# Keynesian Inefficiency and Optimal Policy: A New Monetarist Approach

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**Abstract**

A simple model of monetary/labor search is constructed to study Keynesian indeterminacy and optimal policy. In the model, economic agents have trouble splitting the surplus from exchange appropriately, and we consider monetary and fiscal policies that correct this Keynesian inefficiency. A Taylor rule does not imply determinacy, nor does it support an efficient outcome. Optimal policies yield an efficient and determinate allocation of resources, but equilibrium policy actions, wages, and prices are indeterminate at the optimum.

## 1 Introduction

Keynesian ideas are hard to ignore. In modern macroeconomics, the menu cost models of the 1980s (Mankiw 1985, Blanchard and Kiyotaki 1987), coordination failure models (Bryant 1983, Diamond 1982, Cooper and John 1988), dynamic models with multiple equilibria (e.g. Farmer and Guo 1994), and New Keynesian models (Clarida, Gali, and Gertler 1999, Woodford 2003) all have some claim to the moniker "Keynesian." While these models may appear to be quite different, they share a common source of inefficiency – it is difficult for economic agents to agree on the terms of exchange in decentralized trading.

This idea, that the social inefficiency traced back to Keynes's *General Theory* is principally a problem of inefficient terms of exchange in decentralized trading, is captured in a nice way by Farmer (2012). In this paper, our goals are to extend Farmer's ideas to a New Monetarist environment with monetary exchange, and to consider what policy can do to correct Keynesian inefficiency in such an environment.

We simplify Farmer's framework and extend it in important ways. We start with a static version of Farmer (2012). Economic agents choose between two activities, work and production, but must search for a trading partner. In a successful match, output results, and the worker and producer split the output between them and consume. Not all would-be workers and producers are matched in equilibrium, so there are unemployed workers and unfilled vacancies. In typical labor search models, e.g. Mortensen and Pissarides (1994), workers and firms split the surplus from a match according to some bargaining protocol, e.g. Nash bargaining. However, we can imagine a world – Farmer's Keynesian world – where a matched worker and producer in our model have difficulty splitting the surplus. Then, there exists a continuum of equilibria, indexed by wages and labor market tightness. In general, an equilibrium with a high (low) wage is associated with low (high) labor market tightness. In equilibrium, economic agents are indifferent at the outset between seeking a match as a worker or a producer. If the wage is high, then work is attractive relative to production, so the labor market is not tight and it is relatively more difficult to find a match as a worker than as a producer. In this static model, the equilibrium can be suboptimal, in that labor market tightness is too high or too low. Indeed, the unemployment rate could be too high or too low.

The full-blown model integrates the basic static model in a dynamic Lagos-Wright (2005) framework, in which money is on one side of each transaction. Others have worked with models that have labor search and monetary search, including Shi (1999), who constructs a large-household model with labor and monetary search frictions, and Berentsen, Menzio and Wright (2011) who work with a model of Lagos-Wright (2005) search and Mortensen-Pissarides (1994) search. The model constructed here is much more tractable than either the Shi or Berentsen-Menzio-Wright models, thus making it a good vehicle for extracting the main ideas. In our dynamic model, successful matches involve a worker, a producer, and a consumer, with the worker and producer producing output which they do not wish to consume, but which they can exchange with the consumer for money. Just as in the static model, we can use a straightforward bargaining approach (sequential bilateral bargaining) to determine an equilibrium. This model has some standard properties, if we solve it in the usual way. For example, there is a long-run positive relationship between the inflation rate and the unemployment rate. Higher money growth increases the inflation rate, thus reducing the consumer's surplus from exchange. In equilibrium the unemployment and vacancy rates rise. As well, in the absence of bargaining inefficiencies, a Friedman rule is an optimal prescription for monetary policy.

In the dynamic model, we include fiscal policy, in the form of a subsidy paid to producers who match successfully. This gives the government a second policy tool that will affect the relative surpluses from production, work, and consumption. In an equilibrium with conventional bargaining, a higher subsidy increases the surplus of producers in a productive match, and this acts to reduce the unemployment rate and increase the vacancy rate.

The important component of the paper is the analysis of the model under Keynesian-type assumptions. To start, we take an approach similar to Farmer (2012), and assume that there are no forces to determine how a matched worker, producer, and consumer split the surplus for exchange. In the dynamic model, there is a three-dimensional indeterminacy. The quantity of money that trades for one unit of labor in decentralized trade is indeterminate; the quantity of money that trades for the output in a match is indeterminate; and there is indeterminacy in the path for the price of money in terms of goods in centralized trading, as is standard in monetary models. In the set of equilibria, a high wage is associated with high worker surplus, with low labor market tightness, and with low goods market tightness. Also, a high product price is associated with high labor market tightness and low goods market tightness. By high (low) labor market tightness, we mean a large (small) quantity of producers searching relative to workers, and high (low) goods market tightness is characterized by a large (small) quantity of consumers searching relative to workers.

What is the role of policy given the Keynesian indeterminacy in the model? We first ask whether a Taylor rule (Taylor 1993), according to which the nominal interest rate responds to the inflation rate and the output gap, will perform well. Though the model captures what we think is the essence of Keynesian inefficiency, a Taylor rule does not yield determinacy, and it does not in general support an efficient equilibrium allocation.

Second, we look for optimal policies when there is indeterminacy. In this case, an optimal policy picks out a unique optimal allocation, but prices and wages are indeterminate. Basically, policy responds passively so that economic agents split the surplus from trade in an efficient way, in spite of the fact that these agents are unmotivated to divide up the surplus efficiently on their own.

The paper proceeds as follows. In the second section, a basic static search model is presented. This is then extended in the third section to include monetary exchange, using a Lagos-Wright (2005) approach. The fourth section is a conclusion.

# 2 The Static Model

This is a simplified version of a search model in Farmer (2012), which differs from standard Mortensen-Pissarides labor search models (e.g. Mortensen and Pissarides 1994) in that there is not a perfectly elastic supply of firms. There is a single period, and a continuum of agents with unit mass, each of whom maximizes consumption during the period. An individual agent can choose one of two activities, work or production. Output $y$ of the consumption good is produced if there is a match between a worker and a producer. Letting $x$ and $z$ denote the masses of workers and producers, respectively, who choose to search for a match, the quantity of successful matches is given by

$$l = \psi(x, z),$$

where $\psi(\cdot, \cdot)$ is the matching function. Assume that $\psi(\cdot, \cdot)$ is strictly increasing in both arguments, twice continuously differentiable, homogenous of degree 1, and has the properties $\psi(0, z) = 0$ for $z \geq 0$ and $\psi(x, 0) = 0$ for $x \geq 0$.

In a match, let $w$ denote the wage, with the producer receiving surplus $y - w$. The probabilities of achieving a match, for a worker and a firm, respectively, are $\frac{\psi(x,z)}{x}$ and $\frac{\psi(x,z)}{z}$. In equilibrium, each agent must face the same expected payoff to becoming a worker or a producer, which gives

$$\frac{\psi(x,z)w}{x} = \frac{\psi(x,z)(y-w)}{z},$$

or, defining labor market tightness by $\theta \equiv \frac{z}{x}$,

$$w = \frac{y}{1+\theta}. \tag{1}$$

## 2.1 Conventional Solution

If we follow a typical approach in the search literature, for example Mortensen and Pissarides (1994), a matched worker and producer bargain over $w$. There are several alternative bargaining solutions, including Nash bargaining, but here we will use Kalai bargaining (Kalai 1977). Letting $\alpha$ denote the worker's share of the surplus in a match, with $0 \leq \alpha \leq 1$, Kalai bargaining gives

$$w = \alpha y, \tag{2}$$

and then from (1) and (2), we obtain

$$\theta = \frac{1-\alpha}{\alpha}. \tag{3}$$

We can then calculate the unemployment rate $u$ as the number of workers who search but fail to achieve a match, divided by the number who search, or

$$u = \frac{x - \psi(x,z)}{x} = 1 - \psi(1,\theta), \tag{4}$$

which is decreasing in $\theta$. Similarly, the vacancy rate $v$ is

$$v = \frac{z - \psi(x,z)}{z} = 1 - \psi(\frac{1}{\theta}, 1),$$

so $v$ is increasing in $\theta$. Therefore the vacancy/unemployment ratio $\frac{v}{u}$ is increasing in $\theta$.

Aggregate welfare is increasing in the quantity of aggregate output in equilibrium, as the expected utilities of all agents are identical in equilibrium, so expected utility for each agent is expected consumption for an individual, which is equal to aggregate output. Then, letting $W$ denote aggregate welfare, we have

$$W = y\psi(x,z) = yx\psi(1,\theta),$$

since the matching function is homogeneous of degree 1. But, since $x + z = 1$, we then have

$$W = \frac{y\psi(1,\theta)}{1+\theta}. \tag{5}$$

Therefore, optimal labor market tightness, $\theta^*$, solves the first-order condition

$$\psi_2(1,\theta^*) - \psi_1(1,\theta^*) = 0, \tag{6}$$

where (6) uses the homogeneity-of-degree-one property of the matching function.

In general, equilibrium labor market tightness will be suboptimal, unless bargaining proceeds according to a "Hosios rule" whereby, from (3),

$$\alpha = \alpha^* \equiv \frac{1}{1+\theta^*}$$

## 2.2 Indeterminacy

The approach in Farmer (2012) is to assume that there are no forces in the model that determine how the surplus from a successful match is to be split. We drop the bargaining solution, in which case equation (1) yields a continuum of equilibrium solutions for $w$ and $\theta$. This might seem an uninteresting approach to generating indeterminacy – why not drop any equation determining equilibrium in a well-specified economic model, and argue that we have a "deep" theory based on indeterminacy? But in models with random matching, the terms of exchange are typically determined by some arbitrary trading protocol, for example Nash bargaining (e.g. Lagos-Wright 2005), take-it-or-leave-it offers , Kalai bargaining (as in the previous subsection), or competitive search (e.g. Rocheteau and Wright 2005). As emphasized in Hu et al. (2009), there is no economic theory that allows us to choose among these trading protocols, and key results can be sensitize to the choice of protocol by the modeler. Hu et al. (2009) propose a more flexible approach, in line with the notion in this paper that we can treat the terms of exchange as indeterminate.

With indeterminacy, note that the right-hand side of (1) is a strictly decreasing function of $\theta$, so an equilibrium with high wages is associated with low labor market tightness. In equilibrium, economic agents are indifferent between searching for a match as a worker and searching as a producer. If the wage is high, then the surplus received by workers is high and the surplus received by producers is low. Thus, if agents are to be indifferent between searching as a worker or searching as a producer, it must be more difficult to find a match as a worker than as a producer, so labor market tightness must be low.

Further, from (5), if we differentiate the expression on the right-hand side of (5) with respect to $\theta$, we get

$$\frac{\partial W}{\partial \theta} = \frac{y}{(1+\theta)^2}\left[\psi_2(1+\theta) - \psi(1,\theta)\right] = \frac{y}{(1+\theta)^2}\left[\psi_2(1,\theta) - \psi_1(1,\theta)\right],$$

and, since $\psi(\cdot,\cdot)$ is homogeneous of degree 1, $\frac{\partial W}{\partial \theta} > 0$ for $\theta < \theta^*$, and $\frac{\partial W}{\partial \theta} < 0$ for $\theta > \theta^*$. Therefore, welfare and output are maximized in the equilibrium where $\theta = \theta^*$. Further, if we compare alternative equilibria, the unemployment rate is monotonically decreasing in $\theta$, but as we increase $\theta$ output increases and then decreases. Thus, the relationship between the unemployment rate and aggregate output across equilibria is non-monotonic.

# 3 A Dynamic New Monetarist Model

The next step is to take the static model of the previous section, and extend it to include monetary exchange. Time is indexed by $t = 0, 1, 2, ...$, and each period, as in Lagos and Wright (2005), is divided into two subperiods, denoted the centralized market (CM) and decentralized market (DM), respectively. There is a continuum of infinite-lived agents with unit mass, each of whom has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t (C_t - H_t + c_t),$$

where $0 < \beta < 1$, $C_t$ denotes consumption in the CM, $H_t$ labor supply in the CM, and $c_t$ is consumption in the DM. In the CM, each agent has available a technology that permits one unit of perishable CM consumption goods to be produced for each unit of labor input.

## 3.1 Decentralized Market (DM)

Before he or she enters the DM, an agent must decide whether to be a worker, a producer, or a consumer. Then, in the DM, a successful match occurs when a worker, a producer, and a consumer all meet so that the worker and producer can supply output $y$ to the consumer. Any worker and producer who match cannot consume their own output, and output is perishable. Letting $x_t$, $z_t$, and $k_t$ denote the masses of the population who choose to be workers, producers, and consumers, respectively, the number of matches $l_t$ is determined by the matching function

$$l_t = \chi(x_t, z_t, k_t),$$

where the function $\chi(\cdot, \cdot, \cdot)$ has properties identical to $\psi(\cdot, \cdot)$, except with three arguments instead of two.

In a match among a worker, a producer, and a consumer, credit is not feasible as there is no memory - an agent does not have access to the histories of other agents. Exchange is possible using money, however, and the consumer exchanges $m_t$ units of money (in units of the $t+1$ CM consumption good) for $y$ units of goods. The worker receives $w_t \leq m_t$ units of real money balances, and the producer receives the residual, $m_t - w_t$.

Assume that, in the CM when production and consumption take place, that an agent does not know whether he or she will have a successful match in the subsequent DM, but that each consumer learns this at the end of the period, and that this information becomes public knowledge at that time. The government engages in a simple fiscal policy, which is a subsidy to matched producers, of $s_t$ in units of $t+1$ goods, provided in the CM of period $t+1$. Consumers are able to share money, in that they can write contingent contracts prior to learning whether they achieve a successful match or not, and so only consumers in successful matches will carry money with them into the DM. Matched consumers leave the CM with $m_t$ units of money (in units of the $t+1$ CM consumption good) and unmatched consumers hold no money.

### 3.2 Centralized Market (CM)

In the CM, all agents are together in one location and can trade money for goods on a Walrasian market where the price of money in terms of goods is $\phi_t$. Agents observe $\phi_t$ in the Walrasian market, but cannot observe the actions of other agents. In the CM, each agent pays a lump-sum tax $\tau_t$ to the government. Let $M_t$ denote the quantity of money when the Walrasian market opens in the CM. In period 0, agents receive the first-period money stock as a transfer, so $\tau_0 = -\phi_0 M_0$. Then, the government budget constraint for periods $t = 1, 2, 3, ...,$ is given by

$$\phi_t(M_t - M_{t-1}) + \tau_t - s_t\chi(x_t, z_t, k_t) = 0.$$

In equilibrium, each agent in the CM must be indifferent among the three alternative activities in the succeeding DM, so similar to (1),

$$\frac{\chi(x_t, z_t, k_t)}{x_t}\beta w_t = \frac{\chi(x_t, z_t, k_t)}{z_t}\beta(m_t - w_t + s_t) = \frac{\chi(x_t, z_t, k_t)}{k_t}\left(y - \frac{m_t\phi_t}{\phi_{t+1}}\right),$$

or

$$\theta_t w_t = m_t - w_t + s_t, \tag{7}$$

$$\sigma_t \beta w_t = y - \frac{m_t\phi_t}{\phi_{t+1}}, \tag{8}$$

where $\theta_t \equiv \frac{z_t}{x_t}$ or *labor market tightness*, and $\sigma_t \equiv \frac{k_t}{x_t}$ or *goods market tightness.*

### 3.3 Conventional Solution

Just as in the static model, we use Kalai bargaining, but in stages. First, the producer and the worker bargain over how to split the surplus $\beta m_t$ between them. The worker receives surplus $\beta w_t$, and the producer receives the remainder, $\beta(m_t - w_t)$. The worker's share of the worker/producer surplus is $\frac{\alpha}{\alpha+\delta}$, where $0 < \alpha < \alpha + \delta < 1$, so according to the bargaining rule, the wage is

$$w_t = \frac{\alpha m_t}{\alpha + \delta}. \tag{9}$$

The producer then treats the bargaining solution (9) as given, and bargains with the consumer concerning the quantity of money $m_t$ to be exchanged by the consumer for the producer/worker output $y$. The producer/worker surplus is $\beta m_t$, and the consumer's surplus is $y - \beta m_t$. The total surplus $y$ is split according to

$$\beta m_t = (\alpha + \delta)y, \tag{10}$$

so (9) and (10) give

$$\beta w_t = \alpha y, \tag{11}$$

and the worker's, producer's, and consumer's shares of total surplus are then $\alpha$, $\delta$, and $1 - \alpha - \delta$, respectively. Then, (7), (8), (11), and (10) give

$$\theta_t = \frac{\delta y + \beta s_t}{\alpha y} \tag{12}$$

$$\sigma_t = \frac{1 - \frac{(\alpha+\delta)\phi_t}{\beta\phi_{t+1}}}{\alpha} \tag{13}$$

Further, money demand equals money supply in the CM, so from (10),

$$\frac{(\alpha+\delta)y\chi(1,\theta_t,\sigma_t)\phi_t}{\beta(1+\theta_t+\sigma_t)\phi_{t+1}} = \phi_t M_t, \tag{14}$$

and then (12), (13), and (14) solve for $\{\theta_t, \sigma_t, \phi_t\}_{t=0}^{\infty}$.

Suppose that the money stock grows at a constant rate, i.e. $M_{t+1} = \mu M_t$, for $t = 1, 2, 3, ...$, that $s_t = s$ for all $t$, and confine attention to stationary equilibria where $\theta_t = \theta$, $\sigma_t = \sigma$ for all $t$, and $\phi_t$ grows at a constant rate. Then, from (10)-(14), we have $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ and

$$\theta = \frac{\delta y + \beta s}{\alpha y} \tag{15}$$

$$\sigma = \frac{1 - \frac{(\alpha+\delta)\mu}{\beta}}{\alpha} \tag{16}$$

Then, using (14), we can solve for prices

$$\phi_t = \frac{(\alpha+\delta)y\chi(1,\theta,\sigma)\mu}{\beta(1+\theta+\sigma)M_t}, \tag{17}$$

and output in the DM is given by

$$Y^{DM} = \frac{y\chi(1,\theta,\sigma)}{(1+\theta+\sigma)} \tag{18}$$

Then, from (15), note that labor market tightness $\theta$ is invariant to money growth as this does not affect the relative payoffs to workers and producers. However, from (16), higher money growth causes a decrease in $\sigma$, which represents goods market tightness. Higher inflation reduces the ex ante surplus of consumers, and so reduces the mass of consumers searching relative to producers and workers. A higher subsidy $s$ has no effect on goods market tightness, but from (15) this causes labor market tightness to increase.

As in the static model, we calculate the unemployment rate given the probability for a worker of achieving a match. The unemployment rate is given by

$$u = \frac{x - \chi(x,z,k)}{x} = 1 - \chi(1,\theta,\sigma) = 1 - \frac{\chi\left(\alpha, \delta + \frac{\beta s}{y}, 1 - (\alpha+\delta)\frac{\mu}{\beta}\right)}{\alpha},$$

which is increasing in the money growth rate, in standard fashion. Higher inflation results in an increase in the mass of economic agents who choose to search for work, and to an increase in the mass of producers searching, but there are fewer consumers with whom to match. The result is that a larger fraction of workers goes unmatched, or unemployed. Note also that a higher subsidy for producers reduces unemployment, as this increases the fraction of producers searching relative to workers and consumers.

Similarly, the vacancy rate is given by

$$v = \frac{z - \chi(x, z, k)}{z} = 1 - \chi\left(\frac{1}{\theta}, 1, \frac{\sigma}{\theta}\right) = 1 - \chi\left(\frac{\alpha y}{\delta y + \beta s}, 1, \frac{y - y(\alpha+\delta)\frac{\mu}{\beta}}{\delta y + \beta s}\right).$$

Therefore $v$ is increasing in the money growth factor, and increasing in the subsidy $s$. An increase in the money growth factor reduces the ex ante surplus for consumers, so that fewer consumers search relative to producers and workers, the result being a higher vacancy rate - more producers are ultimately not matched. An increase in the subsidy to producers increases the incentive to become a producer, more producers search relative to workers and consumers, and this also increases the vacancy rate.

What is output in the CM each period? From (17), this is the quantity

$$Y^{CM} = \frac{[(\alpha+\delta)y\mu + \beta s]\,\chi(1, \theta, \sigma)}{\beta(1+\theta+\sigma)},$$

so if we add total output in the CM and the DM each period, we obtain, using (18),

$$Y = Y^{DM} + Y^{CM} = \frac{[(\alpha+\delta)\mu y + \beta y + \beta s]\,\chi(1, \theta, \sigma)}{\beta(1+\theta+\sigma)}.$$

What is an optimal policy in the dynamic model? First, note that a social planner who could choose $\theta$ and $\sigma$ would pick these quantities to maximize the number of matches in the DM, i.e. this planner would solve

$$\max_{\theta,\sigma} \frac{\chi(1, \theta, \sigma)}{1+\theta+\sigma}.$$

Therefore, letting $\theta^*$ and $\sigma^*$ denote the optimal quantities of labor market tightness and goods market tightness, the first-best optimum is the solution to

$$\chi_2(1, \theta^*, \sigma^*) - \chi_1(1, \theta^*, \sigma^*) = 0 \tag{19}$$

$$\chi_3(1, \theta^*, \sigma^*) - \chi_1(1, \theta^*, \sigma^*) = 0 \tag{20}$$

Thus, if there is a monetary policy and a fiscal policy that supports $\theta = \theta^*$ and $\sigma = \sigma^*$ as an equilibrium, then such a set of policies would be optimal. Using (15) and (16), we can solve for an unconstrained optimal monetary and fiscal policy:

$$\mu^* = \frac{\beta(1-\alpha\sigma^*)}{\alpha+\delta} \tag{21}$$

$$s^* = \frac{y(\theta^*\alpha - \delta)}{\beta} \tag{22}$$

But, arbitrage requires that $\mu \geq \beta$ in equilibrium (an implicit nominal interest rate which is nonnegative), so the set of policies given by (21) and (22) is optimal if and only if $\mu^* \geq \beta$, or

$$\sigma^* \leq \frac{1-\alpha-\delta}{\alpha} \tag{23}$$

But, if (23) does not hold, then the optimal money growth factor is $\mu^* = \beta$ (Friedman rule), and from (16), at the optimum

$$\sigma = \frac{1-\alpha-\delta}{\alpha},$$

and $s^*$ solves

$$s^* = \arg\max_s \left[\frac{\chi(\alpha y, \delta y + \beta s, (1-\alpha-\delta)y)}{\beta s + y}\right]$$

Suppose that we eliminate the role for money in this model, by removing anonymity in the decentralized market, so that credit arrangements are possible (and assuming that limited commitment constraints do not bind). Further, suppose that there are no bargaining inefficiencies, so that

$$\alpha = \frac{1}{1+\sigma^*+\theta^*}, \tag{24}$$

$$\delta = \frac{\theta^*}{1+\sigma^*+\theta^*}, \tag{25}$$

then the credit equilibrium is efficient. Then, if (24) and (25) hold, which is a version of the Hosios rule in our model, the optimal policy in the stationary monetary equilibrium we focus on above is $s = 0$ and $\mu = \beta$. Thus, any deviation of the optimal policy from a standard Friedman rule is due to bargaining inefficiencies.

## 3.4 Indeterminacy

Following a similar approach to what was done with the static model, suppose now that the surplus in a match is not split according to any particular bargaining protocol. Market-clearing (money demand equals money supply in the CM) gives

$$\frac{m_t \chi(1, \theta_t, \sigma_t)}{(1+\theta_t+\sigma_t)} = \phi_{t+1} M_t \tag{26}$$

Then, we can describe an equilibrium as a sequence $\{w_t, m_t, \theta_t, \sigma_t, \phi_t\}_{t=0}^{\infty}$ solving (7), (8) and (26) with

$$m_t > 0,\ w_t > 0,\ \theta_t > 0,\ \sigma_t > 0,$$

and

$$\frac{\phi_{t+1}}{\phi_t} \leq \beta, \tag{27}$$

where the latter condition states that the implicit nominal interest rate must be nonnegative in equilibrium.

To illustrate the nature of the indeterminacy, and to compare this with the conventional solution, suppose that the money growth factor is a constant, $\mu$, and restrict attention to stationary equilibria where real quantities are constant, i.e. $w_t = w$, $m_t = m$, $\theta_t = \theta$, $\sigma_t = \sigma$, and $\phi_t M_t = \omega$ for all $t$. Then, from (7), (8) and (26), we get

$$\theta = \frac{m - w + s}{w}, \tag{28}$$

$$\sigma = \frac{y - \mu m}{\beta w}. \tag{29}$$

In (28) and (29) there is a two-dimensional indeterminacy, as we have two equations that must solve for the four unknowns $m$, $w$, $\theta$, and $\sigma$. Note that equilibria with higher $w$ are associated with lower $\theta$ and lower $\sigma$, i.e. if more surplus goes to workers, then the labor and goods markets are less tight. Equilibria with higher $m$ are associated with higher $\theta$ and lower $\sigma$ so that, if more surplus goes to workers and producers, vis-a-vis consumers, then the labor market is tighter and the goods market is less tight. Note that in this example we are ignoring standard monetary indeterminacy, by focusing on equilibria where the inflation rate is equal to the money growth rate. Thus, there is in general another dimension to the indeterminacy.

### 3.4.1 A Taylor Rule

The role of the Taylor rule in New Keynesian models is nicely summarized in Woodford (2001). In New Keynesian models, Taylor rules are typically used to obtain determinacy of equilibrium, and under some conditions can be derived as approximations to optimal monetary policy rules. Principally though, in line with Taylor (1993), proponents of such rules argue that they: (i) fit the data well; and (ii) perform well, by some criteria, in mainstream macroeconomic models.

How would we specify a Taylor rule in our model, and what would happen if the central bank in our model were to adhere to such a rule? The Taylor rule specifies that deviations of the short-term nominal rate of interest should depend positively on the deviation of the inflation rate from some target rate, and on some measure of the "output gap." First, in this model we could determine the nominal interest rate by pricing a nominal bond that trades in the current $CM$ and is a claim to one unit of money in the $CM$ next period. For reasons that we would need to flesh out in model where nominal bonds played an important role (as is not the case here), this claim cannot be traded in the $DM$. Given linear utility, the gross nominal interest rate is then

$$R_t = \frac{\phi_t}{\phi_{t+1}\beta} \tag{30}$$

How would we measure the output gap in this model? We know from above that welfare-maximizing output is given by

$$Y^* = \frac{y\chi(1, \theta^*, \sigma^*)}{1 + \theta^* + \sigma^*}, \tag{31}$$

where $\theta^*$ and $\sigma^*$ are, respectively, optimal labor market tightness and goods market tightness, determined by (19) and (20). Then, in line with Taylor (1993), and writing the policy rule in multiplicative form rather than in logs,

$$R_t = \max\left\{\left(\frac{\phi_t}{\phi_{t+1}\pi^*}\right)^{\rho}\left[\frac{\chi(1, \theta_t, \sigma_t)\,(1 + \theta^* + \sigma^*)}{(1 + \theta_t + \sigma_t)\,\chi(1, \theta^*, \sigma^*)}\right]^{\eta}\frac{\pi^*}{\beta}, 1\right\} \tag{32}$$

In equation (32), $\pi^*$ is the target gross inflation rate, $\frac{1}{\beta}$ is the constant gross real interest rate, $\rho > 1$, and $\eta > 0$. As well, we have accounted for the zero lower bound on the nominal interest rate. In this model, since the real rate is constant (due to linear utility), with the nominal interest rate determined by a pure Fisher effect, the Taylor rule becomes a rule determining the inflation rate. Substituting for $R_t$ on the left-hand side of (32) using (30), and simplifying, we obtain

$$\frac{\phi_t}{\phi_{t+1}} = \pi^*\left[\frac{(1 + \theta_t + \sigma_t)\,\chi(1, \theta^*, \sigma^*)}{\chi(1, \theta_t, \sigma_t)\,(1 + \theta^* + \sigma^*)}\right]^{\nu}, \tag{33}$$

where $\nu = \frac{\eta}{\rho - 1} > 0$. Thus, from (33), the central bank conducts monetary policy so that the deviation of the inflation rate from the target rate is decreasing in the quantity of output. Note from (30) and (33) that, so long as $\pi^* \geq \beta$, we have $R_t \geq 1$ and the zero lower bound on the nominal interest rate will not be violated.

Then, substituting for the policy rule in (8) using (33), we obtain

$$\sigma_t \beta w_t = y - m_t, \tag{34}$$

What we are hoping to get from the Taylor rule are two things: determinacy and optimality. If we followed a New Keynesian approach, we would settle for the former, and hope to come close on the latter. To give the simple Taylor rule the best chance, we could allow fiscal policy to be very sophisticated. From equation (7), the fiscal authority can set $s_t$ so as to choose any value for labor market tightness $\theta_t$ that it wants. In particular, the optimal policy rule for the fiscal authority takes the form

$$s_t = w_t(1 + \hat{\theta}_t) - m_t, \tag{35}$$

where $\hat{\theta}_t$ is desired labor market tightness. This is a sophisticated fiscal policy rule, in that it depends on how the surplus from exchange is split up in the $DM$, i.e. on $w_t$ and $m_t$. Then, in equation (34), if the fiscal authority follows the policy rule given by (35), then

$$\sigma_t \beta w_t = y - m_t \pi^*\left[\frac{\left(1 + \hat{\theta}_t + \sigma_t\right)\chi(1, \theta^*, \sigma^*)}{\chi(1, \hat{\theta}_t, \sigma_t)\,(1 + \theta^* + \sigma^*)}\right]^{\nu}. \tag{36}$$

We can then construct an equilibrium as follows. Fiscal policy, from (35) determines $\theta_t = \hat{\theta}_t$. Then, choose any $(w_t, m_t)$ satisfying $\beta(w_t + m_t) \leq y$ , $w_t \geq 0$, and $m_t \geq 0$ (each matched agent in the $DM$ receives nonnegative surplus), and such that there is at least one solution $\sigma_t = \hat{\sigma}_t$ with $\hat{\sigma}_t \geq 0$ that solves (36). Then, from (26), the money stock in period $t$ is given by

$$M_t = \frac{m_t \chi(1, \hat{\theta}_t, \hat{\sigma}_t)}{\phi_{t+1}(1 + \hat{\theta}_t + \hat{\sigma}_t)}, \tag{37}$$

so monetary policy is conducted to accommodate whatever nominal money demand arises (the quantity on the left-hand side of (37)), given the Taylor rule.

Thus, in terms of determinacy, the Taylor rule does not solve the problem. In general, given fiscal policy there is indeterminacy in $\{w_t, m_t, \phi_t\}_{t=0}^{\infty}$ . In fact, even given $m_t$ and $w_t$, equation (36) implies that there can be two solutions for $\sigma_t$. Further, on the optimality front, the Taylor rule cannot in general achieve an efficient equilibrium with $\theta_t = \theta^*$ and $\sigma_t = \sigma^*$.

#### 3.4.2 Optimal Policy Rules with Bargaining Indeterminacy

Suppose that we think more broadly about optimal policy rules, in the case where $\{w_t, m_t\}_{t=0}^{\infty}$ is in general indeterminate. As a first step in determining optimal monetary and fiscal policy, substitute for $\phi_{t+1}$ in (8) using (26), obtaining

$$\sigma_t \beta w_t = y - \frac{\phi_t M_t (1 + \theta_t + \sigma_t)}{\chi(1, \theta_t, \sigma_t)}, \tag{38}$$

Then, we can back out optimal policy rules directly from (7) and (38). Optimal policy rules are

$$M_t = \frac{(y - \sigma^* \beta w_t) \chi(1, \theta_t, \sigma_t)}{\phi_t (1 + \theta_t + \sigma_t)} \tag{39}$$

$$s_t = (\theta^* + 1) w_t - m_t \tag{40}$$

Note, in (39) and (40), that the money stock $M_t$ and the subsidy $s_t$ respond to $w_t$, $m_t$, $\theta_t$, $\sigma_t$, and $\phi_t$, which are endogenous. In equilibrium, labor market tightness and goods market tightness, $\theta_t$ and $\sigma_t$, respectively, are determinate, with $\theta_t = \theta^*$ and $\sigma_t = \sigma^*$ for all $t$. But $\{w_t, m_t, \phi_t\}_{t=0}^{\infty}$ is indeterminate, as is $\{M_t, s_t\}_{t=0}^{\infty}$. Therefore, what optimal policy does here is to respond passively to endogenous variables so that wages and prices are irrelevant for economic welfare. This optimal policy may not be unique.

This is quite different from some standard Keynesian approaches. For example, in New Keynesian thought, optimal policy is typically about using policy to manipulate relative prices to remove distortions, and in models with multiple equilibria, we typically seek policies that eliminate indeterminacy altogether. A Taylor rule, thought by some to be robust to alternative model specifications, certainly does not have good properties in this model which, it could be argued, captures the flavor of the key Keynesian ideas. Further, while the optimal policy rule we have constructed eliminates the key indeterminacy, and maximizes welfare, there is residual indeterminacy in prices – but many Keynesians seem focussed on relative price distortions as the source of Keynesian inefficiency. Obviously Keynesian inefficiency need not lead us to the policies that Keynesians typically prescribe.

# 4 Conclusion

The model constructed in this paper captures labor search and monetary search in a simple and tractable model that is useful for non-Keynesian work, though the model was used here to exposit, extend, and evaluate some Keynesian ideas. Central to how the model works is that organizing the production of output, working to produce output, and shopping for output all require time, and it is convenient to model production and sales as a matching process. The model includes monetary exchange, so that we can address how monetary and fiscal policy matter for the allocation of resources.

In the model, economic agents have difficulty determining the allocation of the surplus from exchange among themselves. Inefficiencies arise which monetary and fiscal policy can potentially correct. But how should we correct those inefficiencies? The Taylor rule, which has become a key element in New Keynesian analysis, does not have good properties here – it does not eliminate indeterminacy, nor does it yield an optimal equilibrium allocation. But an optimal policy, while it eliminates real indeterminacy and maximizes welfare, does not eliminate indeterminacy in prices. Rather, policy accommodates the paths of wages and prices in such a way as to eliminate inefficiency. While this model appears to capture something of the essence of Keynesian ideas about inefficiency, it does not yield the policy conclusions that Keynesians typically advocate.

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