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MONOPOLISTIC COMPETITION: BEYOND THE CONSTANT ELASTICITY OF SUBSTITUTION

EVGENY ZHELOBODKO
Novosibirsk State University, 630090, Novosibirsk, Russia, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Russia

SERGEY KOKOVIN
Novosibirsk State University and Sobolev Institute of Mathematics, 630090, Novosibirsk, Russia, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Russia

MATHIEU PARENTI
CORE-UCLouvain, 1348 Louvain la Neuve, Belgium, Paris School of Economics (Paris 1), 75013 Paris, France, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Russia

JACQUES-FRANÇOIS THISSE
CORE-UCLouvain, 1348 Louvain la Neuve, Belgium, National Research University Higher School of Economics, 190008, Saint-Petersburg, Russia, and CEPR

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NOTES AND COMMENTS

MONOPOLISTIC COMPETITION: BEYOND THE CONSTANT ELASTICITY OF SUBSTITUTION

BY EVGENY ZHELOBODKO, SERGEY KOKOVIN, MATHIEU PARENTI, AND JACQUES-FRANÇOIS THISSE[1]

We propose a model of monopolistic competition with additive preferences and variable marginal costs. Using the concept of "relative love for variety," we provide a full characterization of the free-entry equilibrium. When the relative love for variety increases with individual consumption, the market generates pro-competitive effects. When it decreases, the market mimics anti-competitive behavior. The constant elasticity of substitution is the only case in which all competitive effects are washed out. We also show that our results hold true when the economy involves several sectors, firms are heterogeneous, and preferences are given by the quadratic utility and the translog.

KEYWORDS: Monopolistic competition, additive preferences, love for variety, heterogeneous firms.

1. INTRODUCTION

THE CONSTANT ELASTICITY OF SUBSTITUTION (CES) model of monopolistic competition is the workhorse of recent theories of trade, growth, and economic geography. It is also widely applied in empirical trade studies. Yet it is fair to say that this model suffers from major drawbacks. First, preferences lack flexibility because the elasticity of substitution is constant and the same across varieties. Second, prices and markups are not affected by firm entry and market size. This contradicts economic theory, in general, and industrial organization, in particular, which have long stressed the role of entry in the determination of market prices. Third, there is no scale effect, that is, the size of firms is independent of the number of consumers. Such a result runs against empirical evidence. For example, Holmes and Stevens (2004) observed that the correlation sign between firm and market sizes differs in services and manufacturing. Fourth, and last, firms' price and size are independent from the geographical distribution of demand. Yet it is well documented that firms benefit from being closer to their larger markets, with distance accounting for more than half of the overall difference between large plant and small plant shipments (Holmes and Stevens (2012)).

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Thus, we find it both meaningful and important to develop a more general model of monopolistic competition. The CES must be a special case of our setting to assess how our results depart from those obtained under the CES. Moreover, to provide a better description of real world markets than the CES, our setting must also be able to cope with issues highlighted in oligopoly theory, such as the impact of entry and market size on prices and firm size. Developing such a model and studying the properties of the market equilibrium is the main objective of this paper. To achieve our goal, we assume that preferences over the differentiated product are *additively separable* across varieties, but without using specific functional form. Though still restrictive, we show that additive preferences are rich enough to describe a range of market outcomes that is much wider than the CES. In particular, this setting will allow us to deal with various patterns of substitution through the *relative love for variety*, that is, the elasticity of the marginal utility. When consumption is the same across varieties, the relative love for variety is the inverse of the elasticity of substitution.

Though it ignores explicit strategic interactions, our model displays several effects highlighted in industrial organization, and uncovers new results that have empirical appeal. Specifically, we show that the market outcome depends on how the relative love of variety varies with the consumption level. To be precise, the market outcome may obey two opposite patterns. On the one hand, when the relative love for variety increases with consumption, the equilibrium displays the standard *price-decreasing* effects: more firms, a larger market size, or both lead to *lower* market prices because the elasticity of substitution increases. On the other hand, when the relative love for variety decreases, the market generates *price-increasing* effects, that is, a larger number of firms, a bigger market, or both lead to *higher* prices because the elasticity of substitution now decreases. Although at odds with the standard paradigm of entry, this result agrees with several recent contributions in industrial organization (Amir and Lambson (2000), Chen and Riordan (2007)) as well as with empirical studies showing that entry or economic integration may lead to higher markups (Ward, Shimshack, Perloff, and Harris (2002), Badinger (2007)). They should not be viewed, therefore, as exotica. In other words, our paper adds to the literature the idea that what looks like an anti-competitive outcome need not be driven by defense or collusive strategies: it may result from the nature of preferences with well behaved utility functions. In this respect, our analysis provides a possible rationale for contrasting results observed in the empirical literature.

These results rely on the fact that our model involves a *variable elasticity of substitution*, the value of which is determined at the market equilibrium. How this value is determined depends on the behavior of the relative love for variety. We also want to stress that the CES is the dividing line between the above-mentioned two classes of utility functions since the CES does not display any of the effects discussed above. Furthermore, though our setting allows for *variable marginal cost*, we show that the difference in marginal cost behavior does not affect the nature of our results. This sheds new light on models that are commonly used in the empirical literature, as one may expect different estimates of the elasticity of substitution to be obtained with different data sets. We need not assume changing preferences to rationalize this difference. It is sufficient to work with a variable elasticity of substitution.

To highlight the versatility of our model as a building block for broader settings, we briefly discuss three extensions. First, we consider a multisector economy in which the income share spent on the differentiated good varies with the prices set by firms and we show that our main results remain valid. Second, though the argument of Section 3 depends on symmetry assumptions, our modeling strategy keeps its relevance in the case of heterogeneous firms à la Melitz (2003). In particular, we show that, regardless of the cost distribution, the cutoff cost and markup decrease (increase) with the size of the market when the relative love for variety increases (decreases) with consumption. Therefore, according to the nature of preferences, the average productivity and average markup rise or fall. Last, we also show that additive preferences are not as restrictive as they seem to be at first glance, because nonadditive preferences such as quadratic and translog yield a market outcome that inherits the properties of a special additive model.

Related Literature

Using additive preferences, Spence (1976), Dixit and Stiglitz (1977), Kuhn and Vives (1999), and Vives (1999) have derived equilibrium conditions similar to ours in their comparison of the market outcome and the social optimum under increasing returns and product differentiation. Pursuing the same objective as Spence and Vives, Dhingra and Morrow (2012) used the elasticity of the marginal utility to show that the CES is the only utility under which the market delivers the first-best outcome in Melitz-like models. However, the main purpose of these papers is different from what we accomplish here. Our model also shares several similarities with Krugman (1979), who showed how decreasing demand elasticity yields what we call price-decreasing competition, but his approach has been ignored in subsequent works. As observed by Neary (2004, p. 177), this is probably because Krugman's specification of preferences "has not proved tractable." Instead, we show that Krugman's approach is *tractable*.[2] Using the concept of relative love for variety, we provide a complete characterization of the market outcome and of all the comparative statics implications in terms of prices, consumption level, outputs, and mass of firms/varieties.

The next section presents the model. The existence, uniqueness, and properties of a free-entry equilibrium are established in Section 3. Extensions are discussed in Section 4. In Section 5, we conclude by discussing some implications of our model for theoretical and empirical works.

[2]It shoud be noted however that, later on, Neary (2009) suggested a rigorous presentation of Krugman's ideas, which is close to ours.

2. THE MODEL

The economy involves one sector supplying a differentiated good and one production factor—labor. There are L workers and each supplies E efficiency units of labor. The unit of labor is chosen as the numéraire, so that E is both a worker's income and expenditure. The differentiated good is made available as a continuum N of horizontally differentiated varieties indexed by $i \in [0, N]$.

2.1. *Preferences and Demand*

Consumers' preferences are *additively separable*. Given a price mapping $\mathbf{p} = p_{i \leq N}$ and an expenditure value E, every consumer chooses a consumption mapping $\mathbf{x} = x_{i \leq N}$ to maximize her utility subject to her budget constraint,

$$\max_{x_i \geq 0} \mathcal{U} \equiv \int_0^N u(x_i)\, di \quad \text{such that} \quad \int_0^N p_i x_i \, di = E,$$

where $u(\cdot)$ is thrice continuously differentiable, strictly increasing, and strictly concave on $(0, \infty)$. Furthermore, it is well known that additive preferences with $u(0) = 0$ are globally homothetic if and only if $u(x) = x^\rho$ with $0 < \rho < 1$.[3] By using a general utility function u, we thus obviate one of the main pitfalls encountered in many applications of the CES.

The concavity of the utility function u means that consumers are variety lovers: rather than concentrate their consumption over a small mass of varieties, they prefer to spread it over the whole range of available varieties. Indeed, for any given quantity $Q > 0$ of the differentiated good, u is concave if and only if $nu(Q/n)$ increases with n for all $n < N$, that is, consumers are variety lovers. In other words, the consumer has a preference for variety if she is willing to trade a lower consumption against more variety.

It should be clear that decision-making by variety-loving consumers is formally equivalent to decision-making in the Arrow–Pratt theory of risk aversion, the mix of risky assets being replaced with the mix of differentiated varieties. It is well known that there are different ways to measure the degree of risk aversion, so the same holds for measuring consumers' attitude toward variety loving. In this paper, we use the *relative love for variety* (RLV):

$$r_u(x) \equiv -\frac{xu''(x)}{u'(x)} > 0.$$

The reason for this choice is that the RLV allows for the characterization of the market outcome in a simple way. When the RLV takes on a higher (lower)

[3]We assume that $u(0) = 0$. Indeed, $u(0) \neq 0$ implies that the introduction of new varieties affects consumers' well-being when they keep their consumption pattern unchanged. This does not strike us as being plausible.

value, the love for variety is said to be stronger (weaker). Under the constant average risk aversion (CARA) utility $u(x) = 1 - \exp(-\alpha x)$ with $\alpha > 0$, the RLV, which is given by αx, increases with the consumption level (Behrens and Murata (2007)). Under the CES, the RLV is constant and given by $1 - \rho$.

To shed more light on the meaning of the RLV, we can appeal to the elasticity of substitution σ between any given pair of varieties (Nadiri (1982, p. 442)). At a symmetric consumption pattern $x_i = x$, σ is such that

$$r_u(x) = 1/\sigma(x). \tag{1}$$

Because the value of $\sigma(x)$ does not depend on the chosen pair of varieties, *the RLV is the inverse of the elasticity of substitution* associated with the consumption level x. Unlike the CES, where the elasticity of substitution is constant, the value of $\sigma(x)$ varies here with the consumption level. As a result, when preferences display an increasing (decreasing) RLV, consumers perceive varieties as being less (more) differentiated when they consume more. This property has a mirror image that may be expressed as follows: when preferences display an increasing (decreasing) RLV, consumers care less (more) about variety when their consumption level is lower, as reflected by the lower (higher) value of the RLV.

Differentiating the Lagrangian with respect to x_i, we obtain the inverse demand function

$$p_i(x_i) = u'(x_i)/\lambda, \tag{2}$$

where the Lagrange multiplier is

$$\lambda = \frac{\displaystyle\int_0^N x_i u'(x_i)\,di}{E}.$$

In other words, the marginal utility of income varies with the consumption function $x(\cdot)$, the mass of varieties N, and the expenditure E. Because λ acts as a demand shifter, (2) implies that the inverse demand inherits the properties of the marginal utility. In particular, $p_i(x_i)$ is strictly decreasing because u is strictly concave.

Denote by $\mathcal{E}g \equiv (x/g)(dg/dx)$ the elasticity of function g. Because all consumers face the same multiplier, the functional form of any variety's demand is the same across consumers $x(p)$, where the index i is disregarded. This implies that the market demand is given by $Lx(p)$. Furthermore, the elasticity $\mathcal{E}_p$ of the inverse demand $p(x)$ and the elasticity $\mathcal{E}_x$ of the demand $x(p)$ are related to the RLV as

$$\frac{1}{r_u(x)} = -\frac{1}{\mathcal{E}_p(x)} = -\mathcal{E}_x(p).$$

Therefore, the RLV is increasing if and only if the demand for a variety becomes more elastic when the price of this variety rises, which corresponds to the case studied by Krugman (1979). Our analysis also copes with the opposite case.

Thus, as in the CES case, the relative love for variety, the elasticity of substitution, and the price elasticity of a variety's demand can be used interchangeably. Unlike the CES, however, their values vary with consumption level x. Moreover, the relationship $r_u(x) = 1/\sigma(x)$ ceases to hold off-diagonal. Note, finally, that the RLV need not be monotone. As a consequence, the demand elasticity, or the elasticity of substitution, may vary in opposite directions with the consumption level for the same RLV function.

2.2. *Producers*

Each firm produces a single variety and no two firms sell the same variety. Firms share the same cost function (we address the case of heterogeneous firms in Section 4). To operate every firm bears a fixed cost $F > 0$ and a variable cost $V(q)$, which is twice continuously differentiable and strictly increasing. The production cost of a firm supplying the quantity q is thus equal to $C(q) = F + V(q)$ for $q > 0$ and $C(0) = 0$. When $V'' \geq 0$, this class of functions includes L- and U-shaped average cost curves.

Varieties are provided by monopolistically competitive firms. As stressed by Vives (1999), one of the main distinctive features of monopolistic competition is that economic agents' decisions are based on a few aggregate statistics of the distribution of firms' actions. Such a statistic is given here by λ, which is the counterpart of the price index in the CES. Being negligible to the market, each firm accurately treats λ as a parameter but must anticipate its equilibrium value to choose its profit-maximizing strategy. Having done this, the firm behaves like a monopolist on its market. Thus, maximizing profits with respect to price or quantity yields the same equilibrium outcome.

Denoting firm i's revenue by $R(q_i)$, the producer's program is

$$\max_{q_i \geq 0} \pi(q_i) = R(q_i) - C(q_i) \equiv \frac{u'(q_i/L)}{\lambda} q_i - V(q_i) - F. \tag{3}$$

2.3. *Equilibrium*

Since all firms face the same Lagrange multiplier, the solutions to the first-order condition for profit maximization,

$$u'(q_i/L) + (q_i/L)u''(q_i/L) = \lambda V'(q_i), \tag{4}$$

are the same across firms. Let $q_i = \bar{q}$ for all $i \in [0, N]$ be such a solution.

The above condition can be rewritten in terms of the markup M as

$$M(\bar{q}) \equiv \frac{p(\bar{q}/L) - V'(\bar{q})}{p(\bar{q}/L)} = r_u(\bar{q}/L) < 1. \tag{5}$$

In other words, at the profit-maximizing output, *the markup of a firm is equal to the RLV*. Consequently, when the RLV is increasing (decreasing), a higher consumption per capita of the differentiated product leads to a higher (lower) markup. The markup is constant if and only if the utility is given by the CES, regardless of the properties of the variable cost function $V(q)$.

The solution $\bar{q}$ is the unique maximizer of the profit function if $\pi(q_i)$ is strictly quasi-concave. This is so when the second-order condition for profit maximization is satisfied at any solution to the first-order condition. Differentiating (4) with respect to q_i, dividing the resulting expression by (4), and rearranging terms yields the condition

$$\bigl[2 - r_{u'}(q_i/L)\bigr]r_u(q_i/L) - \bigl[1 - r_u(q_i/L)\bigr]r_C(q_i) > 0 \quad \text{for all } q_i \geq 0, \tag{6}$$

where $r_C \equiv -qC''/C'$. Though (6) is a priori obscure, it condenses the usual conditions that the demand and cost functions must satisfy for a monopolist's profit function to be quasi-concave. More precisely, (6) holds when the inverse demand is not "too" convex for the marginal revenue to be increasing and the variable cost is not "too" concave for the marginal cost to decrease at a higher rate than the marginal revenue. When the marginal cost is constant, (6) is equivalent to $r_{u'} < 2$.[4] In what follows, we assume that (6) always holds.

Since the market outcome must be symmetric, the zero-profit condition is given by

$$\pi(q) = R(q) - C(q) = 0.$$

A *free-entry equilibrium* (FEE) is defined by an output $\bar{q}$ such that no firm finds it profitable to change its output, a mass of firms $\bar{N}$ that satisfies labor market clearing,

$$\bar{N}C(\bar{q}) = LE, \tag{7}$$

and the value $\bar{\lambda}$ of the Lagrange multiplier such that the zero-profit condition holds. Variety market clearing implies

$$\bar{x} = \bar{q}/L \tag{8}$$

and the equilibrium price is given by

$$\bar{p} = C'(\bar{q})/\bigl[1 - r_u(\bar{x})\bigr]. \tag{9}$$

[4]This argument highlights the role played by the third derivative of the utility for the existence and uniqueness of the equilibrium through $r_{u'}$.

Because $R(L\bar{x}) = C(L\bar{x})$, a FEE is characterized by the equilibrium conditions (4), (8), and

$$(10) \qquad \mathcal{E}_R(\bar{x}) = \mathcal{E}_C(L\bar{x}),$$

where the elasticity of $R(L\bar{x})$ is independent of L. In other words, at a FEE (if any), *the elasticity of a firm's revenue is equal to the elasticity of its cost*. Solving (10) determines $\bar{x}$, whence (8) yields $\bar{q}$, which together with (7) gives $\bar{N}$. Given (6), any solution $\bar{x} > 0$ is associated with a FEE.

Because $\mathcal{E}_R$ and $\mathcal{E}_C$ do not depend on E, the consumption per capita $\bar{x}$ is independent of the expenditure level. It then follows from (5) that $\bar{q}$, $\bar{p}$, and $\bar{M}$ are also independent of E. By contrast, (7) shows that $\bar{N}$ is a linear function of E.

Before proceeding, it is worth comparing the equilibrium condition (10) to the optimality condition that characterizes the first-best outcome x^* (Vives (1999)):

$$(11) \qquad \mathcal{E}_u(x^*) = \mathcal{E}_C(Lx^*).$$

Under the CES, the two expressions are identical because $\mathcal{E}_u(x) = \mathcal{E}_R(x) = \rho$. Therefore, in a one-sector economy where consumers have CES preferences, the first-best and the market outcomes are identical. Dhingra and Morrow (2012) extended this result to heterogeneous firms. On the other hand, as shown by Dixit and Stiglitz (1977) and Vives (1999), this relationship ceases to hold in a multisector economy: the market equilibrium does not distort output, but it does undersupply variety.

Given the similarities between (11) and (10), it is natural to ask how the RLV and Vives' preference for variety are related. Because the derivative of $nu(Q/n)$ with respect to n is equal to $u(Q/n)(1 - \mathcal{E}_u)$, Vives proposed to measure the preference for variety by $1 - \mathcal{E}_u \geq 0$. Using the utility $u(x) = x + 2\ln(1 + x)$, we can readily verify that the RLV is an inverted U-shape, whereas Vives' preference for variety is increasing. The two concepts are thus independent. The need for different concepts to measure the love for variety can be explained as follows: *the planner cares about the elasticity of utility, whereas firms care about the elasticity of demand*.

3. THE MARKET OUTCOME

3.1. *Existence and Uniqueness of a FEE*

(i) A FEE exists if and only if the two loci $\mathcal{E}_R(x)$ and $\mathcal{E}_C(Lx)$ intersect at least once. Assume

$$(12) \qquad 0 \leq \mathcal{E}_C(0) < \mathcal{E}_R(0) < \infty, \quad \mathcal{E}_R(\infty) < \mathcal{E}_C(\infty).$$

The functions $\mathcal{E}_R(x)$ and $\mathcal{E}_C(Lx)$ being continuous, the intermediate value theorem implies that (10) has a positive and finite solution x. Because $\mathcal{E}_R(x) = \mathcal{E}_C(Lx) > 0$ at any intersection point, it must be $r_u(x) < 1$ at any such point.

The inequalities (12) are satisfied under fairly common assumptions. Indeed, $\mathcal{E}_C$ is increasing when the marginal cost is constant (with $\mathcal{E}_C(0) = 0$ and $\mathcal{E}_C(\infty) = 1$) or increasing. Furthermore, it follows from $\mathcal{E}_R(x) = 1 - r_u(x)$ that $\mathcal{E}_R(0) > 0$ is equivalent to $r_u(0) < 1$. This inequality rules out the case of a market outcome where firms choose to sell a zero quantity at an infinite price; in particular, $r_u(0) = 0$ when there is a chocke price. Similarly, $r_u(\infty) = 0$ when there is a saturation point. Under these circumstances, the inequality $\mathcal{E}_R(\infty) < \mathcal{E}_C(\infty)$ holds.

(ii) We now prove that the FEE is unique. The condition (10) may be rewritten as

$$pq - V(q) = \frac{V'(q)}{1 - r_u(q/L)}q - V(q) = F. \tag{13}$$

Under (6), the left-hand side of this expression is increasing and, thus, the above equation has at most one solution. To sum up, (12) and the continuity of profits are used to show the existence of a FEE, whereas the quasi-concavity of profits (6) allows proving uniqueness. These conditions are similar to those usually made for a monopolist to have a single profit-maximizing output.

Last, using the solution to (13) and the labor market clearing condition, we obtain the equilibrium mass of firms $\bar{N}$.

Proposition 1 comprises a summary.

PROPOSITION 1: *If* (6) *and* (12) *hold*, *then a unique FEE exists*. *Furthermore*, *the FEE satisfies the conditions*

$$\mathcal{E}_R(\bar{x}) = \mathcal{E}_C(L\bar{x}), \quad \bar{M} = r_u(\bar{x}), \quad \bar{q} = L\bar{x}, \quad \bar{N} = EL/C(\bar{q}).$$

The determination of the FEE is illustrated in Figure 1 when the RLV is increasing.

3.2. *Market Size*

In this subsection, we study the impact of a larger market on the FEE by increasing L from L_1 to L_2. For simplicity in exposition, we assume that V is weakly convex, which implies that $\mathcal{E}_C$ is increasing (details are given in Appendix A in the Supplemental Material (Zhelobodko, Kokovin, Parenti, and Thisse (2012))). Note, however, that all our results hold true when V is not too concave.

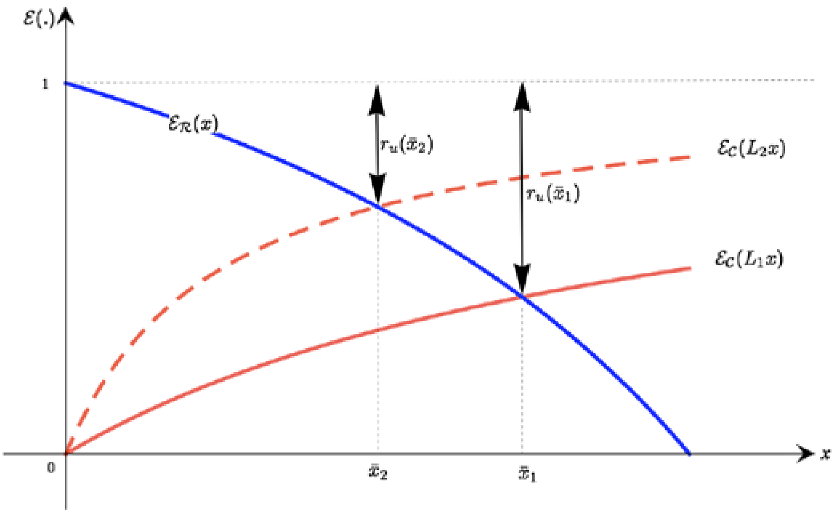

FIGURE 1.——Impact of L on the FEE under increasing RLV.

Consumption per capita

Assume that the market size increases from L_1 to L_2. The curve $\mathcal{E}_R(x) = 1 - r_u(x)$ is independent of L, whereas the curve $\mathcal{E}_C(Lx)$ is shifted leftward by an increase in L. Therefore, as illustrated by Figure 1, the two curves intersect at a value of x that is smaller than its initial equilibrium value: $d\bar{x}/dL < 0$. Indeed, keeping constant the equilibrium mass of firms $\bar{N}_1$ prevailing when the market size is L_1, the incumbents face a higher demand when the market size grows to L_2. This invites entry, and thus the new equilibrium mass of firms $\bar{N}_2$ exceeds $\bar{N}_1$. In this case, in a market providing more variety, consumers trade a lower consumption of each variety against a more diversified basket of varieties. Moreover, when the RLV is increasing (decreasing), the lower consumption level implies that consumers' love for variety becomes weaker (stronger), which leads to a mild (sharp) decline in consumption (see Proposition 1).

Output

Consider now the impact of L on output. When the RLV is increasing, it follows from the above result that $r_u(\bar{x}_2) < r_u(\bar{x}_1)$. Thus, the inverse demand is less elastic at $\bar{x}_2$ than at $\bar{x}_1$, which entails a hike in the elasticity of a firm's revenue: $\mathcal{E}_R(\bar{x}_2) > \mathcal{E}_R(\bar{x}_1)$. As illustrated in Figure 1, the equilibrium condition (10) implies that $\mathcal{E}_C(L_2\bar{x}_2) > \mathcal{E}_C(L_1\bar{x}_1)$. The function $\mathcal{E}_C$ being increasing owing to the convexity of $V(q)$, it must be that $\bar{q}_2 = L_2\bar{x}_2 > L_1\bar{x}_1 = \bar{q}_1$. In other words, when the RLV is increasing there is a scale effect associated with a larger market size: $d\bar{q}/dL > 0$. Furthermore, since $\bar{x}$ decreases, this scale effect implies $-1 < \mathcal{E}_{\bar{x}/L} < 0$, that is, $\bar{x}$ decreases less than proportionally with L. A similar argument shows that $d\bar{q}/dL < 0$ and $\mathcal{E}_{\bar{x}/L} < -1$ when the RLV is decreasing.

In the CES case, $\mathcal{E}_R(x) = \rho$ and thus the elasticity of $\bar{x}$ with respect to L is equal to -1 because $\mathcal{E}_C(Lx)$ is shifted proportionally to L. Because $\mathcal{E}_R(q/L)$ is constant and $\mathcal{E}_C(q)$ is independent of L, the equilibrium size of firms is independent of the market size.

It is worth stressing that the above three patterns hold *regardless* of the behavior of the marginal cost function.

Markup and Price

Regarding markups, we have seen that the equilibrium consumption $\bar{x}$ always decreases with L, so that the equilibrium markup $\bar{M} = r_u(\bar{x})$ decreases with L when the RLV is increasing: $d\bar{M}/dL < 0$. Similarly, we have $d\bar{M}/dL > 0$ if and only if $r_u' < 0$, whereas the markup is constant in the CES case.

How does the equilibrium price react to a larger market size when $V(q)$ is weakly convex? Differentiating the zero-profit condition with respect to L and using the convexity of $V(q)$ shows that firms' output and market price always move in opposite directions when L increases. Since the equilibrium output rises (falls) with L when the RLV is increasing (decreasing), the market price must decrease (increase) with L in the former (latter) case regardless of the behavior of the marginal cost function. In other words, under well behaved utilities, a larger market may lead to a lower or higher market price.

The intuition for this result is straightforward when the marginal cost is constant. In this case, $\bar{M}$ and $\bar{p}$ always move in the same direction. When $r_u' > 0$, (1) implies that the elasticity of substitution increases with L. This means that the entry of new firms, hence varieties, implies that consumers view varieties as being less differentiated. This in turn makes competition tougher, thus leading to a lower market price ($d\bar{p}/dL < 0$). This result has a mirror image expressed in terms of love for variety: consumers' weaker love for variety incentivizes firms to compete more fiercely.

On the contrary, when $r_u' < 0$, the elasticity of substitution $\sigma(\bar{x})$ decreases with L. A higher degree of product differentiation implies that competition is relaxed and, thus, the market price is higher ($d\bar{p}/dL > 0$). However, the market price does not become arbitrarily large because the consumption per capita of each variety decreases with L at a rate exceeding 1. Using the above mirror image, we can say that competition is relaxed because consumers display a stronger love for variety at the new equilibrium.

In the CES case with variable marginal cost, the equilibrium price is always constant because both the equilibrium markup and output are independent of L. This implies that an equilibrium outcome in which price and output vary with market size cannot be rationalized by a setting involving a CES utility and a variable marginal cost.

Number of Varieties

We know that $d\bar{N}/dL > 0$. When $r_u' > 0$, the entry of new firms triggered by an increase in market size makes competition tougher and yields a lower market price. Everything else being equal, this lowers profits and slows down the entry process, which stops when the increase in the mass of firms is less proportional than the increase in market size. On the contrary, when $r'_u < 0$, the entry of new firms results in higher prices, which invites more entry. This process will come to an end because individual consumption decreases sharply as the love for variety gets stronger. In this case, the elasticity exceeds 1.[5]

To sum up, we present the next proposition.

PROPOSITION 2: *Assume that* (6) *and* (12) *hold while the variable cost V is weakly convex. Then, if the market size increases, the equilibrium outcome is described by the following three different patterns*:

Elasticity	$r'_u(\bar{x}) > 0$	$r'_u(\bar{x}) = 0$	$r'_u(\bar{x}) < 0$
Price $\bar{p}(L)$	$\mathcal{E}_{\bar{p}} < 0$	$\mathcal{E}_{\bar{p}} = 0$	$0 < \mathcal{E}_{\bar{p}}$
Diversity $\bar{N}(L)$	$0 < \mathcal{E}_{\bar{N}} < 1$	$\mathcal{E}_{\bar{N}} = 1$	$1 < \mathcal{E}_{\bar{N}}$
Consumption $\bar{x}(L)$	$-1 < \mathcal{E}_{\bar{x}} < 0$	$\mathcal{E}_{\bar{x}} = -1$	$\mathcal{E}_{\bar{x}} < -1$
Output $\bar{q}(L)$	$0 < \mathcal{E}_{\bar{q}} < 1$	$\mathcal{E}_{\bar{q}} = 0$	$\mathcal{E}_{\bar{q}} < 0$

This proposition shows that *the properties of the market outcome are determined by the variety-loving attitude of consumers and not by the production conditions*, which can exhibit L- and U-shaped average cost curves.

The contrasted results displayed in Proposition 2 suggest the existence of two regimes with the CES as a borderline case, namely, *price-decreasing* and *price-increasing competition*. In other words, according to the nature of preferences, the market mimics pro- or anti-competitive behavior. Yet, it must be stressed that the same principle stands behind the difference in results: a higher degree of product differentiation softens competition. What the above results show is that the degree of product differentiation may grow or fall with the number of varieties, thus affecting the market price accordingly. The CES is the only function for which entry does not impact the equilibrium price. Therefore, we may safely conclude that the CES is the borderline between two different classes of utility functions, which give rise to price-decreasing or price-increasing competition.

Another peculiar feature of the CES is that the equilibrium size of firms ($\bar{q}$) is independent of the market size. Our results show that *firms' size increases in the price-decreasing regime*. This is because the industry size $\bar{N}\bar{q}$ grows at a lower pace than the market size. On the contrary, firms' size decreases when there is price-increasing competition because the mass of firms rises at a more than proportionate rate. These effects combine to yield a lower firm output. All of this provides a possible reconciliation between the diverging empirical results mentioned in the Introduction. Indeed, Proposition 2 suggests that the

[5]Formally, this result may be proven by using (6) and (7).

sign of the correlation between these two variables depends on the nature of preferences, which need not behave in the same way across goods.

Under the CES, individual welfare always increases with market size. When the RLV increases, this is a fortiori true because the market price decreases and the mass of varieties increases. Things are less clear under a decreasing RLV. Indeed, the mass of firms increases at a high rate but the market price also rises. Totally differentiating the equilibrium utility $\bar{N}u(\bar{x})$ with respect to L shows that this derivative is positive if and only if $\mathcal{E}_{\bar{N}/L} > -\mathcal{E}_u \cdot \mathcal{E}_{\bar{x}/L}$. When this inequality does not hold, the gain stemming from more varieties is outweighed by the loss generated by a higher market price.

4. EXTENSIONS

4.1. *Multisector Economy*

Consider a two-sector economy involving a differentiated good X supplied under increasing returns and monopolistic competition, and a homogeneous good Y supplied under constant returns and perfect competition. Labor is the only production factor and is perfectly mobile between sectors. Each individual is endowed with preferences defined by

$$\max_{X,Y} \mathcal{U} \equiv U(X, Y) = U\left[\int_0^N u(x_i)\, di, Y\right],$$

where U is strictly increasing and concave. Choosing the unit of the homogeneous good for the marginal productivity of labor to be equal to 1 and choosing the homogeneous good as the numéraire, the equilibrium wage is equal to 1. Since profits are zero, the budget constraint is given by

$$\int_0^N p_i x_i\, di + Y = E + Y = 1,$$

where E is now endogenous because competition across firms is affected by the relative preference between the goods X and Y.

The consumer optimization problem is decomposed into two subproblems (note that doing so is not equivalent to the standard two-stage budgeting approach). First, for any given $E < 1$, the lower-tier utility maximization problem is given by

$$\max_{x_i \geq 0} \int_0^N u(x_i)\, di \quad \text{such that} \quad \int_0^N p_i x_i\, di = E,$$

the optimal value of which is the indirect utility

$$v(p, N, E) \equiv N u\left(\frac{E}{Np}\right).$$

Because the equilibrium price ($\bar{p}$), output ($\bar{q}$), and consumption ($\bar{x}$) prevailing in the differentiated sector are independent of the value of E, *they inherit the properties stated in Proposition 2 for any specification of* U. It is worth stressing that this remains true in general equilibrium models involving several sectors supplying differentiated goods.

By contrast, we know that the mass of firms depends on the value of E, which is given by the solution $E(p, N)$ to the upper-tier maximization problem

$$\max_E U\big[v(p, E, N), 1 - E\big].$$

How the equilibrium mass of varieties varies with L is thus more involved than in Section 3, because E now varies with N and p. Therefore, to determine the properties of $\bar{N}$, we need additional assumptions. In the Appendix B in the Supplemental Material, we give sufficient conditions for the utilities U and u to yield an expenditure function $E(p, N)$ that satisfies the properties

$$0 \le \frac{p}{E} \cdot \frac{\partial E}{\partial p} < 1, \qquad \frac{N}{E} \cdot \frac{\partial E}{\partial N} < 1. \tag{14}$$

The interpretation of these conditions has some intuitive appeal. First, if X and Y are complements ($U''_{12} \ge 0$), the second and third inequalities hold. Furthermore, the first inequality means that a higher price for the differentiated good leads consumers to spend more on this good, which seems reasonable. In the Appendix B in the Supplemental Material, we show that *the equilibrium mass of varieties increases with* L *when* (14) *holds*.

4.2. *Heterogeneous Firms*

Firms are heterogeneous in that their variable cost functions $V(q, \theta)$ are parametrized by a firm's efficiency index θ. The parameter θ is distributed according to the continuous density $\gamma(\theta)$ defined on $[0, \infty)$. We assume the marginal cost functions satisfy the Spence–Mirrlees condition: for any given $q \ge 0$, $\partial V/\partial q$ strictly increases with θ. Since $V(0) = 0$, this implies that the monotonicity of the variable cost, $V(q, \theta)$, increases with θ for all $q \ge 0$. In other words, as θ rises, firms are less efficient. Since firms of different types face the same downward-sloping marginal revenue, the monotonicity condition for variable cost functions implies that type θ_1 firms are more profitable than type θ_2 firms if and only if $\theta_1 < \theta_2$. In the special case where $V(q, \theta) = \theta q$, the cost side of our framework is equivalent to that of Melitz (2003). By contrast, when V is not linear, the average productivity is endogenously determined through two different channels, that is, the cutoff efficiency index and the distribution of output across operating firms.

Preferences being defined as in Section 2, the inverse demand function is given by $p_\theta(x_\theta) = u'(x_\theta)/\lambda$ so that a type θ firm solves the same program as in

Section 2. The Spence–Mirrlees condition thus implies that lower θ firms have a greater output, a lower price, and higher profits than higher θ firms.

Assume that a Melitz-like free-entry equilibrium $(\bar{\theta}, \bar{\lambda}, \bar{q}_{\theta\leq\bar{\theta}}, \bar{x}_{\theta\leq\bar{\theta}})$ exists. Using (5), a type θ firm's markup is given by

$$(15) \qquad \bar{M}_\theta = r_u(\bar{x}_\theta) = 1/\sigma(\bar{x}_\theta).$$

Because $\sigma(\bar{x}_\theta)$ measures the elasticity of substitution among type θ varieties, (15) implies that *the elasticity of substitution varies across firm types*. Specifically, when $r_u' > 0$ ($r_u' < 0$), the degree of product differentiation among varieties supplied by low θ firms is lower (higher) than the degree of those provided by high θ firms.

Given (6), for each type θ firm, the maximum operating profits $\pi_o^*(\theta, \lambda; L)$ are well defined and continuous:

$$\pi_o^*(\theta, \lambda; L) \equiv \max_{q\geq 0}\left\{\frac{u'(q/L)}{\lambda}q - V(q, \theta)\right\}.$$

The envelop theorem implies that, for all θ, $\pi_o^*(\theta, \lambda; L)$ is strictly decreasing in λ. As a result, the solution $\bar{\theta}(\lambda; L)$ to $\pi_o^*(\theta, \lambda; L) - F = 0$ is unique. Using the monotonicity condition for variable cost functions, the free entry condition can be rewritten as

$$(16) \qquad \int_0^{\bar{\theta}(\lambda;L)} \left[\pi_o^*(\theta, \lambda; L) - F\right]\gamma(\theta)\, d\theta - F_e = 0,$$

where F_e is the entry cost. Using the zero-profit condition at $\bar{\theta}(\lambda; L)$, the left-hand side of (16) is decreasing in λ. As a consequence, the above equation has a unique solution $\bar{\lambda}$, which in turn determines the equilibrium cutoff $\bar{\theta}(L) = \bar{\theta}(\bar{\lambda}; L)$. In other words, the FEE, if it exists, is unique. The expression (16) also shows that a FEE exists when the fixed production cost F and the entry cost F_e are not too large.

We now study the impact of market size on the cutoff efficiency index. The zero-profit condition at $\bar{\theta}$ implies that

$$\frac{\partial \pi_o^*}{\partial L} + \frac{\partial \pi_o^*}{\partial \theta}\frac{d\bar{\theta}}{dL} + \frac{\partial \pi_o^*}{\partial \lambda}\frac{d\bar{\lambda}}{dL} = 0.$$

Rewriting this expression in terms of elasticity and applying the envelop theorem to each term shows that the elasticity of $\bar{\theta}$ with respect to L is such that

$$\mathcal{E}_{\bar{\theta}} = \frac{r(\bar{q}_{\bar{\theta}}/L) - \mathcal{E}_{\bar{\lambda}}}{\bar{\theta} \cdot \dfrac{\partial V(\bar{\theta})}{\partial \theta}}.$$

Up to a positive factor, the numerator of this expression is equal to

$$\int_0^{\bar{\theta}} \left[r_u(\bar{q}_{\bar{\theta}}/L) - r_u(\bar{q}_\theta/L) \right] \bar{R}(\theta)\gamma(\theta)\, d\theta, \tag{17}$$

where $\bar{R}(\theta)$ is the equilibrium revenue of a type θ firm. The Spence–Mirrlees condition implies that $\bar{q}_\theta$ decreases with θ. As a consequence, (17) is positive (negative) if and only if r_u is decreasing (increasing).

The following proposition is a summary.

PROPOSITION 3: *Assume that* (6) *and* (12) *hold while the variable cost V is weakly convex and satisfies the Spence–Mirrlees condition*. *Regardless of the distribution of θ*, *the cutoff efficiency index decreases* (*increases*) *with market size if* $r_u'(\bar{x}) > 0$ ($r_u'(\bar{x}) < 0$). *Furthermore*, *the cutoff efficiency index is independent of the market size if and only if u is the CES*.

Thus, the distinction between the price-decreasing and price-increasing regimes keeps its relevance even when cost functions differ across firms. Specifically, regardless of the behavior of the marginal cost, *the way the cutoff efficiency index $\bar{\theta}$ changes with market size depends only on the behavior of the RLV*. Under price-decreasing competition, a larger market makes competition tougher, which triggers the exit of the least productive firms. On the contrary, in the price-increasing regime, a larger market softens competition, which allows less productive firms to operate. Last, even when marginal costs are variable, *the cutoff remains unaffected by market size if and only if preferences are CES*.

Constant Marginal Costs

Under variable marginal costs, studying the impact of market size on firms' decisions is hard because many effects are involved. This is why we now consider the literature-based case of constant marginal costs ($V(q, \theta) = \theta q$). Repeating for each type of firm the argument developed in the homogeneous firm case, it is readily verified that the individual consumption $\bar{x}_\theta$ decreases with L for all θ smaller than the cutoff cost $\bar{\theta}$. Therefore, when the RLV is increasing, the markup increases with L for all firms remaining in business. It then follows from (9) that a larger market leads to lower prices as in the case of homogeneous firms. As a consequence, *the average productivity gain generated by a larger market is shared between firms and consumers through a higher average markup and a lower average price* regardless of the distribution of the efficiency index. The opposite holds true when the RLV is decreasing.

4.3. *Nonadditive Preferences*

Our approach can also be extended to cope with nonadditive preferences. To make comparison with the literature easier, we consider homogeneous firms and constant marginal costs. A first example is provided by the quadratic utility in which the subutility of variety i is given by

$$u(x_i, \mathbb{X}) = x_i - \frac{x_i^2}{2} - \gamma x_i \int_0^N x_j \, dj, \tag{18}$$

γ being a positive parameter that expresses the substitutability between variety i and any other variety. The nonadditivity of preferences is reflected by the fact that $u(x_i, \mathbb{X})$ is shifted downward with the total consumption of the differentiated good.

When (18) is nested in a linear utility U, the Lagrange multiplier equals 1 and the inverse demand evaluated is given by $p_i(x_i, \mathbb{X}) = 1 - x_i - \gamma\mathbb{X}$. In this case, varieties compete through the cross-effects $x_i\mathbb{X}$ and not through the budget constraint as in Section 2. For any given $\mathbb{X}$, the elasticity of the above demand is increasing. However, this is not sufficient to imply that the quadratic utility generates price-decreasing competition. Indeed, the market aggregate $\mathbb{X}$ also changes with market size. To account for the full impact of L, we must study the behavior of the RLV at the equilibrium. Rewriting $\mathbb{X}$ in terms of the profit-maximizing condition, the RLV evaluated at the FEE consumption,

$$r_u(\bar{x}) = \frac{\bar{x}}{\bar{x} + c},$$

is increasing.[6] Hence, the market outcome is described by the price-decreasing regime as in Section 4.1.

Regarding the translog expenditure function, Feenstra (2003) showed that the demand for variety i is given by

$$d(p_i; \Lambda_{\text{trans}}, L) = \frac{L}{p_i}(\Lambda_{\text{trans}} - \beta \ln p_i),$$

where Λ_{trans} is a market aggregate treated parametrically by firms and determined at the equilibrium. Under the CARA utility $u(x) = 1 - \exp(-x/\beta)$, the corresponding demand is

$$d(p_i; \Lambda_{\text{cara}}, L) = L(\Lambda_{\text{cara}} - \beta \ln p_i),$$

where $\Lambda_{\text{cara}} \equiv -\beta \ln(\beta\lambda)$ while λ is the marginal utility of income.

Applying the first-order condition, we readily verify that the equilibrium prices $\bar{p}_{\text{trans}}$ and $\bar{p}_{\text{cara}}$ solve, respectively, the equilibrium conditions (see Appendix C in the Supplemental Material)

$$\beta(p - c)^2/p = cf/L, \quad \beta(p - c)^2/p = f/L.$$

[6]This is identical to the RLV obtained under the Stone–Geary preferences used by Simonovska (2010).

Consequently, if the unit of the differentiated product is chosen for $c = 1$, the two prices are equal and thus $\bar{p}_{\text{trans}}$ decreases with market size as $\bar{p}_{\text{cara}}$ does. Therefore, the market outcome under the nonadditive translog behaves like the market outcome under the additive CARA utility. This example highlights the generality of additive preferences in applications of monopolistic competition to various topics in economic theory.

5. CONCLUDING REMARKS

Our purpose was to develop a general, but tractable, model of monopolistic competition that obviates the shortcomings of the CES mentioned in the Introduction. This new setting encompasses different features of oligopoly theory, while retaining most of the tractability of the CES model. Moreover, we have been able to provide a full characterization of the market equilibrium and to derive conditions for the market to display price-decreasing or price-increasing competition under well behaved utility functions. That the properties of the market outcome are characterized through necessary and sufficient conditions in terms of the RLV suggests that this concept is meaningful in conducting positive analyses.

We would be the last to claim that using the CES is a defective research strategy. Valuable theoretical insights have been derived from this model by taking advantage of its various specificities. In this respect, if the world is CES, it is worth stressing that our analysis shows that the assumption of constant marginal cost is not restrictive since the properties of the market outcome remain the same when the marginal cost is variable. However, having shown how peculiar are the results obtained under a CES utility, it is our contention that a "theory" cannot be built on this model.

From the empirical viewpoint, we want to make the following points. Because the elasticity of substitution is likely to vary across space and time, our paper suggests that the estimations performed in many empirical papers lack solid microeconomic foundations. For example, our model calls for a more careful interpretation of CES-based estimations of the gravity equation. In the same vein, several recent empirical trade papers interpret variations in output unexplained by prices as quality differences. Our analysis suggests a complementary explanation: demands are different because the RLV of the corresponding varieties takes on different values.

Furthermore, to test whether the RLV is increasing or decreasing in the neighborhood of the equilibrium, at least two strategies are available. The first one is a direct estimation of the elasticity of substitution at different points in time and/or space. This strategy has been implemented for the CES (Feenstra (1994)). Instead of assuming a constant elasticity across time, we plead for a parametrization of the RLV that captures the fact that the values of the RLV may change over time and/or across space. Using the translog comes down to such a research strategy since it allows for a variable elasticity of substitution.

However, as seen above, such a specification generates price-decreasing competition only and, thus, the possibility of price-increasing competition in some sectors *cannot* be tested. Consequently, there is a need for a more general specification of preferences that encompasses both types of market behavior.

The second strategy consists in determining the behavior of the RLV as implied by the theoretical predictions of our model. One approach is to isolate the effect of market size on the equilibrium output and/or price of specific products for which preferences are more or less homogeneous across particular geographical markets. This could be accomplished by using the research strategy implemented by Asplund and Nocke (2006), who investigated the effect of market size on firm turnover. Note that a cross-sectional comparison of the demand for one variety exported to different countries would also allow identifying the monotonicity of the RLV provided that countries are chosen for the heterogeneity in consumer preferences to be weak.

Last, various "augmented-CES" models have been successfully developed to cope with different issues. Yet, it is hard to figure out how the corresponding results can be reconciled within a unified framework. We believe that the model presented here displays enough versatility to provide a framework in which several of these approaches can be recast.

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Novosibirsk State University, 630090, Novosibirsk, Pirogova 2, Russia, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia; ezhelobodko@gmail.com,

Novosibirsk State University and Sobolev Institute of Mathematics, 630090, Novosibirsk, Pirogova 2 and 4, Russia, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia; skokov7@gmail.com,

CORE-UCLouvain, 34 Voie du Roman Pays, 1348 Louvain la Neuve, Belgium, Paris School of Economics (Paris 1), 106-112 Bd de l'Hôpital, 75013 Paris, France, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia; parenti.mathieu@gmail.com,

and

CORE-UCLouvain, 34 Voie du Roman Pays, 1348 Louvain la Neuve, Belgium, National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia, and CEPR; jacques.thisse@ucloubain.be.

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Econometrica Supplementary Material

SUPPLEMENT TO "MONOPOLISTIC COMPETITION: BEYOND THE CONSTANT ELASTICITY OF SUBSTITUTION"

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BY EVGENY ZHELOBODKO, SERGEY KOKOVIN, MATHIEU PARENTI, AND JACQUES-FRANÇOIS THISSE

IN THIS APPENDIX, we prove the various statements made in our paper. In Appendix A, we study the impact of market size on the FEE. Appendix B is devoted to the multisector economy, while Appendix C shows that equilibrium under the translog behaves like equilibrium under the CARA.

APPENDIX A: THE IMPACT OF MARKET SIZE ON THE FEE

It is readily verified that (6) is equivalent to

$$r_u'x + (r_u - r_C)(1 - r_u) > 0. \tag{A.1}$$

This expression will be used below.

Output. Differentiating (10) leads to

$$\frac{[qV''(\bar{q}) + V'(\bar{q})]C(\bar{q}) - q[V'(\bar{q})]^2}{[C(\bar{q})]^2}\frac{d\bar{q}}{dL} = -r_u'\left(\frac{1}{L}\frac{d\bar{q}}{dL} - \frac{\bar{q}}{L^2}\right).$$

Using

$$V'(\bar{q})\bar{q} = (1 - r_u)C(\bar{q}),$$

we obtain

$$(r_u - r_C)(1 - r_u)\frac{L}{\bar{q}} \cdot \frac{d\bar{q}}{dL} = -r_u'\left(\frac{d\bar{q}}{dL} - \frac{\bar{q}}{L}\right),$$

which amounts to

$$(r_u - r_C)(1 - r_u)\mathcal{E}_{\bar{q}/L} = -r_u'\frac{\bar{q}}{L}(\mathcal{E}_{\bar{q}/L} - 1).$$

Thus, the elasticity of $\bar{q}$ with respect to (w.r.t.) to L is equal to

$$\mathcal{E}_{\bar{q}/L} = \frac{r_u'\bar{x}}{r_u'\bar{x} + (r_u - r_C)(1 - r_u)}.$$

It follows from (6) that the denominator is positive. Consequently, a firm's output increases (decreases) when the RLV is increasing (decreasing). Furthermore, the weak convexity of V implies that

$$\mathcal{E}_{\bar{q}/L} < 1. \tag{A.2}$$

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Consumption per capita. It is readily verified that the elasticity of $\bar{x}$ w.r.t. L can be derived from $\mathcal{E}_{\bar{q}/L}$ as

$$\mathcal{E}_{\bar{x}/L} = \mathcal{E}_{\bar{q}/L} - 1 = -\frac{(r_u - r_C)(1 - r_u)}{r'_u \bar{x} + (r_u - r_C)(1 - r_u)}.$$

Thus, $\bar{x}$ decreases with L when $r_u - r_V > 0$. Observe that this inequality holds when V is convex or not too concave.

Markup. From the comparative statics above, it is straightforward that markups decrease (increase) with L if and only if the RLV is increasing (decreasing).

Price. It follows from (9) that

$$\text{(A.3)} \qquad \frac{d\bar{p}}{dL} = \frac{V'(\bar{q})\bar{q} - C(\bar{q})}{\bar{q}^2} \cdot \frac{d\bar{q}}{dL}.$$

Then, firms' output and market price move in opposite directions with L:

$$\frac{d\bar{p}}{dL} = -r_u \frac{C(\bar{q})}{\bar{q}^2} \cdot \frac{d\bar{q}}{dL}.$$

Number of varieties. The number of varieties $\bar{N}$ is determined by labor market clearing:

$$\bar{N} C(\bar{q}) = L.$$

Thus, the elasticity of $\bar{N}$ w.r.t. L is

$$\mathcal{E}_{\bar{N}/L} + \mathcal{E}_C \cdot \mathcal{E}_{\bar{q}/L} = 1,$$

which amounts to

$$\mathcal{E}_{\bar{N}/L} = 1 - \mathcal{E}_C \cdot \frac{r'_u \bar{x}}{r'_u \bar{x} + (r_u - r_C)(1 - r_u)}.$$

Again, the denominator of the second term is strictly positive by (A.1). Furthermore, at the equilibrium, it must be that $0 < \mathcal{E}_C(L\bar{x}) = 1 - r_u(\bar{x}) < 1$ and, thus, the sign of $\mathcal{E}_{\bar{N}/L} - 1$ is determined by r'_u. Consequently, the elasticity of $\bar{N}$ w.r.t. L is smaller (larger) than 1 if the RLV is increasing (decreasing).

APPENDIX B: THE MULTISECTOR ECONOMY

Properties of the Expenditure Function in the Two-Sector Economy

The following two lemmas provide a rationale for the following assumptions made in Section 4.1:

$$\text{(B.1)} \qquad 0 \le \frac{p}{E} \cdot \frac{\partial E}{\partial p} < 1, \qquad \frac{N}{E} \cdot \frac{\partial E}{\partial N} < 1.$$

Set

$$D \equiv U''_{11} \cdot \big(v'_E\big)^2 - 2U''_{12} v'_E + U''_{22} + U'_1 v''_{EE}.$$

LEMMA 1: *If $U''_{21} \geq 0$, then the elasticity of E w.r.t. N is such that*

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = \frac{-U''_{11} v'_E v + U''_{21}(v + v'_E E) - U''_{22} E}{DE} \leq 0.$$

LEMMA 2: *If $U''_{21} \geq 0$ and the inequality*

$$\text{(B.2)} \qquad \frac{1 - r_u(x)}{\mathcal{E}_u(x)} \leq \frac{U''_{21}(X, Y)X}{U'_2(X, Y)} - \frac{U''_{11}(X, Y)X}{U'_1(X, Y)}$$

hold at a symmetric outcome, then the elasticity of E w.r.t. p is such that

$$\text{(B.3)} \qquad -1 \leq \frac{\partial E}{\partial p} \cdot \frac{p}{E} - 1 = \frac{U'_1 v'_E + U''_{21} E v'_E - E U''_{22}}{DE} \leq 0.$$

REMARK: Under $u(0) = 0$, the indirect utility function

$$v(p, E, N) = N u\left(\frac{E}{pN}\right)$$

is homogeneous of degree 0 w.r.t. (p, E) and of degree 1 w.r.t. (E, N). Therefore, v'_E and v'_p are homogeneous of degree -1 w.r.t. (p, E) and of degree 0 w.r.t. (E, N). Finally, we have $v''_{EE} < 0$.

Let $E(p, N)$ be the unique solution to the first-order condition for the upper-tier utility maximization,

$$\text{(B.4)} \qquad U'_1\big(v(p, E, N), 1 - E\big) v'_E(p, E, N) - U'_2\big(v(p, E, N), 1 - E\big) = 0,$$

where the second-order condition is given by

$$D < 0.$$

Note that $U(v(p, E, N), 1 - E)$ is concave w.r.t. E because U is concave, while the concavity of u implies that of v.

PROOF OF LEMMA 1: Differentiating (B.4) w.r.t. N and solving for $\partial E / \partial N$, we get

$$\frac{\partial E}{\partial N} = -\frac{U''_{11} v'_E v'_N + U'_1 v''_{EN} - U''_{21} v'_N}{D} = -\frac{(U''_{11} v'_E - U''_{21}) v'_N + U'_1 v''_{EN}}{D}.$$

Consequently,

$$\begin{aligned}\frac{\partial E}{\partial N}\cdot\frac{N}{E}-1&=-N\frac{(U''_{11}v'_E-U''_{21})v'_N+U'_1v''_{EN}}{DE}-1\\&=\Big(-U''_{11}\big[v'_ENv'_N+E\big(v'_E\big)^2\big]+U''_{21}\big(Nv'_N+2v'_EE\big)\\&\quad-U'_1\big(Nv''_{EN}+Ev''_{EE}\big)-EU''_{22}\Big)\\&\quad/(DE).\end{aligned}$$

Applying the Euler theorem to v and v', we obtain the equalities

$$\begin{aligned}&-U''_{11}\big[v'_ENv'_N+E\big(v'_E\big)^2\big]=-U''_{11}v'_E\big(Nv'_N+Ev'_E\big)=-U''_{11}v'_Ev,\\&U''_{21}\big(Nv'_N+2Ev'_E\big)=U''_{21}\big(v+Ev'_E\big),\\&-U'_1\big(Nv''_{EN}+Ev''_{EE}\big)=0.\end{aligned}$$

As a result, we have

$$\frac{\partial E}{\partial N}\cdot\frac{N}{E}-1=\frac{-U''_{11}v'_Ev+U''_{21}(v+Ev'_E)-EU''_{22}}{DE}.$$

Since $U''_{21}\geq 0$, the numerator of this expression is positive. Since $D<0$, we have

$$\frac{\partial E}{\partial N}\cdot\frac{N}{E}-1\leq 0.\qquad\qquad \textit{Q.E.D.}$$

PROOF OF LEMMA 2: Differentiating (B.4) w.r.t. p and solving for $\partial E/\partial p$, we get

$$\frac{\partial E}{\partial p}=\frac{-U''_{11}v'_pv'_E-U'_1v''_{Ep}+U''_{21}v'_p}{D},\tag{B.5}$$

which implies

$$\begin{aligned}\frac{\partial E}{\partial p}\cdot\frac{p}{E}-1&=p\frac{-U''_{11}v'_pv'_E-U'_1v''_{Ep}+U''_{21}v'_p}{DE}-1\\&=\Big(-U''_{11}\big[pv'_pv'_E+E\big(v'_E\big)^2\big]-U'_1\big(pv''_{Ep}+Ev''_{EE}\big)\\&\quad+U''_{21}\big(pv'_p+2Ev'_E\big)-EU''_{22}\Big)\\&\quad/(DE).\end{aligned}$$

Applying the Euler theorem to v and v' yields

$$-U''_{11}\left[pv'_p v'_E + E\left(v'_E\right)^2\right] = -U''_{11} v'_E\left(pv'_p + Ev'_E\right) = 0$$

and

$$-U'_1\left(pv''_{Ep} + Ev''_{EE}\right) = U'_1 v'_E > 0.$$

Therefore,

$$\frac{\partial E}{\partial p} \cdot \frac{p}{E} - 1 = \frac{U'_1 v'_E + U''_{21} E v'_E - EU''_{22}}{DE} \leq 0$$

since $U''_{21} \geq 0$. Consequently, the right inequality of (B.3) is proven.

To show that $\partial E/\partial p > 0$, we rewrite (B.4) as

$$\frac{\partial E}{\partial p} = \frac{v'_p}{D}\left(-U''_{11} v'_E - U'_1 \frac{v''_{Ep}}{v'_p} + U''_{21}\right).$$

By definition of v, we have

$$v'_p = -\frac{Eu'}{p^2} < 0, \quad v'_E = \frac{u'}{p}, \quad v''_{Ep} = -\frac{u'}{p^2} - \frac{Eu''}{Np^3}.$$

Since $v'_p/D > 0$, the sign of $\partial E/\partial p$ is the same as that of the bracketed term of (B.5). Substituting these three expressions into (B.5) leads to

$$\begin{aligned}
&-U''_{11} v'_E - U'_1 \frac{v''_{Ep}}{v'_p} + U''_{21} \\
&\quad = -U''_{11} \frac{u'}{p} - U'_1 \frac{-\frac{u'}{p^2} - \frac{Eu''}{Np^3}}{-\frac{Eu'}{p^2}} + U''_{21} \\
&\quad = -\frac{U'_1}{E}\left[\left(\frac{U''_{11} Nu}{U'_1} - \frac{U''_{21} Nu}{U'_2}\right)\frac{Eu'}{Npu} + 1 + \frac{Eu''}{Npu'}\right].
\end{aligned}$$

Using $-U'_1/E < 0$ and $U'_1 v'_E(p, E, N) = pU'_2/u'$, it follows from (B.2) that

$$\left(\frac{U''_{11} Nu}{U'_1} - \frac{U''_{21} Nu}{U'_2}\right)\frac{Eu'}{Npu} + 1 + \frac{Eu''}{Npu'} < 0 \quad \Longrightarrow \quad \frac{\partial E}{\partial p} > 0,$$

which implies the left inequality of (B.3). *Q.E.D.*

The Impact of Market Size on the Mass of Firms in the Two-Sector Economy

We now show that the equilibrium mass of firms decreases with market size. Using the budget constraint and the zero-profit condition yields

$$N\left[F+V\big(\bar{q}(L)\big)\right]=LE\big(\bar{p}(L),N\big).$$

Rewriting this expression in elasticity terms w.r.t. L, we get

$$\mathcal{E}_N+\frac{\bar{q}V'(\bar{q})}{F+V(\bar{q})}\mathcal{E}_q=1+\frac{\partial E}{\partial p}\frac{\bar{p}}{E}\cdot\mathcal{E}_p+\frac{\partial E}{\partial N}\frac{N}{E}\cdot\mathcal{E}_N,$$

which can be rewritten as

$$\text{(B.6)}\qquad \mathcal{E}_N\left(1-\frac{\partial E}{\partial N}\frac{N}{E}\right)=1+\frac{\partial E}{\partial p}\frac{\bar{p}}{E}\cdot\mathcal{E}_p-\frac{\bar{q}V'(\bar{q})}{F+V(\bar{q})}\mathcal{E}_q.$$

The expression (A.3) is equivalent to

$$\text{(B.7)}\qquad \mathcal{E}_p=-r_u\mathcal{E}_q.$$

Using (10) and (B.7), (B.6) implies

$$\begin{aligned}\mathcal{E}_N\left(1-\frac{\partial E}{\partial N}\frac{N}{E}\right)&=1+\frac{\partial E}{\partial p}\frac{\bar{p}}{E}\cdot\mathcal{E}_p-(1-r_u)\mathcal{E}_q\\&=1+\frac{\partial E}{\partial p}\frac{\bar{p}}{E}\cdot\mathcal{E}_p+\frac{1-r_u}{r_u}\mathcal{E}_p\\&>1-\left(\frac{\partial E}{\partial p}\frac{\bar{p}}{E}+\frac{1-r_u}{r_u}\right)r_u=\left(1-\frac{\partial E}{\partial p}\frac{\bar{p}}{E}\right)r_u,\end{aligned}$$

where we have used (A.2) for the inequality. Since the elasticity of E w.r.t. p is smaller than 1 by assumption, the last term in the above expression is positive. Since the elasticity of E w.r.t. N in the first term is also smaller than 1, it must be that

$$\mathcal{E}_N=\frac{dN}{dL}\cdot\frac{L}{N}>0.$$

APPENDIX C: RELATIONSHIP BETWEEN THE TRANSLOG AND CARA MODELS

Under the translog, the profit is given by

$$\text{(C.1)}\qquad \pi(p_i;\Lambda_{\text{trans}},L)-F=(p_i-c)\frac{L}{p_i}(\Lambda_{\text{trans}}-\beta\ln p_i)-F.$$

Differentiating this expression w.r.t. p_i yields

$$\frac{c}{p_i^2}(\Lambda_{\text{trans}} - \beta \ln p_i) - \beta \frac{p_i - c}{p_i^2} = 0.$$

Solving for

$$\Lambda_{\text{trans}} - \beta \ln p_i = \beta \frac{p_i - c}{c},$$

plugging this expression into (C.1), and rearranging terms leads to the equilibrium condition

$$\beta(p-c)^2/(cp) = F/L.$$

Applying the same argument to the CARA model yields the desired expression:

$$\beta(p-c)^2/p = F/L.$$

Novosibirsk State University, 630090, Novosibirsk, Pirogova 2, Russia, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia; ezhelobodko@gmail.com,

Novosibirsk State University and Sobolev Institute of Mathematics, 630090, Novosibirsk, Pirogova 2 and 4, Russia, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia; skokov7@gmail.com,

CORE-UCLouvain, 34 Voie du Roman Pays, 1348 Louvain la Neuve, Belgium, Paris School of Economics (Paris 1), 106-112 Bd de l'Hôpital, 75013 Paris, France, and National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia; parenti.mathieu@gmail.com,

and

CORE-UCLouvain, 34 Voie du Roman Pays, 1348 Louvain la Neuve, Belgium, National Research University Higher School of Economics, 190008, Saint-Petersburg, Ulitsa Soyuza Pechatnikov 16, Russia, and CEPR; jacques.thisse@ucloubain.be.

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