Non-Exclusive Competition in the Market for Lemons*

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Abstract

In order to check the impact of the exclusivity regime on equilibrium allocations, we set up a simple Akerlof-like model in which buyers may use arbitrary tariffs. Under exclusivity, we obtain the (zero-profit, separating) Riley-Rothschild-Stiglitz allocation. Under non-exclusivity, there is also a unique equilibrium allocation that involves a unique price, as in Akerlof (1970). These results can be applied to insurance (in the dual model in Yaari, 1987), and have consequences for empirical tests of the existence of asymmetric information.

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1 Introduction

Adverse selection is widely recognized as a major obstacle to the efficient functioning of markets. This is especially true on financial markets, where buyers care about the quality of the assets they purchase, and fear that sellers have superior information about it. The same difficulties impede trade on second-hand markets and insurance markets. Theory confirms that adverse selection may indeed have a dramatic impact on economic outcomes. First, all mutually beneficial trades need not take place in equilibrium. For instance, in Akerlof's (1970) model of second-hand markets, only the lowest quality goods are traded at the equilibrium price. Second, there may be difficulties with the very existence of equilibrium. For instance, in Rothschild and Stiglitz's (1976) model of insurance markets, an equilibrium fails to exist whenever the proportion of low-risk agents is too high.

Most contributions to the theory of competition under adverse selection have considered frameworks in which competitors are restricted to make exclusive offers. This assumption is for instance appropriate in the case of car insurance, since law forbids to take out multiple policies on a single vehicle. By contrast, competition on financial markets is typically non-exclusive, as each agent can trade with multiple partners who cannot monitor each others' trades with the agent. This paper supports the view that this difference in the nature of competition may have a significant impact on the way adverse selection affects market outcomes. This has two consequences. First, empirical studies that test for the presence of adverse selection should use different methods depending on whether competition is exclusive or not. Second, the regulation of markets plagued by adverse selection should be adjusted to the type of competition that prevails on them.

To illustrate these points, we consider a stylized model of trade under adverse selection. In our model, a seller endowed with some quantity of a good attempts to trade it with a finite number of buyers. The seller and the buyers have linear preferences over quantities and transfers exchanged. In line with Akerlof (1970), the quality of the good is the seller's private information. Unlike in his model, the good is assumed to be perfectly divisible, so that any fraction of the seller's endowment can potentially be traded. An example that fits these assumptions is that of a firm which floats a security issue by relying on the intermediation services of several investment banks. Buyers compete by simultaneously offering menus of contracts, or, equivalently, price schedules.[1] After observing the menus offered, the seller decides of her trade(s). Competition is exclusive if the seller can trade with at most one

[1]As established by Peters (2001) and Martimort and Stole (2002), there is no need to consider more general mechanisms in this multiple-principal single-agent setting.

buyer, and non-exclusive if trades with several buyers are allowed.

Under exclusive competition, our conclusions are qualitatively similar to Rothschild and Stiglitz's (1976). In a simple version of the model with two possible levels of quality, pure strategy equilibria exist if and only if the probability that the good is of high quality is low enough. Equilibria are separating: the seller trades her whole endowment when quality is low, while she only trades part of it when quality is high.

The analysis of the non-exclusive competition game yields strikingly different results. Pure strategy equilibria always exist, both for binary and continuous quality distributions. Aggregate equilibrium allocations are generically unique, and have an all-or-nothing feature: depending of whether quality is low or high, the seller either trades her whole endowment or does not trade at all. Buyers earn zero profit on average in any equilibrium. These allocations can be supported by simple menu offers. For instance, one can construct linear price equilibria in which buyers offer to purchase any quantity of the good at a constant unit price equal to the expectation of their valuation of the good conditional on the seller accepting to trade at that price. While other menu offers are consistent with equilibrium, corresponding to non-linear price schedules, an important insight of our analysis is that this is also the unit price at which all trades take place in any equilibrium.

These results are of course in line with Akerlof's (1970) classic analysis of the market for lemons, for which they provide a fully strategic foundation. It is worth stressing the differences between his model and ours. Akerlof (1970) considers a market for a non-divisible good of uncertain quality, in which all agents are price-takers. Thus, by assumption, all trades must take place at the same price, in the spirit of competitive equilibrium models. Equality of supply and demand determines the equilibrium price level, which is equal to the average quality of the goods that are effectively traded. Multiple equilibria may occur in a generic way.[2] By contrast, we allow agents to trade any fraction of the seller's endowment. Moreover, our model is one of imperfect competition, in which a fixed number of buyers choose their offers strategically. In particular, our analysis does not rely on free entry arguments. Finally, buyers can offer arbitrary menus of contracts, including for instance non-linear price schedules. That is, we avoid any a priori restrictions on instruments. The fact that all trades take place at a constant unit price in equilibrium is therefore no longer an assumption, but rather a consequence of our analysis.

A key to our results is that non-exclusive competition expands the set of deviations

[2]This potential multiplicity of equilibria arises because buyers are assumed to be price-takers. Mas-Colell, Whinston and Green (1995, Proposition 13.B.1) allow buyers to strategically set prices in a market for a non-divisible good where trades are restricted to be zero-one. The equilibrium is then generically unique.

that are available to the buyers. Indeed, each buyer can strategically use the offers of his competitors to propose additional trades to the seller. Such deviations are blocked by *latent* contracts, that is, contracts that are not traded in equilibrium but which the seller finds it profitable to trade at the deviation stage. These latent contracts are not necessarily complex or exotic. For instance, in a linear price equilibrium, all the buyers offer to purchase any quantity of the good at a constant unit price, but only a finite number of contracts can end up being traded as long as the seller does not randomize on the equilibrium path. The purpose of the other contracts, which are not traded in equilibrium, is only to deter cream-skimming deviations that aim at attracting the seller when quality is high. The use of latent contracts has been criticized on several grounds. First, they may allow one to support multiple equilibrium allocations, and even induce an indeterminacy of equilibrium.[3] This is not the case in our model, since aggregate equilibrium allocations are generically unique. Second, a latent contract may appear as a non-credible threat, if the buyer who issues it would make losses in the hypothetical case where the seller were to trade it.[4] Again, this need not be the case in our model. In fact, we construct examples of equilibria in which latent contracts would be strictly profitable if traded.

This paper is closely related to the literature on common agency between competing principals dealing with a privately informed agent. To use the terminology of Bernheim and Whinston (1986), our non-exclusive competition game is a *delegated* common agency game, as the seller can choose a strict subset of buyers with whom she wants to trade. In the specific context of incomplete information, a number of recent contributions use standard mechanism design techniques to characterize equilibrium allocations. The basic idea is that, given a profile of mechanisms proposed by his competitors, the best response of any single principal can be fully determined by focusing on simple menu offers corresponding to direct revelation mechanisms. This allows one to construct equilibria that satisfy certain regularity conditions. This approach has been successfully applied in various delegated agency contexts.[5] Closest to this paper is Biais, Martimort and Rochet (2000), who study competition among principals in a common value environment. In their model, uninformed market-makers supply liquidity to an informed insider. The insider's preferences are quasi-linear, and quadratic with respect to quantities exchanged. Unlike in our model, the insider has no capacity constraint. Variational techniques are used to construct an equilibrium in

[3]Martimort and Stole (2003, Proposition 5) show that, in a complete information setting, latent contracts can be used to support any level of trade between the perfectly competitive outcome and the Cournot outcome.

[4]Latent contracts with negative virtual profits have been for example considered in Hellwig (1983).

[5]See for instance Khalil, Martimort and Parigi (2007) or Martimort and Stole (2007).

which market-makers post convex price schedules. Such techniques do not apply in our model, as all agents have linear preferences, and the seller cannot trade more than her endowment. Instead, we allow for arbitrary menu offers, and we characterize candidate equilibrium allocations in the usual way, that is by checking whether they survive to possible deviations. While this approach may be difficult to apply in more complex settings, it delivers interesting new insights, in particular on the role of latent contracts.

The paper is organized as follows. Section 2 introduces the model. Section 3 focuses on a two-type setting. We show that there always exists a market equilibrium where buyers play a pure strategy. In addition, equilibrium allocations are generically unique. We also characterize equilibrium menu offers, with special emphasis on latent contracts. Section 4 analyzes the general framework with a continuum of sellers' types. Section 5 concludes.

2 The Model

There are two kinds of agents: a single seller, and a finite number of buyers indexed by $i = 1, \ldots, n$, where $n \geq 2$. The seller has an endowment consisting of one unit of a perfectly divisible good that she can trade with the buyers. Let q^i be the quantity of the good purchased by buyer i, and t^i the transfer he makes in return. The set of feasible trades is the set of vectors $((q^1, t^1), \ldots, (q^n, t^n))$ such that $q^i \geq 0$ and $t^i \geq 0$ for all i, and $\sum_i q^i \leq 1$. Thus the quantity of the good purchased by each buyer must be at least zero, and the sum of these quantities cannot exceed the seller's endowment.[6]

The seller has preferences represented by

$$T - \theta Q,$$

where $Q = \sum_i q^i$ and $T = \sum_i t^i$ denote aggregate quantities and transfers. Here θ is a random variable that stands for the quality of the good as perceived by the seller.[7] Each buyer i has preferences represented by

$$v(\theta) q^i - t^i.$$

Here $v(\theta)$ is a deterministic function of θ that stands for the quality of the good as perceived by the buyers.

[6] This differs from the model of Biais, Martimort and Rochet (2000), in which the insider and the market-makers can trade on both sides of the market.

[7] This is another difference with Biais, Martimort and Rochet (2000), where the preferences of the insider can be represented by $\theta Q - \frac{1}{2}\gamma\sigma^2 Q^2 - T$, where γ and σ^2 are common knowledge risk-aversion and volatility parameters.

We will typically assume that $v(\theta)$ is not a constant function of θ, so that both the seller and the buyers care about θ. Gains from trade arise in this common value environment if $v(\theta) > \theta$ for some realization(s) of θ. However, in line with Akerlof (1970), mutually beneficial trades are potentially impeded because the seller is privately informed of the quality of the good at the trading stage. Following standard usage, we shall thereafter refer to θ as to the *type* of the seller.

Buyers compete in menus for the good offered by the seller. As in Biais, Martimort and Rochet (2000), trading is non-exclusive in the sense that the seller can pick or reject any of the offers made to her, and can simultaneously trade with several buyers. The following timing of events characterizes our non-exclusive competition game:

1. Each buyer i proposes a menu of contracts, that is, a set C^i of quantity-transfer pairs $(q^i, t^i) \in [0,1] \times \mathbb{R}_+$ that contains at least the no-trade contract $(0,0)$.[8]

2. After privately learning the quality θ, the seller selects one contract (q^i, t^i) from each of the menus C^i's offered by the buyers, subject to the constraint that $\sum_i q^i \leq 1$.

A pure strategy for the seller is a mapping s that associates to each type θ and each menu profile $(C^1, \ldots, C^n)$ a vector $((q^1, t^1), \ldots, (q^n, t^n)) \in ([0,1] \times \mathbb{R}_+)^n$ such that $(q^i, t^i) \in C^i$ for each i and $\sum_i q^i \leq 1$. We accordingly denote by $s^i(\theta, C^1, \ldots, C^n)$ the contract traded by type θ of the seller with buyer i. To ensure that the seller's problem

$$\sup \left\{ \sum_i t^i - \theta \sum_i q^i : \sum_i q^i \leq 1 \text{ and } (q^i, t^i) \in C^i \text{ for all } i \right\}$$

has a solution for any type θ and menu profile $(C^1, \ldots, C^n)$, we require the buyers' menus to be compact sets.

At a later stage of the analysis, it will be instructive to compare the equilibrium outcomes under non-exclusive competition with those arising under exclusive competition, that is, when the seller can trade with at most one buyer. The timing of this latter game is similar to that presented above, except that stage 2 is replaced by

2'. After privately learning the quality θ, the seller selects one contract (q^i, t^i) from one of the menus C^i's offered by the buyers.

Given a menu profile $(C^1, \ldots, C^n)$, the seller's problem then becomes

$$\sup \left\{ t^i - \theta q^i : (q^i, t^i) \in C^i \text{ for some } i \right\}.$$

[8] As usual, the assumption that each menu must contain the no-trade contract allows one to deal with participation in a simple way.

Throughout the paper, and unless stated otherwise, the equilibrium concept is pure strategy perfect Bayesian equilibrium.

3 The Two-Type Case

In this section, we consider the binary version of our model in which the seller's type can be either low, $\theta = \underline{\theta}$, or high, $\theta = \overline{\theta}$, for some $\overline{\theta} > \underline{\theta} > 0$. Denote by $\nu \in (0,1)$ the probability that $\theta = \overline{\theta}$. We assume that the seller's and the buyers' perceptions of the quality of the good move together, that is $v(\overline{\theta}) > v(\underline{\theta})$, and that it would be efficient to trade no matter the quality of the good, that is $v(\underline{\theta}) > \underline{\theta}$ and $v(\overline{\theta}) > \overline{\theta}$.

3.1 Equilibrium Outcomes

We first characterize the restrictions that equilibrium behavior implies for the outcomes of the non-exclusive competition game. Next, we show that this game always admits an equilibrium in which buyers post linear prices. Finally, we contrast the equilibrium outcomes with those arising in the exclusive competition model.

3.1.1 Aggregate Equilibrium Allocations

Let $\underline{c}^i = (\underline{q}^i, \underline{t}^i)$ and $\overline{c}^i = (\overline{q}^i, \overline{t}^i)$ be the contracts traded by the two types of the seller with buyer i in equilibrium, and let $(\underline{Q}, \underline{T}) = \sum_i \underline{c}^i$ and $(\overline{Q}, \overline{T}) = \sum_i \overline{c}^i$ be the corresponding aggregate equilibrium allocations. To characterize these allocations, one needs only to require that three types of deviations by a buyer be blocked in equilibrium. In each case, the deviating buyer uses the offers of his competitors as a support for his own deviation. This intuitively amounts to pivoting around the aggregate equilibrium allocation points $(\underline{Q}, \underline{T})$ and $(\overline{Q}, \overline{T})$ in the (Q, T) space. We now consider each deviation in turn.

Attracting type $\underline{\theta}$ by pivoting around $(\underline{Q}, \underline{T})$ The first type of deviations allows one to prove that type $\underline{\theta}$ trades efficiently in any equilibrium.

Lemma 1 $\underline{Q} = 1$ *in any equilibrium.*

One can illustrate the deviation used in Lemma 1 as follows. Observe first that a basic implication of incentive compatibility is that, in any equilibrium, $\overline{Q}$ cannot be higher than $\underline{Q}$. Suppose then that $\underline{Q} < 1$ in a candidate equilibrium. This situation is depicted on Figure 1. Point $\underline{A}$ corresponds to the aggregate equilibrium allocation $(\underline{Q}, \underline{T})$ traded by type $\underline{\theta}$, while point $\overline{A}$ corresponds to the aggregate equilibrium allocation $(\overline{Q}, \overline{T})$ traded by type $\overline{\theta}$. The two solid lines passing through these points are the equilibrium indifference curves of type $\underline{\theta}$ and type $\overline{\theta}$, with slopes $\underline{\theta}$ and $\overline{\theta}$. The dotted line passing through $\underline{A}$ is an indifference curve for the buyers, with slope $v(\underline{\theta})$.

—Insert Figure 1 here—

Suppose now that some buyer deviates and includes in his menu an additional contract that makes available the further trade $\underline{AA'}$. This leaves type $\underline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium. Type $\overline{\theta}$, by contrast, cannot gain by trading this new contract. Assuming that the deviating buyer can break the indifference of type $\underline{\theta}$ in his favor, he strictly gains from trading the new contract with type $\underline{\theta}$, as the slope $\underline{\theta}$ of the line segment $\underline{AA'}$ is strictly less than $v(\underline{\theta})$. This contradiction shows that one must have $\underline{Q} = 1$ in equilibrium. The assumption on indifference breaking is relaxed in the proof of Lemma 1.

Attracting type $\underline{\theta}$ by pivoting around $(\overline{Q}, \overline{T})$ Having established that $\underline{Q} = 1$, we now investigate the aggregate quantity $\overline{Q}$ traded by type $\overline{\theta}$ in equilibrium. The second type of deviations allows one to partially characterize the circumstances in which the two types of the seller trade different aggregate allocations in equilibrium. We say in this case that the equilibrium is *separating*. An immediate implication of Lemma 1 is that $\overline{Q} < 1$ in any separating equilibrium. Let then $p = \frac{\underline{T}-\overline{T}}{1-\overline{Q}}$ be the slope of the line connecting the points $(\overline{Q}, \overline{T})$ and $(1, \underline{T})$ in the (Q, T) space. Thus p is the implicit unit price at which the quantity $1-\overline{Q}$ can be sold to move from $(\overline{Q}, \overline{T})$ to $(1, \underline{T})$. By incentive compatibility, p must lie in the interval $[\underline{\theta}, \overline{\theta}]$ in any separating equilibrium. The strategic analysis of the buyers' behavior induces further restrictions on p.

Lemma 2 *In a separating equilibrium, $p < \overline{\theta}$ implies that $p \geq v(\underline{\theta})$.*

In the proof of Lemma 1, we showed that, if $\underline{Q} < 1$, then each buyer has an incentive to deviate. By contrast, in the proof of Lemma 2, we only show that if $p < \min\{v(\underline{\theta}), \overline{\theta}\}$ in a candidate separating equilibrium, then at least one buyer has an incentive to deviate. This makes it more difficult to illustrate why the deviation used in Lemma 2 might be profitable. It is however easy to see why this deviation would be profitable to an entrant or, equivalently, to an inactive buyer that would not trade in equilibrium. This situation is depicted on Figure 2. The dotted line passing through $\overline{A}$ is an indifference curve for the buyers, with slope $v(\underline{\theta})$. Contrary to the conclusion of Lemma 2, the figure is drawn in such a way that this indifference curve is strictly steeper than the line segment $\overline{A}\underline{A}$.

—Insert Figure 2 here—

Suppose now that the entrant offers a contract that makes available the trade $\overline{A}\underline{A}$. This leaves type $\underline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium by trading the aggregate allocation $(\overline{Q}, \overline{T})$ together with the new contract. Type $\overline{\theta}$, by contrast, cannot gain by trading this new contract. Assuming that the entrant can break the indifference of type $\underline{\theta}$ in his favor, he earns a strictly positive payoff from trading the new contract with type $\underline{\theta}$, as the slope p of the line segment $\overline{A}\underline{A}$ is strictly less than $v(\underline{\theta})$. This shows that, unless $p \geq v(\underline{\theta})$, the candidate separating equilibrium is not robust to entry. The assumption on indifference breaking is relaxed in the proof of Lemma 2, which further shows that the proposed deviation is profitable to at least one active buyer.

Attracting both types by pivoting around $(\overline{Q}, \overline{T})$ A separating equilibrium must be robust to deviations that attract both types of the seller. This third type of deviations allows one to find a necessary condition for the existence of a separating equilibrium. When this condition fails, both types of the seller must trade the same aggregate allocations in equilibrium. We say in this case that the equilibrium is *pooling*.

Lemma 3 *If* $E[v(\theta)] > \overline{\theta}$*, any equilibrium is pooling, with*

$$(\underline{Q}, \underline{T}) = (\overline{Q}, \overline{T}) = (1, E[v(\theta)]).$$

The proof of Lemma 3 consists in showing that if $E[v(\theta)] > \overline{\theta}$ in a candidate separating equilibrium, then at least one buyer has an incentive to deviate. As for Lemma 2, this makes it difficult to illustrate why this deviation might be profitable. It is however easy to see why this deviation would be profitable to an entrant or, equivalently, to an inactive buyer that would not trade in equilibrium. This situation is depicted on Figure 3. The dotted line passing through $\overline{A}$ is an indifference curve for the buyers, with slope $E[v(\theta)]$. Contrary to the conclusion of Lemma 3, the figure is drawn in such a way that this indifference curve is strictly steeper than the indifference curves of type $\overline{\theta}$.

—Insert Figure 3 here—

Suppose now that the entrant offers a contract that makes available the trade $\overline{A}\underline{A}'$. This leaves type $\overline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium by trading the aggregate allocation $(\overline{Q}, \overline{T})$ together with the new contract. Type $\underline{\theta}$ strictly gains by trading this new contract. Assuming that the entrant can break the indifference of type $\overline{\theta}$ in his favor, he earns a strictly positive payoff from trading the new contract with both types as the slope $\overline{\theta}$ of the line segment $\overline{A}\underline{A}'$ is strictly less than $E[v(\theta)]$. This shows that, unless $E[v(\theta)] \leq \overline{\theta}$, the candidate equilibrium is not robust to entry. Once again, the assumption on indifference breaking is relaxed in the proof of Lemma 3, which further shows that the proposed deviation is profitable to at least one active buyer.

The following result provides a partial converse to Lemma 3.

Lemma 4 *If $E[v(\theta)] < \overline{\theta}$, any equilibrium is separating, with*

$$(\underline{Q}, \underline{T}) = (1, v(\underline{\theta})) \text{ and } (\overline{Q}, \overline{T}) = (0, 0).$$

The following is an important corollary of our analysis.

Corollary 1 *Each buyer's payoff is zero in any equilibrium.*

Lemmas 1 to 4 provide a full characterization of the aggregate trades that can be sustained in a pure strategy equilibrium of the non-exclusive competition game. While each buyer always receives a zero payoff in equilibrium, the structure of equilibrium allocations is directly affected by the severity of the adverse selection problem.

We shall say that adverse selection is *mild* whenever $E[v(\theta)] > \overline{\theta}$. Separating equilibria are ruled out in these circumstances. Indeed, if the aggregate allocation $(\overline{Q}, \overline{T})$ traded by type $\overline{\theta}$ were such that $\overline{Q} < 1$, some buyer would have an incentive to induce both types of the seller to trade this allocation, together with the additional quantity $1 - \overline{Q}$ at a unit price between $\overline{\theta}$ and $E[v(\theta)]$. Competition among buyers then bids up the price of the seller's endowment to its average value $E[v(\theta)]$ for the buyers, a price at which both types of the seller are ready to trade. This situation is depicted on Figure 4. The dotted line passing through the origin is the equilibrium indifference curve of the buyers, with slope $E[v(\theta)]$.

—Insert Figure 4 here—

We shall say that adverse selection is *strong* whenever $E[v(\theta)] < \overline{\theta}$. Pooling equilibria are ruled out in these circumstances, as type $\overline{\theta}$ is no longer ready to trade her endowment at price $E[v(\theta)]$. However, non-exclusive competition induces a specific cost of screening the seller's type in equilibrium. Indeed, any separating equilibrium must be such that no buyer has an incentive to deviate and induce type $\underline{\theta}$ to trade the aggregate allocation $(\overline{Q}, \overline{T})$, together with the additional quantity $1 - \overline{Q}$ at some mutually advantageous price. To eliminate any incentive for buyers to engage in such trades with type $\underline{\theta}$, the implicit unit price at which this additional quantity $1-\overline{Q}$ can be sold in equilibrium must be relatively high, implying at most an aggregate payoff $\{E[v(\theta)] - \overline{\theta}\}\overline{Q}$ for the buyers. Hence type $\overline{\theta}$ can trade actively in a separating equilibrium only in the non-generic case where $E[v(\theta)] = \overline{\theta}$, while type $\overline{\theta}$ does not trade at all under strong adverse selection. This situation is depicted on Figure 5. The dotted line passing through the origin is the equilibrium indifference curve of the buyers, with slope $v(\underline{\theta})$.

—Insert Figure 5 here—

3.1.2 Equilibrium Existence

We now establish that the non-exclusive competition game always admits an equilibrium. Specifically, we show that there always exists an equilibrium in which all buyers post linear prices. In such an equilibrium, the unit price at which any quantity can be traded is equal to the expected quality of the goods that are actively traded.

Proposition 1 *The following holds*:

(i) Under mild adverse selection, the non-exclusive competition game has an equilibrium in which each buyer offers the menu

$$\{(q,t) \in [0,1] \times \mathbb{R}_+ : t = E[v(\theta)]q\},$$

and thus stands ready to buy any quantity of the good at a constant unit price $E[v(\theta)]$.

(ii) Under strong adverse selection, the non-exclusive competition game has an equilibrium in which each buyer offers the menu

$$\{(q,t) \in [0,1] \times \mathbb{R}_+ : t = v(\underline{\theta})q\},$$

and thus stands ready to buy any quantity of the good at a constant unit price $v(\underline{\theta})$.

In the non-generic case where $E[v(\theta)] = \overline{\theta}$, it is easy to check that there exist two linear price equilibria, a pooling equilibrium with constant unit price $E[v(\theta)]$ and a separating equilibrium with constant unit price $v(\underline{\theta})$. In addition, there exists in this case a continuum of separating equilibria in which type $\overline{\theta}$ trades actively. Indeed, to sustain an equilibrium trade level $\overline{Q} \in (0,1)$ for type $\overline{\theta}$, it is enough that all buyers offer to buy any quantity of the good at unit price $v(\underline{\theta})$, and that one buyer offers in addition to buy any quantity of the good up to $\overline{Q}$ at unit price $E[v(\theta)]$. Both types $\underline{\theta}$ and $\overline{\theta}$ then sell a fraction $\overline{Q}$ of their endowment at unit price $E[v(\theta)]$, while type $\underline{\theta}$ sells the remaining fraction of her endowment at unit price $v(\underline{\theta})$.

3.1.3 Comparison with the Exclusive Competition Model

Our analysis provides a fully strategic foundation for Akerlof's (1970) original intuition: if adverse selection is severe enough, only goods of low quality are traded in any market equilibrium. This contrasts sharply with the predictions of standard models of competition under adverse selection, in which exclusivity clauses are typically assumed to be enforceable at no cost. To see this within the context of our model, let $(\underline{Q}^e, \underline{T}^e)$ and $(\overline{Q}^e, \overline{T}^e)$ be the allocations traded by each type of the seller in an equilibrium of the exclusive competition game. One then has the following result.

Proposition 2 *The following holds*:

(i) Any equilibrium of the exclusive competition game is separating, with

$$(\underline{Q}^e, \underline{T}^e) = (1, v(\underline{\theta})) \text{ and } (\overline{Q}^e, \overline{T}^e) = \frac{v(\underline{\theta}) - \underline{\theta}}{v(\overline{\theta}) - \underline{\theta}}(1, v(\overline{\theta})).$$

(ii) The exclusive competition game admits an equilibrium if and only if

$$\nu \leq \frac{\overline{\theta} - \underline{\theta}}{v(\overline{\theta}) - \underline{\theta}}.$$

Hence, when the rules of the competition game are such that the seller can trade with at most one buyer, the structure of market equilibria is formally analogous to that obtaining in the competitive insurance model of Rothschild and Stiglitz (1976). First, any pure strategy equilibrium must be separating, with type $\underline{\theta}$ selling her whole endowment, $\underline{Q}^e = 1$, and type $\overline{\theta}$ selling less than her whole endowment, $\overline{Q}^e < 1$. The corresponding contracts trade at unit prices $v(\underline{\theta})$ and $v(\overline{\theta})$ respectively, yielding both a zero payoff to the buyers. Second, type $\underline{\theta}$ must be indifferent between her equilibrium contract and that of type $\overline{\theta}$, implying

$$\overline{Q}^e = \frac{v(\underline{\theta}) - \underline{\theta}}{v(\overline{\theta}) - \underline{\theta}}.$$

This contrasts with the separating outcome that prevails under non-exclusivity and strong adverse selection, as type $\underline{\theta}$ then strictly prefers the aggregate equilibrium allocation $(1, v(\underline{\theta}))$ to the no-trade contract selected by type $\overline{\theta}$. An immediate implication of our analysis is thus that the equilibrium allocations under exclusivity cannot be sustained in equilibrium under non-exclusivity. These allocations are depicted on Figure 6. Point $\underline{A}^e$ corresponds to the equilibrium contract of type $\underline{\theta}$, while point $\overline{A}^e$ corresponds to the equilibrium contract of type $\overline{\theta}$. The two solid lines passing through these points are the equilibrium indifference curves of type $\underline{\theta}$ and type $\overline{\theta}$. The dotted line passing through the origin are indifference curves for the buyers, with slope $v(\underline{\theta})$ and $v(\overline{\theta})$.

—Insert Figure 6 here—

As in Rothschild and Stiglitz (1976), a pure strategy equilibrium exists under exclusivity only under certain parameter restrictions. This contrasts with the non-exclusive competition game, which, as shown above, always admits an equilibrium. Specifically, the equilibrium indifference curve of type $\overline{\theta}$ must lie above the indifference curve for the buyers with slope $E[v(\theta)]$ passing through the origin, for otherwise there would exist a profitable deviation attracting both types of the seller. This is the case if and only if the probability ν that the good is of high quality is low enough. Simple computations show that the threshold

$$\nu^e = \frac{\overline{\theta} - \underline{\theta}}{v(\overline{\theta}) - \underline{\theta}},$$

for ν below which an equilibrium exists under exclusivity is strictly above the threshold

$$\nu^{ne} = \max\left\{0, \frac{\overline{\theta} - v(\underline{\theta})}{v(\overline{\theta}) - v(\underline{\theta})}\right\}$$

for ν below which the equilibrium is separating under non-exclusivity. When $0 < \nu < \nu^{ne}$, the equilibrium is separating under both exclusivity and non-exclusivity, and more trade takes place in the former case. By contrast, when $\nu^{ne} < \nu < \nu^e$, the equilibrium is separating under exclusivity and pooling under non-exclusivity, and more trade takes place in the latter case. From an ex-ante viewpoint, exclusive competition leads to a more efficient outcome under strong adverse selection, while non-exclusive competition leads to a more efficient outcome under mild adverse selection.

3.2 Equilibrium Menus and Latent Contracts

We now explore in more depth the structure of the menus offered by the buyers in equilibrium. Our first result provides equilibrium restrictions on the price of all issued contracts.

Proposition 3 *The following holds*:

(i) Under mild adverse selection, the unit price of any contract issued in an equilibrium of the non-exclusive competition game is at most $E[v(\theta)]$.

(ii) Under strong adverse selection, the unit price of any contract issued in an equilibrium of the non-exclusive competition game is at most $v(\underline{\theta})$.

The intuition for this result is as follows. If some buyer offered to purchase some quantity at a unit price above $E[v(\theta)]$ under mild adverse selection, then any other buyer would have an incentive to induce both types of the seller to trade this contract and to sell him the remaining fraction of their endowment at a unit price slightly below $E[v(\theta)]$. Similarly, if some buyer offered to purchase some quantity at a unit price above $v(\underline{\theta})$ under strong adverse selection, then any other buyer would have an incentive to induce type $\underline{\theta}$ to trade this contract and to sell him the remaining fraction of her endowment at a unit price slightly below $v(\underline{\theta})$. As a corollary, one obtains a simple characterization of the price of traded contracts.

Corollary 2 *The following holds*:

(i) Under mild adverse selection, the unit price of any contract traded in an equilibrium of the non-exclusive competition game is $E[v(\theta)]$.

(ii) Under strong adverse selection, the unit price of any contract traded in an equilibrium of the non-exclusive competition game is $v(\underline{\theta})$.

With these preliminaries at hand, we can investigate which contracts need to be issued to sustain the aggregate equilibrium allocations. From a strategic viewpoint, what matters for each buyer is the outside option of the seller, that is, what aggregate allocations she can achieve by trading with the other buyers only. For each buyer i, and for each menu profile $(C^1, \ldots, C^n)$, this is described by the set of aggregate allocations that remain available if buyer i withdraws his menu offer C^i. One has the following result.

Proposition 4 *The following holds*:

(i) Under mild adverse selection, and in any equilibrium of the non-exclusive competition game, the aggregate allocation $(1, E[v(\theta)])$ *traded by both types of the seller remains available if any buyer withdraws his menu offer.*

(ii) Under strong adverse selection, and in any equilibrium of the non-exclusive competition game, the aggregate allocation $(1, v(\underline{\theta}))$ *traded by type* $\underline{\theta}$ *of the seller remains available if any buyer withdraws his menu offer.*

The aggregate equilibrium allocations must therefore remain available even if a buyer deviates from his equilibrium menu offer. The reason is that this buyer would otherwise have an incentive to offer both types to sell their whole endowment at a price slightly below $E[v(\theta)]$ (in the mild adverse selection case), or to offer type $\underline{\theta}$ to sell her whole endowment at price $\underline{\theta}$ while offering type $v(\overline{\theta})$ to sell a smaller part of her endowment on more advantageous terms (in the strong adverse selection case). The flip side of this observation is that no buyer is essential in providing the seller with her aggregate equilibrium allocation. This rules out standard Cournot outcomes in which the buyers would simply share the market and in which all issued contracts would actively be traded by some type of the seller. As an illustration, when there are two buyers, there is no equilibrium in which each buyer would only offer to purchase half of the seller's endowment.

Because of the non-exclusivity of competition, equilibrium in fact involves much more restrictions on menus offers than those prescribed by Proposition 3. For instance, in the mild adverse selection case, there is no equilibrium in which each buyer only offers the allocation $(1, E[v(\theta)])$ besides the no-trade contract. Indeed, any buyer could otherwise deviate by offering to purchase a quantity $\overline{q} < 1$ at some price $\overline{t} \in (E[v(\theta)] - \overline{\theta}(1-\overline{q}), E[v(\theta)] - \underline{\theta}(1-\overline{q}))$. By construction, this is a cream-skimming deviation that attracts only type $\overline{\theta}$, and that yields the deviating buyer a payoff

$$\nu[v(\overline{\theta})\overline{q} - \overline{t}] > \nu\{v(\overline{\theta})\overline{q} - E[v(\theta)] + \underline{\theta}(1 - \overline{q})\},$$

which is strictly positive for $\overline{q}$ close enough to 1. To block such deviations, latent contracts must be issued that are not actively traded in equilibrium but which the seller has an incentive to trade if some buyer attempts to break the equilibrium. In order to play this deterrence role, the corresponding latent allocations must remain available if any buyer withdraws his menu offer. For instance, in the mild adverse selection case, the cream-skimming deviation described above is blocked if the quantity $1 - \overline{q}$ can always be sold at unit price $E[v(\theta)]$ at the deviation stage, since both types of the seller then have the same incentives to trade the contract proposed by the deviating buyer. This corresponds to the linear price equilibria described in Proposition 1.

An important insight of our analysis is that one can also construct non-linear equilibria in which latent contracts are issued at a unit price different from that of the aggregate allocation that is traded in equilibrium.

Proposition 5 *The following holds:*

(i) Under mild adverse selection, for each $\phi \in [\overline{\theta}, E[v(\theta)])$, the non-exclusive competition game has an equilibrium in which each buyer offers the menu

$$\left\{(q,t) \in \left[0, \frac{v(\overline{\theta}) - E[v(\theta)]}{v(\overline{\theta}) - \phi}\right] \times \mathbb{R}_+ : t = \phi q\right\} \cup \{(1, E[v(\theta)])\}.$$

(ii) Under strong adverse selection, for each $\psi \in \left(v(\underline{\theta}), v(\underline{\theta}) + \frac{\overline{\theta} - E[v(\theta)]}{1-\nu}\right]$*, the non-exclusive competition game has an equilibrium in which each buyer offers the menu*

$$\{(0,0)\} \cup \left\{(q,t) \in \left[\frac{\psi - v(\underline{\theta})}{\psi}, 1\right] \times \mathbb{R}_+ : t = \psi q - \psi + v(\underline{\theta})\right\}.$$

This results shows that equilibrium allocations can also be supported through non-linear prices. In such equilibria, the price each buyer is willing to pay for an additional unit of the good is not the same for all quantities purchased. For instance, in the equilibrium for the strong adverse selection case described in Proposition 6(ii), buyers are not ready to pay anything for all quantities up to the level $\frac{\psi - v(\underline{\theta})}{\psi}$, while they are ready to pay ψ for each additional unit of the good above this level. The price schedule posted by each buyer is such that, for any $q < 1$, the unit price $\max\left\{0, \psi - \frac{\psi - v(\underline{\theta})}{q}\right\}$ at which he offers to purchase the quantity q is strictly below $\underline{\theta}$, while the marginal price ψ at which he offers to purchase an additional unit given that he has already purchased a quantity $q \geq \frac{\psi - v(\underline{\theta})}{\psi}$ is strictly above $\underline{\theta}$. As a result of this, the equilibrium budget set of the seller, that is,

$$\left\{(Q,T) \in [0,1] \times \mathbb{R}_+ : Q = \sum_i q^i \text{ and } T \leq \sum_i t^i \text{ where } (q^i, t^i) \in C^i \text{ for all } i\right\},$$

is not convex in this equilibrium. In particular, the seller has a strict incentive to deal with a single buyer. This contrasts with recent work on competition in non-exclusive mechanisms under incomplete information, where attention is typically restricted to equilibria in which the informed agent has a convex budget set in equilibrium, or, what amounts to the same thing, where the set of allocations available to her is the frontier of a convex budget set.[9] In our model, this would for instance arise if all buyers posted concave price schedules. It is therefore interesting to notice that, as a matter of fact, our non-exclusive competition game admits no equilibrium in which each buyer i posts a strictly concave price schedule T^i. The reason is that the aggregate price schedule $\mathcal{T}$ defined by $\mathcal{T}(Q) = \sup\{\sum_i T^i(q^i) : \sum_i q^i = Q\}$ would otherwise be strictly concave in the aggregate quantity traded Q. This would in turn imply that contracts are issued at a unit price strictly above $\mathcal{T}(1)$, which, as shown by Proposition 3, is impossible in equilibrium.

A further implication of Proposition 6 is that latent contracts supporting the equilibrium allocations can be issued at a profitable price. For instance, in the strong adverse selection case, any contract in $\left\{\left[\frac{\psi - v(\underline{\theta})}{\psi}, 1\right) \times \mathbb{R}_+ : t = \psi q - \psi + v(\underline{\theta})\right\}$ would yield its issuer a strictly

[9]See for instance Biais, Martimort and Rochet (2000), Khalil, Martimort and Parigi (2007) or Martimort and Stole (2007). Piaser (2007) offers a general discussion of the role of latent contracts in incomplete information settings.

positive payoff, even if it were traded by type $\underline{\theta}$ only. In equilibrium, no mistakes occur, and buyers correctly anticipate that none of these contracts will be traded. Nonetheless, removing these contracts would break the equilibrium. One should notice in that respect that the role of latent contracts in non-exclusive markets has usually been emphasized in complete information environments in which the agent does not trade efficiently in equilibrium.[10] In these contexts, latent contracts can never be profitable. Indeed, if they were, there would always be room for proposing an additional latent contract at a less profitable price and induce the agent to accept it. In our model, by contrast, type $\underline{\theta}$ sells her whole endowment in equilibrium. It follows from Proposition 3 that there cannot be any latent contract inducing a negative profit to the issuer. In addition, there is no incentive for any single buyer to raise the price of these contracts and make the seller willing to trade them.

Finally, Proposition 6 shows that market equilibria can always be supported with only one active buyer, provided that the other buyers coordinate by offering appropriate latent contracts. Hence non-exclusive competition does not necessarily entail that the seller enters into multiple contracting relationships.

3.3 Discussion

In this two-type framework, the role of latent contracts is to prevent unilateral deviations which only attract the $\overline{\theta}$-type of sellers. A single buyer issues these additional offers anticipating that the $\underline{\theta}$-type will have an incentive to trade them following a cream-skimming deviation from any of his opponents. As suggested in the previous paragraphs, the number of such deviations is possibly very high. Although it is difficult to provide a full characterization of the structure of latent contracts, one can nonetheless argue that an infinite number of latent allocations should be made available at equilibrium.

Proposition 6. *Under both mild and strong adverse selection and in any perfect Bayesian equilibrium of the non-exclusive competition game, an infinite number of latent contracts must remain available if any of buyers withdraws his offers.*

The proof emphasizes that if only a finite number of contracts was offered at equilibrium, there would always be an incentive for a buyer to propose only one contract, accepted by type $\overline{\theta}$ alone, which guarantees him a strictly positive profit.

Remark: Our results can be interpreted in terms of the literature on common agency

[10] See for instance Hellwig (1983), Martimort and Stole (2003), Bisin and Guaitoli (2004) or Attar and Chassagnon (2008).

games, which analyzes the relevance of situations where a number of principals compete through mechanisms in the presence of a single agent. A communication mechanism associates an allocation to every message sent by the agent. In our context, a mechanism proposed by principal (buyer) i is a mapping $\gamma^i : M^i \to C^i$, where M^i is the set of messages available to the agent (seller). We take Γ^i to be the set of mechanisms available to principal i and we denote $\Gamma = \times_{i=1}^n \Gamma^i$. In a common agency game relative to Γ, the agent takes her participation and communication decisions after having observed the array of offered mechanisms $\left(\gamma^1, \gamma^2, \ldots, \gamma^n\right) \in \Gamma$. With reference to such a scenario, Martimort and Stole (2002) and Peters (2001) proved a characterization result: the equilibrium outcomes relative to any set of mechanisms Γ correspond to the outcomes that can be supported at equilibrium in a game where principals offer menus over the allocations induced by Γ.[11]

In our set-up, buyers compete over menus for the trade of a divisible good. Even in a situation where only two types of sellers are considered, it turns out that equilibrium menus should contain an infinite number of allocations. This indeed suggests that to support our Akerlof-like outcomes when competition over mechanisms is considered, a rich structure of communication has to be postulated. That is, an infinite number of messages should be available to the seller; this allows her to effectively act as a coordinating device among buyers, so to guarantee existence of an equilibrium.

In particular, these allocations cannot be supported if buyers were restricted to compete through simple direct mechanisms. In our context, a direct mechanism for buyer i is defined by a mapping $\tilde{\gamma}^i : \Theta \to C^i$. In a direct mechanism game, the allocations offered by any of the buyers are contingent on the seller's private type only. In such a context, it is immediate to verify that for every array of mechanisms $(\tilde{\gamma}^1, \tilde{\gamma}^2, \ldots, \tilde{\gamma}^n)$ proposed by buyers, only a finite set of offers will be available to the seller, which makes impossible to support our equilibrium allocations. That is, direct mechanisms do not provide enough flexibility to buyers to make a strategic use of the seller in deterring cream-skimming deviations.[12] The possibility to support some equilibrium allocations in a common agency game relative to an arbitrary set of indirect mechanisms Γ, but not in the corresponding direct mechanism game, has been acknowledged as a failure of the Revelation Principle in multiprincipal games.[13] We therefore

[11]This is usually referred to as the Delegation Principle (see Martimort and Stole, 2002).

[12]The same difficulty would arise if stochastic direct mechanisms were considered. At any pure strategy equilibrium of a direct mechanism game where buyers are using a stochastic mechanism the seller will communicate before observing the realization of uncertainty. At equilibrium, a finite number of lotteries over allocations will be offered. Bilateral risk-neutrality then makes this situation equivalent to that where only deterministic allocations are proposed. One should however observe that it is problematic to find a rational for stochastic mechanisms in our contexts, given the existence of quantity constraints.

[13]See Peck (1996), Martimort and Stole (2002), and Peters (2001).

exhibit a relevant economic scenario where such a possibility, usually documented in specific game-theoretic examples, takes place.

Contrarily to the exclusive environment, where market equilibria can be characterized through simple direct mechanisms without any loss of generality, the restriction to direct mechanisms turns out to be crucial in our non-exclusive context. In such a case, it is indeed an immediate implication of our analysis that no allocation can be supported at equilibrium in the direct mechanism game.

4 Latent contracts and efficiency in the continuous-type case

The results we have derived so far may depend on the particular two-type setting we have used. It is therefore important to check whether equilibria still exist, and whether they support the Akerlof outcome, in the case when the agent's type is continuously distributed. We would also like to better understand the role and the necessity of latent contracts : are they needed to support equilibria ? Finally it is important to evaluate the second-best efficiency of the equilibrium outcome.

The model remains essentially the same; but we now assume that the type θ has a bounded support $[\underline{\theta}, \bar{\theta}]$, and a distribution characterized by a c.d.f. F, and a p.d.f. f assumed strictly positive on the whole interval. The valuation function $v(.)$ is assumed continuous, but not necessarily monotonic. For convenience, assume that v is defined for all real numbers, even outside $[\underline{\theta}, \bar{\theta}]$. The game we consider is the same as in the previous section : first buyers simultaneously post offers $(C_1, .., C_n)$, and then the seller chooses one contract in each offer. The seller's payoff can be defined as

$$U(\theta) \equiv \sup\{\sum_i t_i - \theta \sum_i q_i; \sum_i q_i \leq 1, \forall\ i\ (q_i, t_i) \in C_i\} \tag{1}$$

Notice that $U(\theta)$ is convex and weakly decreasing. Its derivative is well-defined almost everywhere, and wherever it exists it is equal to $(-Q(\theta))$, that is minus the total quantity sold by type θ.

Let us finally define our equilibrium concept. As in most of the literature, we restrict attention to pure strategies for the buyers, but we allow the seller to randomize. Second we look for equilibria that verify a simple refinement called robustness. In words, a Perfect Bayesian Equilibrium is moreover **robust** if a buyer cannot profitably deviate by adding one contract to its equilibrium subset of offers, assuming that those types of sellers that would strictly loose from trading the new contract do not change their behavior compared to the equilibrium path.

Hence robustness requires that sellers do not play an active role in deterring deviations by buyers if they do not profit from doing so. This requirement was not needed in the study of the two-type case, because we were able to perfectly control the behavior of all types following a deviation. This is more difficult with a continuum of types, and for the sake of simplicity we choose to reinforce the equilibrium concept.

4.1 The monopsony case

As a warming exercise, consider the monopsony case ($n = 1$). Suppose first that the monopsony can only offer to buy one unit, at a price we denote by p. Because only types below p accept this offer, the monopsony's profits are

$$w(p) \equiv \int_{-\infty}^{p} [v(\theta) - p] dF(\theta)$$

¿From our assumptions, w is continuous, is zero below $\underline{\theta}$, and is strictly decreasing above $\bar{\theta}$. It thus admits a maximum value $w^m \geq 0$, that is attained at some point in $[\underline{\theta}, \bar{\theta}]$. To avoid ambiguities, we define the monopsony price p^m as the highest such point.

Let us also define p^* as the supremum of those p such that $w(p) > 0$ (set $p^* = \underline{\theta}$ if this set is empty). Thus p^* is the highest price at which one unit can be profitably bought, and is thus the price that should prevail under competition if buyers are only allowed to buy zero or one unit. In other words, p^* is the Akerlof price, since the equality $w(p^*) = 0$ can be rewritten under the more familiar

$$p^* = E[v(\theta)|\theta < p^*]$$

By definition we know that $w(p) \leq 0$ for $p > p^*$. To avoid discussing multiple equilibria, in the following we assume that

Assumption 1 $w(p) < 0$ *for* $p > p^*$.

It is only slightly more complex to study the case when the monopsony is allowed to offer an arbitrary menu of contracts. Fortunately, and as is well-known from the Revelation Principle, one only has to maximize the monopsony's profit

$$\int [(v(\theta) - \theta)Q(\theta) - U(\theta)] dF(\theta)$$

under the seller's incentive-compatibility (IC) constraints

$$\forall\ \theta \qquad U'(\theta) = -Q(\theta) \ \text{ a.e.} \qquad\qquad Q(.) \text{ is weakly decreasing}$$

and the seller's individual rationality (IR) constraint

$$\forall\ \theta \qquad U(\theta) \geq 0$$

This problem was already solved in Samuelson (1984). The proof given in Appendix confirms that the monopsony cannot benefit from trading quantities that differ from zero and one.

Lemma 1 ***(Samuelson, 1984)*** *The monopsony maximizes its profit by offering to buy one unit at the price p^m.*

4.2 Exclusive competition

Under exclusive competition, the seller can only trade with one buyer, so that we have

$$U(\theta) \equiv \sup\{t - \theta q; \exists\ i\ (q,t) \in C_i\}$$

Recall that in the two-type case results were similar to those derived in the Rotschild-Stiglitz model; in particular equilibria need not exist. In the continuous-type case, non-existence of equilibria turns out to be the rule. To establish this, following Riley (1985, see also 2001) we first show that in any equilibrium the allocation (U^{ne}, Q^{ne}) is uniquely defined. In particular, it must verify

$$U(\theta) = (v(\theta) - \theta)Q(\theta)$$

Thus profits must be zero for each type θ; this strong requirement results from the ability of each buyer to undercut its competitors whenever one contract is profitably sold. This has dramatic consequences; for example, if the surplus from trade $(v(\theta) - \theta)$ is negative at some point θ, then both U^{ne} and Q^{ne} must be zero for all higher values of θ.

The least we can do is thus to assume that $v(\underline{\theta}) > \underline{\theta}$. Since buying more from type $\underline{\theta}$ is strictly profitable, one must have $Q(\underline{\theta}) = 1$. Together with the zero-profit condition above, and the IC constraint $U'(\theta) = -Q(\theta)$, these conditions indeed characterize a unique candidate allocation (U^{ne}, Q^{ne}). But this allocation is as usual threatened by a pooling offer to sell one unit to an interval of types containing $\underline{\theta}$. In fact, due to the simplicity of our model we get a more striking result :

Proposition 1 *Suppose that the density f is continuous and positive at the right of $\underline{\theta}$. If there exists a robust equilibrium of the exclusive competition game with non-zero trade, then there exists $\theta > \underline{\theta}$ such that v is a constant on $[\underline{\theta}, \theta]$.*

The case when v is a constant corresponds to that of a private good : buyers do not care about the type of the seller. As soon as one assumes (rather intuitively) that v is increasing, there cannot be any trade at equilibrium, and in fact equilibria do not exist[14]. Hence the Proposition shows that under exclusive competition non-existence of equilibria is the rule rather than the exception.

4.3 Non-Exclusive competition

By contrast, our first result in this section shows that equilibria always exist under non-exclusivity :

Proposition 2 *Under non-exclusive competition, there exists a robust Perfect Bayesian Equilibrium in which each buyer proposes to buy any quantity at a unit price p^*, and the seller sells one unit to a randomly chosen buyer.*

The intuition for this result is the following. Given the other buyers' offers, any deviating buyer would have to propose unit prices above p^* to attract some types of sellers, and consequently would attract all types below p^*. But all such types would end up selling one unit, either to the deviating buyer or to the other buyers. Consequently they would all choose the most profitable manner to sell one unit. The deviating buyer can thus only hope to sell the same quantity to these types, at a unit price above p^*; but this cannot be profitable, by definition of p^*.

More striking is the fact that equilibria always exist, even when v is not monotonic. In the two-type case, the same result obtained under some intuitive assumptions[15]. These assumptions are not needed here.

Our second result proves that all equilibrium outcomes must support the Akerlof outcome :

Proposition 3 *Under non-exclusive competition, all robust equilibria are such that the aggregate quantity traded is $Q(\theta) = 1$ if $\theta < p^*$, and $Q(\theta) = 0$ if $\theta > p^*$. Buyers get zero-profits.*

[14] Unless the monopsony profit w^m is zero.

[15] Recall that in the two-type case we have assumed $\underline{\theta} < \underline{v} < \bar{v}$ and $\bar{\theta} < \bar{v}$. Relaxing these assumptions is possible, but threatens the existence of equilibria. Indeed, in the (quite exotic) case when $\underline{\theta} < \bar{\theta} < \underline{v}$ and $\bar{v} < \bar{\theta}$, no buyer wants to buy from the highest type. The maximum price is then $\bar{\theta}$, but then buyers get positive profits. In such a case, it can be proven that equilibria do not exist. Interestingly this difficulty disappears when types are continuously distributed.

The intuition here can best be understood in the context of a free-entry equilibrium (a complete proof is given in the Appendix). Suppose that some type $\theta_1 < p^*$ sells an equilibrium quantity $Q_1 < 1$. Because the total quantity sold cannot increase with θ, we can even choose θ_1 such tat $w(\theta_1) > 0$. Then an entrant could offer to buy $1 - Q_1$ at a unit price θ_1. Clearly all types above θ_1 would reject this new offer. On the other hand, θ_1 would be indifferent, which means that by accepting the offer θ_1 would behave optimally; notice that θ_1 would sell one unit. Because types below θ_1 are more eager to sell, they must also choose to sell one, and the entrant's offer ensures this is possible. Therefore all types below θ_1 would accept the new offer. The entrant's profit would then be $w(\theta_1)(1 - Q_1) > 0$, a contradiction since entry would be profitable.

Therefore equilibrium aggregate quantities and transfers are unique. Because $p^m \leq p^*$, there is more trade than in the monopsony case, which does not come as a surprise. Recall that p^* verifies $w(p^*) = 0$, or equivalently

$$p^* = E[v(\theta)|\theta \leq p^*]$$

Hence the equilibrium trades correspond to those that would obtain in the classical Akerlof model. Recall though that our model allows for a divisible good, together with arbitrary tariffs, in an imperfect competition framework. This result thus provides solid game-theoretic foundations to Akerlof's predictions.

4.4 Latent contracts

We also want to examine the role and necessity of latent contracts. As in the two-type case, define the unit price of a contract (q, t) as the ratio t/q, whenever q is positive. One easily gets

Proposition 4 *Under non-exclusive competition, in any robust equilibrium all contracts issued have a unit price equal to or below p^*, and all contracts traded have a unit price equal to p^*.*

This result illustrates how competition disciplines buyers; even though they are allowed to use arbitrary tariffs, at equilibrium they end up trading at a unique price. Even contracts that are not traded must remain below the unit price p^*; otherwise one of the buyers could use such a contract and pivot on it to increase its profits.

This result shows that latent contracts cannot specify two high a unit price, because these contracts can be used by other buyers to make profitable a deviation. On the other hand, latent contracts may also be an impediment to deviations, and thus contribute to support equilibria. Intuitively, consider types below but close to p^*. Because these types are less eager to sell, it is possible to deviate by offering to sell a quantity $1-\varepsilon$ slightly below one, at a price slightly above p^*. If the valuation function v is increasing, then it can be shown that the deviating buyer would get positive profits from these types. What makes non-exclusive competition particular is that other types may be attracted as well. Such low types would accept the offer, and sell their remaining ε to non-deviating buyers, using latent contracts that these buyers have proposed. These arguments show that contracts that allow to trade small quantities at a price close enough to p^* are needed to support equilibria (the proof in Appendix is a bit more involved):

Proposition 5 *Under non-exclusive competition, suppose that v is increasing. Then there exists $q_0 > 0$ such that all quantities below q_0 can be traded thanks to a contract offered at equilibrium.*

This shows that in equilibrium many contracts (in fact a continuum of contracts) must be made available. The same conclusion was derived in the two-type case, though in the strong adverse selection case we were only able to show the necessity of a denumerable number of contracts. A closer examination of the proof to Proposition 5 reveals that the result depends on whether there are at least two types that trade in equilibrium. The strong adverse selection case thus appears as a particular case, mainly because only one type is trading positive quantities at equilibrium.

Still we cannot conclude yet to the necessity of latent contracts to support equilibria. Indeed the contracts characterized in this proposition may be used at equilibrium by some types of sellers. It turns out that equilibria without latent contracts may exist. In fact, we can even build an equilibrium in Direct Revelation Mechanisms. Indeed one can show the existence of a n-tuple of functions $(g_1, \ldots, g_n)$ verifying the following properties :

$$\forall\ i = 1, \ldots, n \qquad g_i([\underline{\theta}, \bar{\theta}]) = [0,1]$$

$$\forall\ i = 1, \ldots, n \qquad \int^{p^*} [v(\theta) - p^*] g_i(\theta) dF(\theta) = 0$$

$$\forall\ \theta < p^*,\ \sum_i g_i(\theta) = 1 \qquad \forall\ \theta > p^*, i\ g_i(\theta) = 0$$

Then each buyer i could propose the following mechanism : tell me your type $\hat{\theta}$, and I will buy from you the quantity $g_i(\hat{\theta})$ at price p^*. The last property ensures that each seller sells one or zero unit, as in the Akerlof outcome; the second property ensures zero-profits, and the first property ensures that all such contracts are indeed traded in equilibrium by at least one type. Therefore there are no latent contracts. Finally these offers indeed form an equilibrium set of offers, from Proposition 2.

To speak frankly, we think that such a construction is artificial, as it requires different types to behave differently when they in fact sell the same aggregate quantity for the same aggregate transfer. The only alternative would be to allow the seller to randomize on the quantities she sells to the different buyers; then strictly speaking there are no latent contracts. Notice however that this candidate is not an equilibrium in Direct Revelation Mechanisms, as the quantity traded by one buyer depends not only on the seller's type, but also on the result of the seller's randomization.

4.5 Assessing efficiency [TO BE COMPLETED]

It is possible to show that the Akerlof outcome is in fact second-best efficient, under some conditions.

5 Conclusion

In this paper, we have studied a simple imperfect competition model of trade under adverse selection. When competition is exclusive, the existence of equilibria is problematic, while equilibria always exist when competition is non-exclusive. In this latter case, aggregate quantities and transfers are generically unique, and correspond to the allocations that obtain in Akerlof's (1970) model. Linear price equilibria can be constructed in which buyers stand ready to purchase any quantity at a constant unit price.

The fact that possible market outcomes tightly depend on the nature of competition suggests that the testable implications of competitive models of adverse selection should be evaluated with care. Indeed, these implications are typically derived from the study of exclusive competition models, such as Rothschild and Stiglitz's (1976) two-type model of insurance markets. By contrast, our analysis shows that more competitive outcomes can be sustained in equilibrium under non-exclusive competition, and that these outcomes can involve a substantial amount of pooling.

These results offer new insights into the empirical literature on adverse selection. For instance, several studies have taken to the data the predictions of theoretical models of insurance provision, without reaching clear conclusions.[16] Cawley and Philipson (1999) argue that there is little empirical support for the adverse selection hypothesis in life insurance. In particular, they find no evidence that marginal prices raise with coverage. Similarly, Finkelstein and Poterba (2004) find that marginal prices do not significantly differ across annuities with different initial annual payments. The theoretical predictions tested by these authors are however derived from models of exclusive competition,[17] while our results clearly indicate that they do not hold when competition is non-exclusive, as in the case of life insurance or annuities. Indeed, non-exclusive competition might be one explanation for the limited evidence of screening and the prevalence of nearly linear pricing schemes on these markets. As a result, more sophisticated procedures need to be designed in order to test for the presence of adverse selection in markets where competition is non-exclusive.

[16] See Chiappori and Salanié (2003) for a survey of this literature.

[17] Chiappori, Jullien, Salanié and Salanié (2006) have derived general tests based on a model of exclusive competition, that they apply to the case of car insurance.

Appendix

Proof of Lemma 1. Suppose instead that $\underline{Q} < 1$, and consider some buyer i. Buyer i can deviate by offering a menu consisting of the no-trade contract and of two new contracts. The first one is

$$\underline{c}^i(\varepsilon) = (\underline{q}^i + 1 - \underline{Q}, \underline{t}^i + (\underline{\theta} + \varepsilon)(1 - \underline{Q})),$$

where ε is some positive number, and is designed to attract type $\underline{\theta}$. The second one is

$$\overline{c}^i(\varepsilon) = (\overline{q}^i, \overline{t}^i + \varepsilon^2),$$

and is designed to attract type $\overline{\theta}$. The key feature of this deviation is that type $\underline{\theta}$ can sell her whole endowment by trading $\underline{c}^i(\varepsilon)$ together with the contracts $\underline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1 - \underline{Q}$ in $\underline{c}^i(\varepsilon)$ is $\underline{\theta} + \varepsilon$, this guarantees her a payoff increase $(1 - \underline{Q})\varepsilon$ compared to what she obtains in equilibrium. When ε is close enough to zero, she cannot obtain as much by trading $\overline{c}^i(\varepsilon)$ instead. Indeed, even if this were to increase her payoff compared to what she obtains in equilibrium, the corresponding increase would be at most $\varepsilon^2 < (1 - \underline{Q})\varepsilon$. Hence type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation. Consider now type $\overline{\theta}$. By trading $\overline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$, she can increase her payoff by ε^2 compared to what she obtains in equilibrium. By trading $\underline{c}^i(\varepsilon)$ instead, the most she can obtain is her equilibrium payoff, plus the payoff from selling the quantity increment $1 - \underline{Q}$ at unit price $\underline{\theta} + \varepsilon$. For ε close enough to zero, $\underline{\theta} + \varepsilon < \overline{\theta}$ so that this unit price is too low from the point of view of type $\overline{\theta}$. Hence type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$-\nu\varepsilon^2 + (1 - \nu)[v(\underline{\theta}) - \underline{\theta} - \varepsilon](1 - \underline{Q})$$

which is strictly positive for ε close enough to zero if $\underline{Q} < 1$. Thus $\underline{Q} = 1$, as claimed. ■

Proof of Lemma 2. Suppose that $p < \overline{\theta}$ in a separating equilibrium, and consider some buyer i. Buyer i can deviate by offering a menu consisting of the no-trade contract and of two new contracts. The first one is

$$\underline{c}^i(\varepsilon) = (\overline{q}^i + 1 - \overline{Q}, \overline{t}^i + (p + \varepsilon)(1 - \overline{Q})),$$

where ε is some positive number, and is designed to attract type $\underline{\theta}$. The second one is

$$\overline{c}^i(\varepsilon) = (\overline{q}^i, \overline{t}^i + \varepsilon^2),$$

and is designed to attract type $\overline{\theta}$. The key feature of this deviation is that type $\underline{\theta}$ can sell her whole endowment by trading $\underline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1-\overline{Q}$ in $\underline{c}^i(\varepsilon)$ is $p+\varepsilon$, this guarantees her a payoff increase $(1-\overline{Q})\varepsilon$ compared to what she obtains in equilibrium. As in the proof of Lemma 1, it is easy to check that when ε is close enough to zero, she cannot obtain as much by trading $\overline{c}^i(\varepsilon)$ instead. Hence type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation. Consider now type $\overline{\theta}$. By trading $\overline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$, she can increase her payoff by ε^2 compared to what she obtains in equilibrium. As in the proof of Lemma 1, it is easy to check that when $p+\varepsilon < \overline{\theta}$, she cannot obtain as much by trading $\underline{c}^i(\varepsilon)$ instead. Hence type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$-\nu\varepsilon^2 + (1-\nu)\{v(\underline{\theta})(\overline{q}^i - \underline{q}^i) - \overline{t}^i + \underline{t}^i + [v(\underline{\theta}) - p - \varepsilon](1-\overline{Q})\},$$

which must be at most zero for any ε close enough to zero. Since $\underline{Q} = 1$ by Lemma 1, summing over the i's and letting ε go to zero then yields

$$v(\underline{\theta})(\overline{Q}-1) - \overline{T} + \underline{T} + n[v(\underline{\theta}) - p](1-\overline{Q}) \leq 0,$$

which, from the definition of p and the fact that $\overline{Q} < 1$, implies that

$$(n-1)[v(\underline{\theta}) - p] \leq 0.$$

Since $n \geq 2$, it follows that $p \geq v(\underline{\theta})$, as claimed. ■

Proof of Lemma 3. Suppose that a separating equilibrium exists, and consider some buyer i. Buyer i can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\tilde{c}^i(\varepsilon) = (\overline{q}^i + 1 - \overline{Q}, \overline{t}^i + (\overline{\theta}+\varepsilon)(1-\overline{Q})),$$

where ε is some positive number, that is designed to attract both types of the seller. The key feature of this deviation is that both types can sell their whole endowment by trading $\tilde{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1-\overline{Q}$ in $\tilde{c}^i(\varepsilon)$ is $\overline{\theta}+\varepsilon$, and since $\overline{\theta} \geq p$, this guarantees both types of the seller a payoff increase $(1-\overline{Q})\varepsilon$ compared to what they obtain in equilibrium. Hence both types trade $\tilde{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$\{E[v(\theta)] - \overline{\theta} - \varepsilon\}(1-\overline{Q}) + (1-\nu)[v(\underline{\theta})(\overline{q}^i - \underline{q}^i) - \overline{t}^i + \underline{t}^i],$$

which must be at most zero for any ε. Since $\underline{Q} = 1$ by Lemma 1, summing over the i's and letting ε go to zero then yields

$$n\{E[v(\theta)] - \overline{\theta}\}(1-\overline{Q}) + (1-\nu)[v(\underline{\theta})(\overline{Q}-1) - \overline{T} + \underline{T}] \leq 0,$$

which, from the definition of p and the fact that $\overline{Q} < 1$, implies that

$$n\{E[v(\theta)] - \overline{\theta}\} + (1 - \nu)[p - v(\underline{\theta})] \leq 0.$$

Starting from this inequality, two cases must be distinguished. If $p < \overline{\theta}$, then Lemma 2 applies, and therefore $p \geq v(\underline{\theta})$. It then follows that $E[v(\theta)] \leq \overline{\theta}$. If $p = \overline{\theta}$, the inequality can be rearranged so as to yield

$$(n - 1)\{E[v(\theta)] - \overline{\theta}\} + \nu[v(\overline{\theta}) - \overline{\theta}] \leq 0.$$

Since $n \geq 2$ and $v(\overline{\theta}) > \overline{\theta}$, it follows again that $E[v(\theta)] \leq \overline{\theta}$, which shows the first part of the result. Consider next some pooling equilibrium, and denote by $(1, T)$ the corresponding aggregate equilibrium allocation. To show that $T = E[v(\theta)]$, one needs to establish that the buyers' aggregate payoff is zero in equilibrium. Let B^i be buyer i's equilibrium payoff, which must be at least zero since each buyer always has the option not to trade. Buyer i can deviate by offering a menu consisting of the no-trade contract and the contract

$$\hat{c}^i(\varepsilon) = (1, T + \varepsilon),$$

where ε is some positive number. It is immediate that both types trade $\hat{c}^i(\varepsilon)$ following buyer i's deviation. The change in payoff for buyer i induced by this deviation is

$$E[v(\theta)] - T - \varepsilon - B^i,$$

which must be at most zero for any ε. Letting ε go to zero yields

$$B^i \geq E[v(\theta)] - T = \sum_j B^j$$

where the equality follows from the fact that each type of the seller sells her whole endowment in a pooling equilibrium. Since this inequality holds for each i and all the B^i's are at least zero, they must all in fact be equal to zero. Hence $T = E[v(\theta)]$, as claimed. ■

Proof of Lemma 4. Suppose first that a pooling equilibrium exists, and denote by $(1, T)$ the aggregate allocation traded by both types in this equilibrium. Then the buyers' aggregate payoff is $E[v(\theta)] - T$. One must have $T - \overline{\theta} \geq 0$ otherwise type $\overline{\theta}$ would not trade. Since the buyers' aggregate payoff must be at least zero in equilibrium, it follows that $E[v(\theta)] \geq \overline{\theta}$, which shows the first part of the result. Next, observe that in any separating equilibrium, the buyers' aggregate payoff is equal to

$$(1 - \nu)[v(\underline{\theta}) - \underline{T}] + \nu[v(\overline{\theta})\overline{Q} - \overline{T}] = (1 - \nu)[v(\underline{\theta}) - p(1 - \overline{Q})] + \nu v(\overline{\theta})\overline{Q} - \overline{T}$$

by definition of p. We claim that $p \geq v(\underline{\theta})$ in any such equilibrium. If $p < \overline{\theta}$, this follows from Lemma 2. If $p = \overline{\theta}$, this follows from Lemma 3, which implies that $\overline{\theta} \geq E[v(\theta)] > v(\underline{\theta})$ whenever a separating equilibrium exists. Using this claim along with the fact that $\overline{T} \geq \overline{\theta Q}$, one obtains that the buyers' aggregate payoff is at most $(E[v(\theta)] - \overline{\theta})\overline{Q}$. Since this must be at least zero, one necessarily has $(\overline{Q}, \overline{T}) = (0, 0)$ whenever $E[v(\theta)] < \overline{\theta}$. In particular, the buyers' aggregate payoff $(1 - \nu)[v(\underline{\theta}) - p]$ is then equal to zero. It follows that $p = v(\underline{\theta})$ and thus $\underline{T} = v(\underline{\theta})$, which shows the second part of the result. ■

Proof of Corollary 1. In the case of a pooling equilibrium, the result has been established in the proof of Lemma 3. In the case of a separating equilibrium, it has been shown in the proof of Lemma 4 that the buyers' aggregate payoff is at most $(E[v(\theta)] - \overline{\theta})\overline{Q}$. As a separating equilibrium exists only if $E[v(\theta)] \leq \overline{\theta}$, it follows that the buyers' aggregate payoff is at most zero in any such equilibrium. Since each buyer always has the option not to trade, the result follows. ■

Proof of Proposition 1. (i) Consider first the mild adverse selection case.

Step 1. Given the menus offered, any best response of the seller leads to an aggregate trade $(1, E[v(\theta)])$ irrespective of her type. Assuming that each buyer trades the same quantity with both types of the seller, all buyers obtain a zero payoff.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $E[v(\theta)]q > t$. However, given the menus offered by the other buyers, the seller always has the option to trade quantity q at unit price $E[v(\theta)]$. She would therefore be strictly worse off trading the contract (q, t) no matter her type. Such a deviation is thus infeasible.

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, an additional contract $(\underline{q}, \underline{t})$ attracts type $\underline{\theta}$ only if $\underline{t} \geq E[v(\theta)]\underline{q}$, since she has the option to trade any quantity at unit price $E[v(\theta)]$. The corresponding payoff for the deviating buyer is then at most $\{v(\underline{\theta}) - E[v(\theta)]\}\underline{q}$ which is at most zero.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq E[v(\theta)]\overline{q}$, since she has the option to trade any quantity at unit price $E[v(\theta)]$. However, type $\underline{\theta}$ can then also weakly increase her payoff by mimicking type $\overline{\theta}$'s behavior. One can therefore construct the seller's strategy in such a way that it is impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Whatever the contract traded by the seller with the deviating buyer, and no matter her type, she can sell to the other buyers the remaining fraction of her endowment at unit price $E[v(\theta)]$. Hence each type of the seller faces the same problem, namely to use optimally the deviating buyer's and the other buyers' offers to sell her whole endowment at the maximum price. One can therefore construct the seller's strategy in such a way that each type selects the same contract from the deviating buyer's menu. By Step 2, this makes such a deviation non profitable. The result follows.

(ii) Consider next the strong adverse selection case.

Step 1. Given the menus offered, any best response of the seller leads to aggregate trades $(1, v(\underline{\theta}))$ for type $\underline{\theta}$ and $(0, 0)$ for type $\overline{\theta}$, and all buyers obtain a zero payoff.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $E[v(\theta)]q > t$. Under strong adverse selection, this however implies that $t - \overline{\theta}q < 0$, so that type $\overline{\theta}$ would be strictly worse off trading the contract (q, t). Such a deviation is thus infeasible.

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, an additional contract $(\underline{q}, \underline{t})$ attracts type $\underline{\theta}$ only if $\underline{t} \geq v(\underline{\theta})\underline{q}$, since she always has the option to trade quantity $\underline{q}$ at unit price $v(\underline{\theta})$. The corresponding payoff for the deviating buyer is then at most zero.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq \overline{\theta}\overline{q}$. However, since $\overline{\theta} > E[v(\theta)] > v(\underline{\theta})$ under strong adverse selection, type $\underline{\theta}$ can then strictly increase her payoff by trading the contract $(\overline{q}, \overline{t})$ and selling to the other buyers the remaining fraction of her endowment at unit price $v(\underline{\theta})$. It is thus impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Given the offers of the other buyers, the most profitable deviations lead to trading some quantity $\overline{q}$ at unit price $\overline{\theta}$ with type $\overline{\theta}$, and trading a quantity 1 at unit price $\overline{\theta}\overline{q} + v(\underline{\theta})(1 - \overline{q})$ with type $\underline{\theta}$. By construction, type $\underline{\theta}$ is indifferent between trading the contract $(1, \overline{\theta}\overline{q} + v(\underline{\theta})(1 - \overline{q}))$ and trading the contract $(\overline{q}, \overline{\theta}\overline{q})$ while selling to the other buyers the remaining fraction of her endowment at unit price $v(\underline{\theta})$. As for type $\overline{\theta}$, she is indifferent between trading the contract $(\overline{q}, \overline{\theta}\overline{q})$ and not trading at all. The corresponding payoff for the deviating buyer is then

$$\nu[v(\overline{\theta}) - \overline{\theta}]\overline{q} + (1 - \nu)[v(\underline{\theta}) - \overline{\theta}\overline{q} - v(\underline{\theta})(1 - \overline{q})] = \{E[v(\theta)] - \overline{\theta}\}\overline{q},$$

which is at most zero under strong adverse selection. The result follows. ■

Proof of Proposition 2.

The proof goes through a series of steps.

Step 1. Each buyer must earn zero profit at equilibrium. If not, there will be at least one buyer i earning a profit strictly smaller than $\frac{1}{2}\left[\nu\big(v(\overline{\theta})\overline{Q}^e - \overline{T}^e\big) + (1-\nu)\big(v(\underline{\theta})\underline{Q}^e - \underline{T}^e\big)\right]$ at any equilibrium. Such a buyer can indeed profitably deviate offering the array of contracts $\{\underline{c}^i(\epsilon), \overline{c}^i(\epsilon)\}$ where $\underline{c}^i(\epsilon) = (\underline{Q}^e, \underline{T}^e + \epsilon)$ and $\overline{c}^i(\epsilon) = (\overline{Q}^e, \overline{T}^e + \epsilon)$, with $\epsilon \in \Big(0, \frac{1}{2}\left[\nu\big(v(\overline{\theta})\overline{Q}^e - \overline{T}^e\big) + (1-\nu)\big(v(\underline{\theta})\underline{Q}^e - \underline{T}^e\big)\right]\Big)$. Both types will trade with the deviating buyer, since any of them can achieve a utility greater than the equilibrium one. In particular, the $\underline{\theta}$-type will trade $\underline{c}^i(\epsilon)$, and the $\overline{\theta}$-type will purchase $\overline{c}^i(\epsilon)$. The deviation is hence profitable for buyer i.

Step 2. There cannot be any pooling equilibrium with both types trading the same allocation (Q^p, T^p). At a pooling equilibrium, one should have $E[v] \geq \overline{\theta}$, otherwise it would not be possible to guarantee participation for both types without incurring a loss; following step 1, one also gets: $(Q^p, T^p) = (1, E[v])$. In such a situation, though, there is always an incentive for any of the inactive buyers, say the $i-th$ one, to propose the contract $c^i = (Q^i, T^i)$ such that $U(\overline{\theta}, c^i) > U(\underline{\theta}, Q^p, T^p)$ and $U(\underline{\theta}, c^i) < U(\underline{\theta}, Q^p, T^p)$. Given the assumptions $\overline{\theta} > \underline{\theta}$ and $v(\overline{\theta}) > v(\underline{\theta})$, there always exists a contract c^i satisfying the former two inequalities and such that $v(\overline{\theta})Q^i - T^i > 0$. That is, c^i constitutes a profitable (cream-skimming) deviation which only attracts the $\overline{\theta}$-type. At equilibrium, we must therefore have separation of types.

Step 3. At any separating equilibrium it must be $v(\overline{\theta})\overline{Q}^e = \overline{T}^e$ and $v(\underline{\theta})\underline{Q}^e = \underline{T}^e$. That is, no cross-subsidization across types takes place. We first remark that if $v(\underline{\theta})\underline{Q}^e - \underline{T}^e > 0$, then any buyer i who is not trading with the $\underline{\theta}$-type will have an incentive to offer the contract $c^i(\epsilon) = (1, \underline{T}^e + (1-\underline{Q}^e)(\underline{T}^e + \epsilon)$, with $\epsilon \in (0, v(\underline{\theta})\underline{Q}^e - \underline{T}^e)$. Clearly, the $\underline{\theta}$-type will purchase $c^i(\epsilon)$. By construction, the deviating buyer i will therefore earn a strictly positive profit. In a similar way, one can show that in the case $v(\overline{\theta})\overline{Q}^e - \overline{T}^e > 0$ any of the buyers who is not trading with the $\overline{\theta}$-type can profitably deviate by offering the array of contracts $\{\underline{c}^i, \overline{c}^i\}$, where $\underline{c}^i$ is the same as before and $\overline{c}^i = (\overline{Q}^e - \delta, \overline{T}^e - \overline{\theta}\delta(1+\epsilon))\}$. If $\delta > 0$ and $\epsilon > 0$ are small enough, then the $\overline{\theta}$-type will select $\overline{c}^i$ while the $\underline{\theta}$-type has an incentive to trade the $\underline{c}^i$ contract, which guarantees that the deviation is profitable.

Step 4. At any separating equilibrium the $\underline{\theta}$-type type will trade the allocation $(\underline{Q}^e, \underline{T}^e) = (1, v(\underline{\theta}))$. Since at equilibrium it must be $\underline{T}^e = v(\underline{\theta})\underline{Q}^e$, it is immediate to verify that if $\underline{Q}^e < 1$, then any of the buyers, say the $i-th$ one, can profitably deviate. Consider the array $\{\overline{c}^i, \underline{c}^i(\epsilon)\}$, where $\overline{c}^i$ is the same as before and $\underline{c}^i(\epsilon) = (1, \underline{T}^e + (1-\underline{Q}^e)(\underline{\theta} + \epsilon))$. The

$\underline{\theta}$-type will always trade the deviating contract and the buyer earns a strictly positive profit for every $\epsilon \in (0, \dfrac{v(\underline{\theta})\underline{Q}^e - \underline{T}^e}{1-\underline{Q}} - \underline{\theta})$.

Step 5. At any separating equilibrium the $\overline{\theta}$ type will trade the allocation $(\underline{Q}^e, \underline{T}^e)$. Suppose not. Then, she must be trading an allocation $(\overline{T}, \overline{Q})$ such that $\overline{T} = v(\bar{(\theta)})\overline{Q}$ and $\overline{T} < \overline{T}^e$.[18] In this case, any of the buyers (say again the $i-th$ one) can profit by offering the array $\{\underline{c}^i, \overline{c}^i(\epsilon)\}$, where $\underline{c}^i$ is the same as before and $\overline{c}^i(\epsilon) = \big(\overline{Q} + \delta, \overline{T} + \delta\overline{\theta}(1+\epsilon)\big)$. If $\delta > 0$ and $\epsilon > 0$ are small enough, then the $\overline{\theta}$-type will select $\overline{c}^i$ while the $\underline{\theta}$-type has an incentive to trade the $\underline{c}^i$ contract, which guarantees that the deviation is profitable.

Step 6. In a final step, we explicitly construct an equilibrium supporting the aforementioned allocations. Consider a situation where every buyer posts the array $\{\big(\underline{Q}^e, \underline{T}^e\big), \big(\overline{Q}^e, \overline{T}^e\big)\}$. It is a best reply for the $\underline{\theta}$ type to trade $\big(\overline{Q}^e, \overline{T}^e\big)$ and for the $\underline{\theta}$ type to trade $\{\big(\underline{Q}^e, \underline{T}^e\big)$. In such a context, we have already argued that none of the buyers can profitably deviate by proposing only one allocation. It remains to show that there is no incentive to deviate by proposing two allocations either. Suppose then that buyer i proposes the array of contracts $\{\big(\underline{Q}', \underline{T}'\big), \big(\overline{Q}', \overline{T}'\big)\}$ anticipating that $\big(\underline{Q}', \underline{T}'\big)$ will be traded by $\underline{\theta}$ and $\big(\overline{Q}', \overline{T}'\big)$ by $\overline{\theta}$. By construction, the deviation must guarantee a strictly positive profit on $\big(\underline{Q}', \underline{T}'\big)$, so to compensate the negative profit earned on $\big(\underline{Q}', \underline{T}'\big)$. To determine the best deviation for any single buyer, one should set:

$$\begin{aligned} \overline{T}' - \overline{\theta}\overline{Q}' &= \overline{T} - \overline{\theta}\overline{Q} \\ \overline{T}' - \underline{\theta}\overline{Q}' &= \underline{T}' - \underline{\theta}, \end{aligned}$$

where the first equality guarantees that the buyer is maximizing the rent earned on the $\overline{\theta}$ type, who is left at his equilibrium utility. The second equality, where $\underline{Q}'$ has been set equal to 1, guarantees that the losses incurred on the $\underline{\theta}$ type are minimized. Considering these equations together, one can indeed express the deviator's payoff in terms of $\overline{Q}'$:

$$(1-\nu)[v(\underline{\theta}) - \underline{T}'] + \nu[v(\overline{\theta})\overline{Q}' - \overline{T}'] = (1-\nu)[v(\underline{\theta}) - \overline{Q}'(1 + \overline{\theta} - \underline{\theta}) - \overline{K}] + \nu[v(\overline{\theta})\overline{Q}' - \overline{\theta}\overline{Q}' - \overline{K}],$$

where $\overline{K} = \overline{T} - \overline{\theta}\overline{Q}$. It is then straightforward to observe that the deviator's profit is strictly decreasing in $\overline{Q}'$, as long as $\nu < \dfrac{v(\overline{\theta}) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}$, which concludes the proof.

[18] If $\overline{T} > \overline{T}^e$ the $\underline{\theta}$-type would have an incentive to trade the $(\overline{T}, \overline{Q})$ allocation, contradicting the assumption of a separating equilibrium.

Proof of Proposition 3. (i) Consider first the mild adverse selection case. Suppose that an equilibrium exists in which some buyer i offers a contract $c^i = (q^i, t^i)$ at unit price $\frac{t^i}{q^i} > E[v(\theta)]$. Notice that one must have $E[v(\theta)] - t^i \geq \overline{\theta}(1 - q^i)$ otherwise c^i would give type $\overline{\theta}$ more than her equilibrium payoff. Similarly, one must have $q^i < 1$ otherwise c^i would give both types more than their equilibrium payoff. Any other buyer j could offer a menu consisting of the no-trade contract and of the contract

$$c^j(\varepsilon) = (1 - q^i, E[v(\theta)] - t^i + \varepsilon),$$

with $0 < \varepsilon < t^i - q^i E[v(\theta)]$. If both c^i and $c^j(\varepsilon)$ were available, both types of the seller would sell their whole endowment at price $E[v(\theta)] + \varepsilon$ by trading c^i with buyer i and $c^j(\varepsilon)$ with buyer j, thereby increasing their payoff by ε compared to what they obtain in equilibrium. Buyer j's equilibrium payoff is thus at least

$$E[v(\theta)](1 - q^i) - \{E[v(\theta)] - t^i + \varepsilon\} = t^i - q^i E[v(\theta)] - \varepsilon > 0,$$

which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. Hence, no contract can be issued at a price strictly above $E[v(\theta)]$.

(ii) Consider next the strong adverse selection case. Suppose that an equilibrium exists in which some buyer i offers a contract $c^i = (q^i, t^i)$ at unit price $\frac{t^i}{q^i} > v(\underline{\theta})$. Notice that one must have $t^i \leq \overline{\theta} q^i$ otherwise c^i would give type $\overline{\theta}$ more than her equilibrium payoff. Similarly, one must have $v(\underline{\theta}) - t^i \geq \underline{\theta}(1 - q^i)$ and $q^i < 1$ otherwise c^i would give type $\underline{\theta}$ more than her equilibrium payoff. Any other buyer j could offer a menu consisting of the no-trade contract and of the contract

$$c^j(\varepsilon) = (1 - q^i, v(\underline{\theta}) - t^i + \varepsilon),$$

where $0 < \varepsilon < \min\{t^i - q^i v(\underline{\theta}), \overline{\theta} - v(\underline{\theta})\}$. If both c^i and $c^j(\varepsilon)$ were available, type $\underline{\theta}$ would sell her whole endowment at price $v(\underline{\theta}) + \varepsilon$ by trading c^i with buyer i and $c^j(\varepsilon)$ with buyer j, thereby increasing her payoff by ε compared to what she obtains in equilibrium. Moreover, since $v(\underline{\theta}) + \varepsilon < \overline{\theta}$, type $\overline{\theta}$ would strictly lose from trading $c^j(\varepsilon)$ with buyer j. Buyer j's equilibrium payoff is thus at least

$$(1 - \nu)\{v(\underline{\theta})(1 - q^i) - [v(\underline{\theta}) - t^i + \varepsilon]\} = (1 - \nu)[t^i - q^i v(\underline{\theta}) - \varepsilon] > 0,$$

which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. Hence, no contract can be issued at a price strictly above $v(\underline{\theta})$. ■

Proof of Corollary 2. (i) Consider first the mild adverse selection case. We know from Proposition 3(i) that no contract is issued, and a fortiori traded, at a unit price strictly above $E[v(\theta)]$. Suppose now that a contract with unit price strictly below $E[v(\theta)]$ is traded in equilibrium. Then, since the aggregate allocation traded by both types is $(1, E[v(\theta)])$, at least one buyer must be trading a contract at a unit price strictly above $E[v(\theta)]$, a contradiction. The result follows.

(ii) Consider next the strong adverse selection case. We know from Proposition 3(ii) that no contract is issued, and a fortiori traded, at a unit price strictly above $v(\underline{\theta})$. Suppose now that a contract with unit price strictly below $v(\underline{\theta})$ is traded in equilibrium. Then, since the aggregate allocation traded by type $\underline{\theta}$ is $(1, v(\underline{\theta}))$, at least one buyer must be trading a contract at a unit price strictly above $v(\underline{\theta})$, a contradiction. The result follows. ■

Proof of Proposition 4. Fix some equilibrium with menu offers $(C^1, \ldots, C^n)$, and let

$$\mathcal{A}^{-i} = \left\{ \sum_{j \neq i} (q^j, t^j) : \sum_{j \neq i} q^j \leq 1 \text{ and } (q^j, t^j) \in C^j \text{ for all } j \neq i \right\}$$

be the set of aggregate allocations that remain available if buyer i withdraws his menu offer C^i. By construction, $\mathcal{A}^{-i}$ is a compact set.

(i) Consider first the mild adverse selection case. Suppose that the aggregate allocation $(1, E[v(\theta)])$ traded by both types does not belong to $\mathcal{A}^{-i}$. Since $\mathcal{A}^{-i}$ is compact, there exists some open set of $[0, 1] \times \mathbb{R}_+$ that contains $(1, E[v(\theta)])$ and that does not intersect $\mathcal{A}^{-i}$. Moreover, any allocation (Q^{-i}, T^{-i}) in $\mathcal{A}^{-i}$ is such that $T^{-i} \leq E[v(\theta)]Q^{-i}$ by Proposition 3(i). Since $\overline{\theta} < E[v(\theta)]$ under mild adverse selection, this implies that $\mathcal{A}^{-i}$ does not intersect the set of allocations that are weakly preferred by both types to $(1, E[v(\theta)])$. By continuity of the seller's preferences, it follows that there exists some positive ε such that the contract $(1, E[v(\theta)] - \varepsilon)$ is strictly preferred by each type to any allocation in $\mathcal{A}^{-i}$. Thus, if this contract were available, both types would trade it. This implies that buyer i's equilibrium payoff is at least ε, which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. The result follows.

(ii) Consider next the strong adverse selection case. Suppose that the aggregate allocation $(1, v(\underline{\theta}))$ traded by type $\underline{\theta}$ does not belong to $\mathcal{A}^{-i}$. Since $\mathcal{A}^{-i}$ is compact, there exists an open set of $[0, 1] \times \mathbb{R}_+$ that contains $(1, v(\underline{\theta}))$ and that does not intersect $\mathcal{A}^{-i}$. Moreover, any allocation (Q^{-i}, T^{-i}) in $\mathcal{A}^{-i}$ is such that $T^{-i} \leq v(\underline{\theta})Q^{-i}$ by Proposition 3(ii). Since $\underline{\theta} < v(\underline{\theta})$, this implies that $\mathcal{A}^{-i}$ does not intersect the set of allocations that are weakly preferred by type $\underline{\theta}$ to $(1, v(\underline{\theta}))$. Since the latter set is closed and $\mathcal{A}^{-i}$ is compact, it follows that there exists a contract $(\overline{q}^i, \overline{t}^i)$ with unit price $\frac{\overline{t}^i}{\overline{q}^i} \in (\overline{\theta}, v(\overline{\theta}))$ such that the allocation $(1, v(\underline{\theta}))$ is strictly preferred by type $\underline{\theta}$ to any allocation obtained by trading the contract $(\overline{q}^i, \overline{t}^i)$ together with some allocation in $\mathcal{A}^{-i}$.[19] Moreover, since $\frac{\overline{t}^i}{\overline{q}^i} > \overline{\theta}$, the contract $(\overline{q}^i, \overline{t}^i)$ guarantees a strictly positive payoff to type $\overline{\theta}$. Thus, if both $(1, v(\underline{\theta}))$ and $(\overline{q}^i, \overline{t}^i)$ were available, type $\underline{\theta}$ would trade $(1, \underline{\theta})$ and type $\overline{\theta}$ would trade $(\overline{q}^i, \overline{t}^i)$. This implies that buyer i's equilibrium payoff is at least $\nu[v(\overline{\theta})\overline{q}^i - \overline{t}^i] > 0$, which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. The result follows. ■

Proof of Proposition 5.

We first introduce some preliminary definitions. Fix an equilibrium with zero-profit, and a buyer i. Let A_{-i} be the set of (aggregate) pairs (t,q) that can be traded with the other buyers. Define

$$z(\theta, q) = \sup\{t' - \theta q' : (q', t') \in A_{-i}, q' \leq q\}$$

In words, $z(\theta, q)$ is the highest payoff a seller of type θ can get from other buyers, if her remaining stock is q. Notice that $z \geq 0$, and z is non-decreasing in q. Also, any trade $(q',$ $t')$ satisfying the constraints can be selected by both types. We can thus write

$$t' - \bar{\theta} q' = t' - \underline{\theta} q' + (\underline{\theta} - \bar{\theta}) q' \geq t' - \underline{\theta} q' + (\underline{\theta} - \bar{\theta}) q$$

because $q' \leq q$. Taking supremums, we get a useful property :

$$z(\bar{\theta}, q) \geq z(\underline{\theta}, q) + (\underline{\theta} - \bar{\theta}) q \tag{2}$$

We consider the situation where buyer i deviates adding to his equilibrium offer some contract (t_0, q_0), designed to attract only type $\bar{\theta}$. To ensure this, we impose the following IC constraints

$$t_0 - \underline{\theta} q_0 + z(\underline{\theta}, 1 - q_0) < U(\underline{\theta})$$

$$t_0 - \bar{\theta} q_0 + z(\bar{\theta}, 1 - q_0) > U(\bar{\theta})$$

Clearly these constraints together require that

[19] This follows directly from the fact that if K is compact and F is closed in some normed vector space X, and if $K \cap F = \emptyset$, then for any vector u in X, $(K + \lambda u) \cap F = \emptyset$ for any sufficiently small scalar λ.

$$\underline{\theta}q_0 - z(\underline{\theta}, 1-q_0) + U(\underline{\theta}) > \bar{\theta}q_0 - z(\bar{\theta}, 1-q_0) + U(\bar{\theta}) \tag{3}$$

The resulting profit is then $\bar{v}q_0 - t_0$, which must be non-positive; and t_0 can be computed from the second IC constraint. Thus we have shown the following implication : if q_0 is such that (3) holds, then

$$(\bar{v} - \bar{\theta})q_0 \leq U(\bar{\theta}) - z(\bar{\theta}, 1-q_0) \tag{4}$$

In the mild adverse selection case, we have $U(\bar{\theta}) = Ev - \bar{\theta}$ and $\bar{U} + \bar{\theta} = \underline{U} + \underline{\theta}$. (4) is false if

$$z(\bar{\theta}, 1-q_0) > Ev - \bar{\theta} - (\bar{v} - \bar{\theta})q_0 \tag{5}$$

Let

$$q_0^* \equiv \frac{Ev - \bar{\theta}}{\bar{v} - \bar{\theta}}$$

Notice that $q_0^* < 1$. Thus, for $q_0 > q_0^*$ the right-hand-side of (5) is negative, and thus (5) holds. Hence (4) is false, and therefore (3) is false:

$$z(\bar{\theta}, 1-q_0) \leq z(\underline{\theta}, 1-q_0) + (\underline{\theta} - \bar{\theta})(1-q_0)$$

But this is the opposite inequality as in (2). We thus have shown that for $q_0 > q_0^*$

$$z(\bar{\theta}, 1-q_0) = z(\underline{\theta}, 1-q_0) + (\underline{\theta} - \bar{\theta})(1-q_0)$$

This is only possible if both types choose to sell $q' = 1 - q_0$ in program z. Letting $T_{-i}(q)$ be the best price offered for a quantity q, we have shown that $T_{-i}(q) - \bar{\theta}q$ is non-decreasing with q, for q low enough. More precisely : there exists a dense set of quantities q in $[0, 1-q_0^*]$, such that $(q, T_{-i}(q))$ belongs to A_{-i}, for some function $T_{-i}(q)$ such that $T_{-i}(q) - \bar{\theta}q$ is non-decreasing with q.

In the strong adverse selection case, we have $\bar{U} = 0$ and $\underline{U} = \underline{v} - \underline{\theta}$. Then the RHS of (4) is zero, and the LHS is positive if q_0 is positive. Therefore (3) cannot hold, and we get

$$\forall\ q_0 > 0 \qquad z(\underline{\theta}, 1-q_0) \geq \underline{v} - \underline{\theta} - (\bar{\theta} - \underline{\theta})q_0$$

Recall also that $z(\underline{\theta}, q) \leq (\underline{v} - \underline{\theta})q$. Thus z must lie between these two bounds :

$$(\underline{v}-\underline{\theta})q \geq z(\underline{\theta}, q) \geq \underline{v}-\underline{\theta}-(\bar{\theta}-\underline{\theta})(1-q)$$

Since these two bounds are equal at $q = 1$, we can also show that there must be a sequence of points in A_{-i} with quantities converging to 1 and transfers converging to $\underline{v}$ (it is enough to draw a graph of the two bounds with q on the horizontal axis, and to consider a non-decreasing z).

Proof of Proposition 6. (i) Consider first the mild adverse selection case.

Step 1. Given the menus offered, any best response of the seller leads to an aggregate trade $(1, E[v(\theta)])$ irrespective of her type. Since $\phi < E[v(\theta)]$, it is optimal for each type of the seller to trade her whole endowment with a single buyer. Assuming that each type of the seller trades with the same buyer, all buyers obtain a zero payoff. Note also that if any buyer withdraws his menu offer, the most the seller can achieve by trading with the other buyers consists in trading with a single buyer.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $E[v(\theta)]q > t$. Since $\phi < E[v(\theta)]$, the highest payoff the seller can achieve by purchasing the contract (q, t) together with some contract in the menu offered by the other buyers is less than the payoff from trading the contract $(1, E[v(\theta)])$, which remains available at the deviation stage. She would therefore be strictly worse off trading the contract (q, t) no matter her type. Such a deviation is thus infeasible.

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, trading an additional contract $(\underline{q}, \underline{t})$ with type $\underline{\theta}$ is profitable only if $v(\underline{\theta})\underline{q} > \underline{t}$. The same argument as in Step 2 then shows that type $\underline{\theta}$ would be strictly worse off trading the contract $(\underline{q}, \underline{t})$ rather than the contract $(1, E[v(\theta)])$, which remains available at the deviation stage. Such a deviation is thus infeasible.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ that is profitable when traded with type $\overline{\theta}$ attracts her only if $\overline{t} + \phi(1-\overline{q}) \geq E[v(\theta)]$, that is, only if she can weakly increase her payoff by trading the contract $(\overline{q}, \overline{t})$ and selling to the other buyers the remaining fraction of her endowment at unit price ϕ. That this is feasible follows from the fact that, when $\overline{t} + \phi(1-\overline{q}) \geq E[v(\theta)]$ and $v(\overline{\theta})\overline{q} > \overline{t}$, the quantity $1-\overline{q}$ is less than the maximal quantity $\frac{v(\overline{\theta})-E[v(\theta)]}{v(\overline{\theta})-\phi}$ that can be traded at unit price ϕ with the other buyers. Moreover, the fact that $\phi \geq \overline{\theta}$ guarantees that it is indeed optimal for type $\overline{\theta}$ to behave in this way at the deviation stage. However, type $\underline{\theta}$

can then also weakly increase her payoff by mimicking type $\overline{\theta}$'s behavior. One can therefore construct the seller's strategy in such a way that it is impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Whatever the contract traded by the seller with the deviating buyer, and no matter her type, she will sell to the other buyers the remaining fraction of her endowment at unit price ϕ. Hence, each type of the seller faces the same problem, namely to use optimally the deviating buyer's and the other buyers' offers to sell her whole endowment at the maximum price. One can therefore construct the seller's strategy in such a way that each type selects the same contract from the deviating buyer's menu. By Step 2, this makes such a deviation non profitable. The result follows.

(ii) Consider next the strong adverse selection case.

Step 1. Given the menus offered, any best response of the seller leads to an aggregate trade $(1, v(\underline{\theta}))$ for type $\underline{\theta}$ and $(0, 0)$ for type $\overline{\theta}$. Since each buyer is not ready to pay anything for quantities up to $\frac{\psi-\theta}{\psi}$ and offers to purchase each additional unit at a constant marginal price ψ above this level, it is optimal for type $\underline{\theta}$ to trade her whole endowment with a single buyer, and all buyers obtain a zero payoff. Note also that if any buyer withdraws his menu offer, the most the seller can achieve by trading with the other buyers consists in trading with a single buyer.

Step 2. No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. This can be shown as in Step 2 of the proof of Proposition 1(ii).

Step 3. No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, trading an additional contract $(\underline{q}, \underline{t})$ with type $\underline{\theta}$ is profitable only if $v(\underline{\theta})\underline{q} > \underline{t}$. Since $\psi > v(\underline{\theta})$, the highest payoff type $\underline{\theta}$ can achieve by purchasing the contract (q, t) together with some contract in the menu offered by the other buyers is less than the payoff from trading the contract $(1, \underline{\theta})$, which remains available at the deviation stage. She would therefore be strictly worse off trading the contract (q, t). Such a deviation is thus infeasible.

Step 4. It follows from Step 3 that a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq \overline{\theta}\overline{q}$. Two cases must be distinguished. If $\overline{q} \leq \frac{v(\theta)}{\psi}$, then type $\underline{\theta}$ can trade the contract $(\overline{q}, \overline{t})$ and sell to some other buyer the remaining fraction of her endowment at price $\psi(1 - \overline{q}) - \psi + v(\underline{\theta})$. The price at which she can sell her whole endowment is therefore at least $(\overline{\theta}-\psi)\overline{q}+v(\underline{\theta})$, which is strictly higher than the price $\underline{\theta}$ that she obtains in equilibrium since $\overline{\theta} > v(\underline{\theta}) + \frac{\overline{\theta}-E[v(\theta)]}{1-\nu} \geq \psi$. If $\overline{q} > \frac{v(\underline{\theta})}{\psi}$, then by trading the contract $(\overline{q}, \overline{t})$, type $\underline{\theta}$ obtains at least a payoff $\frac{(\overline{\theta}-\underline{\theta})v(\underline{\theta})}{\psi}$, which, since $\overline{\theta} > \psi > v(\underline{\theta})$, is more than her equilibrium payoff $v(\underline{\theta}) - \underline{\theta}$. Thus type $\underline{\theta}$ can always strictly increase her payoff by trading the contract $(\overline{q}, \overline{t})$. It is therefore impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. It follows from Steps 3 and 4 that a profitable deviation must involve trading with both types. Given the offers of the other buyers, the most profitable deviations lead to trading some quantity $\overline{q} \leq \frac{v(\underline{\theta})}{\psi}$ at unit price $\overline{\theta}$ with type $\overline{\theta}$, and trading a quantity 1 at unit price $\overline{\theta}\overline{q} + v(\underline{\theta}) - \psi\overline{q}$ with type $\underline{\theta}$. By construction, type $\underline{\theta}$ is indifferent between trading the contract $(1, \overline{\theta}\overline{q} + v(\underline{\theta}) - \psi\overline{q})$ and trading the contract $(\overline{q}, \overline{\theta}\overline{q})$ while selling to the other buyers the remaining fraction of her endowment at price $\psi(1-\overline{q}) - \psi + v(\underline{\theta})$. As for type $\overline{\theta}$, she is indifferent between trading the contract $(\overline{q}, \overline{\theta}\overline{q})$ and not trading at all. The corresponding payoff for the deviating buyer is then

$$\nu[v(\overline{\theta}) - \overline{\theta}]\overline{q} + (1-\nu)\{v(\underline{\theta}) - [\overline{\theta}\overline{q} + v(\underline{\theta}) - \psi\overline{q}]\} = [\nu v(\overline{\theta}) + (1-\nu)\psi - \overline{\theta}]\overline{q},$$

which is at most zero since $\psi \leq v(\underline{\theta}) + \frac{\overline{\theta}-E[v(\theta)]}{1-\nu}$. The result follows. ■

Proof of Lemma 1 : for further reference, we solve here a slightly more general problem, that is parameterized by three elements $(\theta_0,\ \theta_1,\ Q_1)$, with $\theta_0 \leq \theta_1$ and $0 \leq Q_1 \leq 1$. This problem consists in maximizing

$$\int_{-\infty}^{\theta_1} [(v(\theta) - \theta)Q(\theta) - U(\theta)]dF(\theta)$$

under the IC and (IR) constraints, and two additional constraints that we now spell. The first constraint imposes that $Q(\theta) = 1$ if $\theta \leq \theta_0$. The second constraint imposes that $Q(\theta)$ is at least equal to Q_1. Notice that the monopsony problem corresponds to $Q_1 = 0$, $\theta_0 = -\infty$, $\theta_1 = +\infty$.

Using standard techniques, the problem reduces to maximizing

$$\int_{-\infty}^{\theta_1} [v(\theta) - \theta]Q(\theta)dF(\theta) - \int_{-\infty}^{\theta_1} Q(\theta)F(\theta)d\theta$$

under the constraint that Q is weakly decreasing, and our two additional constraints. The objective is linear in Q. Moreover any Q verifying the constraints is a convex combination of functions indexed by $\theta' \geq \theta_0$, such that $Q(\theta) = 1$ if $\theta \leq \theta'$, and $Q(\theta) = Q_1$ if $\theta > \theta'$.

Therefore the buyer cannot loose anything by using such functions; each function corresponds to offering to buy one unit for a transfer θ'.

Hence the problem reduces to maximizing on $\theta' \in [\theta_0, \theta_1]$ the buyer's payoff

$$\int_{-\infty}^{\theta'} [v(\theta) - \theta] dF(\theta) - \int_{-\infty}^{\theta'} F(\theta) d\theta + Q_1 \int_{\theta'}^{\theta_1} [v(\theta) - \theta] dF(\theta) - Q_1 \int_{\theta'}^{\theta_1} F(\theta) d\theta$$

$$= \int_{-\infty}^{\theta'} v(\theta) dF(\theta) - \theta' F(\theta') + Q_1 \int_{\theta'}^{\theta_1} v(\theta) dF(\theta) + Q_1 [\theta' F(\theta') - \theta_1 F(\theta_1)]$$

and thus the buyer's maximum profit is

$$Q_1 w(\theta_1) + (1 - Q_1) \sup_{\theta' \in [\theta_0, \theta_1]} w(\theta') \tag{6}$$

For the monopsony problem under study, setting ($\theta_0 = -\infty$, $\theta_1 = +\infty$, $Q_1 = 0$) proves Lemma 1. Another consequence of (6) will be used when dealing with competition : then aggregate profits cannot exceed the bound in (6). Therefore, if we know that $Q(\theta) = 1$ for $\theta < p^*$, we can apply the formula at ($\theta_0 = p^*$, $\theta_1 = +\infty$, $Q_1 = 0$); Assumption 1 then implies that $Q(\theta) = 0$ for $\theta > p^*$. QED.

Proof of Proposition 1 : Suppose that a robust equilibrium exists, with outcome (U, Q). Let us first prove the following result, that is used repeatedly :

Lemma 2 *Choose $\theta_a < \theta_b$, and suppose that the following property holds at (θ_a, θ_b) :*

$$\exists\, \theta, \theta' \qquad \theta_a < \theta < \theta' < \theta_b \qquad \textit{and} \qquad Q(\theta) > Q(\theta') \tag{7}$$

Define

$$q_0 = \frac{U(\theta_a) - U(\theta_b)}{\theta_b - \theta_a} \qquad t_0 = \frac{\theta_b U(\theta_a) - \theta_a U(\theta_b)}{\theta_b - \theta_a}$$

Then at equilibrium one must have

$$n \int_{\theta_a}^{\theta_b} [v(\theta) q_0 - t_0] dF(\theta) \leq \int_{\theta_a}^{\theta_b} [(v(\theta) - \theta) Q(\theta) - U(\theta)] dF(\theta)$$

Proof of Lemma 2 : since $U'(\theta)$ is equal to $(-Q(\theta))$ almost everywhere, q_0 is computed as an average of the quantities traded; under (7) it must be that $Q(\theta_b) < q_0 < Q(\theta_a)$. Notice moreover that

$$t_0 = U(\theta_a) + \theta_a q_0 = U(\theta_b) + \theta_b q_0$$

Now suppose that principal i deviates by adding the contract $(q_0,\ t_0)$ to his equilibrium offer. For $\theta > \theta_b$, convexity of U implies

$$U(\theta) \geq U(\theta_b) + (\theta - \theta_b)(-Q(\theta_b))$$

and using the definitions of q_0 and t_0 we get

$$U(\theta) \geq t_0 - \theta q_0 + (\theta - \theta_b)(q_0 - Q(\theta_b))$$

which is strictly greater than $t_0 - \theta q_0$. Thus θ strictly prefers his equilibrium trade to trading $(q_0,\ t_0)$; from robustness this implies that following the buyer's deviation θ does not trade $(q_0,\ t_0)$, and does not change his behaviour. The same properties can be shown similarly for all types $\theta < \theta_a$. Finally consider types such that $\theta_a < \theta < \theta_b$. By convexity we have

$$U(\theta_a) \geq U(\theta) + (\theta - \theta_a)Q(\theta)$$
$$U(\theta_b) \geq U(\theta) + (\theta - \theta_b)Q(\theta)$$

and from (7) at least one of these inequalities is strict. Multiplying by well-chosen positive constants and summing, we get

$$U(\theta_a)(\theta_b - \theta) + U(\theta_b)(\theta - \theta_a) > U(\theta)(\theta_b - \theta_a)$$

which reduces to $t_0 - \theta q_0 > U(\theta)$. Hence under robustness all types in $]\theta_a, \theta_b[$ choose to trade $(q_0,\ t_0)$. This establishes that for any principal i the variation in profits is

$$\int_{\theta_a}^{\theta_b} [v(\theta)q_0 - t_0 - b^i(\theta)]dF(\theta)$$

where $b^i(\theta)$ is the expected profit that principal i gets from type θ on the equilibrium path. We get the result by summing over $i = 1..n$, because $\sum_i b^i(\theta) = (v(\theta) - \theta)Q(\theta) - U(\theta)$. *QED.*

Now choose some positive quantity Q_0 that is actually traded by some type θ_0 in $]\underline{\theta}, \bar{\theta}[$. Impose moreover that $Q(.)$ is continuous at θ_0 (this is not a strong restriction, as anyway the non-increasing function Q is continuous almost everywhere). We distinguish two cases, in order to show that the buyers' aggregate profits are zero or below zero when Q_0 is traded.

First case : suppose that θ_0 is the only type to sell Q_0. Then for any $\theta_1 < \theta_0 < \theta_2$, (7) holds at (θ_1, θ_2). We can thus apply Lemma 2, and because $t_0 = U(\theta_1) + \theta_1 q_0$ we obtain

$$n \int_{\theta_1}^{\theta_2} [v(\theta)q_0 - U(\theta_1) - \theta_1 q_0] dF(\theta) \leq \int_{\theta_1}^{\theta_2} [(v(\theta) - \theta)Q(\theta) - U(\theta)] dF(\theta)$$

Since this is valid for any $\theta_1 < \theta_0 < \theta_2$, one can divide this inequality by $(F(\theta_2) - F(\theta_1))$ and compute the limit when both bounds go to θ_0, to get

$$n[v(\theta_0)Q_0 - U(\theta_0) - \theta_0 Q_0] \leq (v(\theta_0) - \theta_0)Q_0 - U(\theta_0)$$

or equivalently

$$(v(\theta_0) - \theta_0)Q_0 - U(\theta_0) \leq 0$$

which indicates that the buyers' aggregate profits from trading Q_0 cannot be positive.

Second case : the only other case is when $Q(\theta) = Q_0$ on some maximum interval (θ_2, θ_3) containing θ_0. Choose (θ_1, θ_4) such that $\theta_1 < \theta_2 < \theta_3 < \theta_4$. One can now apply Lemma 2 at, say, (θ_1, θ_3), and take limits as above when θ_1 goes to θ_2, to obtain

$$n \int_{\theta_2}^{\theta_3} [v(\theta)Q_0 - U(\theta) - \theta Q_0] dF(\theta) \leq \int_{\theta_2}^{\theta_3} [(v(\theta) - \theta)Q_0 - U(\theta)] dF(\theta)$$

or equivalently

$$\int_{\theta_2}^{\theta_3} [(v(\theta) - \theta)Q_0 - U(\theta)] dF(\theta) \leq 0$$

Hence we have established that whatever the quantity traded the buyers' aggregate profits are zero or below zero. Because aggregate profits must be at least zero, this implies that profits are exactly zero for all quantities traded (apart for a negligible subset of θ), as announced.

We can now extend our analysis of the second case : choose some θ' such that $\theta_2 < \theta' < \theta_3$, and apply Lemma 2 at (θ_1, θ'), and take the limit when θ_1 goes to θ_2 to get

$$\int_{\theta_2}^{\theta'} [(v(\theta) - \theta)Q_0 - U(\theta)] dF(\theta) \leq 0$$

Similarly apply Lemma 1 at (θ', θ_4), and take the limit when θ_4 goes to θ_3 to get

$$\int_{\theta'}^{\theta_3} [(v(\theta) - \theta)Q_0 - U(\theta)]dF(\theta) \leq 0$$

Because these two functions of θ' add up to zero, these inequalities imply that they are identically equal to zero. Summarizing, we have shown that $U(\theta) = (v(\theta) - \theta)Q_0$ for almost all $\theta \in [\underline{\theta}, \bar{\theta}]$. Since finally U and v are continuous, then Q must be continuous, and the equality is in fact valid for all θ.

There remains to check deviations in which a buyer proposes to buy one unit from sellers with type below some threshold $\theta_0 > \underline{\theta}$. To attract exactly such types, the price quoted must be equal to $U(\theta_0) + \theta_0$. At equilibrium the resulting profit cannot be positive, so that

$$\int^{\theta_0} (v(\theta) - U(\theta_0) - \theta_0)dF(\theta) \leq 0$$

or equivalently

$$w(\theta_0) \leq U(\theta_0)F(\theta_0)$$

This inequality must be valid for any θ_0. To study it, first recall that by definition $w' = (v - \theta)f - F$, and $U = -(v - \theta)U'$. We thus get $w'U' = -[UF]'$, so that we can compute F from U and w:

$$F(\theta_0)U(\theta_0) = -\int^{\theta_0} U'(\theta)w'(\theta)d\theta$$

Our inequality thus becomes

$$w(\theta_0) + \int^{\theta_0} U'w' \leq 0$$

or equivalently

$$\int^{\theta_0} w'(1 - Q) \leq 0$$

The problem is now that at $\underline{\theta}$ we have $w'(\underline{\theta}) = (v(\underline{\theta}) - \underline{\theta})f(\underline{\theta})$, which is positive by assumption; and w' is continuous, therefore w' is positive on a neighborhood of $\underline{\theta}$. Therefore one must have $Q = 1$ on a neighborhood of $\underline{\theta}$. But when Q is a constant on some interval, each θ in this interval sells the same quantity for a unit price $v(\theta)$. For clear incentive-compatibility reasons, $v(.)$ must then be a constant on this interval. QED.

Proof of Proposition 2 : suppose that all buyers offer to buy any quantity at price p^*, and the seller chooses randomly across buyers. Suppose a buyer i were to deviate. A type

$\theta < p^*$ trading $(q,\, t)$ with i would also sell $1-q$ at price p^* to other buyers. θ's overall payoff would be

$$t - \theta q + p^*(1-q) - \theta(1-q) = t - p^* q + p^* - \theta$$

and should be maximized on the set of contracts $(q,\, t)$ offered by i. But θ does not impact this maximization problem; we can thus safely assume that all types below p^* choose to trade the same contract $(q,\, t)$ with buyer i. Moreover the unit price t/q must be above p^*, otherwise all types would sell zero to i. But then buyer i cannot make any profits from types below p^*. The same conclusion holds for types above p^* : even a monopoly cannot extract any profits from these types (see the formula (6), applied at $(\theta_0 = p^*,\, \theta_1 = +\infty,\, Q_1 = 1)$). QED.

Proof of Proposition 3 : let us first study necessity. Suppose we are given a robust equilibrium, whose outcome include the set $\mathcal{C}_i$ of contracts offered by each buyer $i = 1..n$, and payoffs $U(\theta)$ and total quantity traded $Q(\theta)$ (possibly random) for each type θ of the agent. Let us define $b^i(\theta)$ as the equilibrium expected profit obtained by buyer i from the seller of type θ. Define also θ_0 as the supremum of those types that sell a quantity one (set $\theta_0 = -\infty$ if this set is empty).

Let $\theta_1 > \theta_0$, and let $Q_1 < 1$ be a quantity that this type sells with strictly positive probability. Because Q_1 is possibly traded at equilibrium, there exists $(q_i,t_i)_{i=1..n}$ in $\mathcal{C}_1 \times .. \times \mathcal{C}_n$ such that

$$Q_1 = \sum_i q_i \qquad\qquad U(\theta_1) = \sum_i t_i - \theta_1 Q_1 \tag{8}$$

Choose any buyer i, and consider the following deviation : offer the same subset of contracts $\mathcal{C}_i$ as before, plus the contract $(q_i + 1 - Q_1,\, t_i + \theta_1(1-Q_1))$. The seller reacts to this deviation depending on his type θ.

If $\theta > \theta_1$, then θ strictly prefers $(q_i,\, t_i)$ to the new contract, because its unit price is too low. We can then apply part i) of our robustness refinement to conclude that θ does not change its behavior.

If $\theta < \theta_1$, then θ can choose to trade the new contract, together with the contracts $(q_j,\, t_j)_{j \neq i}$ that are defined in (8). Then θ would sell exactly one unit, and would get a payoff

$$\theta_1(1-Q_1)+t_i+\sum_{j\neq i} t_j-\theta = U(\theta_1)+\theta_1-\theta > U(\theta)$$

because $U(\theta)+\theta$ is strictly increasing on $[\theta_0,\theta_1]$. Since $U(\theta)$ is the best payoff θ can get by rejecting the new contract, we have shown that θ strictly gains by trading the new contract compared to not trading it, and from part ii) of our robustness requirement he must do so.

Now we can compute the change in profits for buyer i, following the deviation. For $\theta \leq \theta_1$, now buyer i gets

$$(q_i+1-Q_1)v(\theta)-t_i-\theta_1(1-Q_1)$$

while at equilibrium i was getting an expected profit $b^i(\theta)$. Therefore the variation in profits can be written

$$\int_{-\infty}^{\theta_1}[(q_i+1-Q_1)(v(\theta)-\theta_1)+\theta_1 q_i-t_i-b^i(\theta)]dF(\theta)$$

and this must be weakly negative (otherwise the deviation would be strictly profitable). Using the definition of w, we obtain

$$(q_i+1-Q_1)w(\theta_1)\leq \int_{-\infty}^{\theta_1}[t_i-\theta_1 q_i+b^i(\theta)]dF(\theta)0$$

Now we can sum over i. Notice that at equilibrium the total profits from θ are a.e.

$$\sum_i b^i(\theta)=(v(\theta)-\theta)Q(\theta)-U(\theta)$$

Using also (8), we get

$$(Q_1+n(1-Q_1))w(\theta_1)\leq \int_{-\infty}^{\theta_1}[(v(\theta)-\theta)Q(\theta)-(U(\theta)-U(\theta_1))]dF(\theta)$$

where $n\geq 2$ is the number of buyers. Let us study the right-hand side integral. We know that $(Q,\,U-U(\theta_1))$ must satisfy the IC and IR constraints, that moreover $Q(\theta)\geq Q_1$, and finally that $Q(\theta)=1$ for $\theta<\theta_0$. Using the expression for the monopoly profits derived in (6), we get that the right-hand-side integral must lie below

$$Q_1 w(\theta_1)+(1-Q_1)\sup_{\theta\in[\theta_0,\theta_1]} w(\theta)$$

Replacing and simplifying since $Q_1 < 1$, we finally get

$$nw(\theta_1) \leq \sup_{\theta \in [\theta_0, \theta_1]} w(\theta) \tag{9}$$

This must hold for all $\theta_1 > \theta_0$, by definition of θ_0. We can take supremums to get

$$n \sup_{\theta_1 > \theta_0} w(\theta_1) \leq \sup_{\theta_1 > \theta_0} \sup_{\theta \in [\theta_0, \theta_1]} w(\theta) = \sup_{\theta \geq \theta_0} w(\theta)$$

and by continuity of w, and because $n \geq 2$, we get

$$\sup_{\theta \geq \theta_0} w(\theta) \leq 0$$

¿From Assumption 1, this implies that $\theta_0 \geq p^*$, so that $Q(\theta) = 1$ for $\theta < p^*$. Applying the result stated in the last paragraph of the proof to Lemma 1, we get that $Q(\theta)$ is equal to one for all $\theta < p^*$, and $Q(\theta)$ is zero above p^*. QED.

Proof of Proposition 4 : we only have to show that all contracts issued have a unit price below p^*. Suppose otherwise, and consider a contract $(q > 0,\ t)$ with a unit price strictly above p^*, offered by one of the buyers. Another buyer could then deviate by adding the contract $C' = (1-q, (p^* - \varepsilon)(1-q))$ to its equilibrium offer, where ε is such that $t - p^* q > \varepsilon(1-q)$. Then clearly types θ above $p^* - \varepsilon$ do not trade this contract, since the unit price is too low. Types below $p^* - \varepsilon$ could trade this contract together with contract $(q,\ t)$ and get

$$t + (p^* - \varepsilon)(1-q) - \theta = p^* - \theta + t - p^* q - \varepsilon(1-q) > p^* - \theta = U(\theta)$$

so that under robustness these types should accept to trade C'. Overall our buyer gets

$$\int^{p^* - \varepsilon} [v(\theta) - p^* + \varepsilon][1-q] dF(\theta) = (1-q) w(p^* - \varepsilon)$$

which is positive for ε small and well-chosen, by definition of p^*. Q.E.D.

Proof of Proposition 5 : Fix an equilibrium with zero-profit, and a buyer i. Let A be the set of (aggregate) pairs (t,q) that can be traded with the other buyers $j \neq i$, and let $\bar{A}$ be its closure (under our compactness assumption, we in fact have $\bar{A} = A$). Define

$$z(\theta, q) = \sup\{t' - \theta q' : (q', t') \in \bar{A}, q' \leq q\}$$

(Notice that the value of z would be unchanged if one were to replace $\bar{A}$ by A in the definition of z.) In words, $z(\theta, q)$ is the highest payoff a seller of type θ can get from other buyers, if her remaining stock is q. Notice that $z \geq 0$, and z is non-decreasing in q. Also, any trade (q', t') satisfying the constraints can be selected by both types. We can thus write, for $\theta < \theta'$,

$$t' - \theta' q' = t' - \theta q' + (\theta - \theta')q' \geq t' - \theta q' + (\theta - \theta')q$$

because $q' \leq q$. Taking supremums, we get a useful property :

$$z(\theta', q) \geq z(\theta, q) + (\theta - \theta')q$$

or equivalently $z(\theta, q) + \theta q$ is non-decreasing with θ. Moreover, if $z(\theta, q) + \theta q = z(\theta', q) + \theta' q$, then there must exist a contract $(1 - q_0, t)$ in $\bar{A}$, that is chosen by both θ and θ'.

Now let $\theta_0 \in]\underline{\theta}, \min(p^*, \bar{\theta})[$, and let $q_0 \in]0, 1[$. Let us distinguish two cases. If $z(\theta, q)$ is a constant on some interval containing θ_0, then we are done, from the remark above. The complementary case is when, for all θ', θ" such that $\theta' < \theta_0 < \theta$" we have

$$z(\theta', 1 - q_0) + \theta'(1 - q_0) < z(\theta_0, 1 - q_0) + \theta_0(1 - q_0) < z(\theta", 1 - q_0) + \theta"(1 - q_0) \qquad (10)$$

In that case buyer i could deviate, by adding to his offer a contract (q_0, t_0) that we now describe. We impose that θ_0 is indifferent between sticking to the previous offer (selling one at price p^*) and trading (q_0, t_0):

$$t_0 - \theta_0 q_0 + z(\theta_0, 1 - q_0) = p^* - \theta_0$$

Now from (10) all types $\theta > \theta_0$ strictly prefer the new offer to selling one unit at price p^*, and all types $\theta < \theta_0$ strictly prefer to sell one unit at price p^*. For types $\theta > p^*$, notice that $z(\theta, 1 - q_0) = U(\theta) = 0$, because such a type gets a zero-payoff at equilibrium. Hence θ accepts the new offer if $t_0 > \theta q_0$, or equivalently $\theta < \theta_1$, where

$$t_0 = \theta_1 q_0 = \theta_0 q_0 + p^* - \theta_0 - z(\theta_0, 1 - q_0)$$

It is easily checked that $\theta_1 \geq p^*$ if and only if $(p^* - \theta_0)(1 - q_0) \geq z(\theta_0, 1 - q_0)$, which we know is true from proposition 4. Overall, we have shown that the deviation (q_0, t_0) attracts all types in some interval $]\theta_0, \theta_1[$, with $\theta_0 < p^* < \theta_1$, that θ_0 and θ_1 are indifferent, and that all other types reject the deviation. Hence the profit realized is

$$\int_{\theta_0}^{\theta_1} [v(\theta)q_0 - t_0]dF(\theta)$$

Now let q_0 go to one. Then $z(\theta_0, 1 - q_0)$ goes to zero (once more this is because $(p^* - \theta_0)(1 - q_0) \geq z(\theta_0, 1 - q_0) \geq 0$), so that θ_1 and t_0 go to p^*. Then the limit of the profit is

$$\int_{\theta_0}^{p^*} [v(\theta) - p^*]dF(\theta)$$

Because $v - p^*$ is assumed to be increasing, and $w(p^*) = \int^{p^*}(v - p^*)dF = 0$, this profit must be positive; but this cannot be true at equilibrium, a contradiction. QED.

References

[1] Akerlof, G.A. (1970): "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84(3), 488–500.

[2] Attar, A., and A. Chassagnon (2008): "On Moral Hazard and Nonexclusive Contracts," Mimeo, Toulouse School of Economics.

[3] Biais, B., D. Martimort, and J.-C. Rochet (2000): "Competing Mechanisms in a Common Value Environment," *Econometrica*, 68(4), 799–837.

[4] Bisin, A., and D. Guaitoli (2004): "Moral Hazard and Nonexclusive Contracts," *RAND Journal of Economics*, 35(2), 306–328.

[5] Hellwig, M. (1983): "On Moral Hazard and Non-Price Equilibria in Competitive Insurance Markets," Discussion Paper No. 109, Sonderforschungsbereich 21, University of Bonn.

[6] Khalil, F.A., D. Martimort, and B. Parigi (2007): "Monitoring a Common Agent: Implications for Financial Contracting," *Journal of Economic Theory*, 135(1), 35–67.

[7] Martimort, D., and L.A. Stole (2003): "Contractual Externalities and Common Agency Equilibria," *Advances in Theoretical Economics*, 3(1), Article 4.

[8] Martimort, D., and L.A. Stole (2007): "Market Participation under Delegated and Intrinsic Common Agency Games," Mimeo, Toulouse School of Economics and University of Chicago Graduate School of Business.

[9] Piaser, G. (2007): "Direct Mechanisms, Menus and Latent Contracts," CREFI-LSF Working Paper No. 07-09, Université du Luxembourg.

[10] Rothschild, M., and J.E. Stiglitz (1976): "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90(4), 629–649.

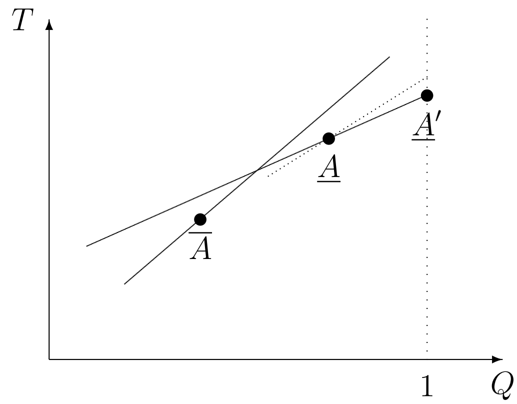

Figure 1 Attracting type $\underline{\theta}$ by pivoting around $(\underline{Q}, \underline{T})$

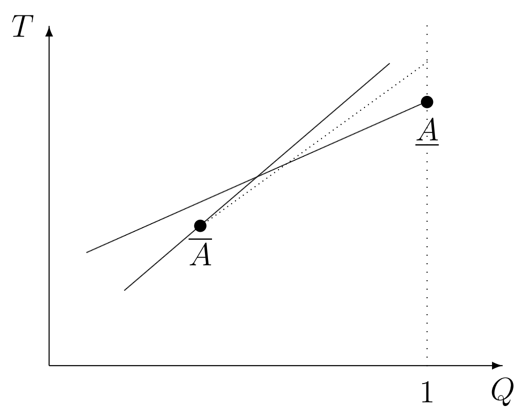

Figure 2 Attracting type $\underline{\theta}$ by pivoting around $(\overline{Q}, \overline{T})$

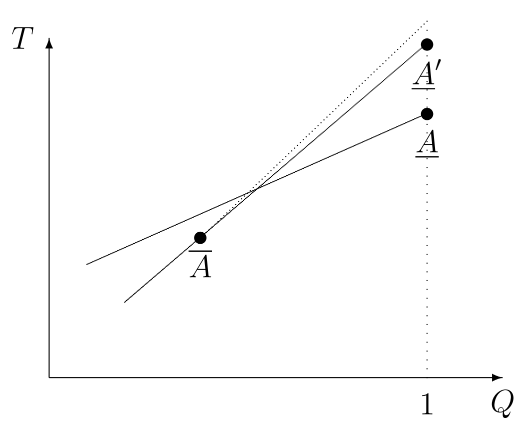

Figure 3 Attracting both types by pivoting around $(\overline{Q}, \overline{T})$

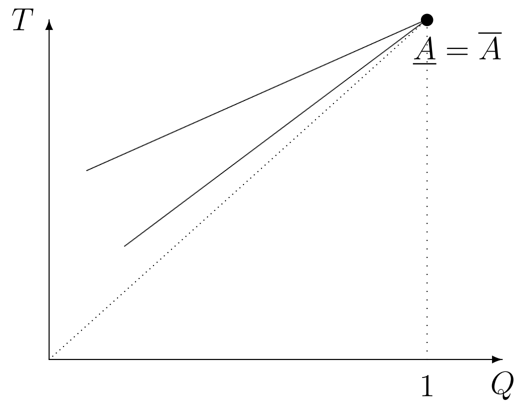

Figure 4 Aggregate equilibrium allocations in the mild adverse selection case

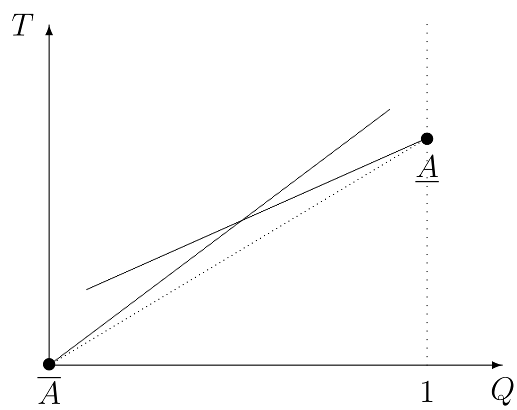

Figure 5 Aggregate equilibrium allocations in the strong adverse selection case

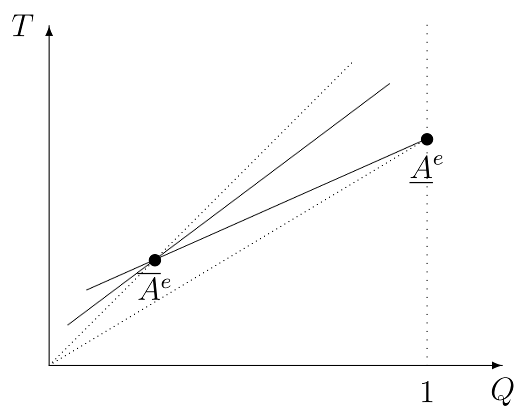

Figure 6 Equilibrium allocations under exclusive competition