

MULTI-PRODUCT FIRMS AND TRADE LIBERALIZATION*

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Abstract

This paper develops a general equilibrium model of international trade that features selection across firms, products and countries. Firms' export decisions depend on a combination of firm "productivity" and firm-product-country "consumer tastes", both of which are stochastic and unknown prior to the payment of a sunk cost of entry. Higher-productivity firms export a wider range of products to a larger set of countries than lower-productivity firms. Trade liberalization induces endogenous reallocations of resources that foster productivity growth both within and across firms. Empirically, we find key implications of the model to be consistent with U.S. trade data.

Keywords: heterogeneous firms, endogenous product scope, love of variety, core competency

JEL classification: F12, F13, L11

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1. Introduction

While multiple-product, multiple-destination firms dominate international trade, comparatively little research examines their production and export decisions or how these decisions are influenced by globalization. We develop a tractable, general equilibrium model of endogenous product and destination-country selection that offers a natural and intuitive explanation for key features of U.S. trade data. Our model and empirical analysis demonstrate the importance of exporters' product and country selection for aggregate trade flows as well as for firm- and industry-level productivity growth in response to trade liberalization.

We emphasize a distinction between three “extensive” margins of trade – the number of exporting firms, the number of exported products and the number of export destinations – and an “intensive” margin of average exports per firm-product-destination. Our empirical evidence reveals that these extensive margins account for much of the observed variation in export shipments across countries and firms, and that they also behave quite differently from the intensive margin. We show that the well-known negative effect of distance on bilateral trade flows, for example, is due primarily to extensive margins: while the number of exporting firms and the number of exported products decline significantly with distance, the relationship between distance and the intensive margin is, if anything, *positive*. In our model, variation in extensive margins is driven by selection, with the most able firms exporting more products to more destinations. As trade costs fall, the weakest firms exit and, within surviving firms, the least-profitable products are dropped. These reactions induce revenue-based productivity growth both across and within firms, with the latter providing an additional source of welfare gains from trade. The model thus provides an intuitive micro-foundation for the much-discussed idea that international trade spurs firms to rationalize production, but one that does not imply money being left on the table prior to liberalization.

Our theoretical setup is straightforward. Following the closed-economy model of Bernard, Redding and Schott (2009), we allow firms to produce multiple products of varying profitability. We assume that firm profitability in a particular product increases with two stochastic and independent draws. The first, for firm “productivity”, governs the amount of labor that must be used to produce a unit of output. The second, for firm-product-country “consumer tastes”, regulates demand for a firm’s product in a particular destination country. We assume both draws are revealed to firms after incurring a sunk cost of entry. If firms decide to enter after having observed these draws, they face fixed and variable costs for each good they choose to supply to a market as well as a fixed cost of serving each market that is independent of the number of goods supplied. On the demand side, we assume consumers possess Dixit-Stiglitz (1977) constant elasticity of substitution preferences.

The model yields several distinctive predictions that find support in U.S. trade data.

The first is that trade liberalization induces firms to shift resources towards their relatively high-profit (i.e., “core-competency”) products.¹ As in the Melitz (2003) model, our model has falling trade costs heightening competition and causing the lowest-productivity firms to exit. In our model, a similar reallocation also occurs within firms, as greater competition induces all surviving firms to drop their least-profitable, domestically sold products. Evidence in favor of this reallocation is found in several existing studies of firm diversification during trade liberalization. Baldwin and Gu (2009), for example, find a sharp decline in product diversification among Canadian firms following the tariff reductions mandated by the Canada-U.S. Free Trade Agreement (CUSFTA). Here, we provide evidence of a similar retrenchment among U.S. manufacturing firms in response to reductions in Canadian tariffs on U.S. imports.

A second set of predictions relates to the response of the extensive and intensive margins of trade to changes in variable trade costs. In our model, increases in the variable trade costs of serving a particular destination country decrease the country-level extensive margins of exports, i.e., the number of exporting firms and the number of exported products. The effect of rising trade costs on the intensive margin of average exports per firm-product-country, in contrast, is ambiguous. On the one hand, higher variable trade costs decrease a given firm’s exports of a given product to a given destination. On the other hand, higher variable trade costs induce firms to export fewer products and serve fewer export destinations with lower values of consumer tastes, raising average exports per firm-product-destination. We investigate the empirical relevance of these predictions cross-sectionally, examining how U.S. exports vary across trading partners using partners’ distance from the United States as a proxy for variable trade costs. In line with the model’s predictions, we find that an increase in distance has a negative and statistically significant association with the extensive margins and a positive but statistically insignificant effect on the intensive margin.

A third set of predictions relates firm productivity to firms’ extensive margins, i.e., the number of products firms export and the number of countries to which they export. In the model, more productive firms have more active extensive margins because they are able to earn higher revenue and therefore cover the fixed costs of exporting at lower levels of consumer demand. Here, too, there is ambiguity with respect to the intensive margin: while higher productivity raises a firm’s exports of a given product to a given destination, it also extends their reach into additional products and destinations with lower values of consumer tastes and therefore lowers exports. We evaluate this implication of the model using cross-sectional data on U.S. firm exports. We find a positive and statistically significant relationship between firms’ estimated productivity and both the number of products they export and the number of export destinations they serve. In contrast, we

¹The concept of core competency appears in both the theory of the firm and the business strategy literature (see for example Porter 1998 and Sutton 2005). As formalized here, core competency refers to firms’ most profitable activity.

find that average exports per product and destination are generally flat with respect to firm productivity.

Our final set of empirical results demonstrate the model's usefulness for understanding how the extensive margins of trade contribute to the extreme inequality of U.S. exports across exporting firms and destinations. We compare the actual distributions of exports across firms and countries to hypothetical distributions that successively eliminate the influence of extensive margins. In both cases, we find substantial changes in the pattern of exports, with exports considerably less concentrated across firms and countries in the hypothetical distributions than in the actual distributions.

Our analysis contributes to three literatures. First, we extend the influential heterogeneous-firm model of Melitz (2003) to incorporate multiple-product, multiple-destination firms as well as heterogeneity within firms. Our effort is related to other generalizations of the Melitz framework, including Eaton, Kortum and Kramarz (2008), who investigate the distribution of export shipments across countries, Chaney (2008), who analyzes the relationship between trade costs, the elasticity of substitution and trade flows, and Bernard, Redding and Schott (2007) and Melitz and Ottaviano (2008a), who study the behavior of heterogeneous firms in settings incorporating endowment-driven comparative advantage and market power, respectively.

Second, our paper is related to the closed-economy industrial organization literature on multiple-product firms. Early research in this literature, reviewed by Bailey and Friedlaender (1982), emphasizes supply-side economies of scope. Subsequent analysis, by Brander and Eaton (1984), Shaked and Sutton (1990), Eaton and Schmidt (1994) and Johnson and Myatt (2003), introduces demand-side forces favoring production of multiple goods as well as strategic interaction between firms. More recently, Klette and Kortum (2004) and Lentz and Mortenson (2005) have developed innovation-based models of multiple-product firms, while Bernard, Redding and Schott (2009) develop a model emphasizing firm heterogeneity and selection.

Finally, our paper relates to more recent open-economy models of multiple-product firms that have appeared in the international trade literature. One strand of this research, e.g., Eckel and Neary (2006), focuses on strategic interaction between a fixed number of symmetric multiple-product firms. Another, which includes Agur (2006) and Feenstra and Ma (2008), examines the relationship between trade liberalization and the proliferation of product variety with multiple-product firms. Several other contributions vary in terms of how firms' extensive margins are determined. Nocke and Yeaple (2006) consider a model in which firms with higher organizational capability produce a larger number of products but where the devotion of this capability to larger numbers of goods raises the (common) cost of producing every product. Melitz and Ottaviano (2008b) entertain a setting where firms choose endogenous ranges of varieties within products subject to a ladder of progressively higher costs as they move away from their ideal variety. Arkolakis

and Muendler (2008) model an environment in which firms supply endogenous ranges of varieties subject to market entry costs that are convex in the number of products exported to a destination. Here, we seek a tractable general equilibrium framework for modeling the distribution of exports across firms, products and destinations that does not impose specific functional forms on probability distributions and which incorporates demand heterogeneity across countries within firms and products.

The remainder of the paper is structured as follows. Section 2 provides motivation for key assumptions of the model. Section 3 develops the model and solves for general equilibrium. Section 4 examines the comparative statics of trade liberalization and the properties of the open-economy equilibrium. Section 5 provides evidence of the model’s consistency with key features of U.S. trade data. Section 6 concludes. Appendices A through C contain data and theoretical appendices.

2. Motivation

Key attributes of the data that we seek to incorporate in our model are variation in exports across firms, variation in exports across products within firms, and variation in exports across countries within firm-products. In our model we interpret this variation as arising from an interaction of productivity that varies across firms and consumer tastes that vary across firm-product-country combinations. In this section we document sources of variation in U.S. exports across firms, products and countries and also show that U.S. firms’ export markets do not appear to follow a strict “hierarchy”. The data used here are described in greater detail in Section 5 and Appendix A.

We evaluate the importance of firm, product and destination-country attributes in U.S. exports by regressing the logarithm of firm-product-country exports on three sets of fixed effects and comparing their explanatory power. Table 1 reports results using data for 2002, but we note that we obtain similar results across the years spanned by our data. Our first step is to use country-product fixed effects to sweep out heterogeneity across countries and products that is common to all firms, such as comparative advantage, trade costs or markets’ and products’ overall size. In our regressions, products correspond to ten-digit Harmonized System categories. As noted in the first row of the table, country-product fixed effects explain 26 percent of the variation in log exports.

The second row of the table investigates the importance of firm attributes via the addition of firm fixed effects. Addition of these fixed effects increases the R^2 of the model to 43 percent, indicating that firm-level factors that are common across products and countries explain roughly 23 percent (17/74) of the remaining variation in log exports. This relative importance of firm factors motivates our use of a firm “productivity” draw. The final regression, reported in the third row of Table 1, uses firm-product rather than firm fixed effects. This regression allows for product variation within firms that is constant across countries. The R^2 of 89 percent highlights the importance of product selection

Table 1: Common and Idiosyncratic Components of Firm-Product Shipments

Fixed Effects	Cumulative R-Squared
Country-Product	0.26
Country-Product + Firm	0.43
Country-Product + Firm-Product	0.89

Notes: Table reports R^2 s of a sequence of OLS regressions of the logarithm of U.S. firm-product-country exports in 2002 on the fixed effects noted in each row.

within firms in explaining U.S. export flows. At the same time, it reveals that roughly 15 percent (11/74) of the variation in U.S. exports that remains after including country-product fixed effects is due to idiosyncratic heterogeneity across countries within firm-products.² This finding motivates our use of a stochastic firm-product-country “taste” draw.³

Further evidence of idiosyncratic heterogeneity across countries within firm-products, which we refer to as “firm-product-country heterogeneity”, comes from considering hierarchies of export markets. If firm-product-country heterogeneity were unimportant, firms’ products would exhibit a strict hierarchy across destinations, i.e., the markets in which their less profitable products appear would be subsets of the markets in which their most profitable product appears, and rankings of firms’ products by size would be identical across all of the markets in which they jointly appear.⁴ We examine each of these implications in turn.

First, for each multiple-product firm in the U.S. export data in 2002, we compare the markets to which it exports its “largest” (i.e., in the model, its most profitable) product to the markets to which it exports all of its other (“smaller”) products. Absent firm-product-country heterogeneity, the destinations to which firms export their smaller products should be subsets of the destinations to which they export their largest product. Empirically, we define a firm’s largest product as the one it sends to the largest number of export destinations, with ties going to the product with the greatest export value. Let n_{kf} be the number of export destinations for smaller product k in firm f and j_{kf} be the number of destinations smaller product k has in common with the firm’s largest product. Then $Nested_{kf} = j_{kf}/n_{kf}$ represents the share of “smaller” product k ’s markets that are

²As might be expected, the explanatory power of firm and firm-product fixed effects declines when single-export-product firms are excluded. Restricting the sample to firms that export ten or more products, for example, yields cumulative R^2 s of 0.28, 0.37 and 0.79, respectively.

³In the model developed below, firm-product-country exports are log linear in the firm “productivity” draw and the firm-product-country “taste” draw, which is consistent with the log linear empirical specification employed here.

⁴In the model developed below, a strict hierarchy of markets is observed in the absence of firm-product-country heterogeneity in “taste.”

common to the firm’s largest product. If firms’ products exhibit a strict hierarchy, one would expect $Nested_{kf} = 1$. Instead, we find a mean value of 0.67 across the 151,204 firm-product observations in our sample, which bootstrap standard errors reveal to be significantly different from unity.⁵

Examination of bilateral correlations of country rankings across “smaller” products within firms conveys a similar message. Preparing the data in the same manner just described, we compute the rank correlations of all firms’ smaller products with their largest product across all of the destinations to which both products are jointly exported. Absent firm-product-country heterogeneity, all bilateral correlations would be unity. Instead, we find an average correlation of 0.17 across all firm-product combinations, with 38 percent of the bilateral correlations being both positive and statistically significant and an additional 24 percent being positive but not significant. This finding of imperfectly correlated within-firm product rankings across destination markets is consistent Munch and Nguyen’s (2008) analogous demonstration of imperfectly correlated firm rankings across export markets in Danish export data.

3. The Model

We consider a world consisting of many countries and many products. Firms decide whether to produce, what products to make, and where to export these products. Products are imperfect substitutes in demand, and within each product firms supply horizontally differentiated varieties. The model can therefore be interpreted as capturing an economy consisting of many products or as capturing an industry (e.g. consumer electronics) with many products (e.g. DVD-players, Televisions and MP3-players) of which firms supply distinct varieties (e.g. Sony, JVC and Panasonic).⁶ For simplicity, we develop the model for the case in which products are symmetric and the world consists of $n + 1$ symmetric countries.⁷

3.1. Preferences and Endowments

The world consists of a home country and a continuum of n foreign countries, each of which is endowed with L_n units of labor that are supplied inelastically with zero disutility. The representative consumer in each country derives utility from the consumption of a continuum of products that we normalize to the interval $[0, 1]$. There is a constant

⁵Averaging $Nested_{kf}$ first across products within firms and then across firms yields a value of 0.59, which bootstrapping also reveals is significantly different from unity. Results are similar if restricted to a sample of firms exporting ten or more products.

⁶We develop a multi-industry version of the model in a web-based technical appendix.

⁷While the model does not in general have a closed form solution with asymmetric countries and products, it is possible to characterize the equilibrium numerically if specific functional forms for firm productivity and consumer tastes are assumed. With Pareto distributions for firm productivity and consumer tastes, a number of analytical results are possible for asymmetric countries.

elasticity of substitution across products so that the utility function for the representative consumer in country j takes the standard Dixit-Stiglitz (1977) form:

$$U_j = \left[\int_0^1 C_{jk}^\nu dk \right]^\frac{1}{\nu}, \quad 0 < \nu < 1, \quad (1)$$

where k indexes products. Within each product, a continuum of firms produce horizontally differentiated varieties of the product. Hence C_{jk} is a consumption index, which also takes the constant elasticity of substitution form, and depends on the varieties consumed from each country in the world:

$$C_{jk} = \left[\int_0^{n+1} \int_{\omega \in \Omega_{ijk}} [\lambda_{ijk}(\omega) c_{ijk}(\omega)]^\rho d\omega di \right]^\frac{1}{\rho}, \quad 0 < \rho < 1, \quad (2)$$

where i and j index countries, ω indexes varieties of product k supplied from country i to country j and Ω_{ijk} denotes the endogenous set of these varieties.⁸ The demand parameter $\lambda_{ijk}(\omega) \geq 0$ captures the strength of the representative consumer's tastes for firm variety ω and reflects demand heterogeneity, as discussed further below. While not central to our results, we make the natural assumption that the elasticity of substitution across varieties within products is greater than the elasticity of substitution across products: $\sigma \equiv \frac{1}{1-\rho} > \kappa \equiv \frac{1}{1-\nu} > 1$. Similarly, we assume for simplicity that the elasticity of substitution across varieties within products, $\sigma \equiv \frac{1}{1-\rho}$, is the same for all products. The corresponding price index dual to (2) is:

$$P_{jk} = \left[\int_0^{n+1} \int_{\omega \in \Omega_{ijk}} \left(\frac{p_{ijk}(\omega)}{\lambda_{ijk}(\omega)} \right)^{1-\sigma} d\omega di \right]^\frac{1}{1-\sigma}. \quad (3)$$

As products are symmetric, we suppress from here forward the implicit dependence on products k . Furthermore, countries are also symmetric, and the only difference between the domestic market and each export market is that a common value of trade costs has to be incurred for each export market. Therefore, instead of indexing variables in terms of country of production, i , and market of consumption, j , we distinguish between the domestic market, d , and each export market, x , except where otherwise indicated.

3.2. Production Technology

The specification of entry and production follows Melitz (2003). However, we augment that model to allow firms to manufacture multiple products and to allow for demand heterogeneity across products and countries. There is a competitive fringe of potential

⁸Our model focuses on firms that supply multiple products for final consumption. While vertical integration provides another reason why firms can produce multiple products (intermediate and final), many firms supply multiple products for final consumption.

firms who are identical prior to entry. In order to enter, firms must incur a sunk entry cost of $f_{ei} > 0$ units of labor in country i . Incurring the sunk entry cost creates a firm brand and a blueprint for one horizontally differentiated variety of each product that can be produced using this brand.⁹ Firms are defined by their production technology (productivity φ) and product attributes that influence demand (consumer tastes λ). Both productivity and consumer tastes are uncertain prior to entry. Thus, only once the sunk entry cost has been incurred does a firm observe its productivity, φ , and consumer tastes for each of its products k in each of the countries j , λ_{jk} .¹⁰

Firm productivity, $\varphi \in [0, \infty)$, is common across all of a firm's products and is drawn from a continuous distribution $g(\varphi)$ with cumulative distribution function $G(\varphi)$. In contrast, consumer tastes for a firm's varieties, $\lambda_{jk} \in [0, \infty)$, vary across both products k and countries j , and are drawn separately for each product and country from a continuous distribution $z(\lambda)$ with cumulative distribution function $Z(\lambda)$. This conceptualization captures the idea that some firm characteristics (productivity) enhance profitability for the firm as a whole, while other firm characteristics (product attributes) affect profitability unevenly across both products and countries. Our formulation allows for demand heterogeneity across countries for a given product, as well as demand heterogeneity across products for a given country.¹¹

To make use of law of large numbers results, we make the simplifying assumption that the productivity and consumer tastes distributions are independent across firms. For the same reason, we also assume that the productivity and consumer tastes distributions are independent of one another and that the consumer tastes distributions are independent across products and countries. While these simplifying assumptions provide a tractable way to introduce heterogeneity across products and countries within firms, it is straightforward to extend the analysis to introduce dependence in consumer tastes. For example, the model can be augmented with a common component of consumer tastes, which varies across products within firms but is the same across countries. This common component of consumer tastes plays a similar role in the model to firm productivity, which is the same across both countries and products within firms.¹² Similarly, the model can be augmented

⁹Our framework can be also extended to allow for endogenous measures of varieties within each product, introducing the potential for strategic interaction since firms are no longer of measure zero relative to each product.

¹⁰As the focus of our analysis is the cross-section distribution of exports across firms, products and countries, we abstract from stochastic variation over time in firm productivity and consumer tastes, and hence from steady-state adding and dropping of products and destinations. However, the model can be extended to capture these dynamics by incorporating features from the closed-economy model of Bernard, Redding and Schott (2008), as shown in the web-based technical appendix.

¹¹The three features that are important for our analysis are (a) variation in profitability across products within firms, (b) variation in profitability across countries within products within firms, and (c) variation in average profitability across firms. While our formulation captures these features in an intuitive and tractable way, one could also generate idiosyncratic variation in firm-product-country profitability from interactions between firm, product and country characteristics.

¹²See the web-based technical appendix for further discussion of this extension. A special case of our

with common components of consumer tastes for groups of related products within firms. More generally, explicit correlation in consumer tastes draws can be allowed, although at the cost of reduced analytical tractability.

While these extensions complicate the analysis, they do not alter the model's central mechanism of selection within firms, which is driven by heterogeneity across products within firms. Even with our simplifying assumptions, a firm's profitability is correlated across products and countries, because higher productivity raises a firm's profitability across all products and countries. These correlations are, however, imperfect because of the stochastic variation in consumer tastes.

Once the sunk cost has been incurred, and productivity and consumer tastes are observed, firms decide whether to enter and what products to make. Labor is the sole factor of production.¹³ We assume that firms face fixed costs of supplying each market, which are $F_x > 0$ for each foreign market and $F_d > 0$ for the domestic market. These market-specific fixed costs capture among other things the costs of building distribution networks. In addition, we assume that firms face fixed costs of supplying each product to a market, which are $f_x > 0$ for each foreign market and $f_d > 0$ for the domestic market. These product and market-specific fixed costs capture the costs of market research, advertising and conforming to foreign regulatory standards for each product. As more products are supplied to a market, total fixed costs rise, but average fixed costs fall, since the fixed cost of serving each market is spread over a larger number of products.

In addition to these fixed costs, there is also a constant marginal cost of production for each product that depends on firm productivity, such that $q_{jk}(\varphi, \lambda_{jk})/\varphi$ units of labor are required to produce $q_{jk}(\varphi, \lambda_{jk})$ units of output of product k for market j . Finally, we allow for variable costs of trade, such as transportation costs, which take the standard "iceberg" form, whereby a fraction $\tau > 1$ of a variety must be shipped in order for one unit to arrive in a foreign country.¹⁴ We note that to simplify the characterization of a firm's decision to supply a market, we have set fixed production costs to zero and have assumed instead a fixed cost of supplying each market.¹⁵ For simplicity, we have also assumed that the fixed costs of serving each market are incurred in terms of labor in the

model is where there is a single firm-product draw that is the same for all countries, which corresponds to perfect correlation of consumer tastes across countries.

¹³The multiple-industry version of the model considered in the web-based technical appendix also introduces multiple factors of production, which gives rise to Heckscher-Ohlin based comparative advantage, as in the single-product heterogeneous-firm model of Bernard, Redding and Schott (2007).

¹⁴For evidence on the magnitude of overall trade costs, see Anderson and van Wincoop (2004) and Hummels (2001). For evidence on the fixed costs of exporting, see Roberts and Tybout (1997), Bernard and Jensen (2004), Eaton, Kortum and Kramarz (2008).

¹⁵In our setting with demand heterogeneity across countries, a firm can choose to export a product but not supply it domestically, as found empirically by Iacovone and Javorcik (2008). Setting fixed production costs equal to zero simplifies the firm's problem, because each market can be considered separately. While introducing fixed production costs is straightforward, as shown in Section B3. of the appendix, it complicates the analysis by introducing interdependence across markets.

country of production, although it is straightforward to instead consider the case where they are incurred in the market supplied.

3.3. Firm-Product Profitability

Demand for a product variety depends on the own-variety price, the price index for the product and the price indices for all other products. If a firm is active in a product market, it manufactures one of a continuum of varieties and so is unable to influence the price index for the product. At the same time, the price of firm's variety in one product market only influences the demand for its varieties in other product markets through the price indices. Therefore, the firm's inability to influence the price indices implies that its profit maximization problem reduces to choosing the price of each product variety separately to maximize the profits derived from that product variety.¹⁶ This optimization problem yields the standard result that the equilibrium price of a product variety is a constant mark-up over marginal cost:

$$p_x(\varphi, \lambda_x) = \tau p_d(\varphi, \lambda_d) = \tau \frac{1}{\rho} \frac{w}{\varphi}, \quad (4)$$

where equilibrium prices in the export market are a constant multiple of those in the domestic market due to the variable costs of trade; λ_d varies across products with idiosyncratic realizations of consumer tastes; for the same reason λ_x varies across both products and export markets. We choose the wage in one country as the numeraire, which together with country symmetry implies $w = 1$ for all countries.

Substituting for the pricing rule, equilibrium revenue in each export market and the domestic market are respectively:

$$r_x(\varphi, \lambda_x) = \tau^{1-\sigma} \left(\frac{\lambda_x}{\lambda_d} \right)^{\sigma-1} r_d(\varphi, \lambda_d), \quad r_d(\varphi, \lambda_d) = E(\rho P \varphi \lambda_d)^{\sigma-1}, \quad (5)$$

where E denotes aggregate expenditure on a product in each market. The equilibrium profits from a product in each export market and the domestic market are therefore:

$$\pi_x(\varphi, \lambda_x) = \frac{r_x(\varphi, \lambda_x)}{\sigma} - f_x, \quad \pi_d(\varphi, \lambda_d) = \frac{r_d(\varphi, \lambda_d)}{\sigma} - f_d. \quad (6)$$

From equations (5) and (6), firm productivity enters the equilibrium revenue and profit functions in exactly the same way as consumer tastes, because prices are a constant mark-up over marginal costs and demand exhibits a constant elasticity of substitution.¹⁷

¹⁶The structure of our model eliminates strategic interaction within or between firms. This choice of model structure enables us to focus purely on the implications of introducing multi-product firms into a model of industry equilibrium with ongoing entry, exit and firm heterogeneity.

¹⁷Therefore differences in consumer tastes have the same effect on equilibrium firm revenue as differences in productivity. As a result, λ has an equivalent interpretation as a component of firm productivity that is specific to products and countries. While consumer tastes are perhaps a more natural source of heterogeneity across countries, the determination of general equilibrium under this alternative interpretation remains unchanged.

Additionally, equation (5) implies that the relative revenue from two varieties of the same product within a given market depends solely on relative productivity and consumer tastes:

$$r_{ij}(\varphi'', \lambda'') = (\varphi''/\varphi')^{\sigma-1} (\lambda''/\lambda')^{\sigma-1} r_{ij}(\varphi', \lambda'). \quad (7)$$

Similarly, as countries are symmetric, equation (5) implies that the relative revenue derived from two varieties of the same product with the same values of productivity and consumer tastes in the export and domestic markets depends solely on variable trade costs: $r_x(\varphi, \lambda')/r_d(\varphi, \lambda') = \tau^{1-\sigma}$.

A firm with a given productivity φ and consumer taste draw λ decides whether or not to supply a product to a market based on a comparison of revenue and fixed costs for the product. For each firm productivity φ , there is a **zero-profit cutoff for consumer tastes** for the domestic market, $\lambda_d^*(\varphi)$, such that a firm supplies the product domestically if it draws a value of λ_d equal to or greater than $\lambda_d^*(\varphi)$. This value of $\lambda_d^*(\varphi)$ is defined by the following zero-profit condition:

$$r_d(\varphi, \lambda_d^*(\varphi)) = \sigma f_d. \quad (8)$$

Using this product zero-profit cutoff condition for each firm productivity (8) together with relative variety revenues in (7), $\lambda_d^*(\varphi)$ can be expressed relative to its value for the lowest productivity supplier, $\lambda_d^*(\varphi_d^*)$:

$$\lambda_d^*(\varphi) = \left(\frac{\varphi_d^*}{\varphi} \right) \lambda_d^*(\varphi_d^*), \quad (9)$$

where φ_d^* is the lowest productivity at which a firm supplies the domestic market. As a firm's own productivity increases, its zero-profit cutoff for consumer tastes falls because higher productivity ensures that sufficient revenue to cover product fixed costs is generated at a lower value of consumer tastes. In contrast, an increase in the lowest productivity at which a firm supplies the domestic market, φ_d^* , or an increase in the zero-profit consumer tastes cutoff for the lowest productivity supplier, $\lambda_d^*(\varphi_d^*)$, raises a firm's own zero-profit consumer tastes cutoff. The reason is that an increase in either φ_d^* or $\lambda_d^*(\varphi_d^*)$ enhances the attractiveness of rival firms' products, which intensifies product market competition, and hence increases the value for consumer tastes at which sufficient revenue is generated to cover product fixed costs.

Following a similar line of reasoning, there is an **exporting cutoff for consumer tastes** $\lambda_x^*(\varphi)$ for each firm productivity φ , such that the firm will export the product if it draws a value of λ_x equal to or greater than $\lambda_x^*(\varphi)$. This value of $\lambda_x^*(\varphi)$ is defined by an analogous zero-profit condition:

$$r_x(\varphi, \lambda_x^*(\varphi)) = \sigma f_x. \quad (10)$$

Using this product exporting condition for each firm productivity together with relative variety revenues in (7), the product exporting cutoff for each firm productivity, $\lambda_x^*(\varphi)$, can be expressed relative to the product exporting cutoff for the lowest productivity exporter, $\lambda_x^*(\varphi_x^*)$:

$$\lambda_x^*(\varphi) = \left(\frac{\varphi_x^*}{\varphi} \right) \lambda_x^*(\varphi_x^*), \quad (11)$$

where φ_x^* is the lowest productivity at which a firm exports. Following a parallel logic as above, a firm's product exporting cutoff is increasing in its own productivity but is decreasing in the lowest productivity at which a firm exports, φ_x^* , and is decreasing in the product exporting cutoff of the lowest productivity exporter, $\lambda_x^*(\varphi_x^*)$.

As consumer tastes are independently and identically distributed across the unit continuum of products, the law of large numbers implies that the fraction of products supplied to the domestic market by a firm with a given productivity φ equals the probability of drawing a consumer taste above $\lambda_d^*(\varphi)$, that is $[1 - Z(\lambda_d^*(\varphi))]$. Similarly, the fraction of products exported by a firm with a given productivity φ equals the probability of drawing a consumer taste above $\lambda_x^*(\varphi)$, that is $[1 - Z(\lambda_x^*(\varphi))]$. Finally, as consumer tastes are also independently and identically distributed across the continuum of countries, the law of large numbers implies that the fraction of foreign countries to which a given product is exported equals $[1 - Z(\lambda_x^*(\varphi))]$.

The relative values of the exporting and zero-profit cutoffs for consumer tastes depend on fixed and variable trade costs. From relative revenue in the domestic and export markets (5), the zero-profit cutoff condition (8) and the exporting cutoff condition (10), $\lambda_x^*(\varphi) > \lambda_d^*(\varphi)$ for values of fixed and variable trade costs for which $\tau^{\sigma-1}(f_x/f_d) > 1$. For parameter values satisfying this inequality, there is probabilistic selection into export markets within firms: a firm is more likely to supply a product domestically than to export the product. Nevertheless heterogeneity in demand across countries implies that even for parameter values satisfying this inequality, a sufficiently high consumer taste draw in an export market, combined with a sufficiently low consumer taste draw in the domestic market, can induce a firm to export the product but not supply the product to the domestic market.¹⁸

3.4. Firm Profitability

Having examined equilibrium revenue and profits from each product, we now turn to the firm's equilibrium revenue and profits across the continuum of products as a whole. As consumer tastes are independently distributed across the unit continuum of symmetric products, the law of large numbers implies that a firm's expected revenue across the unit

¹⁸For empirical evidence of firms exporting products not supplied to the domestic market, see Iacovone and Javorcik (2008).

continuum of products equals its expected revenue for each product. Expected revenue for each product is a function of firm productivity φ and equals the probability of drawing a consumer taste above the cutoff times expected revenue conditional on supplying the product. Therefore total firm revenue across the unit continuum of products in the domestic and export markets is:

$$r_j(\varphi) = \int_{\lambda_j^*(\varphi)}^{\infty} r_j(\varphi, \lambda_j) z(\lambda_j) d\lambda_j, \quad j \in \{d, x\}.$$

Similarly, total profits in the domestic and export markets equal expected profits from each product minus the relevant fixed costs:

$$\pi_j(\varphi) = \int_{\lambda_j^*(\varphi)}^{\infty} \left(\frac{r_j(\varphi, \lambda_j)}{\sigma} - f_j \right) z(\lambda_j) d\lambda_j - F_j \quad j \in \{d, x\}. \quad (12)$$

Consider the relationship between expected profits from each product and firm productivity in (12). Equilibrium revenue from each product within the domestic market, $r_d(\varphi, \lambda_d)$, is increasing in firm productivity and consumer tastes. Hence the lower a firm's productivity, φ , the higher its zero-profit consumer tastes cutoff, $\lambda_d^*(\varphi)$, and the lower its probability of drawing a consumer tastes high enough for a product to be profitable. Therefore firms with lower productivities have lower expected profits from individual products and supply a smaller fraction of products to the domestic market, $[1 - Z(\lambda_d^*(\varphi))]$. For sufficiently low firm productivity, the excess of domestic market revenue over product fixed costs in the small range of profitable products falls short of the fixed cost of supplying the domestic market, F_d . The requirement that total profits in the domestic market are greater than or equal to zero therefore defines a zero-profit cutoff for firm productivity, φ_d^* , such that only firms that draw a productivity equal to or greater than φ_d^* enter:

$$\pi(\varphi_d^*) = 0. \quad (13)$$

Combining this firm zero-profit condition with the product zero-profit condition (8), we can determine the zero-profit cutoff for consumer tastes for the lowest productivity firm, $\lambda_d^*(\varphi_d^*)$. Using these two zero-profit conditions together with total firm profits (12) and relative variety revenues (7), we obtain:

$$\int_{\lambda_d^*(\varphi_d^*)}^{\infty} \left[\left(\frac{\lambda_d}{\lambda_d^*(\varphi_d^*)} \right)^{\sigma-1} - 1 \right] f_d z(\lambda_d) d\lambda_d = F_d, \quad (14)$$

which determines a unique equilibrium value of $\lambda_d^*(\varphi_d^*)$ as a function of the fixed cost of supplying the domestic market, F_d , and other parameters.

A directly analogous line of reasoning applies for each export market. For sufficiently low firm productivity, the excess of export market revenue over product fixed exporting costs in the small range of profitable products falls short of the fixed costs of serving a

foreign destination, F_x . The requirement that total firm profits from an export market are greater than or equal to zero therefore defines an exporting cutoff for firm productivity, φ_x^* , such that only firms that draw a productivity equal to or greater than φ_x^* export to each foreign destination:

$$\pi_x(\varphi_x^*) = 0. \quad (15)$$

Combining this firm exporting condition with the product exporting condition (10), we can determine the exporting cutoff for consumer tastes for the lowest productivity exporter, $\lambda_x^*(\varphi_x^*)$. Using these two zero-profit conditions together with total firm profits from the export market (12) and relative variety revenues (7), we obtain:

$$\int_{\lambda_x^*(\varphi_x^*)}^{\infty} \left[\left(\frac{\lambda_x}{\lambda_x^*(\varphi_x^*)} \right)^{\sigma-1} - 1 \right] f_x z(\lambda_x) d\lambda_x = F_x, \quad (16)$$

which determines a unique equilibrium value of $\lambda_x^*(\varphi_x^*)$ as a function of the fixed cost of serving an export market, F_x , and other parameters.

The value of the exporting productivity cutoff, φ_x^* , relative to the zero-profit productivity cutoff, φ_d^* , depends on fixed and variable trade costs. From relative revenue in the domestic and export markets (5), relative revenue within the same market for different values of productivity and consumer tastes (7), the product zero-profit cutoff condition (8) and the product exporting cutoff condition (10), the exporting and zero-profit productivity cutoffs are related as follows:

$$\varphi_x^* = \Gamma \varphi_d^*, \quad \Gamma \equiv \tau \left(\frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}} \left(\frac{\lambda_d^*(\varphi_d^*)}{\lambda_x^*(\varphi_x^*)} \right), \quad (17)$$

where $\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi_x^*)$ were determined as a function of the fixed cost of serving the domestic market, F_d , and the fixed cost of serving an export market, F_x , above.

For values of fixed and variable trade costs in (17) for which $\Gamma > 1$, there is selection into export markets across firms, where only the most productive firms export and lower-productivity firms serve only the domestic market.¹⁹ Therefore, although idiosyncratic differences in product demand across countries can induce a firm to export a product but not supply it to the domestic market, the law of large numbers implies that these idiosyncratic differences in demand average out across products. As a result, for the parameter values satisfying the inequality in (17), no firm ever serves an export market without also serving the domestic market.

As for simplicity we have developed the model for the case of symmetric countries, the exporting productivity cutoff is the same for each export market. Therefore each firm either serves only the domestic market or both the domestic market and all export

¹⁹A large empirical literature finds evidence of selection into export markets across firms, e.g., Bernard and Jensen (1995, 1999) and Roberts and Tybout (1997).

markets. Clearly, introducing country asymmetries would result in different exporting productivity cutoffs for each export market, so that more productive firms would export to more destinations. Nonetheless, even with symmetric countries, the number of export markets to which a particular product is exported varies with firm productivity. More productive firms have lower consumer-taste cutoffs and therefore export a given product to more countries.

3.5. Free Entry

Having examined a firm's *ex post* decision whether to supply markets, we now consider the firm's *ex ante* decision whether to incur the sunk cost of entry. Firms from the competitive fringe decide whether or not to enter based on a comparison of the expected value of entry and the sunk entry cost. We assume that there is a constant exogenous probability of firm death, δ , due to *force majeure* events beyond the firm's control, which generates ongoing firm entry and exit. Therefore the expected value of entry, V_i , is the *ex ante* probability of successful entry multiplied by average firm profits conditional on entry, $\bar{\pi}_i$, and discounted by the probability of firm death. Free entry requires:

$$V = \frac{[1 - G(\varphi_d^*)] \bar{\pi}}{\delta} = f_e, \quad (18)$$

where the *ex ante* probability of successful entry is $[1 - G(\varphi_d^*)]$. Expected profits conditional on entry, $\bar{\pi}_i$, equal expected profits in the domestic market plus the probability of exporting times the number of foreign markets times expected profits in each export market:

$$\bar{\pi} = \int_{\varphi_d^*}^{\infty} \pi_d(\varphi) \frac{g(\varphi)}{1 - G(\varphi_d^*)} d\varphi + \frac{1 - G(\varphi_x^*)}{1 - G(\varphi_d^*)} n \int_{\varphi_x^*}^{\infty} \pi_x(\varphi) \frac{g(\varphi)}{1 - G(\varphi_x^*)} d\varphi.$$

Using total firm profits in the domestic and export markets (12), product profits in the domestic and export markets (6) and the relationship between relative variety revenues (7), the free entry condition can be written solely in terms of the zero-profit and exporting cutoffs for productivity and consumer tastes:

$$V = \frac{1}{\delta} \int_{\varphi_d^*}^{\infty} \left[\int_{\lambda_d^*(\varphi)}^{\infty} \left[\left(\frac{\lambda_d}{\lambda_d^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_d z(\lambda_d) d\lambda_d - F_d \right] g(\varphi) d\varphi \quad (19)$$

$$+ \frac{n}{\delta} \int_{\varphi_x^*}^{\infty} \left[\int_{\lambda_x^*(\varphi)}^{\infty} \left[\left(\frac{\lambda_x}{\lambda_x^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_x z(\lambda_x) d\lambda_x - F_x \right] g(\varphi) d\varphi = f_e,$$

where the first term captures expected profitability in the domestic market and the second term captures expected profitability in export markets.

3.6. Goods and Labor Markets

The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e , and a constant mass of firms producing, M . The mass of firms supplying a given product to the domestic market, M_d , is a constant fraction of the mass of firms, M , which is determined by the probability of supplying a product to the domestic market, $[1 - Z(\lambda_d^*(\varphi))]$. Similarly, the steady-state mass of firms supplying a given product to each export market, M_x , is a constant fraction of the mass of firms, M , which is determined by the probability of exporting a product, $[1 - Z(\lambda_x^*(\varphi))]$. The steady-state equilibrium also features stationary distributions of firm productivity and consumer tastes in the domestic and export markets, which are determined by the zero-profit and exporting cutoffs, φ_d^* , φ_x^* , $\lambda_d^*(\varphi)$ and $\lambda_x^*(\varphi)$.

We begin by characterizing the mass of firms and the fraction of these firms that supply a given product to the domestic market and each export market. For the mass of firms to be constant in steady-state, the mass of successful entrants that draw a productivity sufficiently high to produce must equal the mass of existing firms that die, which yields the following steady-state stability condition:

$$[1 - G(\varphi_d^*)] M_e = \delta M. \quad (20)$$

This steady-state stability condition can be combined with the free entry condition (18) to show that total payments to labor used in entry equal aggregate profits: $M\bar{\pi} = M_e f_e = L_e$. Total payments to labor used in production, on the other hand, equal aggregate revenue minus aggregate profits, $L_p = R - M\bar{\pi}$. Combining these two expressions, it follows that total payments to labor equal aggregate revenue: $R = L$. Thus the labor market clears:

$$L_p + L_e = \bar{L}, \quad (21)$$

Having solved for aggregate revenue, the mass of firms can be determined as follows:

$$M = \frac{R}{\bar{r}}, \quad (22)$$

where average revenue can be expressed solely in terms of the zero-profit and exporting cutoffs for productivity and consumer tastes, as shown in equation (36) in the appendix.

Of the mass of firms with productivity φ , the fraction $[1 - Z(\lambda_d^*(\varphi))]$ supply a given product to the domestic market, while the fraction $[1 - Z(\lambda_x^*(\varphi))]$ supply the given product to each export market. Hence the mass of firms supplying a product to the domestic market is:

$$M_d = \left[\int_{\varphi_d^*}^{\infty} [1 - Z(\lambda_d^*(\varphi))] \left(\frac{g(\varphi)}{1 - G(\varphi_d^*)} \right) d\varphi \right] M, \quad (23)$$

while the mass of firms supplying a product to each export market is:

$$M_x = \left[\int_{\varphi_x^*}^{\infty} [1 - Z(\lambda_x^*(\varphi))] \left(\frac{g(\varphi)}{1 - G(\varphi_x^*)} \right) d\varphi \right] M, \quad (24)$$

where heterogeneity in consumer tastes across countries implies that a firm can export a given product to one foreign country but not to another.

Using the equilibrium pricing rule (4) and country symmetry, the aggregate price index for each product can be written as a function of the mass of firms supplying each product to the domestic and export markets, as well as the prices of varieties with a weighted average of firm productivity and consumer tastes in the domestic and export markets:

$$P = \left[M_d \left(\frac{1}{\rho \tilde{\varphi}_d} \right)^{1-\sigma} + n M_x \left(\frac{\tau}{\rho \tilde{\varphi}_x} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (25)$$

where the weighted average in the domestic market, $\tilde{\varphi}_d$, is pinned down by the zero-profit cutoffs of productivity and consumer tastes, φ_d^* and $\lambda_d^*(\varphi)$, as specified in Section B1. of the appendix. Similarly, the weighted average in the export market, $\tilde{\varphi}_x$, depends on the exporting cutoffs of productivity and consumer tastes, φ_x^* and $\lambda_x^*(\varphi)$, as also specified in Section B1. of the appendix.

3.7. General Equilibrium

The open-economy general equilibrium is referenced by a vector of six variables: the zero-profit productivity cutoff, φ_d^* , the exporting productivity cutoff, φ_x^* , the zero-profit consumer tastes cutoff of the least productive firm, $\lambda_d^*(\varphi_d^*)$, the exporting consumer tastes cutoff of the least productive exporter, $\lambda_x^*(\varphi_x^*)$, the price index for each product, P , and aggregate revenue, R . All other endogenous variables can be written as functions of these six elements of the equilibrium vector.

Proposition 1 *There exists a unique open-economy equilibrium referenced by the sextuple $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*), P, R\}$.*

Proof. See Appendix. ■

In Proposition 1, we characterize general equilibrium for arbitrary continuous distributions of firm productivity and consumer tastes, $g(\varphi)$ and $z(\lambda)$. In the web-based technical appendix, we report solutions for the special case of Pareto distributions of firm productivity and consumer tastes, which we use below in examining the model's comparative statics.

4. Properties of the Open-Economy Equilibrium

One of the central features of our model is heterogeneity and self-selection within firms. This heterogeneity exists in a firm's exports across products within a given country and across countries for a given product, and in both cases reflects idiosyncratic variation in demand for a firm's varieties. As a result of this heterogeneity, trade liberalization results

in reallocations of resources within firms that influence standard revenue-based measures of firm productivity.

Proposition 2 (*Core Competencies*) *The opening of the closed economy to trade results in **within-firm resource reallocation**: all surviving firms drop products with low values of consumer tastes from the domestic market, which raises their revenue-based productivity. In addition, high-productivity surviving firms begin to export, and therefore add products with high values of consumer tastes in the export market, which further raises their revenue-based productivity.*

Proof. See Appendix. ■

The key to understanding this result is recognizing that opening a closed economy to trade has uneven effects across firms depending on whether or not they begin to export and uneven effects across products within firms depending on whether or not these products begin to be exported. Since there is a positive *ex ante* probability of drawing a productivity high enough to export, the opening of trade increases the expected value of entry. As a result, there is increased entry, which enhances product-market competition in the domestic market and raises the zero-profit productivity cutoff φ_d^* below which firms exit. This rise in φ_d^* elevates the average productivity of the varieties supplied by competing firms, strengthens product-market competition, and so induces surviving firms to drop lower-consumer-tastes products.

This concentration on core competencies that follows the opening of trade leads to a change in the composition of firm output that increases revenue-based firm productivity. Marginally viable low-consumer-tastes products are dropped; output of all surviving products for the domestic market contracts; and entry into exporting generates new output for export markets of higher-consumer-tastes products. Each of these responses shifts the composition of firm output towards higher-consumer-tastes products that have higher revenue per unit of labor input. Therefore, even though firm productivity and consumer tastes are themselves parameters that are fixed by assumption, revenue-based firm productivity rises as a result of the within-firm reallocation of resources across products.²⁰

This within-firm reallocation following the opening of trade contrasts with the standard model of heterogeneous firms and trade, in which firms are implicitly restricted to production of just a single product. Our results also contrast with the predictions of an alternative framework in which products are randomly allocated to firms. In such a

²⁰We follow Foster, Haltiwanger and Syverson (2008) in differentiating between revenue- and quantity-based measures of productivity. Since both firm varieties and products are horizontally differentiated from one another, the revenue-based measures of productivity typically used in empirical work are the theoretically appropriate measures for examining the impact of trade liberalization on firm productivity. As discussed in footnote 17, consumer tastes and firm productivity have the same effects on equilibrium revenue, which implies that they have the same effects on revenue-based measures of productivity, as shown in Section B2 of the Appendix.

model, only those firms who happen to have products with a value of consumer tastes above the old and below the new zero-profit cutoff would drop those products after the opening of trade. In contrast, in our model, each firm chooses optimally the range of products to supply, and therefore all firms focus on their core competencies in higher consumer tastes products after the opening of trade. This optimal choice of the range of products to supply has systematic implications for the extensive and intensive margins of trade, which we examine further below.

While we have developed the core competencies prediction of the model in the context of opening the closed economy to international trade, related but more nuanced predictions hold for reductions in variable trade costs in the open-economy equilibrium of the model.²¹

Proposition 3 *Reductions in variable trade costs also result in within-firm reallocation that leads surviving multiple-product firms to focus on their core competencies:*

- (a) *surviving firms that continue to supply only the domestic market drop products with low values of consumer tastes, which raises their revenue-based productivity,*
- (b) *surviving firms that enter the export market drop products with low values of consumer tastes from the domestic market and add products with higher values of consumer tastes in the export market, which raises their revenue-based productivity,*
- (c) *surviving exporters drop products with low values of consumer tastes from the domestic market and add products in the export market. As these products added in the export market have higher values of consumer tastes than those dropped from the domestic market, but have lower values of consumer tastes than those previously exported, the effect on their revenue-based productivity is ambiguous.*

Proof. See Appendix. ■

The dropping of products from the domestic market following reductions in variable trade costs is a source of revenue-based productivity growth for all firms. However, firms that enter the export market experience an additional source of revenue-based productivity growth. New exporters not only drop lower-consumer tastes products from the domestic market, but also expand output of higher-consumer tastes products for the export market, which further shifts the composition of firm output towards products with higher revenue per unit of labor input. This prediction is consistent with a number of empirical studies that find significant increases in revenue-based productivity in the year of transition to exporting (see, for example, Bernard and Jensen 1999).

For continuing exporters, the implications of reductions in variable trade costs for productivity are more subtle. Continuing exporters experience revenue-based productivity growth from the expansion of output at previously exported products, which shifts

²¹While we concentrate on reductions in variable trade costs, reductions in the fixed costs of exporting products have similar effects, except where otherwise indicated, as long as there remains selection into export markets: $\lambda_x^*(\varphi) > \lambda_d^*(\varphi)$ and $\varphi_x^* > \varphi_d^*$.

the composition of output towards higher-consumer tastes products. But continuing exporters also begin to export products that have lower values of consumer tastes than those previously exported, which reduces average revenue per unit of labor input in export markets. Combining these two effects, it is ambiguous whether reductions in variable trade costs lead to higher or lower revenue-based productivity growth at continuing exporters than at non-exporting firms. This is consistent with empirical findings of no significant differences in revenue-based productivity growth between continuing exporters and non-exporters (see for example Bernard and Jensen 1999).

Whereas the previous two propositions have concentrated on the model's core competencies predictions and their impact on firm revenue-based productivity, we now examine more closely the model's predictions for the relationship between the extensive and intensive margins and variable trade costs.

Proposition 4 *A reduction in variable trade costs:*

- (a) **increases** aggregate exports through the share of products exported to a given country by incumbent exporters (*within-firm product extensive margin*),
- (b) **increases** aggregate exports through the share of countries to which a given product is exported by incumbent exporters (*within-firm country extensive margin*),
- (c) **increases** aggregate exports through the share of firms that export (*across-firm extensive margin*),
- (d) has an **ambiguous** effect on aggregate exports through average exports per firm-product-country (*intensive margin*).

Proof. See Appendix. ■

The intuition for these results is as follows. A reduction in variable trade costs reduces the price of products in export markets, which with elastic demand increases revenue and variable profits in export markets. As a result, some lower-consumer-tastes products that were previously only supplied to the domestic market can now be profitably exported. Therefore reductions in variable trade costs raise aggregate exports through the share of products exported to a given country and the share of countries to which a given product is exported (the *within-firm extensive margins of products and destinations*). Additionally, a reduction in variable trade costs induces some lower productivity firms that previously only served the domestic market to enter export markets. Hence aggregate exports also increase because of a rise in the share of firms that export (the *across-firm extensive margin*).

In contrast, the reduction in variable trade costs has an *ambiguous* effect on average exports per firm-product-country (the *intensive margin*). This result arises from two counteracting forces. On the one hand, the reduction in variable trade costs reduces the price of products that are already exported in each foreign country, which increases average exports per firm-product-country. On the other hand, the reduction in variable

trade costs causes products with lower values of consumer tastes to be exported. As these lower-consumer-taste products are exported in smaller amounts, this reduces average exports per firm-product-country through a change in export composition. The net effect of these two forces is ambiguous and depends on the functional form of the distribution for consumer tastes. For the special case of a Pareto distribution of consumer tastes, the two forces exactly offset one another, so that average exports per firm-product-country are independent of variable trade costs, as shown in the web-based technical appendix.²²

Having characterized the relationship between the margins of trade and variable trade costs, we now turn to examine their relationship with firm productivity.

Proposition 5 *Higher firm productivity φ :*

- (a) **increases** total firm exports through the share of products exported by the firm to a given country (*within-firm product extensive margin*),
- (b) **increases** total firm exports through the share of countries to which a given product is exported by the firm (*within-firm country extensive margin*),
- (c) has an **ambiguous** effect on total firm exports through the intensive margin of average exports per firm-product-country.

Proof. See Appendix. ■

The intuition for these comparative statics follows a similar logic as for the reduction in variable trade costs. More-productive firms charge lower prices for their products, which with elastic demand increases revenue and variable profits for a given value of consumer tastes. As a result, more productive firms find it profitable to export a product at a lower value of consumer tastes, and hence export more products to each country (the *within-firm product extensive margin*) and export each product to more countries (the *within-firm country extensive margin*). Both extensive margin responses induce endogenous variation across firms in the share of exports in firm output.

In contrast, higher firm productivity has an ambiguous effect on average exports per firm-product-country (the *intensive margin*). On the one hand, higher firm productivity increases exports of a given product with a given value of consumer tastes for a given destination, which increases average exports per firm-product-country. On the other hand, higher firm productivity causes products with lower values of consumer tastes to be exported. As these lower-consumer-taste products are exported in smaller amounts, this reduces average exports per firm-product-country through a change in export composition.

²²While a reduction in the fixed costs of exporting increases aggregate exports through the extensive margins, it has a different effect on the intensive margin from a reduction in variable trade costs. As the fixed costs of exporting fall, a lower value of consumer tastes is required to generate sufficient revenue to profitably export, which unambiguously reduces average exports per firm-product-country through a change in export composition, as shown in the web-based technical appendix for a Pareto distribution of consumer tastes.

Again the net effect of these two counteracting forces depends on the functional form of the distribution for consumer tastes. For the special case of a Pareto distribution of consumer tastes, the two forces exactly offset one another, so that average exports per firm-product-country are independent of firm productivity, as shown in the web-based technical appendix.²³

As the model's ambiguous predictions for the intensive margin arise from changes in export composition, they have an immediate corollary. The model implies that exports of a given product with a given value of consumer tastes to a given destination (which holds export composition constant) increase more rapidly with firm productivity than average exports per product to the same destination. We examine this further implication of the model in our empirical analysis below.

5. Empirical Evidence

In this section, we compare key predictions of the model with U.S. trade data. We also provide a detailed examination of the distribution of export shipments across products and destinations within firms and present evidence on the quantitative importance of the extensive margins of trade for the distribution of U.S. exports across firms and trade partners. We begin with a brief description of our data, deferring a more detailed summary to Appendix A below.

5.1. Data Description

Our analysis makes use of two datasets: the U.S. Linked/Longitudinal Firm Trade Transaction Database (LFTTD) and the U.S. Census of Manufactures (CMF). The LFTTD captures all U.S. international trade transactions from 1992 through 2004 and is derived from customs forms. For each export transaction, we observe the identity of the firm responsible for the export, the export product's ten-digit Harmonized System (HS) product classification, the value shipped, the date of the shipment and the destination country.

The quinquennial CMF collects information on manufacturing establishments' inputs and output in each census year. We use this information along with industry price deflators provided by Bartelsman et al. (2000), to compute manufacturing establishments' revenue-based labor productivity and total factor productivity (TFP).²⁴ The CMF also tracks

²³The lack of correlation between the intensive margin and firm productivity requires both a Pareto distribution of consumer tastes and a product fixed cost of exporting, f_x , that is independent of consumer tastes, λ_x . Even with a Pareto distribution of consumer tastes, the correlation between the intensive margin and firm productivity is negative if f_x is increasing in λ_x , and is positive if f_x is decreasing in λ_x , as shown in the web-based technical appendix.

²⁴We measure firm TFP as the shipment-weighted average TFP of its plants. Plant TFP in a given census year is measured relative to other plants in its main industry in percentage terms using the multi-factor superlative index number of Caves et al. (1982). This index accounts for plants' use of capital, production workers, non-production workers and materials. Plant shipments, capital and materials are

the set of five-digit Standard Industrial Classification (SIC) categories manufacturing establishments produce over time. We use this information to examine changes in firm scope between census years. We link manufacturing establishments in the CMF to firms in the LFTTD using the bridge developed by Bernard, Jensen and Schott (2009).

We interpret the approximately 8,000 ten-digit HS and 1,500 five-digit SIC codes used to classify exports and production as discrete partitions of the model's continuum of products, which become coarser as one increases the level of aggregation. With this interpretation, the model provides a natural explanation for the coexistence of single- and multiple-product firms. We think of firms producing a single product as those whose range of products falls within a single classification code. Multiple-product firms, on the other hand, are those whose product range is wide enough to span several classification codes.

5.2. *Core Competencies*

One of our central theoretical results in Propositions 2 and 3 is that trade liberalization induces firms to focus on their core competencies, which raises revenue-based firm productivity. As a large empirical literature has provided evidence of firm revenue-based productivity growth following trade liberalization, we concentrate on the model's more novel prediction that trade liberalization leads to a focusing on core competencies through reallocations of resources within firms.²⁵

In the model, trade liberalization induces firms to drop products with low values of consumer tastes from the domestic market, but induces them to add products with high values of consumer tastes in the export market. In a setting without firm-product demand heterogeneity across countries, the range of products exported is a subset of the range of goods produced and sold at home for parameter values for which there is selection into export markets. Therefore, without firm-product-country demand heterogeneity, the model yields the unambiguous prediction that trade liberalization reduces the range of products produced by a firm.

In contrast, in our setting with demand heterogeneity across countries, trade liberalization can induce a firm to add new products in export markets that it does not supply to the domestic market. These are products with high values of consumer tastes in export markets but low values of consumer tastes in the domestic market, and their addition can increase the range of products produced by a firm. Therefore, with demand heterogeneity, trade liberalization has an ambiguous effect on the number of products produced, depend-

deflated according to the four-digit SIC deflator of its major industry using deflators provided by Bartelsman et al. (2000). Wages are deflated by the U.S. consumer price index available at www.bls.gov. A plant's main industry is the four-digit SIC in which it has the largest value of shipments.

²⁵For evidence on firm productivity growth following trade liberalization, see among others Tybout and Westbrook (1995), Bernard, Jensen and Schott (2006), Pavcnik (2002), Treffer (2004), Fernandes (2008), and Park et al. (2008).

ing on whether the deletion of products from the domestic market exceeds the addition of new products in export markets that are not supplied domestically. Thus, evidence that the number of products produced falls following trade liberalization is consistent with the focusing on core competencies emphasized by the model, even though with demand heterogeneity focusing on core competencies does not necessarily imply a fall in the number of products produced.

To examine the impact of trade liberalization on the number of products firms produce, we use the Canada-U.S. Free Trade Agreement (CUSFTA) as a natural experiment. CUSFTA, signed in 1988, came into effect on January 1, 1989 and involved substantial tariff reductions for a number of goods (see, for example, Trefler 2004). In contrast to many trade liberalizations in developing countries, CUSFTA involved a clearly defined change in trade policy that did not come as part of larger package of reforms. Additionally, the mechanism behind the focusing on core competencies in our model is an expansion of export opportunities, and CUSFTA was a reciprocal liberalization that enhanced export opportunities for firms in both Canada and the United States. Consistent with our model, Baldwin and Gu (2009) find a sharp decline in product diversification among Canadian firms following implementation of CUSFTA. Here, we provide evidence of a similar response among U.S. firms.

Our empirical analysis combines data from the CMF on the number of five-digit SIC goods firms produced in the years 1987 and 1992, before and after the introduction of CUSFTA, with data on reductions in Canadian tariffs on U.S. manufacturing imports measured at the four-digit SIC level over this period.²⁶ We measure a firm's exposure to CUSFTA as the domestic-shipment weighted average tariff reduction in the four-digit SIC industries in which it was active in 1987,

$$\Delta \text{Tariff}_f = \frac{\sum_i v_{fi}^{87} (\Delta \text{Tariff}_i)}{\sum_i v_{fi}^{87}},$$

where f and i index firms and four-digit SIC industries, respectively; v_{fi}^{87} represents firm domestic shipments in industry i in 1987; and ΔTariff_i is the percentage point change in the Canadian tariff rate on U.S. manufacturing imports in industry i between 1989 and 1992.²⁷ Across industries, the mean tariff reduction is 4.0 percentage points with a standard deviation of 2.9 percentage points. Across surviving firms in our data, the mean and standard deviation of ΔTariff_f is 3.1 and 2.4 percentage points, respectively. Note

²⁶Five-digit SIC products are defined consistently across years as in Bernard, Redding and Schott (2008). As the LFTTD data on exports are not available until 1992, our analysis of the impact of CUSFTA is restricted to the CMF, which records the number of five-digit SIC products U.S. manufacturing firms produce in 1987 and 1992.

²⁷Industry-level Canadian tariff data are from Trefler (2004) and are available from 1989 to 1992. We note that we obtain – as expected for a largely reciprocal liberalization – similar results when using U.S. tariff changes on Canadian four-digit SIC imports over the same period.

that in the regressions below we use $-\Delta \text{Tariff}_f$ as a covariate so that increases in this variable represent greater tariff reductions, i.e., greater increases in export opportunities.

Our empirical strategy involves a “differences-in-differences” specification, in which we compare the number of products produced by firms before and after CUSFTA (first difference) in industries experiencing above versus below-median Canadian tariff reductions (second difference). This “differences-in-differences” specification can be represented as the following regression:

$$\text{Products}_{ft} = \beta (\text{Post}_t \times \text{Exposure}_f) + \eta_f + d_t + u_{ft} \quad (26)$$

where f , again, denotes firms, Products_{ft} is the number of five-digit SIC products produced by a firm in 1987 and 1992; Post_t is a dummy variable which equals zero in 1987 prior to CUSFTA and one in 1992 afterwards; Exposure_f is a dummy variable which equals one if a firm experienced above-median Canadian tariff reductions between 1989 and 1992 and zero otherwise; η_f are firm fixed effects that control for unobserved heterogeneity in the number of products across firms; d_t are time dummies that control for common macro shocks; and u_{ft} is a stochastic error.²⁸

As we have two cross-sections of data in 1987 and 1992, the fixed effects specification in (26) has an equivalent representation in first differences. Taking first differences in (26), the left-hand side variable becomes the change in the number of products between the two years, the right-hand side variable becomes the Exposure_f dummy variable for whether a firm experienced above or below-median Canadian tariff reductions, the firm fixed effects η_f difference out, and the time dummies d_t become a constant. We cluster the standard errors in this first differences specification by firms’ main four-digit SIC industry to account for the fact that our firm-specific measure of exposure to CUSFTA is constructed using four-digit SIC data on Canadian tariff reductions.

Results are reported in the first row of Table 2. In column (1), we find that firms experiencing above-median Canadian tariff reductions reduce the number of products they produce relative to firms experiencing below-median Canadian tariff reductions. In columns (2) and (3), we show that this result is robust to including additional controls for major firms’ four-digit industry and log 1987 employment as a measure of initial firm size.

In the second row, we replace the number of products on the left-hand side of (26) with an alternative measure of firm diversification used by Baldwin and Gu (2009). This “entropy” measure is defined as $\sum_i s_{fkt} \ln(s_{fkt})$, where s_{fkt} represents the share of firm shipments accounted for by five-digit SIC product k . It captures the extent to which a firm’s output is skewed towards its largest rather than its smallest products. Estimating

²⁸We find similar results using alternative cutoffs, e.g., comparing firms experiencing Canadian tariff reductions above the 75th percentile to those experiencing Canadian tariff reductions below the 25th percentile. Results are also robust to including the number of products firms produce in 1987, and to using the log difference in the number of products produced rather than levels.

Table 2: U.S. Manufacturing Firm Diversification During the Canada-U.S. Free Trade Agreement

	[1]	[2]	[3]
Change in Products	-0.059 0.015	-0.624 0.101	-0.572 0.096
Change in Entropy	0.011 0.003	0.156 0.026	0.153 0.026
Firm Observations	66,472	66,472	66,472
Major Industry Dummy Variables	No	Yes	Yes
Log 1987 Employment	No	No	Yes

Notes: Table reports mean difference in noted variable between surviving firms experiencing above- and below-median changes in Canadian export opportunities from 1987 and 1992. Each cell reports the mean difference and associated standard error from a separate OLS regression. Change in products refers to change in number of five-digit SIC categories produced in the United States. Change in entropy is defined in the text. Change in export opportunities refers to the output-weighted average change in Canadian tariffs across the four-digit SIC industries produced by the firm. Standard errors are clustered according to firms' main four-digit SIC industry. Additional covariates are included as noted.

the regression specification again in first differences, column (1) shows that firms experiencing above-median Canadian tariff reductions exhibit a rise in entropy, i.e., an increased concentration of production in their largest products, relative to firms experiencing below-median Canadian tariff reductions. Columns (2) and (3) show that this finding is robust to controlling for major four-digit industry and initial firm size. Overall, both sets of empirical results provide support for the idea that trade liberalization induces firms to focus on their core competencies.²⁹

5.3. Margins of Trade and Trade Costs

Proposition 4 relates the extensive and intensive margins of exports to variable trade costs. Here, we use a “gravity equation” specification to examine empirically how the margins of trade vary across countries using their distance from the United States as a proxy for variable trade costs.³⁰

We begin by decomposing aggregate U.S. exports to each destination country c (x_c) into the number of firms exporting to the destination (f_c), the number of ten-digit HS products exported to the destination (p_c), the fraction of firm-product combinations with positive exports which we refer to as the “density” of trade (d_c), and the average value of

²⁹Additional empirical support comes from Iacovone and Javorcik (2008), who find a decline in the number of goods produced and a rise in the number of goods exported at Mexican firms following the North American Free Trade Agreement (NAFTA) that superseded CUSFTA in 1994. For further supportive evidence using import penetration data, see Bowen and Wiersema (2005) and Liu (2006).

³⁰Distance is a widely-used proxy for trade costs in the large gravity equation literature, as reviewed by Disdier and Head (2008). For direct evidence on the relationship between trade costs and distance, see, for example, Hummels (2001) and Limao and Venables (2001).

exports per firm-product-country conditional on positive exports (\bar{x}_c),

$$x_c = f_c p_c d_c \bar{x}_c, \quad (27)$$

where $d_c = o_c / (f_c p_c)$, $\bar{x}_c = x_c / o_c$, and o_c is the number of firm-product observations with positive exports for country c .³¹ In this specification, the density extensive margin captures the extent to which firm-product cells for a given export destination are “filled-in”. It ranges from $\min\{1/f_c, 1/p_c\}$ to unity.

We regress total exports to trade partners, as well as each component of total exports, on the great-circle distance of trade partners from the United States. To control for the potential influence of market size, we also include trade partners’ GDP,

$$\ln Z_c = \zeta_0 + \zeta_1 \ln distance_c + \zeta_2 \ln GDP_c + \varepsilon_c, \quad (28)$$

where $Z_c \in \{x_c, f_c, p_c, d_c, \bar{x}_c\}$.³² For brevity, we report regression results using 2002 data, but note that results for other years are similar.

The first column of Table 3 echoes the well-known result that destination-country exports decline with distance and increase with market size. Results in the next four columns decompose these overall effects into the contributions of the extensive and intensive margins. Consistent with the model, the number of exporting firms and the number of exported products decline with distance. In contrast, density rises with distance. This result is also intuitive: as the number of exporting firms and the number of exported products both increase, the number of possible firm-product observations expands multiplicatively. If each firm is active in a limited subset of products, the number of firm-product observations with positive trade expands less than proportionately. As a result, density is negatively correlated with the firm and product extensive margins, and hence with aggregate U.S. exports. By summing the coefficients for density and number of products, we find that the number of products with positive exports per firm, o_c/f_c , declines with distance. Finally, in contrast to the strong negative effect of distance on the extensive margins, we find that distance has a *positive* but statistically insignificant effect on the intensive margin of average exports per firm and product to a country. Re-estimating the same specification for other years, we find that the coefficient on distance for the intensive margin is positive but statistically insignificant throughout. This ambiguous pattern of results is consistent with the ambiguous intensive-margin predictions of the theory.

³¹Including a term for the density of trade augments the gravity specification considered in Bernard, Jensen, Redding, and Schott (2007) and isolates the intensive margin.

³²Distance data are from CEPII (see Mayer and Zignago 2006). GDP data are from the World Bank’s World Development Indicators database. In contrast to many gravity equation specifications, specification (28) includes importer but not exporter GDP, because with data on only U.S. exports, exporter characteristics are controlled for in the regression constant.

Table 3: Gravity and the Margins of U.S. Exports

	$\ln(\text{Value}_c)$	$\ln(\text{Firms}_c)$	$\ln(\text{Products}_c)$	$\ln(\text{Density}_c)$	$\ln(\text{Intensive}_c)$
$\ln(\text{Distance}_c)$	-1.37 0.17	-1.17 0.15	-1.10 0.15	0.84 0.13	0.05 0.10
$\ln(\text{GDP}_c)$	1.01 0.04	0.71 0.03	0.55 0.03	-0.48 0.03	0.23 0.02
Constant	7.82 1.83	0.52 1.59	3.48 1.55	-2.20 1.37	6.03 1.07
Observations	175	175	175	175	175
R^2	0.82	0.76	0.68	0.66	0.37

Notes: Table reports results of country-level OLS regressions of U.S. exports or their components on trading-partners' GDP and great-circle distance (in kilometers) from the United States. Standard errors are noted below each coefficient. Data are for 2002.

5.4. Margins of Trade and Firm Productivity

Our final set of theoretical results, from Proposition 5, link firms' extensive margins to their productivity. In this section we examine the extent to which these margins increase with estimates of firm productivity and with other variables that, in the model, are monotonically related to firm productivity.

The first five columns of Table 4 summarize the results of regressing the log number of ten-digit HS products exported by firms on a series of covariates using OLS. The second five columns report analogous regressions using the log number of destinations served by firms as the dependent variable. As only the CMF contains the information on inputs needed to estimate firms' total factor productivity (TFP) and labor productivity (domestic shipments per worker), used in the first two columns of each panel, for consistency we restrict our analysis throughout the table to the subset of exporting firms from the LFTTD that also appear in the CMF. Data are for 1997, the latest year for which merged data is available.³³ All regressions include dummies for firms' main four-digit SIC industry to focus on variation across firms within the same major industry, and we also cluster standard errors on this dimension of the data. We note that measuring TFP for multiple-product firms is problematic unless data on inputs, outputs and prices is available at the firm-product level (see also De Loecker 2007). Since this requirement is *not* met by our data, we also consider total firm exports and other variables that in the model are related monotonically to firm productivity.

The first two columns of each panel examine the relationship between firms' extensive margins and their revenue-based TFP and log labor productivity. Consistent with the

³³We note that we find results very similar to those reported in the final three columns of each panel when we re-estimate the same specification on the larger set of firms that appear in the LFTTD, both in 1997 and across years.

model, we find positive and statistically significant correlations in all four regressions.

Table 4: Correlation of U.S. Firms' Extensive and Intensive Margins

	ln(Products _{<i>i</i>})					ln(Countries _{<i>i</i>})				
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
TFP _{<i>i</i>}	0.071					0.076				
	0.022					0.022				
ln(Output _{<i>i</i>} /Worker _{<i>i</i>})		0.474					0.426			
		0.019					0.020			
ln(Exports _{<i>i</i>})			0.384					0.347		
			0.004					0.006		
ln(Largest Product _{<i>i</i>})				0.345					0.329	
				0.003					0.006	
ln(5 th -Largest Product _{<i>i</i>})					0.405					0.345
					0.004					0.005
Constant	1.894	0.436	-3.022	-2.300	0.405	1.292	-0.797	-3.141	-2.714	-1.733
	0.006	0.096	0.053	0.061	0.004	0.006	0.101	0.072	0.078	0.051
Observations	27,987	27,987	27,987	27,987	16,215	27,987	27,987	27,987	27,987	16,215
R ²	0.13	0.18	0.69	0.56	0.50	0.21	0.24	0.60	0.55	0.53

Notes: Table reports results of firm-level OLS regressions of the log number of ten-digit HS products exported by the firm, or log the number of destination countries served by the firm, on noted covariates. All regressions include dummies for firms' main four-digit SIC industry, and we standard errors are clustered on this dimension of the data. Results in columns 5 and 10 are restricted to firms exporting at least five products. Data are for 1997.

The last three columns of each panel of Table 4 consider the relationship between firms' product and country extensive margins and less-direct manifestations of firm productivity. In the model, variation across firms in the size of their largest product to a particular destination is due solely to variation in firm-level productivity. With a continuum of symmetric products and independent and identical consumer taste distributions, each firm draws the same distribution of consumer tastes across products within a destination. Thus, while the identities of firms' largest (or second-largest, etc.) products will vary across firms, their size, conditional on firm-level productivity, will not. As a result, more productive firms have greater exports of their largest product than less productive firms supplying the same destination. Additionally, with a continuum of symmetric countries in the model, more productive firms also have greater total exports of their largest product and greater total exports across all products and destinations.³⁴

The last three columns of both panels of Table 4 show strong and statistically significant positive correlations between firms' extensive margins and their total exports, the exports of their largest product across destinations and the exports of their fifth-largest

³⁴In practice, products and countries can be asymmetric, which implies that a firm could have high total exports of its largest product and high total exports across all products either because it has a high productivity or because it happens to draw high consumer tastes for large countries and products. Nonetheless, even with asymmetric products and countries, exports of a given product to a given country are monotonically increasing in firm productivity. Therefore, even in the presence of asymmetries, exports of a firm's largest product and total firm exports are positively correlated with firm productivity.

product across destinations. Results are similar using other-ranked products (e.g., the second-largest). We note that these relationships need not be positive. Diseconomies of scope, for example, could provide more productive firms with greater incentives to specialize in their most profitable products and destinations, potentially inducing negative relationships in Table 4.

We also investigate a related prediction of the model, which has exports of a given firm-product – e.g., the firm’s largest product – increasing more rapidly with productivity than average exports per product. This gap arises from the differences in export composition between less- and more-productive firms highlighted in Proposition 5. As productivity rises, exports of a given product increase but the firm extends itself into additional products with lower values of consumer taste.

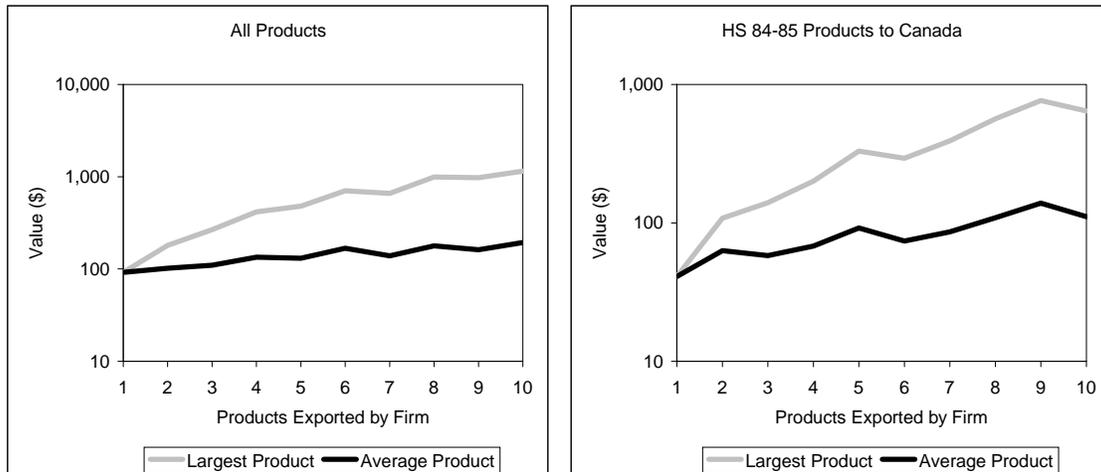
To examine this prediction, we again use cross-sectional data on firms exporting different numbers of products. The left panel of Figure 1 displays the mean size of firms’ largest export product versus the mean size of their average export product across firms exporting up to ten products.³⁵ The right panel reports the same statistics but for a much narrower sample restricted to firms’ exports of Machinery and Electrical products (HS 84-85) to Canada. Vertical axes in both panels use logarithmic scales, and both figures are truncated at ten products to conform with Census disclosure requirements. Data are for 2002, but we note that we find similar patterns in other years as well as for other combinations of countries and products. Consistent with the compositional effects emphasized in the model, both panels show that largest-product exports increase sharply with the number of products exported, while average exports per product increase more modestly.³⁶

5.5. *Within-Firm Heterogeneity*

A key feature of our model is heterogeneity in export shipments across products and destinations within firms. As examined above, this variation has aggregate implications for both revenue-based productivity and the relationship between the extensive and intensive margins of trade. In this section, we present additional evidence on within-firm product and destination heterogeneity and the extent to which it conforms to the predictions of a Pareto distribution. For brevity we report results for firms exporting ten products or exporting to ten destinations, but note that findings are similar for firms producing up to nine products and serving up to nine destinations, where we again restrict the analysis to ten or less products and destinations to conform with Census disclosure requirements.

³⁵Firms exporting up to ten products account for roughly 85 percent of U.S. exporting firms and 10 percent of U.S. export value, respectively.

³⁶As discussed in footnote 23, Pareto-distributed consumer tastes and product fixed costs of exporting that are independent of consumer tastes together imply that average exports per product are uncorrelated with the number of products exported. This implication is at odds with the moderate but statistically significant rise in average exports per product displayed in Figure 1.



Notes: Figures display the mean value of firms' largest and average export product across all firms exporting the noted number of products. The left panel displays trends for all exports. The right panel is restricted to exports of Machinery and Electrical products (HS 84-85) to Canada. Data are for 2002. Y-axes are logged.

Figure 1: Exports of Firms' Largest and Average Products by Number of Products Firms Export, 2002.

As before, data are for 2002, but results are comparable across years.

The first two columns of Table 5 report the average share of each product and each destination in total firm exports for firms exporting ten ten-digit HS products and serving ten destinations, respectively. As apparent from the first two columns, the distribution of exports across products and destinations is highly skewed, with the largest of a firm's products or destinations accounting for roughly one half of its exports. A similar skewness in the distribution of exports across products is observed even within destinations, as shown in the third column of the table, which restricts analysis to firms' exports to Canada. Comparable skewness is also apparent within firms' Machinery and Electrical product exports to Canada, in the fourth column.

In the model, exports are log linear in consumer tastes and only vary across a firm's products within destinations as a result of heterogeneity in consumer tastes. Under the assumption of a Pareto distribution of consumer tastes, the model implies a linear relationship between the log rank of products and their log share of firm exports within destinations.³⁷ To provide evidence on this relationship, we estimate an OLS regression of the log rank of products exported to Canada on their log share of firm exports to Canada using the data reported in the third column of Table 5. The fitted and actual

³⁷If the distribution of firm exports across products within destinations is Pareto with minimum value k and shape parameter a , we have $\text{Prob}(x > x') = (k/x')^a$, where x denotes exports. Taking logarithms in this expression and rearranging terms yields the following relationships: $\log(\text{Rank}_p) = A - a \log(x_p) = B - a \log(\text{Share}_p)$, where Rank_p is the rank of x_p , $\text{Share}_p = x_p/X$, and A , B and $X = \sum_p x_p$ are constants.

Table 5: Distribution of Firm Exports Across Products and Destinations, 2002

	Rank	Export	Export	Products	HS 84-85
		Products	Destinations	Exported to Canada	Products Exported to Canada
% of Firm Exports	1	49.0	47.7	47.4	47.9
	2	18.6	19.5	19.4	19.3
	3	10.5	11.1	11.1	11.0
	4	6.7	7.1	7.0	7.0
	5	4.6	4.9	4.8	4.7
	6	3.4	3.4	3.4	3.3
	7	2.5	2.4	2.5	2.4
	8	1.9	1.7	1.9	1.8
	9	1.5	1.2	1.5	1.4
	10	1.1	0.9	1.1	1.1

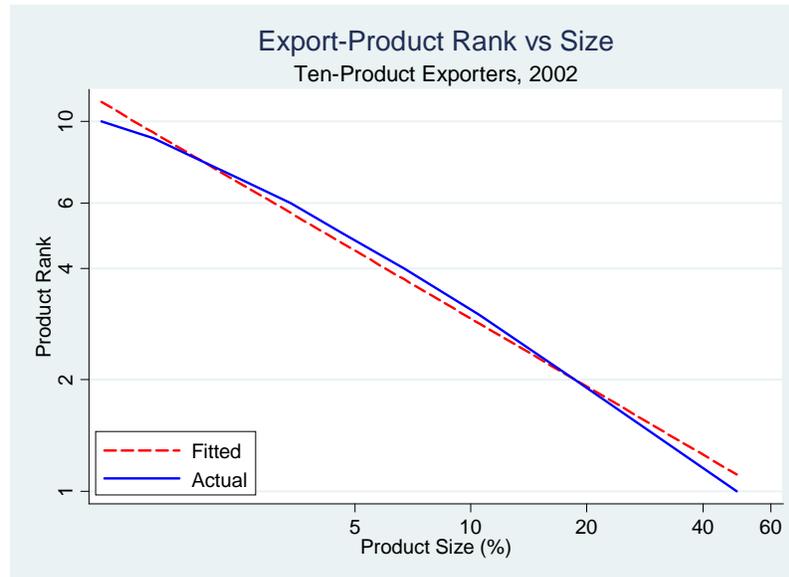
Notes: First two columns display mean percent of firm exports represented by product or country with noted rank (from high to low) across firms exporting ten ten-digit HS products or exporting to ten destinations, respectively, in 2002. Third and fourth columns restricted to firms exporting ten products to Canada, and ten Machinery and Electrical products (HS 84-85) to Canada, respectively. Sample sizes across the four columns are 2791, 1641, 983 and 322 firms, respectively.

values for log rank and log share are displayed in Figure 2. As indicated in the figure, actual values lie above the regression line in the middle of the distribution and below the regression line in the tails, implying thinner tails than the Pareto distribution. Including a quadratic term in the log share of firm exports in the regression, we find that the null hypothesis of linearity implied by a Pareto distribution is rejected at conventional levels of statistical significance.

We note that we find a similar pattern of results for firms exporting between four and nine products to Canada, and for firms' exports to other destinations. We also find similar rejections of a Pareto distribution for the distribution of exports across products for all destinations (first column of Table 5), the distribution of exports across destinations for all products (second column of Table 5), and the distribution of exports across products within HS codes 84-85 for Canada (fourth column of Table 5).³⁸

Taken together, these results suggest that the rejection of a Pareto distribution is a feature of the distribution of firm sales across products within destinations and is not driven by aggregation across destinations or products. Furthermore, the distribution of exports within firms exhibits the same features as the distribution of sales across firms in the firm-size literature, which also finds evidence of thinner tails than a Pareto distribution (see, for example, Rossi-Hansberg and Wright 2007).

³⁸These results are consistent with those in Arkolakis and Muendler (2008), who find departures from a Pareto distribution in the upper and lower tails of the distribution of exports within destinations in Brazilian data.



Notes: The solid line plots average within-firm product rank (low to high) against within-firm product size across the 1641 firms exporting ten products to Canada. The fitted line is the estimated relationship from an ordinary-least-squares regression of log product rank on log product size.

Figure 2: Within-Firm Export Product Size versus Product Rank for Firms Exporting Ten Products to Canada in 2002

5.6. Quantitative Importance of the Extensive Margins

One of the more striking features of micro data on international trade is the overwhelming importance of multiple-product, multiple-destination firms. U.S. firms exporting five or more ten-digit Harmonized System (HS) products, for example, account for 98 percent of total U.S. exports, while firms exporting to five or more destinations represent 93 percent of exports. Moreover, these activities are highly correlated: 92 percent of U.S. exports are due to firms that export five or more products while also exporting to five or more destinations.³⁹

In our framework, extensive margins magnify inequality in the firm-size and country-size export distributions compared with a setting in which these margins do not exist. Relative to less-productive firms, more-productive firms export more products to a given destination, export a given product to more destinations, and export more value per given product-country and level of consumer taste. Analogously, countries with lower variable trade costs receive more products from a given firm, receive a given product from more firms, and receive more value per given firm-product and level of consumer taste.

³⁹These figures are reported by Bernard, Jensen and Schott (2009). Bernard, Redding and Schott (2009) and Goldberg, Khandelwal, Pavcnik and Topalova (2008) show that multiple-product firms account for correspondingly large shares of production.

Here, we quantify the importance of firms' product and country selection for the firm-size distribution by constructing counterfactual distributions that sequentially eliminate their extensive margins. These counterfactuals are based on a decomposition of total firm exports (x_f) into the number of countries to which a firm exports (c_f), the number of products which the firm exports (p_f), the density of firm exports ($d_f = o_f/(c_f p_f)$) and average exports per country-product conditional on positive exports (\bar{x}_f):

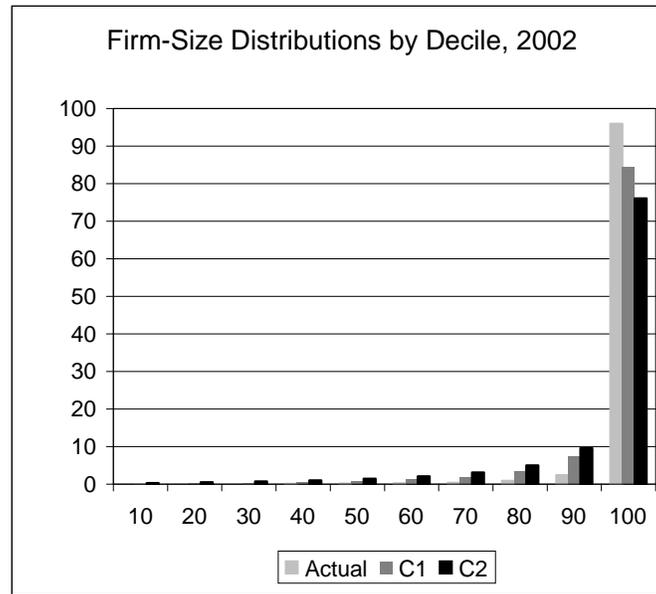
$$x_f = c_f p_f d_f \bar{x}_f \quad (29)$$

where $\bar{x}_f = x_f/o_f$ and o_f is the number of country-product observations with positive exports for firm f . In our first counterfactual ($C1$), we exclude the country extensive margin by assuming that all firms export to just a single country. That is, we use equation (29) to recalculate total firm exports for all firms by using $c_f = 1$ in place of its actual value. In our second counterfactual ($C2$), we further assume that firms export only a single product, setting both $p_f = 1$ and $d_f = 1$.⁴⁰

Actual and counterfactual firm export distributions across firm-export-size deciles are reported in Figure 3. As indicated in the figure, excluding extensive margins results in considerably less concentrated distributions of exports across firms. Comparing the actual, $C1$ and $C2$ distributions, we find that the top 10 percent of firms account for 96 percent of actual exports but only 84 percent of hypothetical exports when the country margin is eliminated and only 76 percent of exports when both extensive margins are eliminated. To explain the extreme concentration of actual exports, standard models of heterogeneous firms and trade require either a highly-skewed distribution of productivity and/or a high elasticity of substitution. Figure 3 suggests that endogenous product and destination-market selection can rationalize this concentration with less skewed productivity distributions or lower elasticities of substitution. As a result of the positive correlation between firms' productivity and their extensive margins, a given skewness of the productivity distribution and a given elasticity of substitution results in more inequality of exports across firms than in conventional settings.

To quantify the impact of the extensive margins of trade on the distribution of exports across U.S. export destinations, Figure 4 plots U.S. export destinations' total export value against the intensive-margin component of that value, i.e., x_c versus \bar{x}_c from the decomposition of country exports in (27). While by construction actual exports are greater than or equal to the contribution of the intensive margin, the ratio of actual exports to the intensive margin increases substantially with the value of actual exports. As the axes of the figure are in logarithms, this finding implies that the percentage contribution of the extensive margin to actual exports increases with the value of actual exports. An immediate corollary of this finding is that the pattern of actual U.S. exports across countries is

⁴⁰As predicted by our model, the number of countries to which a firm exports is highly skewed: roughly two-thirds of firms export to a single country. We also find skewness in the number of products firms export: roughly two-fifths of firms export a single product to a single country.



Notes: Figure plots actual and two counter-factual distributions of U.S. exports across firm-size deciles, from low to high. The first counter-factual distribution (C1) assumes firms serve only a single export destination. The second counter-factual distribution (C2) further assumes that firms export only a single product. Firms are re-ranked according to size for each distribution. Data are for 2002.

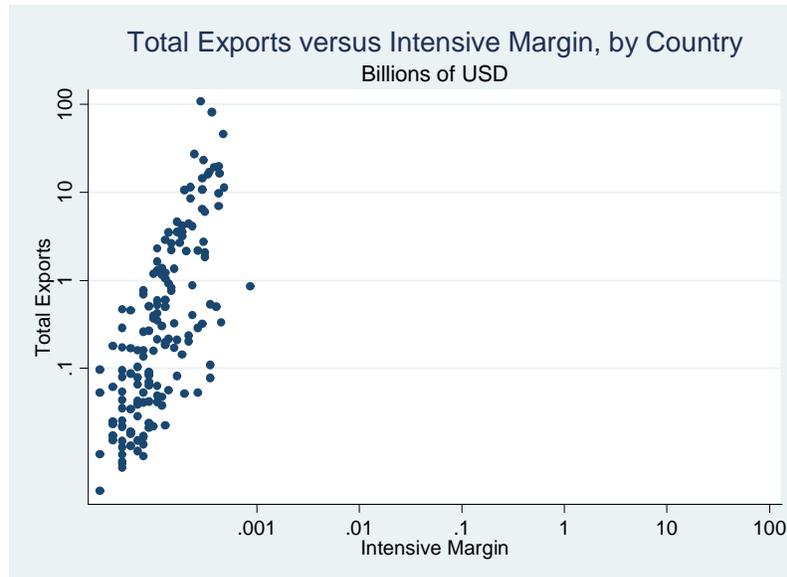
Figure 3: Actual versus Counterfactual Firm-Size Distributions, 2002

substantially different from what it would be with only intensive-margin variation. The greater contribution of the extensive margin at higher values of actual exports magnifies inequality in the distribution of U.S. exports across countries.

The data displayed in Figure 4 exhibit a starkly different pattern from the predictions of representative-firm new trade theory models such as Krugman (1980), which predict that all of the response to differences in trade costs across countries occurs on the intensive margin. Thus, Figure 4 reinforces the message that the types of extensive-margin adjustments emphasized by our model are quantitatively important in the aggregate as well as across firms.

6. Conclusion

Firms exporting multiple products to multiple destinations dominate production and trade but have received comparatively little theoretical attention. This paper develops a tractable general equilibrium model of multiple-product, multiple-destination firms that is consistent with key features of the distribution of exports across firms, products and countries. In contrast to existing approaches, our model incorporates idiosyncratic variation



Notes: Figure displays U.S. export destinations' total export value against their intensive-margin of average exports per firm-product observation. Sample restricted to countries receiving exports from at least 100 U.S. firms. Data are for 2002.

Figure 4: The Contribution of the Intensive Margin to U.S. Exports, by Destination Country

in exports within firms across both products and destinations through heterogeneity in consumer tastes. Incorporating such heterogeneity within firms not only brings the model closer to the data, but also has important aggregate implications for the determinants of trade and the impact of trade liberalization on firm and industry productivity.

We show that U.S. trade data provide strong empirical support for key predictions of the model. Across countries, we find that the negative effect of distance on bilateral trade is accounted for entirely by the extensive margins, with distance having, if anything, a positive association with the intensive margin. Across firms, we find that more productive firms export more products to more destinations, that trade liberalization is accompanied by firms focusing on their core competencies, and that the export value of firms' largest products increase more rapidly with the number of products exported than average exports per product, consistent with firm productivity influencing export composition.

More broadly, our analysis suggests that the reallocation of resources in response to events such as trade liberalization may be even more important than hitherto thought insofar as it occurs within as well as across firms. Relatedly, our findings point to the role of the within-firm organization of economic activity across products and countries in influencing firm, industry and aggregate outcomes.

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A Data Appendix

Data on firm’s domestic output, domestic factor use and the number of five-digit SIC goods produced domestically are from the U.S. Census of Manufactures (CMF). Data on export value, ten-digit HS export products and export destinations is from the Linked/Longitudinal Firm Trade Transaction Database (LFTTD).

A1. *Census of Manufactures (CMF) Data*

Manufacturing Censuses are conducted every five years and we make use of data from the 1987, 1992 and 1997 censuses. The sampling unit for each Census is a manufacturing “establishment”, or plant, and the sampling frame in each Census year includes information on the mix of products produced by the plant. Very small manufacturing plants (referred to as Administrative Records) are excluded from the analysis unless otherwise noted because data on their mix of products are unavailable. Because product-mix decisions are made at the level of the firm, we aggregate the data to that level for our analysis.⁴¹

Our definition of “product” is based upon 1987 Standard Industrial Classification (SIC) categories, which segment manufacturing output generally according to its end use. We refer to five-digit SIC categories as products or goods.⁴² In our sample, aggregate manufacturing contains 1848 products. For each firm in each Census year, we record the set of products in which the firm produces. We also observe firms’ total and product-level output. There are an average of 141,561 surviving firms in each Census year for which such extensive-margin adjustments can be observed. For more detail, see Bernard, Redding and Schott (2009).

A2. *Linked/Longitudinal Firm Trade Transaction Database (LFTTD)*

This dataset has two components. The first, foreign trade data assembled by the U.S. Census Bureau and the U.S. Customs Bureau, captures all U.S. international trade transactions between 1992 and 2004 inclusive. For each flow of goods across a U.S. border, this dataset records the product classification and the value. Products in the LFTTD are tracked according to ten-digit Harmonized System (HS) categories, which break exported goods into 8572 products. The second component of the LFTTD is the Longitudinal

⁴¹Firm identifiers are derived from firms’ legal identities, and firms can consist of one or many establishments. Census uses an annual Company Organization Survey both to determine how new firms are organized and to keep track of changes in incumbent firms’ ownership structure over time, e.g., the buying and selling of plants, the creation of new plants or the closing of existing plants.

⁴²SIC categories undergo minor revisions in each census year but experienced a major revision in 1987. Census uses an internally generated concordance to map product codes collected in censuses after 1972 to the 1987 revision. To be conservative, we drop the roughly 1 percent of five-digit codes (representing roughly 5 percent of total value) that do not appear in the 1987 to 1997 censuses.

Business Database (LBD) of the U.S. Census Bureau, which records annual employment and survival information for most U.S. establishments.⁴³ Employment information for each establishment is collected in March of every year and we aggregate the establishment data up to the level of the firm. Matching the annual information in the LBD to the transaction-level trade data yields the LFTTD. We note that our ability to match trade transactions to firms is imperfect: across 1992 to 2004, we match transactions representing 76 and 82 percent of export and import value, respectively. For further details about the construction of the dataset, see Bernard, Jensen and Schott (2009).

A3. Other Data Sources

Industry-level Canadian tariff data are from Treffer (2004). To match Canadian SIC manufacturing industries to U.S. SIC manufacturing industries we use a concordance developed by Statistics Canada. Using this concordance, we can observe tariff changes for 40 percent (174) of U.S. manufacturing industries representing 50 percent of total manufacturing shipments in 1987. For industries where tariff information is missing, we assign the average of the two-digit SIC industry in which it belongs.

B Theory Appendix

B1. Weighted Average Productivity

Weighted-average productivity in the domestic market is:

$$\tilde{\varphi}_d \equiv \left[\frac{1}{1 - G(\varphi_d^*)} \int_{\varphi_d^*}^{\infty} (\varphi \tilde{\lambda}_d(\varphi))^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, \quad (30)$$

where $\tilde{\lambda}_d(\varphi)$ denotes weighted-average consumer tastes in the domestic market for a firm with productivity φ :

$$\tilde{\lambda}_d(\varphi) = \left[\frac{1}{1 - Z(\lambda_d^*(\varphi))} \int_{\lambda_d^*(\varphi)}^{\infty} \lambda_d^{\sigma-1} z(\lambda_d) d\lambda_d \right]^{\frac{1}{\sigma-1}}.$$

Similarly, weighted-average productivity in the export market is:

$$\tilde{\varphi}_x \equiv \left[\frac{1}{1 - G(\varphi_x^*)} \int_{\varphi_x^*}^{\infty} (\varphi \tilde{\lambda}_x(\varphi))^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, \quad (31)$$

⁴³This dataset excludes the U.S. Postal Service and firms in agriculture, forestry and fishing, railroads, education, public administration and several smaller sectors. See Jarmin and Miranda (2002) for an extensive discussion of the LBD and its construction.

where $\tilde{\lambda}_x(\varphi)$ denotes weighted-average consumer tastes in the export market for a firm with productivity φ :

$$\tilde{\lambda}_x(\varphi) = \left[\frac{1}{1 - Z(\lambda_x^*(\varphi))} \int_{\lambda_x^*(\varphi)}^{\infty} \lambda_x^{\sigma-1} z(\lambda_x) d\lambda_x \right]^{\frac{1}{\sigma-1}}.$$

B2. Revenue-Based Productivity

Following standard revenue-based measures of productivity (see for example Klette and Griliches 1996 and De Loecker 2006), we deflate a firm's revenue from an individual product by the aggregate price index for that product. The firm's real revenue from a product in market j is therefore:

$$\frac{r_j(\varphi, \lambda_j)}{P_j} = \frac{p_j(\varphi, \lambda_j) q_j(\varphi, \lambda_j)}{P_j}.$$

Using the inverse CES demand curve to substitute for $p_j(\varphi, \lambda_j)$, we obtain:

$$\frac{r_j(\varphi, \lambda_j)}{P_j} = \lambda_j^{\frac{\sigma-1}{\sigma}} q_j(\varphi, \lambda_j)^{\frac{\sigma-1}{\sigma}} \left(\frac{E_j}{P_j} \right)^{\frac{1}{\sigma}}.$$

Using the production technology and taking logarithms, we obtain:

$$\log r_j(\varphi, \lambda_j) - \log P_j = \frac{\sigma-1}{\sigma} \log(\varphi \lambda_j) + \frac{\sigma-1}{\sigma} \log l_{vj}(\varphi, \lambda_j) + \frac{1}{\sigma} \log \left(\frac{E_j}{P_j} \right), \quad (32)$$

where $l_{vj}(\varphi, \lambda_j)$ denotes variable labor input and E_j/P_j captures aggregate product market conditions for market j .

From the ‘‘revenue production function’’ (32), the revenue-based measure of a firm's productivity in a product for market j is $\varphi \lambda_j$, which captures both true productivity, φ , and demand, λ_j . Given data on revenue and inputs by firm, product and market, and given appropriate instruments for variable labor input, $l_{vj}(\varphi, \lambda_j)$, and controls for aggregate product market conditions, E_j/P_j , a firm's revenue-based productivity in a product and market can be estimated from (32).

Following standard empirical methods for productivity aggregation, we define productivity for the firm as a whole as the revenue-share weighted average of productivity for each product and market:

$$\begin{aligned} \theta &\equiv \varphi \left[\int_{\lambda_d^*(\varphi)}^{\infty} \tilde{r}_d(\varphi, \lambda_d) \lambda_d d\lambda_d + n \int_{\lambda_x^*(\varphi)}^{\infty} \tilde{r}_x(\varphi, \lambda_x) \lambda_x d\lambda_x \right], \\ &= \varphi \int_{\lambda_d^*(\varphi)}^{\infty} \tilde{r}(\varphi, \lambda) \lambda d\lambda \end{aligned} \quad (33)$$

where $\tilde{r}_j(\varphi, \lambda_j) = r_j(\varphi, \lambda_j) z(\lambda_j) / r(\varphi)$ for $j \in \{d, x\}$ and $\tilde{r}(\varphi, \lambda) = [r_d(\varphi, \lambda) + nr_x(\varphi, \lambda)] z(\lambda) / r(\varphi)$.

While the exposition in this section of the appendix concentrates on measures of revenue-based productivity derived from production function estimation, similar results hold for other revenue-based measures such as labor productivity. Although our assumption of constant elasticity of substitution preferences and monopolistic competition implies that prices are inversely related to the firm productivity draw φ , revenue-based labor productivity is monotonically increasing in φ because of fixed production costs. As φ rises, variable labor input and output rise, but prices fall, leaving revenue per variable input unchanged. Nonetheless, as φ rises and variable labor input and revenue increase, the fixed labor input is spread over more units of revenue, which raises revenue-based labor productivity.

B3. Fixed Production Costs

To simplify the characterization of a firm's decision to supply a market, we set fixed production costs to zero in the main text and instead assume a fixed cost of supplying each market. In this section of the Appendix, we show that the introduction of fixed production costs is straightforward, but merely complicates the analysis without adding additional insight, because it introduces interdependence across markets.

In a setting with demand heterogeneity across countries and fixed production costs, the decision to whether supply a product to an individual market depends on whether that product is produced, which depends on profitability across all markets. In the presence of a product fixed production cost of $\vartheta > 0$, a firm's production and market supply decisions can be characterized as follows. First, denote the (endogenous) set of products produced by a firm by $\Psi(\omega) \subseteq [0, 1]$, where we index firms by ω because firms with same productivity φ can produce different sets of products depending on their idiosyncratic draws for consumer tastes. Product k is supplied to market j by firm ω with productivity φ and consumer tastes λ_{jk} if:

$$r_{jk}(\varphi, \lambda_{jk}, \omega) \geq \sigma f_j, \quad \text{and} \quad k \in \Psi(\omega). \quad (34)$$

Second, denote the (endogenous) set of markets to which product k can be profitably supplied by firm ω by $\Phi_k(\omega) \subseteq [0, n+1]$. Product k is produced by firm ω with productivity φ and realizations of consumer tastes λ_{jk} that vary across markets j if:

$$\int_{j \in \Phi_k(\omega)} \left[\frac{r_{jk}(\varphi, \lambda_{jk}, \omega)}{\sigma} - f_j \right] dj \geq \vartheta. \quad (35)$$

Together these two equations characterize the firm's market supply and production decisions. Intuitively, when there are fixed production costs, a firm will only supply a product to a market if it both draws a consumer tastes above the product cutoff (34) for that market and also can generate sufficient variable profits across all markets to cover fixed production costs (35). The analysis remains exactly as in the main text except that in

addition to the product cutoff condition being satisfied ((8) for the domestic market and (10) for each export market), we also require the profitable production condition (35) to be satisfied.

B4. Proof of Proposition 1

Proof. As the model has a recursive structure, the determination of general equilibrium is straightforward. We begin by determining $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$. The zero-profit cutoff condition for the least productive firm (14) determines $\lambda_d^*(\varphi_d^*)$ independently of the other equations of the model. Similarly, the exporting cutoff condition for the least productive exporter (16) determines $\lambda_x^*(\varphi_x^*)$ independently of the other equations of the model. Having characterized $\{\lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$, φ_d^* can be determined from the free entry condition independently of the other equations of the model. Substituting for $\lambda_d^*(\varphi)$, $\lambda_x^*(\varphi)$, and φ_x^* using (9), (11) and (17), the free entry condition (19) can be written solely in terms of φ_d^* and the already determined $\lambda_d^*(\varphi_d^*)$. As $\varphi^* \rightarrow 0$, $V \rightarrow \infty$, while as $\varphi^* \rightarrow \infty$, $V \rightarrow 0$. Moreover, V is monotonically decreasing in φ_d^* . Therefore the free entry condition alone determines a unique equilibrium value of φ_d^* . Having characterized $\{\lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*), \varphi_d^*\}$, φ_x^* follows immediately from (17). Given $\{\lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*), \varphi_d^*, \varphi_x^*\}$, we can determine $\lambda_d^*(\varphi)$ for all $\varphi \geq \varphi_d^*$ and $\lambda_x^*(\varphi)$ for all $\varphi \geq \varphi_x^*$ from (9) and (11). We next determine the price index and aggregate revenue for each product $\{P, R\}$. Aggregate revenue for the economy as a whole can be determined from the steady-state stability and free entry conditions, which as discussed in the main text imply $R = L$. The price index for each product, P , depends on weighted average productivity and the mass of firms in the domestic and export markets (equation (25)). Having characterized $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$, and hence the functions $\lambda_d^*(\varphi)$ and $\lambda_x^*(\varphi)$, weighted average productivity in the domestic and export market follow from (30) and (31). To determine the mass of firms supplying each product to the domestic and export markets, note that average firm revenue can be written as a function of $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$ alone using (7):

$$\begin{aligned} \bar{r} = & \int_{\varphi_d^*}^{\infty} \left[\int_{\lambda_d^*(\varphi)}^{\infty} \left(\frac{\lambda_d}{\lambda_d^*(\varphi)} \right)^{\sigma-1} \sigma f_d z(\lambda_d) d\lambda_d \right] \left(\frac{g(\varphi)}{1-G(\varphi_d^*)} \right) d\varphi \\ & + \left(\frac{1-G(\varphi_x^*)}{1-G(\varphi_d^*)} \right) \int_{\varphi_x^*}^{\infty} \left[\int_{\lambda_x^*(\varphi)}^{\infty} \left(\frac{\lambda_x}{\lambda_x^*(\varphi)} \right)^{\sigma-1} \sigma f_x z(\lambda_x) d\lambda_x \right] \left(\frac{g(\varphi)}{1-G(\varphi_x^*)} \right) d\varphi \end{aligned} \quad (36)$$

The mass of firms follows immediately from aggregate revenue and average revenue: $M = R/\bar{r}$. Combining the mass of firms with $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$ and the functions $\lambda_d^*(\varphi)$ and $\lambda_x^*(\varphi)$ determines the mass of firms supplying a given product to the domestic market, M_d , in (23) and the export market, M_x , in (24). This completes the determination of the product price index. Revenue for each product follows immediately from the CES

revenue function, the product price index and aggregate revenue. This completes the characterization of the equilibrium sextuple $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*), P, R\}$. ■

B5. Proof of Proposition 2

Proof. The free entry condition in the open economy (19) takes the same form as in the closed economy except that the expression for the expected value of entry, V , includes an additional positive term (in the closed economy $\varphi_x^* \rightarrow \infty$ and $\lambda_x^*(\varphi) \rightarrow \infty$). From the proof of Proposition 1, V can be written solely in terms of φ_d^* and $\lambda_d^*(\varphi_d^*)$, where from (14) $\lambda_d^*(\varphi_d^*)$ is unchanged by the opening of trade. As V is monotonically decreasing in φ_d^* , it follows that φ_d^* must be higher in the open economy than in the closed economy in order to equate V with the unchanged sunk entry cost.

As $\lambda_d^*(\varphi) = (\varphi_d^*/\varphi) \lambda_d^*(\varphi_d^*)$ is monotonically increasing in φ_d^* , and $\lambda_d^*(\varphi_d^*)$ is unchanged by the opening of trade, the opening of trade raises $\lambda_d^*(\varphi)$ for each value of φ , and induces firms to drop low-consumer-tastes products from the domestic market. In addition, some surviving firms begin to export for finite values of φ_x^* and $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi) \lambda_x^*(\varphi_x^*)$ in the open economy. As there is selection into export markets, these new entrants into exporting are high-productivity firms, $\varphi_x^* > \varphi_d^*$, and the products that they add in export markets have high consumer tastes, $\lambda_x^*(\varphi) > \lambda_d^*(\varphi)$.

The effect of the opening of trade on firm revenue-based productivity (33) depends on the change in the firm revenue share of products with different values of consumer tastes. Under autarky, the firm revenue share of products with consumer tastes $\lambda \in [\lambda_d^{*A}(\varphi), \infty)$ is:

$$\tilde{r}^A(\varphi, \lambda) = \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{\int_{\lambda_d^{*A}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda}, \quad \lambda \in [\lambda_d^{*A}(\varphi), \infty), \quad (37)$$

where $\tilde{r}(\varphi, \lambda) \equiv r(\varphi, \lambda) z(\lambda) / r(\varphi)$ is defined as above; the superscript A denotes autarky; and the superscript T will be used to denote the open economy below. To characterize the firm revenue share of products in the open economy, consider non-exporters and exporters in turn.

(a) For non-exporters, products with consumer tastes $\lambda \in [\lambda_d^{*A}(\varphi), \lambda_d^{*T}(\varphi))$ are dropped from the domestic market and hence experience a decline in their share of firm revenue. In contrast, products with consumer tastes $\lambda \in [\lambda_d^{*T}(\varphi), \infty)$ experience a rise in their share of firm revenue. Therefore the distribution $\tilde{r}(\varphi, \lambda)$ in the open economy first-order stochastically dominates that in the closed economy, and revenue-based productivity (33) for non-exporters rises.

(b) For exporters, products with consumer tastes $\lambda \in [\lambda_d^{*A}(\varphi), \lambda_d^{*T}(\varphi))$ are dropped from the domestic market and hence experience a decline in their share of firm revenue. Products with consumer tastes $\lambda \in [\lambda_d^{*T}(\varphi), \lambda_x^{*T}(\varphi))$ experience an ambiguous change in

their share of firm revenue:

$$\begin{aligned}\tilde{r}^T(\varphi, \lambda) &= \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{\int_{\lambda_d^{*A}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda - \Theta_1}, \quad \lambda \in [\lambda_d^{*T}(\varphi), \lambda_x^{*T}(\varphi)], \\ \Theta_1 &\equiv \int_{\lambda_d^{*A}(\varphi)}^{\lambda_d^{*T}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda - n\tau^{1-\sigma} \int_{\lambda_x^{*T}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda.\end{aligned}$$

As the sign of Θ_1 is in general ambiguous, $\tilde{r}^T(\varphi, \lambda) \geq \tilde{r}^A(\varphi, \lambda)$ for $\lambda \in [\lambda_d^{*T}(\varphi), \lambda_x^{*T}(\varphi)]$. Finally, products with consumers tastes $\lambda \in [\lambda_x^{*T}(\varphi), \infty)$ experience a rise in their share of firm revenue:

$$\begin{aligned}\tilde{r}^T(\varphi, \lambda) &= \frac{[1 + n\tau^{1-\sigma}] (\varphi\lambda)^{\sigma-1} z(\lambda)}{[1 + n\tau^{1-\sigma}] \int_{\lambda_d^{*A}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda - \Theta_2}, \quad \lambda \in [\lambda_x^{*T}(\varphi), \infty), \\ \Theta_2 &\equiv \left[\begin{aligned} &[1 + n\tau^{1-\sigma}] \int_{\lambda_d^{*A}(\varphi)}^{\lambda_d^{*T}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda \\ &+ \int_{\lambda_d^{*T}(\varphi)}^{\lambda_x^{*T}(\varphi)} n\tau^{1-\sigma} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda \end{aligned} \right] > 0.\end{aligned}$$

where we have re-written the denominator of $\tilde{r}^T(\varphi, \lambda)$ in a different form. As $\Theta_2 > 0$, $\tilde{r}^T(\varphi, \lambda) > \tilde{r}^A(\varphi, \lambda)$ for $\lambda \in [\lambda_x^{*T}(\varphi), \infty)$.

Irrespective of whether the revenue share of products with intermediate consumer tastes $\lambda \in [\lambda_d^{*A}(\varphi), \lambda_d^{*T}(\varphi)]$ rises or falls, the difference between the open economy value of $\tilde{r}(\varphi, \lambda)$ and the closed economy value goes from being negative at low values of $\lambda \in [\lambda_d^{*A}(\varphi), \lambda_d^{*T}(\varphi)]$ to being positive at high values of $\lambda \in [\lambda_x^{*T}(\varphi), \infty)$. This is a sufficient condition for the distribution $\tilde{r}(\varphi, \lambda)$ in the open economy to first-order stochastically dominate that in the closed economy. Therefore revenue-based firm productivity (33) for exporters rises. ■

B6. Proof of Proposition 3

Proof. We first characterize $d\varphi_d^*/d\tau$. From the free entry condition (19), define $\Upsilon = V - f_e$. By the implicit function theorem, $d\varphi_d^*/d\tau = -(d\Upsilon/d\tau) / (d\Upsilon/d\varphi_d^*)$. Substituting for φ_x^* , $\lambda_d^*(\varphi)$ and $\lambda_x^*(\varphi)$ in (19) using (9), (14), (11), (16), and (17), we obtain $dV/d\tau < 0$ and $dV/d\varphi_d^* < 0$. Therefore, we have established that $d\varphi_d^*/d\tau < 0$.

We next characterize $d\varphi_x^*/d\tau$. Differentiating with respect to τ in equation (17), we obtain:

$$\left(\frac{d\varphi_x^*}{d\tau} \frac{\tau}{\varphi_x^*} \right) = 1 + \left(\frac{d\varphi_d^*}{d\tau} \frac{\tau}{\varphi_d^*} \right), \quad (38)$$

It follows that to establish $d\varphi_x^*/d\tau > 0$, it suffices to show that $(d\varphi_d^*/d\tau) / (\tau/\varphi_d^*) > -1$. To do so, we again use the implicit function theorem to evaluate $d\varphi_d^*/d\tau = -(d\Upsilon/d\tau) / (d\Upsilon/d\varphi_d^*)$. Additionally, equations (9), (14), (11), (16), and (17) imply the following: $d\lambda_x^*(\varphi)/d\tau = \lambda_x^*(\varphi)/\tau$, $d\lambda_x^*(\varphi)/d\varphi_d^* = \lambda_x^*(\varphi)/\varphi_d^*$, $d\varphi_x^*/d\tau = \varphi_x^*/\tau$ and $d\varphi_x^*/d\varphi_d^* = \varphi_x^*/\varphi_d^*$. Combining

these results with $d\varphi_d^*/d\tau = -(d\Upsilon/d\tau)/(d\Upsilon/d\varphi_d^*)$, we obtain $(d\varphi_d^*/d\tau)/(\tau/\varphi_d^*) > -1$. Therefore we have established that $d\varphi_x^*/d\tau > 0$.

Since $\lambda_d^*(\varphi) = (\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)$, where $\lambda_d^*(\varphi_d^*)$ is invariant to τ , and since $d\varphi_d^*/d\tau < 0$, we have established that $d\lambda_d^*(\varphi)/d\tau < 0$. Additionally, since $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\lambda_x^*(\varphi_x^*)$, where $\lambda_x^*(\varphi_x^*)$ is invariant to τ , and since $d\varphi_x^*/d\tau > 0$, we have established that $d\lambda_x^*(\varphi)/d\tau > 0$. The remainder of the proof follows a similar structure as for Proposition 2 above. Therefore, to conserve space, we sketch the remainder of the proof here and present the complete derivation in the web-based technical appendix.

(a) As $d\lambda_d^*(\varphi)/d\tau < 0$, reductions in variable trade costs induce domestic firms to drop low consumer tastes products from the domestic market, which raises their revenue-based productivity (33).

(b) As $d\lambda_d^*(\varphi)/d\tau < 0$, reductions in variable trade costs also induce new exporters to drop low-consumer-tastes products from the domestic market, which raises their revenue-based productivity (33). In addition, new exporters add high consumer tastes products in the export market (since $\lambda_x^*(\varphi) > \lambda_d^*(\varphi)$), which further raises their revenue-based productivity.

(c) As $d\lambda_d^*(\varphi)/d\tau < 0$, reductions in variable trade costs also induce continuing exporters to drop low consumer tastes products from the domestic market, which raises their revenue-based productivity (33). In addition, as $d\lambda_x^*(\varphi)/d\tau > 0$, reductions in variable trade costs induce continuing exporters to add additional products in export markets. As these additional products have low values of consumer tastes relative to the products exported prior to the reduction in variable trade costs, adding these additional products can reduce the revenue-based productivity of continuing exporters. The overall change in the revenue-based productivity of continuing exporters is therefore ambiguous. ■

B7. Proof of Proposition 4

Proof. **(a)** and **(b)**: The share of products exported to a given country by existing exporters with productivity $\varphi \geq \varphi_x^*$ is $[1 - Z(\lambda_x^*(\varphi))]$. Similarly, the share of countries to which a given product is exported by existing exporters of that product is $[1 - Z(\lambda_x^*(\varphi))]$. Additionally, from (11) and (16), we have $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\lambda_x^*(\varphi_x^*)$ where $\lambda_x^*(\varphi_x^*)$ is invariant to τ . Therefore, to establish **(a)** and **(b)**, it suffices to show $d\varphi_x^*/d\tau > 0$, which has already been established in the proof of Proposition 3. Hence reductions in variable trade costs reduce φ_x^* , reduce $\lambda_x^*(\varphi)$, and raise $[1 - Z(\lambda_x^*(\varphi))]$.

(c) and **(d)**: The share of firms that export equals $\chi \equiv [1 - G(\varphi_x^*)]/[1 - G(\varphi_d^*)]$. Since the proof of Proposition 3 established $d\varphi_d^*/d\tau < 0$ and $d\varphi_x^*/d\tau > 0$, it follows that $d\chi/d\tau < 0$. Hence reductions in variable trade costs increase the share of firms that export, χ .

(e) Average exports per product-country for an existing exporter are:

$$\bar{r}_x(\varphi) = \frac{1}{1 - Z(\lambda_x^*(\varphi))} \int_{\lambda_x^*(\varphi)}^{\infty} \left(\frac{\lambda_x}{\lambda_x^*(\varphi)} \right)^{\sigma-1} \sigma f_x z(\lambda_x) d\lambda_x. \quad (39)$$

Since $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi) \lambda_x^*(\varphi_x^*)$, where $\lambda_x^*(\varphi_x^*)$ is invariant to τ , and since $d\varphi_x^*/d\tau > 0$, we have established that $d\lambda_x^*(\varphi)/d\tau > 0$. Now note that from (39):

$$\frac{d\bar{r}_x(\varphi)}{d\tau} = \left[\underbrace{\frac{dZ(\lambda_x^*(\varphi))/d\lambda_x^*(\varphi)}{1 - Z(\lambda_x^*(\varphi))}}_{>0} \bar{r}_x(\varphi) - \frac{(\sigma - 1)}{\lambda_x^*(\varphi)} \bar{r}_x(\varphi) - \frac{\sigma f_x z(\lambda_x^*(\varphi))}{1 - Z(\lambda_x^*(\varphi))} \right] \underbrace{\frac{d\lambda_x^*(\varphi)}{d\tau}}_{>0},$$

which is in general ambiguous in sign depending on the value of $\lambda_x^*(\varphi)$ and the functional form of the cumulative distribution function for consumer tastes. ■

B8. Proof of Proposition 5

Proof. (a) and (b): The share of products exported to a given country by existing exporters with productivity $\varphi \geq \varphi_x^*$ is $[1 - Z(\lambda_x^*(\varphi))]$. Similarly, the share of countries to which a given product is exported by existing exporters is $[1 - Z(\lambda_x^*(\varphi))]$. As from (11) $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi) \lambda_x^*(\varphi_x^*)$ is monotonically decreasing in φ , and as $Z(\lambda)$ is a continuous cumulative distribution function that is increasing in λ , it follows that $[1 - Z(\lambda_x^*(\varphi))]$ is increasing in φ , which establishes parts (a) and (b) of the proposition.

(c) Average exports per product-country for an existing exporter are (39). Therefore:

$$\frac{d\bar{r}_x(\varphi)}{d\varphi} = \left[\underbrace{\frac{dZ(\lambda_x^*(\varphi))/d\lambda_x^*(\varphi)}{1 - Z(\lambda_x^*(\varphi))}}_{>0} \bar{r}_x(\varphi) - \frac{(\sigma - 1)}{\lambda_x^*(\varphi)} \bar{r}_x(\varphi) - \frac{\sigma f_x z(\lambda_x^*(\varphi))}{1 - Z(\lambda_x^*(\varphi))} \right] \underbrace{\frac{d\lambda_x^*(\varphi)}{d\varphi}}_{<0},$$

which is in general ambiguous in sign depending on the value of $\lambda_x^*(\varphi)$ and the functional form of the cumulative distribution function for consumer tastes. ■