Banks, Liquidity Management and Monetary Policy*

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Preliminary:

Abstract

We develop a new framework to study the implementation of monetary policy through the banking system. Banks make loans by issuing deposits. Loans are illiquid and, therefore, cannot be used to settle transfers of deposits. Instead, banks use central bank reserves for settlements but they may end short of reserves. This possibility induces a tradeoff between profiting from more loans against more liquidity risk exposure.

Monetary policy alters this tradeoff and consequently affects aggregate credit and interest rates. In turn, banks also react to shocks that alter the distribution of payments, induce bank equity losses, increase capital requirements, and cause contractions in the loans demand. We study how the effectiveness of monetary policy varies with these shocks. We calibrate our model to study, quantitatively, why have banks increased their liquidity holdings but not increased lending despite the policy efforts of recent years.

Keywords: Banks, Monetary Policy, Liquidity, Capital Requirements

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1 Introduction

The conduct of monetary policy around the world is changing. The past five years have witnessed banking systems that bore unprecedented financial losses and subsequent freezes in interbank markets. To protect themselves against a potential insolvency, banks cut back on their lending to the private sector. In response, the central banks of the US and Europe have reduced policy rates to almost zero, injected equity to the banking system and continuously purchased private paper in an open attempt to preserve financial stability and reinvigorate lending. However, in reaction to these unprecedented policy interventions, banks seem to have, for the most part, accumulated central bank reserves without renewing their lending activities as intended.[1] Why? Can central banks do more about this? These remain open questions.

Not surprisingly, the role of banks in the transmission of monetary policy has been at the center of policy debates. Unfortunately, there are few modern macroeconomic models that take into account that monetary policy is implemented through the banking system, as occurs in practice. Instead, most macroeconomic models assume that Central Banks control interest rates or the flow of credit directly and abstract from how the transmission of monetary policy may depend on the conditions of banks. This paper presents a model that contributes to filling this gap.

We use our model to answer a number of theoretical issues. What type of shocks can induce banks to hold more reserves and lend less? How does the transmission of monetary policy depend on the decisions of commercial banks? How does its strength vary with shocks to the banking system? In addition, we exploit the lessons derived from this theoretic framework to investigate, quantitatively, why are banks not lending despite all the policy efforts.

Our model is able to contrast different hypotheses that are informally discussed in policy and academic circles. Through the lens of the model, we evaluate the plausibility of the following hypothesis:

Hypothesis 1 - Bank Equity Losses: We study the hypothesis that the lack of lending responds to an optimal behavior by banks given the substantial equity losses suffered in 2008.

Hypothesis 2 - Interbank Uncertainty: We also investigate if banks hold more reserves because they faced more uncertainty about potential costs of accessing the interbank market.

Hypothesis 3 - Capital Requirements: We analyze if the expected path of capital requirements are leading banks to hold more reserves and simultaneously lend less.

Hypothesis 4 - Weak Demand: Finally, we study if banks behave as if they face a weaker effective demand for loans. This hypothesis encompasses a direct shock to the demand for credit or a lack of borrowers that meet credit standards which leads to a weaker *effective* demand for loans.

We calibrate our model and fit it with shocks associated with each hypothesis. We use the its

[1] As is well known, the Bank of Japan had been facing similar issues since the early nineties.

predictions to uncover which shocks are consistent with less lending in times when reserves have increased by several multiples. Our model suggests that a combination of shocks best fits the data. Overall, the model favors an early increase in disruptions in the interbank market followed by a substantial contraction in loan demand.

The Mechanism. The building block of our model is a liquidity management problem. Liquidity management is recognized as one of the fundamental problems in banking and can be explained as follows. When a bank grants a loan, it simultaneously creates demand deposits —or credit lines. These deposits can be used by the borrower to perform transactions at any time. Granting a loan is profitable because a higher interest is charged on the loan than what is paid on deposits. However, more lending relative to a given amount of central bank reserves increases a bank's liquidity risks. When deposits are transferred out of a bank, that bank must transfer reserves to other banks in order to settle transactions. Central bank reserves are critical to clear settlements because, as occurs in practice, loans cannot be sold immediately. Thus, the lower the reserve holdings of a bank, the more likely it is to be short of reserves in the future. This is a source of risk because the bank must incur in expensive borrowing from other banks —or the Central Bank's discount window— if it ends short of reserves. This friction —the liquidity mismatch— induces a trade-off between a profiting from lending against additional liquidity risks. Bank lending reacts to monetary policy because its policy instruments alter this tradeoff.

We introduce this liquidity management problem and an interbank market into a tractable dynamic general-equilibrium model with rational profit-maximizing banks. Bank liquidity management is captured through a portfolio problem with non-linear returns that depend on the bank's reserve position. We use this to study the effects of shocks to banks affect that their aggregate lending and reserve holdings.

Implementing Monetary Policy. In the model, a Central Bank is equipped with various tools. A first set of instruments are discount rates and interests on reserves which influence the costs of being short of reserves. A second set are reserve requirements, open-market operations and direct lending to banks. This latter set of instruments, alters the effective aggregate amount of reserves in the system. All of these instruments carry real effects by tilting the liquidity management tradeoff. Macroeconomic effects result from their indirect effect on aggregate lending —and interest rates. However, as much as a Central Bank can influence bank decisions, the shocks associated with each hypothesis limit the power of monetary policy.

Testable Implications. The model delivers a rich set of descriptions. For individual banks, it explains the behavior of their reserve ratio, their leverage ratio and their dividend policies. Aggregating across banks provides descriptions for aggregate lending, interbank lending volumes and excess reserves. In general equilibrium, this yields predictions for interbank and non-interbank borrowing and lending rates. The model also describes other financial indicators for banks. For example, the return on loans, the return on equity, dividend ratios and book and market equity values. Moreover, the model also yields predictions for the evolution of the financial sector's equity. At the macroeconomic level, the model also explains the evolution of an endogenous money multiplier. We use this rich descriptions to identify the shocks associated with hypotheses 1-4. This allows us to shed light on which of the four hypotheses best fits the patterns we have seen since the 2008-2009 financial crisis.

Organization. The paper is organized as follows. The following section discusses where the model fits in the literature. Section 2, presents the model and some theoretical results. Section 3 presents a calibration exericse. We study the steady state and policy functions under that calibration in Section 5. We study transitional dynamics after an unexpected shocks associated with each hypothesis in section 6. Finally, in Section 7 we evaluate and discuss the plausibility each hypotheses.

1.1 Related Literature

There is a tradition in macroeconomics that dates back at to least Bagehot (1873) which stresses the importance of analyzing monetary policy in conjunction with banks. A classic mechanical framework to study policy with a full description of households, firms and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was abandoned from macroeconomics for many years. Until the Great Recession, the macroeconomic effects of monetary policy and its implementation through banks were analyzed independently.[2]

In the aftermath of the crisis, however, there have been numerous calls for constructing models with an explicit role for banks. [3] Some early steps have been taken by Gertler and Karadi (2009) and Curdia and Woodford (2009). In those models, shocks to bank equity —coupled with leverage constraints— propagate because they interrupt financial intermediation and increase spreads. The focus of those papers to explain the effects of policies that recapitalizes banks. In contrast, policy effects in our model arise from differences in the liquidity of assets.This relates our model to classic models of bank liquidity management and monetary policy.[4] Our contribution to bring the classic insights from the liquidity-management literature into a modern general-equilibrium dynamic model that can be used for the analysis of policy and banking crises.

We share common elements with recent work by Brunnermeier and Sannikov (2012). Brunnermeier and Sannikov (2012) also introduce inside and outside money into a dynamic macro model. Their focus is on the real effects of monetary policy through the redistributive effects of inflation.

[2] This was a natural simplification by the literature. In the US, the behaviour of banks did not seem to matter for monetary policy. In fact, the banking industry was among the most stable industries in terms of returns and the pass-through from policy tools to aggregate conditions had little variability.

[3] See for example Woodford (2010) and Mishkin (2011).

[4] Classic papers that study static liquidity management —also called reserve management— by individual banks are Poole (1968) and Frost (1971). There are many modern textbooks for practitioners that deal with liquidity management. For example, Saunders and Cornett (2010) and Duttweiler (2009) provide managerial and operations research perspectives. Many modern banking papers have focused on bank runs. See for example Diamond and Dybvig (1983), Allen and Gale (1998), Ennis and Keister (2009), or Holmstrm and Tirole (1998). Gertler and Kiyotaki (2013) is a recent paper that incorporates bank runs into a dynamic macroeconomic model.

In that sense, their model is closer to Gertler and Karadi (2009) and Curdia and Woodford (2009) because the distribution of wealth affects the extent of financial frictions. In our setup, outside money does not circulate outside the banking sector. Instead, central bank reserves only serve as an instruments to settle payments among banks.

The use of reserves for precautionary motives also places the model close to Stein (2012) and Stein et al. (2013). Those papers study the effects of an increase in the supply of reserves given an exogenous demand for short-term liquid assets. Williamson (2012) studies an environment where assets of different maturity have different properties as mediums of exchange. Our paper builds on earlier insights from two papers in the money-search literature. In particular, Cavalcanti et al. (1999) provide a theoretical foundation to our setup because reserves there emerge as disciplining device to sustain credit creation under moral-hazard and guarantee the circulation of deposits. In turn, we model an interbank market building on earlier work by Afonso and Lagos (2012). That paper models the Fed-funds market as an over-the-counter market where illiquidity costs arise endogenously. Our market for reserves is a simplified version of that model. Our paper also relates to Corbae and D'Erasmo (2013a, 2013a) who study a quantitative model of banking industry dynamics with heterogeneity. Their focus is on the strategic interaction between small and large fringe banks and on the effects of capital requirements on the level of concentration and competition in the banking industry.

2 The Model

The description of the model begins with a partial-equilibrium dynamic model of banks. The goal is to derive the supply of loans and the demand for reserves given an exogenous demand for loans, central bank policies and aggregate shocks. We later on derive a demand for loans and deposits from the real sector to close the model.

2.1 Environment

Time is discrete, is indexed by t and there is an infinite horizon. Each period is divided into two stages: a lending stage (l) and a balancing stage (b). The economy is populated by a continuum of competitive banks whose identity is denoted by z. Banks face an exogenous demand for loans and a vector of shocks that we describe later. There is an exogenous deterministic monetary policy chosen by the monetary authority which we refer to as the Fed. There are three types of assets, deposits, loans and central bank reserves. Deposits and loans are denominated in real terms. Reserves are denominated in nominal terms. Deposits play the role of a numeraire.

Banks. A bank's preferences over real dividend streams $\{DIV_t\}_{t\geq 0}$ are evaluated via an expected utility criterion:

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t U\left(DIV_t\right)$$

where $U\left(DIV\right) \equiv \frac{DIV^{1-\gamma}}{1-\gamma}$ and DIV_t is the banker's consumption at date t.[5] Banks hold a portfolio of loans, B_t, and central bank reserves, C_t, as part of their assets. Demand deposits, D_t, are their only form of liabilities. These holdings are the individual state variables of a bank.

Loans. Banks make loans during the lending stage. The flow of new loan issuances is I_t. These loans constitute a promise to repay the bank $I_t\left(1-\delta\right)\delta^n$ in period $t+1+n$ for all $n \geq 0$, in units of numeraire. Thus, loans promise a geometrically decaying stream of payments as in the Leland-Toft model —see Leland and Toft (1996). We denote by B_t the stock of loans held by a banks at time t. Given the structure of payments, the stock of loans has a recursive representation:

$$B_{t+1} = \delta B_t + I_t.$$

When giving out a loan, banks give out the borrower demand deposits which amount to $q_t^l I_t$ where q_t is the price of loans. Banks take q_t as given.Consequently, the bank's immediate accounting profits are $\left(1-q_t^l\right) I_t$.

A first key feature of our model is that bank loans are illiquid —they cannot be sold or bought— during the balancing stage.[6] The lack of a liquid market for loans in the balancing stage can be rationalized by several market frictions. For example, loans may be illiquid assets if banks specialize in particular customers or if they face agency frictions.[7]

Demand Deposits. Deposits are callable on demand and earn a real gross interest rate $R^D = \left(1+r^d\right)$. Behind the scenes, banks enable transactions between third parties. When they obtain a loan, borrowers receive deposits. This means that banks make loans —a liability for the borrower— by issuing their own liabilities —an asset ultimately held by a third party. This swap of liabilities enables borrowers to purchase of goods because deposits are effective mediums of exchange.The holder of those deposits after the transaction is made may, in turn, transfer those funds again to the accounts of others, make payments, and so on.

A second key feature of the environment is that banks face a random withdrawals during the balancing stage. In particular, a bank receives a withdrawal of $\omega_t D_t$ where $\omega_t \sim F_t\left(\cdot\right)$ with support

[5]Introducing curvature into the objective function is important. This assumption generates smooth dividends and slow-moving bank equity, as observed empirically. Similar preferences are often found in the corporate finance literature. One way to rationalize these preferences is through undiversified investors that hold bank equity. Alternatively, agency frictions may induce equity adjustment costs.

[6]The assumption that loans can be sold during the lending stage allows us to reduce the state space. In particular, it is not necessary to keep track of the composition but only the size of bank balance sheets thanks to this assumption. Dispending this assumption leads would require to keep track of a non-degenerate cross-sectional distribution for reserves, deposits and loans.

[7]Diamond (1984) and Williamson (1987) introduce specialized monitoring technologies. Holmstrom and Tirole (1997) build a model where bankers must hold a stake on the loans because of moral hazard. Finally, Bolton and Freixas (2009) introduce a differentiated role for different bank liabilities following from asymmetric information.

in $(-\infty, 1]$. Here, F_t is is the time-varying cumulative distribution for withdrawals.. For simplicity, we assume F_t is common to all banks.[8] When ω_t is positive (negative), the bank loses (receives) deposits. The shock ω_t captures the idea above that deposits are constantly circulating when payments are executed. The complexity of these transactions is approximated by the random process of ω_t. For simplicity, we assume that deposits do not leave the banking sector:

Assumption 1 (Deposit Conservation)**.** *Deposits remain within the banking system:* $\int_{-\infty}^{1} \omega_t dF_t(\omega) = 0, \forall t$.

This assumption implies that there are no withdrawals of reserves outside banking system.[9]

When a an amount of deposits is transferred across banks, the receptor bank absorbs a liability issued by another bank. Therefore, this transaction needs to be settled with the transfer of an asset. Since bank loans are illiquid, deposit transfers are settled with reserves. Thus, the illiquidity of loans induces a demand for reserves. The stock of deposits held by a bank is altered as borrowers repay their loans over time or as banks issue deposits to buy loans or reserves from other banks during the lending stage.

Reserves. Reserves are special assets. They are issued by the Fed and used by banks to settle transactions. Banks can buy or sell reserves during the lending stage. However, during the balancing stage, they can only borrow or lend reserves in the interbank market —details below. We denote by p_t be the price of reserves in terms of deposits. This term is also the inverse of the price level because deposits are in real terms.

By law, banks must hold a minimum amount of reserves within the balancing stage. In particular, the law states that $p_t C_t \geq \rho D_t(1 - \omega_t)/R^D$ where $\rho \in [0, 1]$ is a reserve requirement chosen by the Fed.[10] The case $\rho = 0$ requires banks to finish with a positive balance of reserves —banks cannot issue these liabilities. Given the reserve requirement, if ω_t is large, reserves may be insufficient to settle the outflow of deposits. In turn, banks that receive a large unexpected inflow will hold reserves in excess of the requirement.

To meet reserve requirements or allocate reserves in excess, banks can lend and borrow each other or from the Fed. These trades constitute the interbank market. As part of its toolbox, the Fed chooses two policy rates: a lending rate, r_t^{DW}, and a borrowing rate, r_t^{ER}. The borrowing rate —the interest on excess reserves —is the interest paid by the Fed to banks who deposit excess reserves at the Fed. The lending rate —or discount window— is the rate at which the Fed lends reserves to banks in deficit. These rates satisfy $r_t^{DW} > r_t^{ER}$ and paid within the period with

[8] We could assume that F is a function of the bank's liquidity or leverage ratio. This would add complexity to the bank's decisions but would not break any aggregation result. This tractability is lost if F_t is a function of the bank's size.

[9] This assumption can be relaxed without problem to allow for a demand for currency or system wide bank-runs at an extreme.

[10] Some operating frameworks compute reserve balances over a maintenance period. Bank choices in our model would correspond to averages over that period.

deposits.[11] If banks wish not to borrow or lend from the Fed, they can always do these operations with other banks.

Interbank Market. We assume that the interbank market for reserves is a directed over-the-counter (OTC) market.[12] This interbank market works the following way. After the realization of the withdrawal shocks, banks end with either positive or negative balances of relative to their reserve requirements. A bank that wishes to lend a dollar in excess can place a lending order. A bank that needs to borrow a dollar to patch its deficit can place a borrowing order. Thus, orders are placed on a per-unit basis as in Atkeson et al. (2012). Orders are directed to either the borrowing or lending sides of the market. After orders are directed to either side, a dollar in excess is randomly matched with a dollar in deficit. Once a match is realized, the lending bank can transfer the unit overnight. Banks use Nash bargaining to split the surplus of the dollar transfer.

In the bargaining problem that emerges, the outside option for the lending bank is to deposit the dollar at the Fed earning r^{ER}. For the bank in deficit, the outside option is the discount window rate r^{DW}. Because the principle of the loan —the dollar itself— is returned by the end of the period, banks bargain only about the net rates.We call that term the Fed funds rate, r^{FF}.

The bargaining problem for a match is:

Problem 1 (Interbank-market bargaining problem)

$$\max_{r^{FF}} \left(m_l r_t^{DW} - m_l r^{FF}\right)^{\xi} \left(m_b r^{FF} - m_b r_t^{ER}\right)^{1-\xi}.$$

In the objective function, m_l is the marginal utility of the bank lending reserves and m_b the corresponding term for banks borrowing.The first order condition of this problem is:

$$\frac{\left(r^{FF} - r_t^{ER}\right)}{\left(\left(1 + r_t^{DW}\right) - \left(1 + r^{FF}\right)\right)} = \frac{(1-\xi)}{\xi}.$$

This condition yields an implicit solution for r^{FF}. Since $(1-\xi)/(\xi)$ is positive, it is clear that r^{FF} will fall within the Fed's corridor of interest rates, $\left[r_t^{ER}, r_t^{DW}\right]$.

Now, not every order is necessarily matched. Instead, the probabilities that a lending orders meets borrowing order depends on the relative masses on either side of the market. In particular, γ^- is the probability that a deficit dollar is matched with a surplus dollar. We denote M^+ the mass of lending orders and M^- the mass of borrowing orders. The probability that a borrowing order finds a lending order is given by $\gamma^- = \min\left(1, M^+/M^-\right)$. Conversely, the probability that a lending order finds a borrowing order is $\gamma^+ = \min\left(1, M^-/M^+\right)$. The probabilities will affect the average cost of being short or long of reserves.

[11] This determines what in practice is know as the corridor system. In practice, there is an additional wedge between these two rates associated with the stigma from borrowing from the Fed.

[12] The idea that of modeling the interbank market is inherited from work by Afonso and Lagos (2012), Ashcraft and Duffie (2013) and Duffie (2012).

In this framework, there are a couple of conventions. First, if an order does not find a match, it does not lose the opportunity to lend (borrow) from (to) the Fed. Second, banks cannot places orders beyond its reserves needs or excess. Finally, interests are paid with deposits —this is just a convention since all assets are liquid during the lending stage.

Bank Equity and Payouts. The market value of equity is defined as $E_t = q_t B_t + p_t C_t - D_t$. This term evolves depending on prices and the realization of bank profits. Finally, profits are realized during and dividend payouts occur during the lending stage.

2.2 Timing, Laws of Motion and Bank Problems

This section shows expresses the model recursively. Thus, we drop time subscripts from now on. We adopt the following notation: If Z is a variable at the beginning of the period, $\tilde{Z}$ is its value by the end of the lending stage and the beginning of the balancing stage. Similarly, Z' is its value by the end of the balancing stage and the beginning of the following period. The aggregate state includes: all policy decisions by the Fed, the distribution of withdrawal shocks, F, and the demand for loans —to be specified below. This aggregate state is summarized in the vector X. We denote by V^l and V^b the bank's value function during the lending and balancing stages.

Lending Stage. Banks enter the lending stage with reserves, C, loans, B, and deposits, D. The bank chooses dividends, DIV, loan issuances, I, and purchases of reserves, φ.[13] The evolution of deposits follows:

$$\frac{\tilde{D}}{R^D} = D + qI + DIV + \varphi p - B(1-\delta).$$

Several actions affect this evolution. First, deposits increase when the bank credits qI deposits in the accounts of borrowers —or whomever they trade with. Second, banks pay dividends to shareholders with deposits. Third, the bank issues $p\varphi$ deposits to buy φ reserves. Finally, deposits fall by $B(1-\delta)$ because loans are amortized with deposits.

At the end of the lending stage reserves are the sum of the previous stock plus purchases of reserves, $\tilde{C} = C + \varphi$. Loans evolve according to $\tilde{B} = \delta B + I$. Banks choose $\{I, DIV, \varphi\}$ subject to these laws of motion and a capital requirement constraint. The capital requirement constraint imposes an upper bound, κ, on the stock of deposits relative to equity —marked-to-market. The bank's problem in the lending stage is:[14]

[13] The purchase of reserves φ occurs during the lending stage. Thus, this is a different flow than the flow that follows from loans in the interbank market which occurs during the balancing stage.

[14] On the technical side, the capital requirement constraint bounds the bank's problem and prevents a Ponzi-scheme. It is important to note that if the bank arrives to a node with negative equity, the problem is not well defined. However, when choosing its policies, the bank will make decisions such that it is guaranteed that it doesn't run out of equity. Implicitly, it is assumed that if it violates any constraint, the bank goes bankrupt.

Problem 2 *In the lending stage, banks solve:*

$$\begin{aligned} V^l(C,B,D;X) &= \max_{I,DIV,\varphi} U(DIV) + \mathbb{E}\left[V^b(\tilde{C},\tilde{B},\tilde{D};\tilde{X})\right] \\ \frac{\tilde{D}}{R^D} &= D + qI + DIV + p\varphi - B(1-\delta) \\ \tilde{C} &= C + \varphi \\ \tilde{B} &= \delta B + I \\ \frac{\tilde{D}}{R^D} &\leq \kappa\left(q\tilde{B} + p\tilde{C} - \frac{\tilde{D}}{R^D}\right); \tilde{B},\tilde{C},\tilde{D} \geq 0. \end{aligned}$$

Balancing Stage. During the balancing stage, banks place orders in the interbank market or at the Fed. Loans remain unchanged. However, the withdrawal $\omega\tilde{D}$ shifts the holdings of deposits and reserves. Let x be the reserve deficit. Given that withdrawals are settled with reserves, this deficit is:

$$x = \rho\underbrace{\left(\frac{\tilde{D}-\omega\tilde{D}}{R^D}\right)}_{\substack{\text{End-of-Stage} \\ \text{Desposits}}} - \underbrace{\left(\tilde{C}p - \frac{\omega\tilde{D}}{R^D}\right)}_{\substack{\text{End-of-Stage} \\ \text{Reserves}}}.$$

Given the structure of the OTC market described earlier, a bank with reserve surplus obtains a return of r^{FF} if it lends a unit of reserves in the interbank market and r^{ER} if it lends to the Fed. Since for any Nash-bargaining parameter $r^{FF} > r^{ER}$, banks always attempt to lend first in the interbank market. Thus, they place lending orders for every dollar in excess. In equilibrium, only a fraction γ^+ of those orders are matched and earn a return of r^{FF}. The rest earns the Fed's borrowing rate r^{ER}. Thus, the average return on excess reserves is:

$$\chi_l = \gamma^+ r^{FF} + \left(1-\gamma^+\right) r_t^{ER}$$

Analogously, banks in deficit try to first borrow from other banks before than from the Fed because $r^{FF} < r_t^{DW}$. The analogous cost of reserve deficits is:

$$\chi_b = \gamma^- r^{FF} + \left(1-\gamma^-\right) r_t^{DW}.$$

The difference between χ_l and χ_b is an endogenous wedge between the marginal value of excess reserves and costs of reserve deficits.The simple rule that characterizes orders in the interbank market problem, yields a value function for the bank during the balancing stage:

Problem 3 *The value of the Bank's problem during the balancing stage is:*

$$
\begin{aligned}
V^b(\tilde{C},\tilde{B},\tilde{D};\tilde{X}) &= \beta\mathbb{E}\left[V^l(C',B',D';X')|\tilde{X}\right] \\
D' &= \tilde{D}(1-\omega)+\chi(x) \\
B' &= \tilde{B} \\
x &= \rho\left(\frac{\tilde{D}-\omega\tilde{D}}{R^D}\right)-\left(\tilde{C}p-\frac{\omega\tilde{D}}{R^D}\right) \\
C' &= \tilde{C}-\frac{\omega\tilde{D}}{p}.
\end{aligned}
$$

Here χ represents a illiquidity cost, the return/cost of excess/deficit of reserves:

$$
\chi(x)=\begin{cases} \chi_l x \; if \;\; x\leq 0 \\ \chi_b x \;\; if \;\; x>0 \end{cases}
$$

We can collapse the problem of a bank for the entire period through a single Bellman equation that by substituting V^b into V^l:

Problem 4 *The bank's problem during the lending stage is:*

$$
\begin{aligned}
V^l(C,B,D,X) &= \max_{\left\{I,DIV,\tilde{C},\tilde{D}\right\}\in\mathbb{R}^4_+} U\left(DIV\right)... \\
& \quad +\beta\mathbb{E}\left[V^l\left(\tilde{C}-\frac{\omega'\tilde{D}}{p},\tilde{B},\tilde{D}+\chi\left(\frac{(\rho+\omega'\left(1-\rho\right))\tilde{D}}{R^D}-\tilde{C}p\right)\right);X'|X\right] \\
\frac{\tilde{D}}{R^D} &= D+qI+DIV_t+p\varphi-B(1-\delta) && (1) \\
\tilde{B} &= \delta B+I && (2) \\
\tilde{C} &= \varphi+C && (3) \\
\frac{\tilde{D}}{R^D} &\leq \kappa\left(\tilde{B}q+\tilde{C}p-\frac{\tilde{D}}{R^D}\right).
\end{aligned}
$$

The following section provides a characterization of this problem.

2.3 Characterization of Bank Problem

The recursive problem of banks can be characterized through a single state variable, the banks' equity value after loan amortizations, $E\equiv pC+B(1-\delta+\delta q)-D$.To show this, we clear out I and φ from the laws of motion of loans and reserves, equations (2) and (3), and substitute out I and φ into the law of motion for deposits, equation (1). After substitutions, the evolution of deposits takes the form of a budget constraint:

$$q\tilde{B}+\tilde{C}p+DIV-\tilde{D}=E.$$

In this budget constraint E are the value of the bank's available resources which are predetermined. We use an updating rule for E that depends on the bank's current decisions to express the bank's value function through a single-state variable:

Proposition 1 (Single-State Representation)

$$\begin{aligned}
V(E) &= \max_{\tilde{C},\tilde{B},\tilde{D},DIV\in\mathbb{R}_{+}^{4}} U(DIV)+\beta\mathbb{E}\left[V(E')|X\right] \\
E &= q\tilde{B}+p\tilde{C}+DIV-\frac{\tilde{D}}{R^{D}} \\
E' &= \left(q'\delta+1-\delta\right)\tilde{B}+p'\tilde{C}-\tilde{D}-\chi\left(\frac{(\rho+\omega'\left(1-\rho\right))\tilde{D}}{R^{D}}-\tilde{C}p\right) \\
\frac{\tilde{D}}{R^{D}} &\leq \kappa\left(\tilde{B}q+\tilde{C}p-\frac{\tilde{D}}{R^{D}}\right).
\end{aligned} \tag{4}$$

This problem resembles a standard consumption-savings subject to a leverage constraint. Dividends play the role of consumption, the bank's savings are allocated into loans, $\tilde{B}$, and reserves, $\tilde{C}$, and it can lever its position issuing deposits $\tilde{D}$.[15] Its choice is subject to a capital requirement constraint —the leverage constraint. The budget constraint is linear in E and the objective is homothetic. Thus, by the results in Alvarez and Stokey (1998), the solution to this problem exists, is unique, and policy functions are linear in equity. Formally,

Proposition 2 (Homogeneity—γ) *The value function $V(E;X)$ satisfies*

$$V(E;X)=v\left(X\right)E^{1-\gamma}$$

where $v\left(\cdot\right)$ satisfies

$$v\left(X\right)=\max_{\tilde{c},\tilde{b},\tilde{d},div\in\mathbb{R}_{+}^{4}} U(div)+\beta\mathbb{E}\left[v\left(X'\right)|X\right]\mathbb{E}_{\omega'}\left(e'\right)^{1-\gamma} \tag{5}$$

[15] From here on, we use the terms cash and reserves interchangeably. We acknoweledge that in the context of non-depositary instutitions, cash may mean holdings of desposits.

subject to

$$\begin{aligned}
1 &= q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D} \\
e' &= (q'\delta + (1-\delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - p\tilde{c}) \\
\frac{\tilde{d}}{R^D} &\leq \kappa\left(q\tilde{b} + \tilde{c}p - \frac{\tilde{d}}{R^D}\right)
\end{aligned}$$

In the expression above, $\mathbb{E}_{\omega'}$ is the expectation under F.

According to this proposition, the policy functions in (4) can be recovered from (5) by scaling them by equity, i.e., if c^* is the solution to (5), we have that $C = Ec^*$, and the same applies for the rest of the policy functions. An important implication is that two banks with different equity are scaled versions of a bank with one unit of equity. This also implies that the distribution of equity is not a state variable, but rather only the aggregate value of equity. Moreover, although there is no invariant distribution for bank equity —the variance of distribution grows over time—, the model yields predictions about the cross-sectional dispersion growth.

An additional useful property of the bank's problem is that it satisfies portfolio separation. In particular, the choice of dividends can be analyzed independently —through consumption savings problem with a single asset— from the portfolio choices between deposits, reserves and loans. We use the principle of optimality to break the Bellman equation (5) into two components.

Proposition 3 (Separation) *The value function $v(\cdot)$ defined in (5) solves:*

$$v(X) = \max_{div \in \mathbb{R}_+} U(div) + \beta \mathbb{E}[v(X')|X]\,\Omega(X)^{1-\gamma}(1-div)^{1-\gamma}. \tag{6}$$

Here $\Omega(X)$ is the value of the certainty-equivalent portfolio value of the bank. $\Omega(X)$ is the outcome of the following liquidity-management portfolio problem:

$$\begin{aligned}
\Omega(X) &\equiv \max_{w_b, w_c, w_d} \left\{\mathbb{E}_{\omega'}\left[R^B_X w_b + R^C_X w_c - \ w_d R^D_X - R^\chi_X(w_d, w_c)\ \right]^{1-\gamma}\right\}^{\frac{1}{1-\gamma}} \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa(w_b + w_c - w_d)
\end{aligned} \tag{7}$$

with $R^B_X \equiv \frac{q'\delta+(1-\delta)}{q}$, $R^C_X \equiv \frac{p'}{p}$, $R^\chi_X \equiv \chi((\rho + \omega'(1-\rho))\,w_d - w_c)$.

Once we solve the policy functions of this portfolio problem, we can reverse the solution for $\tilde{c}, \tilde{b}, \tilde{d}$ that solve (5) via the following formulas: $\tilde{b} = (1-div)\,w_b/q$, $\tilde{c} = (1-div)\,w_c/p$ and $\tilde{d} = (1-div)\,w_d R^D$.

The maximization problem that determines $\Omega(X)$ consists of choosing portfolio shares among assets of different risk, liquidity, and return. This problem is a liquidity-management portfolio problem with the objective of maximizing the certainty equivalent return on equity: $R^E(\omega'; w_b, w_d, w_c) \equiv R^B w_b + R^C w_c - R^D w_d - R^\chi(w_d, w_c, \omega')$. This portfolio problem is not a standard portfolio problem as it features non-linear returns that result from the joint determination of the return on reserves and deposits —given the illiquidity of loans and the reserve requirements. The return on loans is the sum of the coupon payment plus resale price: $R^B \equiv (\delta q + (1-\delta))/q$. The return on reserves and deposit components of the portfolios is determined jointly and depend on the withdrawal shock ω. These returns can be separated into an independent return and a joint return component that follows from the characteristics of the interbank market. The independent return on reserves is $R^C \equiv p'/p$. This return, captures the revaluation component —given that deposits are denominated in real terms. Hence, R^C is the inverse of inflation found monetary models.[16] The independent return of deposits is the interest on deposits, R_X^D. The joint return component of reserves and deposits is the potential cost —or benefit— of running out of reserves. This illiquidity cost is given by:

$$R_X^\chi(w_d, w_c, \omega') \equiv \chi\left(\left(\rho + (1-\rho)\,\omega'\right) w_d - w_c\right).$$

The risk and return of each assets varies with aggregate variables X. Thus, the solutions to the liquidity-management portfolio problem are time varying —outside steady state. The solution for the dividend rate and marginal values of bank equity satisfy a system of equations:

Proposition 4 (Solution for dividends and bank value) *Given the solution to the portfolio problem 7 the dividend ratio and value of bank equity are given by:*

$$div(X) = \frac{1}{1 + \left[\beta(1-\gamma)\mathbb{E}\left[v(X')\,|X\right]\Omega^*(X)^{1-\gamma}\right]^{1/\gamma}}$$

and

$$v(X) = \frac{1}{1-\gamma}\left[1 + \left(\beta(1-\gamma)\Omega^*(X)^{1-\gamma}\,\mathbb{E}\left[v(X')\,|X\right]\right)^{\frac{1}{\gamma}}\right]^{\gamma}.$$

The policy functions of banks determine the loans supply and demand for reserves. This concludes the partial equilibrium analysis of the bank's portfolio decisions. We now describe the demand for loans and the actions of the Fed.

2.4 Loan Demand

We consider a demand for loans of CES form increasing in the loan price —i.e., decreasing in the yield:

$$q_t = \Theta_t\left(I_t^D\right)^{\epsilon}, \epsilon > 0, \Theta_t > 0. \quad (8)$$

where ϵ is the inverse of the semi-elasticity of credit demand with respect to the price and Θ_t are possible demand shifters. In the quantitative analysis, consider shocks to Θ_t to evaluate the

[16] For most of the analysis we set $p' = p$, so that reserves yield a return equal to zero.

extent of shocks to aggregate demand for loans —hypothesis 4. In the appendix we obtain this demand from a static problem of a firm that needs to borrow working capital loans, but clearly there are many ways to obtain similar demand functions.

2.5 The Fed's Balance Sheet and its Operations

This section describes the Fed's balance sheet and how the Fed implements monetary policy. The Fed's financial structure is similar that of commercial banks with the exception that the Fed doesn't issue deposits as liabilities, but issues reserves instead. As part of its assets, the Fed holds commercial bank deposits, D_t^{Fed}, and private sector loans, B_t^{Fed}. As liabilities, the Fed issues M_t^0 reserves —high power money. The Fed's assets and liabilities satisfy the following laws of motion:

$$\begin{aligned} M0_{t+1} &= M0_t + \varphi_t^{Fed} \\ \frac{D_{t+1}^{Fed}}{R^D} &= D_t^{Fed} + p_t\varphi_t^{Fed} + (1-\delta)\, B_t^{Fed} - q_t I_t^{Fed} + \chi_t^{Fed} - T_t \\ B_{t+1}^{Fed} &= \delta B_t^{Fed} + I_t^{Fed}. \end{aligned}$$

The laws of motion for these state variables are very similar to the laws of motion for banks. Here, φ_t^{Fed} represents the Fed's purchase of deposits by issuing reserves to commercial banks. Its deposits are affected by the purchase or sale of loans, I_t^{Fed}, and the coupon payments of previous loans, $(1-\delta)\, B_t^{Fed}$. In addition, the Fed's deposists vary with, T_t, the transfers to or from the fiscal authority —the analogue of dividends.[17] Finally, χ_t^{Fed} represents the Fed's income revenue that stems from its participation in the Fed funds market:

$$\chi_t^{Fed} = \underbrace{r_t^{DW}\left(1-\gamma^-\right) M^-}_{\substack{\text{Earnings from}\\ \text{Discount Loans}}} - \underbrace{r_t^{ER}\left(1-\gamma^+\right) M^+}_{\substack{\text{Losses from}\\ \text{Interest Payments}\\ \text{on Excess Reserves}}}.$$

The balance sheet constraint of the Fed is obtained by replacing the law of motion of $M0_t$ and B_t^{Fed} into the law of motion for deposits:

$$p_t\left(M_{t+1}^0 - M_t^0\right) + (1-\delta)\, B_t^{Fed} + \chi_t^{Fed} = D_{t+1}^{Fed}/R^D - D_t^{Fed} + q_t\left(B_{t+1}^{Fed} - \delta B_t^{Fed}\right) + T_t. \tag{9}$$

Expressed in real terms, the Fed's balance sheet is the following identity:

$$\underbrace{E_t^{Fed}}_{\substack{\text{Fed}\\ \text{equity}}} = \underbrace{D_t^{Fed} + ((1-\delta) + q_t\delta)\, B_t^{Fed}}_{\text{Assets}} - \underbrace{p_t M_t^0}_{\text{Liabilities}}.$$

[17]In the Appendix we assume that this transfers are in turn lump-sum transfers to household. This assumption guarantees that these don't affect the demand for loans.

where E_t^{Fed} is the Fed's equity. The Fed has a monopoly over the supply of reserves, M_t^0 and alters this quantity through several operations.

Unconventional Open-Market Operations. Since there are no government bonds, only unconventional monetary operations are available.[18] An unconventional OMO involves the purchase of loans and the issuance of reserves. This operation has no effects on the stock of commercial bank deposits held by the Fed. To keep the amount of deposits constant, the Fed issues M0 buying deposits from banks but then sells those deposits to purchase loans. Thus, an unconventional OMO satisfies:

$$p_t \Delta \varphi_t^{Fed} - q_t \Delta I_t^{Fed} = 0$$

where $\Delta \varphi_t^{Fed}$ and ΔI_t^{Fed} are the changes in deposits and loans corresponding to the OMO. Thus, the effect on the supply of reserves is $\Delta M0_t = \frac{q_t}{p_t} \Delta I_t^{Fed}$.

Open-Market Liquidity Facilities. Liquidity facilities are swaps of liabilities of the Fed for deposits in a way that keeps $M0_t$ constant —a change in $\Delta \varphi_t^{Fed}$ without an offsetting ΔI_t^{Fed}.

Fed Profits and Transfers. In equilibrium, the Fed can return profits or losses. These operational results follow from the return on the Fed's loans and its profits/losses in the interbank market χ_t^{Fed}. We assume that the Fed transfers losses or profits immediately.

Fed Targets. For the analysis of the transitional dynamics we assume that the Fed chooses a target for the Fed funds rate r_t^{FF} —effectively setting r_t^{DW} and r_t^{ER}— and a value for $M0_t$. They choice is such that it maintains price stability: $p_t = p$. We alter this assumption when we study the Fed's actions during the crisis.

2.6 Market Clearing, Evolution of Bank Equity and Equilibrium

Bank Equity Evolution. Define $\bar{E}_t \equiv \int_0^1 E_t(z)\,dz$, the aggregate of equity in the banking industry. The equity of an individual bank evolves according to $E_{t+1}(z) = e_t(\omega) E_t(z)$. Here, $e_t(\omega)$ is the growth rate of bank equity of a bank with withdrawal shock ω.The measure of equity holdings at each bank is denoted by Γ_t. Since the model is scale invariant, we only keep track of the evolution of average equity, $\int_0^1 E_t(z)\,dz$, which by independence grows at rate $E_\omega[e_t]$.[19]

Loans Market. Market clearing in the loans market requires us to equate the loans demand I_t^D to the supply of new loans by banks and the Fed. Hence, equilibrium must satisfy: $I_t^D \equiv (q_t/\Theta_t)^{\frac{1}{\epsilon}} = B_{t+1} - \delta B_t + B_{t+1}^{Fed} - \delta^T B_t^{Fed}$.

[18]Incorporating Treasury Bills (T-Bills) and conventional open market operations into our model is relatively straight forward. If T-Bills are illiquid in the balancing stage, T-Bills and loans become perfect substitutes from banks perspective and the model becomes equivalent to our baseline model, with an additional market clearing condition for T-Bills. If T-Bills are perfectly liquid, we can show that banks that have a deficit in reserves sell first their holdings of Treasuries before accessing the interbank market. In the intermediate case where T-Bills are imperfect substitutes, the price of T-Bills would depend on the distribution of assets in the economy.

[19]A limiting distribution for Γ_t is not well defined unless one adapts the process for equity growth.

Money Market. Reserves are not leant outside the banking system —there is no use of currency in the model. This implies that the aggregate holdings of reserves during the lending stage must equal the supply of reserves by the Fed:

$$\int_0^1 \tilde{c}_t(z) E_t(z) dz = M_t^0 \longrightarrow \tilde{c}_t \bar{E}_t = M_t^0.$$

Interbank Market. The equilibrium conditions for the interbank market depend on γ^+ and γ^-, the probability of matches in the reserve market. These probabilities, in turn, depend on M^- and M^+, the mass of reserves in deficit and surplus. During the lending stage, banks are identical replicas of each other —scaled by equity. Thus, for every value of $E_t(z)$, there's an identical distribution of banks short and long of reserves. The shock that leads to $x = 0$ is $\omega^* = \left(\tilde{C}/p - \rho\tilde{D}\right)/(1-\rho)$. This implies that the mass of reserves in deficit in is given by:

$$M^- = \mathbb{E}\left[x(\omega)|\omega > \omega^*\right]\left(1 - F\left(\frac{\tilde{C}/p - \rho\tilde{D}}{(1-\rho)}\right)\right)\bar{E}_t$$

and the mass of surplus reserves is,

$$M^+ = \mathbb{E}\left[x(\omega)|\omega < \omega^*\right] F\left(\frac{\tilde{C}/p - \rho\tilde{D}}{(1-\rho)}\right)\bar{E}_t.$$

Money Aggregate. Deposits constitute the monetary creation by banks, $M_t^1 \equiv \int_0^1 \tilde{d}_t(z) E_t(z) dz$. The endogenous money multiplier is $\mu_t = \frac{M_t^1}{M_t^0}$.

The definition of equilibrium is:

Definition. *Given M_0, D_0, B_0, a competitive equilibrium is a sequence of bank policy rules $\left\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, div_t\right\}_{t\geq 0}$, bank values $\{v_t\}_{t\geq 0}$, government policies $\left\{\rho_t, D_{t+1}^{Fed}, B_{t+1}^{Fed}, M_{t+1}^0, T_t, \kappa_t, r_t^{ER}, r_t^{DW}\right\}_{t\geq 0}$, aggregate shocks $\{\Theta_t, F_t\}_{t\geq 0}$, measures of equity distributions $\{\Gamma_t\}_{t\geq 0}$, measures of reserve surpluses and deficits $\{M^+, M^-\}_{t\geq 0}$ and prices $\left\{q_t, p_t, r_t^{FF}\right\}_{t\geq 0}$, such that: (1) Given price sequences $\left\{q_t, p_t, r_t^{FedFunds}\right\}_{t\geq 0}$ and policies $\left\{\rho_t, D_t^{Fed}, B_{t+1}^{Fed}, M_{t+1}^0, \kappa_t, r_t^{ER}, r_t^{DW}\right\}_{t\geq 0}$, the policy functions $\left\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, div_t\right\}_{t\geq 0}$ are solutions to Problem 4. Moreover, v_t is the value in Proposition 3. (2) The money market clears: $\tilde{c}_t \bar{E}_t = M_t$. (3) The loan market clears: $I_t^D = \Theta_t^{-1} q_t^{\frac{1}{\epsilon}}$, (4) Γ_t evolves consistently with $e_t(\omega)$, (5) the masses $\{M^+, M^-\}_{t\geq 0}$ are also consistent with policy functions and the sequence of distributions F_t. All the policy functions of Problem 4 satisfy $X = xE$.*

2.7 Theoretical Analysis

This section provides a further characterization of the equilibrium. We first study decisions of banks through their portfolio problems. We then derive a liquidity premium earned by reserves.

To provide further intutiont, we study equilibria when banks are risk neutral and when there are no deposit withdrawals. We finally discuss a version of the zero-lower bound that emerges in this model.

Bank Portfolio Problem. Fix any given state X. To shorten the notation, we suppress the argument X from the returns and portfolio weights in Problem 7 leaving this reference as implicit. Given scale invariance, we focus on a bank with one unit of equity. We begin rewriting Problem 7 substituting the budget constraint:

$$\max_{\{w_d,w_c\}} \left(\mathbb{E}_{\omega'}\left[\left(\underbrace{R^B}_{\text{Return to Equity}} \underbrace{-\left(R^B - R^C\right) w_c}_{\text{Cash Opportunity Cost}} + \underbrace{\left(R^B - R^D\right)}_{\text{Arbitrage}} w_d - \underbrace{R^{\chi}\left(w_d, w_c, \omega'\right)}_{\text{Liquidity Cost}}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$$

subject to $w_d \leq \kappa$. The value of the problem is $\Omega(X)$. The objective of the problem is clear: if banks hold all their equity on loans, they obtain R^B. Issuing additional deposits yields an arbitrage opportunity when the spread between return on loans and return on deposits is positive. In turn, holding reserves bears an opportunity cost which equals the spread between the return on loans and the return on reserves. The choice of (w_d, w_c) jointly determines the liquidity cost given a shock ω'.

Liquidity Premium. In the context of our model, reserves have a liquidity premium relative to loans. To see this, we derive the first order conditions from the problem above. The conditions for w_c and w_d are respectively:

$$R^b - R^C = \frac{\mathbb{E}_{\omega'}\left[\left(R^E_{\omega'}\right)^{-\gamma} R^{\chi}_c\right]}{\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}} \tag{10}$$

$$R^D - R^C = \frac{\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}\left(1 - R^{\chi}_c\left(\rho + \omega'\left(1-\rho\right)\right)\right) + \mu}{\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}} \tag{11}$$

We can use these expressions to derive a liquidity premium, i.e., a difference between the return on loans and cash. Rearranging (10) and using the stochastic discount factor $m' \equiv \operatorname{div}\left(X'\right) \frac{R^E_{\omega'} E(1-\operatorname{div}(X))}{E \operatorname{div}(X)}$ we obtain:

$$\begin{aligned}
\underbrace{R^B - R^C}_{\text{Cash Opportunity Cost}} &= -\frac{\mathbb{E}_{\omega'}\left[m' \cdot R^{\chi}_c\left(w_d, w_c, \omega'\right)\right]}{\mathbb{E}_{\omega'}\left[m'\right]} \\
&= \underbrace{\mathbb{E}_{\omega'}\left[R^{\chi}_c\left(w_d, w_c, \omega'\right)\right]}_{\text{Direct Liquidity Effect}} - \underbrace{\frac{\mathbb{COV}_{\omega'}\left[m' \cdot R^{\chi}_c\left(w_d, w_c, \omega'\right)\right]}{\mathbb{E}_{\omega'}\left[m'\right]}}_{\text{Liquidity Risk Premium}}.
\end{aligned}$$

The return on reserves has two terms, a direct return, R^C, and an additional additive term that follows from the reduction in liquidity risk for the bank, $R^{\chi}_c\left(w_d, w_c, \omega'\right)$. The expression above says that the excess return on loans, $R^B - R^C$, the opportunity cost of holding reserves, equals the additional benefit of holding reserves $R_c^{\chi}(w_d, w_c, \omega')$ which is adjusted by the risk-premium associated with the withdrawal shocks. A similar expression can be derived for the spread between loans and deposits when the capital requirement is not binding.

$$\underbrace{R^B - R^D}_{\text{Arbitrage}} = \underbrace{\mathbb{E}_{\omega'}\left[R_d^{\chi}(w_d, w_c, \omega')\right]}_{\text{Direct Liquidity Effect}} - \underbrace{\frac{\mathbb{COV}_{\omega'}\left[m' \cdot \left(R_d^{\chi}(w_d, w_c, \omega') - R^D(\omega')\right)\right]}{\mathbb{E}_{\omega'}[m']}}_{\text{Liquidity Risk Premium}}.$$

This expression states that the arbitrage on loans by borrowing with deposits is limited by the direct expected increase in liquidity costs $\mathbb{E}_{\omega'}\left[R_d^{\chi}(w_d, w_c, \omega')\right]$ adjusted by the liquidity risk premium on deposits. Notice also, that when the capital requirement constraint is binding, there is a larger excess return between loans and deposits. This expression is similar to standard asset-pricing equations and can be potentially used to obtain measures of the increase in the perceived liquidity risk.

Risk-Neutral Bankers. From Liquidity

$$R_X^{\chi}(w_d, w_c, \omega') \equiv \chi\left(\left(\rho + (1-\rho)\,\omega'\right) w_d - w_c\right).$$

To make further progress in the analysis, we consider various polar cases to illustrate the mechanism in the model. To simplify notation we eliminate the subscript X that indexes the aggregate variables. An analysis of the liquidity management problem Ω yields the following proposition.

Proposition 5 (Theoretical Characterization) *We have the following proposition:*

1. *Binding Capital Requirement: If $R^D < R^C$ then the capital requirement constraint is binding.*

2. *Full/Partial insurance: If $R^D \leq R^C$ either the capital requirement constraint is binding or$\chi(\omega') \in [-\infty, 1]$*

3. *Non-monetary equilibrium: If $r^{DW} = r^{ER} = 0$, and $R^D > R^C$, we have that $C = 0$.*

4. *If $r^{DW} = r^{ER} = 0$, $R^B > R^D \;\Leftrightarrow$ the capital requirement constraint binds.*

The intuition for these results are as follows. First, if reserves provide a larger return for every possible realization of the withdrawal shock, banks will lever up against to their capital requirement constraint. The second result says that if the independent return on return on cash is at least as high as the return on deposits, banks will full insure as long as their capital requirement is not binding. If the capital requirement is binding, banks will expose to illiquidity losses for some states of nature. The third result implies that if the return on deposits is at least as high as the return on reserves, and there is no liquidity role for cash, the holdings of cash must equal zero.

3 Calibration

We use information from individual commercial bank Call Reports collected by the Federal Deposit Insurance Corporation (FDIC) to calibrate our model. We define a period in our model to be one quarter.

3.1 Dispersion of Deposit Growth

Calibrating our model requires an empirical counterpart for the random-withdrawal process for deposits, F_t. We use information from Call Reports to obtain an estimate of the evolution of deposit withdrawals. The Data Call Reports present balance-sheet information for all commercial banks in the US. Publicly available data spans all the quarters from 1990 until 2011. The Appendix, provides more details on how we construct the data we report in this section and other aspects of the data that are relevant for the paper.

We take the stance of calibrating F_t using information from the volatility of Total Deposits. To justify this choice, we need first to discuss Figure 1. The bars in the figure contain pre-crisis sample (2000Q1-2007Q4) information. The solid line that shifts to the left reports Great Recession (post-crisis) (2008Q1-2010Q4) information. The units of observation in Figure 1 correspond to quarter-bank observations on Total Deposits. The histogram plots the empirical frequencies of cross-sectional deviations of the growth rates of each bank quarter from the average growth rate for a given quarter in the sample.

In our model, banks feature the same growth rates of equity as the average bank unless they experience a withdrawal shock. Thus, in the model, a bank showing an increase in deposit growth higher than the mean is a bank experiencing an inflow of deposits, and viceversa. Hence, the deviations from average growth rates have a one-to-one map to the withdrawal shocks. Banks in our model have only one form of liability, demand deposits. In practice, commercial banks have other forms of liabilities that include bonds, long-term deposits (savings deposits) and other variable income securities. For this reason, we must be careful in our choice of the data counterpart of F_t. We choose total deposits as our counterpart.

In the Appendix, we show that Total Deposits in the data are substantially less volatility than Demand Deposits. Second, Total Deposits feature a trend which is consistent with the growth of bank liabilities whereas this is not the case for demand deposits. In practice, total deposits may include savings for short periods of time, or may also be removed from a bank at a cost.

Given the substantial variation in the volatility of total deposits observed in Figure 1 we believe this is a relevant measure to capture liquidity risk. Thus, we use this empirical histogram of quarterly deviations of today deposits, to calibrate F_t, the process for withdrawal shocks. Given the empirical density estimated, we fit a logistic distribution $F(\omega, \mu^\omega, \sigma^\omega)$ with $\mu = -0.00038204$ and $\sigma^\omega = 0.0246$. We conduct a Kolmogorov-Smirnov goodness-of-fit hypothesis test and do not

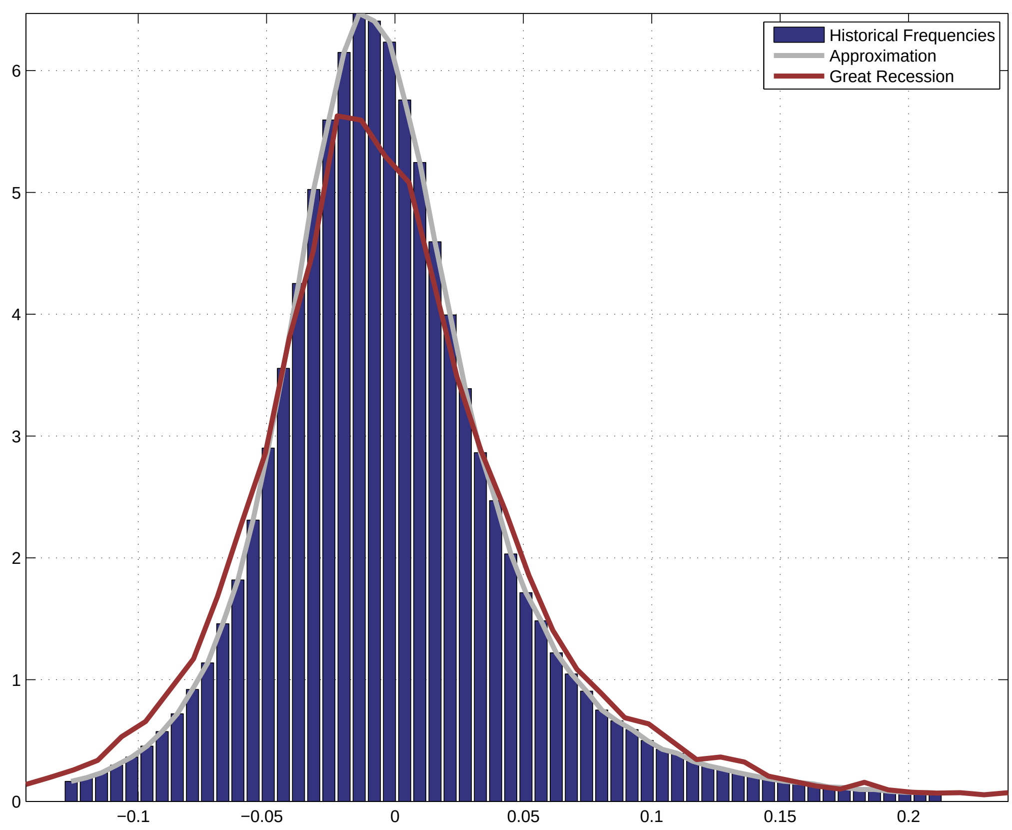


Figure 1: Cross-Sectional Distribution of Deviation from Cross-Sectional Average Growth Rates

reject that the empirical distribution is logistic with a 50 percent confidence.

Our model also predicts that the behavior of equity should be perfectly correlated with the behavior of deposits. We report this correlation in Appendix ??. We find that the correlation is positive, as suggested by our model significantly lower from one which is reasonable given that equity captures variations in the prices of securities, credit risks and operating costs that we do not include in the model. The data appendix discusses this point as well as the validity of the growth independence assumption. A final thing to note is that the variation in deposit growth has shifted to the left.

3.2 Parameter Values

The values of all parameters are listed in Table 1. We need to assign values to eleven parameters $\left\{\kappa, \beta, \delta, \gamma, \epsilon, \rho, r^{DW}, r^{ER}, r^{d}\right\}$. We set the capital requirement and the reserve requirement according to standard regulatory measures. In particular, we set $\kappa = 10$, which corresponds to a required Capital Ratio of 9 percent, and $\rho = 5$ percent. The risk aversion is set to $\gamma = 0.5$.

We set $r^{ER} = 0$, which is the pre-crisis interest rate on reserved paid by the Federal Reserve. The interest rate on discount window is set to be 2.5 percent annually, which delivers a Fed funds rate of 1.25 percent. The interest rate on deposits is set to $r^{d} = 0$. For now, we set $\delta = 0$ so that loans become one-period loans. The value of the loan demand elasticity given by the inverse of ϵ is set to 1.8, which is an estimate of the loan demand elasticity by Bassett et al. (2010). Finally, we set the discount factor so as to match a return on equity of 8 percent a year. This implies $\beta = 0.98$.

Table 1: Parameter Values

	Value
Capital requirement	$\kappa = 10$
Discount factor	$\beta = 0.985$
Risk aversion	$\gamma = 0.5$
Loan Maturity	$\delta = 0.$
Liquidity Requirement	$\rho = 0.05$
Loan Demand Elasticity	$1/\epsilon = 1.8$
Discount window rate (annual)	$r^{DW} = 2.5\%$

4 Liquidity Management and Portfolio Allocation

We start by analyzing the equilibrium portfolio at the stochastic steady state and investigate the effects of withdrawal shocks over the banks' balance sheets. The equilibrium portfolio corresponds to the solution of the Bellman equation 1 evaluated at the loan price that clears the loans market, according to condition 8, and the equilibrium probability of matching in the inter-bank market.

The left panel of Figure 2 shows the probability distribution of the deficit in cash in the balancing stage, and the penalty associated with each level of deficit. The penalty χ has a kink at zero, due to the fact that the discount window rate is larger than the interest rate on excess reserves. This asymmetry in the return for cash is a crucial feature of our model. Notice that the distribution of the cash deficit inherits the distribution of the withdrawal shock, as the cash deficit depends linearly on the withdrawal realization. Because in equilibrium, there is on average excess surplus, the distribution's mean is to the right of zero. That is, it is relatively more likely that a bank will end up with positive surplus.

The right panel of Figure 2 shows the distribution of equity growth as a function of the withdrawal realization. In equilibrium, the banks that experience positive withdrawal shocks will increase the size of their equity whereas those that experience negative withdrawal shocks will tend to shrink. Because of the non-linear penalty that inflicts relatively higher losses when adverse withdrawal shock hits the bank, the distribution of equity growth is skewed to the left. In particular, there is a fat tail with probabilities of losing about 2 percent of equity in a given period, while the probability of growing more than 1 percent in a period is close to nil.

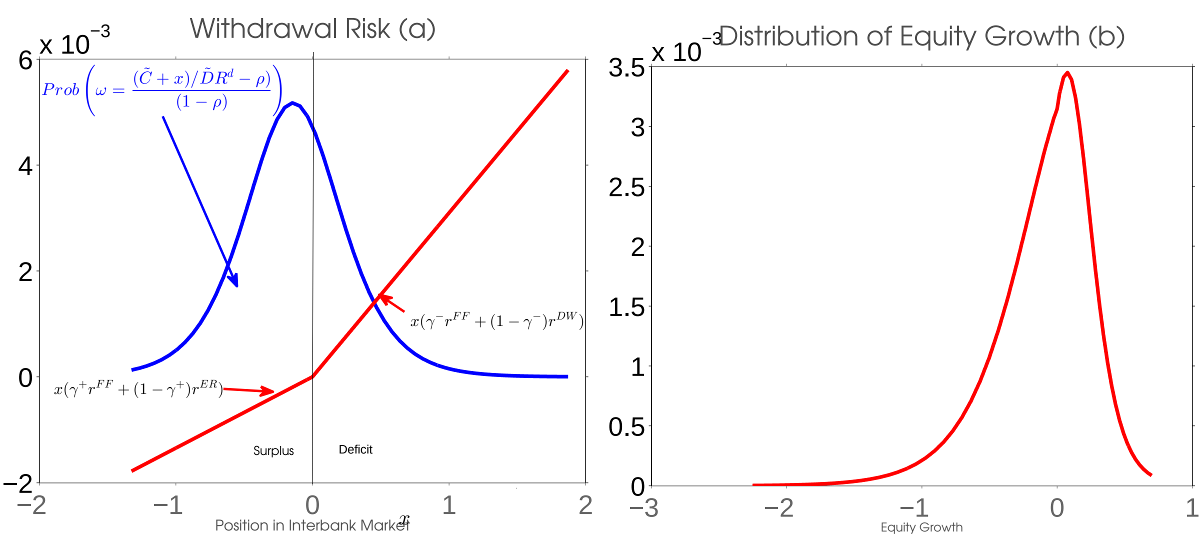


Figure 2: Portfolio Choices and Effects of Withdrawal Shocks

5 Policy Functions - Prices Given

We start with a partial equilibrium analysis of the model by showing banks policy functions at different loan prices. Figure 3 reports decisions for cash, loans, dividend, as well as liquidity and leverage ratios, the value of the asset portfolio, liquidity risk, expected returns and expected equity growth for different levels of loan prices. These policies correspond to the solution to the Bellman equation (4) for different values of loan prices q and leaving the probability of a match in the interbank markets at its steady state value. The solid dots in Figure 3 corresponds to the values associated with the steady state price of loans.

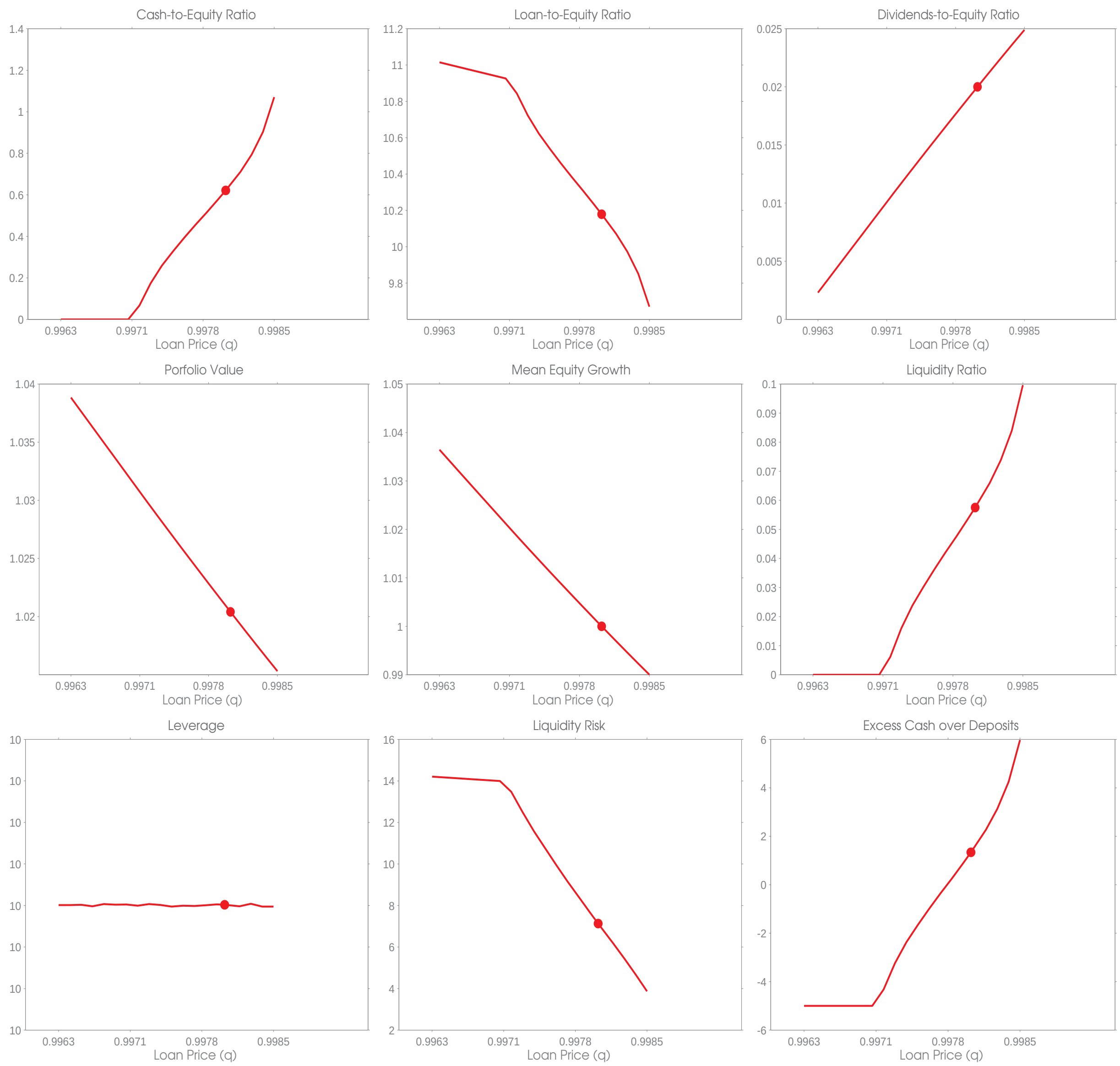


Figure 3: Policy function for different Loan Prices

A first observation that emerge from Figure 3 is that the supply of loans is decreasing in the loan price whereas dividends and cash ratios are increasing in the loan price. As loan prices decrease, loans become relatively more profitable leadings banks to keep a lower fraction of its assets in relatively low return assets, i.e., cash. Moreover, banks cut on dividend rate payments to allocate more funds to loan issuances and experience higher equity growth. The exposure to liquidity risk, measured as the standard deviation of the cost from rebalancing the portfolio χx, is also decreasing in loan prices, reflecting the fact that banks' asset portfolio becomes relatively more illiquid when loan prices decrease. Finally, notice that for a sufficiently low price of loans, the non-negativity constraint on cash becomes binding.

6 Transitional Dynamics

This section studies the transitional dynamics of the economy in response to different shocks associated with hypotheses 1-5. The shocks we consider are equity losses, a tightening of capital requirements, increases in the dispersion of withdrawals, changes in the discount window and interest on reserves, shocks that mimic disruptions in interbank markets and shocks to the for loans. Shocks are unanticipated upon arrival at $t = 0$ but their paths are deterministic for $t > 0$. In each exercise, the Fed's has a zero inflation target.

6.1 Equity Losses

We begin with a shock that translates into a sudden unexpected decline in bank equity. This shock captures an unexpected rise in non-performing loans, security losses or to off-balance sheet items that are left out of the model.[20] Figure 4 illustrates the response of the economy to equity losses of 2 percent. Recall that all bank policy functions are linear in equity. Thus, holding prices fixed, this shock leads to a proportional 2 percent decline in the quantity of loans supplied and reserve holdings. However, the contraction in loans supply does generate a drop in loan prices on impact —a movement along the demand for loans. This reduction further reduces the supply of credit: the capital requirement constraint applies to marked-to-market equity, which falls with prices. The reduction in q_t also leads to an increase in loan returns throughout the transition. As a consequence of the higher profitability on loans, reserve holdings fall relatively more than loans. Banks shift their portfolios towards loans desireably expose themselves to more liquidity risk. The overall return to the banks portfolio also increases. With this, dividends fall as banks their opportunity cost increases. The increase in bank returns and lower dividends leads to a gradual recovery of original equity losses. As equity recovers, the economy converges to the initial steady state where the returns to equity are paid as dividends. The transition is quick. The effects of the shock cannot be observed after six quarters.

[20] One way to incorporate this explicitly in the model would be to consider an aggregate shock to the default rate on loans. To the extent that equity is the only state variable, the analysis of the transitional dynamics is analogue to studying the evolution of the model with a richer structure on loans.

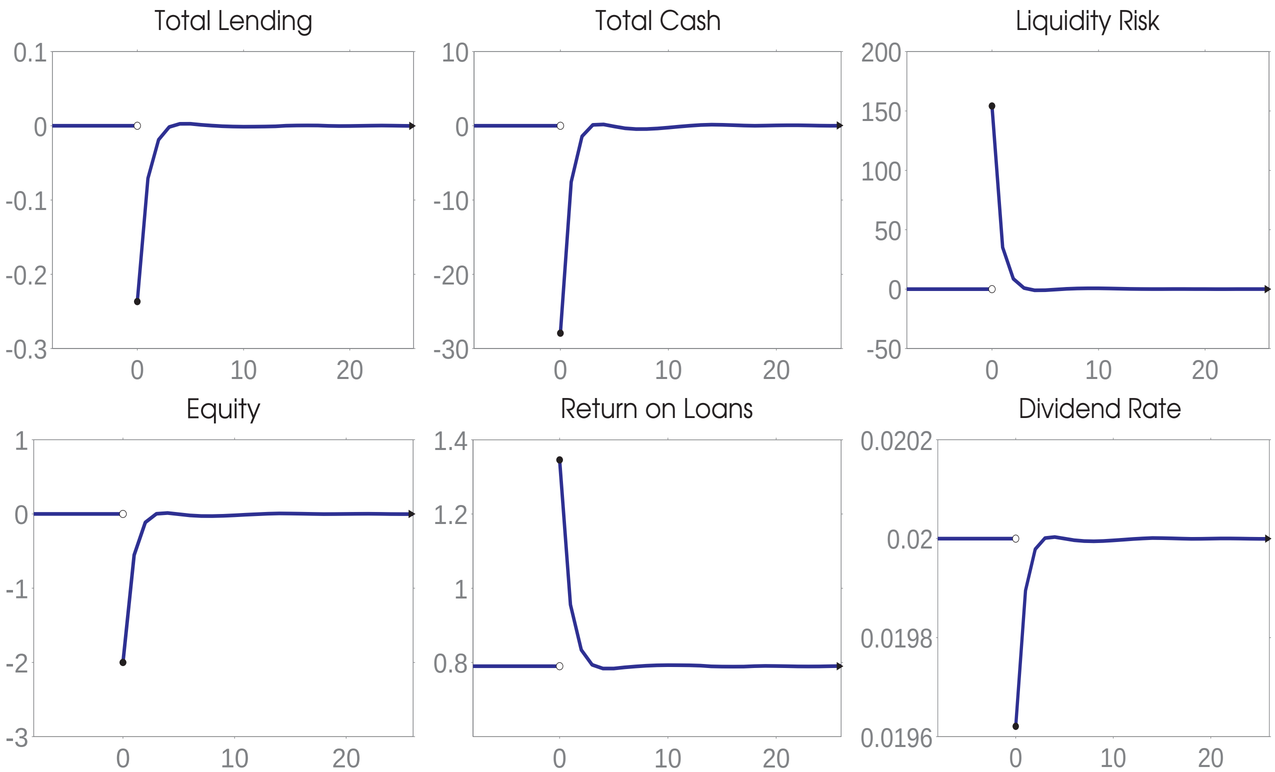


Figure 4: Impulse Response to Equity Losses

6.2 Capital Requirements

The effects of sudden and permanent tightening of capital requirements, i.e., a reduction in κ are shown in Figure 5. The shock is a 10 percent decrease in κ which is associated with a 1 percent increase in the capital ratio of banks for the calibrated level of leverage. The short-run behavior of the transition is very similar to the behavior after equity losses. As with equity losses, the contraction in capital requirements reduces the supply of loans in the amount of the decrease in κ —κ is bidinding in steady state. In this case, there is "second round" tightening in the capital requirement constraint as q_t decreases.

On impact, reserve holdings fall for two reasons. First, the increase in the return on loans increases that follows from the contraction in loans supply increases the opportunity cost of holding reserves. Second, because banks have lower leverage, this reduces the liquidity risk premium —an effect not present in the polar cases analyzed before. Having a lower liquidity risk premium implies that banks expose to more liquidity risk. The increased return on loans leads banks to reduce dividends, which contribute to the build up of their equity over time. In the long-run, equity converges to a higher value such that the return to the banks portfolio is the same as in the initial steady state. The holdings of reserves relative to loans change at the new steady state because of the reduction in the liquidity risk premium. The return on loans converges to a lower value, similarly, as the risk-premium falls with less leverage.

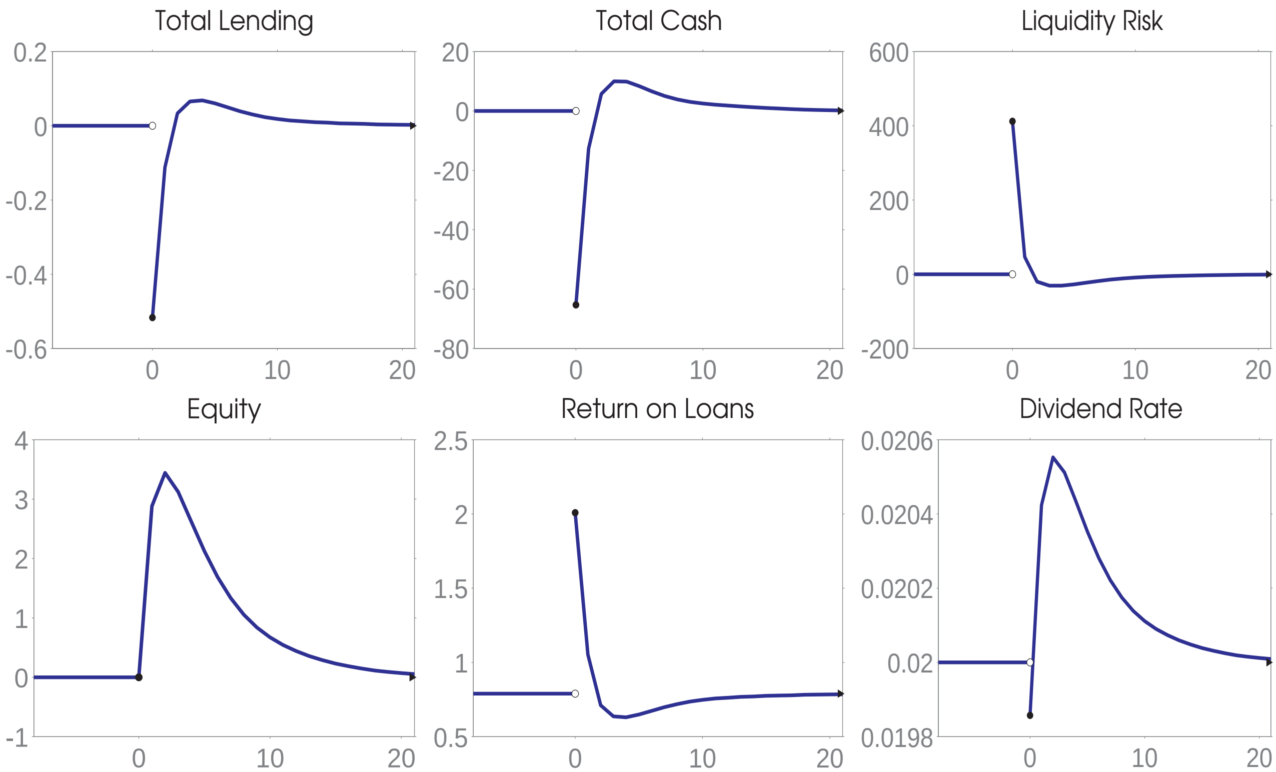


Figure 5: Impulse Response to a Tightening in Capital Requirements

6.3 Withdrawal Uncertainty (Bank Runs)

Here we study an increase in the probability of a bank-run of 5 percent. In other words, we consider an increase in the probability that all the deposits are withdrawn from a bank —that is $\omega = 1$. We maintain the assumption that deposits are not withdrawn from the banking system as a whole.[21] We assume that the increase in this probability follows a deterministic AR(1) process such that the shock lives for about 2 years. The effects of this shock are illustrated in Figure 6.

The risk of a bank-run generates an increase in liquidity risk, leading banks to hoard cash. Notice that liquidity risk is still about 3 times as large as in the initial steady state. Notice that this occurs eventhough banks hold more reserves and the Fed supplies reserves to keep the price constant. The reason for the increase in liquidity costs follows from the asymmetry in the liquidity cost function and the response of the return on loans. Indeed, the increase in liquidity risk spills over to the loans market. Higher liquidity costs induce a decline in the supply of loans. In equilibrium, this leads to an increase in the price of loans and a decline in the aggregate volume of lending —a movement along the loans demand schedule. The increase in the return weakens the desire to hold reserves to reduce the exposure to liquidity risks.

In tandem, banks respond to the risk of a bank-run by cutting dividend payments. Although higher liquidity costs are associated with lower returns, the contraction in loans supply generates

[21] Thus we adjust F accordingly.

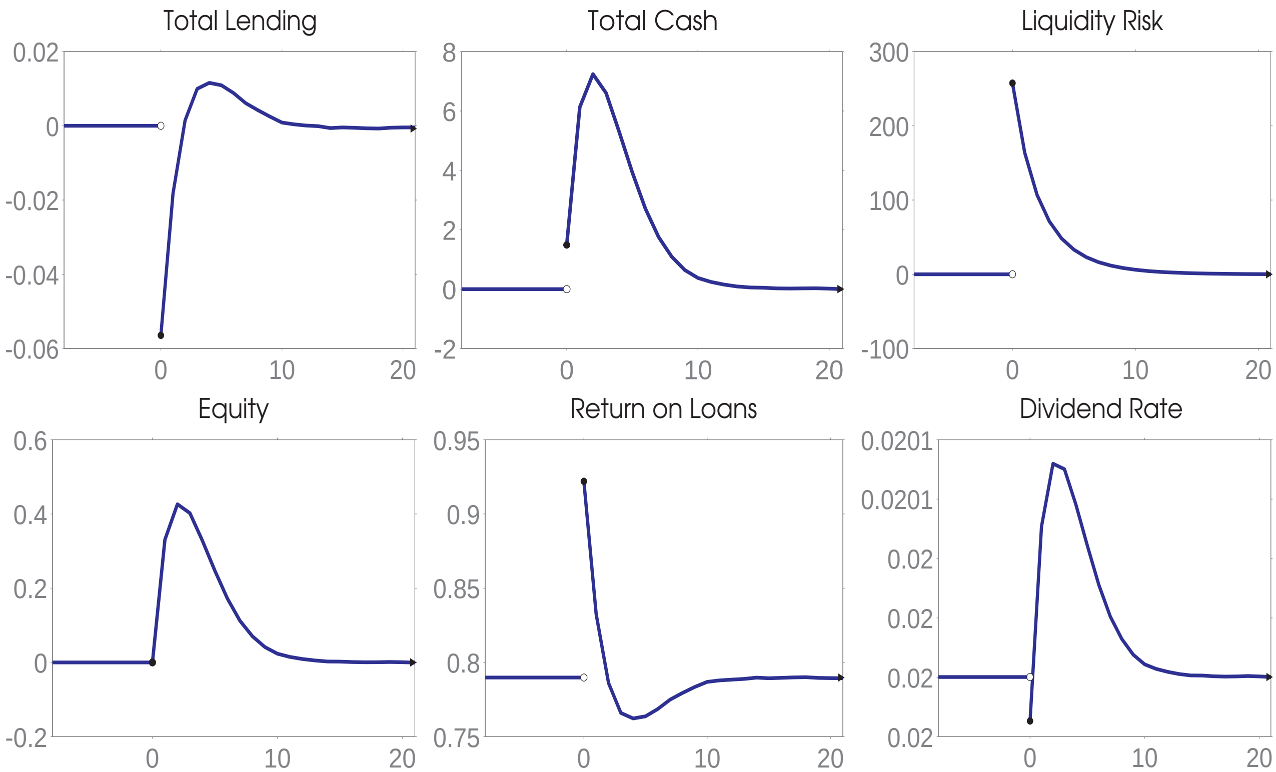


Figure 6: Impulse Response to a Bank-Run Shock

an increase in expected bank returns that leads to an increase in bank returns. The reduction in dividend payments and returns on equity lead to an increase in equity over time. As equity grows, this mitigates the fall in lending ratios. Eventually lending raises above the steady state value because the effect on bank equity eventaully compensates for the portfolio effect. Thus, once the bank-run shock practically dies, the banking sector has a level of equity which is above the steady state value.

6.4 Interbank Market Shutdown

The effects of a shutdown in the interbank market can be studied through a shock that forces the probability of a match in the interbank markets to zero —under this shock, the interbank market is not in equilibrium.[22] Hence, hence reserves are borrowed or lent only to the Fed. In particular, banks that face a reserve deficit borrow directly from the Fed at r^{DW}. Thus, the shock increases liquidity costs. The effects of the interbank market freeze is shown in Figure 7. Overall, the effects are similar to the bank-run shock we study above.

[22]A recent macroeconomic model of endogenous interbank market freezing due to asymmetric information with one-period lived banks is Boissay et al. (2013).

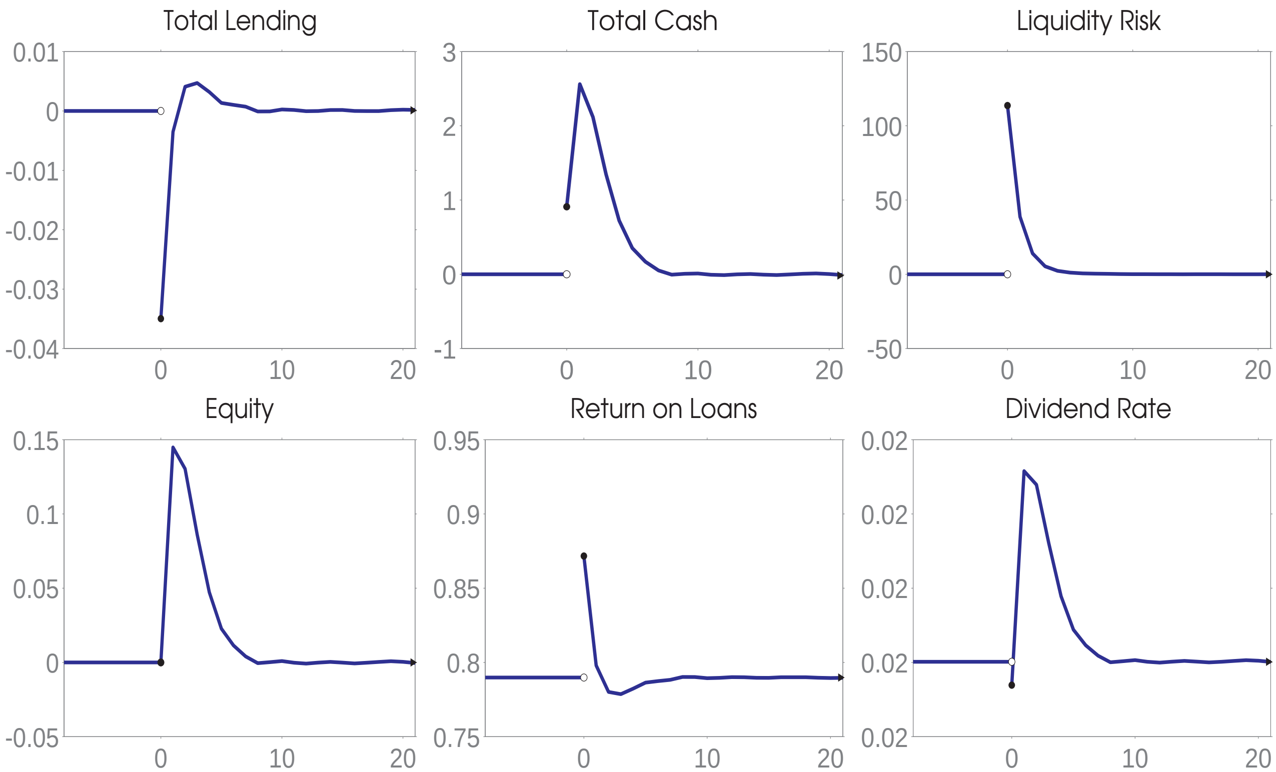


Figure 7: Impulse Response to Freezing in Inter-bank Markets

6.5 Credit Demand

The effects of negative credit demand shock are captured through a decline in Θ_t, the log-intercept of the loans demand. In the microfoundation we provide in the appendix, this shock captures a decline in total factor productivity or an increase in labor market distoritions that reduce the demand for working capital by firms.[23] Figure 8 illustrates the effects of a negative temporary shock to credit demand. We assume the shock follows deterministic AR(1) process that lasts for about 7 years.

The effects of credit demand shocks contrast sharply with the effect of the shocks considered above because all of the shocks above cause a contraction in the supply of loans. On impact, the shock causes a decline in the return to loans. This leads banks to shift their portfolios towards reserves as the opportunity cost of making loans is lower. As a result, the liquidity risk drops almost entirely. Initially, banks also respond paying higher dividends due to the overall decline in the return of their portfolios. The reduction in returns and increments in dividend payments brings equity significally below steady state. As the credit demand shock dies out —around a year and a half later—, the economy follows a similar transition as with the shock to equity slowly increasing lending rates and reducing dividend rates until equity reaches its steady state level.

[23]More broadly, a credit demand shock could also have a financial origin, e.g., a decline in the value of firm's or household collateral that limit their ability to borrow. Moreover, this shock is also isomorphic to a reduction in credit-quality if banks are well diversified.

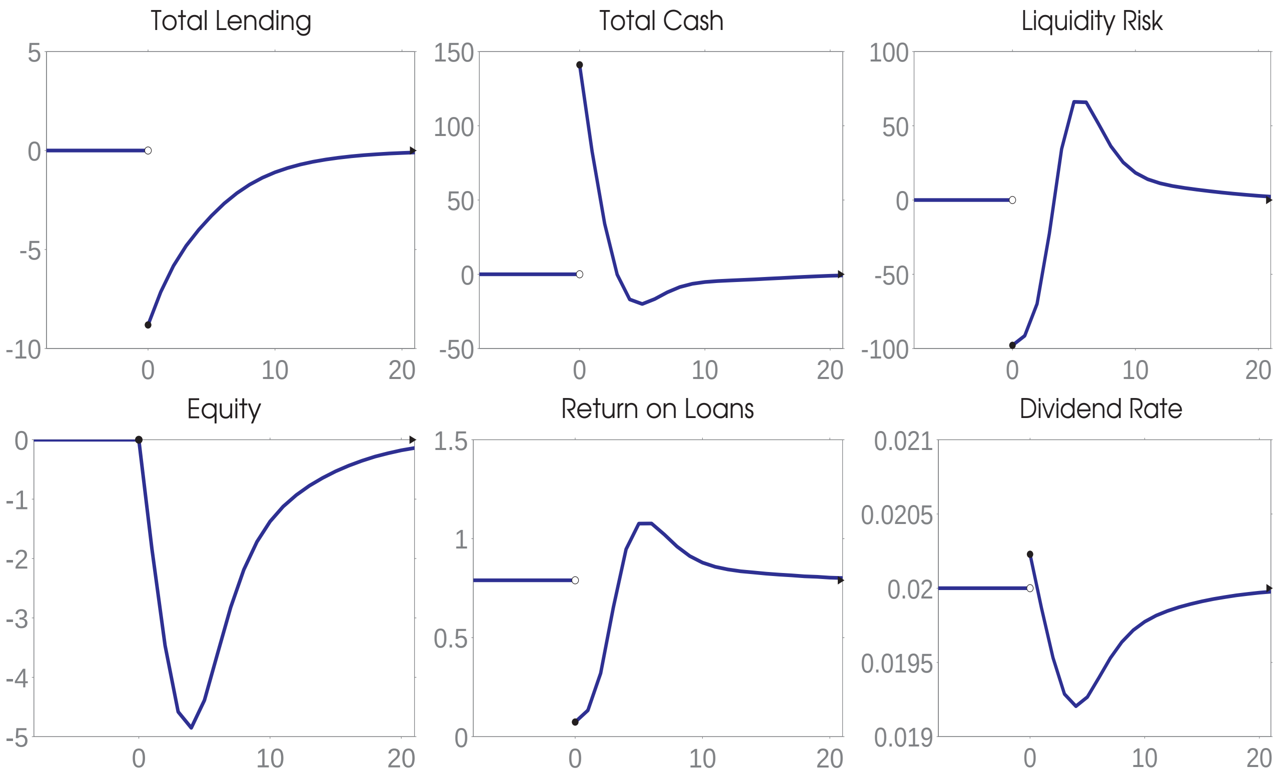


Figure 8: Impulse Response to Credit Demand Shock

6.6 Discount Window and Interest on Reserves

We now consider the effects of interest rate policy shocks. In the experiment, we consider a shock to the discount window rate of 100bps —in annualized terms— but open market operations keep inflaiton constant. The effects of a rise in the discount window rate are shown in Figure 9. Banks respond to the increase in the discount window rate by reducing lending. The policy effects are very similar to the effects of shocks that increase the liquidity costs.

The final shock we consider in our impulse responses is a shock to the interest on excess reserves that raises from 0 to 10bps. This shock is in line with the recent shift in policy by Federal Reserve. The effect of this policy are illustrated in Figure 10. As the central bank pays interest on excess reserves, cash becomes relatively more attractive, and banks reallocate their portfolio from loans to cash.[24]

[24]Notice that liquidity risk does not decline despite the increase in cash holdings by banks. This occurs because the increase in the interest rate on excess reserves, leads to larger differences in returns between banks on surplus and deficit.

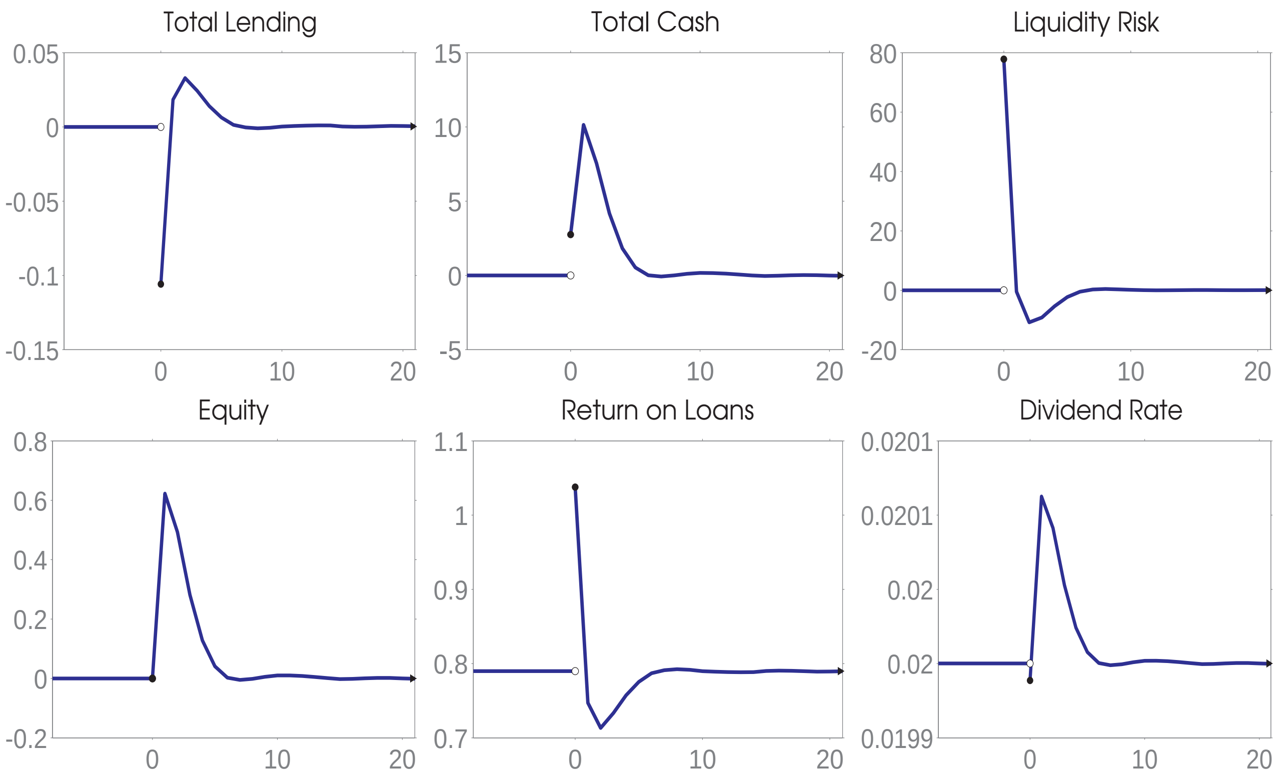


Figure 9: Impulse Response to Rise in Discount Window

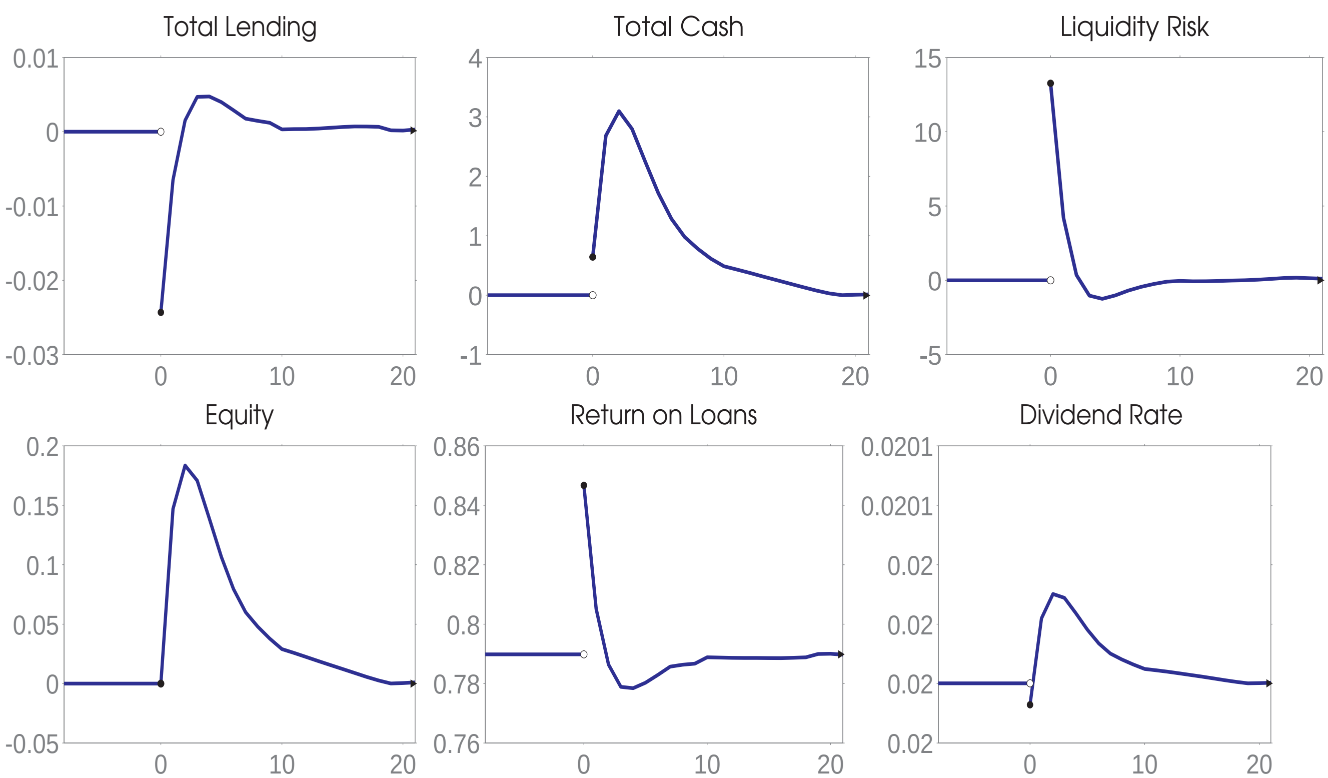


Figure 10: Impulse Response to Interest on Reserves

7 US Financial Crises

[INCOMPLETE]

This section seeks to explore the possible driving forces behind the pattern of the banking sector during the US financial crisis. Building on the analysis from the previous section, we explore if there are plausible shocks within our model that can lead to contraction with similar features to the one experienced by the US economy.

The first four panel of Figure 11 plots the evolution of leverage, liquidity, return on assets, dividend payments, presented as simple and weighted averages according to the level of assets. In addition, the two inferior panels show the VIX and bank lending. As has been extensively documented, during the financial crises there has been a significant increase in overall volatility and losses in financial assets, decline in bank lending, reduction in dividend payments, sharp deleveraging and a substantial increase in holdings of liquid assets.

8 Conclusions

The majority of macroeconomic models tha study monetary policy have developed independently from models in the banking literature —e.g. Diamond and Dybvig (1983). In particular, there is no modern macroeconomic framework to study the implementation of monetary policy through the banking system and, in particular, through the liquidity management of banks —as occurs in practice.[25] The crisis, however, has revealed an urgency to have a model that allows to study the conjunction between monetary policy and banking.

This paper is a first attempt to fill this gap. We used this model to understand the effects of various shocks on the power of monetary policy. In our application, we argued that a combination of an interbank market freez first, and a later decline in the demand for loans seemed the most plausible story to explain the increase in the holding of reserves and the decline in lending post 2008. We argue that this combination of shocks is suggestive of phenomenon by which an initial contraction in the supply of loans beguets a following contraction in the effective loans demand.

We believe the model can be used to answer a number of related questions. Among these, the model can be used to study the Fed's exit strategy from quantitative easing. It can also be used to analyze changes in policy tools. Moreover, the model can be used to estimate the fiscal costs of the Fed's policies of recent years. Similarly, the model can be used to analyze the effects of the policies recently undertaken by the European Central Bank. For open economies, the model can be extended to analyze interventions in the exchange rate market. An extension that breaks aggregation may allow to study the cross-sectional responses of banks depending their liquidity

[25] In doing so, the profession has lacked an explicit modeling of monetary policy through the financial system, and for good reasons. For a long time it did not seem to make much difference, monetary policy seemed to be carried with ease. In fact, the banking industry showed very stable indicators for equity growth, leverage, dividends and interest premia. Thus, the financial sector can appeared irrelevant for monetary policy of a long stretch of time.

and leverage ratios. We hope that the answers to this questions will provide a guide for policy in practice.

References

Afonso, Gara and Ricardo Lagos, "Trade Dynamics in the Market for Federal Funds," 2012. Mimeo, NY Fed. 5, 8

Allen, Franklin and Douglas Gale, "Optimal Financial Crises," *The Journal of Finance*, 1998, *53* (4), 1245–1284. 4

Alvarez, Fernando and Nancy L Stokey, "Dynamic Programming with Homogeneous Functions," *Journal of Economic Theory*, 1998, *82* (1), 167 – 189. 12, 47

Atkeson, Andrew G, Andrea L Eisfeldt, and Pierre-Olivier Weill, "The market for otc credit derivatives," 2012. Working Paper, UCLA. 8

Bagehot, Walter, *Lombard Street: A description of the money market*, Wiley, 1873. 4

Bassett, William F, Mary Beth Chosak, John C Driscoll, and Egon Zakrajsek, "Identifying the macroeconomic effects of bank lending supply shocks," 2010. Mimeo, Federal Reserve Board. 22

Boissay, Frédéric, Fabrice Collard, and Frank Smets, "Booms and systemic banking crises," 2013. Mimeo, European Central Bank. 28

Bolton, Patrick and Xavier Freixas, "Corporate Finance and the Monetary Transmission Mechanism," *The Review of Financial Studies*, 2009, *19* (3), 829–870. 6

Brunnermeier, Markus K. and Yuliy Sannikov, "The I-Theory of Money," 2012. Mimeo, Princeton. 4

Cavalcanti, Ricardo, Andres Erosa, and Ted Temzelides, "Private Money and Reserve Management in a Random Matching Model," *Journal of Political Economy*, 1999, *107* (5), pp. 929–945. 5

Corbae, Dean and Pablo D'Erasmo, "Capital Requirements in a Quantitative Model of Banking Industry Dynamics," 2013. 5

_ and _ , "A Quantitative Model of Banking Industry Dynamics," 2013. Mimeo, Wisconsin. 5

Curdia, Vasco and Michael Woodford, "Conventional and Unconventional Monetary Policy," 2009. Mimeo, NY Fed. 4, 5

Diamond, Douglas W., "Financial Intermediation and Delegated Monitoring," *The Review of Economic Studies*, 1984, *51* (3), pp. 393–414. 6

Diamond, Douglass W. and Phillip H. Dybvig, "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy*, 1983, *91* (3), 401–419. 4, 32

Duttweiler, Rudolf, *Managing Liquidity in Banks: A Top Down Approach*, John Wiley & Sons Ltd, 2009. 4

Ennis, Huberto M and Todd Keister, "Bank runs and institutions: The perils of intervention," *The American Economic Review*, 2009, *99* (4), 1588–1607. 4

Frost, Peter A., "Banks' Demand for Excess Reserves," *Journal of Political Economy*, 1971, *79* (4), pp. 805–825. 4

Gertler, Mark and Nobuhiro Kiyotaki, "Banking, Liquidity and Bank Runs in an Infinite Horizon Economy," 2013. Mimeo, NYU. 4

_ **and Peter Karadi**, "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 2009. 4, 5

Gurley, John G. and Edward S. Shaw, *Money in a Theory of Finance* 1964. 4

Holmstrm, Bengt and Jean Tirole, "Private and Public Supply of Liquidity," *The Journal of Political Economy*, 1998, *106* (1), pp. 1–40. 4

Holmstrom, Bengt and Jean Tirole, "Financial Intermediation, Loanable Funds, and the Real Sector," *The Quarterly Journal of Economics*, 1997, *112* (3), pp. 663–691. 6

Leland, Hayne E. and Klaus Bjerre Toft, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *The Journal of Finance*, 1996, *51* (3), pp. 987–1019. 6

Mishkin, Frederic S., "Over the Cliff: From the Subprime to the Global Financial Crisis," *The Journal of Economic Perspectives*, 2011, *25* (1), pp. 49–70. 4

Poole, William, "Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy," *The Journal of Finance*, 1968, *23* (5), pp. 769–791. 4

Saunders, Anthony and Marcia Cornett, *Financial Institutions Management: A Risk Management Approach*, McGraw-Hill/Irwin Series in Finance, Insurance and Real Estate, 2010. 4

Stein, Jeremy C., "Monetary Policy as Financial Stability Regulation," *The Quarterly Journal of Economics*, 2012, *127* (1), pp. 57–95. 5

Stein, Jeremy, Robin Greenwood, and Samuel Hanson, "A Comparative-Advantage Approach to Government Debt Maturity," 2013. Mimeo. 5

Williamson, Stephen, "Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach," 2012. forthcoming, American Economic Review, 2012. 5

Williamson, Stephen D., "Costly Monitoring, Loan Contracts, and Equilibrium Credit Rationing," *The Quarterly Journal of Economics*, 1987, *102* (1), pp. 135–146. 6

Woodford, Michael, "Financial Intermediation and Macroeconomic Analysis," *The Journal of Economic Perspectives*, 2010, *24* (4), pp. 21–44. 4

A Additional Figures

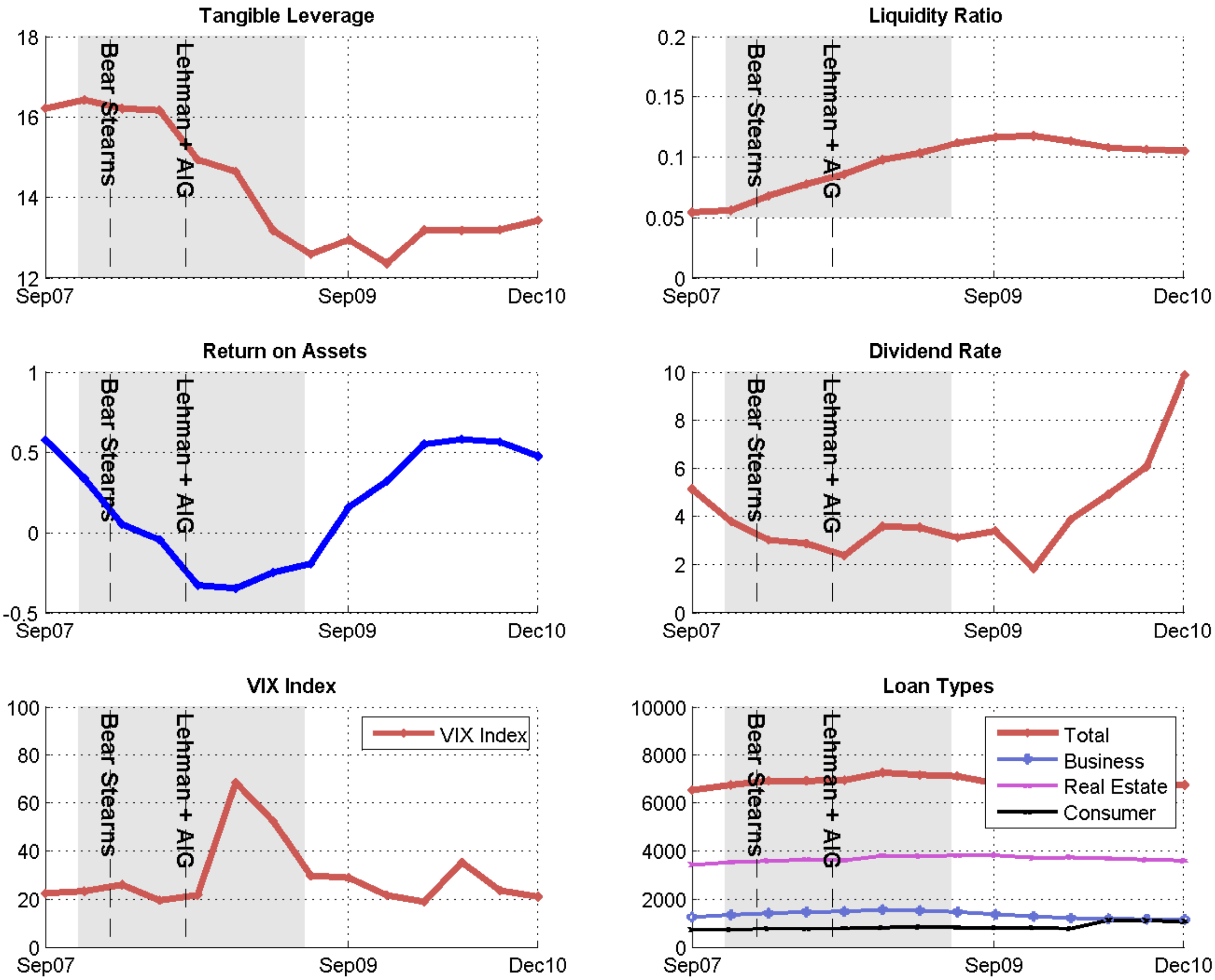


Figure 11: Evolution of Key Ratios for the banking sectors during the last decade.

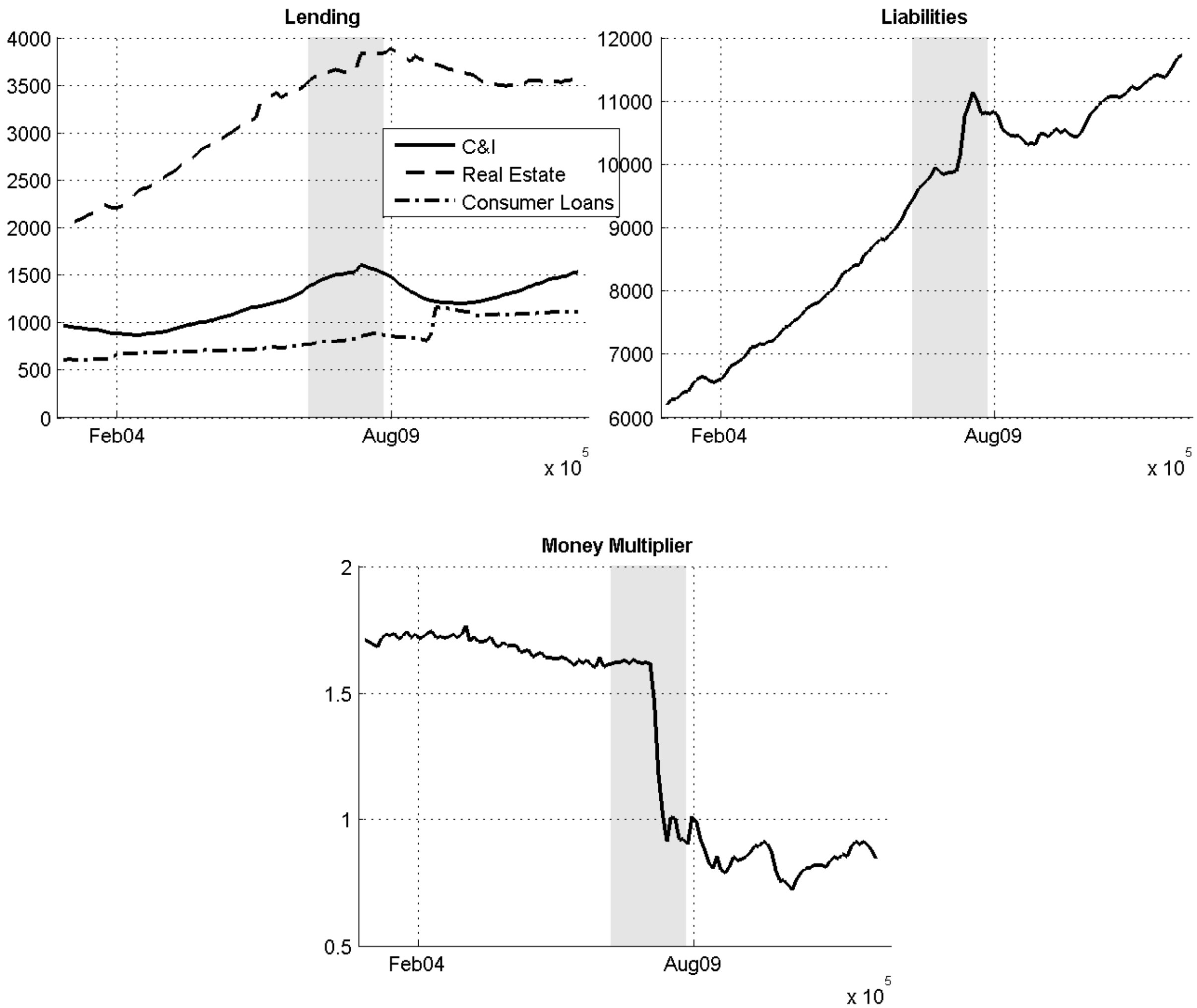


Figure 12: **Commercial Bank Assets: 2002-2012.** The figure shows several measures of commercial bank lending.

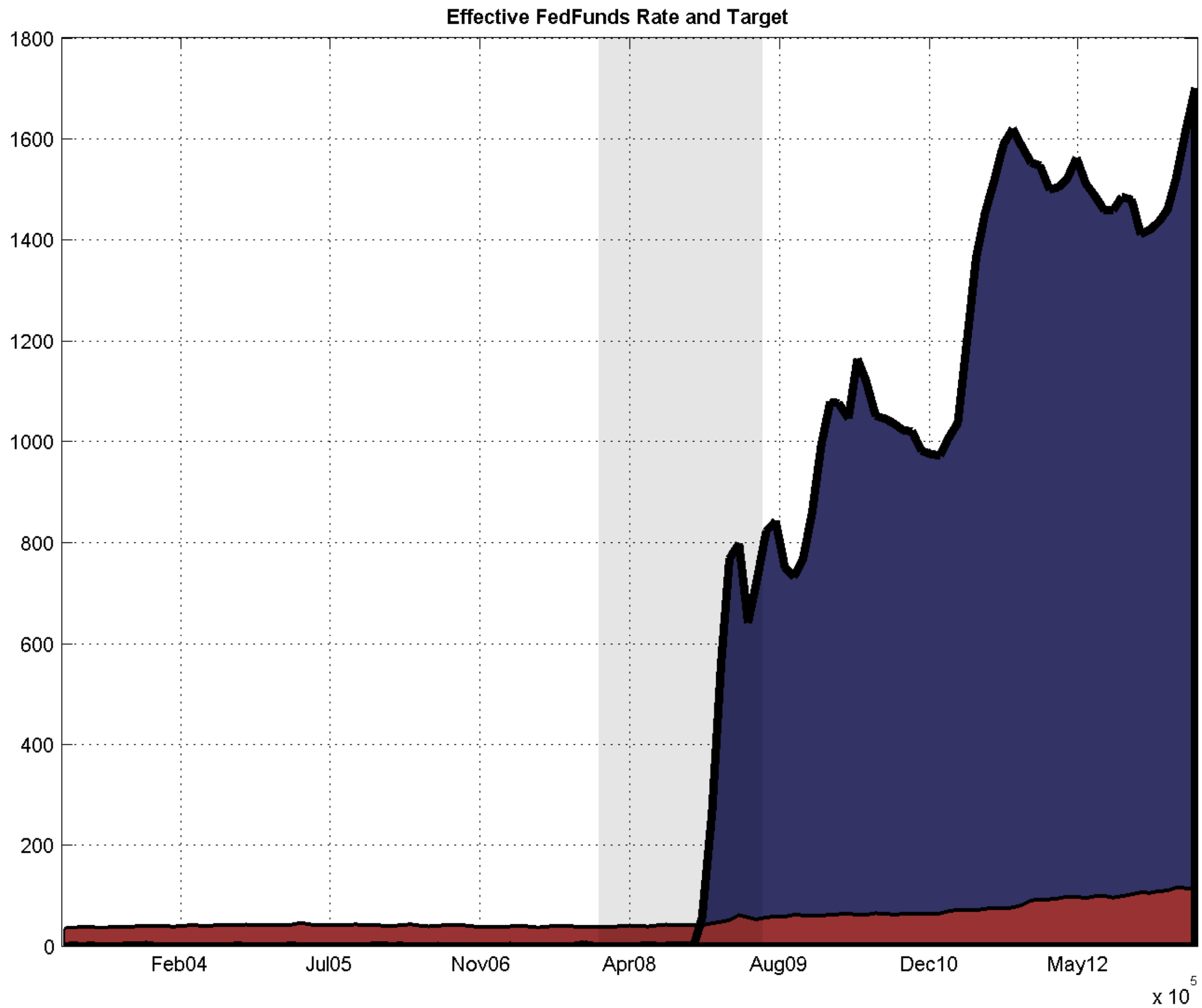


Figure 13: **Federal Reserve Assets: 2002-2012.** The figure shows the evolution of the Commercial Bank excess (blue) and required (red) reserve holding.

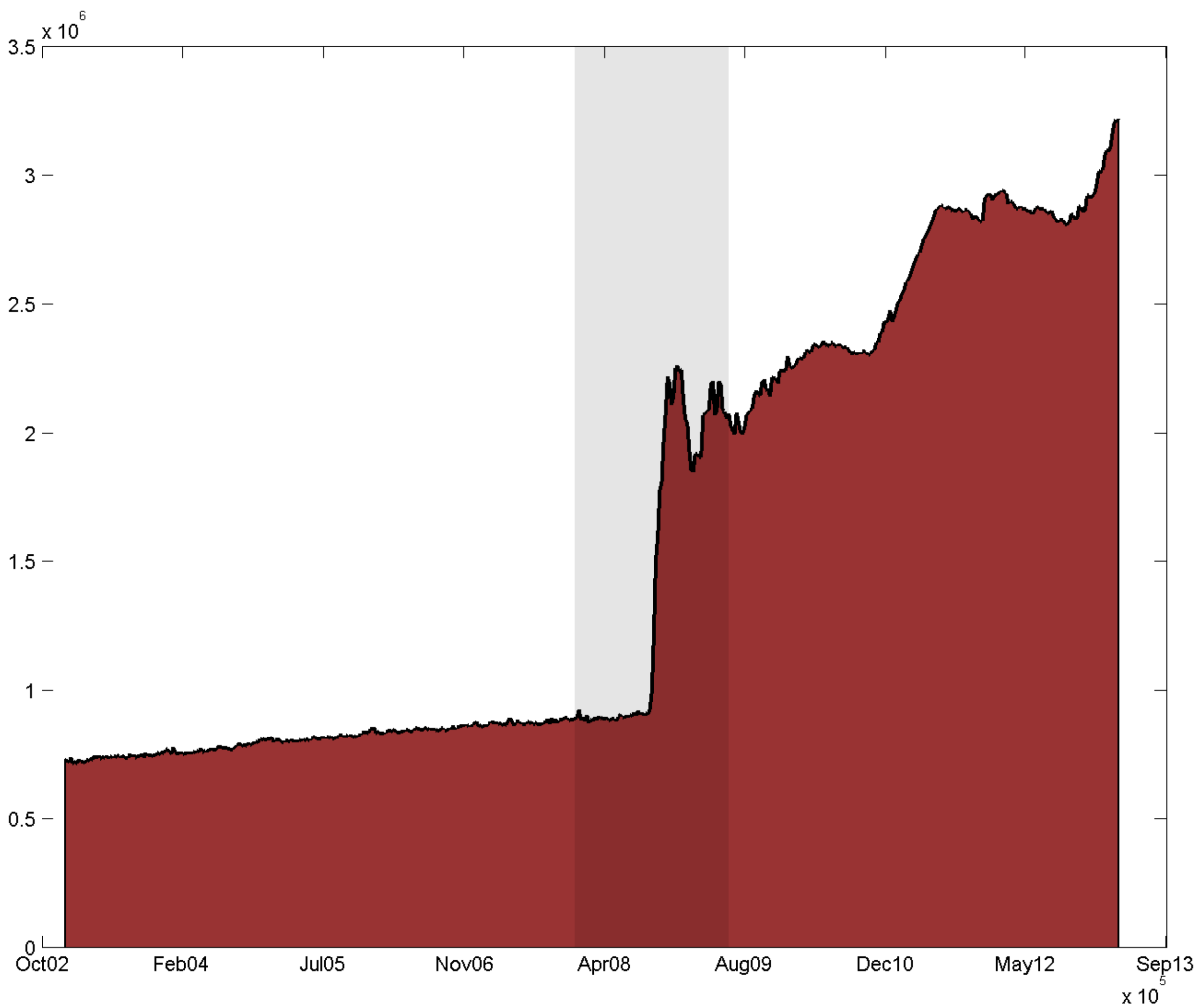
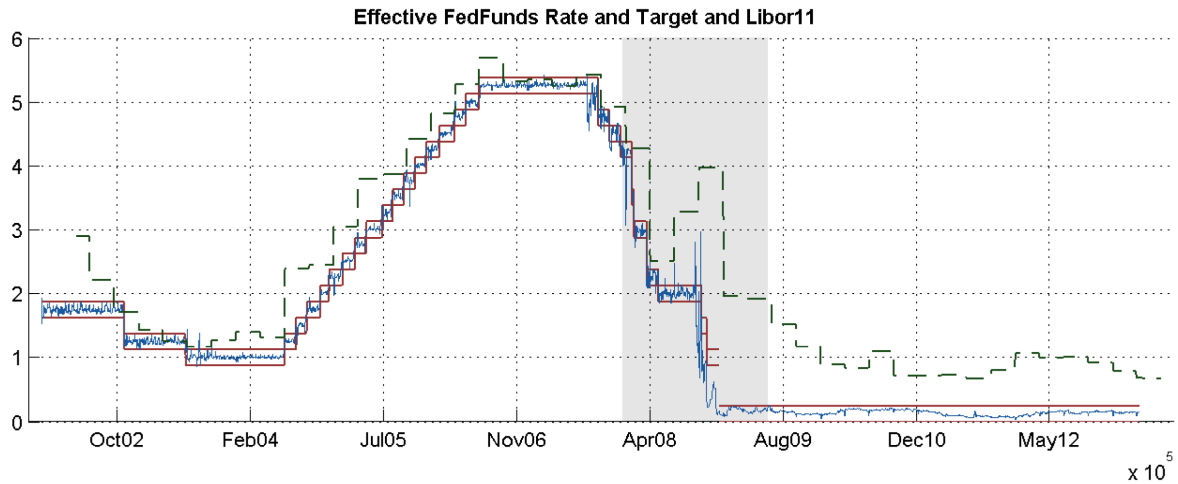


Figure 14: **Federal Reserve Assets: 2002-2012.** The figure shows the expansion in the FED's asset holdings. The magnitudes are in Millions of US$.



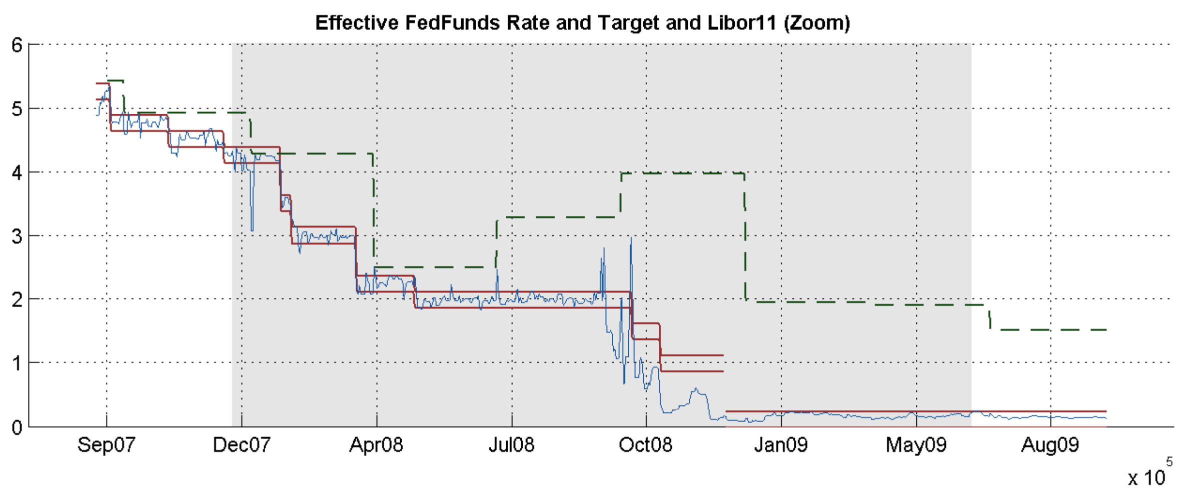


Figure 15: **Fed Funds Rate 2002-2012.** The figures plot the evolution of the FED Funds rate, the 11-month Libor rate and the overnight lending and borrowing rates.

B Proofs

B.1 Proof of Propositions 1, 2 and 3

This section provides a proof of the optimal policies described in Section 3.4. The proof of Proposition 1 is straight forward by noticing that once E is determined, the banker does not care how he came-up with those resources. The proof of Propositions 2 and 3 is presented jointly and the strategy is guess and verify. Let X be the aggregate state. We guess the following. $V(E;X) = v(X) E^{1-\gamma}$ where $v(X)$ is the slope of the value function, a function of the aggregate state that will be solved for implicitly. Policy functions are given by: $DIV(E;X) = div(X) E$, $\tilde{B}(E;X) = \tilde{b}(E;X) E$, $\tilde{D}(E;X) = \tilde{d}(E;X) E$ and $\tilde{C}(E;X) = \tilde{c}(E;X) E$.

B.1.1 Proof of Proposition 2

Given the conjecture for functional form of the value function, the value function satisfies:

$$
\begin{aligned}
V(E;X) \quad &= \quad \max_{DIV,\tilde{C},\tilde{B},\tilde{D}} U(DIV) + \beta\mathbb{E}\left[v(X')\left(E'\right)^{1-\gamma})|X\right] \\
\text{Budget Constraint:} \quad & \quad E = q\tilde{B} + \tilde{C}p + DIV - \frac{\tilde{D}}{R^D} \\
\text{Evolution of Equity} \quad &: \quad E' = \left(q'\delta + (1-\delta)\right)\tilde{B} + \tilde{C}p' - \tilde{D} - \chi((\rho + \omega'(1-\rho))\frac{\tilde{D}}{R^D} - p\tilde{C}) \\
\text{Capital Requirement} \quad &: \quad \frac{\tilde{D}}{R^D} \leq \kappa(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D})
\end{aligned}
$$

where the form of the continuation value follows from our guess. We can express all of the constraints in the problem as linear constraints in the ratios of E. Dividing all of the constraints by E, we obtain:

$$
\begin{aligned}
1 \quad &= \quad div + q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{1+r_d} \\
E'/E \quad &= \quad \left(q'\delta + 1 - \delta\right)\tilde{b} + \tilde{c}p' - \tilde{d} - \chi((\rho + \omega'(1-\rho))\tilde{D} - p\tilde{C}) \\
\frac{\tilde{d}}{R^D} \quad &\leq \quad \kappa(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D})
\end{aligned}
$$

where $div = DIV/E$,$\tilde{b} = \tilde{B}/E, \tilde{c} = \tilde{C}/E$ and $\tilde{d} = \tilde{D}/E$. Since, E is given at the time of the decisions of B,C,D and DIV, we can express the value function in terms of choice of these ratios.

Substituting the evolution of E' into the objective function, we obtain:

$$\begin{aligned}
V(E;X) &= \max_{div,\tilde{c},\tilde{b},\tilde{d}} U(divE) + \beta\mathbb{E}\left[v(X')\left(R(\omega,X,X')E\right)^{1-\gamma})|X\right] \\
1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\
\frac{\tilde{d}}{R^D} &\leq \kappa(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D})
\end{aligned}$$

where we use the fact that E' can be written as:

$$E' = R(\omega,X,X')E$$

where $R(\omega,X,X')$ is the realized return to the bank's equity and defined by:

$$R(\omega,X,X') \equiv (q(X')\delta + (1-\delta))\tilde{b} + (1+r(X'))\tilde{c} - (1+r_d)\tilde{d} - \chi((\rho+\omega'(1-\rho))(1+r_d)\frac{\tilde{d}}{R^D} - p\tilde{c}).$$

We can do this factorization for E because the evolution of equity on hand is linear in all the term where prices appear. Moreover, it is also linear in the penalty χ also. To see this, observe that $\chi\left((\rho+\omega'(1-\rho))\frac{\tilde{D}}{R^D} - p\tilde{C}\right) = \chi\left((\rho+\omega'(1-\rho))\frac{\tilde{d}E}{R^D} - \tilde{c}E\right)$ by definition of $\left\{\tilde{d},\tilde{c}\right\}$. Since, $E \geq 0$ always, we have that

$$(\rho+\omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C} \leq 0 \leftrightarrow \left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) \leq 0.$$

Thus, by definition of χ,

$$\begin{aligned}
\chi((\rho+\omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C}) &= \begin{cases} E\underline{\chi}\left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) & \text{if } \left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) \leq 0 \\ E\overline{\chi}\left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) & \text{if } \left((\rho+\omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) > 0 \end{cases}. \\
&= E\chi\left((\rho+\omega'(1-\rho))\tilde{d} - \tilde{c}\right).
\end{aligned}$$

Hence, the evolution of $R(\omega,X,X')$ is a function of the portfolio ratios b, c and d but not of the level of E. With this properties, we can factor out, $E^{1-\gamma}$ from the objective because it is a constant when decisions are made. Thus, the value function may be written as:

$$\begin{aligned}
V(E;X) &= E^{1-\gamma}\left[\max_{div,\tilde{c},\tilde{b},\tilde{d}} U(div) + \beta\mathbb{E}\left[v(X')R(\omega,X,X')^{1-\gamma})|X\right]\right] && (12) \\
1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\
\tilde{d} &\leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{d}) && (13)
\end{aligned}$$

Then, let an arbitrary $\tilde{v}(X)$ be the solution to:

$$\begin{aligned}\tilde{v}(X) &= \max_{div,\tilde{c},\tilde{b},\tilde{d}} U(div) + \beta\mathbb{E}\left[\tilde{v}(X')\,R(\omega,X,X')^{1-\gamma})|X\right] \\ 1 &= div + q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{R^D} \\ \frac{\tilde{d}}{R^D} &\leq \kappa(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D})\end{aligned}$$

We now shows that if $\tilde{v}(X)$ exists, $v(X) = \tilde{v}(X)$ verifies the guess to our Bellman equation. Substituting $v(X)$ for the particular choice of $\tilde{v}(X)$ in (12) allows us to write $V(E;X) = \tilde{v}(X)\,E^{1-\gamma}$. Note this is true because maximizing over $div, \tilde{c}, \tilde{b}, \tilde{d}$ yields a value of $\tilde{v}(X)$. Since, this also shows that $div, \tilde{c}, \tilde{b}, \tilde{d}$ and independent of E, and $DIV = divE$, $\tilde{B} = \tilde{b}E$, $\tilde{C} = \tilde{c}E$ and $\tilde{D} = \tilde{d}E$.

B.1.2 Proof of Proposition 3

We have from Proposition 2 that

$$\begin{aligned}v(X) &= \max_{\tilde{c},\tilde{b},\tilde{d},div\in\mathbb{R}^4_+} U(div) + \beta\mathbb{E}\left[v(X')\,|X\right]..... \\ &\mathbb{E}_{\omega'}\left((q'\delta + (1-\delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho))\,\tilde{d}/R^D - p\tilde{c})\right)^{1-\gamma}\end{aligned}$$

subject to

$$\begin{aligned}1 &= q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D} \\ \frac{\tilde{d}}{R^D} &\leq \kappa\left(q\tilde{b} + \tilde{c}p - \frac{\tilde{d}}{R^D}\right)\end{aligned}$$

Now define:

$$w_b \equiv \frac{\tilde{b}q}{(1-div)},\ w_c \equiv \frac{\tilde{c}p}{(1-div)} \text{ and } w_d \equiv \frac{\tilde{d}}{R^D(1-div)}.$$

and collecting terms on $1 = q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D}$, we obtain:

$$div + (1-div)(w_b + w_c + w_d) = 1 \Leftrightarrow .w_b + w_c - w_d = 1$$

Then using the definition of w_b, w_c, w_d have that $v(X)$

$$\begin{aligned}
v(X) &= \max_{w_b, w_c, w_d div} U(div) + \beta \mathbb{E}\left[v(X')\,|X\right](1-div)^{1-\gamma} \ldots \\
& \mathbb{E}_{\omega'}\left\{\frac{q'\delta + (1-\delta)}{q} w_b + p' w_c - \ w_d(R^D) - \chi((\rho + \omega'(1-\rho))\ w_d - w_c\ )\right\}^{1-\gamma} \\
& s.t. \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa(w_b + w_c - w_d)
\end{aligned}$$

Using the definition of returns, we can define portfolio value as:

$$\begin{aligned}
\Omega^*(X) &\equiv \max_{w_b, w_c, w_d} \left\{\mathbb{E}_{\omega'}\left(R^B w_b + R^C w_c - \ w_d R^D - R^\chi(w_d, w_c)\ \right)^{1-\gamma}\right\}^{\frac{1}{1-\gamma}} \\
& s.t \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa(w_b + w_c - w_d)
\end{aligned}$$

Since, the solution to $\Omega(X)$ is the same for any div and using the fact that X is deterministic,

$$v(x) = \max_{w_b, w_c, w_d div} U(div) + \beta \mathbb{E}\left[v(X')\,|X\right](1-div)^{1-\gamma} \Omega^*(X)^{1-\gamma}$$

which is the formulation in Proposition 3.

For $\gamma \to 1$, the objective becomes:

$$\Omega(X) = \exp\left\{\mathbb{E}_\omega\left[\log\left(R(\omega, X, X')\right)\right]\right\}.$$

Q.E.D

B.2 Proof of Proposition 4

Taking first-order conditions on (5) and using the CRRA functional form for $U(\cdot)$, we obtain:

$$div = \left(\beta \mathbb{E} v(X')\,|X\right)^{-1/\gamma} \Omega^*(X)^{-(1-\gamma)/\gamma}(1-div)(1-\gamma)$$

and therefore one obtains:

$$div = \frac{1}{1 + \left[\beta \mathbb{E}\left[v(X')\,|X\right](1-\gamma)\Omega^*(X))^{1-\gamma}\right]^{1/\gamma}}.$$

Substituting this expression for dividends, one obtains a functional equation for the value function:

$$\begin{aligned} v(X) &= \frac{1/(1-\gamma)}{\left(1+\left[\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}} \\ &\quad +\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\left[\frac{\left[\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\right]^{\frac{1}{\gamma}}}{\left(1+\left[\beta\mathbb{E}\left[v(X')\,|X\right](\Omega^*(X))^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)}\right]^{1-\gamma} \end{aligned}$$

Therefore, we obtain the following functional equation:

$$v(X) = DA\left[1+\left(\beta(1-\gamma)\Omega^*(X)^{1-\gamma}\,\mathbb{E}\left[v(X')\,|X\right]\right)^{\frac{1}{\gamma}}\right]^{\gamma}.$$

We can treat the right hand side of this functional equation as an operator. This operator will be a contraction depending on the values of $\left(\beta(1-\gamma)(\Omega^*(X))^{1-\gamma}\right)^{\frac{1}{\gamma}}$. Theorems in Alvarez and Stokey (1998) guarantee that this operator satisfies the dynamic programming arguments.

In a non-stochastic steady state we obtain:

$$v^{ss} = \frac{1}{1-\gamma}\left(\frac{1}{1-\left(\beta\Omega^{*1-\gamma}\right)^{\frac{1}{\gamma}}}\right)^{\gamma}.$$

and

$$div^{ss} = \frac{1}{1+\left[\beta\left(\frac{1}{1-\left(\beta\Omega^{*1-\gamma}\right)^{\frac{1}{\gamma}}}\right)^{\gamma}\Omega^*(X)^{1-\gamma}\right]^{1/\gamma}}.$$

B.3 Proof of Proposition 5

1. Suppose that the capital requirement is not binding. Then we have that

$$\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}\left(R^C-R^D\right) = \mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}\left(\chi'\left(\rho+\omega'(1-\rho)\right)-1\right) \leq 0$$

where the last inequality follows from $\omega \in [-\infty, 1]$ and implies that $R^D > R^C$ is a contradiction.

2. Suppose the capital requirement is not binding and there is ω_0 such that $\left(\rho+\omega\prime_0(1-\rho)\right)w_d - w_c > 0$

$$0 = \mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}\left(R^C-R^D\right) = \mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma}\left(\chi'\left(\rho+\omega'(1-\rho)\right)-1\right) < 0$$

where the last inequality follows from the fact that $\chi' \geq 0$ and $\chi' > 0\ \forall x > 0$, and implies a contradiction

3. Assuming there is an interior solution for C, i.e. $C > 0$, we have that

$$\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma} R^C = \mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma} R^D - \frac{\mu}{1-\gamma} > \mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma} R^C$$

   where the last inequality follows from $\mu \geq 0$ and $R^C > R^D$ and implies a contradiction.

4. The proof follows from combining:

$$\begin{aligned} R^b\mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma} &= \mathbb{E}_{\omega'}\left(R^E_{\omega'}\right)^{-\gamma} R^D - \frac{\mu}{1-\gamma} \\ R^B &= R^D - \frac{\mu}{\mathbb{E}_{\omega'}\left(R^E\right)^{-\gamma}(1-\gamma)} \end{aligned}$$

C Evolution of Bank Equity Distribution

Because the economy displays equity growth, equity is unbounded and thus, the support of this measure is the positive real line. Let $\mathcal{B}$ be the Borel σ-algebra on the positive real line. Then, define as $Q_t(e, \boldsymbol{E})$ as the probability that an individual bank with current equity e transits to the set $\boldsymbol{E}$ next period. Formally $Q_t : \mathbb{R}_+ \times \mathcal{B} \to [0, 1]$, and

$$Q(e, \boldsymbol{E}) = \int_{-1}^{1} \mathbb{I}\left\{e_t\left(\omega\right) e \in \boldsymbol{E}\right\} F\left(d\omega\right)$$

where $\mathbb{I}$ is the indicator function of the event in brackets. Then Q is a transition function and the associated T^* operator for the evolution of bank equity is given by:

$$\Gamma_{t+1}\left(\boldsymbol{E}\right) = \int_{0}^{1} Q(e, \boldsymbol{E})\Gamma_{t+1}\left(e\right) de.$$

The distribution of equity is fanning out and the operator is unbounded. Gibrat's law shows that for t large enough Γ_{t+1} is approximated well by a log-normal distribution. Moreover, by introducing more structure into the problem, we could easily obtain a Power law distribution for $\Gamma_{t+1}\left(\boldsymbol{E}\right)$. We will use this properties in the calibrated version of the model.

D Data Analysis

E Algorithm

E.1 Steady State

1. Guess prices for loans q and for the probability of a match in the interbank market γ^{-}, γ^{+}
2. Solve bank optimization problem 5. Compute value of the bank and dividend payments.
3. Compute associated average equity growth and average surplus in the interbank market.
4. If equity growth equals zero and the conjectured probability of a match in the interbank market is consistent with the average surplus, stop. Otherwise, adjust and continue iterating.

Algorithm to solve transition dynamics in baseline model

E.2 Transitional Dynamics

1. Guess a sequence of loan prices q_t and for the probability of a match in the interbank market $\gamma_t^{-}, \gamma_t^{+}$
2. Solve by backward induction banks's dynamic programming problem using 5 for banks' portfolio and 4 for value function and dividend rates
3. Compute growth rate of equity and average surplus in interbank markets
4. Compute price implied by aggregate sequence of loans resulting from (2) and (3), and the probability of a match according to average surpluses computed in (3)
5. If the conjectured price equal effective price from (4) and the average surplus computed in (4) are consistent with the guessed sequences, stop. Otherwise, continue iterating until convergence.

F Derivation of Loan Demand

Section Pending.