

Back to Basics: Private and Public Investment in Basic R&D and Macroeconomic Growth*

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Still Preliminary and Incomplete...

Abstract

This paper introduces endogenous technical change through basic and applied research in a growth model. Basic research differs from applied research in two significant ways. First, significant advances in technological knowledge come through basic research rather than applied research. Second, these significant advances could potentially be applicable to multiple industries. Since these applications are not immediate, the innovating firm cannot exploit all the benefits of the basic innovations for production. We analyze the impact of this appropriability problem on firms' basic research incentives in an endogenous growth framework with private firms and an academic sector. After characterizing the equilibrium, we estimate our model using micro level data on research expenditures and behavior by French firms. We then decompose the aggregate growth by the source and type of innovation. Moreover, we quantitatively document the size of the underinvestment in basic research and consider various research policies to alleviate this inefficiency. Our analysis highlights the need for devoting a larger fraction of GDP for basic academic research, as well as higher subsidy rates for private research.

Keywords: Innovation, basic research, applied research, research and development, government spending, endogenous growth, spillover.

JEL classification: O31, O38, O40, 043, 047, L78

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1 Introduction

This paper focuses on the distinct features of basic and applied research in the context of macroeconomic growth. Basic research is defined by the NSF as a “systematic study to gain more comprehensive knowledge or understanding of the subject under study without specific applications in mind. Although basic research may not have specific applications as its goal, it can be directed to fields of current or potential interest. This focus is often the case when performed by industry or mission-driven federal agencies”. Applied research is, on the other hand, defined as a “systematic study to gain knowledge or understanding to meet a specific, recognized need. In industry, applied research includes investigations to discover new scientific knowledge that has specific commercial objectives with respect to products, processes, or services”.¹ This distinction raises many policy-relevant questions.

What are the key roles of basic and applied research for productivity growth? What are the incentives of private firms to do basic research? How does academic research contribute to innovation and productivity growth? Given the distinct natures of academic and corporate research, what are the appropriate government policies to promote economic growth through technological progress? These questions have long been at the heart of industrial policy debates.

In many countries national funds allocated to basic research have been among the top items in governments’ policy agenda. For instance, in a recent report by the US Congress Joint Economic Committee, it is argued that despite its value to society as a whole, basic research is underfunded by private firms precisely because it is performed with no specific commercial applications in mind. The level of federal funding for basic research is “worrisome” and it must be increased to overcome the underinvestment in basic research (JEC, 2010) . However the report also complains about the lack of available studies on the subject to measure this underinvestment and the lack of available data.²

Although the different characteristics of basic and applied research on the one hand and academic and corporate research on the other hand have been widely recognized to be of first-order importance by policymakers, these issues have received insufficient attention from the economic literature on productivity and economic growth. In particular, the endogenous growth literature pioneered by Romer (1986, 1990), Lucas (1988), Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom, Anant and Dinopolous (1990) has mainly considered a uniform type of (applied) research and with a few exceptions³ overlooked basic

¹<http://www.nsf.gov/statistics/seind10/c4/c4s.htm#sb2>

²http://jec.senate.gov/public/?a=Files.Serve&File_id=29aac456-fce3-4d69-956f-4add06f111c1

³Some paper have modeled public basic research by government institutions. See for example, Aghion Howitt

research investment by private firms.

A satisfactory framework for the study of the above listed questions must allow for different types of research investments and model the related incentives explicitly. We strongly believe that firm-level studies would greatly contribute to our understanding of these questions. Our goal in this project is to take a first step towards developing this theoretical framework, to estimate it using micro-level data, and to discuss the effects of different research policies on productivity and growth. To the best of our knowledge, this would be the first study to model private investment into basic and applied research simultaneously as well as government investment into basic research in an endogenous growth framework and the first study on basic research that combines a structural model with micro-level evidence.

Our analysis proceeds in three steps. We first document some important empirical facts on basic research. Second, motivated by those empirical facts we propose a simple multi-industry framework with firms and an academic sector. Firms conduct both basic and applied research whereas the academic sector mainly focuses on basic research.⁴ In our model, basic research generates fundamental technological innovations and potential spillovers for subsequent applied innovations.⁵ Applied research on the other hand, advances these new technologies further. After characterizing the equilibrium of the model, we estimate the structural parameters. Finally, we use the estimated model to *(i)* assess the extent of underinvestment in basic and applied research, *(ii)* determine the contribution of innovations by the academic sector and private firms to aggregate growth, and *(iii)* study the implications of several important R&D policies.

Our analysis highlights the central role played by basic research in economic growth. Two distinguishing features of basic research are that it generates significant advances in technological knowledge and it is likely to generate substantial externalities. Indeed the scientific knowledge generated by basic research can potentially be of use in more than a single field (Nelson, 1959; Rosenberg, 1990; Dasgupta and David, 1994). Ideally, in order to capture the full return from this new scientific knowledge in industries where it could have an application but in which the innovating firm is not present, the innovator must first patent and then license or sell the innovation to other firms in those industries. However, the applications of basic scientific advances are often not immediate and firms are often only able to transform them into

(2009), Morales, Gersbach et al (2008), Narin et al. 1997, Mansfield 1998, Martin 1998, Park (1998)

⁴Trajtenberg et al. (1992) show that academic research is more basic and less frequently patented.

⁵By fundamental innovation, we mean the major technological improvements that generate a much bigger contribution to the aggregate knowledge stock in a society relative to the rest of innovations, and also have long-lasting spillover effects on the quality of subsequent innovations within the same field.

patentable applications in their own industries. This is the well-known *appropriability problem* of basic research which is initially discussed in Nelson (1959) and again in Rosenberg (1990) and Dasgupta and David (1994). Hence, firms operating in more industries will have a greater probability of being able to directly utilize more facets of a basic innovation. As Nelson (1959) puts it, “It is for this reason that firms which support research toward the basic-science end of the spectrum are firms that have *fingers in many pies*” (pp. 302). Note that the key concept that we are emphasizing here is not firm size per se, but the diversity of its products, which is also highly related to the width of its technological base. A broader range of industries increases the potential share of the total benefit that the innovating firm can enjoy.

We provide empirical evidence consistent with Nelson’s hypothesis. In the theoretical framework, we will explicitly allow for spillovers across industries. The range of industries a firm is present in will be one of the crucial determinants of its basic research investment. In our estimation, we find strong micro-level evidence of this pattern.

An important contribution of our study is to introduce academic basic research into a structural growth model and quantify their contribution to the economic growth process. Our results suggest that this effect is very sizable and around 25% of growth is attributable to academic basic research.

Another important contribution is the analysis of different research policies that are commonly considered by policymakers. The first policy of interest is a research subsidy to private firms. This policy has been analyzed by various studies (Aghion and Howitt, pp 487), yet the findings have been mixed. These results come from models with a single type of research. Once the distinction between basic and applied research is introduced, the results can differ greatly. We show that in an economy with both types of research efforts, the major underinvestment is in basic research due to its very sizable spillovers. In this environment subsidizing overall private research is less effective since this policy would oversubsidize applied research which is relatively less underinvested. Therefore the welfare improvement from a uniform subsidy is limited unless the policymaker is able to discriminate between types of research projects at the firm level, which is considered to be quite impractical.

Another policy that we consider in this paper is the provision of funding to academic institutions. Although academic research could benefit society through other channels, our focus in this project is its effect on new employment and product creation. This policy itself improves the basic research level in the society yet does not promote applied research directly. Therefore a policy that combines both private subsidy and public funds to academic research can govern the distinct nature of basic and applied research. Our quantitative analysis shows

that the decentralized economy achieves a 56% consumption equivalent to the social planner's optimum. Using only a uniform research subsidy, we can increase this number to 75%, and through combining a uniform subsidy with an increased share of GDP allocated to academic research funding we can further increase it to 96%.

Before we introduce our theoretical framework, we provide some empirical facts to motivate our modeling approach.

2 Empirical Facts

Despite the vast literatures in industrial organization and endogenous growth focusing on applied research, the literature on firms' basic research decisions has been very thin and empirical studies are even rarer. Notable exceptions include Mansfield (1980, 1981), Griliches (1986) and Link (1981) which empirically document the positive contribution of basic research to firms' productivity. Existing studies on basic research have been mainly theoretical and devoted their attention to academic/public research as the source of basic research and ignored the *private* side of it (Segerstrom, 1998; Aghion and Howitt, 1996, Morales, 2004). We believe that part of the reason for this outcome, as argued in one Congressional report (JEC 2010), has been the lack of firm-level data on distinct types of private research. Our empirical evidence contributes to the literature on innovating firms (eg. Klette and Kortum (2004) Lentz and Mortensen (2008), Akcigit and Kerr (2010), Acemoglu and Cao (2010)), characterizing the innovation and firm dynamics of R&D conducting firms.

In this paper we use unique data on the French economy combining information not only on product markets and R&D characteristics of individual firms, but also on firm ownership status for the period 2000-2006. R&D information comes from an annual survey conducted by the French Ministry of Research that covers a large, representative cross-section of innovating French firms. Details regarding data sources are provided in section 4.3.

The next section presents the main empirical facts emerging from this data.

2.1 Basic and Applied Research

First we document that private firms' investment in basic research forms a non-negligible fraction of both total private research spending and total basic research spending.⁶ Table 1

⁶Public statistics define basic research expenditures following the Frascati Manual: "Investment into basic research is undertaken either for pure scientific interest or to bring a theoretical contribution to the resolution of technical problems." Alternatively: "The objective of applied research is to identify the potential applications of results from fundamental research or to find new solutions to a precisely identified problem."

reports official statistics from the Ministry of Research on public investment in basic research and private applied and basic research investments for the period 2000-2006 in France. Private

Table 1: EXPENDITURES INTO BASIC R&D

Year	Private		Public	Private	Basic
	Basic	Applied	Basic	<u>Basic</u> <u>Applied</u>	<u>Private</u> <u>Public</u>
2000	802	7005	6425	.11	.13
2001	795	7748	6786	.1	.12
2002	959	8899	7037	.11	.14
2003	1092	8928	7133	.12	.15
2004	1175	9482	7338	.12	.16
2005	1227	9469	7331	.13	.17
2006	1213	10278	7755	.12	.16

Notes: Official R&D expenditures into basic R&D in Million of Euros for the period 2000-2006. Source: Ministry of Research.

spending on basic research amounted to an average of 1 billion Euros per year as opposed to 8.3 billion on applied research for the period 2000-2006. During the same period, public expenditures on basic research represented an average of 7 billion Euros per year in France. This implies that more than 11% of private research is spent on basic research. More importantly, almost 15% of total basic research in the economy is undertaken by private entities.⁷

The picture that emerges therefore hints at a significant involvement of the private sector in undertaking basic scientific research. Thus, ignoring the private incentives behind basic research might prevent economists and policymakers from designing more effective policies for productivity growth.

Table 2 documents the link between firm size, in terms of total employment, and unconditional basic research intensity using our sample of innovating firms from the R&D survey.⁸ Basic research intensity is defined as the ratio of total firm investment into basic research divided by total firm investment into applied research.⁹

Except for the category of micro-firms we find a positive relationship between basic research intensity and employment. Indeed, basic research intensity increases from an average of .056 for small and medium-sized firms to .068 for large and .085 for very large firms. This means that on average basic research represents 8.5% of applied research in very large firms.¹⁰ Micro-firms

⁷Similarly, Howitt (2003) using a NSF survey finds that around 22% of all basic research in the US during the period 1993-1997 was performed by private enterprises.”

⁸Firm total employment is defined as the aggregate employment of all the divisions and subsidiaries of a firm. Similarly basic and applied research are defined as the total investment of all the divisions and subsidiaries into basic and applied research respectively.

⁹Employment categories are constructed on the basis of firm classifications established by the Ministry of Research (Dhont-Peltrault and Pfister, 2009).

¹⁰Note that figures computed on the basis of the R&D survey exclude observations falling outside of a multiple

Table 2: BASIC RESEARCH INTENSITY AND SIZE

	Total Employment (X)			
	$X \leq 10$	$10 < X \leq 249$	$249 < X \leq 4999$	$4999 < X$
Basic Research Intensity	.078 (.22)	.056 (.18)	.068 (.2)	.085 (.2)
N	[1835]	[7185]	[4091]	[593]

Note: The table reports the mean of *Basic Research Intensity* as a function of *Total employment*. *Total Employment* categories are constructed on the basis of a firm classification established by the Ministry of Research (2009). Standard errors are reported in parenthesis and the number of observations in square brackets.

constitute the exception to this pattern but are in many dimensions not directly comparable to more established firms.¹¹

One possible explanation for heterogeneous levels of basic research investment amongst firms is the presence of certain unobservable characteristics that affect a firm’s return to basic research. For instance, one might think that high profit margin firms are the ones that are investing more heavily in basic research. In this scenario, firms with higher basic research intensity would be observed to have higher profitability. However, this is not what we observe in the data. Table 3 reports mean basic research intensity as a function of firm profitability. Firm profitability is defined as an employment weighted mean of individual establishments return on assets. Profitability categories are defined on the basis of quartiles of the distribution. We find no clear relation between profitability and basic research intensity. On average the

Table 3: BASIC RESEARCH INTENSITY AND PROFITABILITY

	Profitability - ROA			
	1 st Quartile	2 nd Quartile	3 rd Quartile	4 th Quartile
Basic Research Intensity	.062 (.18) [3425]	.064 (.19) [3426]	.066 (.19) [3426]	.064 (.2) [3426]

Note: The table reports the mean of *Basic Research Intensity* as a function of a firms *ROA*. Standard errors are reported in parenthesis and the number of observations in square brackets.

least profitable firms in our sample have a basic research intensity of .062. This ratio seems only marginally lower when compared to the basic research intensity of the most profitable firms. In addition, the increase in basic research intensity is non-monotonic in profitability categories since firms in the third quartile have a basic research intensity of .066.

of five times the interquartile range in terms of total R&D to sales, total basic to total applied research investment as well as return on assets.

¹¹Amongst other reasons, because of special fiscal and administrative regulations applying to them. Moreover total investment into basic research by micro-firms represents less than .5% of total private investment into basic research.

2.2 Basic R&D Intensity and Multi-Industry Presence

Next we focus on private incentives for basic research. More specifically, we test Nelson’s hypothesis that the main basic research investors would be those firms that have *fingers in many pies*. According to this argument, as the range of its products and industries gets more diversified, a firm’s incentive for investing into basic research relative to applied research should increase due to better appropriability of potential knowledge spillovers.

Figure 1 plots average basic research intensity against the total number of distinct 1 digit SIC codes in which the firm is present together with a simple linear fit of the data.¹²

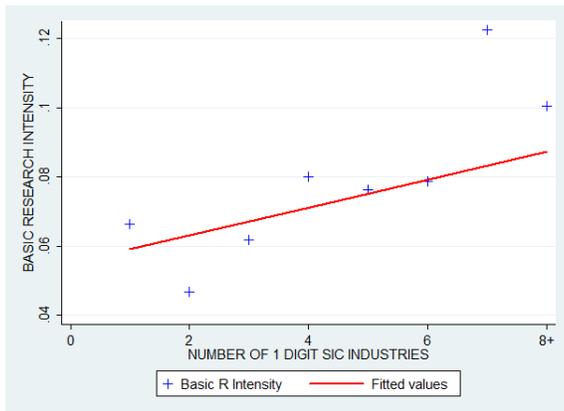


Figure 1: BASIC RESEARCH INTENSITY AND NUMBER OF INDUSTRIES

As shown, a simple linear fit of the data suggests a positive and statistically significant relationship between the two variables.

In the appendix Table A1 provides further multivariate evidence about the relationship between multi-industry presence and basic research intensity. Table A1 estimates a Tobit model correlating a firm’s basic research intensity with its multi-industry presence.

[TABLE A1 HERE]

To measure multi-industry presence, we count how many distinct SIC codes a firm is present in. However, at the very disaggregate level of the SIC classification, we run the risk of counting in the same way very diverse and very similar industries. Therefore we successively define industries at 4,3,2 and 1 digit SIC levels but consider as our benchmark case distinct 1 digit SIC codes. Other controls include the size of the firm as measured by its total employment, nationality, profitability, and market share. In all alternative specifications basic research

¹²Note that thanks to the combination of the LIFI and EAE data we are able identify multiple SIC codes for both business groups and multidivisional firms.

intensity is increasing in the number of industries. According to the benchmark estimation, presence in an additional industry increases a firm's basic research intensity by 3 percentage points on average.

Alternatively, the technological base of a firm can be proxied through the firm's patent portfolio since patents are closely related to technologies that firms use. Therefore, in Table A2 (see appendix) we alternate our proxy for the technological base by using patent classifications instead of SICs.¹³

[TABLE A2 HERE]

Again estimates are positive and statistically significant, a broader technological base being associated with higher investment into basic research relative to applied research.

We thus find correlations that are supportive of Nelsons' hypothesis about the link between multi-industry presence and relative research incentives. We check in non-reported regressions that these correlations are economically significant and robust to (i) alternative statistical models and estimation methods, (ii) alternative specifications of both the dependent and right hand side variables of interest, (iii) a variety of potential confounding factors. In addition we also provide evidence suggesting that firm scope impacts basic research intensity, and not the reverse.

2.3 Firm Size and Multi-Industry Distributions

Another stylized fact emerging from the data is the extent of multi-industry presence of French firms. Figure 2 plots the empirical distribution of the number of distinct 1-digit SIC codes in which firms operate. On average firms are present in 2 distinct industries as defined by 1-digit

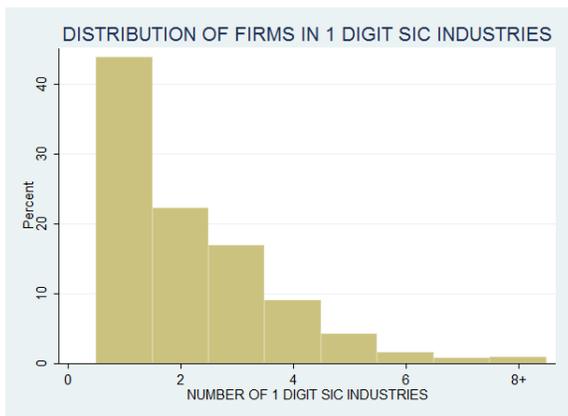


Figure 2: DISTRIBUTION OF FIRMS BY NUMBER OF INDUSTRIES

¹³Note that patent information is only available for the period 2000-2003.

SIC codes. Although nearly 44% of the firms are operating in only one industry, the remaining firms occupy a large spectrum of industries.

Another moment that we would like to replicate for the French economy is the firm size distribution (FSD). An extensive literature has studied the FSD in the US and other economies. Consistent with that literature, our data also documents a very highly skewed firm size distribution for French firms. Additionally, the relationship between number of industries and firm size is of special interest in our study. We find a strong positive correlation between these two variables. Figure 3 plots the FSD for the data from 2000-2006, while Figure 4 shows the distribution of firm size broken down by the number of industries the firm operates in.

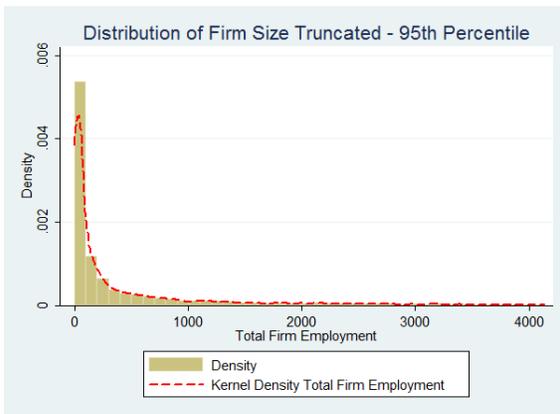


Figure 3: FIRM SIZE DISTRIBUTION

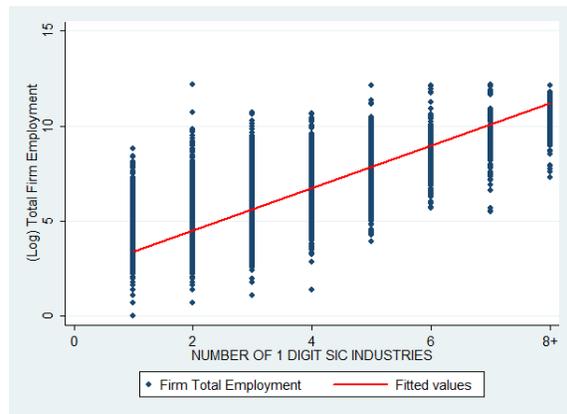


Figure 4: EMPLOYMENT BY # OF INDUSTRIES

Overall, we can summarize the above analysis into a list of stylized facts that highlight the most salient features of the data:

Fact 1 *A significant fraction of innovating firms invest in basic research (27%).*

Fact 2 *Private basic research investment forms a significant portion of total basic research (>15%) and total private research (>10%).*

Fact 3 *Firm size is positively correlated with basic research intensity.*

Fact 4 *Firm profitability is not significantly correlated with basic research intensity.*

Fact 5 *A large fraction of firms operate in multiple industries (56%).*

Fact 6 *Nelson's hypothesis is supported: basic research intensity is positively correlated with multi-industry presence.*

Fact 7 *Multi-industry presence is positively correlated with firm size.*

Fact 8 *The firm size distribution is highly skewed.*

The next section uses the insights from the micro-data in order to construct a theoretical framework to evaluate the impact of alternative innovation policies on aggregate growth and welfare

The remainder of the paper is organized as follows. Section 3 introduces the theoretical model, section 4 estimates the model, section 5 evaluates alternative R&D policies and section 6 concludes.

3 Model

This section introduces a theoretical framework that has three distinct features relative to the standard endogenous growth models. First, the model distinguishes between basic and applied research investment, the key difference being the spillover effects associated with the former. Second, the model features a multi-industry framework that will allow us to discuss the cross-industry spillover of basic research. Finally, we also introduce academic institutions as another source of basic research in the economy. This last feature will allow us to discuss the contribution of academia to economic growth. The proposed theoretical framework will be consistent with the stylized facts described in the previous section. We will then use this framework to evaluate alternative R&D policies.

3.1 Preferences

Consider the following continuous time economy. There is a representative household with constant relative risk aversion preferences given by

$$U = \int_0^{\infty} \exp(-\delta t) \frac{C_t^{1-\gamma} - 1}{1-\gamma} dt \quad (1)$$

where $\delta > 0$ is the discount factor, $\gamma \geq 1$ is the inverse of the intertemporal elasticity of substitution or the coefficient of relative risk aversion, and C_t is the final good consumption at time t . The household consists of individuals of measure 1 who supply their one unit of labor

inelastically. The household's objective is to maximize (1) subject to the following budget constraint

$$C_t + \dot{A}_t \leq r_t A_t + W_t - T_t$$

where A_t , W_t and T_t are total asset holdings, labor income and lump-sum tax in terms of the final good units and r_t is the interest rate. For notational convenience, the time subscripts will henceforth be suppressed.

Production is divided into three major components: Downstream, midstream and upstream sectors. The household consumes the final good Z which is produced in the downstream sector by infinitely many competitive firms that combine inputs from M different industries according to the constant elasticity of substitution production function,

$$Z = \left[\sum_{i=1}^M Y_i^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where Y_i is the aggregate output from industry $i \in \mathcal{I}$ and \mathcal{I} is the set of industries. To map this structure to reality, the reader can think of each i as a different Standard Industrial Classification (SIC) code. We normalize the price of the final good P_t to 1. Since there is no capital and all costs are in terms of labor units, the resource constraint of the economy is simply $Z = C$.

Next we turn to the midstream production of Y_i . Each industry aggregate output Y_i is produced by a monopolist i that combines inputs from a continuum of product lines (quality ladders) of measure 1. Let y_{ij} denote the production of intermediate good j in industry i . Aggregate output in industry i is produced according to the following production function

$$\ln Y_i = \int_0^1 \ln \left(\sum_{f \in \mathcal{F}} q_{ijf} y_{ijf} \right) dj \quad (2)$$

where q_{ijf} is the quality of intermediate good $j \in [0, 1]$ and f is the patent holder for that quality. This structure implies that the total measure of product lines in the economy is M .

In the upstream sector, firm f who owns the patent for q_{ijf} has the right to produce y_{ijf} . The specification in (2) implies that in product line j of industry i , the *quality-adjusted* products of any two firms $f, f' \in \mathcal{F}$ are perfect substitutes. Each intermediate good is produced according to a linear production function

$$y_{ijf} = \varphi l_{ijf} \quad (3)$$

where $\varphi > 0$ is the labor productivity and l_{ijf} is production workers employed. Let us denote the wage rate in the economy by w in terms of the final good. The above specification (3)

implies that each product y_{ijf} has a constant marginal cost of production $w/\varphi > 0$, which is proportional to the wage rate in the economy. Given these specifications, in any product line j , the product that yields the highest quality per dollar will capture the market through limit pricing at the previous leader's marginal cost.

Firm f owns $n_f \in \mathbb{Z}_+$ product lines in any given industry i . Therefore we will denote the portfolio of firm f in industry i by a multiset¹⁴

$$\mathbf{q}_{if} = \{q_{if}(1), q_{if}(2), \dots, q_{if}(n)\}.$$

Similarly firm f can operate in multiple industries ($m_f \geq 1$). Let \mathcal{I}_f and \mathcal{F}_i denote the set of industries in which firm f operates and set of firms that operate in industry i .

3.2 Research and Innovation

3.2.1 Private Firms

Firms innovate by investing in two types of research: *basic* and *applied*. In accordance with Nelson's (1959) description, significant advances in technological knowledge come through basic innovation in our model.

Basic and Applied Research Technologies Innovation through *basic research* introduces a new generation of technical knowledge. Let $\bar{q}_{ij}(t) \equiv \max_{f \in \mathcal{F}} q_{ijf}(t)$ be the best quality of product j at time t . When firm f produces a basic innovation that has a direct application in industry $i \in \mathcal{I}_f$ and product line j , firm f utilizes this basic knowledge on a product in j and patents this new high-value product. As a result, firm f improves $\bar{q}_{ij}(t - \Delta t)$ by η

$$q_{ijf}(t) = (1 + \eta) \cdot \bar{q}_{ij}(t - \Delta t). \quad (4)$$

and generates per-period profit of $\pi(\eta)$ where $\eta > 0$. Moreover basic research features potential spillovers: *within-industry* and *cross-industry*. Within-industry spillover will be explained in the following paragraph and cross-industry spillover will be introduced in the next subsection.

Applied research, on the other hand, makes use of the *within-industry* spillover from basic research and builds on the existing latest basic technological knowledge in the product line. The contribution of each applied innovation to the technology stock is a function of how depreciated the latest basic technology is. For instance, if the latest technology (say touchscreen technology) is relatively new, then building on it through applied research (which results in

¹⁴A multiset is a generalization of a set which can contain more than one instances of the same member. For instance, let $j \neq j'$. Then a multiset \mathbf{q}_{if} can contain $q_{if}(j), q_{if}(j') \in \mathbf{q}_{if}$ and $q_{if}(j) = q_{if}(j')$.

introducing iPhone, for instance) yields a higher per-period return than building on an old technology.¹⁵ If the latest basic technology is undepreciated, a successful applied innovation will benefit from it and improve the latest quality $\bar{q}_{ij}(t - \Delta t)$ of the product line by η as in (4). If the latest basic technology of the product line is depreciated, a successful applied innovation will improve the latest quality only by $\lambda < \eta$ such that $q_{ijf}(t) = (1 + \lambda) \cdot \bar{q}_{ij}(t - \Delta t)$. We assume that a new basic technology depreciates at a Poisson rate $\zeta > 0$. As a result, $1/\zeta$ is simply the intensity of the within-industry spillover. We will denote the share of product lines with an undepreciated latest basic technology in industry i by $\Psi_i \in [0, 1]$.

Firm f with n_{if} products in industry i chooses the arrival rate of applied (A_{if}) and basic (B_{if}) research through employing researchers. Let

$$H_a(A_{if}, n_{if}) \equiv n_{if} h_a(A_{if}/n_{if}) \text{ and } H_b(B_{if}, n_{if}) \equiv n_{if} h_b(B_{if}/n_{if}) \quad (5)$$

denote the number of researchers that firm f needs to hire in order to generate the Poisson flow rates of A_{if} and B_{if} . The above specifications which are standard in this class of models (See Klette and Kortum, 2004; Lentz and Mortensen, 2008, Acemoglu et al., 2010) capture the idea that a firm's knowledge capital facilitates innovation.¹⁶ In (5), $h_a(\cdot) : [0, a^{\max}] \rightarrow \mathbb{R}_+$ and $h_b(\cdot) : [0, b^{\max}] \rightarrow \mathbb{R}_+$ are strictly increasing, convex, differentiable, continuous functions that satisfy: $h_a(0) = h'_a(0) = h'_b(0) = 0$, $h'_a(a^{\max}) = h'_b(b^{\max}) = \infty$ for some $a^{\max}, b^{\max} \in \mathbb{R}_+$. Let us define $a_{if} \equiv A_{if}/n_{if}$ and $b_{if} \equiv B_{if}/n_{if}$ as the applied and basic *innovation intensities*. In addition, basic research features an instantaneous fixed cost $h_f^b > 0$ in terms of labor units that firm f draws instantaneously from a distribution $\mathcal{F}(h^b)$ each period before making its basic research decision. This implies that firms that draw a cost above a certain threshold will not invest in any basic research at that instant t .

As a result, we can summarize the cost of doing applied and basic research as,

$$C_a(a_{if}, n_{if}) = w n_{if} h_a(a_{if}) \text{ and } C_b(b_{if}, n_{if}) = w n_{if} \left[h_f^b + h_b(b_{if}) \right] \quad (6)$$

where w denotes the wage rate in the economy.

Both applied and basic research are *directed* towards particular industries but *undirected* within those industries. In other words, once a firm chooses A_{if} and B_{if} , the realization of innovations will take place within industry i_f .

¹⁵The reader can predict at this stage that there will be underinvestment in basic R&D due to the fact that firms do not internalize their positive externality on subsequent innovators who use the same new technology. This is part of the real-life reason why societies need universities to do basic research. Without them, the equilibrium investment in basic R&D will not be socially optimal. We will introduce the academic sector below in section 3.2.2.

¹⁶It also simplifies the analysis by making the problem proportional to the number of product lines.

Cross-Industry Spillover from Basic Research In addition, basic research features an additional element of uncertainty due to random spillovers into other industries. When basic research by firm f in industry i is successful, the resulting new scientific knowledge will be applied by firm f on a product to increment the productivity of a random product in the target industry. The novel feature of basic research is that it is likely to generate substantial externalities that would result in significant advances in other fields (Nelson, 1959). Therefore we will assume that a basic innovation $\eta_{ijf}(t)$ that is produced by firm f at time t in industry i applies to another industry with probability $p \in (0, 1)$ which is simply the measure of cross-industry spillover. Let $\mathbf{1}_{\eta_{ijf}}^{i'}$ be an indicator function that takes a value of 1 if the basic innovation in product line j by firm f in industry i has an application in industry i' as well and 0 otherwise. Then we have

$$\Pr\left(\mathbf{1}_{\eta_{ijf}}^{i'} = 1\right) = \begin{cases} \frac{p}{M-1} & \text{if } i' \neq i \\ 1 & \text{if } i' = i \end{cases} . \quad (7)$$

The spillover of basic research has the same implication in industry i' and increases the innovation size of a random product line j' to η . This new innovation will be utilized by the same firm f if it is already present in i' . Otherwise the production potential of this innovation will be used by the incumbent firm in j' .¹⁷ We provide these details next.

When a firm generates basic knowledge, it can turn this into an immediate application only in the sectors that it is present. Nelson (1959) observes that in order to capture the full return from new basic scientific knowledge in industries where a firm is not present but the knowledge could have an application, the innovating firm must first patent and then license or sell the innovation to other firms in those industries. However, the applications of significant scientific advances are often *not immediate* and firms can turn them into patentable applications mostly in their own industries due to their expertise and experience in them. Analogous to this real-life observation, the innovating firm f in our model will utilize the basic scientific knowledge on a new product in industry i' if $\mathbf{1}_{\eta_{ijf}}^{i'} = 1$ and $i' \in \mathcal{I}_f$. Otherwise if $\mathbf{1}_{\eta_{ijf}}^{i'} = 1$ and $i' \notin \mathcal{I}_f$, the new basic knowledge will be applied by a random firm f' in i' on a new product j' in order to generate a profit of $\pi(\eta)$.

Let m_f denote the number of industries in which firm f is present. Then the probability of a utilized spillover for firm f is

$$\rho_{m_f} \equiv \frac{p(m_f - 1)}{M - 1} \in [0, 1)$$

¹⁷Firms investing into basic research face *appropriability problems* because applications from this type of research are often not immediate and firms are only able to transform them into patentable applications in their own industries. The reader is referred to the discussion of Nelson's hypothesis (1959) in the introduction.

Note that this structure highlights the well-known *appropriability problem* of basic research. There is a significant chance that the new basic knowledge will be relevant to multiple industries, but it is not always clear that a firm will be in a position to exploit all of these avenues of production and patenting. However, firms operating in more industries will have a greater probability of being able to directly utilize all facets of a basic innovation. As Nelson puts it, firms that have *fingers in many pies* have a higher probability of utilizing the return to basic research. A broad technological base increases the probability of benefiting from the basic research outcome.

3.2.2 Academic Sector

One of the main tasks of the academic sector in an economy is to conduct the necessary research to generate the basic scientific knowledge that will also be part of the engine for subsequent applied innovations and growth. In our model, the academic sector will be the other source of basic knowledge creation. We assume that the academic sector consists of a measure U of research labs per industry each of which is index by $u \in \mathcal{U}$. Each lab receives an instantaneous transfer of $\hat{T}_u = \hat{T}$ from the government to finance its research. We assume that the academic sector operates with the same basic research technology to (6),

$$\tilde{C}_b(d_u) = w \left[\bar{h}^b + h_b(d_u) \right].$$

In this specification d_u is the basic innovation flow of lab u and \bar{h}^b is the fixed cost which we assume to be equal to the average of private firms' fixed cost $\bar{h}^b = \int h^b d\mathcal{F}(h^b)$. This specification implies that the government can affect the basic research output through its funds allocated to the academic sector such that

$$d_u = h_b^{-1} \left(\frac{\hat{T}/(MU)}{w} - \bar{h}^b \right) \quad \text{and} \quad \tilde{d}_u = U d_u \quad (8)$$

where \tilde{d}_u is the academic basic innovation flow per product line. In this economy, resources allocated to the academic sector \hat{T} is a tool controlled by the policymaker. Similar to the basic innovation by the private firms, each of the d_u scientific knowledge by the academic sector has an immediate application in a random industry i and product line j . In addition to i , the same basic knowledge will have an immediate application in another industry $i' \neq i$ and line j' with probability $p \in (0, 1)$. These basic technologies will be used in production by the incumbent firms in j and j' . This implies that the latest qualities in j and j' increase by η and the profit of the incumbent firm becomes $\pi(\eta)$.

At this point, it would be useful to express the total rate of basic innovation that will benefit a producer other than the original innovator,

$$s = \frac{1}{M} \left((1+p) \int_{\mathcal{U}} d_u du + \int_{\mathcal{F}} (p - \rho_{m_f}) b_f df \right). \quad (9)$$

Here the spillover is the sum of academic basic innovation (first integral) and the sum of unappropriated portion $(1 - \rho_{m_f})$ of the spillover from basic research b_f (second integral).

3.3 Entry and Exit

Entry There is a set of potential entrants of measure χ per industry. In order to enter, entrant firms invest in applied research at the cost $h_a(\hat{a})$ in terms of the labor units and innovate a new single product in the economy at a flow rate of $\hat{a} > 0$. This new product is realized with the same probability across industries. The equilibrium applied innovation flow rate of a potential entrant is determined by

$$\max_{\hat{a}} \{ \hat{a} \mathbb{E}_{ij} V(i, j) - wh_a(\hat{a}) \} \quad (10)$$

where $\mathbb{E}V(i, j)$ is the expected value of entering into industry i and innovating on a product line j .

Exit In this economy, firm exit happens in two ways. First, firms get replaced by their competitors through *creative destruction* which happens at the rate τ_i in industry i . When a firm loses all of its product lines, it exits the market.

Second, each incumbent firm f receives a destructive shock at the rate κ . When this happens, the firm shuts-down its production operations and exits the economy. Then each of its product lines are sold to another firm within the same industry at price $\nu_i \geq 0$. This way, each firm in the economy randomly receives a buy-out opportunity.

Buy-Outs We assume that the probability of receiving a buy-out option is proportional to the number of products that the firm has within the same industry and that the price paid is ν_i . As a result, the arrival rate of a new buy-out opportunity to firm f in industry i is $n_{if}\kappa$. This can be thought of as a simple way of modeling mergers and acquisitions in our framework. The quality of production of the acquired firm will be a function of the share of high quality product lines in the economy. Let this share be denoted by $\tilde{\Psi}$.

Industry Expansion Expanding into new markets have advantages, such as better appropriation of research investments and having more access to troubled firms. Therefore firms spend additional resources in order to be able to expand into new industries. Specifically firm f hires $h_e(e_f)$ workers in order to be able to penetrate into a new industry. This effort results with an expansion at the Poisson flow rate e_f .

This concludes the economic environment of the model except the labor market clearing condition. At this point, we find it useful to summarize in the following table the arrival rates in this economy.

NAME	GENERATED BY	RATE	EXPLANATION
1) applied innovation	firm f	a_f	used for production by f
2) basic innovation	firm f	b_f	used for production by f
3) spillovers of basic innovation	firm f	pb_f	used for production by f or some f'
4) academic basic innovation	academic lab u	d_u	used for production by some f
5) spillovers from academic basic innv	academic lab u	pd_u	used for production by some f'
6) outside applied innovation	outsider f	\hat{a}	used by entrant f
7) unappropriated spillover	firms and labs	s	computed from 3) and 5)
8) firm destruction/buy-out	exogenous	κ	product line bought by f
9) expansion	firm f	e_f	firm f expands if successful

3.4 Labor Market

The labor market clearing condition of this economy has to satisfy the following inequality

$$1 \geq \sum_{\mathcal{I}} \left[\int_{\mathcal{F}_i} \int_{\mathcal{J}} n_{if} \left(l_{ijf} + h_a(a_{ijf}) + h_b(b_{ijf}) + \mathbf{1}_{(b_f > 0)} h_f^b + h_e(e_f) \right) djdf + h_a(\hat{a}) \right] + \int_{\mathcal{U}} [\bar{h}_b + h_b(d_u)] du. \quad (11)$$

In each product line, the production takes place only by the latest innovator. Therefore demand for production workers come only from the technological leaders in each product line which is represented as the first term inside the parenthesis. On the other hand, demand for researchers comes from incumbents, outsiders and academic labs. Each active firm hires researchers to conduct applied research (second term) and basic research (third and fourth terms). Firms also hire workers to seek opportunities for industry expansions. The demand for research workers by the entrants is captured by the last term in the square bracket. Finally, academic sector employs additional researchers for the independent projects and this demand is expressed by the last two terms in (11). Labor market clearing condition requires that the total demand for labor should be less than or equal to the total labor supply in the economy which is 1 in this case.

3.4.1 Government

The government simply imposes a lump-sum tax T on the household to finance its spending \hat{T} on academic sector. We assume that the government follows a balanced budget such that $M\hat{T}_t = T_t \forall t$ where M is simply the measure of research labs.

This concludes the description of the model. The next section characterizes the equilibrium of this economy.

3.5 Equilibrium

Throughout, we focus on Symmetric Markov Perfect Equilibria (SMPE) where all allocations across the industries are the same and the strategies are only a function of the pay-off relevant state variables. Let $\tau^{\max} \equiv a^{\max}(1 + \chi) + b^{\max}(1 + p)$ be an upper bound for the creative destruction rate.¹⁸ Let us also define $\mathcal{Z} \equiv \{0, 1, \dots, M\} \times \mathbb{Z}_+^2 \times [0, \tau^{\max}] \times \mathbb{R}_+^3 \times [0, 1]^2$. Focusing on SMPE will allow us to express all strategies only as a function of $z \equiv (m, n_\eta, n_\lambda, \tau, Z, w, r, \Psi, \tilde{\Psi}) \in \mathcal{Z}$ and drop the firm, product line and industry identities (f, j, i) . Let $\Gamma_{m,n}^*$ denote the equilibrium share of *firms* that own n product lines and operates in m industries such that

$$\sum_{m=1}^M \sum_{n=1}^{\infty} \Gamma_{m,n}^* = 1.$$

Similarly we define $\tilde{\Gamma}_m^*$ as the share of *product lines* that are owned by m -industry firms such that

$$\tilde{\Gamma}_m^* = \frac{F^*}{M} \sum_{n=1}^{\infty} \Gamma_{m,n}^* n$$

and $\sum_{m=1}^M \tilde{\Gamma}_m^* = 1$. Since $F^* \Gamma_{m,n}^*$ is the mass of m -industry firms with n product lines, $F^* \Gamma_{m,n}^* n$ is the total number of product lines held by these firms. Summing those values over n and dividing by the total mass of products M then yields the share of products owned by m -industry firms $\tilde{\Gamma}_m^* \in [0, 1]$.

Definition 1 (Symmetric Allocation) Let $\left[T(t), \hat{T}(t) \right]_{t \geq 0}$ be the policy sequence. A **symmetric allocation** for this economy is defined as a sequence of pricing and production decisions $[p_z(t), y_z(t)]_{t \geq 0}$, a sequence of research and expansion decisions for incumbents $[a_z(t), b_z(t), e_m(t)]_{t \geq 0}$ and outsiders $[\hat{a}(t)]_{t \geq 0}$, a sequence of academic sector's research flows $[d(t)]_{t \geq 0}$, sequences of midstream sector's price and output and downstream sector's output $[P(t), Y(t), Z(t)]_{t \geq 0}$, wage rate $[w(t)]_{t \geq 0}$, interest rate $[r(t)]_{t \geq 0}$, sequences of distributions of

¹⁸ τ^{\max} is computed as the sum of maximum arrival rates of a , b and c , taking into account their spillovers.

product line type, and product lines in terms of their owners types $[\Psi(t), \tilde{\Psi}(t), \{\Gamma_{m,n}(t)\}_{m,n}]_{t \geq 0}$ and a sequence of firm measure $[F(t)]_{t \geq 0}$.

Our focus will be on symmetric steady-state where all aggregate variables grow at the same rate g . Let $Q_i(t) \equiv \exp\left(\int_0^1 \ln(q_{ij}(t)) dj\right)$ be the quality index of industry i . We will assume that all industries start with the same quality index $Q_i(0) = Q_0 \forall i$. Since we are focusing on steady state equilibrium, we will normalize all growing variables by Z/M , the average final product per industry, and denote these variables with “ $\tilde{\cdot}$ ” on top, such as $\tilde{X} \equiv XM/Z$. Similarly we define $\tilde{z} \equiv (m, n_\eta, n_\lambda, \tau, \tilde{w}, r, \Psi, \tilde{\Psi})$. Finally we denote all the equilibrium values with a star “ $*$ ”.

Before we move on to its characterization, we first define the equilibrium.

Definition 2 (Symmetric Steady-State Markov Perfect Equilibrium) *For any given policy (T, \hat{T}) , a symmetric steady-state Markov Perfect Equilibrium (SSSMPE) is given by a time-invariant tuple*

$$\{\tilde{Y}^*, P^*, y_z^*, p_z^*, a_z^*, b_z^*, e_z^*, \hat{a}^*, d^*, \tau^*, \Psi^*, \tilde{\Psi}^*, \Gamma_{m,n}^*, \tilde{w}^*, r^*, F^*, g^*\}$$

such that (i) each industry’s aggregate output \tilde{Y}^* and P^* solve the maximization problem of the midstream monopolist, (ii) upstream monopolist’s price and output decisions p_z^* and y_z^* solve its profit maximization problem, (iii) upstream monopolist’s research and expansion efforts a_z^* , b_z^* , and e_z^* maximize its expected firm value, (iv) outsider’s research effort \hat{a}^* maximizes the outsider firm’s expected value (v) academic basic innovation rate d^* is consistent with policy \hat{T} , (vi) creative destruction rate τ^* , the invariant distributions $\tau^*, \Psi^*, \tilde{\Psi}^*, \Gamma_{m,n}^*, F^*$ and the growth rate g^* are consistent with the research and expansion efforts $a_z^*, b_z^*, \hat{a}^*, e_z^*$ (vii) labor share \tilde{w}^* clears the labor market, and (viii) the interest rate r^* satisfies the household’s Euler equation.

3.5.1 Households

The standard maximization of the household leads to the usual Euler equations for CRRA preferences

$$\frac{\dot{Z}^*}{Z^*} = g^* = \frac{r^* - \delta}{\gamma} \quad (12)$$

3.5.2 Profits

Given these specifications, we can express the maximization of the final good sector,

$$\max_{Y_i \in \mathcal{I}} \left\{ \left[\sum_{m=1}^M Y_i \frac{\varepsilon-1}{\varepsilon} \right]^{\frac{\varepsilon}{\varepsilon-1}} - \sum_{m=1}^M P_m Y_m \right\}$$

where Y_i and P_i denote the production and price of the aggregate good of industry i . The optimal choice by the final good producer determines the demand for aggregate output of industry i

$$Y_i^* = Z^* / P_i^{*\varepsilon}. \quad (13)$$

Let the marginal cost of producing the industry aggregate Y_i be denoted by \hat{P}_i in terms of the final good. Taking the demand for her product as given, downstream monopolist i maximizes her profit as

$$\max_{P_i, Y_i} \left\{ (P_i - \hat{P}_i) Y_i \right\} \text{ subject to (13).}$$

As a result, the optimal price is a constant mark-up over the marginal cost

$$P_i^* = \frac{\varepsilon}{\varepsilon - 1} \hat{P}_i^*, Y_i^* = Z^* \left(\frac{\varepsilon - 1}{\varepsilon} \right)_i^\varepsilon \hat{P}_i^{*\varepsilon} \text{ and } \Pi_i^* = \left[\frac{\varepsilon - 1}{\hat{P}_i} \right]^{\varepsilon - 1} Z^* \varepsilon^{-\varepsilon}. \quad (14)$$

The cost minimization problem for the monopolist i is

$$\min_{y_{i,j} \in [0,1]} \int_0^1 p_{i,j} y_{i,j} dj \text{ subject to (2)}$$

where $y_{i,j}$ is the production of good j in industry i . Note that the specification in (2) requires the same expenditure on each product j , $p_{i,j} y_{i,j} = Y_i \hat{P}_i$. This, together with (14) yield the following relationship between prices

$$p_{i,j}^* y_{i,j}^* \varepsilon / (\varepsilon - 1) = P_i^* Y_i^*. \quad (15)$$

Next we turn to the upstream sector and zoom into industry i , product line j . Let q_{ijf} and $q'_{ijf'}$ be the highest and second highest quality goods in product line j , respectively. Since the second highest quality producer can steal the market if and only if the price is higher than its marginal cost, the highest quality producer will set the price equal to the closest competitor's marginal cost as follows,

$$\frac{q_{ijf}}{p_{ijf}^*} = \frac{q'_{ijf'}}{p_{ijf'}^*} = q'_{ijf'} \frac{\varphi}{w^*}$$

which yields

$$p_{ijf}^* = p_k^* = (1 + k) \frac{w^*}{\varphi}, \quad k \in \{\eta, \lambda\} \quad (16)$$

This expression implies that the price mark-up is an increasing function of the innovation size λ_k . Since the highest quality producer will price others out of the submarket, we can drop f subscripts from now on and label the only producer by j .

Symmetry across industries implies $P_i^* Y_i^* = Y^*/M$. Combining this with (15) and (16), we express the equilibrium quantity of each intermediate good as

$$y_{i,j}^* = y_k^* = \frac{\varphi(\varepsilon - 1)}{\tilde{w}^* \varepsilon (1 + k)}, \quad k \in \{\eta, \lambda\} \quad (17)$$

Note that the endogenous variables are only a function of the size of the latest innovation. Using (16), (17), the profit is simply

$$\tilde{\pi}_k^* = \frac{(\varepsilon - 1)k}{\varepsilon(1 + k)}, \quad k \in \{\eta, \lambda\} .$$

Then Y^* from (2) and (17) is simply

$$Y^* = \frac{Q\varphi(\varepsilon - 1)}{\tilde{w}^* \varepsilon (1 + \eta)^{\Psi^*} (1 + \lambda)^{1 - \Psi^*}}$$

where $Q_i \equiv \exp\left(\int_0^1 \ln q_{ij} dj\right)$ is the quality index of industry i . Similarly the final output in symmetric equilibrium ($Q_i = Q$ and $F_i(k) = F(k)$) is

$$Z^* = \frac{M^{\frac{\varepsilon}{\varepsilon - 1}} Q \varphi(\varepsilon - 1)}{\tilde{w}^* \varepsilon (1 + \eta)^{\Psi^*} (1 + \lambda)^{1 - \Psi^*}} . \quad (18)$$

So far we have solved the production decisions of the firms. In order to find the optimal research decisions, we express the value functions of the firms in the next section.

3.5.3 Value Functions

Let $n_{\eta f}$ and $n_{\lambda f}$ denote the number of product lines that firm f owns with η and λ step sizes, respectively. Our focus is on the symmetric equilibrium to turn the problem into a tractable version. Without the symmetry assumption, the model becomes quite intractable without much new insights in terms of the key points of the model. In this case, instead of focusing on the number of products in a particular industry, we need only to keep track of the number of products of each type and the number of industries that the firm is present in. Due to our symmetry assumptions, we have $\Psi_i^* = \Psi^*$ and $\tau_i^* = \tau^*$, $\forall i \in \mathcal{I}$ in equilibrium. For notational simplicity, we will drop the firm subscripts f . The firm takes the equilibrium product share $\tilde{\Psi}^*$, probability of high quality applied innovation Ψ^* , wage rate \tilde{w}^* , creative destruction rate τ^* , and interest rate r^* as given. Then we can express the normalized value function as

$$\begin{aligned}
r^* \tilde{V}(m, n_\eta, n_\lambda) = & \tag{19} \\
\max_{\substack{a_m, b_m, \\ e_m}} & \left\{ \begin{aligned}
& n_\eta \tilde{\pi}_\eta + n_\lambda \tilde{\pi}_\lambda - (n_\eta + n_\lambda) \tilde{w}^* [h_a(a_{mn}) + h_b(b_{md}) + \mathbf{1}_{(b_m > 0)} h^b] + h_e(e_{mn}) \\
& + (n_\eta + n_\lambda) a_m \left[\Psi^* \tilde{V}(m, n_\eta + 1, n_\lambda) + (1 - \Psi^*) \tilde{V}(m, n_\eta, n_\lambda + 1) - \tilde{V}(m, n_\eta, n_\lambda) \right] \\
& + (n_\eta + n_\lambda) b_{md} (1 + \rho_m) \left[\tilde{V}(m, n_\eta + 1, n_\lambda) - \tilde{V}(m, n_\eta, n_\lambda) \right] \\
& + n_\lambda s^* \left[\tilde{V}(m, n_\eta + 1, n_\lambda - 1) - \tilde{V}(m, n_\eta, n_\lambda) \right] \\
& + (n_\eta + n_\lambda) \kappa \left[\tilde{\Psi}^* \tilde{V}(m, n_\eta + 1, n_\lambda) + (1 - \tilde{\Psi}^*) \tilde{V}(m, n_\eta, n_\lambda + 1) - \tilde{V}(m, n_\eta, n_\lambda) - \nu \right] \\
& + \tau^* n_\eta \left[\tilde{V}(m, n_\eta - 1, n_\lambda) - \tilde{V}(m, n_\eta, n_\lambda) \right] \\
& + \tau^* n_\lambda \left[\tilde{V}(m, n_\eta, n_\lambda - 1) - \tilde{V}(m, n_\eta, n_\lambda) \right] \\
& + e_m \left[\tilde{V}(m + 1, n_\eta, n_\lambda) - \tilde{V}(m, n_\eta, n_\lambda) \right] \\
& + \kappa \left[(n_\eta + n_\lambda) \nu - \tilde{V}(m, n_\eta, n_\lambda) \right]
\end{aligned} \right\}
\end{aligned}$$

where s is the unutilized spillovers from basic research as defined in (9). Intuitively, there is discounting at the rate r . The first line simply subtracts the instantaneous research expenditures from operating profits to obtain the net instantaneous profits. The second line expresses the change in firm value due to applied innovation. Notice that with applied innovation, we must form an expectation about how big the innovation size is going to be using the share of undepreciated product line Ψ . The third line expresses the change in firm value due to basic innovation at the initial and the spillover industries. Note that firm will be able to use this innovation also in another industry with probability ρ_m . The fourth line shows the change in firm value due to utilizing an aggregate spillover which arrives at the rate s . The fifth line describes the change in value due to buyouts where firm has to take again into account the distribution of product line types $\tilde{\Psi}$. The sixth and seventh lines describe the change in firm value due to creative destruction which happens at the rate τ . The eighth line is the change due to new industry expansion and the last line describes the change in firm value due to exogenous firm level destruction in which case firm is compensated by ν per-product line.

The following proposition characterizes the value function and the equilibrium research decisions. The value function is found to be linear in the number of products (but not in number of industries) and linearly separable between production value and the value associated with option value of being in more industries.

Proposition 1 *The the value function in (19) can be expressed as*

$$\tilde{V}(m, n_\eta, n_\lambda) = \beta_\eta n_\eta + \beta_\lambda n_\lambda + (n_\eta + n_\lambda) \beta_m \tag{20}$$

where $\beta_\eta^* \equiv \frac{\tilde{\pi}_\eta}{r^* + \tau^* + \kappa}$ and $\beta_\lambda^* \equiv \frac{\tilde{\pi}_\lambda + s^* \beta_\eta^*}{r^* + \tau^* + \kappa + s^*}$ and

$$(r^* + \tau^* + \kappa)\beta_m = \max_{\substack{a_m, b_m, \\ e_m, \mathbf{h}_m^b}} \left\{ \begin{array}{l} a_m [\Psi^* \beta_\eta + (1 - \Psi^*) \beta_\lambda + \beta_m] - \tilde{w}^* h_a(a_m) \\ + b_m (1 + \rho_m) [\beta_\eta + \beta_m] - \tilde{w}^* [h_b(b_m) + \mathbf{1}_{(h^b < \mathbf{h}_m^b)} h^b] \\ + e_m (\beta_{m+1} - \beta_m) - \tilde{w}^* h_e(e_m) \\ + \kappa [\tilde{\Psi}^* \beta_\eta + (1 - \tilde{\Psi}^*) \beta_\lambda] \end{array} \right\}. \quad (21)$$

Moreover, the optimal research decisions would be the solutions of the following equations

$$a_m^* = h_a'^{-1} \left(\frac{\Psi^* \beta_\eta + (1 - \Psi^*) \beta_\lambda + \beta_m}{\tilde{w}^*} \right) \quad (22)$$

$$b_m^* = \begin{cases} h_b'^{-1} \left(\frac{(1 + \rho_m) [\beta_\eta + \beta_m]}{\tilde{w}^*} \right) & \text{if } h_f^b < \mathbf{h}_m^{b*} \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

$$e_m^* = h_e'^{-1} \left(\frac{\beta_{m+1} - \beta_m}{\tilde{w}^*} \right) \quad (24)$$

$$\mathbf{h}_m^{b*} = b_m^* (1 + \rho_m) [\beta_\eta + \beta_m] - \tilde{w}^* h_b(b_m^*) \quad (25)$$

Proof. See Appendix. ■

A couple of key observations are in order. First, the value of a firm in (20) is determined by the interaction of three factors: the production value of a single product line $(\beta_\eta, \beta_\lambda)$, the option value (β_m) , and the firm scale (n_η, n_λ) . Second, the production value of a product line which is equivalent to the discounted sum of future profits flows is increasing in the instantaneous profit $\tilde{\pi}$ (which it self is increasing in innovation of size λ) and decreasing in the rate of creative destruction τ^* and exogenous destruction rate κ due to an increased frequency of product line loss. Third, the option value associated with each product line reflects the fact that each additional product line expands the firm's capacity for research (both basic and applied, a_m and b_m), and investment in industry expansion (e_m), in addition to increasing the rate at which the firm received buyout offers for new product lines (κ).

The first order conditions reflect the incentives associated with each type of research. In (22), the incentive for applied research comes from the interaction between the expected innovation size and the option value associated with gaining a new product line and the marginal cost of innovation. As is critical in our analysis, (23) reflects the fact that there may be additional gains from basic research through utilized spillover, the probability of which is increasing with number of industries (ρ_m) . Note that the fixed cost structure will replicate the observed pattern on the extensive margin of basic research seen in the data. Firms that draw a high fixed cost (d) will find it profitable not to invest in basic research. Finally, in (24), firms will

invest effort in expansion depending upon the additional option value generated from standing in an additional industry.

Next, we turn to the characterization of various invariant distributions of the model.

3.5.4 Invariant Distributions

To solve for the labor market equilibrium, we need to solve for various invariant distributions. The first one is the share of product lines $\Psi \in [0, 1]$ which are not depreciated and building on them generates a high return with a step size η . This share helps our firms to form their expectations about the innovation step size that they will have through applied research. The second one is the share of product lines $\tilde{\Psi} \in [0, 1]$ which are still producing high quality product lines with step size η . Note that Ψ and $\tilde{\Psi}$ are not the same since the exogenous depreciation flow ζ does not affect the current production in the product line and has an impact only on the next innovating firm in that product line. The final one is the share of firms $\Gamma_{m,n}$ that operate in m industries and has n product lines and the share of product lines $\tilde{\Gamma}_m^*$ whose owner operates in m industries. This distribution is determined by equating the inflows and outflows for each state (m, n) . This occurs when the flows from a number of underlying processes are balanced, such as successful applied or basic innovation (with or without spillovers), industry expansion, creative destruction, buyout opportunities, and exogenous firm destruction. Due to its lengthy derivation, the exact forms of the flow equations and further details governing the invariant distribution of $\Gamma_{m,n}^*$ is provided in the appendix. Solving for the invariant $\Gamma_{m,n}^*$ and $\tilde{\Gamma}_m^* = \frac{F^*}{M} \sum_{n=1}^{\infty} \Gamma_{m,n}^* n$ allows us to determine the aggregate creative destruction rate

$$\tau^* = \tilde{a}^* + \sum_{m=1}^M \left[a_m^* + (1 + \rho_m) b_m^* \mathcal{F}(\mathbf{h}_m^{b*}) \right] \tilde{\Gamma}_m^*$$

where $\tilde{a} = \hat{a}\chi$ is the aggregate applied innovation rate by outsiders, $\sum_m a_m^* \tilde{\Gamma}_m^*$ is the aggregate applied innovation rate by incumbents, and $\sum_m (1 + \rho_m) b_m^* \mathcal{F}(\mathbf{h}_m^{b*}) \tilde{\Gamma}_m^*$ is the aggregate basic innovation flow from incumbents. Two observations are in order. First, that in the event of a spillover, a firm will acquire a new product line from another incumbent only if that spillover falls in an industry that he operates in, which occurs with probability ρ_m . Second, only firms that draw a sufficiently low fixed cost will find it profitable to invest in basic research, which happens with probability $\mathcal{F}(\mathbf{h}_m^{b*})$.

For a given equilibrium wage rate w^* , we can express the basic innovation flow from each academic research lab as

$$d^* = h_b^{-1} \left(\frac{\hat{T}/U}{w^*} - \bar{h}^b \right) \quad \text{and} \quad \tilde{d}^* = U d^*$$

Since academic labs operate regardless of their fixed cost realization, they use the remainder of their labor on the variable cost of basic research. Next we can determine the rate at which firms receive an unexploited basic research spillover,

$$s^* = (1 + p) \tilde{d}^* + \sum_{m=1}^M (p - \rho_m) \mathcal{F}(\mathbf{h}_m^{b*}) b_m^* \tilde{\Gamma}_m^*$$

which happens through all academic basic innovation and the unutilized portion of private basic research.

Now we can write the flow equation that determines the invariant share of product lines Ψ^* which will generate an innovation of step size η to the next innovating firm,

$$\Psi^* \zeta = (1 - \Psi^*) (1 + p) \left(\tilde{d}^* + \sum_{m=1}^M \mathcal{F}(\mathbf{h}_m^{b*}) b_m^* \tilde{\Gamma}_m^* \right).$$

The left hand side expresses the outflow of product lines that depreciate at the rate ζ . On the contrary, the right hand side shows the product lines that used to be depreciated and just receive a new basic technology. This happens when academic labs innovate at the aggregate rate \tilde{d}^* and firms do successful basic research. Note that each basic innovation will create another new product in addition to its initial application with probability p .

The shares of product lines with high quality production and probability of high quality step size satisfy the following inequality $\tilde{\Psi}^* > \Psi^*$. Let us define the difference, that is, products that are currently producing at a high quality, but will become low quality upon the next applied innovation, as $\Delta_\Psi \equiv \tilde{\Psi}^* - \Psi^*$. The inflow into Δ_Ψ happens only from the product lines $(\tilde{\Psi}^* - \Delta_\Psi)$ that are producing with η and are not yet hit by the depreciation shock ζ . Outflow from Δ_Ψ happens when a new basic or applied innovation occurs on that product line since the former turns the product line into a high quality product line and the latter simply reduces the production step size to λ . As a result the flow equation reads as $(\tilde{\Psi}^* - \Delta_\Psi) \zeta = \Delta_\Psi (\tau^* + s^*)$ which implies

$$\tilde{\Psi}^* = \Psi^* \left[1 + \frac{\zeta}{\tau^* + s^*} \right].$$

Finally we compute the equilibrium measure of firms F^* . The total number of product lines owned by firms has to be equal to the total measure of product lines M such that $\sum_{m=1}^M \sum_{n=1}^{\infty} F^* \Gamma_{m,n}^* n = M$. Solving for F^* delivers the equilibrium measure of firms as

$$F^* = \frac{M}{\sum_{m=1}^M \sum_{n=1}^{\infty} \Gamma_{m,n}^* n}.$$

3.5.5 Labor Market

Having determined the invariant distributions, next we characterize the labor market equilibrium. Equations (3) and (17) determine the labor demand from a particular product line as

$$l_{ij}^* = l_k^* = \frac{(\varepsilon - 1)}{\tilde{w}^* \varepsilon (1 + k)}, \quad k \in \{\eta, \lambda\}. \quad (26)$$

As a result, we can express the total demand for production workers as

$$L^{prod*} \equiv \frac{(\varepsilon - 1)}{\tilde{w}^* \varepsilon} \left(\frac{\tilde{\Psi}^*}{1 + \eta} + \frac{1 - \tilde{\Psi}^*}{1 + \lambda} \right).$$

The total labor demand consists of demand for the production workers and other exploratory expenditures as in (11). This condition can be rewritten as

$$\frac{1}{M} = \left\{ \begin{array}{c} \overbrace{\sum_{m=1}^M \left\{ \left[h_a(a_m^*) + \mathcal{F}(\mathbf{h}_m^b) \left[h_b(b_m^*) + \mathbb{E} \left(h_f^b \mid h_f^b < \mathbf{h}_m^b \right) \right] \right] \tilde{\Gamma}_m^* + \sum_{n=1}^{\infty} h_e(e_m^*) \Gamma_{m,n}^* \frac{F^*}{M} \right\}}^{\text{incumbents' non-production labor demand}} \\ + \underbrace{\frac{(\varepsilon - 1)}{\tilde{w}^* \varepsilon} \left(\frac{\tilde{\Psi}^*}{1 + \eta} + \frac{1 - \tilde{\Psi}^*}{1 + \lambda} \right)}_{\text{incumbents' production}} + \underbrace{\chi h_a(\tilde{a}^*)}_{\text{outsiders}} + \underbrace{U \left[h_b(d^*) + \bar{h}^b \right]}_{\text{academic}} \end{array} \right\}.$$

The first term on the first row of the right hand side consists of labor demand for applied and basic research and expansion efforts. Note that only $\mathcal{F}(\mathbf{h}_m^b)$ fraction of the state- m firms will invest in basic research. If the condition becomes slack, the wage rate will decrease and the demand for production workers (first term on the right side) will increase until the equality is reset.

3.5.6 Aggregate Growth

Finally we derive the growth rate and the interest rate of the economy. Recall from (18) that the final output is proportional to the quality index. This implies that in steady state equilibrium we have

$$g^* = \frac{\dot{Z}^*}{Z^*} = \frac{\dot{Q}^*}{Q^*}.$$

The next proposition provides the exact expression for the growth rate.

Proposition 2 *The equilibrium steady state growth rate of this economy is equal to*

$$\begin{aligned} g^* &= \ln(1 + \eta)^{(1+p)(\tilde{d}^* + \bar{b}^*) + (\tilde{a}^* + \tilde{a}^*)\Psi^*} (1 + \lambda)^{(\tilde{a}^* + \tilde{a}^*)(1 - \Psi^*)} \\ &\cong \underbrace{(\tilde{a}^* + \tilde{a}^*) \lambda}_{\text{direct applied}} + \underbrace{(\tilde{d}^* + \bar{b}^*) \eta}_{\text{direct basic}} + \underbrace{p(\tilde{d}^* + \bar{b}^*) \eta}_{\text{cross-industry spillover}} + \underbrace{(\tilde{a}^* + \tilde{a}^*) \Psi^* (\eta - \lambda)}_{\text{within-industry spillover}} \end{aligned} \quad (27)$$

where $\bar{a}^* = \sum_{m=1}^M a_m^* \tilde{\Gamma}_m^*$ and $\bar{b}^* = \sum_{m=1}^M b_m^* \tilde{\Gamma}_m^*$.

This proposition makes it clear that the equilibrium growth rate is determined by basic and applied innovation rate and their relevant innovation sizes. Note that basic innovation contributes to growth in three distinct ways. First, it has a direct effect through its direct applications in the initial product line that it applies. The second effect comes through its spillovers across industries which happens with probability p . Finally it impacts the subsequent applied research within the same industry. Each new basic innovation has a long-lasting impact until its effect vanishes at the rate ζ . If basic innovation did not have any effect on the subsequent innovations ($\zeta \rightarrow \infty$), the equilibrium share of product lines with a step size η would vanish ($\Psi^* = 0$) and the within industry contribution of basic research would disappear. Also note that basic innovation makes each applied innovation more productive by increasing the effective step size of applied innovation by $\eta - \lambda$. In section 4.5.2, we quantify these distinct effects for French economy.

Finally, we find the equilibrium value of the interest rate r^* from the household's intertemporal decision

$$r^* = \delta + \gamma g^*$$

3.5.7 Welfare

Our focus so far has been on steady-state equilibria (mainly because of the very challenging nature of transitional dynamics in this class of models). In our quantitative analysis, we continue to focus on steady states and thus look at steady-state welfare. In a steady-state equilibrium, welfare at time $t = 0$ can be written as

$$W(C_0^*, g^*) = \int_0^\infty \exp(-\delta t) \frac{(C_0^* \exp(g^* t))^{1-\gamma} - 1}{1-\gamma} dt = \frac{1}{1-\gamma} \left[\frac{Z_0^{*(1-\gamma)}}{\delta - g^*(1-\gamma)} - \frac{1}{\delta} \right]. \quad (28)$$

where we use the facts that the utility is as in (1) and all output is consumed $C_0^* = Z_0^*$. Note that $\delta > g^*(1-\gamma)$, which is guaranteed by the Euler equation for the household (12) and the fact that $\gamma \geq 1$. We assume that the initial quality index is given by $Q(0) = Q_0$. Then we use (18) to find the initial value of final output as

$$Z_0^* = \frac{M^{\frac{\varepsilon}{\varepsilon-1}} Q_0 \varphi(\varepsilon - 1)}{\tilde{w}^* \varepsilon (1 + \eta)^{\Psi^*} (1 + \lambda)^{1-\Psi^*}} \quad (29)$$

Note that the welfare can be expressed purely as an increasing function of the initial consumption (Z_0^*) and the growth rate (g^*). However, the fixed labor supply in the model

creates a tradeoff between the two. Allocating more researchers for innovation increases the growth rate, but decreases the amount of production workers and hence the output and initial consumption, and vice versa.

From a cross-sectional perspective, (29) implies that initial consumption is reduced when the average markup in the economy is increased (higher Ψ^*), which is captured by the denominator. Moreover, the static component of welfare is increasing in the number of industries (M) due to consumer's love for variety. Finally, when the labor share increases, the marginal cost of production increases, so that production, and hence the initial consumption, goes down.

From a dynamic perspective, allocating more researchers into research will increase the growth rate as in proposition 2, which will lead to higher welfare for any given initial consumption. The policies that we are going to analyze will navigate the tradeoff between the two dimensions of welfare.

3.5.8 Planner's Problem

One of the main goals of this study is to characterize the inefficiencies in the economy and discuss the relevant R&D policies to address these inefficiencies. The inefficiency in the decentralized economy arises from two major sources, misallocation of resources into production and research. In this analysis, we will consider a hypothetical social planner and compare the outcome of our decentralized economy to the planner's optimum. One could potentially consider a planner who controls both production and research. However, as our focus in this paper is on R&D and related policies and not policies directly related to production (productions subsidies, for instance), we will consider a social planner who controls research decisions but is still constrained by markups. In other words, firms will continue to make their own production decisions. We will refer to this as the *constrained first best* (CFB).

At the CFB, it is optimal to set $e_m = 0$ for all m , since all spillovers are internalized from the planner's perspective. Thus all firms will operate in a single industry. Additionally, it is optimal to have $\hat{a}_f = a_f = a$, as both entrants and incumbents operate the same research technology and concavity of the production function dictates that the planner equalize the applied research rates across firms. The planner will choose a , b , d , and \mathbf{h}_f (the cutoff fixed cost for undertaking basic research) to maximize welfare in equation (28) subject to the following

constraints

$$\begin{aligned}
P_f &= \mathbb{P}[h_f < \mathbf{h}_f] \\
\Psi\zeta &= (1 - \Psi)(1 + p)(Ud + P_fb) \\
\tilde{\Psi} &= \Psi \left[1 + \frac{\zeta}{(1 + \chi)a + (1 + p)(P_fb + Ud)} \right] \\
g &= a(1 + \chi)[(1 - \Psi)\lambda + \Psi\eta] + (1 + p)(Ud + P_fb)\eta \\
\tilde{w} &= \frac{\tilde{\Psi}/(1 + \eta) + (1 - \tilde{\Psi})/(1 + \lambda)}{\frac{1}{M} - (1 + \chi)h_a(a) - P_f[h_b(b) + \mathbb{E}[h_f|h_f < \mathbf{h}_f]] - U[h_b(d) + \bar{h}^b]}
\end{aligned}$$

Since the planner is constrained by the markups, it must take into account the effect of its actions on the distribution of markups as encompassed in the second and third lines above. Similarly, the planner takes into account the effect of its actions on the growth rate (fourth line) and wage rate (fifth line).

In this model, there will be major types of inefficiencies arising from research investment. First, the aggregate level of research investment will be below the efficient level both due to the presence of uninternalized spillovers and the fact that monopoly power may not be sufficient to provide the proper incentives for research as a result of creative destruction and exogenous shocks. Second, the composition of resources employed for research will not be optimal, even if the level of investment matches that of the planner. As a result of multi-industry presence, firms will have differing research intensities, although the planner chooses exactly the same level of research investment across firms. The R&D policies we will consider will typically close the gap in welfare by aligning the levels, yet the composition of resources employed for research will remain as a source of inefficiency.

In the following section, we will estimate the decentralized economy and compare it to the planner's solution. Then we will consider various R&D policies that can reduce the welfare gap between the two. One method of quantifying the difference between the CFB solution and a particular allocation is by considering consumption equivalents. We consider changes in welfare by considering the fractional consumption equivalent, α . That is, welfare in the decentralized economy $W(C_0^*, g^*)$ is the same in an economy with the CFB growth rate g^{SP} and a fraction α of the CFB initial consumption C_0^{SP} , such that

$$W(C_0^*, g^*) = W(\alpha C_0^{SP}, g^{SP}).$$

Lemma 1 *Let \tilde{w}^{SP} , $\tilde{\Psi}^{SP}$, and g^{SP} denote the outcome of the social planner's problem and \tilde{w}^* , $\tilde{\Psi}^*$, and g^* the equilibrium values from the decentralized economy. Then the consumption*

equivalent is given by

$$\alpha = \left[\frac{\tilde{w}^{SP}}{\tilde{w}^*} \right] \cdot \left[\frac{1 + \eta}{1 + \lambda} \right]^{\tilde{\psi}^{SP} - \tilde{\psi}^*} \cdot \left[\frac{\delta + g^*(\sigma - 1)}{\delta + g^{SP}(\sigma - 1)} \right]^{\frac{1}{\sigma - 1}}$$

4 Quantitative Analysis

We use the Generalized Method of Moments (GMM) to estimate our model. The next section provides details on the estimation method. Before we proceed, however, we need to specify the functional forms for the research cost functions and the distribution of the fixed cost of basic research $\mathcal{F}(h^b)$. We will assume that:

$$h_a(x) = \xi_1^a x^{\xi_2^a}, \quad h_b(x) = \xi_1^b x^{\xi_2^b} \quad \text{and} \quad h_e(x) = \xi_1^e x^{\xi_2^e}$$

where $\xi_1^a, \xi_1^b, \xi_1^e > 0$ and $\xi_2^a, \xi_2^b, \xi_2^e > 1$. We will also assume that the fixed costs are drawn from a lognormal distribution with mean μ and variance σ^2 . As a result, the set of parameters of the model is $\{\gamma, \delta, \varepsilon, \varphi, \eta, \lambda, p, \zeta, \chi, \kappa, \mu, \sigma, U, \xi_1^a, \xi_2^a, \xi_1^b, \xi_2^b, \xi_1^e, \xi_2^e\}$.

We have some preset parameters. Since labor productivity does not have a direct effect on key moments, including profits, we normalize to $\varphi = 1$. Following the macro-consumption literature, we take the intertemporal elasticity of substitution as $\gamma = 2$. The discount factor δ together with the growth rate g^* and γ determines the interest rate in our model economy. Our benchmark growth rate will be around 1.5%. The interest rate in France for the period was around 3.5%. Therefore it is reasonable to set the discount factor to $\delta = 0.01$. However, appendix C replicates all the exercises for $\rho = 0.05$ as robustness checks. Finally, we pick the CES parameter to be $\varepsilon = 2$.¹⁹ Our moments are typically insensitive to the curvature parameter of the expansion production function ξ_2^e , therefore we simply take a quadratic cost function, but estimate its scale parameter ξ_1^e . This leaves us with the following vector of parameters to be estimated:

$$\theta_{14 \times 1} \equiv \left[\eta \quad \lambda \quad p \quad \zeta \quad \chi \quad \kappa \quad \mu \quad \sigma \quad U \quad \xi_1^a \quad \xi_2^a \quad \xi_1^b \quad \xi_2^b \quad \xi_1^e \right]' \in \Theta$$

where Θ is the set of parameter vectors.

For the period that we consider, there was existing government support for R&D activities. Our dataset contains information on the publicly funded portion of private R&D. On average, 10% of private R&D was funded publicly. Therefore in our estimation, we introduce a uniform

¹⁹A reasonable way to calibrate this parameter would be to target the labor share of the national income account, which is approximately 70%. We will do this in a future estimation.

subsidy to the total R&D spending of the firm $\psi = 0.10$, such that a firm of type (m, n, h_f^b) pays

$$(1 - \psi)n\tilde{w}^*(h_a(a_m^*) + \mathbf{1}_{(h_f^b < \mathbf{h}_m^{b*})}[h_b(b_m^*) + h_f^b])$$

in net research costs. The government has a balanced budget every period, so that the sum of total subsidies and academic funding must be equal to the tax revenues, that is

$$T = \hat{T} + \psi\tilde{w}^* \sum_m \left(h_a(a_m^*) + \mathcal{F}(\mathbf{h}_m^{b*}) \left[h_b(b_m^*) + \mathbb{E} \left(h_f^b \mid h_f^b < \mathbf{h}_m^{b*} \right) \right] \right) \tilde{\Gamma}_m^*.$$

4.1 Estimation Method

In our dataset, for each firm f and each time period t , we have a vector of N observables from the actual data $\mathbf{y}_{ft}^A \equiv [y_{ft}^1 \ \dots \ y_{ft}^N]_{N \times 1}'$ including the number of industries the firm is present in, sales, profits, and labor costs. Let the entire dataset be denoted by \mathbf{y} .

We use GMM for the estimation. Define $\Lambda(\mathbf{y})$ and $\Lambda(\theta)$ to be, respectively, the vectors of real data moments (generated from \mathbf{y}) and equilibrium model moments (generated for some vector of parameters θ). Our proposed estimator minimizes a quadratic form of the difference between these two vectors

$$\hat{\theta} = \arg \min_{\theta \in \Theta} [\Lambda(\theta) - \Lambda(\mathbf{y})] \cdot W \cdot [\Lambda(\theta) - \Lambda(\mathbf{y})]$$

where W is the weighting matrix. We use a diagonal weighting matrix with diagonal elements

$$W_{ii} = 1/\Lambda_i(y)^2$$

This is asymptotically consistent under fairly general conditions, but not efficient²⁰. The standard error of our parameter estimates, given any weighting matrix W is:

$$\hat{Q} = \left(\hat{G}W^{-1}\hat{G} \right)^{-1} \cdot \hat{G}W^{-1}\hat{\Omega}W^{-1}\hat{G} \cdot \left(\hat{G}W^{-1}\hat{G} \right)^{-1}$$

where $\hat{G} = \frac{\partial \Lambda(\theta)}{\partial \theta} \Big|_{\theta=\hat{\theta}}$. We estimate the covariance matrix by bootstrapping the data at the firm level. See Bloom (2008) and Lentz and Mortensen (2008) for further description and usage. In our estimation, we use 26 moments, which are described next.

4.2 Target Moments and Identification

In this section we explain the moments that are used to identify our parameters.

²⁰In the efficient case where $W = \hat{\Omega}^{-1}$ this becomes simply $\hat{Q} = \left(\hat{G}\hat{\Omega}\hat{G} \right)^{-1}$.

Basic Research Intensity by Number of Industries We define basic research intensity as the ratio of spending on basic research to spending on applied research. Since the effect of multi-industry presence on this quantity is of critical importance to our model, we have one moment for each $m \in \{1, \dots, M\}$. Given a set of parameters and an equilibrium of the model, this moment's analytic value for a given m is

$$\Lambda(1-8) = \frac{\int \left[h_b(b_m^*) + h^b \mathbf{1}_{(h^b < \mathbf{h}_m^*)} \right] \mathcal{F}(h^b)}{h_a(a_m^*)}$$

In our estimation, we use $M = 10$. However, in the data there are only a handful of firms with $m > 8$, so we have one moment for each $m \in \{1, \dots, 7\}$ and a final moment which is averaged over $m \in \{8, 9, 10\}$. The way in which this moment increases with m identifies the cross-industry spillover parameter p in our model. Additionally, it provides us with some identification power for the basic research cost parameters (ξ_1^b, ξ_2^b) .

Average Basic and Applied Research Ratio We also have moments to track the mean levels of research spending in the economy. The first is the ratio of applied research to sales, and the second is the ratio of basic research to sales. Since sales and research spending are both proportional to n , these figures will be the same across firms with the same m . Thus the moments is given by

$$\begin{aligned} \Lambda(9) &= \sum_m \tilde{w} h_a(a_m) \cdot \Gamma_m \\ \Lambda(10) &= \sum_m \tilde{w} \mathcal{F}(\mathbf{h}_m^b) \left[h_b(b_m^*) + \mathbb{E} \left(h_f^b \mid h_f^b < \mathbf{h}_m^b \right) \right] \cdot \Gamma_m \end{aligned}$$

Conditional on innovation rates, these moments give us information on the research production functions parameters.

Private to Public Basic Research Ratio The rate of academic innovation d is estimated as a parameter. To determine this, we look at the ratio of private basic research spending to public basic research spending. The moment is given by

$$\Lambda(11) = \frac{\sum_m \mathcal{F}(\mathbf{h}_m^b) \left[h_b(b_m^*) + \mathbb{E} \left(h_f^b \mid h_f^b < \mathbf{h}_m^b \right) \right] \cdot \Gamma_m}{h_b(d) + \bar{h}^b}$$

This moment will allow us to identify the flow rate of academic innovation d , as well as inform us as to the level of private basic research spending.

Distribution of m We track two moments relating to the distribution of m , the mean and mean squared. They are given by

$$\Lambda(12) = \sum_m m \cdot \Gamma_m \quad \text{and} \quad \Lambda(13) = \sum_m m^2 \cdot \Gamma_m$$

These moments identify the scale parameter governing the cost of expansion (ξ_1^e) as well as the mass of potential outside entrants (χ). The cost parameters determine the rates at which firms expand into new industries, while the mass of entrants determines the aggregate creation rate of single-industry firms. Together, these factors determine the equilibrium distribution of multi-industry presence.

Profitability Firm profitability is defined as the ratio of profits to sales. As a result of the logarithmic aggregation in each industry, sales for any given product are the same. So total sales for a firm is just the number of products times a constant, which in this case is $\varepsilon(1 - \varepsilon)^{-1} Y M^{-1}$. For a given product, profitability is then simply $k/(1+k)$ where $k \in \{\eta, \lambda\}$. Thus mean profitability is given by

$$\Lambda(14) = \tilde{\Psi} \cdot \left(\frac{\eta}{1 + \eta} \right) + (1 - \tilde{\Psi}) \cdot \left(\frac{\lambda}{1 + \lambda} \right)$$

where $\tilde{\Psi} \in (0, 1)$ is the share of product lines with the current technology η . This moment, in conjunction with the employment growth and labor dispersion moments, helps us determine the step size parameters η , λ , and ζ .

Employment Since the mass of workers is normalized to 1 in our model, we cannot target the mean employment of firms. However, we can capture the distribution of workers by looking at the dispersion of employment, that is, the ratio of the mean square to the square of the mean. Calculating this moment (number 15) requires knowledge of the joint distribution of step size, n , and m , which is prohibitive to calculate, so we simulate this moment to a high degree of accuracy. This provides further information on the three step size parameters.

Firm Growth We have moments for sales and employment growth amongst firms. Both are calculated conditional on the firm not exiting, since we do not observe the last period's growth rate for exiting firms. The expression for conditional sales growth is given by

$$\Lambda(16) = \frac{\sum_m [a_m + (1 + \rho_m)b_m - \tau \cdot \mathbf{1}(n > 1) + \kappa] \cdot [1 - \tau \cdot \mathbf{1}(n > 1) - \kappa] \cdot \Gamma_{n,m}}{1 - \tau \sum_m \Gamma_{m,1} - \kappa}$$

In the case of wage bill growth, we focus only on single-industry firms for computational reasons. The expression for this moment (number 17) has a cumbersome expression which we omit here. It is similar in structure to the above equation but also depends on the step size parameters and distributions. The sales growth primarily informs on the rate of exogenous destruction κ , while employment growth helps identify the step size parameters.

Exit Rate As exit occurs when firms either receive the exogenous destruction shock or lose their last product, the predicted exit rate will be

$$\Lambda(18) = \kappa + \tau \cdot \sum_m \Gamma_{m,1}$$

This moment serves primarily to determine the value of the rate of exogenous destruction κ , as well as the mass of outside entrants χ , since the size of the pool of entrants affects the rate creative destruction and hence the exit rate of single-product firms.

Extensive Margin of Basic Research Investment by Number of Industries We use the share of positive basic research spending by each m to identify the mean μ and variance σ^2 of the fixed cost distribution basic research. This is simply the probability that the idiosyncratic fixed cost draw is less than the cutoff for a certain m

$$\Lambda(19 - 26) = \mathcal{F}(\mathbf{h}_m^b).$$

4.3 Data

Empirical investigation on the relationship between R&D investment and multi-market activity of a firm requires reliable and extensive information not only on product markets and on R&D characteristics of individual firms, but also on firm ownership status. The latter allows us to identify the product markets to which the firm is linked via its business group. We obtain this information from three different data-sets.

R&D Information Information about R&D investment comes from the annual R&D Survey conducted by the French Ministry of Research. The R&D survey comes in annual waves of cross-sectional data, where the same firms are not necessarily sampled year after year (Mairesse and Mohnen, 2010). The survey covers a representative sample of French firms of more than 20 employees investing into R&D. However firms with less than .8 Million Euros of R&D investment fill out a shorter and simplified survey. The survey includes extensive information

about the financing of R&D. It not only breaks down R&D investment according to the source of the funds, but also provides its allocation to different types of R&D. More specifically all firms are asked to report their R&D investment into basic and applied research.

Multi-Market Activity The identification of business group structures is based on a yearly survey by INSEE called “Enquete Liasons Financieres” (LIFI). It covers all economic activities but restricts its attention to firms which either employ more than 500 employees, or generate more than 60 Million Euros of revenues, or hold more than 1.2 Million Euros of traded shares. However since 1998 the survey is crossed with information from Bureau Van Dijk and thus covers almost the whole economy. The LIFI survey contains information which makes it a unique data set to study the relationship between multi-market activity and investment into basic research. Besides providing information on direct financial links between firms, it also accounts for indirect stakes and cross-ownerships when identifying the head of the group. This is important as it allows to precisely reconstruct the group structure even in the presence of pyramids. This feature allows us to obtain a reliable account of the structure of business groups in the French economy and, as a consequence, reliable measures of our key variable, the multi-market presence of business groups.

Since each firm itself can be active in several markets, we cross the data set with an extensive yearly survey by the Ministry of Industry (“Enquete Annuelle des Entreprises”). The survey is filled by French firms with more than 20 workers and contains information not only on the different markets in which a firm operates but also information on market dedicated sales for each segment. The data covers the vast majority of French firms and spans over the period 2000-2006.

Balance Sheet Information We use the firm- and industry-level data sets based on accounting data extracted again extracted from the EAE files. The data also includes unique firm identifiers allowing us to match it to the R&D and LIFI data.

4.4 Computer Algorithm Outline

An equilibrium of this model can be summarized by a vector of four variables $(x_a^*, x_b^*, x_s^*, \tilde{w}^*)$ satisfying four equations:

$$\begin{aligned} x_a^* &= \sum_m a_m^* \tilde{\Gamma}_m^* + \tilde{a}^* & \text{and} & & x_b^* &= \sum_m (1 + \rho_m) b_m^* \tilde{\Gamma}_m^* \\ x_s^* &= \sum_m (p - \rho_m) b_m^* \tilde{\Gamma}_m^* + (1 + p)d & \text{and} & & \frac{1}{M} &= L_{prod}^* + L_{res}^* \end{aligned}$$

Where a_m^* , b_m^* , e_m^* , and \tilde{a}^* are chosen in accordance with the optimality conditions put forth in Proposition 1, $\tilde{\Gamma}^*$ satisfies the flow equations, and L_{prod}^* and L_{res}^* represent labor demanded for production and research. The first three equations impose consistency on the levels of applied and basic research and spillover. The final equation ensures labor market clearing.

Given a vector of candidate equilibrium variables, we can then evaluate the above equations by taking the following steps:

1. Calculate Ψ and $\tilde{\Psi}$ using x_a , x_b , and x_s .
2. Calculate g using x_a , and x_b .
3. Using g , find r from σ and δ .
4. Now calculate β_η and β_λ .
5. Solve for β_m , a_m , b_m , e_m recursively.
6. Impose an upper bound on n and find the steady state $\Gamma_{m,n}$ using the power method (or your preferred method, e.g. an eigensolver).
7. Plug these into the equations above.

We use a Powell-Hybrid equation solver to solve this set of equations for a given set of parameters. To minimize the GMM objective function, we perform a search over the joint parameter and equilibrium variable space, minimizing the objective function subject to the equilibrium constraints. For this we use a Nelder-Mead (simplex) algorithm.

4.5 Estimation Results of Baseline Economy

Table 4 reports the values of the estimated structural parameters and their asymptotic standard errors.

TABLE 4: PARAMETER ESTIMATES

Parameter	Value	Std. Error	Description	Equation
p	0.18126	0.00412	Cross-industry Spillover of Basic Research	(7)
η	0.39419	0.0168	High Quality Step Size	(4)
α	0.099611	0.00861	Low Quality Step Size	(4)
χ	0.016093	0.000470	Mass of Outside Entrants	(10)
U	1.2532	0.172	Mass of Academic Labs	(8)
ξ_2^a	1.50966	0.0284	Curvature of Applied Research Cost Function	(6)
ξ_2^b	1.62520	0.0117	Curvature of Basic Research Cost Function	(6)
ξ_1^a	0.35854	0.0354	Scale of Applied Research Cost Function	(6)
ξ_1^b	5.52095	0.312	Scale of Basic Research Cost Function	(6)
ξ_1^c	0.0051604	0.000243	Scale of Expansion Cost Function	(19)
κ	0.0058871	0.000412	Exogenous Destruction Rate	(19)
μ_f	7.0706	0.0121	Mean of Basic Research Fixed Cost	(19)
σ_f	0.42082	0.00454	Std. Dev. of Basic Research Fixed Cost	(19)
ζ	0.043695	0.0107	Depreciation Rate	(19)

4.5.1 Parameter Estimates

One of most important estimate of our model is the cross-industry spillover parameter $p = 0.18$ which measures the probability of having an additional immediate application of the basic innovation. This estimate affects the extent to which basic innovations contributes to cross-sectional growth. In equilibrium, firms operate in two industries on average. Therefore the firms have on average a $1/9$ probability of internalizing a given horizontal spillover. Given the estimate value of p , the internalized spillover is $0.02 (= (1/9) \times .18)$. In other words with probability 0.16 , each successful basic innovation will have an application that will not be utilized by the main innovator.

The estimated innovation size of basic research is $\eta = 0.39$ and the innovation size of each new applied innovation is $\lambda = 0.10$. As a result, each basic innovation contributes to the cross-sectional growth almost 4.6 times more than an applied innovation ($((1 + p) \eta / \lambda \approx 4.65)$).

On the other hand, each basic innovation has also a within industry spillover. The depreciation rate is estimated to be $\zeta = 0.04$ which indicates that a basic innovation affects the subsequent innovations in the same product line for almost 25 years on average.

The elasticity of applied innovation counts with respect to the research dollars spent is estimated to be $0.66 (= 1/\xi_2^a)$ and similarly the elasticity of basic innovation with respect to the basic research investment is $0.62 (= 1/\xi_2^b)$. A single elasticity has been estimated for the US economy by several papers (Pakes and Griliches (1984), Hall et al. (1988) and Kortum (1992, 1993)) and the estimates are varying between 0.1 and 0.6. The fact that the elasticity is smaller than 1 suggests that there is more investment in duplicative research efforts as the research investment increases (Kortum, 1993). Therefore, our estimate suggests the there is a smaller fraction of duplicative research efforts in France than in the US for a given proportional

increase in research investment.

4.5.2 Endogenous Variables

The following table provides equilibrium values for some of the important endogenous variables in the model

\bar{a}^*	\tilde{a}^*	\bar{d}^*	\bar{b}^*	\bar{e}^*	$\bar{\rho}^*$	s^*	Ψ^*	g^*	r^*	\tilde{w}	α^*
8.1	0.13	0.37	0.20	0.91	1.74	0.44	13.2	1.40	3.51	48.9	56.4

As the table indicates, most applied innovation in the economy is done by incumbents. Approximately 98% of applied innovations come from incumbent firms. In any given period, about 8% of incumbents produce an applied innovation. On the basic research side, around 64% of basic innovations come from the academic sector. Also note that the arrival rate of basic innovation is significantly smaller than that of applied innovation. Applied innovations arrive 14 times more frequently than basic innovations. However, each basic innovation will have associated additional contributions to economic growth which we analyze in more detail below.

In the table above, $\bar{\rho}^* \left(= \mathbb{E}_m \left[\frac{p(m-1)}{M-1} \right] \right)$ is the average probability of a firm producing an appropriable cross-industry spillover on top of a basic innovation. Our estimates show that this probability is roughly 2%. Since the probability of a cross-industry spillover is $p \cong 18\%$, only 10% ($\bar{\rho}^*/p$) of spillovers are utilized by the innovating firm. In other words, 90% of cross-industry basic innovation spillovers benefit other firms. Each company in the economy receives one of these spillovers either from another company or from the academic sector at a total rate of $s^* \cong 0.4\%$. Thus, a firm on average receives two times as many basic innovations from cross-industry spillovers as it does from its own production of them.

The estimate for Ψ^* captures the within industry spillovers from basic research. Thanks to this spillover, 13% of applied innovations have a larger contribution ($\eta - \lambda$) to growth than they otherwise would. The resulting improvements lead to an aggregate growth rate of 1.4% per annum for the economy as a whole, which we analyze in the next section.

Growth Decomposition In expression (27), we show that research efforts contribute to aggregate growth along several dimensions. The following table decomposes these effects ac-

ording to their source and their immediate impact.

GROWTH DECOMPOSITION BY RESEARCH TYPE

Direct Applied	Direct Basic	Cross-Industry	Within Industry
$(\bar{a}^* + \tilde{a}^*) \lambda$	$(\tilde{d}^* + \bar{b}^*) \eta$	$p (\tilde{d}^* + \bar{b}^*) \eta$	$(\bar{a}^* + \tilde{a}^*) \Psi^* (\eta - \lambda)$
0.82% (58%)	0.22% (16%)	0.04% (3%)	0.32% (23%)

Overall 58% of total growth comes from the direct effect of applied innovations, while 16% of growth comes from the direct effect of basic innovation. In addition to the direct effect, basic research contributes to overall growth through cross-industry spillovers, which account for 3% of total growth, and through within industry spillovers, which account for 23% of total growth. In sum, 62% percent of the contribution of basic innovation to growth comes through its two types of spillovers, which are not fully internalized by innovating firms.

GROWTH DECOMPOSITION BY INSTITUTION

Entrants	Incumbent Applied	Incumbent Basic	Academic Sector
$\tilde{a}^* [\Psi^* \eta + (1 - \Psi^*) \lambda]$	$\bar{a}^* [\Psi^* \eta + (1 - \Psi^*) \lambda]$	$(1 + p) \bar{b}^* \eta$	$(1 + p) \tilde{d}^* \eta$
0.02% (1%)	1.12% (80%)	0.10% (7%)	0.17% (12%)

Our estimates indicate that entrants play a relatively small direct role in overall growth. While in the US, startups are thought to play a large role in overall growth, these numbers suggests that this may not be the case in France. We also recognize that without direct data on entry rates, our estimate relies largely on inference from other aspects of the data, which are listed in section 4.5.3. More interestingly, the contribution of the academic sector to overall growth is 12%. Note that this is a lower bound for the total contribution of the academic sector because this figure mainly captures the instantaneous contribution, but not the effects on subsequent applied research. To give an idea of size of the dynamic contribution, keeping the innovation rates of entrants and outsiders the same, we recompute Ψ^* excluding the contribution from academic basic innovation and recalculate the implied growth contributions from the first two columns. In this case we find that growth falls to 1.20%. This implies that the dynamic effects of academic innovation account for 14% ($= (1.40 - 1.20)/1.40$) of total growth, which increases the overall contribution of the academic sector to 26%.

Finally, we conclude this section by providing a decomposition of the labor force utilization. This is informative because in our model, the only input for both production and innovation is labor. Therefore, it is important to observe the allocation of labor amongst its various possible uses in order to understand the mechanism behind the current results and the upcoming policy analysis.

LABOR DECOMPOSITION BY ACTIVITY (IN PERCENTAGES)

Production	Applied Incumbent	Basic Incumbent	Expansion	Academic	Entrants
89.1	8.5	0.7	≈ 0	1.6	0.1

In our benchmark economy, 90% of labor is used for production, and 10% is employed for innovation activities. Amongst this 10% percent, roughly 30% of researchers are engaged in innovation activities with additional spillovers. The policies that will consider will not only govern the split of labor resources between production and research, but also the composition within research across different types of innovation, since uninternalized spillovers are one of the main sources of inefficiency.

4.5.3 Goodness of Fit and Untargeted Moments

Table 5 contains the values of moments from the actual data and our estimated model.

TABLE 5: MOMENTS USED IN ESTIMATION

#	Description	Model	Data
1	Mean basic intensity, $m = 1$	0.0301493	0.0665769
2	Mean basic intensity, $m = 2$	0.0374873	0.0466243
3	Mean basic intensity, $m = 3$	0.0458843	0.0616927
4	Mean basic intensity, $m = 4$	0.0553412	0.0801807
5	Mean basic intensity, $m = 5$	0.0658307	0.0763586
6	Mean basic intensity, $m = 6$	0.0772940	0.0782929
7	Mean basic intensity, $m = 7$	0.0896397	0.1224092
8	Mean basic intensity, $m = 8+$	0.1237437	0.1013674
9	Mean m	1.8700803	2.209294
10	Mean square m	6.9224227	6.975635
11	Mean profitability	0.0420897	0.059547
12	Mean firm growth	0.0443639	0.0410197
13	Mean exit rate	0.0311700	0.0273
14	Mean applied R&D	0.0829785	0.0747405
15	Mean basic research	0.0031294	0.0036469
16	Private/public basic research	0.1955858	0.1516633
17	Pos. basic research, $m = 1$	0.1943180	0.2431939
18	Pos. basic research, $m = 2$	0.2304994	0.2307943
19	Pos. basic research, $m = 3$	0.2695655	0.2702586
20	Pos. basic research, $m = 4$	0.3111376	0.3482786
21	Pos. basic research, $m = 5$	0.3547701	0.4183674
22	Pos. basic research, $m = 6$	0.3999500	0.4517544
23	Pos. basic research, $m = 7$	0.4460875	0.5508475
24	Pos. basic research, $m = 8+$	0.5607084	0.6803279
25	Labor dispersion	10.8754705	32.79106
26	Employment growth	0.0555172	0.0846145

The results indicate that the model generates firm and industry dynamics similar to those in the data. In line with stylized fact 1, a significant fraction of innovating firms invest in basic research. In particular, 23% of firms are investing in basic research, which was 27% in the data. We also capture the positive relationship between the extensive margin of basic research and multi-industry presence, as evidenced in rows 17-24 and Figure 5.

In addition, the ratio of private basic research investment to public basic research investment is 20%, while it is 15% in the data. This is partly driven by our assumption in the model that academic labs operate regardless of their fixed cost draw, whereas private firms are optimizing in this dimension. However, the fixed cost level is set to ensure the proper

proportion of private firms engage in basic research, hence the model is doing well on private firms' extensive margin at the expense of a 5 percentage point deviation from this target. In our model, the share of basic research in total private research investment is 8%. This number is in line with stylized fact 2, which reports 10% for this figure.

Stylized fact number 3 states that there is a positive correlation between basic research intensity and firm size. This is captured in our model as shown in Figure 6.

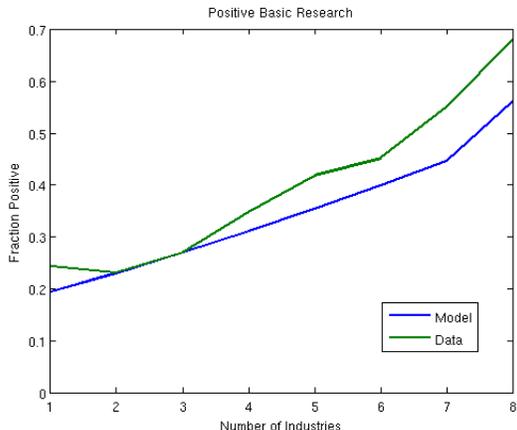


Figure 5: FIRM SIZE DISTRIBUTION

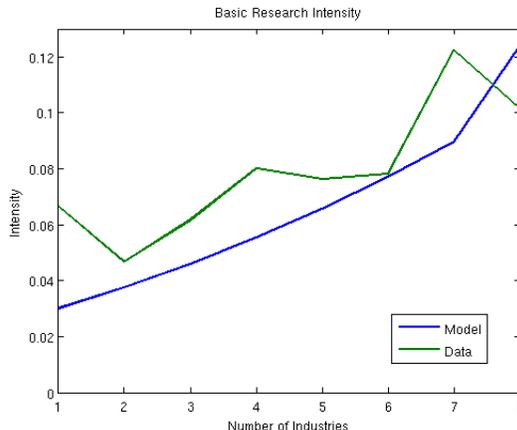


Figure 6: EMPLOYMENT BY # OF INDUSTRIES

The fourth stylized fact regards the correlation between profitability and basic research intensity. The data does not show a significant correlation between these two values. The same implication emerges from our model. In the baseline model, the correlation between profitability and basic research intensity is XXX. This result emerges because basic research investment is determined through instantaneous idiosyncratic shocks which are history independent, whereas profitability is a direct function of the history of step size realizations.

In the data, firms operate on average in 1.9 industries compared to 2.2 in our model. Furthermore, we find the mean squared in the model to be 6.9, which is in accordance with the observed data (7.0).

Stylized fact number 6 was one of the primary motivations for introducing multi-industry presence into our model. As explained previously in the text, multi-industry presence plays an important role in increasing basic research incentives, by allowing a greater potential to internalize the positive spillover from basic research. In our reduced form analysis, we found a significant and positive correlation between multi-industry presence and basic research intensity. This has been the key moment to identify the cross-industry spillover parameter. Our model successfully generates this positive correlation, with a value of 0.011. Rows 1 through

8 report the basic research intensity for different technological bases in the data and model. The slope of the fitted linear line in the data is 0.008.

Our model naturally generates a positive correlation between multi-industry presence and firm size, which has been documented empirically in stylized fact 7. This arises since both of these moments are strongly correlated with firm survival. In the model, we find a correlation of 0.62 between the log of firm sales and multi-industry presence. In the data, this value is 0.73.

Stylized fact 8 documents a well known feature the data, which is documented extensively in the literature. In our model, we capture this fact with a skewness of the firm size distribution of 2.85. This value is 3.07 in the data.

The table above reports some additional moments that are not captured by the stylized facts. For instance, the mean profitability is 5.9% in the data, yet our model predicts a value of 4.2%. The prime determinants of profitability are the step sizes for basic and applied innovation. However, these also affect the investment levels for both types of research, since this increases the return to successful innovation. Therefore, the step size parameters are set to compromise between hitting the profitability moment and the research investment moments.

In our model, firms grow at a rate of 4.4% on average. This is close to the empirical counterparts, which is 4.1%. The exit rate predicted by our model is 3.1%. This value is similar to its empirical counterpart 2.7%.

We are targeting three additional moments regarding research investments. The first two are the average ratios of applied and basic research investment to firm sales. The model slightly overshoots applied research (8.3% vs 7.5%) and closely predicts the level of basic research (0.31% vs 0.36%). The third moment that we are targeting is the ratio of private to public basic research investment. In our estimation, we take academic funding as percentage of GDP as given. Then we let the model economy determine overall private spending. There are many other moments that we target that directly determine private basic research spending. Therefore, hitting this moment comes at the expense of the overall fit.

All of these components of the economy determine the aggregate growth rate. Our model matches the observed growth rate closely. Our model economy grows at a rate of 1.4%, while the French economy grew at an average rate of 1.5% during the period studied (2000-2006).

Finally, this growth rate, in conjunction with the discount factor and the CRRA parameter, determine the interest rate in our economy. Our model predicts an economy interest rate of 3.5%. This figure closely reflects interest rates of the French economy during the sample period. Indeed, European Central Bank interest rates on deposit facilities which banks may

use to make overnight deposits, varied between 3.25% in July 2000 and 2.5% in December 2006. Another comparison figure of interest, perhaps more closely related to the French economy, is the interest paid on 10 year French Treasury Bonds. On average the interest paid on such government bonds was about 4.3% during the sample period and was about 3.4% in 2005.

We will now focus on the social planner's problem to quantify the underinvestment in our model economy. Then we will turn to various policies that could address this inefficiency.

4.5.4 Quantifying the Social Planner's Optimum

In this section, we are going to provide the solution to the social planner's problem described in section 3.5.8. Recall that we are considering a planner who controls the research labs of firms but not the firms' production decisions. The following table summarizes these results

C_0^*/C_0^{SP}	α^{SP}	$1 + k^{SP}$	L_{prod}^{SP}	L_{bas}^{SP}	L_{app}^{SP}	\bar{a}^{SP}
148	100	128	60.2	14.5	23.6	16.3
\tilde{a}^{SP}	\tilde{d}^{SP}	\bar{b}^{SP}	$\mathcal{F}(\mathbf{h}_m^{b,SP})$	Ψ^{SP}	g^{SP}	
0.26	1.60	1.60	100	49.4	5.75	

One thing to note in this class of model is the following. If the economy functions with perfect competition, this removes all incentives for R&D, and as a result, all labor is allocated for production, so that the initial consumption is maximal and the growth rate is zero. Any plan to increase welfare will necessarily have to increase growth at the expense of initial consumption by allocating more workers to research activities. As is well known, the patent mechanism is one way to provide these incentives to hire researchers and innovate. However, the patent mechanism is not guaranteed to provide the proper incentives for research. This fact reflects itself in the table above. Although firms enjoy full monopoly power over the results of their innovations, a large fraction of the labor force is employed for production. Even though the planner allocates roughly 40% of labor to research activities, the decentralized economy employs only 11% of workers for research. As a result, the initial consumption is 48% higher in the decentralized economy, relative to the planner's choice. By construction, the consumption equivalent parameter α^{SP} is equal to one. The insufficiency of the patent mechanism also reflects itself in the planner's choice of the rate of applied research, which is roughly 17% at the optimum, while this number was only 8% in the decentralized economy.

The planner not only optimizes over the level of workers employed for research, but also the composition of those workers amongst the various types of research. The planner internalizes all types of spillovers arising from basic research. Therefore, the aggregate flow of basic research

(excluding spillovers) rises to 3.1%, while it was only 0.6% in the decentralized economy. This happens both on the intensive and extensive margin of basic research. In fact, the planner finds it optimal to employ nearly all research labs, regardless of their fixed cost draw. As a result, a larger fraction of product lines operate with high technology (60%, as opposed to 20% earlier). Consequently, growth rises to 5.7% from 1.4%. Overall, the decentralized economy's welfare corresponds to an 56% consumption equivalent with respect to the social planner's optimum.

The following policy experiments will try to bridge this gap.

5 Policy Analysis

In this section, we analyze the impact of different types of research subsidies. Given our distinction between basic and applied research, it seems natural to propose different subsidy policies for different types of research spending. However, such a policy would generate a moral hazard problem since firms would have an incentive to misreport the type of research they undertake, which is almost impossible to verify for a policymaker. In the previous section, we provided the solution to the social planner's problem as a benchmark. We next consider a hypothetical case where the policymaker can observe the type of research project and subsidize it accordingly. Secondly, to imitate real-world research subsidies, we consider the case in which only uniform subsidies to all research types are implemented. We then consider varying the amount of funding going to academic research labs. Finally, we allow for both uniform subsidies and changes in academic research funding.

5.0.5 R&D Policies

Type-Dependent (TD) Research Subsidy Assume first that the policymaker can distinguish between different types of research efforts and accordingly provide differentiated subsidy rates. Let $\psi_a \in [0, 1)$ and $\psi_b \in [0, 1)$ denote the applied research and basic research subsidy rates respectively. The share of GDP allocated to academic research (\hat{T}/Z) is kept constant at 1% by the policymaker. Total spending by firm f of type (m, n, h^b) is then simply

$$n(1 - \psi_a) \tilde{w}^{TD} h_a(a_{mn}^{TD}) + n(1 - \psi_b) \tilde{w}^{TD} \mathbf{1}_{(h^b < h_m^{TD})} \left[h_b(h_{mn}^{TD}) + h^b \right]$$

The first-order conditions for the value function in (19) become:

$$\Psi^{TD} \beta_\eta^{TD} + (1 - \Psi^{TD}) \beta_\lambda^{TD} + \beta_m^{TD} = (1 - \psi_a) \tilde{w}^{TD} h'_a(a_m^{TD}) \quad (30)$$

$$(1 + \rho_m)(\beta_\eta^{TD} + \beta_m^{TD}) = (1 - \psi_b) \tilde{w}^{TD} h'_b(b_m^{TD}) \quad (31)$$

In these expressions, the left side indicate the expected return to research investment and the right side is the relevant marginal cost. Note that an increase in the subsidy rate (ψ_a or ψ_b) reduces of research cost for the firm and leads to an increase in the research effort as a result. The advantage of this policy is clearly that the policymaker can target a particular type of research to correct the underinvestment in it. The following table reports the optimal subsidy rates under this policy. As there are spillovers associated with basic research that are not internalized, one would expect that the optimal subsidy to basic research would be higher.

TYPE-DEPENDENT RESEARCH SUBSIDY (IN PERCENTAGES)								
ψ_a	ψ_b	\hat{T}/Z	C_0^{TD}/C_0^{SP}	α^{TD}	$1 + \bar{k}^{TD}$	L_{prod}^{TD}	L_{bas}^{TD}	L_{app}^{TD}
64	83	0.8	112	82.5	128	67.5	12.1	20.4
\bar{a}^{TD}	\tilde{a}^{TD}	\tilde{d}^{TD}	\bar{b}^{TD}	$\mathcal{F}(\mathbf{h}_m^{b,TD})$	Ψ^{TD}	g^{TD}	\tilde{w}^{TD}	
14.0	0.21	0.18	1.80	≈ 100	35.7	3.87	.609	

Our quantitative results confirm this expectation. Since the underinvestment is mainly in basic research, the optimal type-dependent subsidy dictates a much larger subsidy rate for it, namely $\psi_b = 83\%$ and $\psi_a = 64\%$. As a result, the researcher share and the equilibrium growth rate increase. This happens at the expense of a lower initial consumption, which happens through allocating a lower share of workers to production (76% as opposed to 89% in the baseline economy), but overall, welfare increases. The consumption equivalent rises to 82% with respect to the social planner's optimum, which is a 26 percentage point improvement over the baseline economy.

As discussed above, this policy is hard to implement in the real world due to the moral hazard problem. Therefore we focus on a policy providing a uniform subsidy across firms and research types.

Uniform Private (UP) Research Subsidy With this policy, the government subsidizes a fraction $\psi \in [0, 1)$ of each firm's total research investment, keeping the share of funds allocated to academic research constant. This implies that the total spending by firm f is simply $n(1 - \psi)\tilde{w}^{UP} \left(h_a(a_{mn}^{UP}) + \mathbf{1}_{(h^b < \mathbf{h}_m^{UP})} [h_b(h_{mn}^{UP}) + h^b] \right)$. In that case the first-order conditions for the value function in (19) become

$$\Psi^{UP} \beta_\eta^{UP} + (1 - \Psi^{UP}) \beta_\lambda^{UP} + \beta_m^{UP} = (1 - \psi) \tilde{w}^{UP} h'_a(a_m^{UP}) \quad (32)$$

$$(1 + \rho_m)(\beta_\eta^{UP} + \beta_m^{UP}) = (1 - \psi) \tilde{w}^{UP} h'_b(b_m^{UP}) \quad (33)$$

Note that such a policy subsidizes not only basic research in (33), but also applied research in (32). Figure 7 plots the steady-state welfare, growth, researcher share in the labor market, and the initial consumption against the subsidy rate.

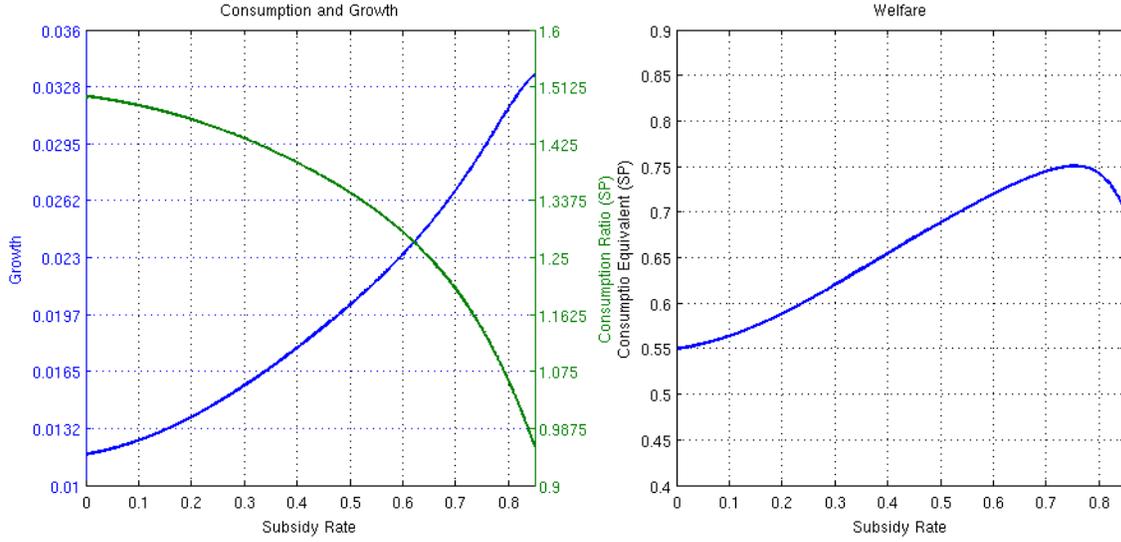


Figure 7: EFFECTS OF UNIFORM SUBSIDY

The following table summarizes the results of the optimal subsidy rate (75%).

UNIFORM RESEARCH SUBSIDY (IN PERCENTAGES)							
ψ	\hat{T}/Z	C_0^{UP}/C_0^{SP}	α^{UP}	$1 + \bar{k}^{UP}$	L_{prod}^{UP}	L_{bas}^{UP}	L_{app}^{UP}
75	0.8	114	75.0	128	68.5	4.73	26.7
\bar{a}^{UP}	\tilde{a}^{UP}	\bar{d}^{UP}	\bar{b}^{UP}	$\mathcal{F}(\mathbf{h}_m^{b,UP})$	Ψ^{UP}	g^{UP}	\tilde{w}^{UP}
17.1	0.27	0.15	0.85	99.5	21.1	3.27	0.627

Our analysis of the baseline economy and the planner's economy documented a significant underinvestment in research overall and misallocation between the different types of research. A uniform subsidy then attempts to align the former constrained by an inability to affect the latter. Although the optimal type dependent basic subsidy is 83%, the optimal uniform subsidy is only 75%, due to cross-subsidization of applied research whose optimal level was only 64%.

Under this policy, we are allocating a larger fraction of the labor force to research relative to the baseline economy. Overall, the researcher share goes up to 31% from 11%. This reduces the initial consumption to 77% of the baseline, yet increases the growth rate to 3.3%. In sum, there is a sizable welfare gain from this policy, producing a consumption equivalent of 75% with respect to the social planner's optimum, a 19 percentage point increase from the baseline.

Although the underinvestment in basic research is sizable as evident from the 83% subsidy rate in the previous section, uniform policy partially makes up for the underinvestment and provides 75% subsidy which is 8 percentage point below the optimal basic research subsidy rate. The main lesson to be drawn from this is that a uniform research subsidy should take into account its negative welfare consequences through its oversubsidization of applied research.

Finding a feasible method to differentiate basic and applied research is essential for better innovation policies.

Optimal Academic Fraction of GDP (AC) In this section we will look for the optimal public funding level for academic research as a fraction of GDP (\hat{T}/Z) keeping the baseline subsidies fixed. This is particularly important because, as was documented in the baseline estimation, almost 26% of growth is due to the academic sector. Moreover, basic research produces many uninternalized spillovers and the academic sector is the major source of this type of research in the economy (64% of total basic research). The following table summarizes the results of the optimal academic policy.

ACADEMIC RESEARCH POLICY (IN PERCENTAGES)

ψ	\hat{T}/Z	C_0^{AC}/C_0^{SP}	α^{AC}	$1 + \bar{k}^{AC}$	L_{prod}^{AC}	L_{bas}^{AC}	L_{app}^{AC}
10	10.0	120	83.8	128	72.3	18.4	9.32
\bar{a}^{AC}	\tilde{a}^{AC}	\tilde{d}^{AC}	\bar{b}^{AC}	$\mathcal{F}(\mathbf{h}_m^{b,AC})$	Ψ^{AC}	g^{AC}	\tilde{w}^{AC}
8.67	0.14	3.34	0.0002	0.08	47.5	3.67	0.544

The advantage of this policy is that it can more directly target the type of research that is mainly underinvested in. As the above table indicates, without changing the private subsidy, this policy can increase both growth and welfare significantly. The optimal fraction of GDP allocated to academic research turns out to be 10.0%. Under this policy 28% of the labor force is allocated to research, however, contrary to the uniform policy case, most of these researchers are working on research projects with high potential for spillovers. The resulting economy has a growth rate of 3.7%. Compared to the uniform, this policy is able to increase the growth rate by 0.4 percentage points while increasing initial consumption.

Our results highlight the special role of academic research in overall growth and show the complementarities present between public and private research. Allocating more resources to academic research not only has a direct effect on growth, but an indirect effect by making private research more productive. However, one should also note that this particular policy alone cannot make up for the underinvestment in research on the part of the private sector. Therefore, the next policy experiment is of particular importance.

Optimal Feasible Policy: Uniform Subsidy and Academic Budget (AU) Our final policy experiment combines both of the feasible policies that have been considered thus far individually. We will allow both the uniform subsidy rate and the academic funding rate to be chosen by the policymaker. The advantage of considering both types of policies is to introduce more freedom to control the incentives for both types of research in a largely separate way. In

particular, ψ and \hat{T}/Z are going to be the choice variables in this exercise. The following table contains the results of this experiment

ACADEMIC AND UNIFORM RESEARCH POLICY							
ψ	\hat{T}/Z	C_0^{AU}/C_0^{SP}	α^{AU}	$1 + \bar{k}^{AU}$	L_{prod}^{AU}	L_{bas}^{AU}	L_{app}^{AU}
73	8.5	99	95.8	128	59.5	13.9	26.5
\bar{a}^{AU}	\tilde{a}^{AU}	\tilde{d}^{AU}	\bar{b}^{AU}	$\mathcal{F}(\mathbf{h}_m^{b,AU})$	Ψ^{AU}	g^{AU}	\tilde{w}^{AU}
17.6	0.28	2.6	0.39	81.1	44.6	5.51	0.677

By using the level of academic funding to reach the proper share of researchers, the policymaker is able to lower the uniform subsidy, thus reducing needless cross-subsidization of applied research. Under the current policy 27% of the labor force is allocated to research. This time around, the composition of workers between applied and basic research is closer to the social optimum. By allocating 5.9% of GDP to academic research and subsidizing private research at a rate of 56%, the policymaker can do even better than the hypothetical case of a type-dependent research subsidy. The growth rate goes up 3.4%, which was only 1.6% in the baseline economy and 3.1% with type-dependent research subsidies. The resulting economy achieves a consumption equivalent of 94% with respect to the social planner's optimum. This is a significant improvement over all the feasible policies we have considered so far, surpassing the consumption equivalent of the next best (academic funding alone) by 2.7 percentage points.

To summarize our findings, recall that the baseline economy, which itself featured a 10% uniform research subsidy, had a 15% lower consumption equivalent than the social optimum. We first considered the most widely discussed policy, which is a uniform subsidy. Using this tool optimally could only partially closed this gap by 3 percentage points. This was mainly due to the fact that the policy could not distinguish between the research types with different spillover and productivity implications. Considering a policy combination that governs both academic and private research could generate a significant improvement, reducing this gap by 9 percentage points. The first main conclusion to be drawn for innovation policy is the importance of recognizing different types of innovations and the impact of policies on these types of research. The second is that it is important to take into account both the direct and indirect effects of academic research on productivity growth when considering growth and innovation policies.

6 Conclusion

In this paper, we studied the distinct roles of basic research and applied research in the growth process, the incentives of firms to invest in these distinct types of research, the contribution

of the academic sector to economic growth, and different government policies to undo the inefficiencies arising from basic innovation spillovers. Our theoretical framework consisted of a general equilibrium model with firms and an academic sector. As opposed to the previous literature, we explicitly allowed firms to invest in both types of research activities. The major difference between basic and applied research is that the former generates potential spillovers in the form of (a) innovation that apply to industries that are not in the activity scope of the firm, and (b) benefits to subsequent applied research within the same industry. Firms' presence in multiple industries determines their ability to appropriate cross-industry spillovers, which in turn determines their incentive to invest in basic research. The heterogeneity induced by our multi-industry framework allowed us to identify some key structural parameters including the probability of cross-industry spillover.

We estimated the model using detailed data on French firms conducting research in the period 2000-2006. Using a large number of empirical moments, our model provided an overall good fit to the data, capturing important stylized facts. We then quantified the contributions of different actors and research types to aggregate growth. For instance, we found that 26% of growth came from the academic sector, and 42% came from basic research investment in the economy. We then solved the social planner's problem and quantified the inefficiency present in the decentralized equilibrium. In particular, we found that the baseline economy achieved a 56% consumption equivalent with respect to the planner's optimal allocation.

Finally, we considered several policies to remedy this inefficiency. We showed that a combination of feasible policies, the uniform research subsidy and share of academic funding in GDP could improve the consumption equivalent by 40 percentage points over the baseline. The advantage of this policy is that it can affect both types of research differentially. While the subsidy lifts both basic and applied research levels, the increase in academic funding brings basic research further towards its optimal level.

The main conclusions we draw are that the impact of basic and applied research on aggregate growth is very distinct. Basic research generates a significant amount of spillovers and R&D policies could be made more effective if designed with an eye to this fact. Using uniform R&D subsidies in an attempt to correct the underinvestment in basic research could harm the economy by oversubsidizing applied research. In fact, complementing uniform subsidies with a stronger support for the academic sector could greatly improve welfare. Our model draws particular attention to the role of academic institutions in the growth process. We hope that the framework presented in this paper will be useful for developing further models to study other important issues such as optimal patent policies.

7 Bibliography

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8 Appendix

8.1 A - Theoretical Appendix

Proof of Proposition 1. Now we propose the following functional form for $\tilde{V}(m, n_\eta, n_\lambda) = \beta_\eta n_\eta + \beta_\lambda n_\lambda + (n_\eta + n_\lambda) \beta_m$. Substitute this into (19)

$$r^* [\beta_\eta n_\eta + \beta_\lambda n_\lambda + (n_\eta + n_\lambda) \beta_m] = \max_{\substack{a_m, b_m, \\ e_m}} \left\{ \begin{array}{l} n_\eta \tilde{\pi}_\eta + n_\lambda \tilde{\pi}_\lambda - (n_\eta + n_\lambda) \tilde{w}^* [h_a(a_{mn}) + h_b(b_{md}) + \mathbf{1}_{(b_m > 0)} h^b] + h_e(e_{mn}) \\ + (n_\eta + n_\lambda) a_m [\Psi^* \beta_\eta + (1 - \Psi^*) \beta_\lambda + \beta_m] \\ + (n_\eta + n_\lambda) b_{md} (1 + \rho_m) [\beta_\eta + \beta_m] + n_\lambda s^* [\beta_\eta - \beta_\lambda] \\ + (n_\eta + n_\lambda) \kappa [\tilde{\Psi}^* \beta_\eta + (1 - \tilde{\Psi}^*) \beta_\lambda + \beta_m - \nu] \\ + \tau^* n_\eta [-\beta_\eta - \beta_m] + \tau^* n_\lambda [-\beta_\lambda - \beta_m] + e_m (n_\eta + n_\lambda) [\beta_{m+1} - \beta_m] \\ + \kappa [(n_\eta + n_\lambda) \nu - [\beta_\eta n_\eta + \beta_\lambda n_\lambda + (n_\eta + n_\lambda) \beta_m]] \end{array} \right\}.$$

Now equating the coefficients of n_η , n_λ and $(n_\eta + n_\lambda)$ we get

$$\begin{aligned} r^* \beta_\eta n_\eta &= \{n_\eta \tilde{\pi}_\eta - \tau^* n_\eta \beta_\eta - \kappa \beta_\eta n_\eta\} \\ r^* \beta_\lambda n_\lambda &= \{n_\lambda \tilde{\pi}_\lambda + n_\lambda s^* [\beta_\eta - \beta_\lambda] - \tau^* n_\lambda \beta_\lambda - \kappa \beta_\lambda n_\lambda\} \\ r^* (n_\eta + n_\lambda) \beta_m &= \max_{\substack{a_m, b_m, \\ e_m}} \left\{ \begin{array}{l} -(n_\eta + n_\lambda) \tilde{w}^* [h_a(a_{mn}) + h_b(b_{md}) + \mathbf{1}_{(b_m > 0)} h^b] + h_e(e_{mn}) \\ + (n_\eta + n_\lambda) a_m [\Psi^* \beta_\eta + (1 - \Psi^*) \beta_\lambda + \beta_m] \\ + (n_\eta + n_\lambda) b_{md} (1 + \rho_m) [\beta_\eta + \beta_m] \\ + (n_\eta + n_\lambda) \kappa [\tilde{\Psi}^* \beta_\eta + (1 - \tilde{\Psi}^*) \beta_\lambda + \beta_m] \\ - \tau^* (n_\eta + n_\lambda) \beta_m + e_m (n_\eta + n_\lambda) [\beta_{m+1} - \beta_m] \\ - \kappa (n_\eta + n_\lambda) \beta_m \end{array} \right\} \end{aligned}$$

which in turn imply

$$\begin{aligned} r^* \beta_\eta &= \tilde{\pi}_\eta / (r^* + \tau^* + \kappa) \\ r^* \beta_\lambda &= (\tilde{\pi}_\lambda + s^* \beta_\eta) / (r^* + s^* + \tau^* + \kappa) \\ r^* \beta_m &= \max_{\substack{a_m, b_m, \\ e_m}} \left\{ \begin{array}{l} -\tilde{w}^* [h_a(a_{mn}) + h_b(b_{md}) + \mathbf{1}_{(b_m > 0)} h^b] + h_e(e_{mn}) \\ + a_m [\Psi^* \beta_\eta + (1 - \Psi^*) \beta_\lambda + \beta_m] + b_{md} (1 + \rho_m) [\beta_\eta + \beta_m] \\ + \kappa [\tilde{\Psi}^* \beta_\eta + (1 - \tilde{\Psi}^*) \beta_\lambda] + e_m [\beta_{m+1} - \beta_m] \end{array} \right\} \end{aligned}$$

■

Proof of Proposition 2. Recall that the final output is propositional to the quality index Q in (18). After Δt period, the quality index becomes

$$Q(t + \Delta t) = \left\{ \begin{array}{l} +(1 + p) \left(\tilde{d}^* \Delta t + \tilde{b}^* \Delta t + (\tilde{a}^* \Delta t + \tilde{a}^* \Delta t) \Psi^* \right) \ln(1 + \eta) \\ + (\tilde{a}^* \Delta t + \tilde{a}^* \Delta t) (1 - \Psi^*) \ln(1 + \lambda) \\ + Q(t) \end{array} \right\}.$$

This follows from the fact that $(1+p) \left(\tilde{d}^* \Delta t + \bar{b}^* \Delta t \right) + (\bar{a}^* \Delta t + \tilde{a}^* \Delta t) \Psi^*$ fraction of the product lines are going to experience an η improvement, $(\bar{a}^* \Delta t + \tilde{a}^* \Delta t) (1 - \Psi^*)$ fraction is going to experience a λ improvement and the remaining product lines will experience no improvement at all. After some simple algebra we get

$$\lim_{\Delta t \rightarrow 0} \frac{Q(t + \Delta t) - Q(t)}{\Delta t} = (1+p) \left(\tilde{d}^* + \bar{b}^* + (\bar{a}^* + \tilde{a}^*) \Psi^* \right) \ln(1 + \eta) + (\bar{a}^* + \tilde{a}^*) (1 - \Psi^*) \ln(1 + \lambda).$$

The last line of the proposition uses the fact that $\ln(1+x) \approx x$ when x is small. ■

Derivation of Invariant $\Gamma_{m,n}^*$. We assume that when a firm loses its last product in a particular industry, it maintains a foothold there, in the sense that it still receives buyout offers and can still directly utilize basic research relevant to that industry. When a firm loses all of its products or receives a destructive shock, it ceases to exist. We wish to find the joint distribution over the number of industries a firm is in and how many product lines it owns. For notational convenience, let us denote the basic research flow from m -industry firms by $\hat{b}_m^* \equiv \mathcal{F}(\mathbf{h}_m^{b*}) b_m^*$. The flow equation for firms in m industries with n products is

$$\begin{array}{l} \text{OUTFLOW} \qquad \qquad \qquad \text{INFLOW} \\ \left[\begin{array}{c} a_1^* + \hat{b}_1^* + \tau^* + \kappa \\ + e_1^* + \kappa \end{array} \right] \Gamma_{1,1}^* = \tilde{a}^* + 2\tau^* \Gamma_{1,2}^* \\ \left[\begin{array}{c} a_m^* + \hat{b}_m^* + \tau^* + \kappa \\ + e_m^* + \kappa \end{array} \right] \Gamma_{m,1}^* = 2\tau^* \Gamma_{m,2}^* + e_{m-1}^* \Gamma_{m-1,1}^* \text{ for } m \geq 2 \\ \left(\begin{array}{c} n \left(a_m^* + \hat{b}_m^* + \tau^* + \kappa \right) \\ + e_m^* + \kappa \end{array} \right) \Gamma_{m,2}^* = \left\{ \begin{array}{c} \left(a_m^* + \hat{b}_m^* (1 - \rho_m) + \kappa \right) \Gamma_{m,n-1}^* \\ + 3\tau^* \Gamma_{m,n+1}^* + e_{m-1}^* \Gamma_{m-1,n}^* \end{array} \right\} \text{ for } m \geq 1 \\ \left(\begin{array}{c} n \left(a_m^* + \hat{b}_m^* + \tau^* + \kappa \right) \\ + e_m^* + \kappa \end{array} \right) \Gamma_{m,n}^* = \left\{ \begin{array}{c} (n-1) \left(a_m^* + \hat{b}_m^* (1 - \rho_m) + \kappa \right) \Gamma_{m,n-1}^* \\ + (n-2) \rho_m \hat{b}_m^* \Gamma_{m,n-2}^* \\ + (n+1) \tau^* \Gamma_{m,n+1}^* + e_{m-1}^* \Gamma_{m-1,n}^* \end{array} \right\} \text{ for } n \geq 3, m \geq 1 \end{array}$$

where we use the convention $\Gamma_{m,-1}^* = \Gamma_{m,0}^* = 0$ and $e_0^* = 0$. The first line equates the outflows from $(m=1, n=1)$ which happens once the firm loses its product at the rate $\tau^* + \kappa$, acquires a new product line at the rate κ , innovates a new good at the rate $a_1^* + \hat{b}_1^*$ on average or expands into a new industry at the rate e_1^* . On the other hand, inflow happens from outsiders at the rate \tilde{a}^* and from the firms with 2 products which lose one of their products at the rate $2\tau^*$. Similar reasonings apply to the next lines. ■

8.2 B - Reduced Form Appendix

TABLE A.1: BASIC RESEARCH INTENSITY AND MULTI-MARKET ACTIVITY - TOBIT MODEL

	1 Digit SIC Level (1)	2 Digit SIC Level (2)	3 Digit SIC Level (1)	3 Digit SIC Level (2)	4 Digit SIC Level (1)	4 Digit SIC Level (2)
Log # of Industries	0.169*** (0.02)	0.041*** (0.00)	0.132*** (0.01)	0.032*** (0.00)	0.111*** (0.01)	0.027*** (0.00)
Constant	-0.549*** (0.03)	-0.546*** (0.03)	-0.547*** (0.03)	-0.542*** (0.03)	0.099*** (0.01)	0.024*** (0.00)
Year FE	YES	YES	YES	YES	YES	YES
Organization FE	YES	YES	YES	YES	YES	YES
N	13708	13708	13708	13708	13708	13708

Notes: Pooled data for the period 2000-2006. Estimates are obtained using censored Tobit estimation. Estimates in columns (1) relate to $\partial E(y^*|x)/\partial x$, i.e. the marginal effect of the regressors with respect to the latent variable mean. Estimates in columns (2) relate to $\partial E(y|x, y > 0)/\partial x$, i.e. the marginal effect of the regressors with respect to the censored variable mean, and are evaluated at the sample mean $x = \bar{x}$. Robust standard errors clustered at the firm level in parentheses. See appendix for the definition of variables.

TABLE A.2: BASIC RESEARCH INTENSITY AND MULTI-MARKET ACTIVITY - TOBIT MODEL

	1 Digit SIC Level		2 Digit SIC Level		3 Digit SIC Level		4 Digit SIC Level	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Log # of Industries	0.127*** (0.02)	0.031*** (0.01)	0.107*** (0.02)	0.026*** (0.00)	0.092*** (0.02)	0.022*** (0.00)	0.081*** (0.02)	0.020*** (0.00)
Total Employment	0.011* (0.01)	0.003* (0.00)	0.007 (0.01)	0.002 (0.00)	0.005 (0.01)	0.001 (0.00)	0.005 (0.01)	0.001 (0.00)
Foreign HQ	-0.052*** (0.02)	-0.013*** (0.00)	-0.046** (0.02)	-0.011** (0.00)	-0.044** (0.02)	-0.011** (0.00)	-0.044** (0.02)	-0.011** (0.00)
Profitability	0.025 (0.02)	0.006 (0.00)	0.026 (0.02)	0.006 (0.00)	0.026* (0.02)	0.006* (0.00)	0.026* (0.02)	0.006* (0.00)
Market Share	0.261** (0.10)	0.063** (0.02)	0.251** (0.10)	0.061** (0.02)	0.257** (0.10)	0.062** (0.02)	0.257** (0.10)	0.062** (0.03)
Constant	-0.581*** (0.04)		-0.561*** (0.04)		-0.554*** (0.04)		-0.549*** (0.04)	
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Organization FE	YES	YES	YES	YES	YES	YES	YES	YES
N	13704	13704	13704	13704	13704	13704	13704	13704

Notes: Pooled data for the period 2000-2006. Estimates are obtained using censored Tobit estimation. Estimates in columns (1) relate to $\partial E(y^*|x)/\partial x$, i.e. the marginal effect of the regressors with respect to the latent variable mean. Estimates in columns (2) relate to $\partial E(y|x, y > 0)/\partial x$, i.e. the marginal effect of the regressors with respect to the censored variable mean, and are evaluated at the sample mean $x = \bar{x}$. Robust standard errors clustered at the firm level in parentheses. See appendix for the definition of variables.

8.3 C - Quantitative Appendix

SOME KEY ENDOGENOUS VARIABLES (IN PERCENTAGES)

\bar{a}^*	\tilde{a}^*	\tilde{d}^*	\bar{b}^*	\bar{e}^*	$\bar{\rho}^*$	s^*	Ψ^*	g^*	r^*	\tilde{w}	α^*
5.8	0.10	0.39	0.04	0.11	0.22	0.47	10.5	0.88	6.76	0.471	87.0

SOCIAL PLANNERS OPTIMUM (IN PERCENTAGES)

C_0^*/C_0^{SP}	α^{SP}	$1 + \bar{k}^{SP}$	L_{prod}^{SP}	L_{bas}^{SP}	L_{app}^{SP}	\bar{a}^{SP}
118	100	126	75.5	9.9	13.5	11.3
\tilde{a}^{SP}	\tilde{d}^{SP}	\bar{b}^{SP}	$\mathcal{F}(\mathbf{h}_m^{b,SP})$	Ψ^{SP}	g^{SP}	
0.18	1.54	1.23	100	42.9	3.88	

TYPE-DEPENDENT RESEARCH SUBSIDY (IN PERCENTAGES)

ψ_a	ψ_b	\hat{T}/Z	C_0^{TD}/C_0^{SP}	α^{TD}	$1 + \bar{k}^{TD}$	L_{prod}^{TD}	L_{bas}^{TD}	L_{app}^{TD}
50	74	0.8	105	93.2	126	79.6	8.2	12.2
\bar{a}^{TD}	\tilde{a}^{TD}	\tilde{d}^{TD}	\bar{b}^{TD}	$\mathcal{F}(\mathbf{h}_m^{b,TD})$	Ψ^{TD}	g^{TD}	\tilde{w}^{TD}	
10.0	0.16	0.32	1.38	≈ 100	31.1	2.72	0.520	

UNIFORM RESEARCH SUBSIDY (IN PERCENTAGES)

ψ_a	ψ_b	\hat{T}/Z	C_0^{TD}/C_0^{SP}	α^{TD}	$1 + \bar{k}^{TD}$	L_{prod}^{TD}	L_{bas}^{TD}	L_{app}^{TD}
59	59	0.8	109	91.0	126	82.3	3.8	13.9
\bar{a}^{TD}	\tilde{a}^{TD}	\tilde{d}^{TD}	\bar{b}^{TD}	$\mathcal{F}(\mathbf{h}_m^{b,TD})$	Ψ^{TD}	g^{TD}	\tilde{w}^{TD}	
11.2	0.18	0.31	0.58	90.0	19.4	2.20	0.522	

ACADEMIC RESEARCH POLICY (IN PERCENTAGES)

ψ_a	ψ_b	\hat{T}/Z	C_0^{TD}/C_0^{SP}	α^{TD}	$1 + \bar{k}^{TD}$	L_{prod}^{TD}	L_{bas}^{TD}	L_{app}^{TD}
10	10	5.0	109	94.9	126	82.6	10.3	7.1
\bar{a}^{TD}	\tilde{a}^{TD}	\tilde{d}^{TD}	\bar{b}^{TD}	$\mathcal{F}(\mathbf{h}_m^{b,TD})$	Ψ^{TD}	g^{TD}	\tilde{w}^{TD}	
7.2	0.12	2.26	0.0	0.12	37.9	2.60	0.487	

ACADEMIC AND UNIFORM RESEARCH POLICY (IN PERCENTAGES)

ψ_a	ψ_b	\hat{T}/Z	C_0^{TD}/C_0^{SP}	α^{TD}	$1 + \bar{k}^{TD}$	L_{prod}^{TD}	L_{bas}^{TD}	L_{app}^{TD}
58	58	4.7	99.1	97.2	126	74.8	9.3	15.9
\bar{a}^{TD}	\tilde{a}^{TD}	\tilde{d}^{TD}	\bar{b}^{TD}	$\mathcal{F}(\mathbf{h}_m^{b,TD})$	Ψ^{TD}	g^{TD}	\tilde{w}^{TD}	
12.5	0.20	1.99	0.20	50.5	37.3	3.68	0.546	