Regulation of Liquidity Risk*

Enrico Perotti
U. Amsterdam, DSF and CEPR

Javier Suarez
CEMFI and CEPR

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Abstract

This paper discusses liquidity regulation when short-term funding enables credit growth but generates negative systemic risk externalities. It focuses on the relative merit of price versus quantity rules, showing how they target different incentives for risk creation.

When banks differ in credit opportunities, a Pigovian tax on short-term funding is efficient in containing risk and preserving credit quality, while quantity-based funding ratios are distorsionary. Liquidity buffers are either fully ineffective or similar to a Pigovian tax with deadweight costs. Critically, they may be least binding when excess credit incentives are strongest.

When banks differ instead mostly in gambling incentives (due to low charter value or overconfidence), excess credit and liquidity risk are best controlled with net funding ratios. Taxes on short-term funding emerge again as efficient when capital or liquidity ratios keep risk shifting incentives under control. In general, an optimal policy should involve both types of tools.

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1 Introduction

The recent crisis has provided a clear rationale for the regulation of banks' refinancing risk, a critical gap in the Basel II framework. This paper studies the effectiveness of different approaches to liquidity regulation.

The basic trade off of short-term funding is that rapid expansion of credit may only be funded by attracting short-term funding (for instance, because such investors do not need to be very informed about new credit choices), but at the cost of creating refinancing risk which may lead to disruptive liquidity runs (Diamond and Dybvig, 1983). Because of fire sales or counterparty risk externalities, each bank's funding decision has an impact on the vulnerability of other banks, causing a negative externality. Even if the individual bank's funding decision is made taking into account the bank's exposure to refinancing risk, it will fail to internalize its system-wide implications (Perotti and Suarez, 2009). Because of the wedge between the net private value of short-term funding and its social cost, absent regulation banks will rely excessively on short-term funding. A prime example is the massive build up in overnight credit (repo) during 2002-2007, which grew explosively to a volume over ten trillion dollars (Gordon, 2009). Its rapid deflation forced an unprecedented liquidity support by central banks, threatening their capability to keep control over the money supply.

We assess the performance of Pigovian taxes (aimed at equating private and social liquidity costs) and quantity regulations in the context of a model in the tradition of Weizman (1974), recognizing how the regulator is constrained in its ability to target individual bank characteristics. The model shows how the industry response to regulation depends on the composition of bank characteristics. Depending on the dominant source of heterogeneity, the socially efficient solution may be attained with Pigovian taxes, quantity regulations or a combination of both.

The model recognizes that banks differ in their credit ability and their incentives to take risk. Banks earn decreasing returns to expand credit to their (monitored) borrowers, so better banks naturally lend more. Shareholders of less capitalized banks gain from investing in poor gambles, since they retain the upside and shift downside risk to the public safety net.[1] To facilitate the discussion, we first analyze the impact of regulation under either

[1] An alternative view of gambling incentives is that it is driven by self-interested and overconfident man-

form of bank heterogeneity.[2] We assume banks differ in capacity to lend profitably and in incentives to care about their solvency. When banks differ only in their credit assessment capability (or their investment opportunities), a simple flat-rate Pigovian tax on short-term funding (possibly scaled up by the systemic importance of each bank, e.g. to incorporate contribution to counterparty risk) implements the efficient allocation. Liquidity risk levies allow better banks to lend more than others.

In this context, a net stable funding ratio or a liquidity coverage ratio (such as those proposed by the Basel Committee on Banking Supervision in December 2009) may improve over the unregulated equilibrium but are generally not optimal. An optimal quantity-based regulation would require precise measures of individual bank characteristics, most of which are unobservable.[3]

A net stable funding ratio, by effectively imposing an upper limit on short-term debt, reduces overall liquidity risk, but redistributes liquidity risk across banks in an inefficient manner. While banks with better credit opportunities will be constrained, the reduced systemic risk actually encourages banks with low credit ability (for whom the requirement is not binding) to expand.

Liquidity coverage ratios which require banks to hold fractional reserves of liquid assets against short-term funding turn out to be very ineffective.[4] When the yield on liquid assets equals the cost of short-term liabilities (roughly the case in normal times, and certainly prior to the crisis), buffers offer no liquidity risk improvement as there is no net cost to stacking liquidity. Banks will increase their short-term funding and their liquidity holdings enough to keep their "net" short-term funding (the difference) at the same level as in the unregulated equilibrium. The only effect is an artificial demand for liquid assets—traditionally kept in money market mutual funds rather than banks—that might be redirected to banks following

agers, which view excessive risks as profitable

[2] By introducing heterogeneity in the cross-section of banks, we model explicitly the response elasticities which are critical in the seminal paper by Weitzman (1974) to determine the relative performance of quantity versus price regulation.

[3] Quantity requirements may be easily targeted to measures of the "systemic importance" of each bank (size, interconnectedness, capitalization, etc.) but certainly not to unobservables such as measures of banks' credit opportunities.

[4] Liquid assets which can be sold at no fire-sale loss in a crisis are essentially cash, central bank reserves, and treasury bills.

the new requirement.

When the spread between liquid asset yields and bank borrowing costs is positive, a liquidity requirement operates as a tax on short-term funding. [5] In principle, choosing the exact requirement enables the regulator to achieve the same effect as a Pigovian tax (although the spread loss may be seen as a deadweight cost). In a dynamic setup, the tax rate will fluctuate with market conditions, so the regulatory requirement would need to be adjusted to avoid procyclical effects.

Introducing a different source of variation across banks, namely solvency incentives (correlated with charter value or any other determinant of risk-taking tendencies) alters the results radically. Low charter value banks have incentives to gamble to shift risk to the deposit insurance provider (Keeley, 1990). We show that decisions driven by such gambling incentives are not properly deterred by levies, while quantity constrains are more effective. Both short-term funding limits (e.g. a net stable funding ratio) and capital requirements can contain risk shifting by limiting the scale of lending. Levies will not be very effective because the most gambling-inclined banks will also be the most inclined to pay the tax and expand their risky lending. In this case, quantity instruments such as net funding or capital ratios are best to contain excess credit expansion.

Our analysis identifies the relative merits of price versus quantity instruments, and suggests that combining them may be adequate for the simultaneous control of gambling incentives and systemic risk externalities. If, as suggested in prior literature (e.g. Hellmann, Murdock, and Stiglitz, 2000), strengthening capital requirements is an effective strategy for the control of gambling incentives, the case for levies on short-term funding gets reinforced.

Other considerations may qualify the recommendation for the use of one instrument or the other. For instance, levies may be less costly to adjust than ratios. First, they might be easier to move for purely institutional reasons (e.g. if regulatory ratios are embedded in some law or international agreement while the levies are, at least partly, under control of a macroprudential authority). More importantly, changes in the levies may have better dynamic properties than changes in quantitative requirements in the presence of adjustment costs at bank level: they imply less of a concern on frictions that affect banks capability

[5] The tax rate will equal the product of the buffer requirement per unit of short-term funding times the interest spread.

to adjust the quantities in their balance sheets on short notice. Similarly, changes in levies are less likely to induce procyclicality, since the Pigovian "tax rate" is directly controlled by the regulator rather than implicitly set by the interaction of some (controlled) quantitative requirement and the (freely fluctuating) market price of the required resource (namely, capital, liquid assets or stable funding). For preventive policy, controlling time varying liquidity risk may then be best achieved by a combination of stable ratios and variable levies.

The rest of the paper is organized as follows. Section 2 describes some related literature and some recent evidence on liquidity risk. Section 3 describes the baseline model. Section 4 characterizes the unregulated equilibrium. Section 5 finds the socially optimal allocation. In Section 6, we discuss the possibility of restoring efficiency with a Pigovian tax on short-term funding. Section 7 considers alternative quantity-based regulations. In Section 8 we analyze the implications of introducing gambling incentives as a second dimension of bank heterogeneity. Section 9 discusses further implications and extensions of the analysis. Section 10 concludes the paper.

2 Evidence from the crisis and related research

The crisis of 2007-2008 has been described as a wholesale bank crisis, or a repo run crisis (Gorton, 2009). The rapid withdrawing of short-term debt was responsible for propagation of shocks across investors and markets (Brunnermeier, 2009). Brunnermeier and Oemhke (2010) show that creditors have an incentive to shorten their loan maturity, so as to pull out in bad times before other creditors can. This, in turn, causes a lender race to shorten maturity, leading to excessively short-term financing. The consequences are formalized in Martin, Skeie and von Thadden (2010), where increased collective reliance on repo funding weakens solvency constraints, and produce repo runs. Acharya and Viswanathan (2010) model the sudden drying up of liquidity when banks need to refinance short-term debt in bad times. As low asset price increase incentives for risk shifting, investors may rationally refuse refinancing to illiquid banks.

While the role of liquidity risk in the crisis has been evident from the beginning, more precise evidence is now emerging. Acharya and Merrouche (2009) show that UK banks with more wholesale funding and fire sale losses in 2007-08 contributed more to the transmission of shocks to the interbank market. A concrete measure of the role of short-term debt played in the credit boom and its demise comes from the explosive rise of repo (overnight) financing in the last years, and its rapid deflation since the panic (Gorton, 2009). Repo funding evaporated in the crisis, leading to bursts of front running in the sales of repossessed securities.

Various proposals seek private or public clearing arrangements to limit the effects of runs. Acharya and Sabri (2010) argue for the establishment of a Repo Resolution Authority to take over repo positions in a systemic event, paying out a fraction of their claims and liquidating the collateral in an orderly fashion. This would force investors to bear any residual loss. On the opposite front, Gorton (2009) has proposed stopping fire sales of seized collateral by a blanket state guarantee, while Gorton and Metrick (2010) propose creating special vehicles they call narrow banks to hold such assets, backed by a public guarantee. Farhi and Tirole (2009) show how ex post liquidity support policy induces a strategic complementarity for bank leverage and liquidity risk decisions. Such an ex post policy is forced by distress associated with liquidity runs (such as by counterparty risk and fire sales). If in a systemic run there is no choice but to provide liquidity, this implies a loss of public control over the money supply, which becomes fully endogenous to private funding choices. Thus measures are necessary to contain the private creation of liquidity risk.

Finally, and perhaps most importantly, systemic crises are the source of important fiscal and real losses not fully internalized by those who make decisions leading to the accumulation of systemic risk (Laeven and Valencia, 2010), making a clear case for the regulation of the underlying externalities.

Various proposal seeks to identify such systemic risk factors at the level of individual intermediaries (Acharya et al, 2009; Adrian and Brunnermeier, 2009) . Adrian and Brunnermeier (2009) estimate a new measure, termed CoVar, and conclude that uninsured short term funding is a most significant determinants of individual risk contribution. More generally, it is important to estimate an aggregate measure of systemic risk, on which to calibrate any price or quantity regulation.

[TO BE COMPLETED]

3 The model

Consider a one-period economy where agents are risk neutral. The banking system is made up of a continuum of heterogenous banks. Owners choose the amount of lending, and thus short term funding, to maximize expected profits.

We initially assume that banks differ only in credit opportunities θ,[6] distributed with positive density $f(\theta)$ over $[0,1]$. Assuming w.l.o.g. that all banks of class θ behave symmetrically, the lending and short-term funding decision of each θ bank is denoted by $x(\theta) \in [0,\infty)$.

The expected NPV associated with a decision x by a bank of class θ can be written as

$$v(x, X, \theta) = \pi(x, \theta) - \varepsilon(x, \theta)c(X), \tag{1}$$

$\pi(x, \theta)$ is the NPV generated by lending x if no liquidity crisis occurs, X is the aggregate systemic risk caused by the sum of individual short term funding decisions, and $\varepsilon(x, \theta)c(X)$ is the loss in a crisis. The multiplicative decomposition of crisis losses has two terms: $\varepsilon(x, \theta)$ captures the individual contribution of each bank's funding decision x (given individual characteristic θ),while $c(X)$ captures the effect of cumulative funding decisions on systemic crisis costs.

We assume that $\pi(x, \theta)$ is increasing and differentiable in both x and θ, strictly concave in x, and with a positive cross derivative, $\pi_{x\theta} > 0$, so that a larger θ (better credit opportunities) implies a higher return from lending. To guarantee interior solutions in x and monotone comparative statics with respect to θ, we also assume that $\varepsilon(x, \theta)$ is increasing, differentiable, weakly convex in x, and non-increasing in θ, with $\varepsilon_{x\theta} \leq 0$. Finally, we assume individual liquidity risk decisions cumulate, so $c(X)$ is increasing, differentiable, and weakly convex in X.

Here $\pi(x, \theta)$ captures the profitability of lending, $\varepsilon(x, \theta)$ captures the probability that the bank faces losses by refinancing problems in a crisis, and $c(X)$ are net liquidation losses incurred in such an event. $c(X)$ is increasing in X due to the impact on distressed sales from other troubled banks (e.g. under some cash-in-the-market pricing logic or simply because

[6] In Section 8, we introduce a second dimension of bank heterogeneity directed to capture differences in banks' gambling incentives.

the alternative users of the liquidated assets face decreasing returns).[7] So θ can be taken as a measure of a bank's credit ability.

Our key results are robust to essentially any specification of the aggregator $X = g(\{x(\theta)\})$, where $\{x(\theta)\}$ is the schedule of the short-term funding used by banks in class $\theta \in [0, 1]$ and $\partial g/\partial x(\theta) \geq 0$ for all θ. For concreteness we focus on the case in which aggregate systemic liquidity risk is the simple sum of all individual decisions:[8]

$$X = g(\{x(\theta)\}) = \int_0^1 x(\theta)f(\theta)d\theta. \tag{2}$$

In Section 9, we will discuss how to adapt our main results to the case in which banks also differ in a "systemic importance" factor that affects the weight of the contribution of their short-term funding to X.

We assume that all investors, except bank owners, have the opportunity to invest their wealth at exogenously given market rates and provide funding at competitive terms, hence obtaining a zero NPV from dealing with the banks. Then, the total NPV generated by banks (and appropriated by their owners) constitutes the natural measure of social welfare W in this economy. Formally,

$$W(\{x(\theta)\}) = \int_0^1 v(x(\theta), X, \theta)f(\theta)d\theta = \int_0^1 [\pi(x(\theta), \theta) - \varepsilon(x(\theta), \theta)c(X)]f(\theta)d\theta. \tag{3}$$

Notice that the short-term funding decision x' of any bank of class θ' determines, via $\varepsilon(x', \theta')$, the vulnerability of that very bank to a systemic crisis, and also, via $c(X)$, the likelihood and/or costs of a systemic crisis to all other banks.

4 Equilibrium

In an unregulated competitive equilibrium each bank chooses x so as to maximize its own expected NPV, $v(x, X, \theta)$, taking X as given. So an *unregulated competitive equilibrium* is a pair $(\{x^e(\theta)\}, X^e)$ that satisfies:

[7] Of course, an increasing $c(X)$ may also partly reflect that X increases the very probability of a systemic crisis. For example, the more vulnerable banks' funding structures are, the more likely it is that asset-side shocks such as a housing market bust or a stock market crash get transformed into a systemic liquidity shock.

[8] Notice that the linearity of X does not necessarily apply to the "natural" measure of each bank's short-term liabilities (e.g. dollar value of outstanding short-term liabilities), since x may represent any monotonic transformation of the relevant natural measure (e.g. the logistic transformation of the ratio of short-term liabilities to total assets).

1. $x^e(\theta) = \arg\max_x\{\pi(x,\theta) - \varepsilon(x,\theta)c(X^e)\}$ for all $\theta \in [0,1]$,

2. $X^e = \int_0^1 x^e(\theta)f(\theta)d\theta$.

Let $y(\theta, X)$ be the value of x that satisfies the first order condition for an interior privately optimal choice of x given θ and X. This function is implicitly defined by:

$$\pi_x(y(\theta, X), \theta) - \varepsilon_x(y(\theta, X), \theta)c(X) = 0. \tag{4}$$

Given the assumed properties of the relevant functions involved above, the implicit function theorem implies that $y(\theta, X)$ is increasing in θ and decreasing in X. Thus the equilibrium value of X can be found as the fixed point of the auxiliary function $h(X) = \int_0^1 y(\theta, X)f(\theta)d\theta$, which is continuously decreasing in X, implying, by standard arguments, that the fixed point $X^e = h(X^e)$, if it exists, is unique. Existence only requires $h(0) > 0$. Furthermore, the existence of an "interior" equilibrium (with $x^e(\theta) > 0$ for all $\theta > 0$) can be guaranteed by assuming that:

$$\pi_x(0, 0) - \varepsilon_x(0, 0)c(X) \geq 0, \tag{5}$$

for a sufficiently large X.[9] This condition says that even in the presence of large funding risk, all banks (except perhaps those with the lowest valuation for short-term funding, $\theta = 0$) would have $\pi_x(0, \theta) - \varepsilon_x(0, \theta)c(X) > 0$ and, thus, be willing to obtain at least some small positive amount of short-term funding.[10]

For future comparison, let us notice that an interior equilibrium allocation will obviously satisfy

$$\pi_x(x^e(\theta), \theta) - \varepsilon_x(x^e(\theta), \theta)c(X^e) = 0 \tag{6}$$

with $X^e = \int_0^1 x^e(\theta)f(\theta)d\theta$, for all $\theta \in [0, 1]$. As shown below, the presence of systemic risk externalities will make the conditions defined by (6) incompatible with social efficiency.

[9] To obtain most of the results below, we need not constrain attention to interior equilibria, but dealing with the possibility of corner solutions involving $x(\theta) = 0$ for some θ would make the presentation unnecessarily cumbersome.

[10] Recall that we have assumed $\pi_{x\theta} > 0$ and $\varepsilon_{x\theta} \leq 0$.

5 The social planners' problem

The socially optimal allocation of short-term funding across banks can be found be maximizing social welfare W taking into account the influence of each individual bank funding strategy on X. Formally, a *socially optimal allocation* can be defined as a pair $(\{x^*(\theta)\}, X^*)$ that satisfies:

$$(\{x^*(\theta)\}, X^*) = \arg \max_{(\{x(\theta)\},X)} \int_0^1 [\pi(x(\theta),\theta) - \varepsilon(x(\theta),\theta)c(X^*)]f(\theta)d\theta \quad \text{s.t.:} \quad \int_0^1 x(\theta)f(\theta)d\theta = X^*. \tag{7}$$

After substituting the constraint in the objective function, one can also find the social optimum as:

$$\{x^*(\theta)\} = \arg \max_{\{x(\theta)\}} \int_0^1 [\pi(x(\theta),\theta) - \varepsilon(x(\theta),\theta)c(\int_0^1 x(z)f(z)dz)]f(\theta)d\theta \tag{8}$$

and, recursively, $X^* = \int_0^1 x^*(\theta)f(\theta)d\theta$.

The first order conditions that characterize the solution to the social planner's problem define the system of equations:

$$\pi_x(x^*(\theta),\theta) - \varepsilon_x(x^*(\theta),\theta)c(X^*) - E_z(\varepsilon(x^*(z),z))c'(X^*) = 0 \tag{9}$$

for all $\theta \in [0,1]$, where $E_z(\varepsilon(x^*(z),z)) = \int_0^1 \varepsilon(x^*(z),z)f(z)dz$. Relative to the conditions for individual bank optimization given in (6), the conditions in (9) add a third, negative term reflecting the marginal external costs associated with each $x(\theta)$. The cost relevant for a bank of class θ is made of two multiplicative factors: the average vulnerability of all the banks in the system to a systemic crisis, $E_z(\varepsilon(x^*(z),z))$, and the marginal effect of aggregate funding risk on systemic crisis costs, $c'(X^*)$.

The assumptions adopted in Section 3 guarantee the existence of a unique socially optimal allocation. To guarantee that such an allocation is "interior" (satisfying $x^*(\theta) > 0$ for all $\theta > 0$) we may need a condition tighter than (5). For instance, having $\pi_x(0,0) \to \infty$ and finite derivatives with respect to x and X for the functions $\varepsilon(x,\theta)$ and $c(X)$, respectively.

Clearly, the interior equilibrium allocation characterized by (6) does not satisfy (9) due to having both $E_z(\varepsilon(x^e(z),z)) > 0$ and $c'(X^e) > 0$. Even accounting for situations involving $x^*(\theta) = 0$ or $x^e(\theta) = 0$ for low values of θ, the following proposition can be generally established:

Proposition 1 *The presence of systemic externalities associated with banks funding decisions,* $c'(X) > 0$*, makes the equilibrium allocation socially inefficient and characterized by an excessive aggregate funding risk* $X^e > X^*$*. Indeed, in an interior equilibrium, we have* $x^e(\theta) > x^*(\theta)$ *for all* θ*.*

Intuitively, the systemic externalities associated with banks' short-term funding decisions create a positive wedge between the social and the private marginal costs of using short-term funding. Banks only internalize the implications of the funding choices for their own vulnerability to refinancing risk, without considering their contribution to all other banks' systemic risk exposure and costs. Their standard marginal reasoning when privately optimizing on x make them choose an amount larger than socially optimal.

6 The Pigovian tax: an efficient solution

As in the standard textbook discussion on the treatment of negative production externalities, the social efficiency of the competitive equilibrium can be restored by imposing a Pigovian tax: by taxing the activity causing the externality at a rate equal to the wedge between the social marginal cost and the private marginal cost of the activity (evaluated, if applicable, at the anticipated socially optimal allocation). In our case, this will boil down to setting a flat tax per unit of short-term funding equal to

$$\tau^* = E_z(\varepsilon(x^*(z), z))c'(X^*). \tag{10}$$

Obviously, the introduction of a tax on short-term funding will alter the first order condition relevant for banks' optimization in the competitive equilibrium with taxes.

Formally, we can define a *competitive equilibrium with taxes* $\{\tau(\theta)\}$ as a pair $(\{x^\tau(\theta)\}, X^\tau)$ satisfying:

1. $x^\tau(\theta) = \arg\max_x\{\pi(x,\theta) - \varepsilon(x,\theta)c(X^\tau) - \tau(\theta)x\}$ for all $\theta \in [0,1]$,

2. $X^\tau = \int_0^1 x^\tau(\theta)f(\theta)d\theta$.

The first order conditions for the private optimality of each $x^\tau(\theta)$ imply

$$\pi_x(x^\tau(\theta),\theta) - \varepsilon_x(x^\tau(\theta),\theta)c(X^\tau) - \tau(\theta) = 0 \tag{11}$$

for all $\theta \in [0, 1]$. And it is immediate to see that the flat tax schedule $\tau(\theta) = \tau^*$, with the tax rate defined as in (10), will make $(\{x^{\tau^*}(\theta)\}, X^{\tau^*}) = (\{x^*(\theta)\}, X^*)$, implementing the socially optimal allocation as a competitive equilibrium.

To set the reference rate τ^* properly, it is of course necessary that the regulator knows the functions that characterize the economy (including the density of the parameter θ that captures banks' heterogeneity) and is, hence, able to compute the socially optimal allocation that appears in (10).

An important practical difficulty when regulating heterogeneous agents is that the particulars of the regulation applicable to each agent may depend on information that its private to the agent. This problem does not affect the efficient Pigovian tax τ^*, which is the same for all values of θ. The following proposition summarizes the key results of this section.

Proposition 2 *When banks differ in the marginal value they can extract from short-term funding, the socially optimal allocation can be reached as a competitive equilibrium by charging banks a* flat *Pigovian tax τ^* on each unit of short-term funding.*

7 Other regulatory alternatives

Pigovian taxation is frequently described as a price-based solution to the regulation of externalities. Such description emphasizes the capacity of the tax solution to decentralize the implementation of the desired allocation as a market equilibrium. The polar alternative is to go for a "centralized" quantity-based solution in which each regulated agent (bank) is directly mandated to choose its corresponding quantity (short-term funding) in the optimal allocation ($x^*(\theta)$ in the model).

In the context of our model, pure quantity-based regulation would require detailed knowledge by the regulator of individual marginal value of short-term funding for each bank (i.e., the derivatives $\pi_x(x, \theta)$ and $\varepsilon_x(x, \theta)$, which vary with θ and appear in (9)). Possibly due to the strong informational requirements that this implies, none of the alternatives for liquidity regulation considered in practice these days opts for directly setting individualized quantity prescriptions such as $x^*(\theta)$.

The alternatives to Pigovian taxes actually under discussion are ratio-based regulations, i.e. regulations that consist on forcing banks to have some critical accounting ratios above or below some regulatory minima or maxima. To be sure, some proposals include making the regulatory bounds functions of individual characteristics of each bank, such as size, interconnectedness, capitalization, etc. but none of the considered characteristics (except perhaps those referring to the regional or sectorial specialization of some banks) seem targeted to control for the heterogeneity in banks' capacity to extract value from short-term funding. These qualifiers can be rather rationalized as an attempt to capture what, in an extension discussed in Section 9, we describe as the *systemic importance* of each bank (the relative importance of the contribution of its short-term funding to the systemic risk measure X).

The most seriously considered ratio-based proposals for the regulation of liquidity are those contained in a consultative paper of the BCBS on the topic issued in December 2009. This document puts forward two new regulatory ratios: a *liquidity coverage ratio*, similar in format and spirit to one already introduced by the Financial Services Authority in the UK in October 2009, and a more innovative *net stable funding ratio.* To facilitate the discussion, we analyze each of these instruments as if it were introduced in isolation, starting with the last one, whose potential effectiveness for the regulation of funding maturity is somewhat less ambiguous.

7.1 A stable funding requirement

The net stable funding requirement calls banks to hold some accounting ratio of "stable funding" (i.e. equity, customer deposits, and other long-term or "stable" sources of funding) to "non-liquid assets" above some regulatory minimum. To translate this to our model, where banks' assets and stable sources of funding have been so far taken as exogenously fixed, we can think of this requirement as equivalent to imposing an upper limit $\overline{x}$ to the short-term debt that the bank can issue. In a more general version of our model, the effective upper limit applicable to each bank could be considered affected by prior decisions of the bank regarding the maturity and liquidity structure of its assets, its retail deposits base, its level of capitalization, etc. But here, for simplicity, one can see these issues as a possible interpretation of the comparative statics of $\overline{x}$.

The introduction of a minimum stable funding requirement has then the implication of adding an inequality constraint of the type $x \leq \overline{x}$ to the private optimization problem of the banks. Formally, a *competitive equilibrium with a stable funding requirement* parameterized by $\overline{x}$ can be defined as a pair $(\{x^{\overline{x}}(\theta)\}, X^{\overline{x}})$ satisfying:

1. $x^{\overline{x}}(\theta) = \arg\max_{x \leq \overline{x}} \{\pi(x,\theta) - \varepsilon(x,\theta)c(X^{\overline{x}})\}$ for all $\theta \in [0,1]$,

2. $X^{\overline{x}} = \int_0^1 x^{\overline{x}}(\theta) f(\theta) d\theta$.

Since the preference for short-term funding is strictly increasing in θ, we may have up to three possible configurations of equilibrium. For $\overline{x} \geq x^e(1)$, the stable funding requirement will not be binding for any bank (since $\theta = 1$ identifies the banks with the highest incentives to use short-term funding), and the equilibrium will then coincide with the unregulated competitive equilibrium characterized in Section 4. For $\overline{x} \leq x^e(0)$, the stable funding requirement will be binding for all banks (since $\theta = 1$ identifies the banks with the lowest incentives to use short-term funding), implying $x^{\overline{x}}(\theta) = \overline{x} < x^e(\theta)$ for all θ and, hence, $X^{\overline{x}} = \overline{x} E_\theta(w(\theta)) < X^e$. For $\overline{x} \in (x^e(0), x^e(1))$, the stable funding requirement will be binding for at least the banks with the largest θs and perhaps for all banks. To see the latter, notice that inducing the limit choice of $x^{\overline{x}}(\theta) = \overline{x} < x^e(\theta)$ to the banks with relatively large θs will push $X^{\overline{x}}$ below X^e, but this, in turn, will push the banks with relatively low θs into choices of $x^{\overline{x}}(\theta) > x^e(\theta)$, possibly (but not necessarily) inducing some or even all of them to also hit the regulatory limit $\overline{x}$.

It is then obvious that, in general, a sufficiently tight stable funding requirement $\overline{x} < x^e(1)$ can reduce the equilibrium measure of aggregate systemic risk $X^{\overline{x}}$ relative to the unregulated equilibrium X^e, thus moving it closer to its value in the socially optimal allocation X^*. The induced allocation will, however, be necessarily inefficient. The reason for this is that the reduction in the activities that generate negative externalities comes at the cost of distorting the allocation of short-term funding across bank classes: (i) constraining the banks with relatively higher valuation for short-term funding to the common upper limit $\overline{x}$, and (ii) encouraging the banks with relatively low valuation for short-term funding to use more of it than it would be socially optimal (since they will choose $x^{\overline{x}}(\theta) > x^e(\theta)$, but $x^e(\theta) > x^*(\theta)$ for all θ). In fact, there is no guarantee that introducing a $\overline{x}$ that simply bring $X^{\overline{x}}$ closer to X^* improves, in welfare terms, over the unregulated equilibrium.

Proposition 3 *A binding net stable funding requirement will affect the measure of aggregate systemic risk X in the same direction as the efficient arrangement (i.e. will reduce X) but it will also redistribute short-term funding inefficiently from banks that value it more to banks that value it less, so that the socially optimal allocation cannot be reached and the improvement in social welfare is not guaranteed.*

The socially optimal choice of $\overline{x}$ (i.e. the "second best" allocation attainable if $\overline{x}$ is the only available instrument for liquidity regulation) can be defined as follows:

$$\begin{aligned} \overline{x}^{SB} = \arg \max_{(\overline{x}, X^{\overline{x}})} \quad & \int_0^{\overline{\theta}} [\pi(y(\theta, X^{\overline{x}}), \theta) - \varepsilon(y(\theta, X^{\overline{x}}), \theta) c(X^{\overline{x}})] f(\theta) d\theta + \int_{\overline{\theta}}^1 [\pi(\overline{x}, \theta) - \varepsilon(\overline{x}, \theta) c(X^{\overline{x}})] f(\theta) d\theta \\ \text{s.t.:} \quad & \int_0^{\overline{\theta}} y(\theta, X^{\overline{x}}) f(\theta) d\theta + \overline{x}[1 - F(\overline{\theta})] = X^{\overline{x}}, \end{aligned} \tag{12}$$

where $\overline{\theta}$ satisfies $y(\overline{\theta}, X^{\overline{x}}) = \overline{x}$, the function $y(\theta, X)$ is defined as in (4), and $F(\theta)$ is the cumulative distribution function associated with $f(\theta)$.

The first order conditions that characterize an interior solution to the above second best social planner's problem can be written after some algebra (and after taking the constraint of the problem and the definition of $y(\theta, X)$ into account) as

$$\int_{\overline{\theta}}^1 [\pi_x(\overline{x}, \theta) - \varepsilon_x(\overline{x}, \theta) c(X^{\overline{x}})] f(\theta) d\theta - E_\theta(\varepsilon(x^{\overline{x}}(\theta), \theta)) c'(X^{\overline{x}}) \frac{dX^{\overline{x}}}{d\overline{x}} = 0, \tag{13}$$

where

$$\frac{dX^{\overline{x}}}{d\overline{x}} = \frac{1 - F(\overline{\theta})}{1 - \int_0^{\overline{\theta}} y_X(\theta, X^{\overline{x}}) f(\theta) d\theta} \in [0, 1]. \tag{14}$$

To gain some intuition on the trade-offs behind the socially optimal choice of $\overline{x}$, it is convenient to compare (13) with the condition for first best efficiency in (9). First, (9) applies point-wise, defining an efficient $x^*(\theta)$ for each θ; in contrast, (13) is just one equation that determines a common $\overline{x}$ trading off costs and benefits that are "averaged" over all the θs. The terms in the integral that appears in (13) resemble the first two terms in the left hand side of (9), but the ones "averaged" here correspond to the set of high θs only, for which the requirement $\overline{x}$ is binding.[11] The second term in (13) and the third in (9) reflect the marginal externality caused by changing $\overline{x}$ and each $x^*(\theta)$, respectively. The relevant difference is due

[11] For lower values of θ, the first order conditions for the individually optimizing decisions make the relevant terms equal to zero.

to the presence of $dX^{\overline{x}}/d\overline{x}$ in (13): as shown in (14), this term captures the fact that raising $\overline{x}$ increases by the same amount the short-term funding of the constrained banks (whose proportion $1 - F(\overline{\theta}) < 1$ appears in the numerator) but has the partially offsetting effect of reducing (in response to the very rise in $X^{\overline{x}}$) the use of short-term by the unconstrained banks (which explains the denominator, where $y_X < 0$).

This comparison evidences the rather limited second best nature (relative to the efficient, flat Pigovian tax) of the regulatory solution based on establishing a stable funding requirement.

7.2 A liquidity requirement

The liquidity coverage ratio described by the BCBS in December 2009 requires banks to back their use of short-term funding with the holding of high-quality liquid assets, i.e. assets that could be easily sold, presumably at no fire-sale loss, in case of a crisis. In its original description this requirement responds to the motivation of providing each bank with its own liquidity buffer, which, presumably might also expand the liquidity available in the system in case of a crisis (on top of that possibly provided by the lender of last resort).

Specifically, it is proposed that banks estimate the refinancing needs that they would accumulate if the functioning of money markets or other conventional borrowing sources were disrupted for some specified period (one month) and keep enough high-quality liquid assets so as to be able to confront the situation with their sale.[12] Qualifying assets would essentially be cash, central bank reserves and treasury bonds.

How can we capture this requirement in the context of our model? Leaving details aside, the liquidity requirement can be seen as a requirement to back some minimal fraction $\phi < 1$ of each bank's short-term funding x with the holding of qualifying liquid assets m, thereby introducing the constraint $m \geq \phi x$. Additionally, the impact of m on the banks objective function could be taken into account by considering the following extended value function:

$$v(x, m, X, \theta) = \pi(x - m, \theta) - \varepsilon(x - m, \theta)c(\widehat{X}) - \delta m, \tag{15}$$

[12] Or by posting them as collateral at the central bank's discount window.

where

$$\widehat{X} = \int_0^1 [x(\theta) - m(\theta)] f(\theta) d\theta, \tag{16}$$

and $\delta = r_b - r_m \geq 0$ is the difference between the bank's short-term borrowing rate r_b and the yield r_m of the qualifying liquid assets. This formulation credits for both the individual and the systemic "buffering" role of the liquid assets by making each bank's individual vulnerability factor $\varepsilon(x - m, \theta)$ a function of its "net" short-term funding and by redefining the systemic risk measure $\widehat{X}$ as the banks' aggregate "net" short-term funding positions.

The other terms in (15) capture the NPV generated in the absence of a systemic crisis. Our formulation is based on assuming that the former function $\pi(x, \theta)$ captured the NPV generated by the bank's core lending or investment activity, which does not include investing in the qualifying liquid assets. The new first argument of $\pi(x - m, \theta)$ is justified by the fact that if a part m of the resources obtained as short-term funding x is invested in liquid assets, the net amount available for core banking activities becomes $x - m$. The funds m invested in liquid assets yield a (risk-free) rate r_m but have a cost equal to the bank's short-term borrowing rate $r_b \geq r_m$. So the spread $\delta = r_b - r_m \geq 0$ is the net direct cost of holding liquid assets.[13]

In this extended framework, social welfare can be written as:

$$W(\{x(\theta), m(\theta)\}) = \int_0^1 [\pi(x(\theta) - m(\theta), \theta) - \varepsilon(x(\theta) - m(\theta), \theta) c(\widehat{X}) - \delta m(\theta)] f(\theta) d\theta, \tag{17}$$

where the presence of $-\delta m(\theta)$ implies considering banks' direct costs of holding liquidity as a deadweight loss.[14]

A *competitive equilibrium with a liquidity requirement* parameterized by ϕ can be defined as a pair $(\{(x^\phi(\theta), m^\phi(\theta))\}, \widehat{X}^\phi)$ satisfying:

1. $(x^\phi(\theta), m^\phi(\theta)) = \arg\max_{m \geq \phi x} \{\pi(x - m, \theta) - \varepsilon(x - m, \theta) c(\widehat{X}^\phi) - \delta m\}$ for all $\theta \in [0, 1]$,

2. $\widehat{X}^\phi = \int_0^1 (x^\phi(\theta) - m^\phi(\theta)) f(\theta) d\theta$.

[13] Having $\delta < 0$ would create an arbitrage opportunity for the banks: they could attain unlimited value by borrowing unlimitedly in order to just invest unlimitedly in liquid assets.

[14] This view is consistent with having assumed that investors provide (short-term) funding to the banks at competitive market rates and thus make zero NPV when doing so. In this context, $\delta > 0$ is a premium that compensates for (unmodeled) utility losses derived from either the risk or the lower liquidity of an investment in bank liabilities.

It is immediate to see that the liquidity requirement can be taken as generally binding (necessarily so if $\delta > 0$ and binding without loss of generality if $\delta = 0$). This allows as to reformulate banks' optimization problem in terms of the sole choice of *net* short-term funding $\widehat{x}(\theta) = x(\theta) - m(\theta)$: the binding liquidity constraint allows us to write $m(\theta)$ as $\frac{\phi}{1-\phi}\widehat{x}(\theta)$. Hence, equilibrium can be redefined as a pair $(\{\widehat{x}^{\phi}(\theta)\}, \widehat{X}^{\phi})$ satisfying:

1. $\widehat{x}^{\phi}(\theta) = \arg\max_{\widehat{x}}\{\pi(\widehat{x}, \theta) - \varepsilon(\widehat{x}, \theta)c(\widehat{X}^{\phi}) - \frac{\delta\phi}{1-\phi}\widehat{x}\}$ for all $\theta \in [0, 1]$,
2. $\widehat{X}^{\phi} = \int_0^1 \widehat{x}^{\phi}(\theta)f(\theta)d\theta$.

We will proceed with the analysis by looking first at the case in which the net cost of holding liquid assets is zero ($\delta = 0$) and then at the case in which it is positive ($\delta > 0$).

7.2.1 The case in which holding liquidity is costless ($\delta = 0$)

The following proposition establishes a somewhat shocking result for the relevant case in which the spread δ is zero (roughly the case in "normal times", when banks are perceived as essentially risk-free borrowers):

Proposition 4 *With $\delta = 0$, the competitive equilibrium with a liquidity requirement $\phi < 1$ involves the same amount of net short-term funding and, hence, the same level of systemic risk as the unregulated equilibrium. That is, it involves $x^{\phi}(\theta) - m^{\phi}(\theta) = x^e(\theta)$ and $\widehat{X}^{\phi} = X^e$.*

The proof of this proposition follows immediately from the equivalence, when $\delta = 0$, between the equilibrium conditions for $(\{\widehat{x}^{\phi}(\theta)\}, \widehat{X}^{\phi})$ and those for $(\{x^e(\theta)\}, X^e)$ (see Section 4). Hence, the only effect of the liquidity requirement relative to the unregulated equilibrium is to induce an artificial demand $M^{\phi} = \frac{\phi}{1-\phi}E_{\theta}(x^e(\theta))$ for the qualifying liquid assets and a spurious increase in banks' "gross" short-term funding, which becomes $E_{\theta}(x^{\phi}(\theta)) = E_{\theta}(x^e(\theta)) + M = \frac{1}{1-\phi}E_{\theta}(x^e(\theta))$.

Therefore, when the direct net cost δ of each unit of liquidity that the requirement forces banks to hold is zero (not implausible in "normal times"), the liquidity coverage ratio totally fails to bring the equilibrium allocation any closer to the socially optimum than in the unregulated scenario. Banks respond to regulation by increasing their short-term funding and their liquidity holding so as to make their "net" short-term funding as high as in the unregulated equilibrium. The artificial demand for high-quality liquid assets may imply that liquid assets kept somewhere else in the financial system (e.g. money market mutual funds) prior to imposing the ratio end up kept by banks after imposing the ratio. However the systemic risk generated by the banks does not change.

7.2.2 The case in which holding liquidity is costly ($\delta > 0$)

When the direct net unit cost of holding liquidity, δ, is positive, the implications are quite different. The equilibrium conditions for $(\{\widehat{x}^{\phi}(\theta)\}, \widehat{X}^{\phi})$ become analogous to those associated with a competitive equilibrium with taxes in which $\tau(\theta) = \frac{\delta\phi}{1-\phi}$ (see Section 6):

Proposition 5 *With $\delta > 0$, the competitive equilibrium with a liquidity requirement $\phi < 1$ involves the same individual net short-term funding decisions and aggregate systemic risk as a competitive equilibrium with a tax on short-term funding with rate $\tau(\theta) = \frac{\delta\phi}{1-\phi}$ for all θ.*

For a given $\delta > 0$, the implicit "tax rate" described above moves from zero to infinity as the liquidity requirement ϕ moves from zero to one. Thus the regulator can *seemingly* replicate the effects of any flat tax (including the efficient Pigovian tax τ^* of Section 6) by setting $\phi = \frac{\tau}{\delta+\tau}$. However, banks' demand for the qualifying liquid assets would be $m^{\phi}(\theta) = \frac{\phi}{1-\phi}\widehat{x}^{\phi}(\theta) = \frac{\tau}{\delta}x^{\tau}(\theta)$ (implying an aggregate demand $M^{\phi} = \frac{\tau}{\delta}X^{\tau}$) and their *gross* short-term funding would be $x^{\phi}(\theta) = x^{\tau}(\theta) + m^{\phi}(\theta) = \frac{\delta+\tau}{\delta}x^{\tau}(\theta) > x^{\tau}(\theta)$ (implying $X^{\phi} = E_{\theta}(x^{\phi}(\theta)) = X^{\tau} + M^{\phi} = \frac{\delta+\tau}{\delta}X^{\tau} > X^{\tau}$ at the aggregate level). Importantly, the total direct net costs of holding liquidity would cause a deadweight loss of $\delta m^{\phi}(\theta) = \tau x^{\tau}(\theta)$ to each bank. Not surprisingly, the aggregate deadweight loss $\delta M^{\phi} = \tau X^{\tau}$ equals the tax revenue that the "replicated" tax on short-term funding could have raised.

The presence of the deadweight loss $\tau^* X^*$ implies that the liquidity requirement that *seemingly* replicates the Pigovian solution ($\phi^* = \frac{\tau^*}{\delta+\tau^*}$) is not socially efficient.

Proposition 6 *With $\delta > 0$, replicating the net short-term funding allocation and aggregate systemic risk of the efficient allocation using a liquidity requirement $\phi^* = \frac{\tau^*}{\delta+\tau^*}$ is feasible, but entails a deadweight loss $\tau^* X^* > 0$.*

Actually, ϕ^* will not generally be optimal even from a second best perspective. Except in the non-generic situation in which the efficient Pigovian tax τ^* happens to be at a critical point of the Laffer curve τX^τ. This is because moving the liquidity requirement marginally away from ϕ^* (in one direction) will reduce the deadweight loss δM^ϕ, while other components of social welfare will not change (since they are maximized precisely with $\phi = \phi^*$).

For a given spread $\delta > 0$, the socially optimal liquidity requirement will be some $\phi^{SB} = \frac{\tau^{SB}}{\delta+\tau^{SB}}$ whose associated "implicit tax rate" τ^{SB} satisfies:

$$\begin{aligned} \tau^{SB} = \quad & \arg\max_{\tau\geq 0} \int_0^1 [\pi(x^\tau(\theta),\theta) - \varepsilon(x^\tau(\theta),\theta)c(X^\tau) - \tau x^\tau(\theta)] f(\theta)d\theta \\ & \text{s.t.:} \quad x^\tau(\theta) = \arg\max_x \pi(x,\theta) - \varepsilon(x,\theta)c(X^\tau) - \tau x \text{ for all } \theta \\ & \qquad\quad \int_0^1 x^\tau(\theta) f(\theta)d\theta = X^\tau. \end{aligned} \tag{18}$$

The formulation of this optimization problem exploits the analogy explained above, which conveniently allows us to write the deadweight loss suffered by each bank as $\tau x^\tau(\theta)$, which is actually independent of δ and will end up making the solution τ^{SB} also independent of δ. Notice that the constraints in the optimization problem are simply the conditions that define an equilibrium with a tax τ on short-term funding (see Section 6).

Typically, the optimal liquidity requirement ϕ^{SB} will be inferior to ϕ^*, implying more short-term funding for each bank and, hence, more aggregate systemic risk than in the first best allocation. The intuition for this is that moving away from the unregulated equilibrium allocation by increasing ϕ will typically monotonically increase the aggregate deadweight loss δM^ϕ, while the remaining marginal benefits of moving towards the first best allocation decline towards zero as ϕ approaches ϕ^*.[15]

Interestingly, the writing of the problem as in (18) makes clear that τ^{SB} does not depend on δ, implying that the total variation of $\phi^{SB} = \frac{\tau^{SB}}{\delta+\tau^{SB}}$ with respect to δ is just given by the partial derivative

$$\frac{\partial \phi^{SB}}{\partial \delta} = \frac{-\tau^{SB}}{(\delta + \tau^{SB})^2} < 0.$$

Hence, if the regulator wants to implement the second best allocation described above (or to seemingly replicate the efficient Pigovian tax), it should be ready to move the imposed liquidity requirement ϕ^{SB} (or ϕ^*) in response to the fluctuations in the spread δ. In practice, moving ϕ and the implied adjustments in quantities may be a source of trouble. On the one hand, authorities will have to be effective in changing ϕ in due course. On the other hand, frequent and sudden changes ϕ might produce changes in M^ϕ that, for reasons left outside

[15] The result might be reversed if δM^ϕ became decreasing in ϕ somewhere before reaching ϕ^*.

the model (such as monetary stability) might not be admissible. This might be especially so if δ approaches zero, in which case the prescriptions for ϕ^{SB} (or ϕ^*) imply that M^ϕ would tend to infinity.

These last predictions suggest, however, that treating δ as exogenously given (as we did so far) might not be appropriate for, at least, the last type of discussion. The high demand for liquid assets and the large gross short-term borrowing needs of the banks that follow the increase in ϕ might eventually produce upward pressure on δ, so that the limit with $\delta = 0$ is not relevant for the implementation of ϕ^{SB} (or ϕ^*). Looking at the situation in which δ is endogenously determined in equilibrium by the interaction of demand and supply (in the markets for liquid assets and banks' short-term debt) constitutes a possible interesting extension of our analysis.

8 Risk-shifting and the case for quantity regulation

In this section we extend the model to address formally one of the main criticisms to the proposal of a Pigovian approach to liquidity risk regulation. Such criticism is based on the "robustness" of the price-based approach to modeling mistakes and, specifically, to the possibility of having some "crazy" or just particularly risk-inclined banks that, for the sake expanding their risky lending are willing to pay large amounts of the established tax so as to use large amounts of short-term funding.

In our baseline formulation, banks that like to take more short-term funding are those that can extract more expected NPV from it. In such formulation, the considered dimension of heterogeneity makes banks with larger θ essentially more valuable, privately and, if properly regulated, also socially. We will now denote that dimension of heterogeneity by θ_1 and introduce a second dimension of heterogeneity, $\theta_2 \in [0, 1]$, intended to capture differences in banks' inclination towards risk-taking.[16] The joint distribution of (θ_1, θ_2) will be described

[16] The literature has identified several sources of such differences. Corporate governance arrangements may affect the severity of the conflicts of interest between shareholders and debtholders, making the former more or less capable to ex post expropriate the former by shifting risk (Jensen and Meckling, 1976). In the case of banks, risk-shifting problems are exacerbated by the existence of safety net guarantees (e.g. deposit insurance) provided at risk-insensitive rates. In such a setup, banks' charter values reduce excessive risk-taking (Keeley, 1990). Capital requirements (especially if risk-based) generally improve the alignment of incentives between the bankers and other stakeholders (Holmstrom and Tirole, 1997) and can specifically

by the density function $f(\theta_1, \theta_2)$.

To capture heterogeneity in banks' risk-shifting inclinations formally, we are going to treat θ_2 as a parameter that determines the fraction of the losses incurred by a bank during a crisis which are *not* internalized by its owners but passed (without compensation) to other stakeholders (e.g. the deposit insurer). We then assume each bank, when privately deciding on x, only considers the fraction $1 - \theta_2$ of $\varepsilon(x, \theta)c(X)$ as an expected value loss, leaving the remaining fraction θ_2 to other stakeholders. Hence, the social welfare measure $W(\{x(\theta)\})$ must now explicitly consider, in addition to the NPV appropriated by the bank owners, the losses $-\theta_2\varepsilon(x, \theta)c(X)$ passed on to other bank stakeholders.

So the new objective function for banks is:

$$v(x, X, \theta_1, \theta_2) = \pi(x, \theta_1) - (1 - \theta_2)\varepsilon(x, \theta_1)c(X), \tag{19}$$

while social welfare is given by:

$$W(\{x(\theta_1, \theta_2)\}) = \int_0^1 \int_0^1 [v(x(\theta_1, \theta_2), X, \theta_1, \theta_2) - \theta_2\varepsilon(x(\theta_1, \theta_2), \theta_1)c(X)] f(\theta_1, \theta_2) d\theta_1 d\theta_2, \tag{20}$$

where

$$X = g(\{x(\theta_1, \theta_2)\}) = \int_0^1 \int_0^1 x(\theta_1, \theta_2) f(\theta_1, \theta_2) d\theta_1 d\theta_2. \tag{21}$$

Plugging (19) into (20), social welfare can be written as

$$W(\{x(\theta_1, \theta_2)\}) = \int_0^1 \int_0^1 [\pi(x(\theta_1, \theta_2), \theta_1) - \varepsilon(x(\theta_1, \theta_2), \theta_1)c(X)] f(\theta_1, \theta_2) d\theta_1 d\theta_2, \tag{22}$$

which is conceptually identical to (3).

8.1 Gambling as the sole source of heterogeneity

To highlight our key argument, suppose that the variation due to θ_1, whose implications we have already discussed in prior sections, is shut down by fixing $\theta_1 = \overline{\theta}_1$ for all banks. So residual bank heterogeneity is due to θ_2 only. How is the unregulated equilibrium determined? And the socially optimal allocation? How do they differ? How should $x(\theta_2)$ be regulated?

attenuate the risk-shifting problem (Hellmann, Murdock, and Stiglitz, 2000).

Without restating all the relevant definitions (which will follow mechanically from the adaptation of those already presented for the baseline model), the answers to the questions above can be found by comparing the first order conditions satisfied by bank decisions, $x^{ee}(\theta_2)$, and the systemic risk measure, X^{ee}, in an interior unregulated equilibrium, with the conditions satisfied by their counterparts, $x^{**}(\theta_2)$ and X^{**}, in an interior socially optimal allocation. Similarly to (6), the unregulated equilibrium objects satisfy:

$$\pi_x(x^{ee}(\theta_2), \overline{\theta}_1) - (1-\theta_2)\varepsilon_x(x^{ee}(\theta_2), \overline{\theta}_1)c(X^{ee}) = 0, \tag{23}$$

while, similarly to (9), in the socially optimal allocation we must have:

$$\pi_x(x^{**}(\theta_2), \overline{\theta}_1) - \varepsilon_x(x^{**}(\theta_2), \overline{\theta}_1)c(X^{**}) - E_z(\varepsilon(x^{**}(z), \overline{\theta}_1))c'(X^{**}) = 0, \tag{24}$$

in both cases for all θ_2. From these conditions, it is immediate to conclude that $x^{ee}(\theta_2)$ is increasing in θ_2 (that is, banks with greater risk-shifting inclinations tend to use more short-term funding) while $x^{**}(\theta_2)$ is independent of θ_2 and, hence, equal to a constant $\overline{x}^{**}$ (since, for any given x, θ_2 determines the distribution of value across bank stakeholders but not the total marginal value of short-term funding).

By simple comparison of the two sets of conditions, it is now obvious that the efficient Pigovian tax schedule is

$$\tau^{**}(\theta_2) = \theta_2\varepsilon_x(x^{**}(\theta_2), \overline{\theta}_1)c(X^{**}) + E_z(\varepsilon(x^{**}(z), \overline{\theta}_1))c'(X^{**}),$$

where the first term is new relative to (10) and reflects that risk shifting incentives produce additional discrepancies between the private and social costs of expanding banks' short-term funding. In contrast to the pure systemic externality term (identical to what we had in the baseline model), the first term depends on θ_2. Hence, the efficient Pigovian tax schedule is not flat and cannot be enforced without detailed knowledge of each bank's risk-shifting inclination. A flat tax on short-term funding will not implement the first best allocation.

Now, however, proper quantity regulation can do a great job. Specifically, a net stable funding requirement that effectively imposes the first best quantity $\overline{x}^{**}$ as a limit to each bank's use of short-term funding would implement the first best. It is easy to see that the regulatory constraint will be binding for all θ_2. As for liquidity requirements, the rather negative conclusions obtained in the baseline analysis would still apply: with $\delta = 0$, a liquidity requirement is as ineffective as it was there, while with $\delta > 0$ its effect is very similar to (but has worse welfare properties than) a flat tax on short-term funding. And a flat tax on short-term funding is not a good solution in this environment!

Our conclusions can then be summarized as follows:

Proposition 7 *If gambling incentives constitute the only source of heterogeneity across banks, a stable funding requirement $\overline{x} = \overline{x}^{**}$ implements the socially efficient allocation while no flat-rate tax on short-term funding can do it. A liquidity requirement has the same short-comings as in the baseline model and is, then, either ineffective (if $\delta = 0$) or very similar (but with larger deadweight costs) than the flat-rate tax solution (if $\delta > 0$).*

8.2 Generalizing the analysis

The analysis of the general case in which both θ_1 and θ_2 exhibit significant variation across banks is complicated and unlikely to yield very clear-cut results, if anything because first best efficiency will not be generally attainable using instruments that are not explicitly contingent on θ_1 or θ_2. The analysis of simple instruments will necessarily be based on their second best performance.

Using a continuity argument and building on the polar cases already analyzed above, we can say that a flat tax on short-term funding will tend to perform better than a stable funding requirement if θ_1 is the dominant source of heterogeneity, i.e. if it has ample variation and, specifically, sufficient density at its upper tail, producing sufficiently many banks with value-generating motives to use short-term funding at a larger scale. The opposite will be true if θ_2 is the dominant source of variation, in this case producing sufficiently many banks whose main reason for wanting to use short-term funding at large scale is risk-shifting. For instance, if the banking system had a small group of gambling banks and an ample majority of non-gambling banks, a stable funding requirement might be helpful to control the otherwise excessive short-term funding that the former would like to use.

But, continuing with the example, one can also anticipate possible advantages from combining the instruments. Suppose, in particular, that there were some additional diversity due to θ_1 that affects, mainly, the banks in the non-gambling group. Then it might be socially valuable to introduce a complementary tax on short-term funding so as to further graduate the contribution of this group of banks to systemic externalities.

Going beyond the pure regulation of short-term funding, capital requirements—the most important regulatory instrument in banking—can be seen as a way to directly influence gambling incentives and, hence, the distribution of θ_2. Strengthening capital requirements, by ensuring shareholders internalize a larger part of the lower tail of the returns generated by the banks, will tend to shift the probability distribution of θ_2 towards lower values, making it more concentrated. This allows us to predict that in a scenario with stronger capital regulation there is greater room for having a tax on short-term funding as part of the second best regulatory mix.

9 Discussion and robustness

9.1 Dealing with heterogeneity in systemic importance

Suppose that factors such as interconnectedness, lack of susbtitutability, centrality or size makes some banks more "systemically important" than other in the very sense that the per-unit contribution of their short-term funding to the systemic risk measure X is larger than for other banks. Suppose in particular that systemic importance is captured by a new dimension of heterogeneity θ_3 which only enters significantly into the equations of the economy through the following extended measure of systemic risk:

$$X = \int_0^1 \int_0^1 w(\theta_3) x(\theta_1, \theta_3) f(\theta_1, \theta_3) d\theta_1 d\theta_3,$$

where $w(\theta_3)$ is the systemic risk weight of the banks of class θ_3.

Extending our characterization of competitive equilibria (unregulated or with taxes) and the socially optimal allocation to deal with this case is immediate. Moreover, it can be shown that decentralizing the socially optimal allocation as a competitive equilibrium with taxes will only require setting $\tau(\theta_3) = \tau^* w(\theta_3)$, where $\tau^* = E_z(\varepsilon(x^*(z), z))c'(X^*)$ is a reference rate set exactly like in (10), except because z should now be interpreted as the vector (z_1, z_3) of individual bank characteristics. So the presence of heterogenous systemic importance simply leads to the need to consider each bank's systemic importance measure $w(\theta_3)$ in scaling up the reference tax rate τ^*. But $\tau(\theta_3)$ preserves the key property of being not directly dependent on the individual value of each bank's lending opportunities as measured by θ_1.

10 Conclusions

We have developed a formal analysis of the relative performance of realistic price-based and quantity-based approaches to the regulation of systemic externalities associated with banks' short-term funding. The analysis suggests that, if the return to the lending (or investment) activities undertaken by the banks using this funding is heterogeneously distributed across banks (or, similarly, over time), a Pigovian tax on short-term funding will dominate a net stable funding ratio or a liquidity coverage ratio. If some (poorly capitalized or low charter value) banks have strong gambling incentives and expand their activity as a way to shift risk to outside stakeholders (e.g. the deposit insurer), quantity requirements may have better properties. In general terms, an optimal regulatory design may combine price and quantity-based instruments, and the emphasis on each of them will depend on what is the dominant dimension of heterogeneity across banks (or variation over time).

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