

Unanticipated Effects of Anticipated Monetary Policy Shocks*

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Abstract

In the recent financial crisis, interest rates may have been lower than what was motivated by inflation developments in order to boost fragile financial markets. Sometimes such events occur without warning and are short-lived. In these cases, treating them as an unexpected development may well be a good approximation. At other times, events that will impact interest rates may unfold before rates are actually changed. In these cases, we would be better off thinking about them as anticipated monetary policy shocks. If agents in the economy are forward-looking, the real effects may look quite different depending on if the shock was anticipated or not. Recent contributions (see Schmitt-Grohé and Uribe [5]) have made this point concerning real shocks. The first contribution of this paper is to explore the distinction between anticipated and unexpected *nominal* shocks, whereas previous contributions have focused on real shocks. The second contribution is that we use data on expectations in order to better identify plausible monetary policy shocks - expected as well as unexpected. The third contribution of the paper is to explore the small open economy aspects of anticipated and unanticipated monetary policy shocks.

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*The views, analysis, and conclusions in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

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Contents

1	Introduction	1
2	Analytical Example - A simple model to fix ideas	4
2.1	Model	4
2.1.1	The effects of a monetary policy shock	6
2.1.2	News processes	7
2.2	A simulation exercise	8
3	A small-scale empirical model	10
3.1	Intermediate input goods	11
3.1.1	Production of the Domestic Homogeneous Good	11
3.1.2	Production of Imported Intermediate Goods	12
3.2	Production of Final Consumption Goods	14
3.3	Production of Final Export Goods	15
3.4	Households	17
3.4.1	Household Consumption Decision	17
3.4.2	Financial Assets and Interest Rate Parity	17
3.5	Wage Setting and Employment Frictions	19
3.6	Monetary Policy	19
3.7	Fiscal Authorities	20
3.8	Foreign Variables	20
3.9	Resource Constraints	21
3.9.1	Resource Constraint for Domestic Homogeneous Output	21
3.9.2	Trade Balance	22
3.10	Exogenous Shock Processes	22
4	Introducing Anticipated Shocks	23
5	Estimating Anticipated Shocks	24
5.1	Data	24
5.2	Measures of policy expectations	25
5.3	Measurement equations	27
5.4	Calibration	28
5.5	Estimation results	30
5.5.1	Effects of anticipated shocks during the financial crisis	32
6	Conclusions	44
	Appendix	49
6.1	Scaling of Variables	49
6.2	First order conditions for domestic homogenous good price setting	50
6.3	First order conditions for export good price setting	52
6.4	Demand for domestic inputs in export production	52
6.5	First order conditions for import good price setting	53
6.6	Wage setting conditions	54
6.7	Output and aggregate factors of production	59
6.8	Restrictions across inflation rates	61
6.9	Endogenous Variables of the Model	61
6.10	Complete Loglinear Model	61
6.11	Marginal Posterior Densities	72
6.12	Forecast Error Variance Decomposition	76

1. Introduction

There has been much progress in recent years in the development of dynamic, stochastic general equilibrium (DSGE) models for the purpose of monetary policy analysis. These models have been shown to fit aggregate data well. They have been shown to do better or as well as simple atheoretical statistical models at forecasting the data outside of the sample of data on which they were estimated.

A key assumption in these models is how monetary policy is conducted. A significant fraction of the variation in central bank policy actions reflects policy makers' systematic responses to variations in the state of the economy. This systematic component is typically formalized with the concept of an instrument rule, or reaction function. Simple instrument rules, such as e.g. Taylor rules, are often used to describe monetary policy behavior. They are also used to "close" policy models and allow the rational expectations equilibrium to be determined. As a practical matter, it is recognized that not all variations in central bank policy can be accounted for as a reaction to the state of the economy. The unaccounted variation is formalized with the notion of a monetary policy shock or an unsystematic policy reaction. There are different views on how this unsystematic part of monetary policy is generated (See e.g. a discussion in Christiano, Eichenbaum and Evans [8]) but much effort has gone into calibrating policy models to give a wanted response to such a shock. In a model without nominal frictions, monetary policy has no real effects. Hence, it could be argued that it is perfectly natural in a model where monetary policy do have real effects to focus on how the economy behaves when the key object of study changes unexpectedly.¹ Indeed, if one goes back 30 years or so, *only* unexpected monetary policy variations had any effects at all (Lucas, Sargent etc.). In contrast, in today's benchmark models of monetary policy, the key focus is on the role that monetary policy plays to manage expectations through, for example, policy rules or alternatively through a targeting rule. The latter feature raises the possibility that the way the economy reacts depends on whether the deviation from the rule was expected or not.

Let us for a moment consider what interpretation one should give to a "monetary policy shock". A naive interpretation is that it represents inefficient idiosyncrasies of the policy makers. A more plausible view - we think - is that it captures all that the rule does not. That is to say, sometimes there are "unusual" events which a rule that works well "most of the time" does not capture. For example, in the recent financial crisis, interest rates may have been lower than what was motivated by inflation developments in order to boost fragile financial markets. Sometimes these events occur

¹This is also in the spirit of a suggestion made by R.E. Lucas [16].

without warning and are short-lived. In these cases, treating them as an unexpected development may well be a good approximation. At other times, events that will impact interest rates may unfold before rates are actually changed. In these cases, we would be better off thinking about them as expected monetary policy shocks. If agents in the economy are forward-looking, the real effects may look quite different depending on if the shock was expected or not. Recent contributions (see Schmitt-Grohé and Uribe [5]) have made this point concerning real shocks.² For example, if the fiscal authorities pre-announce that the tax on durable-goods consumption will be lowered next year, it is likely that the economy would see a large drop in consumption - awaiting that lower tax - and then a spike. In contrast, if the authorities lowers the tax unexpectedly, consumption would instead rise on impact and be unaffected before.

The first contribution of this paper is to explore the distinction between anticipated and unexpected *nominal* shocks, whereas previous contributions have focused on real shocks. Interesting questions include: how are estimates of nominal frictions affected by the introduction of expected policy shocks? We know that estimates of "structural" parameters such as the degree of indexation of prices is sensitive to how expectations are treated. In the learning literature, a typical finding (Milani [25]) is that learning itself introduces inflation persistence even if the Phillips curve is inherently forward-looking. Leeper and Walker [15] e.g. show that a gradual diffusion of news about future productivity can reconcile theory and data without relying heavily on other forms of internal propagation. Slow dispersion of news complements other forms of propagation and is able to generate positive co-movements among macroeconomic aggregates in response to a news shock.

Expected shocks and unexpected shocks have different impulse response functions. This, at least in principle, allows identifying which type of shocks that best describes the dynamics of the data by estimating the model while allowing both types of shocks. A practical problem is that identification in modern DSGE models is often problematic as it is (Canova and Sala [18]), and hence adding more sources of flexibility may only worsen the problem. We propose a strategy by adding data on expectations in order to better identify plausible monetary policy shocks - expected as well as unexpected - and this is the second contribution of the paper.

The third contribution of the paper is to explore the (small) open economy aspects of monetary policy. In a small open economy interest rate differentials and risk premia are important determi-

²Many recent papers document the importance of news shocks. See e.g. Beaudry and Portier ([1], [3]), Schmitt-Grohé and Uribe [5], Jaimovich and Rebelo [6], Christiano, Ilut, Motto, and Rostagno [7], Fujiwara, Hirose, and Shintani [10], Barsky and Sims [11].

nants of especially the exchange rate and inflation through an interest rate parity condition. We allow for the possibility of anticipated risk premia shocks as a possible explanation of expected interest rate differentials between domestic and foreign interest rates. At the moment, short and long market rates as well as implied forward rates are very low in many of the developed countries around the world. In the Eurozone, an initial increase of the policy rate is not expected for at least one year. The policy rates in the United Kingdom and the United States are also expected to remain low for a long time to come. One possible interpretation of this relates to a systematic monetary policy response where market participants believe that the probability of a “double dip” abroad is high and therefore have a much more gloomy view of GDP growth and inflation. Another possibility is that the crisis in certain countries has subdued the future growth potential or led to an increase in precautionary saving, which in turn reduces the economies’ so-called neutral equilibrium interest rate. A further possibility relates to the unsystematic part of monetary policy, and is that the measures to facilitate the supply of credit implemented by central banks around the world have pushed down interest rates for longer maturities more than is justified by lower expectations of future policy rates. It may also play a role that there has recently been considerable demand for safe assets such as government bonds in countries with relatively sound public finances, or what is known as a flight to quality. If this is what lies behind the low long-term interest rates, implied forward rates may provide a picture of expectations of future interest rates driven by anticipated risk premia.

Preliminary results show that the importance of expected future monetary policy shocks in explaining the data is relatively modest. There is some indication that anticipation for 2 and 3 quarters ahead is somewhat more important than anticipation at longer horizons. What is even more interesting is that anticipated risk premium shocks seem to be of much larger importance compared to anticipated monetary policy shocks. Here it is longer term expectations which comes out as more important. Moreover, adding data on implied forward rates as a measure of expected monetary policy gives the same general picture, i.e. that anticipated monetary policy shocks are less important in explaining the data than anticipated risk premium shocks. This seems to be caused by opposing policy forces. Anticipation of a higher instrument rate at a distant future depresses the economy and lowers inflation today. Systematic monetary policy responds to this development by lowering the instrument rate to mitigate the low inflation and stabilize the economy. These forces cancel each other out and the effects of anticipated shocks comes in general out as negligible. This does not hold for anticipated risk premia. Here anticipated risk premia creates inflation and a

higher interest rate. These shocks are able to explain co-movement in the data and comes out from the estimation exercise as much more important. Anticipated risk premium shocks explain around 30 percent of the variation in the implied forward rates data which we use in the estimation of the model.

The outline of the sections of the paper is as follows. We first set up an analytical simple (closed economy) example to illustrate the issues: three equation model with anticipated and unanticipated shocks. We conduct a Monte Carlo experiment where we simulate data from this simple analytical example with both types of shocks and estimate the model with anticipated shocks and illustrate the effect on estimates of key parameters related to nominal rigidity. In the next section we set up an empirically relevant small open economy new-keynesian model in order to examine and estimate the prevalence and effects of unexpected and anticipated foreign and domestic monetary policy shocks as well risk premia. The model is based on Christiano, Eichenbaum and Evans [32] and Adolfson, Laséen, Lindé and Villani [27]. In the estimation subsection we add measures of policy expectations based market expectations in order to better distinguish between expected and unexpected policy and risk premia shocks. The final section offers some conclusions.

2. Analytical Example - A simple model to fix ideas

In this section, we examine the equilibrium dynamics associated with a monetary policy news shock in a simple economic environment. The simplicity allows us to identify the exact role played by news shocks. We are also able to clearly demonstrate how different types of news processes alter dynamics. The results and conclusions reached in this section hopefully extend to the more sophisticated model of section 3.

2.1. Model

We consider a standard Basic New Keynesian Model (NKM) described in detail in King and Wolman [21], Woodford [20], Gali [19] and Gali and Gertler [22]. The model has become the workhorse for the analysis of monetary policy in recent years. The two most important elements of the NKM are: first, imperfect competition in the goods market. Each firm produces a differentiated good for which it sets the price. Second, only a fraction of firms can reset their prices in any given period. In the NKM, aggregate spending is determined by the behavior of the representative household, which seeks to smooth consumption over time by investing its savings in one-period government

bonds. This optimizing behavior results in the following log-linearized aggregate demand equation, referred to as the dynamic IS equation:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n) + \varepsilon_t^x, \quad (2.1)$$

where y_t denotes (log) output, i_t is the short-term nominal interest rate, $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation between t and $t + 1$ (with p_t denoting the log of the price level). According to dynamic IS equation, fluctuations in the short-term real interest rate gap, i.e. $(i_t - E_t \pi_{t+1} - r^n)$, induce deviations of output from its expected future value, $E_t y_{t+1}$, where the operator E_t denotes households' expectation of future values conditional on the information available today.

On the supply side, intermediate goods firms set prices according to the current and expected future evolution of marginal costs and demand conditions. Profit-maximizing behavior results in the following (log-linearized) aggregate supply or New Keynesian Phillips curve equation:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_t^\pi, \quad (2.2)$$

where the parameter σ corresponds to the coefficient of relative risk aversion, β is the household's discount factor, and κ is a coefficient that is inversely related to the degree of price rigidities θ .³ $x_t = y_t - y_t^n$ is the output gap (i.e. the log deviation of output from its natural level, y_t^n). We follow Gali [19] and assume that (log) natural output follows a stationary $AR(1)$ process in first differences; that is,

$$\Delta y_t^n = \rho_y \Delta y_{t-1}^n + \varepsilon_t^{y^n},$$

where $\rho_y \in [0, 1)$ and $\{\varepsilon_t^x, \varepsilon_t^{y^n}, \varepsilon_t^\pi\}$ are a white noise process. One can rewrite (2.1) in terms of the output gap, as follows:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n) + E_t \Delta y_{t+1}^n + \varepsilon_t^x. \quad (2.3)$$

The central bank determines monetary policy by setting the short-term nominal interest rate, i_t , in response to price inflation and the output gap.

$$i_t = \rho + \phi_\pi \pi_t + \phi_x x_t + \varepsilon_t \quad (2.4)$$

where ε_t is a white noise process which is referred to as a monetary policy shock. With this formulation, shocks are unanticipated. This interest rate rule is a variant of the instrument rule proposed by Taylor [23] and [24]. According to this rule, nominal interest rates rise more than one-to-one with inflation and fall in response to output contractions.

³ $\kappa = \lambda \left(\sigma + \frac{(\varphi + \alpha)}{(1 - \alpha)} \right)$, where $\lambda = (((1 - \theta)(1 - \beta\theta))/\theta)\Theta$, $\Theta = (1 - \alpha)/(1 - \alpha + \alpha\epsilon)$, $\rho = -\log(\beta)$.

2.1.1. The effects of a monetary policy shock

The solution to the system of difference equations, i.e. the system of equations (2.2), (2.3) and (2.4) is given by (See Gali [19])

$$\begin{aligned} x_t &= -\frac{1}{(\sigma + \phi_x + \kappa\phi_\pi)}\varepsilon_t, \\ \pi_t &= -\frac{\kappa}{(\sigma + \phi_x + \kappa\phi_\pi)}\varepsilon_t, \\ i_t &= \frac{\sigma}{(\sigma + \phi_x + \kappa\phi_\pi)}\varepsilon_t.^4 \end{aligned} \tag{2.5}$$

Figure 1 shows impulse responses to unexpected monetary policy shock for a baseline calibration where $\beta = 0.99$, $\sigma = 1$, $\phi_\pi = 1.5$, $\phi_x = 0.5/4$, $\kappa = 0.1275^5$

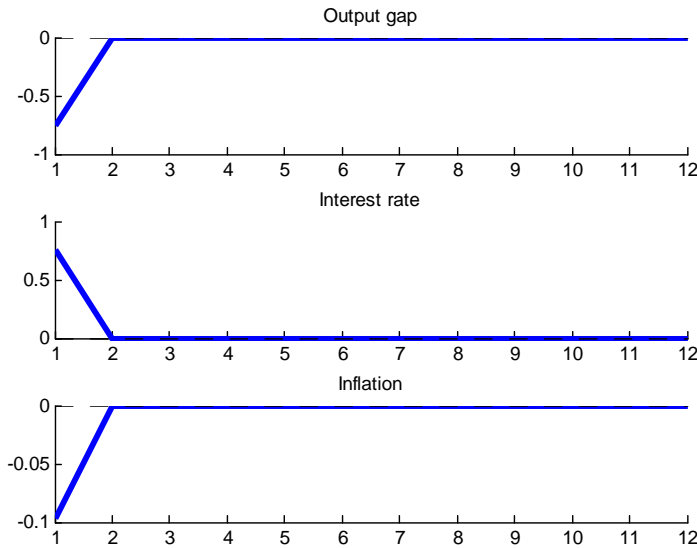


Figure 1: Impulse-response functions to an unexpected monetary policy shock.

The results of this increase in the policy rate are: An increase in the real rate A decrease in output and inflation and the nominal rate goes up but less than the real rate. It is clear from (2.5) that the assumptions about the shock process will be very important for the properties of the solution.

⁵Note that $\kappa = \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$, where $\lambda = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon}$ and φ denotes the Frisch elasticity of labor supply and α is the labor income share. θ is a measure of price stickiness and ε is the demand elasticity. The parameters are assumed to take the following values: $\theta = 2/3$, $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$, $\alpha = 1/3$ and $\varepsilon = 6$. See Gali [19] for a discussion of these values.

2.1.2. News processes

In this section we analyse how different assumptions about the news processes affect the solution and the dynamics of monetary policy shocks. Leeper and Walker [15] analyse different news processes for technology and tax shocks and show that how information enters the economy is crucial for understanding the dynamic impacts of news.

Different types of news processes have been employed in the literature. Research on technology news typically assumes a process without memory, which written for two sequential dates, takes the form

$$\begin{aligned}\varepsilon_{t+1} &= e_{1,t} + e_{2,t-1}, \\ \varepsilon_{t+2} &= e_{1,t+1} + e_{2,t},\end{aligned}\tag{2.6}$$

where $e_{1,t}$ and $e_{2,t}$ are uncorrelated at all leads and lags and drawn from distinct probability distributions. Schmitt-Grohé and Uribe [5] use an expanded version of (2.6) in their estimated DSGE model. An alternative news process allows for memory. Writing it again for two sequential dates, the process is given by

$$\begin{aligned}\varepsilon_{t+1} &= e_t + e_{t-1}, \\ \varepsilon_{t+2} &= e_{t+1} + e_t,\end{aligned}\tag{2.7}$$

where now the e 's are drawn from the same distribution. Leeper, Walker, and Yang [13] employ a version of (2.7) to study tax news. Leeper and Walker [15] show that the two news processes in (2.6) and (2.7) can be obtained from a generalized process. Assume that the news separately evolve according to

$$\varepsilon_t = e_{1,t-1}^x + e_{2,t-2}^x\tag{2.8}$$

$$= \sigma_{11}\eta_{1,t-1}^x + \sigma_{12}\eta_{2,t-1}^x + \sigma_{21}\eta_{1,t-2}^x + \sigma_{22}\eta_{2,t-2}^x\tag{2.9}$$

Each of the shocks in (2.8) is a linear combination of two i.i.d. disturbances

$$\begin{aligned}e_{1,t}^x &= \sigma_{11}\eta_{1,t}^x + \sigma_{12}\eta_{2,t}^x, \\ e_{2,t}^x &= \sigma_{21}\eta_{1,t}^x + \sigma_{22}\eta_{2,t}^x\end{aligned}$$

The $\eta_{i,t}^x$ are assumed to be distributed as standard normal and are uncorrelated at all leads and lags. Innovations are distributed as bivariate normal according to

$$\begin{bmatrix} e_{1,t}^x \\ e_{2,t}^x \end{bmatrix} = N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \sigma_{11}^2 + \sigma_{12}^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_{21}^2 + \sigma_{22}^2 \end{bmatrix} \right]$$

where $\rho = (\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22})/\sigma_1\sigma_2$ and $\sigma_1^2 = \sigma_{11}^2 + \sigma_{12}^2$, $\sigma_2^2 = \sigma_{21}^2 + \sigma_{22}^2$. Temporal correlations between the e 's depend on the properties of the bivariate normal distribution.

2.2. A simulation exercise

In this section we create an identification problem similar to the one that emerges in the economic model analyzed in later sections. We run a monte carlo experiment with data generated from the theoretical model in section 2 with expected policy shocks under alternative assumptions about the news processes in section 2.1.2 estimated with unanticipated and anticipated shocks to illustrate the potential of our empirical strategy to identify the parameters that govern the distributions of the underlying shocks.

We assume that the econometrician can observe three variables, x_t , π_t and i_t and knows the structure of the model. The econometric problem consists in estimating the following eleven parameters $\{\theta, \phi_\pi, \phi_y, \rho_y, \sigma_{y^n}, \sigma_y, \sigma_x, \sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}\}$. Knowledge of the underlying data generating process should allow for the design of a successful econometric strategy to identify the volatilities of the underlying sources of uncertainty. Next, we test this by formally estimating the example economy using Bayesian methods on simulated data for x_t , π_t and i_t . We consider three cases, each representing a different assumption about the news process. The three economies differ in the relative importance of the two underlying news shocks. In one case, the innovations display the pattern given by equation (2.6) above with no memory. In the second case, we assume the more general news process given by equation (2.8) where we allow for both types of news processes. Finally, case 3 represent news process (2.7). In each case, we produce an artificial data set of 500 observations of the observables x_t , π_t and i_t .

Parameter	Case 1	Case 2	Case 3	Description
β	0.99	0.99	0.99	Discount factor
σ	1	1	1	Relative risk aversion
α	1/3	1/3	1/3	Measure of decreasing returns in production
ϵ	6	6	6	Demand elasticity
φ	1	1	1	The inverse of the Frisch elasticity of labor supply
'True' parameters values				
θ	2/3	2/3	2/3	Price stickiness
ϕ_π	1.5	1.5	1.5	Instrument rule - inflation
ϕ_x	0.5/4	0.5/4	0.5/4	Instrument rule - output gap
σ_{y^n}	0.5	0.5	0.5	Std dev Natural output shock
σ_π	0.33	0.33	0.33	Std dev markup shock
σ_x	0.25	0.25	0.25	Std dev output shock
ρ_y	0.5	0.5	0.5	Persistence natural output
σ_{11}	0.1	0.3	1	News shock parameter
σ_{12}	0	0.1	0	News shock parameter
σ_{21}	0	0.1	0.25	News shock parameter
σ_{22}	1	0.3	0	News shock parameter

Table 1. Parameter values.

Estimated parameters values	Posterior Mode			
	Case 1	Case 2	Case 3	
θ	0.6703	0.7033	0.6590	Price stickiness
ϕ_π	1.4943	1.5171	1.5619	Instrument rule - inflation
ϕ_x	0.1524	0.1501	0.1334	Instrument rule - output gap
σ_{y^n}	0.6460	0.5586	0.6256	Std dev Natural output shock
σ_π	0.3151	0.3074	0.3218	Std dev markup shock
σ_x	0.0762	0.0743	0.0721	Std dev output shock
ρ_y	0.4862	0.5246	0.4960	Persistence natural output
σ_{11}	0.0720	0.2959 ⁶	1.0223	News shock parameter
σ_{12}	0	—	0	News shock parameter
σ_{21}	0	—	0.2629	News shock parameter
σ_{22}	1.0806	0.3713	0	News shock parameter

In all cases we do fairly well and recover the true parameter values of $\{\theta, \phi_\pi, \phi_x, \rho_y, \sigma_{y^n}, \sigma_y, \sigma_x\}$. The Bayesian estimation strategy does a relatively good job at uncovering the true values of the parameters in question. In case 1 the posterior modes are, respectively (0.0720, 1.0806), with standard deviations (0.0312, 0.0325). And in case 3, the posterior modes are, respectively (1.0223, 0.2629), with standard deviations (0.0371, 0.0282). However, when it comes to the assumptions about the

⁶It is only possible to estimate the standard deviation of the innovations i.e. $\sigma_1 = \sigma_{11} + \sigma_{12}$, $\sigma_2 = \sigma_{21} + \sigma_{22}$ which is what is reported in the column.

news processes in case 2 we observe that we can't recover the parameters $\{\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}\}$ separately. This is also evident in figure 2 where impulse-response functions for the two innovations $\eta_{1,t}^x$ and $\eta_{2,t}^x$ are displayed. In case 1 and 2 the impulse-responses are distinctly different whereas in case 2 they are not. Hence, some restrictions on the assumptions about the news processes have to be considered in the economic model analyzed in later sections. What is clear from the exercise is that if the news shocks are important driving processes for inflation, gdp and interest rates we should be able to capture this overall phenomenon when we estimate the model.

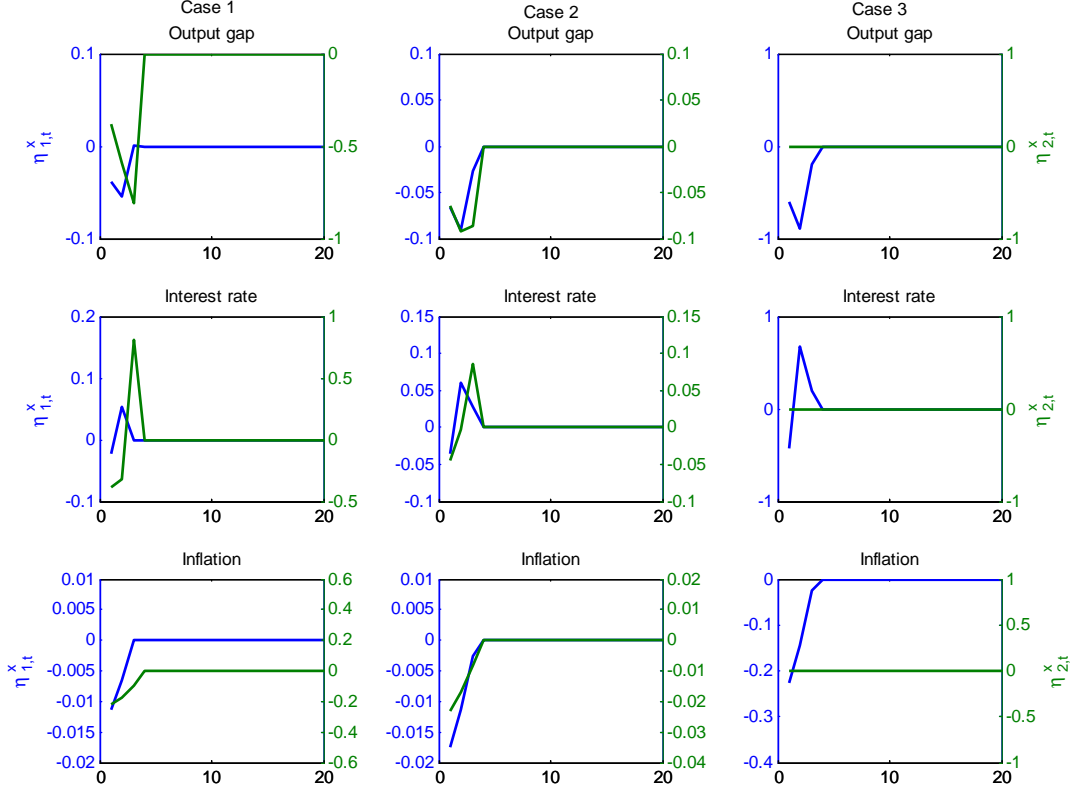


Figure 2: Impulse-response functions for the two innovations $\eta_{1,t}^x$ and $\eta_{2,t}^x$.

3. A small-scale empirical model

In this section we set up a small open economy new-keynesian model. The model is based on Christiano, Eichenbaum and Evans [32] and Adolfson, Laséen, Lindé and Villani [27] from which it inherits most of its open economy structure. The main difference is that we have excluded capital (both physical capital and working capital) in the model. The two final goods, consumption, and exports, are produced by combining the domestic homogenous good with specific imported inputs

for each type of final good. Specialized domestic importers purchase a homogeneous foreign good, which they turn into a specialized input and sell to domestic import retailers. There are two types of import retailers. One uses the specialized import goods to create a homogeneous good used as an input into the production of specialized exports.⁷ Another uses specialized imports to produce a homogeneous input used in the production of consumption goods. Exports involve a Dixit-Stiglitz continuum of exporters, each of which is a monopolist that produces a specialized export good. Each monopolist produces its export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The specialized export goods are sold to foreign, competitive retailers which create a homogeneous good that is sold to foreign citizens.

Below we will describe the production of all these goods.

3.1. Intermediate input goods

3.1.1. Production of the Domestic Homogeneous Good

A homogeneous domestic good, Y_t , is produced using

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_d}} di \right]^{\lambda_d}, \quad 1 \leq \lambda_d < \infty. \quad (3.1)$$

The domestic good is produced by a competitive, representative firm which takes the price of output, P_t , and the price of inputs, $P_{i,t}$, as given.

The i^{th} intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t}) \epsilon_t - z_t \phi, \quad (3.2)$$

where, $\log(z_t)$ is a technology shock whose first difference has a positive mean, $\log(\epsilon_t)$ is a stationary neutral technology shock and ϕ denotes a fixed production cost. In (3.2), $H_{i,t}$ denotes homogeneous labor services hired by the i^{th} intermediate good producer. The firm's marginal cost divided by the price of the homogeneous good is denoted by mc_t . It is given by the first-order condition with respect to labor in the firm's cost minimization problem:

$$mc_t = \tau_t^d \bar{w}_t \frac{1}{\epsilon_t}, \quad (3.3)$$

where $\bar{w}_t = W_t/(z_t/P_t)$. τ_t^d is a tax-like shock, which affects marginal cost, but does not appear in the production function. If there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d , i.e., a markup shock.

⁷This is type of import retailer does not exist in Adolfson, Laséen, Lindé and Villani [27]. It is introduced as in Christiano, Tranandt and Walentin [4].

The i^{th} firm is a monopolist in the production of the i^{th} good and so it sets its price. Price setting is subject to Calvo frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case the price is set according to the following indexation scheme:

$$\begin{aligned} P_{i,t} &= \tilde{\pi}_{d,t} P_{i,t-1}, \\ \tilde{\pi}_{d,t} &\equiv (\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{1-\kappa_d-\varkappa_d} (\check{\pi})^{\varkappa_d}, \end{aligned}$$

where κ_d, \varkappa_d are parameters and $\kappa_d, \varkappa_d, \kappa_d + \varkappa_d \in (0, 1)$, π_{t-1} is the lagged inflation rate, $\bar{\pi}_t^c$ is the central bank's target inflation rate and $\check{\pi}$ is a scalar.⁸

With probability $1 - \xi_d$ the firm can optimize its price and maximize discounted profits,

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} \{P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j}\}, \quad (3.4)$$

subject to the indexation scheme above and the requirement that production equals demand

$$Y_{i,t} = \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d-1}} Y_t, \quad (3.5)$$

where v_t is the multiplier on the household's nominal budget constraint. It measures the marginal value to the household of one unit of profits, in terms of currency. The equilibrium conditions associated with price setting problem and their derivation are reported in section 6.2 in the Appendix.

The domestic intermediate output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t^d + X_t^d \quad (3.6)$$

Here, C_t^d denotes intermediate domestic consumption goods used together with foreign consumption goods to produce the final household consumption good. X_t^d is domestic resources allocated to exports. The determination of consumption, investment and export demand is discussed below.

3.1.2. Production of Imported Intermediate Goods

We now turn to a discussion of imports. Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input (they “brand name” it) and supply that input monopolistically to domestic retailers. There are three types of

⁸ $\check{\pi}$ is a scalar which allows us to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\check{\pi} = 1, \varkappa_d = 1$) or that they index only to the steady state inflation rate (i.e., $\check{\pi} = \bar{\pi}, \varkappa_d = 1$). Note that we get price dispersion in steady state if $\varkappa_d > 0$ and if $\check{\pi}$ is different from the steady state value of π . See Yun (1996) for a discussion of steady state price dispersion.

importing firms: (i) one produces goods used to produce an intermediate good for the production of consumption, (ii) one produces goods used to produce an intermediate good for the production of exports. All importers are subject to Calvo price setting frictions.

Consider (i) first. The production function of the domestic retailer of imported consumption goods is:

$$C_t^m = \left[\int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda^{m,c}}} di \right]^{\lambda^{m,c}},$$

where $C_{i,t}^m$ is the output of the i^{th} specialized producer and C_t^m is the intermediate good used in the production of consumption goods. Let $P_t^{m,c}$ denote the price index of C_t^m and let $P_{i,t}^{m,c}$ denote the price of the i^{th} intermediate input. The domestic retailer is competitive and takes $P_t^{m,c}$ and $P_{i,t}^{m,c}$ as given. In the usual way, the demand curve for specialized inputs is given by the domestic retailer's first order condition for profit maximization:

$$C_{i,t}^m = C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda^{m,c}}{\lambda^{m,c}-1}}.$$

We now turn to the producer of $C_{i,t}^m$, who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $C_{i,t}^m$. The intermediate good producer's marginal cost is

$$\tau_t^{m,c} S_t P_t^*, \tag{3.7}$$

where S_t the exchange rate (domestic currency per unit foreign currency). There is no risk to this firm, because all shocks are realized at the beginning of the period. Also, $\tau_t^{m,c}$ is a tax-like shock, which affects marginal cost but does not appear in the production function. If there are no price and wage distortions in the steady state, $\tau_t^{m,c}$ is isomorphic to a markup shock.

Now consider (ii). The production function of the domestic retailer of imported goods used in the production of an input, X_t^m , for the production of export goods is:

$$X_t^m = \left[\int_0^1 (X_{i,t}^m)^{\frac{1}{\lambda^{m,x}}} di \right]^{\lambda^{m,x}}.$$

The imported good retailer is competitive, and takes output prices, $P_t^{m,x}$, and input prices, $P_{i,t}^{m,x}$, as given. The producer of the specialized input, $X_{i,t}^m$, has marginal cost

$$\tau_t^{m,x} S_t P_t^*.$$

Each of the above two types of intermediate good firms is subject to Calvo price-setting frictions. With probability $1 - \xi_{m,j}$, the j^{th} type of firm can reoptimize its price and with probability $\xi_{m,j}$ it sets price according to:

$$\begin{aligned} P_{i,t}^{m,j} &= \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j}, \\ \tilde{\pi}_t^{m,j} &\equiv \left(\pi_{t-1}^{m,j} \right)^{\kappa_{m,j}} (\bar{\pi}_t)^{1-\kappa_{m,j}-\varkappa_{m,j}} \bar{\pi}_t^{\varkappa_{m,j}}. \end{aligned} \quad (3.8)$$

for $j = c, x$, and $\kappa_{m,j}, \varkappa_{m,j}, \kappa_{m,j} + \varkappa_{m,j} \in (0, 1)$.

The equilibrium conditions associated with price setting by importers are analogous to the ones derived for domestic intermediate good producers and are reported in section 6.5 in the Appendix. The real marginal cost is

$$\begin{aligned} mc_t^{m,j} &= \tau_t^{m,j} \frac{S_t P_t^*}{P_t^{m,j}} \\ &= \tau_t^{m,j} \frac{S_t P_t^* P_t^c P_t}{P_t^c P_t^{m,j} P_t} \\ &= \tau_t^{m,j} \frac{q_t P_t^c}{P_t^{m,j}} \end{aligned} \quad (3.9)$$

for $j = c, x$.

3.2. Production of Final Consumption Goods

Final consumption goods are purchased by households. These goods are produced by a representative competitive firm with the following linear homogeneous technology:

$$C_t = \left[(1 - \omega_c)^{\frac{1}{\eta_c}} \left(C_t^d \right)^{\frac{(\eta_c-1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m \right)^{\frac{(\eta_c-1)}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}. \quad (3.10)$$

using two inputs. The first, C_t^d , is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t . The second input, C_t^m , is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of C_t^m is $P_t^{m,c}$. The representative firm takes the input prices, P_t and $P_t^{m,c}$, as well as the output price of the final consumption good, P_t^c , as given. Profit maximization leads to the following demand for the intermediate inputs (in scaled form):

$$\begin{aligned} c_t^d &= (1 - \omega_c) (p_t^c)^{\eta_c} c_t \\ c_t^m &= \omega_c \left(\frac{P_t^c}{P_t^{m,c}} \right)^{\eta_c} c_t. \end{aligned} \quad (3.11)$$

where $p_t^c = P_t^c/P_t$ and $p_t^{m,c} = P_t^{m,c}/P_t$. The price of C_t is related to the price of inputs by:

$$p_t^c = \left[(1 - \omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}. \quad (3.12)$$

The rate of inflation of the consumption good is:

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t \left[\frac{(1 - \omega_c) + \omega_c (p_t^{m,c})^{1-\eta_c}}{(1 - \omega_c) + \omega_c (p_{t-1}^{m,c})^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}}. \quad (3.13)$$

3.3. Production of Final Export Goods

Total foreign demand for domestic exports is:

$$X_t = \left(\frac{P_t^x}{P_t^*} \right)^{-\eta_f} Y_t^*.$$

In scaled form, this is

$$x_t = (p_t^x)^{-\eta_f} y_t^* \tilde{\mu}_{z,t}. \quad (3.14)$$

Here, Y_t^* is foreign GDP and P_t^* is the foreign currency price of foreign homogeneous goods. P_t^x is an index of export prices, whose determination is discussed below. $\tilde{\mu}_{z,t}$ is an asymmetric technology shock and is described in section 3.8. The goods, X_t , are produced by a representative, competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{\lambda_x}{\lambda_x-1}} di \right]^{\lambda_x-1}. \quad (3.15)$$

where $X_{i,t}$, $i \in (0,1)$, are exports of specialized goods. The retailer that produces X_t takes its output price, P_t^x , and its input prices, $P_{i,t}^x$, as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x} \right)^{\frac{-\lambda_x}{\lambda_x-1}} X_t. \quad (3.16)$$

Combining (3.15) and (3.16), we obtain:

$$P_t^x = \left[\int_0^1 (P_{i,t}^x)^{\frac{1}{1-\lambda_x}} di \right]^{1-\lambda_x}.$$

The i^{th} export monopolist produces its differentiated export good using the following CES production technology:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_x}} (X_{i,t}^m)^{\frac{\eta_x-1}{\eta_x}} + (1 - \omega_x)^{\frac{1}{\eta_x}} (X_{i,t}^d)^{\frac{\eta_x-1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x-1}},$$

where $X_{i,t}^m$ and $X_{i,t}^d$ are the i^{th} exporter's use of the imported and domestically produced goods, respectively. We derive the marginal cost from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$\min \quad \tau_t^x \left[P_t^{m,x} X_{i,t}^m + P_t X_{i,t}^d \right] + \lambda \left\{ X_{i,t} - \left[\omega_x^{\frac{1}{\eta_x}} (X_{i,t}^m)^{\frac{\eta_x-1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} (X_{i,t}^d)^{\frac{\eta_x-1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x-1}} \right\},$$

where $P_t^{m,x}$ is the price of the homogeneous import good and P_t is the price of the homogeneous domestic good. Using the first order conditions of this problem we derive the real marginal cost, mc_t^x :

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x}{q_t p_t^c p_t^x} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}, \quad (3.17)$$

where lower case letters denote scaled variables and where we have used

$$\frac{S_t P_t^x}{P_t} = \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*} = q_t p_t^c p_t^x. \quad (3.18)$$

From the solution to the same problem we also get the demand for domestic inputs for export production:

$$X_{i,t}^d = \left(\frac{\lambda}{\tau_t^x R_t^x P_t} \right)^{\eta_x} X_{i,t} (1-\omega_x) \quad (3.19)$$

The aggregate export demand for the *domestic* homogeneous input good is

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) (\hat{p}_t^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} (p_t^x)^{-\eta_f} Y_t^* \quad (3.20)$$

where \hat{p}_t^x is a measure of the price dispersion, which is not active in this version of the model and hence equal to one (see also section 6.4 in the Appendix).

The aggregate export demand for the *imported* input good is:

$$X_t^m = \omega_x \left(\frac{\left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1-\omega_x) \right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} (\hat{p}_t^x)^{\frac{-\lambda_x}{\lambda_x-1}} (p_t^x)^{-\eta_f} Y_t^* \quad (3.21)$$

The i^{th} export firm takes (3.16) as its demand curve, and sets the price subject to Calvo frictions. With probability ξ_x the i^{th} export good firm cannot reoptimize its price, in which case it update its price as:

$$\begin{aligned} P_{i,t}^x &= \tilde{\pi}_t^x P_{i,t-1}^x, \\ \tilde{\pi}_t^x &= (\pi_{t-1}^x)^{\kappa_x} (\pi^x)^{1-\kappa_x-\varkappa_x} (\check{\pi})^{\varkappa_x}, \end{aligned} \quad (3.22)$$

where $\kappa_x, \varkappa_x, \kappa_x + \varkappa_x \in (0, 1)$.

The equilibrium conditions associated with price setting by exporters that do get to reoptimize their prices are analogous to the ones derived for domestic intermediate good producers and are reported in section 6.3 in the Appendix.

3.4. Households

Household preferences are given by:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln(C_t - bC_{t-1}) - \zeta_t^h A_L \frac{(\varsigma_{i,t})^{1+\sigma_L}}{1+\sigma_L} \right], \quad (3.23)$$

where ζ_t^c is a shock to consumption preferences, ζ_t^h is labor supply shock, $\varsigma_{i,t}$ is hours worked. The household owns the stock of net foreign assets and determines its rate of accumulation.

3.4.1. Household Consumption Decision

The first order condition for consumption is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1} \frac{1}{\mu_{z,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1} \mu_{z,t+1} - bc_t} - \psi_{z,t} p_t^c (1 + \tau_t^c) = 0. \quad (3.24)$$

where

$$\psi_{z,t} = v_t P_t z_t$$

is the marginal value of one unit of the homogenous domestic good at time t .

3.4.2. Financial Assets and Interest Rate Parity

The household does the economy's saving. Period t saving occurs by the acquisition of net foreign assets, A_{t+1}^* , and a domestic asset. This asset pays a nominally non-state contingent return from t to $t+1$, R_t . The first order condition associated with this asset is:

$$-\psi_{z,t} + \beta E_t \frac{\psi_{z,t+1}}{\mu_{z,t+1}} \left[\frac{R_t - \tau_t^b (R_t - \pi_{t+1})}{\pi_{t+1}} \right] = 0, \quad (3.25)$$

where τ_t^b is the tax rate on the real interest rate on bond income (for additional discussion of τ^b , see section 3.7.) In the model the tax treatment of domestic agents' earnings on foreign bonds is the same as the tax treatment of agents' earnings on foreign bonds. The scaled date t first order condition associated with A_{t+1}^* that pays R_t^* in terms of foreign currency is:

$$v_t S_t = \beta E_t v_{t+1} \left[S_{t+1} R_t^* \Phi_t - \tau^b \left(S_{t+1} R_t^* \Phi_t - \frac{S_t}{P_t} P_{t+1} \right) \right]. \quad (3.26)$$

Recall that S_t is the domestic currency price of a unit of foreign currency. On the left side of this expression, we have the cost of acquiring a unit of foreign assets. The currency cost is S_t and this is converted into utility terms by multiplying by the Lagrange multiplier on the household's budget constraint, v_t . The term in square brackets is the after tax payoff of the foreign asset, in domestic currency units. The first term is the period $t + 1$ pre-tax interest payoff on A_{t+1}^* , which is $S_{t+1}R_t^*\Phi_t$. Here, R_t^* is the foreign nominal rate of interest, which is risk free in foreign currency units. The term, Φ_t represents a risk adjustment, so that a unit of the foreign asset acquired in t pays off $R_t^*\Phi_t$ units of foreign currency in $t + 1$. The determination of Φ_t is discussed below. The remaining term pertains to the impact of taxation on the return on foreign assets.⁹

We scale the first order condition, eq. (3.26), by multiplying both sides by $P_t z_t / S_t$:

$$\psi_{z,t} = \beta E_t \frac{\psi_{z,t+1}}{\pi_{t+1} \mu_{z,t+1}} [s_{t+1} R_t^* \Phi_t - \tau_t^b (s_{t+1} R_t^* \Phi_t - \pi_{t+1})], \quad (3.27)$$

where

$$s_t = \frac{S_t}{S_{t-1}}.$$

The risk adjustment term has the following form:

$$\Phi_t = \Phi \left(a_t, E_t s_{t+1} s_t, \tilde{\phi}_t \right) = \exp \left(-\tilde{\phi}_a (a_t - \bar{a}) - \tilde{\phi}_s (E_t s_{t+1} s_t - s^2) + \tilde{\phi}_t \right), \quad (3.28)$$

where, recall,

$$a_t = \frac{S_t A_{t+1}}{P_t z_t},$$

and $\tilde{\phi}_t$ is a mean zero shock whose law of motion is discussed below. In addition, $\tilde{\phi}_a$, $\tilde{\phi}_s$, \bar{a} are positive parameters.

The dependence of Φ_t on a_t ensures, in the usual way, that there is a unique steady state value of a_t that is independent of the initial net foreign assets and capital of the economy. The dependence of Φ_t on the anticipated growth rate of the exchange rate is designed to allow the model to reproduce two types of observations. The first concerns observations related uncovered interest parity. The second concerns the hump-shaped response of output to a monetary policy shock.

⁹If we ignore the term after the minus sign within the set of parentheses, we see that taxation is applied to the whole nominal payoff on the bond, including principal. The term after the minus sign is designed to ensure that the principal is deducted from taxes. The principal is expressed in nominal terms and is set so that the real value at $t + 1$ coincides with the real value of the currency used to purchase the asset in period t . In particular, recall that S_t is the period t domestic currency cost of a unit (in terms of foreign currency) of foreign assets. So, the period t real cost of the asset is S_t / P_t . The domestic currency value in period $t + 1$ of this real quantity is $P_{t+1} S_t / P_t$.

3.5. Wage Setting and Employment Frictions

We now consider the wage setting decision by households. We suppose that the specialized labor supplied by households is combined by labor contractors into a homogeneous labor service as follows:

$$H_t = \left[\int_0^1 (\varsigma_{j,t})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \quad 1 \leq \lambda_w < \infty,$$

where ς_j denotes the j^{th} household supply of labor services. Households are subject to Calvo wage setting frictions as in Erceg, Henderson and Levin [9] (EHL). With probability $1 - \xi_w$ the j^{th} household is able to reoptimize its wage and with probability ξ_w it sets its wage according to:

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \quad (3.29)$$

$$\tilde{\pi}_{w,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w-\varkappa_w)} (\bar{\pi})^{\varkappa_w} (\mu_{z+})^{\vartheta_w}, \quad (3.30)$$

where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0, 1)$. The wage updating factor, $\tilde{\pi}_{w,t+1}$, is sufficiently flexible that we can adopt a variety of interesting schemes.

Consider the j^{th} household that has an opportunity to reoptimize its wage at time t . We denote this wage rate by \tilde{W}_t . This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing \tilde{W}_t , the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[-\zeta_{t+i}^h A_L \frac{(\varsigma_{j,t+i})^{1+\sigma_L}}{1+\sigma_L} + v_{t+i} W_{j,t+i} \varsigma_{j,t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w} \right], \quad (3.31)$$

where τ_t^y is a tax on labor income and τ_t^w is a payroll tax. Also, recall that v_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$\varsigma_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}. \quad (3.32)$$

Here, it is understood that $\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1$ when $i = 0$.

3.6. Monetary Policy

We model monetary policy according to an instrument rule of the following form:

$$\begin{aligned} \ln \left(\frac{R_t}{R} \right) &= \rho_R \ln \left(\frac{R_{t-1}}{R} \right) + (1 - \rho_R) \left[\ln \left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c} \right) + r_\pi \ln \left(\frac{\pi_t^c}{\bar{\pi}_t^c} \right) + r_y \ln \left(\frac{y_t}{y} \right) \right] \\ &+ r_{\Delta\pi} \Delta \ln \left(\frac{\pi_t^c}{\bar{\pi}^c} \right) + r_{\Delta y} \Delta \ln \left(\frac{y_t}{y} \right) + \varepsilon_{R,t}, \end{aligned} \quad (3.33)$$

where the policy parameters are estimated to capture the historical behavior of the Riksbank between 1995 and 2010. Special attention will be given to estimate anticipated monetary policy shocks $\varepsilon_{R,t}$ below.

3.7. Fiscal Authorities

Government consumption expenditures are modeled as

$$G_t = g_t z_t,$$

where g_t is an exogenous stochastic process, orthogonal to the other shocks in the model. We suppose that

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t},$$

where $g = \bar{g}Y$. We set $\bar{g} = 0.3$, the sample average of government consumption as a fraction of GDP.

The tax rates in our model are:

$$\tau_t^k, \tau_t^b, \tau_t^y, \tau_t^c, \tau_t^w.$$

In the current version of the model we set all taxes to zero. Note that we need to set the tax rates on bonds to zero, $\tau^b = 0$, to be able to match the pre-tax real rate on bonds of 2.25% in the data. Setting $\tau^b = 0$ is required to get the interest rate on bonds to be this low, given the high GDP growth rate, log utility of consumption and β not too close to 1. All the tax rates are held constant in the model, implying that there are no stochastic tax shocks.

3.8. Foreign Variables

We assume that the foreign economy is given by a slight modification of the model described in the simple analytical example above. The version used here includes a permanent technology shock and is described in An and Schorfheide [2] and is given in its linearized form by the following equations:

$$\hat{y}_t^* = \hat{y}_{t+1}^* + \hat{g}_t^* - \hat{g}_{t+1}^* - \frac{1}{\tau} \left(\hat{R}_t^* - \hat{\pi}_t^* - \hat{\mu}_{z,t}^* \right), \quad (3.34)$$

$$\hat{\pi}_t^* = \beta \hat{\pi}_{t+1}^* + \kappa (\hat{y}_t^* + \hat{g}_t^*) + \hat{u}_t^*, \quad (3.35)$$

$$\hat{R}_t^* = \rho_{R^*} \hat{R}_{t-1}^* + (1 - \rho_{R^*}) \psi_1 \hat{\pi}_t^* + (1 - \rho_{R^*}) \psi_2 (\hat{y}_t^* + \hat{g}_t^*) + \varepsilon_{R^*t}, \quad (3.36)$$

$$\hat{g}_t^* = \rho_{g^*} \hat{g}_{t-1}^* + \varepsilon_{g^*t}, \quad (3.37)$$

The representation of the stochastic processes driving the foreign variables takes into account that foreign output, Y_t^* , is affected by a world-wide technology disturbances, z_t^* . In particular, our model of Y_t^* is:

$$\ln Y_t^* = \ln y_t^* + \ln z_t^*,$$

where $\log(y_t^*)$ is assumed to be a stationary process and $\ln z_t^* = \ln z_t + \ln \tilde{z}_t$. Hence, we assume that the world-wide technology disturbance consists of a common, $\ln z_t$, as well as a stationary asymmetric component $\ln \tilde{z}_t$.¹⁰ This in turn implies that $\hat{\mu}_{z,t}^* = \hat{\mu}_{z,t} + \hat{\tilde{\mu}}_{z,t}$, where we assume that $\hat{\tilde{\mu}}_{z,t} = \rho_{\hat{\tilde{\mu}}_z} \hat{\tilde{\mu}}_{z,t-1} + \sigma_{\hat{\tilde{\mu}}_z} \varepsilon_{\hat{\tilde{\mu}}_z,t}$.

3.9. Resource Constraints

3.9.1. Resource Constraint for Domestic Homogeneous Output

Resources expressed from the production side defines domestic homogeneous good, Y_t , in terms of aggregate factors of production. The scaled version of the production function (3.2) yields real, scaled GDP:

$$y_t = [\epsilon_t H_t - \phi]. \quad (3.38)$$

where it should be noted that there is no price dispersion ($\hat{p}_t = 1$).

It is convenient to also have an expression that exhibits the uses of domestic homogeneous output. Using (3.6) and (3.20),

$$z_t y_t = G_t + C_t^d + \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) (\hat{p}_t^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} (p_t^x)^{-\eta_f} Y_t^*,$$

or, after scaling by z_t and using (3.11) and (3.14):

$$\begin{aligned} y_t &= g_t + (1 - \omega_c) (p_t^c)^{\eta_c} c_t \\ &+ \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) (\hat{p}_t^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} (p_t^x)^{-\eta_f} y_t^* \tilde{\mu}_{z,t}. \end{aligned} \quad (3.39)$$

where it should be noted that there is no price dispersion ($\hat{p}_t = 1$).

¹⁰We will maintain the assumption that $\mu_z = \mu_{z^*}$, and treat $\ln \tilde{z}_t$ as a stationary shock which measures the degree of asymmetry in the technological progress in the domestic economy versus the rest of the world. By assuming $z_0^* = z_0 = 1$ this implies that the technology levels must be the same in steady state, $\tilde{z}_t = 1$. This specification is similar in spirit to Rabanal, Rubio-Ramirez and Tuesta [38] who study cointegrated technology shocks.

3.9.2. Trade Balance

We begin by developing the link between net exports and the current account. Expenses on imports and new purchases of net foreign assets, A_{t+1} , must equal income from exports and interest from previously purchased net foreign assets:

$$S_t A_{t+1} + \text{expenses on imports}_t = \text{receipts from exports}_t + R_{t-1}^* \Phi_{t-1} S_t A_t^*,$$

where Φ_t is the risk premium defined in (3.12). Expenses on imports correspond to the purchases of the specialized importers in the consumption, investment and export sectors, so that the current account can be written as

$$\begin{aligned} & S_t A_{t+1}^* + S_t P_t^* \left(C_t^m (\hat{p}_t^{m,c})^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + X_t^m (\hat{p}_t^{m,x})^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\ &= S_t P_t^x X_t + R_{t-1}^* \Phi_{t-1} S_t A_t^*, \end{aligned}$$

where $\hat{p}_t^{m,c} = \hat{p}_t^{m,x} = 1$. With price distortions among the imported intermediate goods, the expenses of the homogeneous import goods would be higher for any given value of C_t^m . Writing the current account in scaled form and dividing by $P_t z_t$, we obtain using (3.18)

$$\begin{aligned} a_t + q_t p_t^c \left(c_t^m (\hat{p}_t^{m,c})^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + x_t^m (\hat{p}_t^{m,x})^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\ = q_t p_t^c p_t^x x_t + R_{t-1}^* \Phi_{t-1} s_t \frac{a_{t-1}}{\pi_t \mu_{z^+,t}}, \end{aligned} \quad (3.40)$$

where $a_t = S_t A_{t+1}^* / (P_t z_t)$.

3.10. Exogenous Shock Processes

The structural shock processes in the model are given by the univariate representation

$$\Lambda_t = \rho_\Lambda \Lambda_{t-1} + \sigma_\Lambda \varepsilon_{\Lambda t}, \quad \varepsilon_{\Lambda t} \stackrel{iid}{\sim} N(0, I) \quad (3.41)$$

where $\varsigma_t = \{\varepsilon_{R,t}, \varepsilon_t, \varepsilon_{R^*t}, \tilde{\phi}_t, \varepsilon_{g,t}, \varepsilon_{g^*t}, \mu_{z,t}, \tilde{\mu}_{z,t}, \tau_t^d, \tau_t^{mc}, \tau_t^{mx}, \tau_t^x, \hat{u}_t^*, \zeta_t^c, \zeta_t^h\}$, where $\mu_{z,t} = z_t/z_{t-1}$ and $\tilde{\mu}_{z,t} = \tilde{z}_t/\tilde{z}_{t-1}$, and a hat denotes the deviation of a log-linearized variable from a steady-state level ($\hat{v}_t \equiv dv_t/v$ for any variable v_t , where v is the steady-state level). The τ_t^j , and the \hat{u}_t^* shocks are all assumed to be white noise (that is, $\rho_{\tau^j} = \rho_{u^*} = 0$).

4. Introducing Anticipated Shocks

The model described above is driven by fifteen exogenous forces. We assume that three of these forces, namely, monetary policy $\varepsilon_{R,t}$, foreign monetary policy ε_{R^*t} , and the risk premium shocks $\tilde{\phi}_t$, are subject to anticipated as well as unanticipated innovations. We study a formulation with one to eight-quarter anticipated shocks. This choice is mainly made in order to keep the computational time needed to estimate the model at a manageable level. In general, an innovation that is anticipated 8 periods introduces 8 additional state variables.

We introduce anticipated shocks either as in Case 1 or as in Case 3 in subsection (2.1.2) above. Hence, following Schmitt-Grohe and Uribe [5], the key departure of this paper from standard business-cycle analysis is the assumption that economic agents have an information set larger than one containing current and past realizations of the monetary policy and risk premium news shocks. More specifically we assume that the estimated policy and risk-premium disturbances $\Lambda_t^e = \{\varepsilon_{Rt}, \varepsilon_{R^*t}, \tilde{\phi}_t\}$ evolve according to an $ARMA(8, 8)$ process where ρ controls its persistence, i.e.:

$$\rho\Lambda_t^e = \Sigma\eta_t, \quad \eta_t \stackrel{iid}{\sim} N(0, I). \quad (4.1)$$

The expression for the disturbances includes both unanticipated (Λ_t^e, η_t) and anticipated innovations ($\Lambda_{t-i}^e, \eta_{t-i}$). Each term Λ_{t-i} or η_{t-i} , denotes a "news" shock about future monetary policy or risk premium, which is known to private agents in period $t-i$, but will materialize only i periods ahead. In Schmitt-Grohe and Uribe[5], all shocks were assumed to be uncorrelated but here we generalize the news process to also allow for correlated news i.e. $\rho \neq 0$.

News shocks capture future deviations of monetary policy from the Taylor rule that are credibly announced by the central bank or anticipated by the private sector. Anticipated shocks about future monetary policies affect the expectations about future macroeconomic variables that consumers and firms need to form in order to solve their consumption and price-setting decisions. The surprise shock has the interpretation of a deviation from the Taylor rule that is completely unexpected by the private sector. Thus, the identification of news shocks versus unexpected shocks works through this expectational channel.

To be more concrete, we impose the following structure on monetary policy error term for Case 1:

$$\varepsilon_{Rt} = \sigma^{R1}\eta_{Rt} + \sigma^{R2}\eta_{Rt-1} + \sigma^{R3}\eta_{Rt-2} + \dots + \sigma^{R8}\eta_{Rt-7},$$

For example, η_{Rt-2} is an innovation to ε_{Rt} that materializes in period t , but that agents learn about in period $t-2$. Hence, η_{Rt-2} is in the period $t-2$ information set of economic agents but results only in an actual change in ε_{Rt} in period t . We therefore call η_{Rt-2} a 2-period anticipated innovation to ε_{Rt} . The innovations η_{Rt-i} has mean zero, standard deviation equal to 1, and are uncorrelated across time and across anticipation horizon. Agents in the model are assumed to observe in period t current and past values of the innovations $\eta_{Rt} \cdots \eta_{Rt-7}$ and can therefore forecast future values of ε_{Rt} based on this information.

5. Estimating Anticipated Shocks

To compute the equilibrium decision rules, we proceed as follows. First, we stationarize all quantities determined in period t by scaling with the unit root technology shock z_t . Then, we log-linearize the model around the constant steady state and calculate a numerical (reduced form) solution with the AIM algorithm developed by Anderson and Moore [31]. We start the empirical analysis by estimating the DSGE model, using a Bayesian approach and placing a prior distribution on the noncalibrated structural parameters of the model. Of particular importance among the estimated parameters are those defining the stochastic processes of anticipated innovations. The log-linearized equations are summarized in Appendix 6.10.

5.1. Data

We estimate the model using quarterly Swedish data for the period 1995Q1 – 2010Q3. The vector of 14 observed variables are,

$$\tilde{Y}_t^{CurrentData} = \begin{bmatrix} R_t^{data} & \Delta \ln(W_t/P_t)^{data} & \Delta \ln C_t^{data} & \Delta \ln Y_t^{data} & \Delta \ln Y_t^{*,data} & \pi_t^{*,data} \\ R_t^{*,data} & \hat{H}_t^{data} & \Delta \ln M_t^{data} & \pi_t^{data} & \pi_t^{c,data} & \Delta \ln X_t^{data} \\ \Delta \ln q_t^{data} & \Delta \ln G^{data} & & & & \end{bmatrix}, \quad (5.1)$$

where the repo rate, CPI inflation, GDP deflator inflation, foreign inflation, foreign interest rate, and the hours gap (hours deviation from an hp-trend) are matched in levels. The inflation and interest rates are measured as annualized quarterly rates. The rest of the variables are matched in growth rates measured as quarter-to-quarter log-differences; GDP, consumption, exports, imports, real wage, real exchange rate, government consumption, and foreign output. All real quantities are in per capita terms.

All variables are seasonally adjusted but no other pre-filtering of the data is done (such as demeaning) except for exports, imports and government consumption. Since exports, imports and government consumption grow at substantially different rates compared to output we adjust the mean growth rates of these three series so that they are growing at the same pace as output (i.e., we take out the excess trends in exports and imports and add an extra trend to government consumption). We also extract an obvious outlier in 1997 from the government consumption series.

The data are taken from Statistics Sweden and Sveriges Riksbank (i.e., repo rate and the foreign variables). The foreign variables on output, and inflation are weighted together across Sweden's 20 largest trading partners in 1991 using weights from the IMF. The foreign interest rate is the German 3 month t-bill rate. This rate and the trade weighted rates are very similar. In order to be consistent with the measures of policy expectations which are described in the next section we need to use German data for the interest rate as well.

5.2. Measures of policy expectations

We use forward rates from Sweden and Germany as a crude measure of expected future policy rates to possibly better identify expected monetary policy shocks. Under the perhaps rather heroic assumption of small or negligible term premia forward rates can be interpreted as indicating market expectations of future short interest rates.¹¹ In the absence of explicit forward markets, especially at the medium to long term, implied forward interest rates have to be estimated. Here we use the Nelson-Siegel-Svensson method for this purpose. The method we have used is described in detail in BIS paper No 25 [46]. The data on market expectations of future short interest rates is available in daily and monthly frequency. We convert them to quarterly frequency by computing arithmetic averages over the appropriate time intervals. It is worth noting that the model is estimated on the whole sample period, and do not take account of any learning (which might be important as e.g.

¹¹Calculating monetary policy expectations on the basis of implied forward rates is difficult. This is partly because forward rates also include risk premiums, which means that the measure does not solely reflect expectations of the future policy rate. Another means of measuring monetary policy expectations is to use surveys. These have the advantage that they give estimates of market agents' expectations of the future interest rate without having to take forward premiums into account. In times of financial turmoil, forward premiums often vary substantially. Expectations based on surveys may therefore be a more robust measure of monetary policy expectations during such periods. However, surveys are not without their problems,

either. For instance, the statistical sample is often fairly small and the surveys are not carried out very often. Forward rates and surveys thus have different advantages and disadvantages.

argued by Laubach, Tetlow and Williams [12] and others).

The vector of 14 observed variables extended with data on market expectations of future short interest rates for 8 quarters for Sweden and the rest-of-the-world are given by

$$\tilde{Y}_t^{ForwData} = \begin{bmatrix} R_t^{data} & \Delta \ln(W_t/P_t)^{data} & \Delta \ln C_t^{data} & \Delta \ln Y_t^{data} & \Delta \ln Y_t^{*,data} & \pi_t^{*,data} \\ R_t^{*,data} & \hat{H}_t^{data} & \Delta \ln M_t^{data} & \pi_t^{data} & \pi_t^{c,data} & \Delta \ln X_t^{data} \\ \Delta \ln q_t^{data} & \Delta \ln G_t^{data} & & & & \\ R_{t,t+1}^{data} & R_{t,t+2}^{data} & R_{t,t+3}^{data} & R_{t,t+4}^{data} & R_{t,t+5}^{data} & R_{t,t+6}^{data} \\ R_{t,t+7}^{data} & R_{t,t+8}^{data} & R_{t+1}^{*,data} & R_{t+2}^{*,data} & R_{t+3}^{*,data} & R_{t+4}^{*,data} \\ R_{t+5}^{*,data} & R_{t+6}^{*,data} & R_{t+7}^{*,data} & R_{t+8}^{*,data} & & \end{bmatrix}. \quad (5.2)$$

The implied forward interest rates are displayed in figure 3 from 1995 quarter 2 to 2010 quarter 3. The solid lines are R_t^{data} and $R_t^{*,data}$, and the circles are $R_{t,t+1}^{data} \dots R_{t,t+8}^{data}$ and $R_{t,t+1}^{*,data} \dots R_{t,t+8}^{*,data}$. The figure also displays the difference between the interest rate. There has been a rapid convergence in interest rates over the sample. This convergence was to some extent expected but the actual interest rates converged faster than what was expected. Moreover, interest rate as well as expectations differentials are relatively persistent.

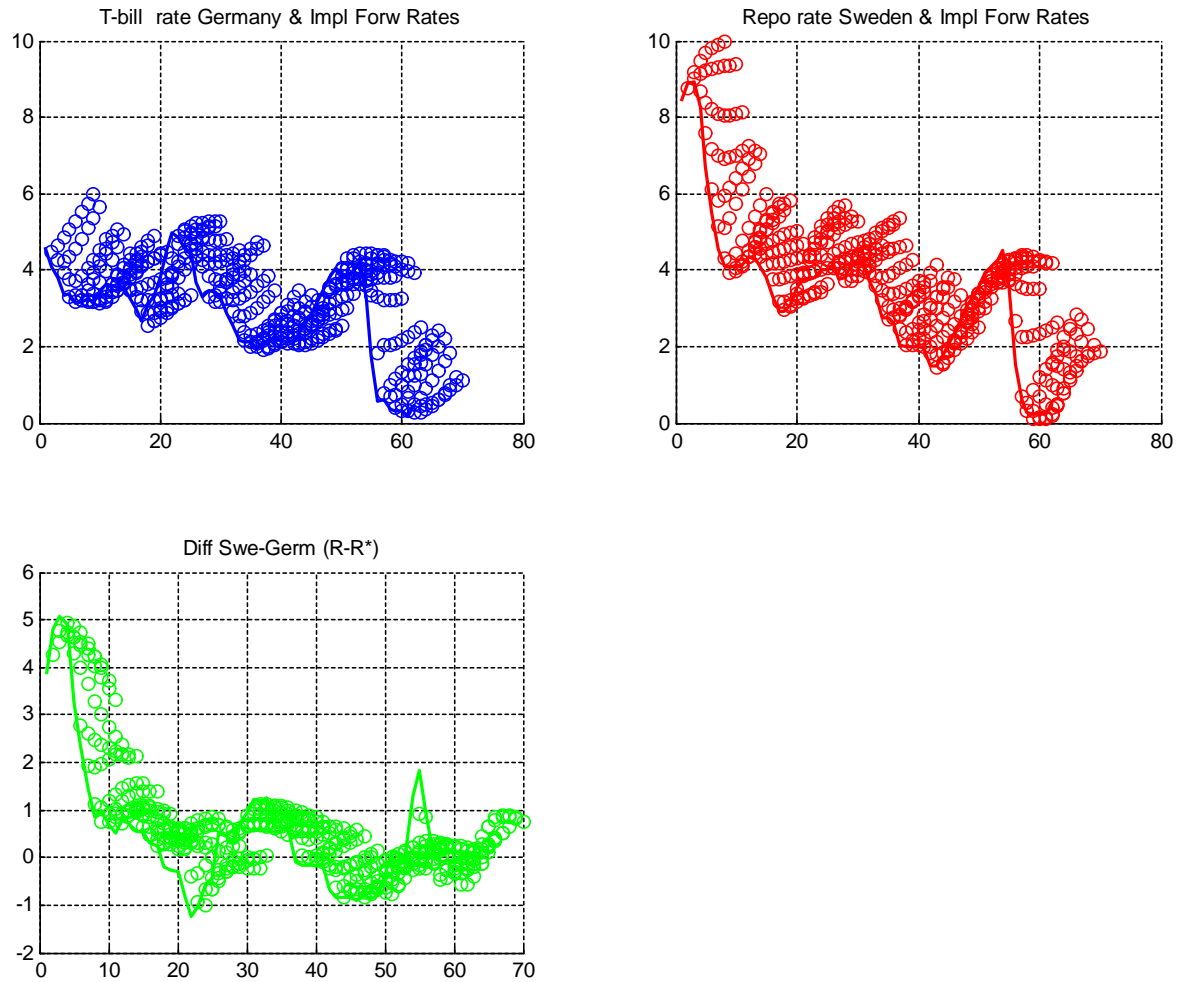


Figure 3. Implied forward interest rates from 1995 quarter 2 to 2010 quarter 3.

5.3. Measurement equations

Below we report the measurement equations we use to link the model to the data. Our data series for inflation and interest rates are annualized in percentage terms, so we make the same transformation for the model variables i.e. multiplying by 400. We match hours worked per capita in terms of deviation from steady state. First differences and deviations from steady state are

written in percentages so model variables are multiplied by 100 accordingly.

$$\begin{aligned}
R_t^{data} &= 400 (R_t - 1) \\
R_t^{*,data} &= 400 (R_t^* - 1) \\
\pi_t^{data} &= 400 \log(\pi_t) + \varepsilon_{\pi,t}^{me} \\
\pi_t^{c,data} &= 400 \log(\pi_t^c) + \varepsilon_{\pi^c,t}^{me} \\
\pi_t^{*,data} &= 400 \log(\pi_t^*) + \varepsilon_{\pi^*,t}^{me} \\
\Delta \ln(W_t/P_t)^{data} &= 100 (\log \mu_{z,t} + w_t - w_{t-1}) + \varepsilon_{w,t}^{me} \\
\Delta \ln C_t^{data} &= 100 (\log \mu_{z,t} + c_t - c_{t-1}) + \varepsilon_{c,t}^{me} \\
\Delta \ln Y_t^{data} &= 100 (\log \mu_{z,t} + y_t - y_{t-1}) + \varepsilon_{y,t}^{me} \\
\Delta \ln Y_t^{*,data} &= 100 (\log \mu_{z,t}^* + y_t^* - y_{t-1}^*) + \varepsilon_{y^*,t}^{me} \\
\Delta \ln M_t^{data} &= 100 (\log \mu_{z,t} + c_t^m - c_{t-1}^m + x_t^m - x_{t-1}^m) + \varepsilon_{M,t}^{me} \\
\Delta \ln X_t^{data} &= 100 (\log \mu_{z,t} + x_t - x_{t-1}) + \varepsilon_{X,t}^{me} \\
\Delta \ln G_t^{data} &= 100 (\log \mu_{z,t} + g_t^m - g_{t-1}^m) + \varepsilon_{G,t}^{me} \\
\Delta \ln q_t^{data} &= 100 (q_t - q_{t-1}) + \varepsilon_{q,t}^{me} \\
\hat{H}_t^{data} &= 100 \left(\frac{H_t - H_t^{trend}}{H_t^{trend}} \right) + \varepsilon_{H,t}^{me} \\
R_{t,t+1}^{data} &= 400 (R_{t+1} - 1) + \varepsilon_{R1,t}^{me} \\
R_{t,t+8}^{data} &= 400 (R_{t+8} - 1) + \varepsilon_{R8,t}^{me} \\
R_{t,t+1}^{*,data} &= 400 (R_{t+1}^* - 1) + \varepsilon_{R^*1,t}^{me} \\
R_{t,t+8}^{*,data} &= 400 (R_{t+8}^* - 1) + \varepsilon_{R^*8,t}^{me}
\end{aligned}$$

" $\varepsilon_{i,t}^{me}$ " denotes the measurement errors for the respective variables. Note that we allow for measurement errors in the market expectations of future short interest. Above we only include data for $t+1$ and $t+8$ to save space but we match the other data in the same way. Since Swedish macro data is measured with substantial noise, we allow for measurement errors in all variables except for the nominal interest rates in Sweden and abroad. The variance of the measurement errors is calibrated so that it corresponds to 10% of the variance in each data series.

5.4. Calibration

The empirical model has a large number of parameters, which makes it desirable to calibrate at least some of them. We choose to calibrate the parameters related to the steady-state values of the

observable quantities, for example the “great ratios” (i.e., C/Y , and G/Y). Table 5.3 shows the calibrated parameters. The discount factor β and the tax rate on bonds τ_b are calibrated to yield a real interest of rate equal to 2.14 percent annually.

Sample averages are used when available, e.g. for the various import shares ω_c, ω_x (obtained from input-output tables), the remaining tax rates, the government consumption share of GDP, η_g , growth rates of technology (using investment prices to disentangle neutral from investment-specific technology) and several other parameters. To calibrate the steady value of the inflation target we simply use the inflation target stated by Sveriges Riksbank.

We let the markup of export good producers λ_x be low so as to avoid double marking up of these goods. All other price markups are set to 1.2, following a wide literature. The indexation parameters $\varkappa^j, j = d, x, mc, mx, w$ are set so that there is no indexation to the inflation target, but instead to $\tilde{\pi}$ which is set equal to the steady state inflation. This implies that we do not allow for partial indexation in this estimation, which would result in steady state price and wage dispersion.

Parameter	Value	Description
β	0.9999	Discount factor
ω_c	0.25	Import share in consumption goods
ω_x	0.35	Import share in export goods
η_g	0.3	Government consumption share of GDP
μ_z	1.005	Steady state growth rate of neutral technology
$\bar{\pi}, \tilde{\pi}$	1.005	Steady state gross inflation target
λ_x	1.05	Export price markup
λ_j	1.2	Price markups, $j = d, mc, mx$
$\tilde{\phi}_a$	0.01	Risk premium dependence on net foreign assets
ϑ_w, κ_w	0	Wage indexation to real growth trend and lagged inflation
\varkappa^j	$1 - \kappa^j$	Indexation to inflation target for $j = d, x, mc, mx, w$
σ_L	2.5	Inverse Frisch elasticity

Table 5.3. Calibrated parameters. Note: The time unit is one quarter.

Throughout the estimation, two observable ratios are chosen to be exactly matched in our steady-state solution and accordingly two corresponding ‘steady-state’ parameters are recalibrated for each (estimated) parameter draw. We set the steady state real exchange rate $\tilde{\varphi}$ to match the export share $P^x X/(PY)$ in the data, and finally we set the disutility of labor scaling parameter A_L to fix the fraction of their time that individuals spend working. The values of these two calibrated parameters (evaluated at the posterior mode) are presented in Table 5.4.

	Parameter description	Calibrated value	Moment	Moment value
$\tilde{\varphi}$	Real exchange rate	0.084	$P^x X/(PY)$	0.44
A_L	Scaling of disutility of work	131.9	H	0.2

Table 5.4. Matched moments and corresponding parameters (evaluated at the posterior mode).

5.5. Estimation results

In total we estimate 45 parameters in the baseline case, of which 6 are parameters for the foreign economy, 9 are AR1-coefficients and 15 are standard deviations of the shocks. The priors and estimated posterior distributions are displayed in Tables 5.5 and 5.6. The location of the prior distribution of the 45(66) estimated parameters corresponds to a large extent to those in Christiano, Trabandt and Walentin [4] and for the foreign economy to those in An and Schorfheide [2]. We are conservative with our choice of prior for the anticipated shocks and we set them close to zero. Hence, the data needs to be informative about the news in order to move these parameters. Three posterior distributions are reported. In the first, labeled “*Baseline*”, we do not estimate any news shocks. The second, labeled “*Current Data*”, shows the results when we use the same data set as in “Baseline” but also estimate the standard deviation of the news shocks. The third, labeled “*Forward Data*”, displays the results with data including market expectations of future short interest rates. The posterior distributions are also displayed in Appendix 6.11.

Figure 4 and figure 5 shows the data (thick red line) used in the estimation and the one-sided Kalman-filtered one-step-ahead predictions from the model (blue dashed) computed at the posterior mode. We see that the model captures the low-frequency fluctuations in the data relatively well for most of the observed variables but misses out on many of the high-frequency movements, especially in the inflation series as well as in exports and imports. In addition, the real wage grows too slowly in the model compared with the data, throughout the sample. One explanation to this is that the real wage is computed using the GDP deflator which is an extremely volatile series. Much of the variance in the data should thus not be attributed to the structural model. Fiscal policy is rudimentary modeled so it is perhaps not so surprising that the model does not capture the growth rate in government consumption so well. For the implicit forward rate variables (in figure 5), the model can explain the data remarkably well which is also evident from figures 6 and 7 which shows the fit of the data in the same way as in figure 3 in the data section above.

There are two important facts to note in the first two posterior distributions pertaining to the estimation of the anticipated shocks. First, it is clear that the importance of expected future

monetary policy shocks in explaining the data is relatively modest. The parameters $\sigma^{R2}, \dots, \sigma^{R8}$ and $\sigma^{R*2}, \dots, \sigma^{R*8}$ are in general very close to zero. There is some indication that anticipation for 2 and 3 quarters ahead is somewhat more important than anticipation at longer horizons with parameters varying between 0.03 and 0.04. What is more interesting is that anticipated risk premium shocks seem to be of much larger importance compared to anticipated monetary policy shocks. Here it is longer term expectations which comes out as more important. Moreover, adding data on implied forward rates as a measure of expected monetary policy gives the same general picture, i.e. that anticipated monetary policy shocks are less important in explaining the data than anticipated risk premium shocks.

Forecast Error Variance Decomposition confirm these results (see Appendix 6.12). It is interesting to note that 8 quarter anticipated risk premium shocks explain around 30 percent of the variation in the implied forward rates but not much of the variation in $\tilde{Y}_t^{CurrentData}$. Further, the asymmetric technology shock is important in explaining the variation in the implied forward rates.

Figures 8, 9 and 10 shows impulse-response functions to anticipated monetary policy shocks, anticipated risk premium shocks and anticipated foreign monetary policy shocks respectively. The red line shows 1 quarter anticipation, the green line shows 4 quarter anticipation and the blue line shows 8 quarter anticipation. The shorter horizon anticipated monetary policy shocks do in general have larger effects than longer anticipation horizon. This seems to be caused by opposing monetary policy forces. Anticipation of a higher instrument rate at a distant future depresses the economy and lowers inflation today. Systematic monetary policy responds to this development by lowering the instrument rate to mitigate the low inflation and stabilize the economy. These forces cancel each other out and the effects of anticipated shocks comes in general out as negligible. This does not hold for anticipated risk premia. Here anticipated risk premia creates inflation and a higher interest rate. These shocks are able to explain co-movement in the data and comes out from the estimation exercise as much more important.

One potentially important observation in the case when we match data on implied forward rates together with current data is that foreign and domestic interest rates and implied forward rates co-move persistently. Persistent co-movement in interest rates is difficult to generate with domestic shocks alone since these only affect domestic interest rates and domestic implied forward rates. Hence, to fit the data, global shocks which explain the joint movement in domestic and foreign interest rates, like the asymmetric technology shock $\hat{\mu}_{z,t}$, can potentially explain more of of the

co-movement in the data than in the case when we only match current data.

5.5.1. Effects of anticipated shocks during the financial crisis

During the financial crisis we have witnessed very low short and long market rates as well as implied forward rates. In the Eurozone, an initial increase of the policy rate is not expected for at least one year. The policy rates in the United Kingdom and the United States are also expected to remain low for a long time to come. It is clear from the results in the section above that anticipated monetary policy shocks do not explain the overall variations in the data to a large extent. It could well be the case that these anticipated monetary policy shocks are more important only during very specific events such as the recent financial crisis. This section looks closer into the contribution of anticipated monetary policy and risk premium shocks during the financial crisis i.e. during 2008 and 2010. Figures 11 – 14 show historical decomposition of the repo rate, the implied forward rate 8 quarters ahead and the foreign economy equivalent.

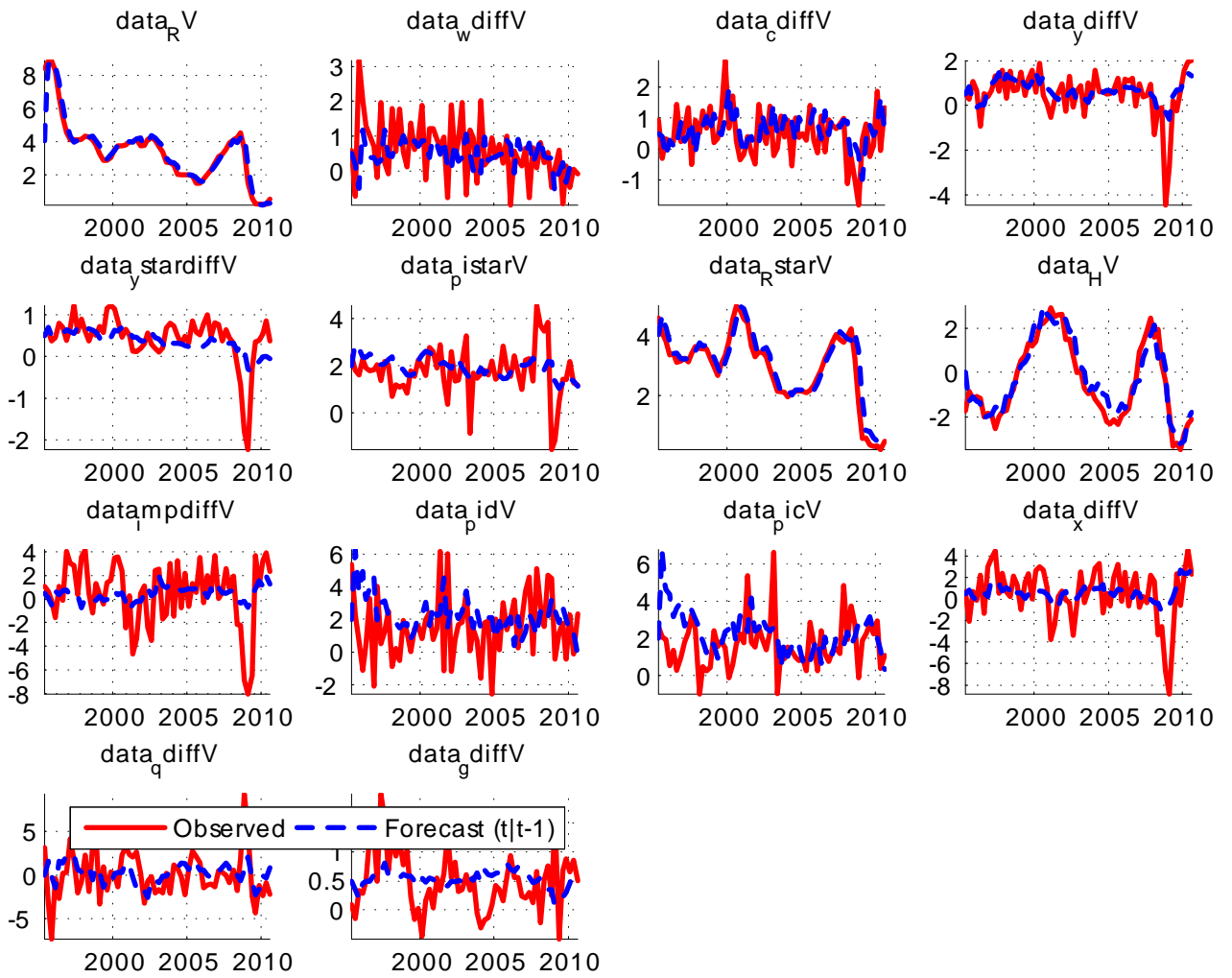
These results confirm the suspicion that anticipated shocks have a larger impact during the period 2007Q1 – 2010Q3 than what is found in the results from the overall variance decompositions. Anticipated shocks have lowered the repo rate with about 75 basis points during the later part of the period. Asymmetric technology shocks seem to be about equally important. However, the most important factors affecting the repo rate during the crisis are not related to the anticipated nominal shocks which we study here.

Table 5.5: Prior and posterior distributions.

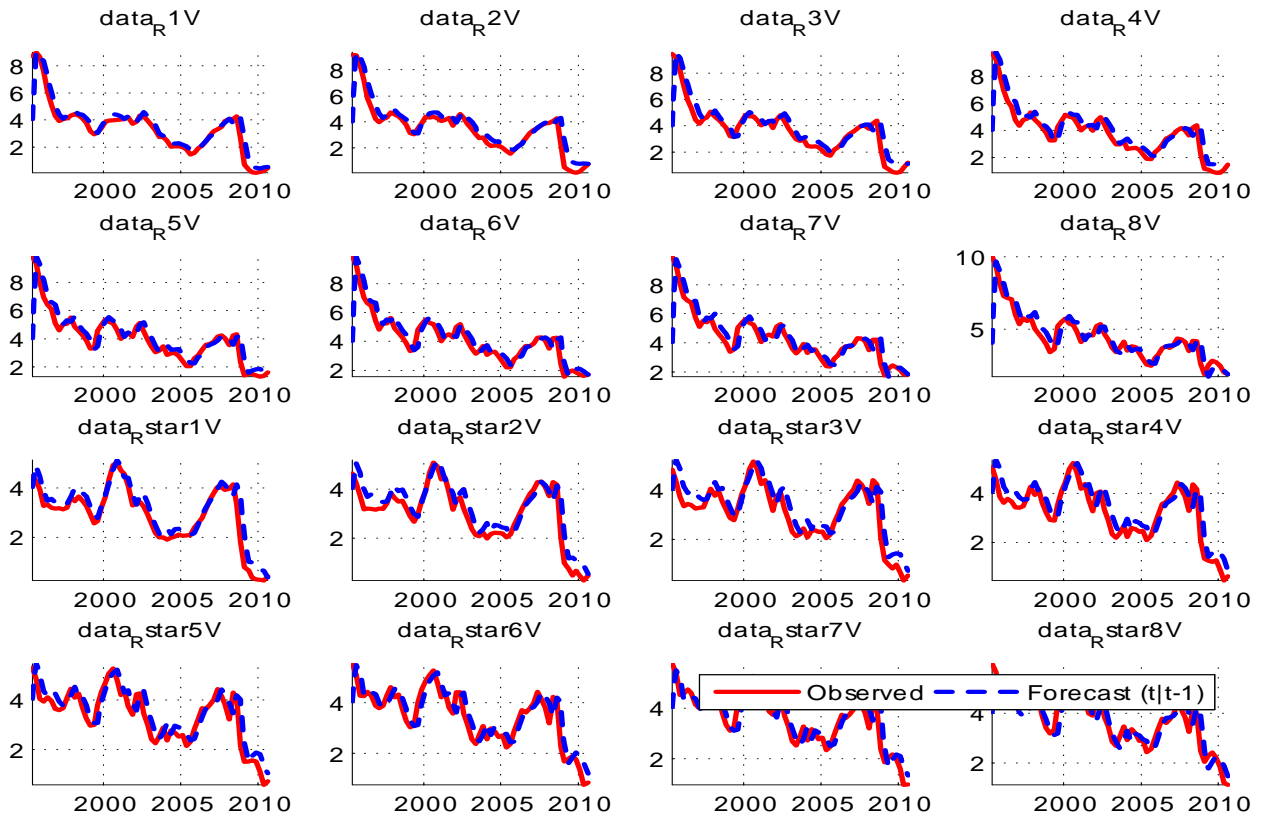
	Distr.	Prior		Posterior Baseline		Posterior Current Data		Posterior Forw. Data	
		Mean	S.d.	Mean	S.d.	Mean	S.d.	Mean	S.d.
b	Beta	0.65	0.15	0.73	0.08	0.72	0.08	0.78	0.06
η_c	Gamma	1.5	0.10	1.32	0.07	1.32	0.08	1.38	0.08
η_f	Gamma	1.5	0.10	1.52	0.10	1.53	0.10	1.56	0.11
η_x	Gamma	1.5	0.10	1.46	0.11	1.46	0.12	1.39	0.10
κ	Beta	0.2	0.10	0.06	0.03	0.08	0.07	0.02	0.01
κ_d	Beta	0.5	0.15	0.36	0.10	0.34	0.10	0.27	0.08
κ_w	Beta	0.5	0.15	0.43	0.14	0.43	0.13	0.47	0.14
ξ_d	Beta	0.75	0.075	0.87	0.04	0.87	0.04	0.87	0.03
ξ_{mc}	Beta	0.75	0.075	0.85	0.03	0.85	0.03	0.84	0.03
ξ_{mc}	Beta	0.75	0.075	0.67	0.07	0.68	0.07	0.67	0.06
ξ_w	Beta	0.75	0.075	0.61	0.08	0.60	0.09	0.35	0.07
ξ_x	Beta	0.75	0.075	0.86	0.03	0.86	0.03	0.85	0.03
ϕ_s	Beta	0.5	0.15	0.45	0.12	0.36	0.11	0.78	0.07
ψ_1	Normal	1.7	0.10	1.67	0.10	1.67	0.10	1.71	0.10
ψ_2	Gamma	0.5	0.25	0.60	0.22	0.47	0.23	0.43	0.16
$r_{\Delta\pi}$	Normal	0.30	0.10	0.07	0.03	0.07	0.03	0.04	0.03
$r_{\Delta y}$	Gamma	0.05	0.025	0.08	0.02	0.07	0.02	0.08	0.01
ρ_z	Beta	0.85	0.075	0.88	0.04	0.89	0.04	0.89	0.04
ρ_g	Beta	0.85	0.075	0.94	0.03	0.94	0.03	0.94	0.03
ρ_{g^*}	Beta	0.80	0.10	0.82	0.10	0.84	0.10	0.92	0.04
ρ_{μ_z}	Beta	0.50	0.15	0.66	0.09	0.67	0.09	0.65	0.09
ρ_{μ_z}	Beta	0.50	0.15	0.92	0.02	0.94	0.02	0.97	0.01
$\rho_{\bar{\phi}}$	Beta	0.85	0.10	0.78	0.08	0.72	0.09	0.60	0.07
ρ_R	Beta	0.85	0.10	0.87	0.02	0.87	0.02	0.93	0.02
ρ_{R^*}	Beta	0.50	0.20	0.85	0.02	0.85	0.02	0.88	0.02
ρ_{c_c}	Beta	0.850	0.075	0.77	0.07	0.78	0.08	0.84	0.07
ρ_{c_h}	Beta	0.850	0.075	0.65	0.10	0.64	0.10	0.99	0.01
r_{π}	Normal	1.70	0.10	1.73	0.10	1.73	0.10	1.68	0.11
Γ_y	Normal	0.125	0.05	0.19	0.04	0.19	0.04	0.19	0.05
σ_{R1}	Invgamma	0.15	2	0.09	0.01	0.06	0.01	0.07	0.01
σ_e	Invgamma	0.50	2	0.51	0.10	0.51	0.10	0.47	0.10
σ_{R^*1}	Invgamma	0.15	2	0.09	0.01	0.07	0.02	0.08	0.01
σ_{ϕ^1}	Invgamma	0.15	2	0.86	0.26	0.17	0.17	1.38	0.25
σ_g	Invgamma	0.15	2	0.46	0.06	0.47	0.06	0.45	0.06
σ_{g^*}	Invgamma	0.15	2	0.11	0.08	0.15	0.11	0.44	0.07
σ_{μ_z}	Invgamma	0.15	2	0.18	0.04	0.18	0.04	0.20	0.04
σ_{μ_z}	Invgamma	0.15	2	0.16	0.03	0.14	0.04	0.13	0.02
$\sigma_{\tau d}$	Invgamma	0.15	2	3.68	2.54	4.41	3.74	3.92	2.53
$\sigma_{\tau mc}$	Invgamma	0.15	2	4.38	2.11	4.46	2.10	3.54	1.32
$\sigma_{\tau mx}$	Invgamma	0.15	2	4.70	2.46	4.88	2.62	4.59	2.10
$\sigma_{\tau x}$	Invgamma	0.15	2	8.72	4.61	9.10	4.73	7.01	3.17
σ_{u^*}	Invgamma	0.15	2	0.23	0.03	0.22	0.03	0.25	0.03
σ_{c_c}	Invgamma	0.15	2	0.37	0.11	0.37	0.10	0.50	0.13
σ_{c_h}	Invgamma	0.15	2	3.01	2.20	2.91	2.28	0.22	0.10
τ	Gamma	2.00	0.50	3.46	0.63	3.46	0.65	4.16	0.63

Table 5.6: Prior and posterior distributions. Anticipated shocks Case 1.

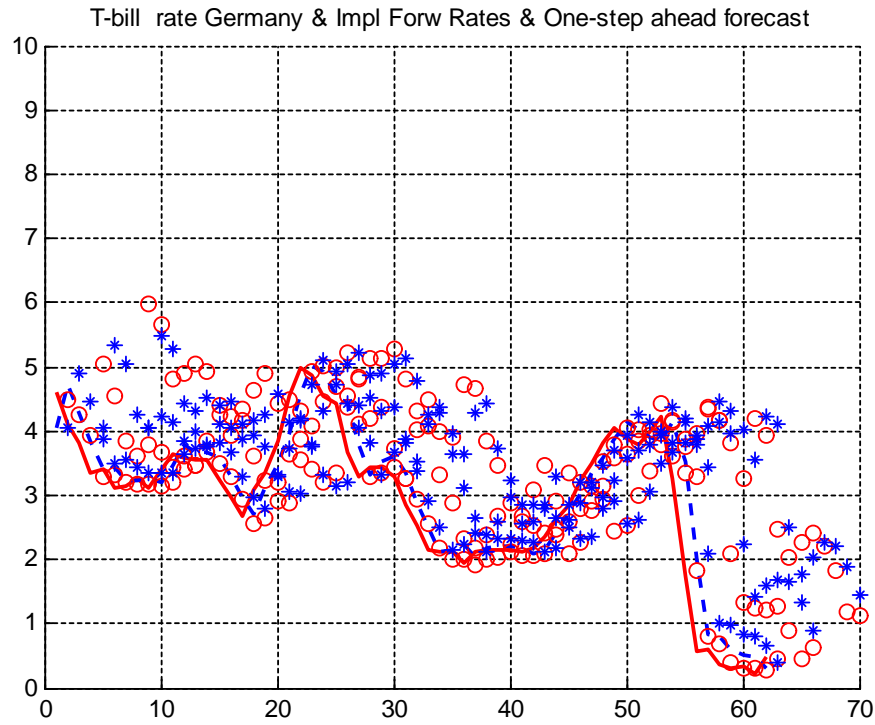
	Distr.	Prior			Posterior Current Data		Posterior Forw. Data	
		Mode	Location	d.f.	Mean	S.d.	Mean	S.d.
		Standard deviation: anticipated domestic monetary policy shocks						
σ^{R2}	Invgamma	0.018	0.025	1	0,029	0,014	0,038	0,016
σ^{R3}	Invgamma	0.018	0.025	1	0,027	0,013	0,032	0,013
σ^{R4}	Invgamma	0.018	0.025	1	0,024	0,011	0,020	0,007
σ^{R5}	Invgamma	0.018	0.025	1	0,025	0,012	0,017	0,005
σ^{R6}	Invgamma	0.018	0.025	1	0,026	0,012	0,016	0,005
σ^{R7}	Invgamma	0.018	0.025	1	0,025	0,012	0,015	0,005
σ^{R8}	Invgamma	0.018	0.025	1	0,027	0,014	0,016	0,05
		Standard deviation: anticipated foreign monetary policy shocks						
σ^{R^*2}	Invgamma	0.018	0.025	1	0,031	0,017	0,020	0,008
σ^{R^*3}	Invgamma	0.018	0.025	1	0,027	0,013	0,030	0,012
σ^{R^*4}	Invgamma	0.018	0.025	1	0,027	0,013	0,026	0,011
σ^{R^*5}	Invgamma	0.018	0.025	1	0,026	0,012	0,017	0,006
σ^{R^*6}	Invgamma	0.018	0.025	1	0,024	0,011	0,015	0,004
σ^{R^*7}	Invgamma	0.018	0.025	1	0,024	0,011	0,014	0,004
σ^{R^*8}	Invgamma	0.018	0.025	1	0,026	0,013	0,016	0,005
		Standard deviation: anticipated risk premia						
$\sigma^{\bar{\phi}2}$	Invgamma	0.018	0.025	1	0,979	0,325	0,080	0,122
$\sigma^{\bar{\phi}3}$	Invgamma	0.018	0.025	1	0,07	0,104	0,087	0,130
$\sigma^{\bar{\phi}4}$	Invgamma	0.018	0.025	1	0,067	0,097	0,070	0,098
$\sigma^{\bar{\phi}5}$	Invgamma	0.018	0.025	1	0,072	0,105	0,053	0,056
$\sigma^{\bar{\phi}6}$	Invgamma	0.018	0.025	1	0,061	0,073	0,058	0,064
$\sigma^{\bar{\phi}7}$	Invgamma	0.018	0.025	1	0,064	0,084	0,052	0,061
$\sigma^{\bar{\phi}8}$	Invgamma	0.018	0.025	1	0,056	0,074	0,614	0,120



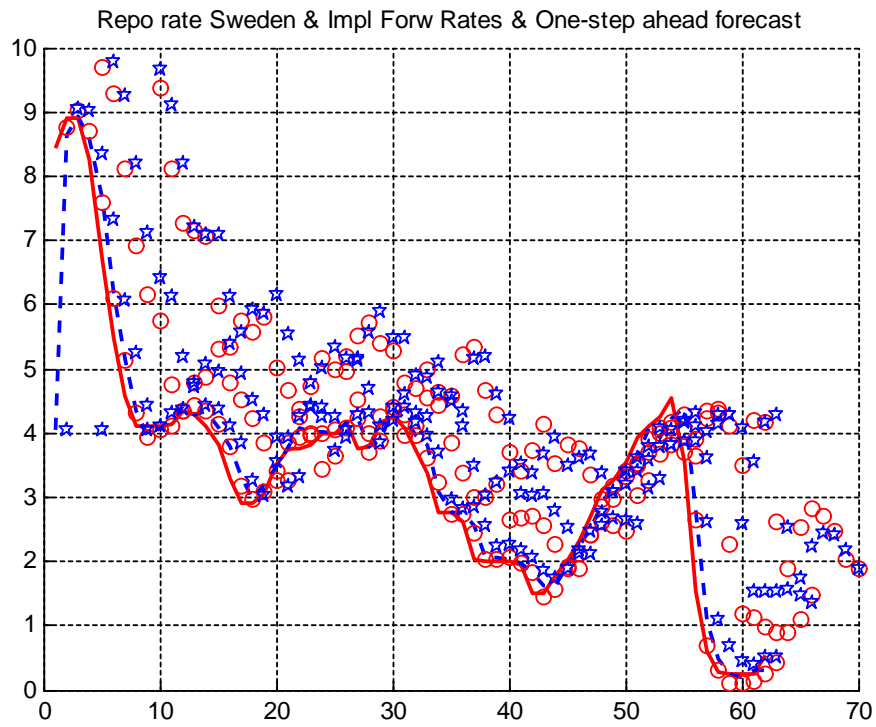
One-step ahead forecast and observed data, $\tilde{Y}_t^{CurrentData}$.



One-step ahead forecast and observed data. Part of $\tilde{Y}_t^{ForwData}$ containing implied forward interest rates 1-8 quarters.



The solid red line is the t-bill rate for Germany and the red circles are implied forward rates. The solid blue line is one-step ahead forecast of the foreign instrument rate and the stars are one-step ahead forecasts the instrument rate one to eight quarters ahead.



The solid red line is the repo rate and the red circles are implied forward rates. The solid blue line is one-step ahead forecast of the instrument rate and the blue pentagrams are one-step ahead forecasts the instrument rate one to eight quarters ahead.

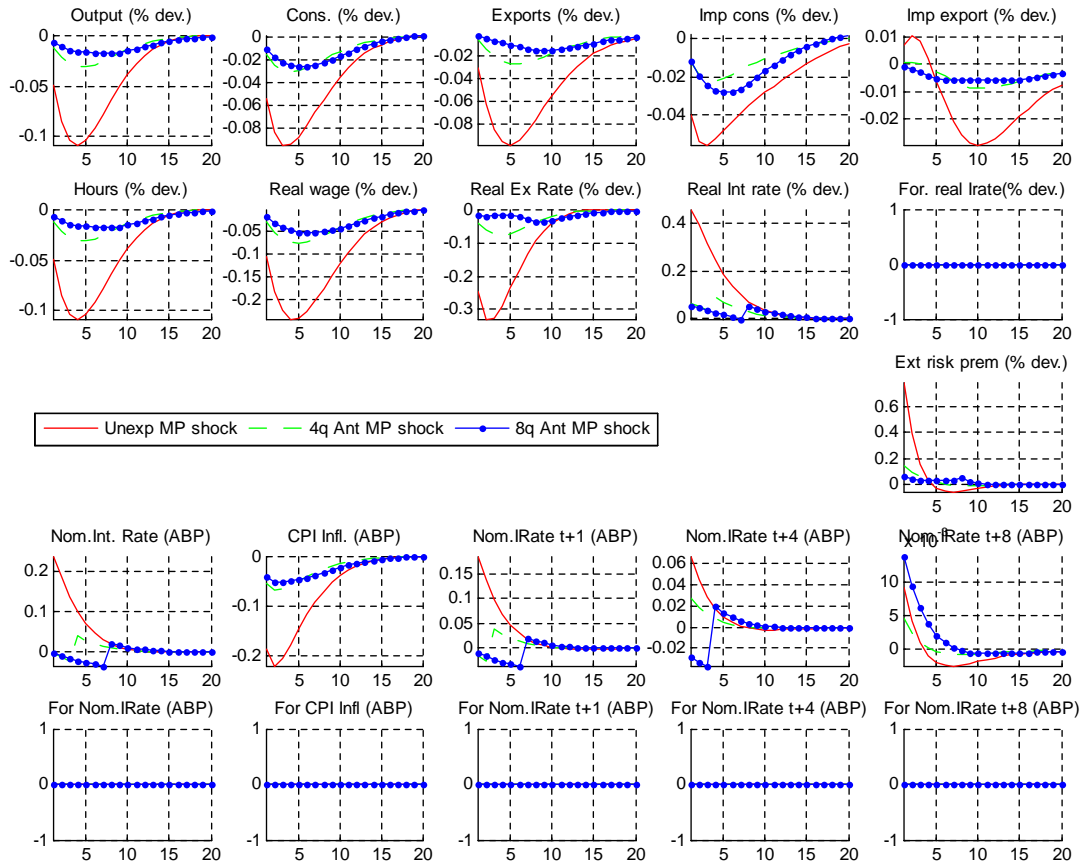


Figure 8: Impulse-response functions to unanticipated and anticipated monetary policy shocks.

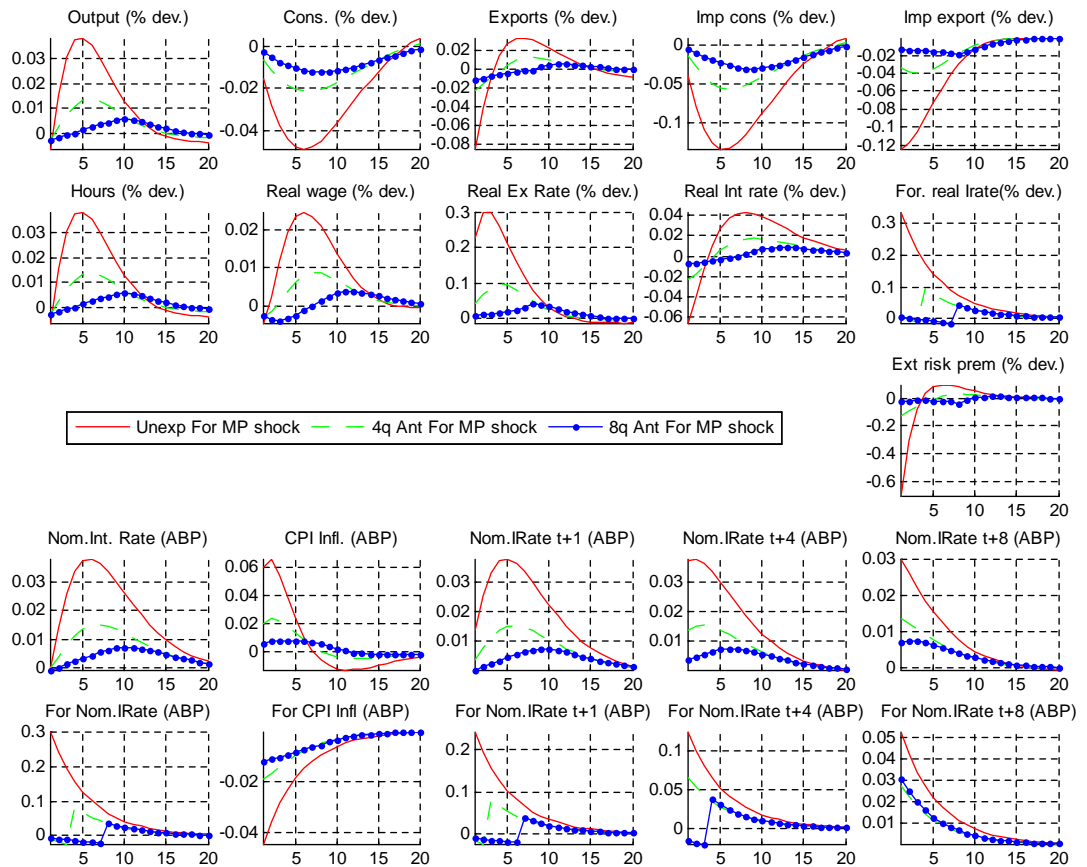


Figure 9: Impulse-response functions to unanticipated and anticipated foreign monetary policy shocks.

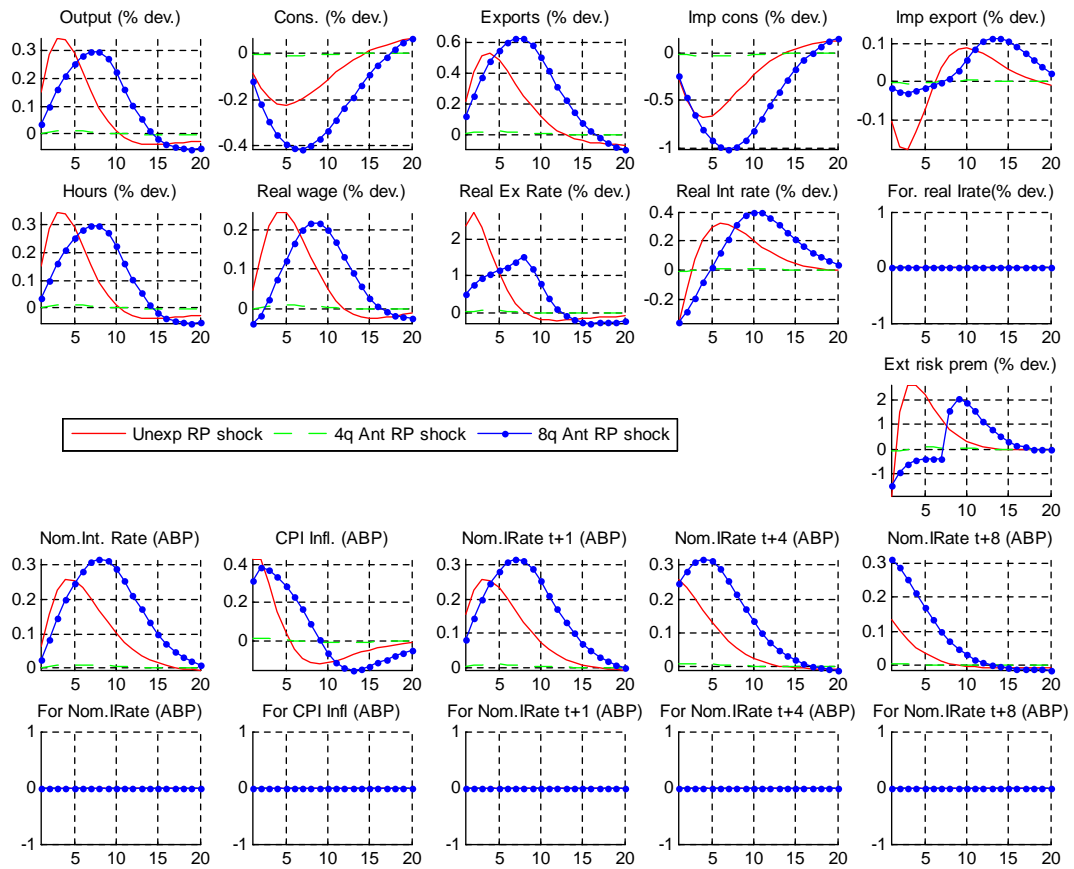


Figure 10: Impulse-response functions to unanticipated and anticipated risk premium shocks.

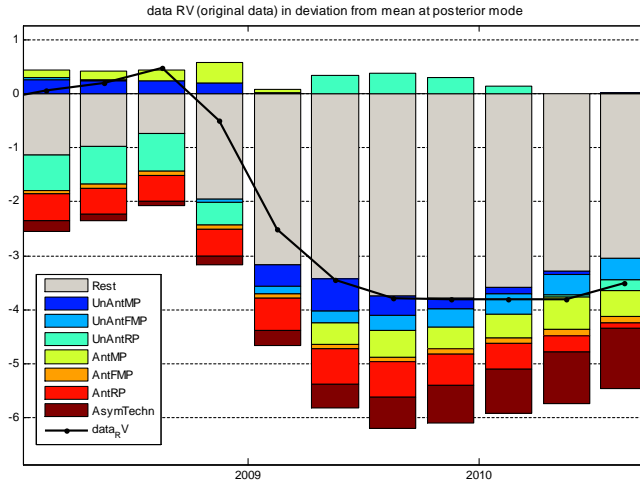


Figure 11: Historical decomposition - Repo rate

The shocks in the model are grouped into eight groups. *AntMP* denotes anticipated monetary policy shocks between 2 – 8 quarter anticipation horizon whereas *AntFMP* denotes anticipated foreign monetary policy shocks also between 2 – 8 quarter anticipation horizon. AsymTechnology is the contribution of the asymmetric technology shock $\tilde{\mu}_{z,t}$.

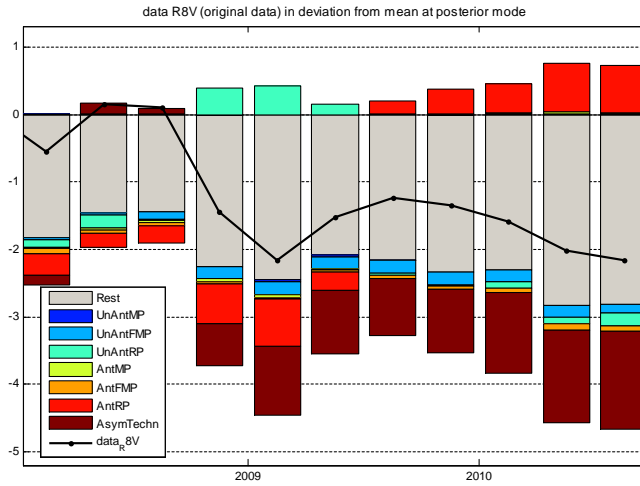


Figure 12: Historical decomposition - Implied Forward Rate 8 quarters ahead

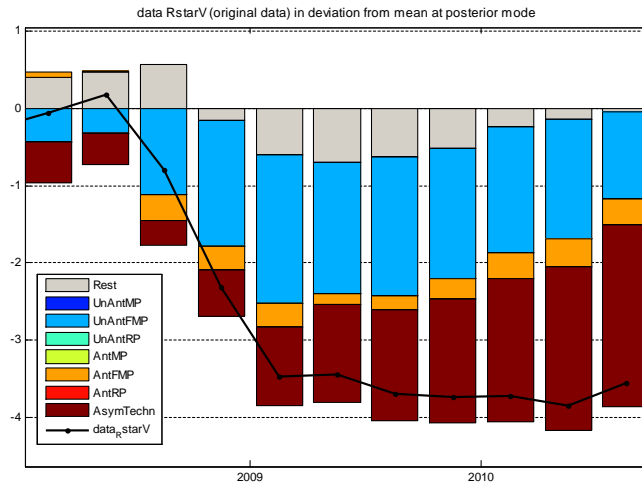


Figure 13: Historical decomposition - Foreign interest rate.

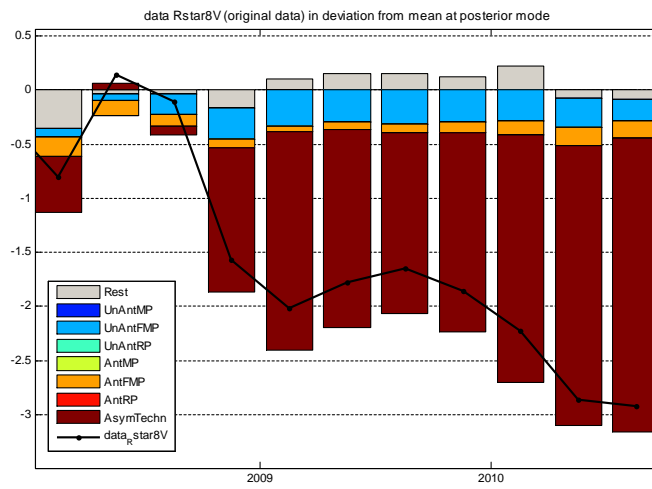


Figure 14: Historical decomposition - Implied Foreign Forward Rate 8 quarters ahead

6. Conclusions

[To be written]

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Appendix

6.1. Scaling of Variables

We adopt the following scaling of variables. The nominal exchange rate is denoted by S_t and its growth rate is s_t :

$$s_t = \frac{S_t}{S_{t-1}}.$$

The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods and C_t are final consumption goods) are scaled by z_t . Government consumption, the real wage and real foreign assets are scaled by z_t . Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled z_t . Also, v_t is the shadow value in utility terms to the household of domestic currency and $v_t P_t$ is the shadow value of one consumption good (i.e., the marginal utility of consumption). The latter must be multiplied by z_t to induce stationarity. \tilde{P}_t is the within-sector relative price of a good. Thus,

$$\begin{aligned} c_t^m &= \frac{C_t^m}{z_t}, c_t^d = \frac{C_t^d}{z_t}, c_t = \frac{C_t}{z_t}, g_t = \frac{G_t}{z_t}, \bar{w}_t = \frac{W_t}{z_t P_t}, a_t \equiv \frac{S_t A_{t+1}^*}{P_t z_t}, \\ x_t^m &= \frac{X_t^m}{z_t}, x_t = \frac{X_t}{z_t}, \psi_{z_t,t} = v_t P_t z_t, w_t = \tilde{W}_t / W_t, (y_t =) \tilde{y}_t = \frac{Y_t}{z_t}, \tilde{p}_t = \frac{\tilde{P}_t}{P_t}. \end{aligned}$$

We define the following inflation rates:

$$\begin{aligned} \pi_t &= \frac{P_t}{P_{t-1}}, \pi_t^c = \frac{P_t^c}{P_{t-1}^c}, \pi_t^* = \frac{P_t^*}{P_{t-1}^*}, \\ \pi_t^x &= \frac{P_t^x}{P_{t-1}^x}, \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}}, \end{aligned}$$

for $j = c, x$. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the ‘CPI’), P_t^* is the price of a foreign homogeneous good, and P_t^x is the price (in foreign currency units) of a final export good.

We define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t . When the price is denominated in foreign currency

units, we divide by P_t^* , the price of the foreign homogeneous good. Thus,

$$\begin{aligned} p_t^{m,x} &= \frac{P_t^{m,x}}{P_t}, \quad p_t^{m,c} = \frac{P_t^{m,c}}{P_t}, \\ p_t^x &= \frac{P_t^x}{P_t^*}, \quad p_t^c = \frac{P_t^c}{P_t}. \end{aligned} \quad (6.1)$$

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case $j = x$, in the production of the final consumption good in the case of $j = c$. When there is just a single superscript the underlying good is a final good, with $j = x, c$ corresponding to exports, and consumption, respectively.

We denote the real exchange rate by q_t :

$$q_t = \frac{S_t P_t^*}{P_t^c}. \quad (6.2)$$

6.2. First order conditions for domestic homogenous good price setting

Substituting eq. (3.5) into eq. (3.4) to obtain, after rearranging,

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} P_{t+j} Y_{t+j} \left\{ \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{1 - \frac{\lambda_d}{\lambda_d - 1}} - m c_{t+j} \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{\frac{-\lambda_d}{\lambda_d - 1}} \right\},$$

or,

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j} P_{t+j} Y_{t+j} \left\{ (X_{t,j} \tilde{p}_t)^{1 - \frac{\lambda_d}{\lambda_d - 1}} - m c_{t+j} (X_{t,j} \tilde{p}_t)^{\frac{-\lambda_d}{\lambda_d - 1}} \right\},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j} \tilde{p}_t, \quad X_{t,j} \equiv \begin{cases} \frac{\tilde{\pi}_{d,t+j} \cdots \tilde{\pi}_{d,t+1}}{\pi_{t+j} \cdots \pi_{t+1}}, & j > 0 \\ 1, & j = 0. \end{cases}$$

The i^{th} firm maximizes profits by choice of the within-sector relative price \tilde{p}_t . The fact that this variable does not have an index, i , reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d - 1} + 1}$, rearranging, and scaling we obtain:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} [\tilde{p}_t X_{t,j} - \lambda_d m c_{t+j}] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z_t, t+j} \tilde{y}_{t+j} X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t^d = \frac{E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} \lambda_d m c_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} X_{t,j}} = \frac{K_t^d}{F_t^d},$$

say, where

$$K_t^d \equiv E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} \lambda_d m c_{t+j}$$

$$F_t^d = E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} X_{t,j}.$$

These objects have the following convenient recursive representations:

$$E_t \left[\psi_{z_t, t} \tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta \xi_d F_{t+1}^d - F_t^d \right] = 0$$

$$E_t \left[\lambda_d \psi_{z_t, t} \tilde{y}_t m c_t + \beta \xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] = 0.$$

Turning to the aggregate price index:

$$P_t = \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}} di \right]^{(1-\lambda_d)} \quad (6.3)$$

$$= \left[(1 - \xi_p) \tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p (\tilde{\pi}_{d,t} P_{t-1})^{\frac{1}{1-\lambda_d}} \right]^{(1-\lambda_d)}$$

After dividing by P_t and rearranging:

$$\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_d} = \left(\tilde{p}_t^d \right)^{\frac{1}{1-\lambda_d}}. \quad (6.4)$$

In sum, the equilibrium conditions associated with price setting for producers of the domestic homogenous good are:

$$E_t \left[\psi_{z_t, t} y_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta \xi_d F_{t+1}^d - F_t^d \right] = 0 \quad (6.5)$$

$$E_t \left[\lambda_d \psi_{z_t, t} y_t m c_t + \beta \xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] = 0, \quad (6.6)$$

$$\hat{p}_t = \left[(1 - \xi_d) \left(\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_d} \right)^{\lambda_d} + \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \hat{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}} \quad (6.7)$$

$$\left[\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_d} \right]^{(1-\lambda_d)} = \frac{K_t^d}{F_t^d} \quad (6.8)$$

$$\tilde{\pi}_{d,t} \equiv (\pi_{t-1})^{\kappa_d} (\tilde{\pi}_t^c)^{1-\kappa_d-\varkappa_d} (\tilde{\pi})^{\varkappa_d} \quad (6.9)$$

When we linearize about steady state and set $\varkappa_d = 0$, we obtain,

$$\begin{aligned} \hat{\pi}_t - \hat{\pi}_t^c &= \frac{\beta}{1 + \kappa_d \beta} E_t (\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^c) + \frac{\kappa_d}{1 + \kappa_d \beta} (\hat{\pi}_{t-1} - \hat{\pi}_t^c) \\ &\quad - \frac{\kappa_d \beta (1 - \rho_\pi)}{1 + \kappa_d \beta} \hat{\pi}_t^c \\ &\quad + \frac{1}{1 + \kappa_d \beta} \frac{(1 - \beta \xi_d)(1 - \xi_d)}{\xi_d} \widehat{m}c_t, \end{aligned} \quad (6.10)$$

where a hat indicates log-deviation from steady state.

6.3. First order conditions for export good price setting

$$E_t \left[\psi_{z_t, t} q_t p_t^c p_t^x x_t + \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x} \right)^{\frac{1}{1-\lambda_x}} \beta \xi_x F_{x, t+1} - F_{x, t} \right] = 0 \quad (6.11)$$

$$E_t \left[\lambda_x \psi_{z_t, t} q_t p_t^c p_t^x x_t m c_t^x + \beta \xi_x \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x} \right)^{\frac{\lambda_x}{1-\lambda_x}} K_{x, t+1} - K_{x, t} \right] = 0, \quad (6.12)$$

$$\hat{p}_t^x = \left[(1 - \xi_x) \left(\frac{1 - \xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x} \right)^{\frac{1}{1-\lambda_x}}}{1 - \xi_x} \right)^{\lambda_x} + \xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x} p_{t-1}^x \right)^{\frac{\lambda_x}{1-\lambda_x}} \right]^{\frac{1-\lambda_x}{\lambda_x}} \quad (6.13)$$

$$\left[\frac{1 - \xi_x \left(\frac{\tilde{\pi}_t^x}{\pi_t^x} \right)^{\frac{1}{1-\lambda_x}}}{1 - \xi_x} \right]^{(1-\lambda_x)} = \frac{K_{x, t}}{F_{x, t}} \quad (6.14)$$

When we linearize around steady state and $\varkappa_{m, j} = 0$, equations (6.11)-(6.14) reduce to:

$$\begin{aligned} \hat{\pi}_t^x &= \frac{\beta}{1 + \kappa_x \beta} E_t \hat{\pi}_{t+1}^x + \frac{\kappa_x}{1 + \kappa_x \beta} \hat{\pi}_{t-1}^x \\ &\quad + \frac{1}{1 + \kappa_x \beta} \frac{(1 - \beta \xi_x)(1 - \xi_x)}{\xi_x} \widehat{m}c_t^x, \end{aligned} \quad (6.15)$$

where a hat over a variable indicates log deviation from steady state.

6.4. Demand for domestic inputs in export production

Integrating eq. (3.19):

$$\begin{aligned} \int_0^1 X_{i, t}^d di &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t} \right)^{\eta_x} (1 - \omega_x) \int_0^1 X_{i, t} di \\ &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t} \right)^{\eta_x} (1 - \omega_x) X_t \frac{\int_0^1 \left(P_{i, t}^x \right)^{\frac{-\lambda_{x, t}}{\lambda_{x, t} - 1}} di}{\left(P_t^x \right)^{\frac{-\lambda_{x, t}}{\lambda_{x, t} - 1}}}. \end{aligned} \quad (6.16)$$

Define \hat{P}_t^x , a linear homogeneous function of $P_{i,t}^x$:

$$\hat{P}_t^x = \left[\int_0^1 (P_{i,t}^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di \right]^{\frac{\lambda_{x,t}-1}{-\lambda_{x,t}}}.$$

Then,

$$\left(\hat{P}_t^x \right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} = \int_0^1 (P_{i,t}^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} di,$$

and

$$\int_0^1 X_{i,t}^d di = \left(\frac{\lambda}{\tau_t^x R_t^x P_t} \right)^{\eta_x} (1 - \omega_x) X_t (\hat{p}_t^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}, \quad (6.17)$$

where

$$\hat{p}_t^x \equiv \frac{\hat{P}_t^x}{P_t^x},$$

and the law of motion of \hat{p}_t^x is given in (6.13).

We now simplify (6.17). Rewriting the second equality in (3.17), we obtain:

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t q_t p_t^c p_t^x} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \frac{S_t P_t^x}{P_t \frac{S_t P_t^*}{P_t^c} \frac{P_t^c}{P_t} \frac{P_t^x}{P_t^*}} \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{1}{1-\eta_x}}.$$

Substituting into (6.17), we obtain:

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x (p_t^{m,x})^{1-\eta_x} + (1 - \omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1 - \omega_x) (\hat{p}_t^x)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} (p_t^x)^{-\eta_f} Y_t^*$$

6.5. First order conditions for import good price setting

$$E_t \left[\psi_{z_t, t} p_t^{m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}} \beta \xi_{m,j} F_{m,j,t+1} - F_{m,j,t} \right] = 0 \quad (6.18)$$

$$E_t \left[\lambda_{m,j} \psi_{z_t, t} p_t^{m,j} m c_t^{m,j} \Xi_t^j + \beta \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} K_{m,j,t+1} - K_{m,j,t} \right] = 0, \quad (6.19)$$

$$\hat{p}_t^{m,j} = \left[(1 - \xi_{m,j}) \left(\frac{1 - \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}}}{1 - \xi_{m,j}} \right)^{\lambda_{m,j}} + \xi_{m,j} \left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}} \hat{p}_{t-1}^{m,j} \right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} \right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}} \quad (6.20)$$

$$\left[\frac{1 - \xi_{m,j} \left(\frac{\hat{\pi}_t^{m,j}}{\bar{\pi}_t^{m,j}} \right)^{\frac{1}{1-\lambda_{m,j}}}}{1 - \xi_{m,j}} \right]^{(1-\lambda_{m,j})} = \frac{K_{m,j,t}}{F_{m,j,t}}, \quad (6.21)$$

for $j = c, x$. Here,

$$\Xi_t^j = \begin{cases} c_t^m & j = c \\ x_t^m & j = x \end{cases}.$$

When we linearize around steady state and $\varkappa_{m,j} = 0$,

$$\begin{aligned} \hat{\pi}_t^{m,j} - \hat{\pi}_t^c &= \frac{\beta}{1 + \kappa_{m,j}\beta} E_t \left(\hat{\pi}_{t+1}^{m,j} - \hat{\pi}_{t+1}^c \right) + \frac{\kappa_{m,j}}{1 + \kappa_{m,j}\beta} \left(\hat{\pi}_{t-1}^{m,j} - \hat{\pi}_{t-1}^c \right) \\ &\quad - \frac{\kappa_{m,j}\beta(1 - \rho_\pi)}{1 + \kappa_{m,j}\beta} \hat{\pi}_t^c \\ &\quad + \frac{1}{1 + \kappa_{m,j}\beta} \frac{(1 - \beta\xi_{m,j})(1 - \xi_{m,j})}{\xi_{m,j}} \widehat{m}c_t^{m,j}. \end{aligned} \quad (6.22)$$

6.6. Wage setting conditions

Substituting eq. (3.32) into the objective function eq. (3.31),

$$\begin{aligned} E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1 + \sigma_L} \\ + v_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w}], \end{aligned}$$

Consider the scaling of variables above, then,

$$\begin{aligned} \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z_t P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z_t P_t} X_{t,i} \\ &= \frac{W_t \left(\tilde{W}_t / W_t \right)}{\bar{w}_{t+i} z_t P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t \right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}, \end{aligned}$$

where

$$\begin{aligned} X_{t,i} &= \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z_t,t+i} \cdots \mu_{z_t,t+1}}, \quad i > 0 \\ &= 1, \quad i = 0. \end{aligned}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \rightarrow \infty$. Thus,

$$X_{t,i} = \frac{(\pi_t^c \cdots \pi_{t+i-1}^c)^{\kappa_w} (\bar{\pi}_{t+1}^c \cdots \bar{\pi}_{t+i}^c)^{(1-\kappa_w-\varkappa_w)} (\bar{\pi}^i)^{\varkappa_w} (\mu_{z_t}^i)^{\vartheta_w}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z_t,t+i} \cdots \mu_{z_t,t+1}}$$

In steady state,

$$\begin{aligned}
X_{t,i} &= \frac{(\bar{\pi}^i)^{\kappa_w} (\bar{\pi}^i)^{(1-\kappa_w-\varkappa_w)} (\check{\pi}^i)^{\varkappa_w} (\mu_{z_t}^i)^{\vartheta_w}}{\bar{\pi}^i \mu_{z_t}^i} \\
&= \left(\frac{\check{\pi}^i}{\bar{\pi}^i} \right)^{\varkappa_w} (\mu_{z_t}^i)^{\vartheta_w - 1} \\
&\rightarrow 0,
\end{aligned}$$

in the no-indexing case, when $\check{\pi} = 1$, $\varkappa_w = 1$ and $\vartheta_w = 0$.

Simplifying using the scaling notation,

$$\begin{aligned}
&E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \\
&+ \nu_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w}],
\end{aligned}$$

or,

$$\begin{aligned}
&E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \\
&+ \psi_{z_t, t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w}],
\end{aligned}$$

or,

$$\begin{aligned}
&E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} w_t^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L)} \\
&+ \psi_{z_t, t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w}],
\end{aligned}$$

Differentiating with respect to w_t ,

$$\begin{aligned}
&E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \lambda_w (1+\sigma_L) w_t^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L) - 1} \\
&+ \psi_{z_t, t+i} w_t^{\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w}] = 0
\end{aligned}$$

Dividing and rearranging,

$$\begin{aligned}
&E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i [-\zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\
&+ \frac{\psi_{z_t, t+i}}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w}] = 0
\end{aligned}$$

Solving for the wage rate:

$$\begin{aligned} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z_t,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}} \\ &= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}} \end{aligned}$$

where

$$\begin{aligned} K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\ F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z_t,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}. \end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}} \right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$\begin{aligned} K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\ &= \zeta_t^h H_t^{1+\sigma_L} + \beta \xi_w \zeta_{t+1}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w-\varkappa_w)} (\check{\pi})^{\varkappa_w} (\mu_{z_t})^{\vartheta_w}}{\pi_{t+1} \mu_{z_t,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \right)^{1+\sigma_L} \\ &\quad + (\beta \xi_w)^2 \zeta_{t+2}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{(\pi_t^c \pi_{t+1}^c)^{\kappa_w} (\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c)^{(1-\kappa_w-\varkappa_w)} (\check{\pi}^2)^{\varkappa_w} (\mu_{z_t}^2)^{\vartheta_w}}{\pi_{t+2} \pi_{t+1} \mu_{z_t,t+2} \mu_{z_t,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \\ &\quad + \dots \end{aligned}$$

or,

$$\begin{aligned} K_{w,t} &= \zeta_t^h H_t^{1+\sigma_L} + E_t \beta \xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w-\varkappa_w)} (\check{\pi})^{\varkappa_w} (\mu_{z_t})^{\vartheta_w}}{\pi_{t+1} \mu_{z_t,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L)} \{ \zeta_{t+1}^h H_{t+1}^{1+\sigma_L} \\ &\quad + \beta \xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{(\pi_{t+1}^c)^{\kappa_w} (\bar{\pi}_{t+2}^c)^{(1-\kappa_w-\varkappa_w)} (\check{\pi})^{\varkappa_w} (\mu_{z_t})^{\vartheta_w}}{\pi_{t+2} \mu_{z_t,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \zeta_{t+2}^h + \dots \} \\ &= \zeta_t^h H_t^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w-\varkappa_w)} (\check{\pi})^{\varkappa_w} (\mu_{z_t})^{\vartheta_w}}{\pi_{t+1} \mu_{z_t,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L)} K_{w,t+1} \\ &= \zeta_t^h H_t^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w} (1+\sigma_L)} K_{w,t+1}, \end{aligned}$$

using,

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_t P_{t+1}}{\bar{w}_t z_t P_t} = \frac{\bar{w}_{t+1} \mu_{z_t,t+1} \pi_{t+1}}{\bar{w}_t} \quad (6.23)$$

Also,

$$\begin{aligned} F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta \xi_w)^i \frac{\psi_{z_t,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1 - \tau_{t+i}^y}{1 + \tau_{t+i}^w} \\ &= \frac{\psi_{z_t,t}}{\lambda_w} H_t \frac{1 - \tau_t^y}{1 + \tau_t^w} \\ &\quad + \beta \xi_w \frac{\psi_{z_t,t+1}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w-\varkappa_w)} (\tilde{\pi})^{\varkappa_w} (\mu_{z_t})^{\vartheta_w}}{\pi_{t+1} \mu_{z_t,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \frac{1 - \tau_{t+1}^y}{1 + \tau_{t+1}^w} \\ &\quad + (\beta \xi_w)^2 \frac{\psi_{z_t,t+2}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \\ &\quad \times \left(\frac{(\pi_t^c \pi_{t+1}^c)^{\kappa_w} (\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c)^{(1-\kappa_w-\varkappa_w)} (\tilde{\pi}^2)^{\varkappa_w} (\mu_{z_t}^2)^{\vartheta_w}}{\pi_{t+2} \pi_{t+1} \mu_{z_t,t+2} \mu_{z_t,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \frac{1 - \tau_{t+2}^y}{1 + \tau_{t+2}^w} \\ &\quad + \dots \end{aligned}$$

or,

$$\begin{aligned} F_{w,t} &= \frac{\psi_{z_t,t}}{\lambda_w} H_t \frac{1 - \tau_t^y}{1 + \tau_t^w} \\ &\quad + \beta \xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{(\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{(1-\kappa_w-\varkappa_w)} (\tilde{\pi})^{\varkappa_w} (\mu_{z_t})^{\vartheta_w}}{\pi_{t+1} \mu_{z_t,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} \left\{ \frac{\psi_{z_t,t+1}}{\lambda_w} H_{t+1} \frac{1 - \tau_{t+1}^y}{1 + \tau_{t+1}^w} \right. \\ &\quad + \beta \xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{(\pi_{t+1}^c)^{\kappa_w} (\bar{\pi}_{t+2}^c)^{(1-\kappa_w-\varkappa_w)} (\tilde{\pi})^{\varkappa_w} (\mu_{z_t})^{\vartheta_w}}{\pi_{t+2} \mu_{z_t,t+2}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} \frac{\psi_{z_t,t+2}}{\lambda_w} H_{t+2} \frac{1 - \tau_{t+2}^y}{1 + \tau_{t+2}^w} \\ &\quad \left. + \dots \right\} \\ &= \frac{\psi_{z_t,t}}{\lambda_w} H_t \frac{1 - \tau_t^y}{1 + \tau_t^w} + \beta \xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1}, \end{aligned}$$

so that

$$F_{w,t} = \frac{\psi_{z_t,t}}{\lambda_w} H_t \frac{1 - \tau_t^y}{1 + \tau_t^w} + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1 - \xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w (\tilde{\pi}_{w,t} W_{t-1})^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w}.$$

Dividing both sides by W_t and rearranging,

$$w_t = \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w}.$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t , and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \hat{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{6.24}$$

where

$$\hat{w}_t \equiv \frac{\hat{W}_t}{W_t}, \quad \hat{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\hat{W}_t = \left[(1 - \xi_w) \left(\tilde{W}_t \right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} \hat{W}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}},$$

so that,

$$\begin{aligned} \hat{w}_t &= \left[(1 - \xi_w) (w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \hat{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1 - \xi_w) \left(\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \hat{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned} \tag{6.25}$$

In addition to (6.25), we have following equilibrium conditions associated with sticky wages:

$$F_{w,t} = \frac{\psi_{z_t,t}}{\lambda_w} \hat{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \frac{1 - \tau_t^y}{1 + \tau_t^w} + \beta \xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t} \right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{6.26}$$

$$K_{w,t} = \zeta_t^h \left(\hat{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \tag{6.27}$$

$$\frac{1}{A_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}. \tag{6.28}$$

Log linearizing these equations about the nonstochastic steady state and under the assumption of $\varkappa_w = 0$, we obtain

$$E_t \begin{bmatrix} \eta_0 \widehat{w}_{t-1} + \eta_1 \widehat{w}_t + \eta_2 \widehat{w}_{t+1} + \eta_3 (\widehat{\pi}_t - \widehat{\pi}_t^c) + \eta_4 (\widehat{\pi}_{t+1} - \rho_{\widehat{\pi}^c} \widehat{\pi}_t^c) \\ + \eta_5 (\widehat{\pi}_{t-1}^c - \widehat{\pi}_t^c) + \eta_6 (\widehat{\pi}_t^c - \rho_{\widehat{\pi}^c} \widehat{\pi}_t^c) \\ + \eta_7 \widehat{\psi}_{z,t} + \eta_8 \widehat{H}_t + \eta_9 \widehat{\tau}_t^y + \eta_{10} \widehat{\tau}_t^w + \eta_{11} \widehat{\zeta}_t^h \\ + \eta_{12} \widehat{\mu}_{z,t} + \eta_{13} \widehat{\mu}_{z,t+1} \end{bmatrix} = 0, \quad (6.29)$$

where

$$b_w = \frac{[\lambda_w \sigma_L - (1 - \lambda_w)]}{[(1 - \beta \xi_w)(1 - \xi_w)]}$$

and

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \\ \eta_{13} \end{pmatrix} = \begin{pmatrix} b_w \xi_w \\ (\sigma_L \lambda_w - b_w (1 + \beta \xi_w^2)) \\ b_w \beta \xi_w \\ -b_w \xi_w \\ b_w \beta \xi_w \\ b_w \xi_w \kappa_w \\ -b_w \beta \xi_w \kappa_w \\ (1 - \lambda_w) \\ -(1 - \lambda_w) \sigma_L \\ -(1 - \lambda_w) \frac{\tau^y}{(1 - \tau^y)} \\ -(1 - \lambda_w) \frac{\tau^w}{(1 + \tau^w)} \\ -(1 - \lambda_w) \\ -b_w \xi_w \\ b_w \beta \xi_w \end{pmatrix}.$$

6.7. Output and aggregate factors of production

Below we derive a relationship between total output of the domestic homogeneous good, Y_t , and aggregate factors of production.

Consider the unweighted average of the intermediate goods:

$$\begin{aligned} Y_t^{sum} &= \int_0^1 Y_{i,t} di \\ &= \int_0^1 [(z_t H_{i,t}) \epsilon_t - z_t \phi] di \\ &= \int_0^1 [z_t^{1-\alpha} \epsilon_t H_{it} - z_t \phi] di \\ &= z_t \epsilon_t \int_0^1 H_{it} di - z_t \phi \end{aligned}$$

where H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital

services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t \epsilon_t H_t - z_t \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\hat{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\hat{P}_t \right)^{\frac{\lambda_d}{1 - \lambda_d}},$$

say, where

$$\hat{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1 - \lambda_d}} di \right]^{\frac{1 - \lambda_d}{\lambda_d}}. \quad (6.30)$$

Dividing by P_t ,

$$\hat{p}_t = \left[\int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_d}{1 - \lambda_d}} di \right]^{\frac{1 - \lambda_d}{\lambda_d}},$$

or,

$$\hat{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1 - \lambda_d}}}{1 - \xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \hat{p}_{t-1} \right)^{\frac{\lambda_d}{1 - \lambda_d}} \right]^{\frac{1 - \lambda_d}{\lambda_d}}. \quad (6.31)$$

The preceding discussion implies:

$$Y_t = (\hat{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \hat{Y}_t = (\hat{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} [z_t \epsilon_t H_t - z_t \phi],$$

or, after scaling by z_t ,

$$y_t = (\hat{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} [\epsilon_t H_t - \phi],$$

We need to replace aggregate homogeneous labor, H_t , with aggregate household labor, h_t . From eq. (6.24) we have $H_t = \hat{w}_t^{-\frac{\lambda_w}{1 - \lambda_w}} h_t$. Plugging this in we obtain:

$$y_t = (\hat{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\hat{w}_t^{-\frac{\lambda_w}{1 - \lambda_w}} h_t \right) - \phi \right].$$

which completes the derivation.

6.8. Restrictions across inflation rates

We now consider the restrictions across inflation rates implied by our relative price formulas. In terms of the expressions in (6.1) there are the restrictions implied by $p_t^{m,j}/p_{t-1}^{m,j}$, $j = x, c$, and p_t^x . The restrictions implied by the relative prices in (6.1), p_t^c , have already been exploited in (??), respectively. Finally, we also exploit the restriction across inflation rates implied by q_t/q_{t-1} and (6.2). Thus,

$$\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t} \quad (6.32)$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \quad (6.33)$$

$$\frac{p_t^x}{p_{t-1}^x} = \frac{\pi_t^x}{\pi_t^*} \quad (6.34)$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t \pi_t^*}{\pi_t^c}. \quad (6.35)$$

6.9. Endogenous Variables of the Model

In the above sections we derived the following 30 equations (note that we do not allow for price and wage dispersion in state price and no time-varying inflation target), which can be used to solve for the following 30 unknowns:

$$\begin{aligned} & mc_t, \pi_t, c_t^m, p_t^c, \pi_t^c, x_t, mc_t^x, \pi_t^x, x_t^m, mc_t^{m,c}, mc_t^{m,x}, \\ & \pi_t^{m,c}, \pi_t^{m,x}, c_t, \psi_{z_t,t}, \bar{w}_t, \pi_w, R_t, y_t, a_t, p_t^{m,x}, p_t^{m,c}, p_t^x, q_t, \Phi_t, H_t, s_t, \\ & y_t^*, \pi_t^* R_t^*. \end{aligned}$$

6.10. Complete Loglinear Model

Marginal cost for domestic goods producers:

$$\widehat{mc}_t = \hat{\tau}_t^d + \hat{w}_t - \hat{\epsilon}_t. \quad (6.36)$$

Domestic price setting equation (no steady state price dispersion and no time-varying inflation target):

$$\hat{\pi}_t^d = \frac{\beta}{1 + \beta\kappa^d} \hat{\pi}_{t+1}^d + \frac{\kappa^d}{1 + \beta\kappa^d} \hat{\pi}_{t-1}^d + \frac{1}{1 + \beta\kappa^d} \frac{(1 - \beta\xi_d)(1 - \xi_d)}{\xi_d} \widehat{mc}_t. \quad (6.37)$$

Demand for imports:

$$\hat{c}_t^m = \eta_c \hat{p}_t^c - \eta_c \hat{p}_t^{mc} + \hat{c}_t. \quad (6.38)$$

Price of consumption goods as a function of the intermediate goods prices:

$$\hat{p}_t^c = \omega_c \left(\frac{p^{mc}}{p^c} \right)^{1 - \eta_c} \hat{p}_t^{mc}. \quad (6.39)$$

CPI inflation:

$$\hat{\pi}_t^c = \frac{(1 - \omega_c) (\pi^d)^{1 - \eta_c} + \omega_c (p^{mc} \pi^d)^{1 - \eta_c}}{(1 - \omega_c + \omega_c (p^{mc})^{1 - \eta_c}) (\pi^c)^{1 - \eta_c}} \hat{\pi}_t^d + \frac{\omega_c (p^{mc} \pi^d)^{1 - \eta_c}}{(1 - \omega_c + \omega_c (p^{mc})^{1 - \eta_c}) (\pi^c)^{1 - \eta_c}} \hat{p}_t^{mc} - \frac{\omega_c (p^{mc} \pi^d)^{1 - \eta_c}}{(1 - \omega_c + \omega_c (p^{mc})^{1 - \eta_c}) (\pi^c)^{1 - \eta_c}} \hat{p}_{t-1}^{mc}. \quad (6.40)$$

Export demand:

$$\hat{x}_t = -\eta_f \hat{p}_t^x + \hat{y}_t^* + \hat{\mu}_{z,t}. \quad (6.41)$$

Marginal cost export goods producers:

$$\widehat{mc}_t^x = \omega_x \left(\frac{\tau^x p^{mx}}{p^c mc^x q p^x} \right)^{1 - \eta_x} \hat{p}_t^{mx} - \hat{q}_t - \hat{p}_t^c - \hat{p}_t^x + \hat{\tau}_t^x. \quad (6.42)$$

Price setting export goods producers:

$$\hat{\pi}_t^x = \frac{\beta}{1 + \beta\kappa^x} \hat{\pi}_{t+1}^x + \frac{\kappa^x}{1 + \beta\kappa^x} \hat{\pi}_{t-1}^x + \frac{1}{1 + \beta\kappa^x} \frac{(1 - \beta\xi_x)(1 - \xi_x)}{\xi_x} \widehat{mc}_t^x. \quad (6.43)$$

Demand for imports for export:

$$\hat{x}_t^m = \hat{x}_t + \left(\frac{\omega_x^{\frac{1}{\eta_x}}}{\left(\frac{x^m (p^x)^{\eta_f}}{y^*} \right)^{\frac{1 - \eta_x}{\eta_x}}} - 1 \right) \hat{p}_t^{mx} \quad (6.44)$$

Marginal cost import consumption goods producers:

$$\widehat{mc}_t^{mc} = \hat{p}_t^c + \hat{q}_t + \hat{\tau}_t^{mc} - \hat{p}_t^{mc}. \quad (6.45)$$

Marginal cost import export goods producers:

$$\widehat{mc}_t^{mx} = \hat{p}_t^c + \hat{q}_t + \hat{\tau}_t^{mx} - \hat{p}_t^{mx}. \quad (6.46)$$

Price setting imported consumption goods producers:

$$\hat{\pi}_t^{mc} = \frac{\beta}{1 + \beta\kappa^{mc}} \hat{\pi}_{t+1}^{mc} + \frac{\kappa^{mc}}{1 + \beta\kappa^{mc}} \hat{\pi}_{t-1}^{mc} + \frac{1}{1 + \beta\kappa^{mc}} \frac{(1 - \beta\xi_{mc})(1 - \xi_{mc})}{\xi_{mc}} \widehat{mc}_t^{mc}. \quad (6.47)$$

Price setting imported export goods producers:

$$\hat{\pi}_t^{mx} = \frac{\beta}{1 + \beta\kappa^{mx}} \hat{\pi}_{t+1}^{mx} + \frac{\kappa^{mx}}{1 + \beta\kappa^{mx}} \hat{\pi}_{t-1}^{mx} + \frac{1}{1 + \beta\kappa^{mx}} \frac{(1 - \beta\xi_{mx})(1 - \xi_{mx})}{\xi_{mx}} \widehat{mc}_t^{mx}. \quad (6.48)$$

First order condition - consumption:

$$\begin{aligned} & \mu_z b \hat{c}_{t-1} + \mu_z (\mu_z - b) \hat{\zeta}_t^c - (\mu_z^2 + \beta b^2) \hat{c}_t - \mu_z b \hat{\mu}_{z,t} + (\mu_z - b) \beta b \hat{\zeta}_{t+1}^c + \mu_z \beta b \hat{c}_{t+1} + \mu_z \beta b \hat{\mu}_{z,t+1} \\ & = p^c c (\mu_z - b)^2 \psi_z \hat{p}_t^c + p^c c (\mu_z - b)^2 \psi_z \hat{\psi}_{z,t} \end{aligned} \quad (6.49)$$

First order condition for domestic bond:

$$\hat{\psi}_{z,t} = \hat{R}_t + \hat{\psi}_{z,t+1} - \hat{\pi}_{t+1}^d - \hat{\mu}_{z,t+1}. \quad (6.50)$$

First order condition for foreign bond:

$$\hat{\psi}_{z,t} = \hat{\psi}_{z,t+1} + \hat{s}_{t+1} + \hat{R}_t^* + \hat{\Phi}_t - \hat{\pi}_{t+1}^d - \hat{\mu}_{z,t+1}. \quad (6.51)$$

Wage setting:

$$\begin{aligned} 0 & = \eta_0 \hat{w}_{t-1} + \eta_1 \hat{w}_t + \eta_2 \hat{w}_{t+1} + \eta_3 \hat{\pi}_t^d + \eta_4 \hat{\pi}_{t+1}^d + \eta_5 \hat{\pi}_{t-1}^c + \eta_6 \hat{\pi}_t^c + \\ & \eta_7 \hat{\psi}_{z,t} + \eta_8 \hat{H}_t + \eta_9 \hat{\zeta}_t^h + \eta_{10} \hat{\mu}_{z,t} + \eta_{11} \hat{\mu}_{z,t+1}. \end{aligned} \quad (6.52)$$

Wage inflation:

$$\hat{\pi}_t^w = \hat{\pi}_t^d + \hat{w}_t - \hat{w}_{t-1} + \hat{\mu}_{z,t}. \quad (6.53)$$

Instrument rule:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) (r_\pi \hat{\pi}_{t-1}^c + r_y \hat{y}_{t-1} + r_q \hat{q}_{t-1}) + r_{\Delta\pi} (\hat{\pi}_t^c - \hat{\pi}_{t-1}^c) + r_{\Delta y} (\hat{y}_t - \hat{y}_{t-1}) + \varepsilon_{R,t}. \quad (6.54)$$

Domestic output, production:

$$\hat{y}_t = \hat{\varepsilon}_t + \hat{H}_t. \quad (6.55)$$

Current account (linearized - denoted with "breve" instead of "hat"):

$$\check{a}_t \frac{1}{p^c q} = p^x x (\hat{x}_t + \hat{p}_t^x + \hat{p}_t^c + \hat{q}_t) + \frac{R^* s}{\mu_z \pi^d p^c q} \check{a}_{t-1} - c^m (\hat{c}_t^m + \hat{p}_t^c + \hat{q}_t) - x^m (\hat{x}_t^m + \hat{p}_t^c + \hat{q}_t). \quad (6.56)$$

Domestic output uses:

$$\hat{y}_t = \bar{g} \hat{g}_t + (1 - \omega_c) (p^c)^{\eta_c} \frac{c}{y \bar{a} r} (\eta_c \hat{p}_t^c + \hat{c}_t) + (1 - \omega_x) \left(1 - \omega_x + \omega_x (p^{m_x})^{1 - \eta_x} \right)^{\frac{\eta_x}{1 - \eta_x}} (p^x)^{-\eta_f} \frac{y^*}{y} \left(\hat{x}_t + \frac{(p^{m_x})^{1 - \eta_x} \eta_x \omega_x}{1 - \omega_x + \omega_x (p^{m_x})^{1 - \eta_x}} \hat{p}_t^{m_x} \right). \quad (6.57)$$

Restrictions across inflation rates implied by relative prices:

$$\hat{p}_t^{m_x} - \hat{p}_{t-1}^{m_x} = \hat{\pi}_t^{m_x} - \hat{\pi}_t^d \quad (6.58)$$

$$\hat{p}_t^{m_c} - \hat{p}_{t-1}^{m_c} = \hat{\pi}_t^{m_c} - \hat{\pi}_t^d \quad (6.59)$$

$$\hat{p}_t^x - \hat{p}_{t-1}^x = \hat{\pi}_t^x - \hat{\pi}_t^* \quad (6.60)$$

$$\hat{q}_t - \hat{q}_{t-1} = \hat{\pi}_t^* + \hat{s}_t - \hat{\pi}_t^c \quad (6.61)$$

External risk premium:

$$\hat{\Phi}_t = \left(1 - \tilde{\phi}_s\right) \hat{s}_{t+1} - \tilde{\phi}_s \hat{s}_t - \tilde{\phi}_a \check{a}_t + \hat{\varepsilon}_{\tilde{\phi},t}. \quad (6.62)$$

Shock processes:

Stationary technology shock:

$$\hat{\epsilon}_t = \rho_\epsilon \hat{\epsilon}_{t-1} + \frac{\sigma_\epsilon}{100} \varepsilon_{\epsilon,t}. \quad (6.63)$$

Domestic marginal cost shock:

$$\hat{\tau}_t^d = \rho_{\tau^d} \hat{\tau}_{t-1}^d + \frac{\sigma_{\tau^d}}{10} \varepsilon_{\tau^d,t}. \quad (6.64)$$

Marginal cost shock, imported consumption goods:

$$\hat{\tau}_t^{mc} = \rho_{\tau^{mc}} \hat{\tau}_{t-1}^{mc} + \frac{\sigma_{\tau^{mc}}}{10} \varepsilon_{\tau^{mc},t}. \quad (6.65)$$

Marginal cost shock, imported export goods:

$$\hat{\tau}_t^{mx} = \rho_{\tau^{mx}} \hat{\tau}_{t-1}^{mx} + \frac{\sigma_{\tau^{mx}}}{10} \varepsilon_{\tau^{mx},t}. \quad (6.66)$$

Marginal cost shock, export goods:

$$\hat{\tau}_t^x = \rho_{\tau^x} \hat{\tau}_{t-1}^x + \frac{\sigma_{\tau^x}}{10} \varepsilon_{\tau^x,t}. \quad (6.67)$$

Shock to the growth rate of permanent technology:

$$\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \frac{\sigma_{\mu_z}}{100} \varepsilon_{\mu_z,t}. \quad (6.68)$$

Shock to the growth rate of permanent foreign technology:

$$\hat{\mu}_{z^*,t} = \hat{\mu}_{z,t} + \hat{\mu}_{z,t}. \quad (6.69)$$

Asymmetric foreign technology shock:

$$\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \frac{\sigma_{\mu_z}}{100} \varepsilon_{\mu_z,t} \quad (6.70)$$

Foreign marginal cost shock:

$$\hat{u}_t^* = \rho_{u^*} \hat{u}_{t-1}^* + \frac{\sigma_{u^*}}{100} \varepsilon_{u^*,t}. \quad (6.71)$$

Shock to consumption preferences:

$$\hat{\zeta}_t^c = \rho_{\zeta^c} \hat{\zeta}_{t-1}^c + \frac{\sigma_{\zeta^c}}{10} \varepsilon_{\zeta^c,t} \quad (6.72)$$

Labor supply shock:

$$\hat{\zeta}_t^h = \rho_{\zeta^h} \hat{\zeta}_{t-1}^h + \frac{\sigma_{\zeta^h}}{10} \varepsilon_{\zeta^h,t}. \quad (6.73)$$

Government consumption shock:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \frac{\sigma_g}{100} \varepsilon_{g,t} \quad (6.74)$$

Foreign Variables:

Output:

$$\hat{y}_t^* = \hat{y}_{t+1}^* + \hat{g}_t^* - \hat{g}_{t+1}^* - \frac{1}{\tau} \left(\hat{R}_t^* - \hat{\pi}_t^* - \hat{\mu}_t \right). \quad (6.75)$$

Inflation:

$$\hat{\pi}_t^* = \beta \hat{\pi}_{t+1}^* + \kappa (\hat{y}_t^* + \hat{g}_t^*) + \hat{u}_t^*. \quad (6.76)$$

Foreign instrument rate

$$\hat{R}_t^* = \rho_{R^*} \hat{R}_{t-1}^* + (1 - \rho_{R^*}) \psi_1 \hat{\pi}_t^* + (1 - \rho_{R^*}) \psi_2 (\hat{y}_t^* + \hat{g}_t^*) + \varepsilon_t^*, \quad (6.77)$$

Government consumption:

$$\hat{g}_t^* = \rho_{g^*} \hat{g}_{t-1}^* + \frac{\sigma_{g^*}}{100} \varepsilon_{g^*,t},$$

Foreign consumption:

$$c_t^* = \hat{y}_t^* - \hat{g}_t^*$$

Real interest rates:

Domestic:

$$\hat{r}_t^r = 400 \left(\hat{R}_t - \hat{\pi}_{t+1}^c \right)$$

Foreign:

$$\hat{r}_t^{r*} = 400 \left(\hat{R}_t^* - \hat{\pi}_{t+1}^* \right)$$

Expected monetary policy shocks - Alternative 1:

$$\begin{aligned} \hat{\varepsilon}_{R,t} &= \hat{\varepsilon}_{R,t-1} + \frac{\sigma^{R1}}{100} e_t^{R1}, \\ \hat{\varepsilon}_{R,t-1} &= \hat{\varepsilon}_{R,t-2} + \frac{\sigma^{R2}}{100} e_t^{R2}, \\ \hat{\varepsilon}_{R,t-2} &= \hat{\varepsilon}_{R,t-3} + \frac{\sigma^{R3}}{100} e_t^{R3}, \\ \hat{\varepsilon}_{R,t-3} &= \hat{\varepsilon}_{R,t-4} + \frac{\sigma^{R4}}{100} e_t^{R4}, \\ \hat{\varepsilon}_{R,t-4} &= \hat{\varepsilon}_{R,t-5} + \frac{\sigma^{R5}}{100} e_t^{R5}, \\ \hat{\varepsilon}_{R,t-5} &= \hat{\varepsilon}_{R,t-6} + \frac{\sigma^{R6}}{100} e_t^{R6}, \\ \hat{\varepsilon}_{R,t-6} &= \hat{\varepsilon}_{R,t-7} + \frac{\sigma^{R7}}{100} e_t^{R7}, \\ \hat{\varepsilon}_{R,t-7} &= \hat{\varepsilon}_{R,t-8} + \frac{\sigma^{R8}}{100} e_t^{R8}, \\ \hat{\varepsilon}_{R,t-8} &= \frac{\sigma^{R9}}{100} e_t^{R9}. \end{aligned}$$

Expected monetary policy shocks - Alternative 2:

$$\begin{aligned} \hat{\varepsilon}_{R,t} &= \frac{\rho^{R1}}{100} \hat{\varepsilon}_{R,t-1} + \frac{\rho^{R2}}{100} \hat{\varepsilon}_{R,t-2} + \frac{\rho^{R3}}{100} \hat{\varepsilon}_{R,t-3} + \frac{\rho^{R4}}{100} \hat{\varepsilon}_{R,t-4} \\ &\quad + \frac{\rho^{R5}}{100} \hat{\varepsilon}_{R,t-5} + \frac{\rho^{R6}}{100} \hat{\varepsilon}_{R,t-6} + \frac{\rho^{R7}}{100} \hat{\varepsilon}_{R,t-7} + \frac{\rho^{R8}}{100} \hat{\varepsilon}_{R,t-8}. \end{aligned}$$

Expected risk-premium shocks:

$$\begin{aligned}
\hat{\varepsilon}_{\tilde{\phi},t} &= \hat{\varepsilon}_{\tilde{\phi},t-1} + \frac{\sigma^{\tilde{\phi}1}}{100} e_t^{\tilde{\phi}1}, \\
\hat{\varepsilon}_{\tilde{\phi},t-1} &= \hat{\varepsilon}_{\tilde{\phi},t-2} + \frac{\sigma^{\tilde{\phi}2}}{100} e_t^{\tilde{\phi}2}, \\
\hat{\varepsilon}_{\tilde{\phi},t-2} &= \hat{\varepsilon}_{\tilde{\phi},t-3} + \frac{\sigma^{\tilde{\phi}3}}{100} e_t^{\tilde{\phi}3}, \\
\hat{\varepsilon}_{\tilde{\phi},t-3} &= \hat{\varepsilon}_{\tilde{\phi},t-4} + \frac{\sigma^{\tilde{\phi}4}}{100} e_t^{\tilde{\phi}4}, \\
\hat{\varepsilon}_{\tilde{\phi},t-4} &= \hat{\varepsilon}_{\tilde{\phi},t-5} + \frac{\sigma^{\tilde{\phi}5}}{100} e_t^{\tilde{\phi}5}, \\
\hat{\varepsilon}_{\tilde{\phi},t-5} &= \hat{\varepsilon}_{\tilde{\phi},t-6} + \frac{\sigma^{\tilde{\phi}6}}{100} e_t^{\tilde{\phi}6}, \\
\hat{\varepsilon}_{\tilde{\phi},t-6} &= \hat{\varepsilon}_{\tilde{\phi},t-7} + \frac{\sigma^{\tilde{\phi}7}}{100} e_t^{\tilde{\phi}7}, \\
\hat{\varepsilon}_{\tilde{\phi},t-7} &= \hat{\varepsilon}_{\tilde{\phi},t-8} + \frac{\sigma^{\tilde{\phi}8}}{100} e_t^{\tilde{\phi}8}, \\
\hat{\varepsilon}_{\tilde{\phi},t-8} &= \frac{\sigma^{\tilde{\phi}9}}{100} e_t^{\tilde{\phi}9}.
\end{aligned}$$

Expected risk-premium shocks - Alternative 2:

$$\begin{aligned}
\hat{\varepsilon}_{\tilde{\phi},t} &= \frac{\rho^{\tilde{\phi}1}}{100} \hat{\varepsilon}_{\tilde{\phi},t-1} + \frac{\rho^{\tilde{\phi}2}}{100} \hat{\varepsilon}_{\tilde{\phi},t-2} + \frac{\rho^{\tilde{\phi}3}}{100} \hat{\varepsilon}_{\tilde{\phi},t-3} + \frac{\rho^{\tilde{\phi}4}}{100} \hat{\varepsilon}_{\tilde{\phi},t-4} \\
&\quad + \frac{\rho^{\tilde{\phi}5}}{100} \hat{\varepsilon}_{\tilde{\phi},t-5} + \frac{\rho^{\tilde{\phi}6}}{100} \hat{\varepsilon}_{\tilde{\phi},t-6} + \frac{\rho^{\tilde{\phi}7}}{100} \hat{\varepsilon}_{\tilde{\phi},t-7} + \frac{\rho^{\tilde{\phi}8}}{100} \hat{\varepsilon}_{\tilde{\phi},t-8}.
\end{aligned}$$

Expected foreign monetary policy shocks:

$$\begin{aligned}
\hat{\varepsilon}_{R^*,t} &= \hat{\varepsilon}_{R^*,t-1} + \frac{\sigma^{R^*1}}{100} e_t^{R^*1}, \\
\hat{\varepsilon}_{R^*,t-1} &= \hat{\varepsilon}_{R^*,t-2} + \frac{\sigma^{R^*2}}{100} e_t^{R^*2}, \\
\hat{\varepsilon}_{R^*,t-2} &= \hat{\varepsilon}_{R^*,t-3} + \frac{\sigma^{R^*3}}{100} e_t^{R^*3}, \\
\hat{\varepsilon}_{R^*,t-3} &= \hat{\varepsilon}_{R^*,t-4} + \frac{\sigma^{R^*4}}{100} e_t^{R^*4}, \\
\hat{\varepsilon}_{R^*,t-4} &= \hat{\varepsilon}_{R^*,t-5} + \frac{\sigma^{R^*5}}{100} e_t^{R^*5}, \\
\hat{\varepsilon}_{R^*,t-5} &= \hat{\varepsilon}_{R^*,t-6} + \frac{\sigma^{R^*6}}{100} e_t^{R^*6}, \\
\hat{\varepsilon}_{R^*,t-6} &= \hat{\varepsilon}_{R^*,t-7} + \frac{\sigma^{R^*7}}{100} e_t^{R^*7}, \\
\hat{\varepsilon}_{R^*,t-7} &= \hat{\varepsilon}_{R^*,t-8} + \frac{\sigma^{R^*8}}{100} e_t^{R^*8}, \\
\hat{\varepsilon}_{R^*,t-8} &= \frac{\sigma^{R^*9}}{100} e_t^{R^*9}.
\end{aligned}$$

Expected foreign monetary policy shocks - Alternative 2:

$$\begin{aligned}\hat{\varepsilon}_{R^*,t} = & \frac{\rho^{R^*1}}{100}\hat{\varepsilon}_{R^*,t-1} + \frac{\rho^{R^*2}}{100}\hat{\varepsilon}_{R^*,t-2} + \frac{\rho^{R^*3}}{100}\hat{\varepsilon}_{R^*,t-3} + \frac{\rho^{R^*4}}{100}\hat{\varepsilon}_{R^*,t-4} \\ & + \frac{\rho^{R^*5}}{100}\hat{\varepsilon}_{R^*,t-5} + \frac{\rho^{R^*6}}{100}\hat{\varepsilon}_{R^*,t-6} + \frac{\rho^{R^*7}}{100}\hat{\varepsilon}_{R^*,t-7} + \frac{\rho^{R^*8}}{100}\hat{\varepsilon}_{R^*,t-8}.\end{aligned}$$

Recursively defined expected instrument rates:

$$\begin{aligned}\hat{R}_t^1 &= \hat{R}_{t+1}, \\ \hat{R}_t^2 &= \hat{R}_{t+1}^1, \\ \hat{R}_t^3 &= \hat{R}_{t+1}^2, \\ \hat{R}_t^4 &= \hat{R}_{t+1}^3, \\ \hat{R}_t^5 &= \hat{R}_{t+1}^4, \\ \hat{R}_t^6 &= \hat{R}_{t+1}^5, \\ \hat{R}_t^7 &= \hat{R}_{t+1}^6, \\ \hat{R}_t^8 &= \hat{R}_{t+1}^7,\end{aligned}\tag{6.78}$$

Recursively defined expected foreign instrument rates:

$$\begin{aligned}\hat{R}_t^{*1} &= \hat{R}_{t+1}^*, \\ \hat{R}_t^{*2} &= \hat{R}_{t+1}^{*1}, \\ \hat{R}_t^{*3} &= \hat{R}_{t+1}^{*2}, \\ \hat{R}_t^{*4} &= \hat{R}_{t+1}^{*3}, \\ \hat{R}_t^{*5} &= \hat{R}_{t+1}^{*4}, \\ \hat{R}_t^{*6} &= \hat{R}_{t+1}^{*5}, \\ \hat{R}_t^{*7} &= \hat{R}_{t+1}^{*6}, \\ \hat{R}_t^{*8} &= \hat{R}_{t+1}^{*7},\end{aligned}\tag{6.79}$$

Measurement equations:

$$\begin{aligned}
R_t^{data} &= 400(R-1) + 400R\hat{R}_t \\
R_t^{*,data} &= 400(R^*-1) + 400R^*\hat{R}_t^* \\
\pi_t^{data} &= 400 \log(\pi^d) + 400\hat{\pi}_t^d + \sigma_{\pi^d}^{me} \varepsilon_{\pi^d,t}^{me} \\
\pi_t^{c,data} &= 400 \log(\pi^c) + 400\hat{\pi}_t^c + \sigma_{\pi^c}^{me} \varepsilon_{\pi^c,t}^{me} \\
\pi_t^{*,data} &= 400 \log(\pi^*) + 400\hat{\pi}_t^* + \sigma_{\pi^*}^{me} \varepsilon_{\pi^*,t}^{me}
\end{aligned}$$

$$\begin{aligned}
\Delta \ln(W_t/P_t)^{data} &= 100(\log \mu_{z,t} + \hat{w}_t - \hat{w}_{t-1}) + \sigma_w^{me} \varepsilon_{w,t}^{me} \\
\Delta \ln C_t^{data} &= 100(\log(\mu_z) + \hat{\mu}_{z,t} + \hat{c}_t - \hat{c}_{t-1}) + \sigma_c^{me} \varepsilon_{c,t}^{me} \\
\Delta \ln Y_t^{data} &= 100(\log(\mu_z) + \hat{\mu}_{z,t} + \hat{y}_t - \hat{y}_{t-1}) + \sigma_y^{me} \varepsilon_{y,t}^{me} \\
\Delta \ln Y_t^{*,data} &= 100(\log(\mu_z) + \hat{\mu}_{z,t}^* + y_t^* - y_{t-1}^*) + \sigma_{y^*}^{me} \varepsilon_{y^*,t}^{me} \\
\Delta \ln M_t^{data} &= 100(\log(\mu_z) + \hat{\mu}_{z,t} + \hat{c}_t^m - \hat{c}_{t-1}^m + \hat{x}_t^m - \hat{x}_{t-1}^m) + \sigma_M^{me} \varepsilon_{M,t}^{me} \\
\Delta \ln X_t^{data} &= 100(\log(\mu_z) + \hat{\mu}_{z,t} + \hat{x}_t - \hat{x}_{t-1}) + \sigma_X^{me} \varepsilon_{X,t}^{me} \\
\Delta \ln G_t^{data} &= 100(\log(\mu_z) + \hat{\mu}_{z,t} + \hat{g}_t - \hat{g}_{t-1}) + \sigma_G^{me} \varepsilon_{G,t}^{me}
\end{aligned}$$

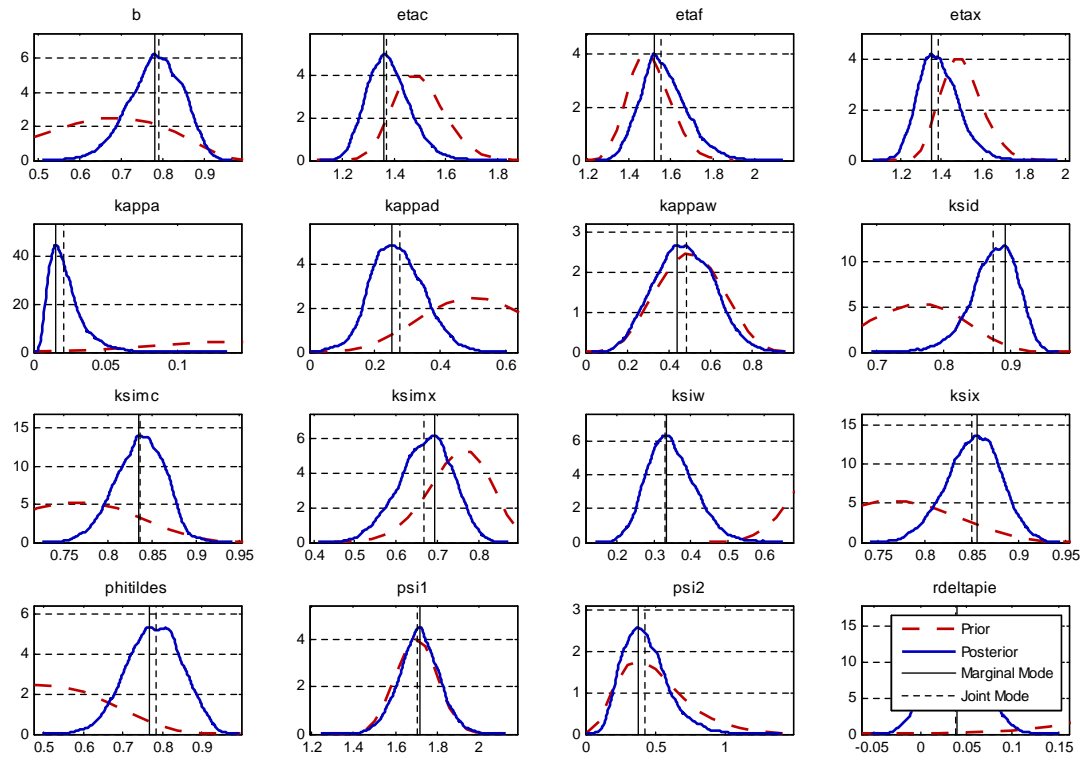
$$\begin{aligned}
\Delta \ln q_t^{data} &= 100(\hat{q}_t - \hat{q}_{t-1}) + \sigma_q^{me} \varepsilon_{q,t}^{me} \\
\hat{H}_t^{data} &= 100\hat{H}_t + \sigma_H^{me} \varepsilon_{H,t}^{me}
\end{aligned}$$

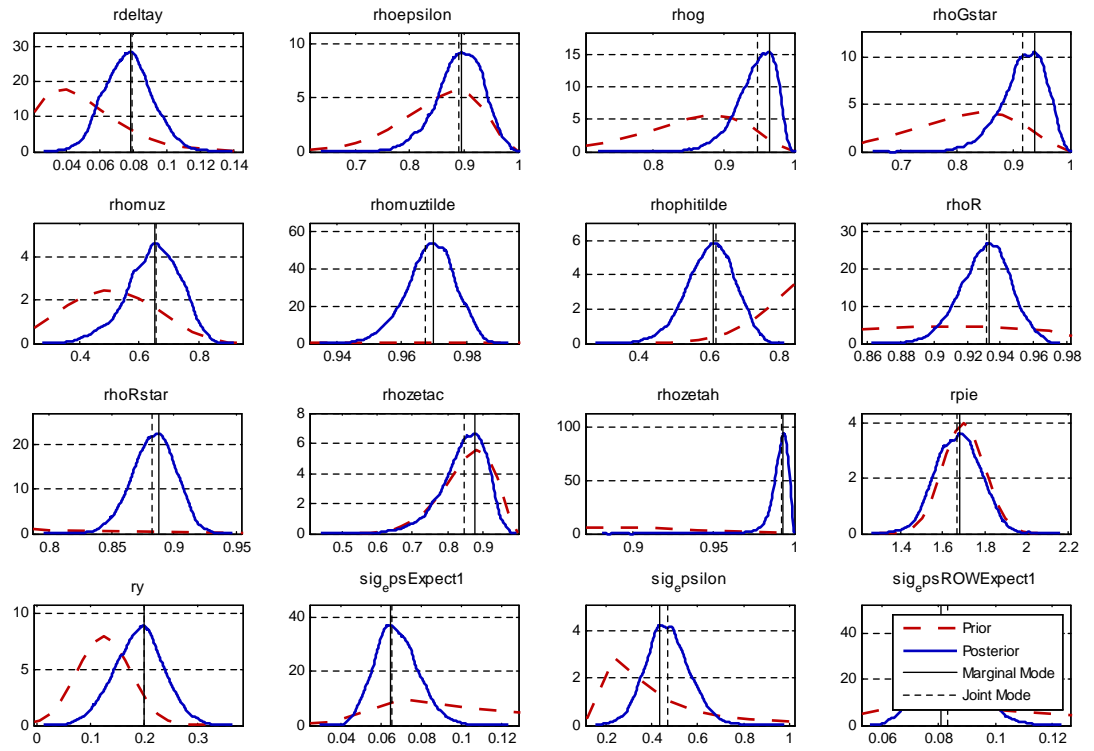
$$\begin{aligned}
R_{t,t+1}^{data} &= 400(R-1) + 400R\hat{R}_t^1 + \sigma^{R1me} \varepsilon_{R1,t}^{me} \\
R_{t,t+2}^{data} &= 400(R-1) + 400R\hat{R}_t^2 + \sigma^{R2me} \varepsilon_{R2,t}^{me} \\
R_{t,t+3}^{data} &= 400(R-1) + 400R\hat{R}_t^3 + \sigma^{R3me} \varepsilon_{R3,t}^{me} \\
R_{t,t+4}^{data} &= 400(R-1) + 400R\hat{R}_t^4 + \sigma^{R4me} \varepsilon_{R4,t}^{me} \\
R_{t,t+5}^{data} &= 400(R-1) + 400R\hat{R}_t^5 + \sigma^{R5me} \varepsilon_{R5,t}^{me} \\
R_{t,t+6}^{data} &= 400(R-1) + 400R\hat{R}_t^6 + \sigma^{R6me} \varepsilon_{R6,t}^{me} \\
R_{t,t+7}^{data} &= 400(R-1) + 400R\hat{R}_t^7 + \sigma^{R7me} \varepsilon_{R7,t}^{me} \\
R_{t,t+8}^{data} &= 400(R-1) + 400R\hat{R}_t^8 + \sigma^{R8me} \varepsilon_{R8,t}^{me}
\end{aligned} \tag{6.80}$$

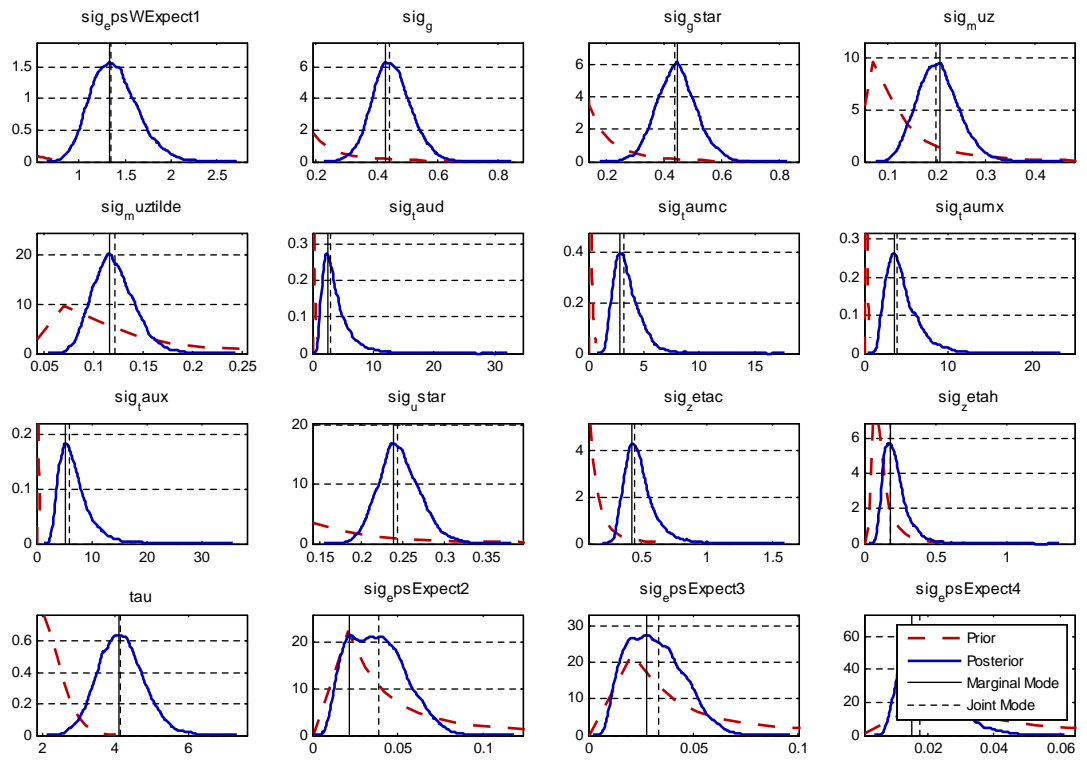
$$\begin{aligned}
R_{t,t+1}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*1} + \sigma^{R^*1me} \varepsilon_{R^*1,t}^{me} \\
R_{t,t+2}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*2} + \sigma^{R^*2me} \varepsilon_{R^*2,t}^{me} \\
R_{t,t+3}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*3} + \sigma^{R^*3me} \varepsilon_{R^*3,t}^{me} \\
R_{t,t+4}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*4} + \sigma^{R^*4me} \varepsilon_{R^*4,t}^{me} \\
R_{t,t+5}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*5} + \sigma^{R^*5me} \varepsilon_{R^*5,t}^{me} \\
R_{t,t+6}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*6} + \sigma^{R^*6me} \varepsilon_{R^*6,t}^{me} \\
R_{t,t+7}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*7} + \sigma^{R^*7me} \varepsilon_{R^*7,t}^{me} \\
R_{t,t+8}^{*,data} &= 400(R^* - 1) + 400R^*\hat{R}_t^{*8} + \sigma^{R^*8me} \varepsilon_{R^*8,t}^{me}
\end{aligned} \tag{6.81}$$

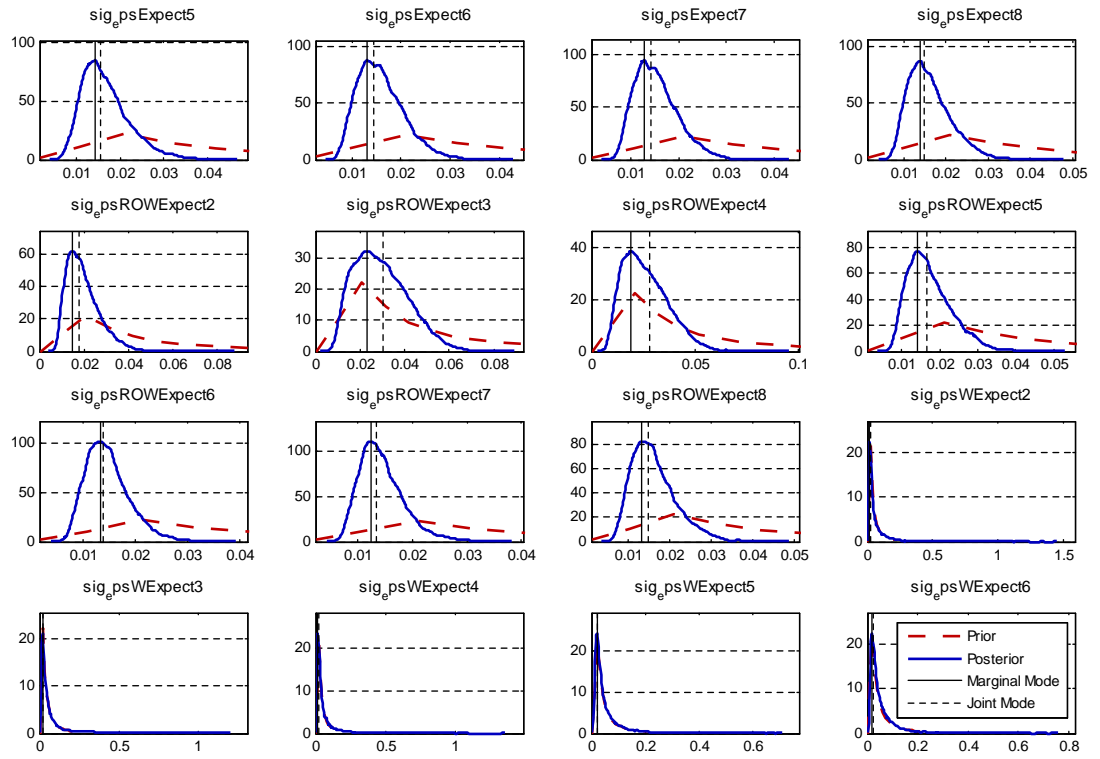
6.11. Marginal Posterior Densities

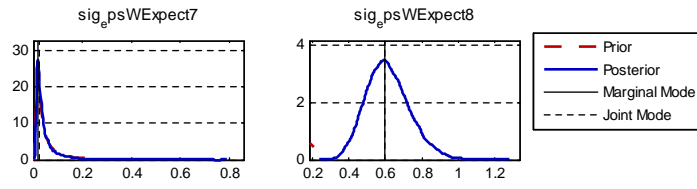
The figures below depict the prior and posterior distributions using the complete $\tilde{Y}_t^{ForwData}$ data set. The results reported are based on a sample of 100,000 draws from the posterior distribution.











6.12. Forecast Error Variance Decomposition

