation. All U.S. inflation forecasts are on headline Consumer Price Index (CPI) index except the density projections from the US-SPF, which are on the annual GDP price inflation. All euro-area inflation forecasts are on the Harmonized Index of Consumer Prices (HICP).

Table 2: Determination of the number of latent factors

Panel A: PCA on residuals

PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	
74.96	93.24	97.60	98.72	99.60	99.75	99.87	99.93	99.96	99.98	100.00	100.00	

Panel B: R^2 coefficients of regressions

EA 1Y	EA 2Y	EA 3Y	EA 5Y	EA 7Y	EA 10Y	US 1Y	US 2Y	US 3Y	US 5Y	US 7Y	US 10Y	Mean
95.62	98.14	99.04	99.10	98.67	96.96	97.81	98.98	98.22	94.44	90.87	83.11	95.91

Panel C: Joint PCA on market-based variances

PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	
79.97	90.17	93.39	95.41	96.51	97.28	98.01	98.61	99.19	99.57	99.88	100.00	

This table summarises the results of our analysis to determine the number of latent factors in our estimation. Panel A summarizes the results of a principal component analysis on residuals stemming from bivariate regressions of area-specific inflation swaps on area-specific realised inflation series. The cumulative percentage of cross-sectional variation accounted for by each component is displayed. Panel B reports the R^2 coefficients stemming from regressions of inflation swaps on realised inflation series of the two economies and on the two first principal components of Panel A. Panel C displays the results of a principal component analysis on risk-neutral variances estimated on inflation floors contracts. Reported figures are expressed in percentage points. Area-specific inflation swaps and market-based variances consist of six different maturities, namely 1, 2, 3, 5, 7, and 10 years. Refer to Appendix 7.6 for further details.

Table 3: Parameter estimates

	Adjust.	Value	St.dev.		Adjust.	Value	St.dev.
$\bar{\pi}^{(1)}$		1.796	–	$\Theta_{1,1}$	$(\times 10^3)$	0.441	*0.401*
$\bar{\pi}^{(2)}$		2.402	–	$\Theta_{2,1}$	$(\times 10^3)$	-1.624	*1.087*
				$\Theta_{3,1}$	$(\times 10^3)$	-1.615	*0.657*
$\delta_1^{(1)}$		1.000	–	$\Theta_{4,1}$	$(\times 10^3)$	-0.971	*0.428*
$\delta_2^{(1)}$		0.282	*0.053*	$\Theta_{1,2}$	$(\times 10^3)$	0.083	*0.085*
$\delta_3^{(1)}$		1.000	–	$\Theta_{2,2}$	$(\times 10^3)$	-2.304	*1.066*
$\delta_4^{(1)}$		0.146	*0.067*	$\Theta_{3,2}$	$(\times 10^3)$	-0.090	*0.311*
$\delta_1^{(2)}$		1.154	*0.576*	$\Theta_{4,2}$	$(\times 10^3)$	-0.362	*0.212*
$\delta_2^{(2)}$		1.000	–				
$\delta_3^{(2)}$		0.577	*0.112*	$\Gamma_{Y,0[1]}$	$(\times 10^3)$	3.530	*5.543*
$\delta_4^{(2)}$		1.000	–	$\Gamma_{Y,0[2]}$	$(\times 10^3)$	24.743	*54.667*
				$\Gamma_{Y,0[3]}$	$(\times 10^3)$	15.234	*11.574*
$\Phi_{Y[1,1]}$		0.983	*0.004*	$\Gamma_{Y,0[4]}$	$(\times 10^3)$	1.890	*19.526*
$\Phi_{Y[2,1]}$		0.004	*0.192*				
$\Phi_{Y[3,1]}$		-0.002	*0.074*	$\Gamma_{Y,1[1,1]}$	$(\times 10^3)$	0.008	*0.065*
$\Phi_{Y[4,1]}$		-0.034	*0.049*	$\Gamma_{Y,1[2,1]}$	$(\times 10^3)$	0.000	*0.019*
$\Phi_{Y[2,2]}$		0.714	*0.039*	$\Gamma_{Y,1[3,1]}$	$(\times 10^3)$	0.209	*1.140*
$\Phi_{Y[3,2]}$		0.005	*0.031*	$\Gamma_{Y,1[4,1]}$	$(\times 10^3)$	1.154	*0.585*
$\Phi_{Y[4,2]}$		-0.090	*0.027*	$\Gamma_{Y,1[1,2]}$	$(\times 10^3)$	1.184	*0.317*
$\Phi_{Y[3,3]}$		0.891	*0.013*	$\Gamma_{Y,1[2,2]}$	$(\times 10^3)$	0.182	*0.087*
$\Phi_{Y[4,3]}$		0.001	*0.011*	$\Gamma_{Y,1[3,2]}$	$(\times 10^3)$	0.424	*0.339*
$\Phi_{Y[4,4]}$		0.934	*0.006*	$\Gamma_{Y,1[4,2]}$	$(\times 10^3)$	0.904	*0.297*
ν_1		0.073	*0.043*				
ν_2		0.301	*0.255*				
$\Phi_{z[1,1]}$		0.991	*0.002*				
$\Phi_{z[2,1]}$	$(\times 10^2)$	0.000	*0.043*				
$\Phi_{z[2,2]}$		0.984	*0.002*				

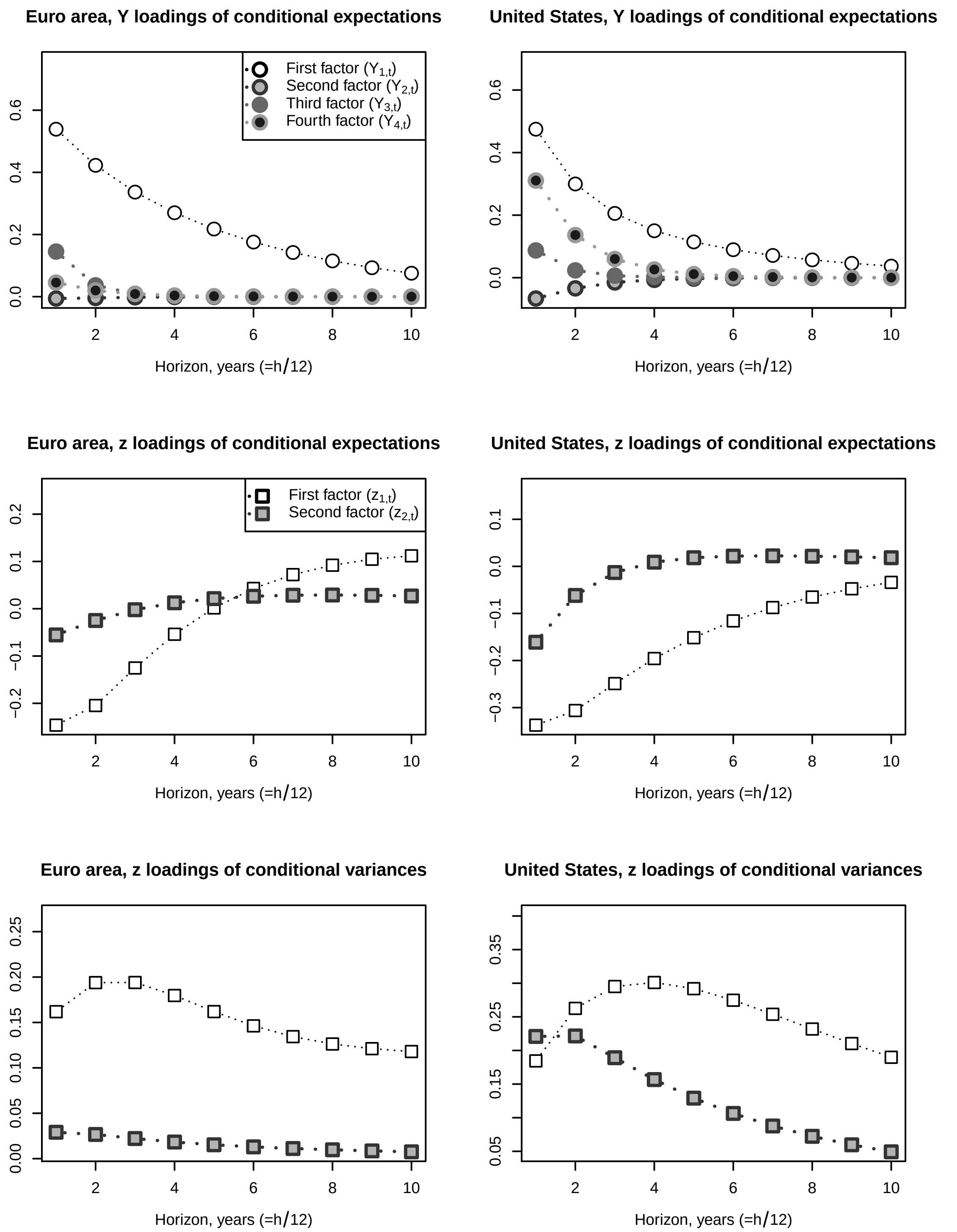
The model is estimated by maximizing the quasi-likelihood stemming from a modified Kalman filter. Standard deviations (in italics) are calculated from the outer product of the log-likelihood gradient, evaluated at the estimated parameter values. For the sake of identification, different elements of δ are set to 1. Superscripts in parentheses indicate the currency areas: 1 for the euro area and 2 for the US.

Table 4: Euro-area and U.S. inflation: Deflation probabilities, comovements, and risk measures

	VIX	*VSTOXX*	*US EPU*	*EA EPU*	*EC ES*	*UM SC*	*EA 1-y IRP*	*EA 5-y IRP*	*US 1-y IRP*	*US 5-y IRP*
Panel A: Comovements										
Joint Deflation Proba., 1-y	(+) 2	(+) 1	(+) 1	(+) 4*	(−) 17*	(−) 9	(−) 12**	(−) 10*	(−) 5*	(−) 9*
Joint Deflation Proba., 5-y	(−) 3	(−) 0	(+) 9*	(+) 39**	(−) 5	(−) 7	(−) 26**	(−) 56**	(−) 2	(−) 4
Inflation Covariance, 1-y	(+) 7*	(+) 6	(+) 18**	(+) 19*	(−) 37**	(−) 57**	(−) 0	(+) 0	(−) 17**	(−) 7*
Inflation Covariance, 5-y	(−) 0	(+) 0	(+) 17**	(+) 43**	(−) 17**	(−) 27**	(−) 15*	(−) 29**	(−) 6	(−) 5
Panel B: United States										
U.S. Deflation proba., 1-y	(+) 18*	(+) 12	(+) 3	(+) 0	(−) 39*	(−) 22*	(−) 3	(+) 0	(−) 19**	(−) 13**
U.S. Deflation proba., 5-y	(+) 0	(+) 1	(+) 16**	(+) 40**	(−) 17**	(−) 25**	(−) 17**	(−) 31**	(−) 9	(−) 8
U.S. Inflation variance, 1-y	(+) 21**	(+) 12**	(+) 1	(−) 5	(−) 16*	(−) 23**	(+) 14	(+) 46**	(−) 9*	(−) 1
U.S. Inflation variance, 5-y	(+) 11**	(+) 9*	(+) 16**	(+) 12	(−) 39**	(−) 59**	(+) 0	(+) 2	(−) 18**	(−) 6
Panel C: Euro area										
E.A. Deflation proba., 1-y	(−) 0	(+) 0	(+) 1	(+) 12**	(−) 3	(−) 1	(−) 33**	(−) 45**	(−) 3	(−) 10
E.A. Deflation proba., 5-y	(−) 2	(−) 0	(+) 10*	(+) 41**	(−) 4	(−) 6	(−) 27**	(−) 59**	(−) 2	(−) 4
E.A. Inflation variance, 1-y	(−) 0	(+) 0	(+) 17**	(+) 42**	(−) 18**	(−) 28**	(−) 14*	(−) 28**	(−) 7	(−) 5
E.A. Inflation variance, 5-y	(−) 1	(−) 0	(+) 14**	(+) 44**	(−) 11*	(−) 17*	(−) 20*	(−) 42**	(−) 3	(−) 4

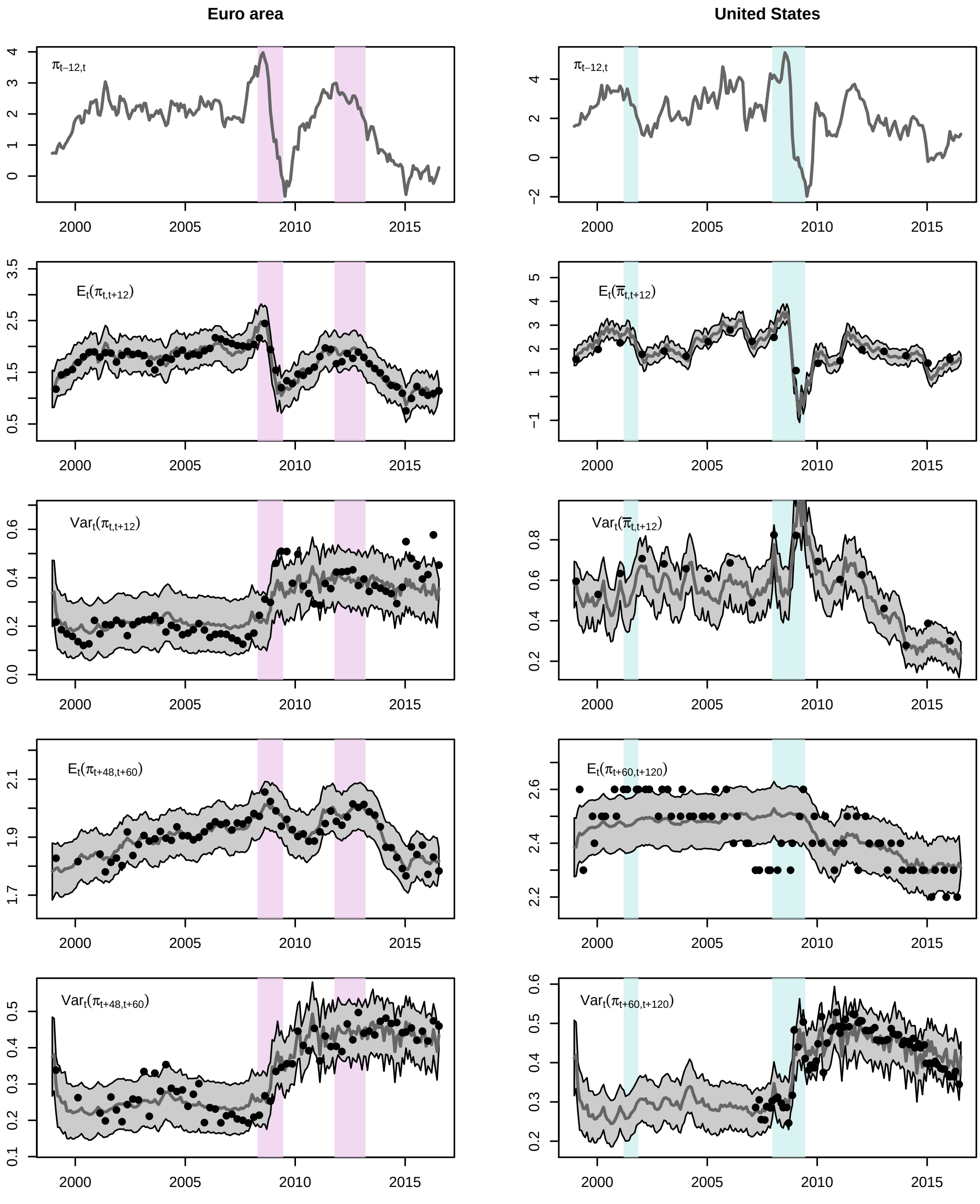
This table reports synthetic results of bivariate regressions of the variables appearing on the first column of the table on those reported in the first row. The sign of the slope is in parentheses. Reported figures are regression R^2s, expressed in percentage points. *, **, and *** denote statistical significance of the slope coefficient at the 10%, 5%, and 1% level, respectively. Standard errors are HAC Newey-West (12 lag) corrected. The number of observations is 203. Variables' abbreviations are as follows: y - year; EC ES - European commission economic sentiment index; UM CS - University of Michigan survey of consumers sentiment index; EPU - economic policy uncertainty; IRP - inflation risk premium.

Figure 1: Factor loadings of expectations and variances of future inflation rates



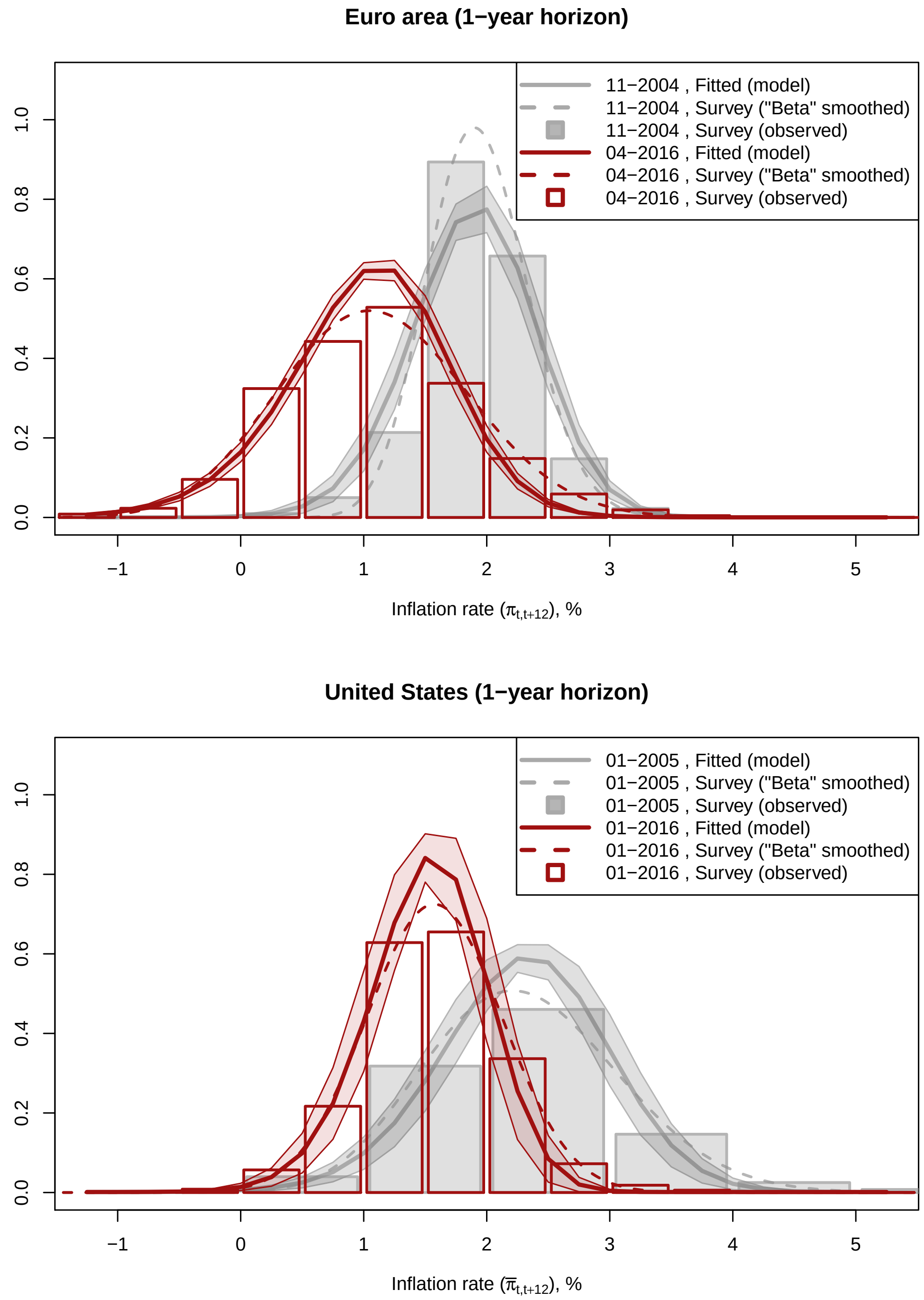
This figure displays, for different horizons $h/12$, where h is measured in months, the entries of vectors $b_h^{(i)}$ and $\beta_h^{(i)}$ appearing in eqs. (9) and (10). In order to facilitate interpretation, these loadings have been multiplied by the marginal standard deviations of the associated factors. That is, the y-coordinates correspond to the effect of a one-standard deviation change in the factors on the conditional level of inflation expectations (or variances for the bottom charts).

Figure 2: Fit of inflation and survey data



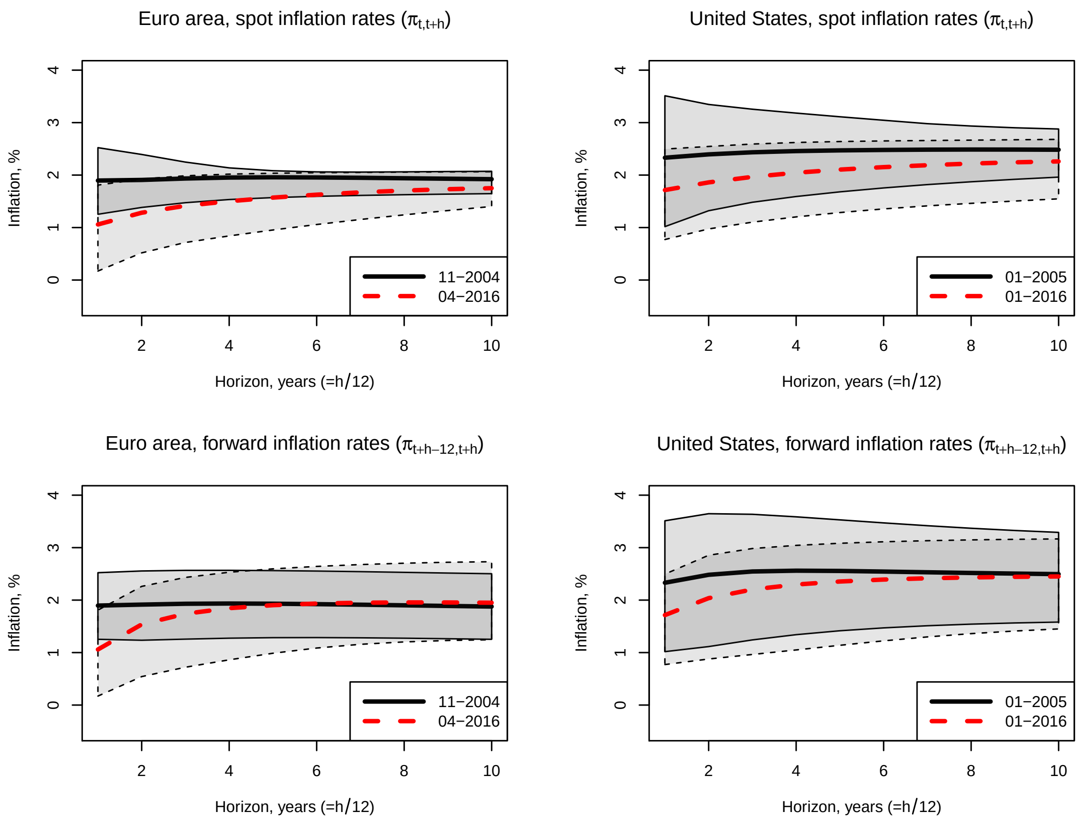
This figure illustrates the fitting properties of the model. The top charts plot realized inflation rates, based on the HICP (in case of the Euro area) and the headline CPI (in case of the U.S.). The dots on the charts at the left-hand side (euro-area results) correspond to the observations available at the ECB-SPF survey. The dots on the charts at the right hand side (U.S. results) are taken from several U.S. surveys: the dots on the $E_t(\bar{\pi}_{t+12})$ and $Var_t(\bar{\pi}_{t+12})$ charts are taken from the US-SPF survey, the dots on the $E_t(\bar{\pi}_{t+60,t+120})$ chart are taken from the long-range BCFF and BCEI surveys, the dots on the $Var_t(\bar{\pi}_{t+60,t+120})$ chart are from the SPD survey. The grey-shaded areas are the 2-standard-deviation confidence intervals. For the sake of readability, this figure does not show the fit of all observed surveys. For the United States charts, the notation $\bar{\pi}_{t+h}$ refers to the US-SPF inflation measure, which is the annual-quarter-average over annual-quarter-average percent change in prices (see Appendix 7.4.1 for details.) Here and on the following charts, pink bars indicate euro-area CEPR recessions, and blue bars indicate U.S. NBER recessions.

Figure 3: Fit of survey distributions



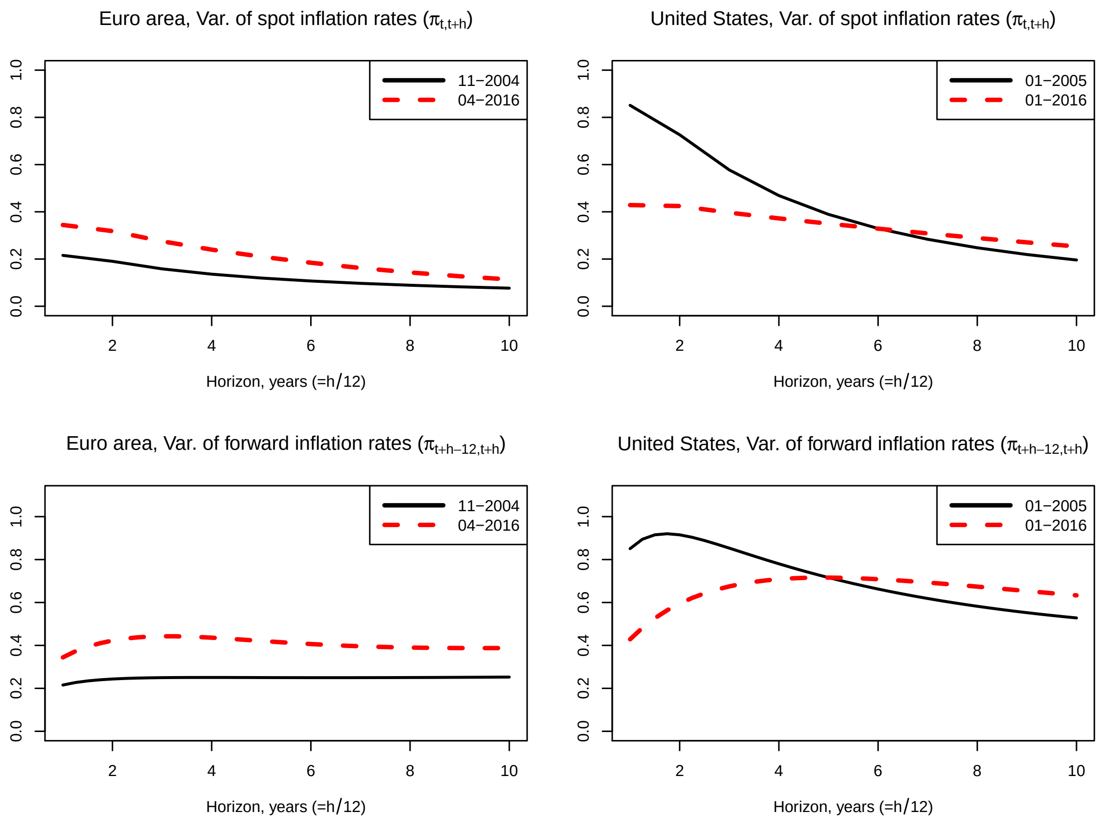
This figure compares the one-year ahead survey-based histograms to their smoothed counterparts and to the one-year ahead model-implied distributions. For the model-implied distributions, two-standard-deviation confidence intervals are reported. These standard deviations reflect uncertainty associated with the estimation of the latent factors X_t. These standard deviations are obtained by applying the delta method on the function relating factors X_t to the conditional cumulative distribution function (c.d.f.) of future inflation. The covariance matrix of the filtered values of X_t stems from the Kalman filter. Appendix 7.3 provides details of the computation of the c.d.f. of future inflation rates.

Figure 4: Term structure of inflation expectations



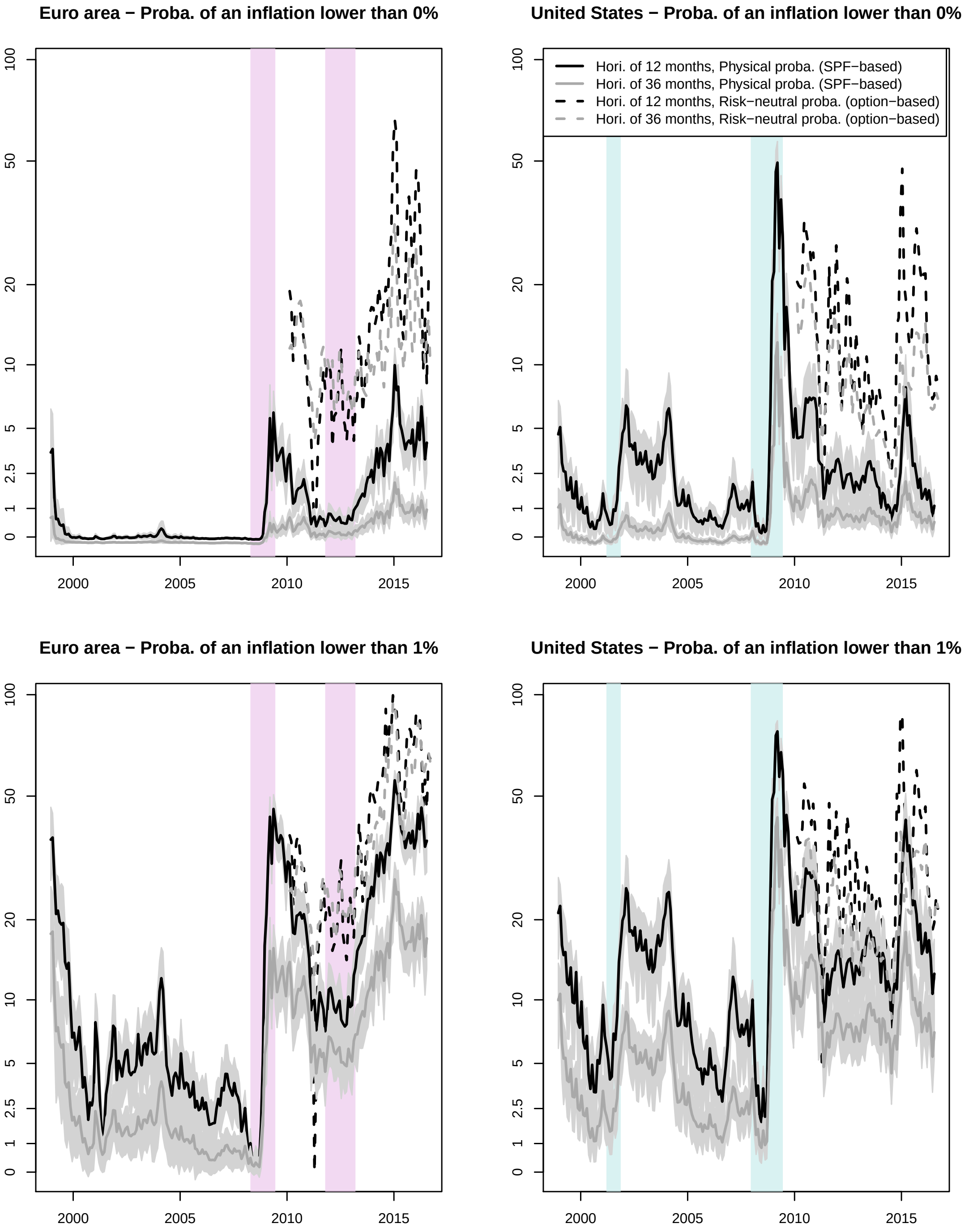
This figure displays the term structure of model-implied expected inflation rates along with the 5th and 95th quantiles associated with the respective conditional distributions. Top two (bottom two) panels display spot inflation (one-year ahead forward inflation) rates up to horizon of 10 years in the Euro area and the Untied States. The quantiles are derived from the closed-form formulas given in Appendix 7.3.

Figure 5: Term structure of inflation uncertainty



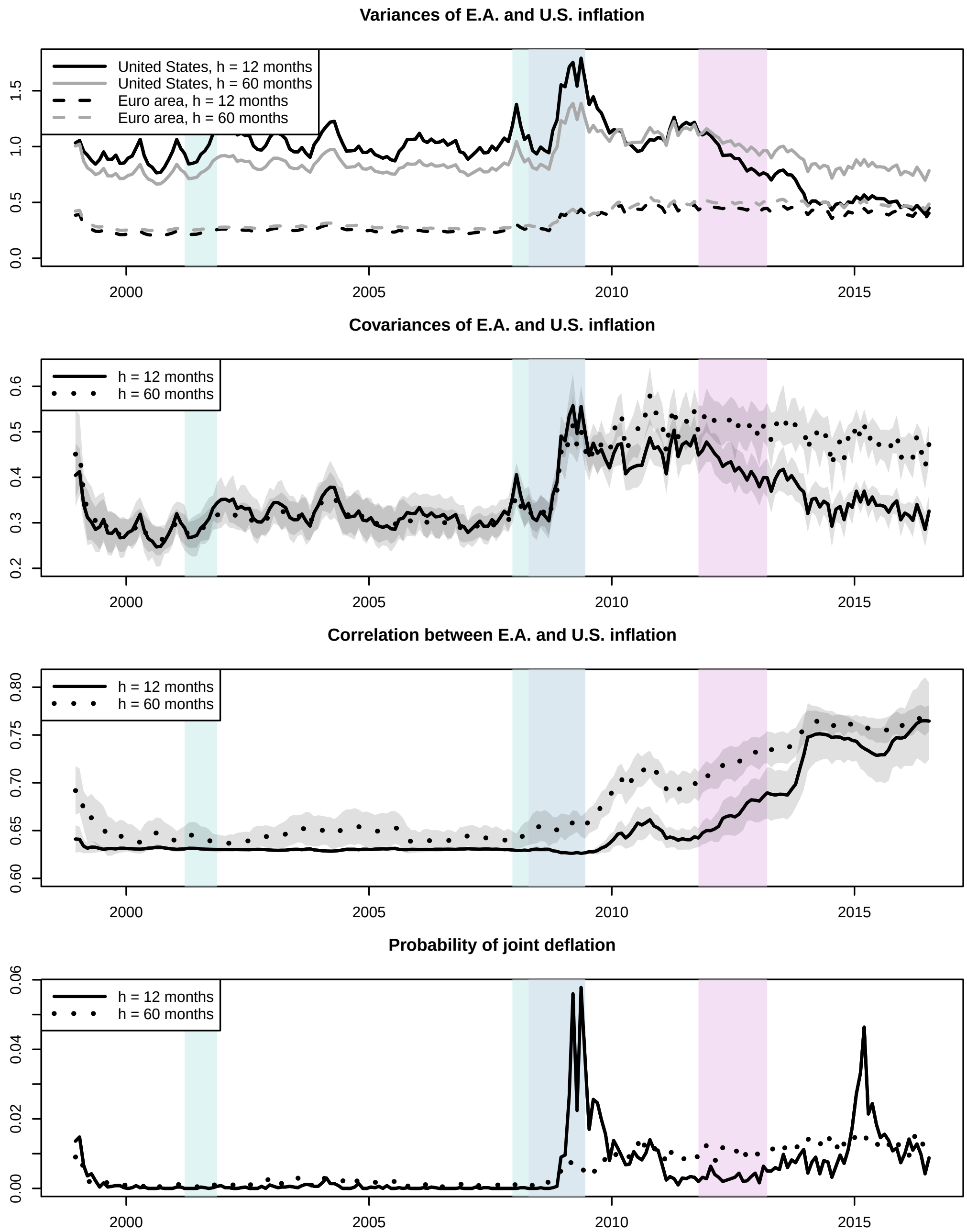
This figure displays the term structure of model-implied variances of the model-implied expected spot and forward inflation rates, which are plotted in Figure 4. Top two panels display the variances (or, inflation uncertainty) of the annualized average spot rates up to horizon $h = 10$ years in the Euro area and the United States. Bottom two panels display the variances of the one-year rates h periods ahead.

Figure 6: Option-based and model-implied probabilities of low inflation



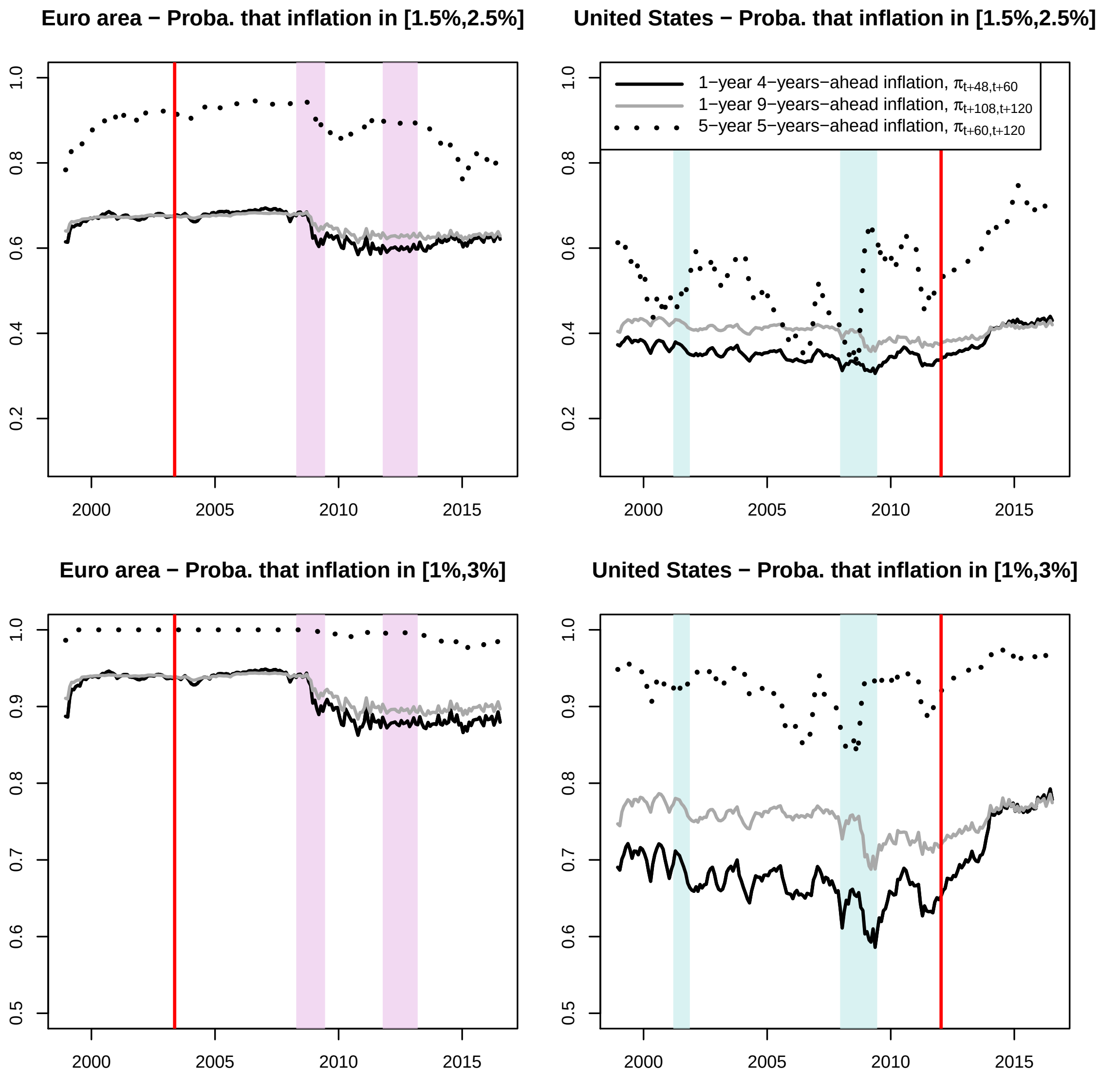
This figure compares model-implied (physical) deflation probabilities (top charts) and probabilities of inflation falling below 1 percent (bottom charts) in both economies to their risk-neutral counterparts. The probabilities are plotted for horizons of 12 and 36 months ahead. The risk-neutral probabilities are based on inflation derivatives, namely zero-coupon inflation swaps and inflation floors.

Figure 7: Expected joint movements of euro area and U.S. inflation rates



The top three panels display the U.S. and euro-area conditional variances, covariances, and correlations for inflation rates for $h = 12$ and $h = 60$ months ahead. The bottom panel shows the joint probabilities of deflation, i.e. $\mathbb{P}(\pi_{t+h}^{(E.A.)} \leq 0, \pi_{t+h}^{(U.S.)} \leq 0 | \underline{S_t^a})$ for the same horizons.

Figure 8: Measure of the anchoring of inflation expectations



This figure displays probabilities that future inflation rates will fall in the two intervals: $I_1 = [1.5\%, 2.5\%]$ (upper plots) and $I_2 = [1\%, 3\%]$ (lower plots). Formally, for an interval I_j, $j \in \{1, 2\}$, they show the time series of the conditional probabilities $\mathbb{P}(\pi^{(i)}_{t+h-m,t+h} \in I_j | \underline{X_t})$. On each plot, three time series are plotted: for two horizons ($h = 60$, 120) and two tenors ($m = 12$, 60). The red vertical lines indicate the months when the ECB and the Federal Reserve announced their inflation objectives, in May 2003 and in January 2012, respectively.