Real-time forecasting of GDP based on a large factor model with monthly and quarterly data*

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Abstract

This paper discusses a factor model for estimating monthly GDP using a large number of monthly and quarterly time series in real-time. To take into account the different periodicities of the data and missing observations at the end of the sample, the factors are estimated by applying an EM algorithm combined with a principal components estimator. We discuss the in-sample properties of the estimator in real-time environments and methods for out-of-sample forecasting. As an empirical application, we estimate monthly German GDP in real-time, discuss the nowcast and forecast accuracy of the model and the role of revisions. Furthermore, we assess the contribution of timely monthly data to the forecast performance.

Key words: Monthly GDP, EM algorithm, principal components, factor models

JEL codes: E37, C53

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1 Introduction

Macroeconomic policy making in real-time faces the problem of assessing the current state of the economy with incomplete statistical information. Important economic variables are released with considerable time lags. As a key indicator of real economic activity, GDP is published at quarterly frequency and with a considerable delay only. For example, in Germany, GDP is released about six weeks after the end of the respective quarter. If policy decisions require information within or right after that quarter, more timely information is needed. One possibility is to use monthly indicators which are often available more timely than GDP and might help to monitor the current state of the economy. Another approach, which will be pursued here, is to employ monthly and quarterly data for estimating and forecasting indicators of monthly GDP.

In this paper, a large-scale factor model with monthly and quarterly data is used to estimate monthly GDP in real-time. Following Stock and Watson (2002a, 2002b), we estimate the factors by principal components. To take into account data with different frequencies (in our case quarterly and monthly data) an expectation-maximisation (EM) algorithm is employed. This algorithm iteratively estimates monthly factors from mixed-frequency data and uses the estimated factors to construct monthly estimates of the quarterly series. The EM algorithm can also take into account other missing data problems, such as different publication dates at the end of the sample. Since different publication lags are typically present in real-time datasets, the proposed EM algorithm is an attractive extension of existing forecast methods based on large datasets.

In practice, monthly forecasts of GDP are often obtained using a state-space framework, see Mariano and Murasawa (2003), Evans (2005), Mittnik and Zadrozny (2005) and Nunes (2005). Other approaches rely on bridge equations in order to relate monthly to quarterly variables, e.g. Rünstler and Sedillot (2003), Baffigi et al. (2004) and Mitchell et al. (2005). A new line of research is based on mixed data-frequency sampling proposed by Ghysels et al. (2006) and Clements and Galvão (2005), where quarterly time series can be directly regressed on monthly indicators. A common feature of bridge equations and the mixed data-frequency sampling approach is that they are typically restricted to a rather limited information set with a small number of variables as regressors, whereas forecasts based on principal components as factors allow for very large datasets. In a large factor model framework, Giannone et al. (2005) investigate the information content of different groups of monthly data to estimate current GDP taking into account the different availability of indicators at the end of the sample. In their model, GDP is transformed to monthly frequency prior to factor estimation, whereas in this paper, quarterly GDP together with other quarterly and monthly indicators are used simultaneously for the estimation of common monthly factors. Another factor model approach is proposed by Altissimo et al. (2006), where a large number of monthly indicators is used to forecast smoothed GDP growth, defined in the frequency domain as including only waves of period larger than one year. We consider a similar application as Bernanke and Boivin (2003) who investigate the real-time forecasting accuracy of the factor model proposed by Stock and Watson (2002a). However, their comparison of alternative forecast procedures is concerned with forecasting monthly variables like inflation and industrial production, whereas our focus is on the estimation of a monthly GDP indicator, which requires modifications of the existing forecasting methods for factor models based on large datasets. In-sample properties of the EM algorithm are discussed in Angelini et al. (2006) and related to other approaches by Marcellino (2006), where Monte Carlo simulations as well as empirical interpolation and backcasting exercises are carried out. Here, we discuss the properties of the EM algorithm with special emphasis on forecasting monthly GDP in real-time situations, where solutions are needed to tackle the mixed-frequency data problem as well as missing observations at the end of the sample. The EM algorithm provides a solution to these issues, as long as timely information from monthly indicators is available. However, as this is the case only for short forecast horizons, we have to employ additional forecasting methods, when forecasts for longer horizons are of interest. Factor forecasting using single-frequency data is comprehensively discussed in Boivin and Ng (2005). However, in many empirical applications with mixed-frequency data and missing values, we have to modify the existing methods. Below, we discuss these modifications and apply them in a real-time forecast exercise.

The empirical application in this paper uses German post-unification real-time data to estimate German GDP, which was taken from the Bundesbank Monthly Bulletin Supplement for seasonal adjusted data. Overall the dataset consists of about 50 time series. The real-time dataset is comparatively small related to other large factor model studies, such as Bernanke and Boivin (2003), but still substantially larger than state-space model approaches or bridge equation models. However, as discussed in Boivin and Ng (2006), the information content of the dataset rather than the number of the time series is important for forecasting in the factor model context, and we will discuss below, whether the model employed here can exploit the medium-sized dataset for forecasting German GDP in an efficient way. The novel dataset employed here is an extended version of the dataset collected in Gerberding et al. (2005a, 2005b) and has not been used in a large factor model context before. In the empirical application below, the vintages of data are used to recursively estimate and forecast monthly GDP in real-time. We compare real-time forecasts with forecasts from using final-vintage data in order to assess the importance of data revisions. In addition, the impact of timely monthly variables on forecast accuracy is investigated using different compositions of the dataset.

The paper is organized as follows. In the section 2, the estimation of factor models with mixed-frequency data is discussed and the estimation method based on the EM algorithm is introduced. Furthermore, the properties of the EM algorithm in real-time environments are discussed and different forecast methods to be used in this context are proposed. Section 3 discusses the details of the real-time dataset. The empirical application of the mixed-frequency factor model to German real-time data is provided in section 4. Section 5 concludes.

2 Factor estimation and forecasting of monthly GDP

This section outlines the estimation procedure for the common factors. Two cases are distinguished, the case without data irregularities and situations with different periodicities of the data. A discussion of the properties of the in-sample estimators follows and motivates the choice of forecasting methods. These methods are discussed in the final subsection below.

2.1 The approximate factor model and its estimation with single-frequency data

Our aim is to model the stationary variables $x_{i,t}$ for observation t with $t = 1, \dots, T$ and from cross-section i for $i = 1, \dots, N$. The index t denotes either monthly or quarterly observations for all variables, so different periodicities are neglected so far. For the ease of exposition, we assume that each variable has expectation zero, although in practice an unknown mean will be accounted for by subtracting the sample means of the variables.

Factor model representation In factor models, the variables $x_{i,t}$ are represented as the sum of two mutually orthogonal components: the common and idiosyncratic components. The common component for each variable is a linear combination of a small number of factors collected in the $r \times 1$ vector F_t common to all variables in the model. The idiosyncratic components $e_{i,t}$ are variable-specific. Thus, we have the representation

$$x_{i,t} = \Lambda_i' F_t + e_{i,t} \tag{1}$$

for $t = 1, \dots, T$ and $i = 1, \dots, N$. Λ_i is a $r \times 1$ vector of factor loadings for the i-th variable. For later use, the following equivalent representations

$$X_t = \Lambda F_t + e_t, \quad X_i = F\Lambda_i + e_i, \quad \text{and} \quad X = F\Lambda' + e \tag{2}$$

are also relevant, see also Bai (2003, p. 140). In these representations, the $N \times 1$ dimensional vector $X_t = (x_{1,t}, \dots, x_{N,t})'$ contains the t-dated observations for all cross-section units, $X_i = (x_{i,1}, \dots, x_{i,T})'$ collects the T observations of variable i and $X = (X_1, \dots, X_N)$

contains all observations. The matrices for the idiosyncratic components are defined accordingly. F is the $T \times r$ matrix containing the stacked time series values of the factors. Finally, $\Lambda = (\Lambda_1, \dots, \Lambda_N)'$ is the $N \times r$ loadings matrix.

Assumptions According to Bai and Ng (2002), the factor model described here is an approximate factor model, as some degree of cross-sectional correlation between units of e_t is allowed for as well as serial correlation over time and a limited degree of heteroscedasticity. The limiting matrices of $N^{-1}\Lambda'\Lambda$ and $T^{-1}F'F$ are assumed to be $r \times r$ positive definite matrices. Following Bai (2003, p. 141), the model allows the factors F_t to be a serially dependent process, for example the VAR process

$$(I - A(L)L)F_t = u_t, \tag{3}$$

where the lag polynomial $A(L) = A_1 + A_2 L + \dots$ has to ensure a stationary process for the factors, hence the roots of $|I - A(z)z| = 0$ have to be inside the unit circle. The residuals u_t are white noise. Note that the dynamics are solely present in the factors, and lags of the factors do not directly enter (1), see Bai and Ng (2002, p. 211). Therefore, (1) represents a static factor model.

Estimation with single-frequency data The factors in this model can be estimated using principal components. Let V be the $N \times r$ matrix of stacked eigenvectors $V = (V_1, \dots, V_r)$ corresponding to the r largest eigenvalues of the sample covariance matrix $T^{-1}\sum_{t=1}^{T} X_t X_t'$. The principal component estimators of the factors and the loading matrix are given by

$$\widehat{F}_t = V'X_t, \;\; \widehat{F} = XV, \;\text{ and }\; \widehat{\Lambda} = V. \tag{4}$$

The asymptotic properties of these estimators are analysed in Stock and Watson (2002b) and Bai (2003). Under the mild assumptions above, the estimators are consistent and asymptotically normally distributed as N and T tend to infinity.

2.2 Estimation based on monthly and quarterly data using the EM algorithm

The EM algorithm is an iterative algorithm for efficient estimation, when data irregularities such as missing observations or mixed frequencies are present.[1] Therefore, the

[1] For a general introduction to the EM algorithm, see Little and Rubin (2002), pp. 166-189, and Ng et al. (2004).

EM algorithm considers two key characteristics of real-time problems. In real-time, policy makers often want to take into account the most recent statistical information for forecasting. However, as statistical information is released with different delays, appropriate methods for forecasting that can tackle this lack of synchronisation are required, since standard techniques are often only appropriate for balanced data. Moreover, the key indicator GDP is a quarterly variable that cannot be easily combined with monthly indicators.

The EM algorithm for the approximate factor model was introduced by Stock and Watson (2002a) to allow for irregularities in large datasets typically used for factor estimation. In the case of mixed frequencies, the iterations of the EM algorithm applied to the factor model proceed as follows: Given an initial monthly estimate of the quarterly data, the full dataset including the initial monthly estimates can be used to extract monthly factors. Based on the factors, improved estimates of the missing monthly observations are obtained. This estimation procedure is repeated until the monthly estimators have converged. In the following, the steps of the estimation procedure are explained in more detail.

Time aggregation matrices One key step during the iterations is the transformation of monthly estimates of observations into a quarterly time series. Another necessary transformation is required, if missing observations occur, e.g. at the end of the sample. To consider these different cases, we distinguish the $T^{\text{obs}} \times 1$ vector of available observations X_i^{obs} for the variable $i \in \{1, \ldots, N\}$ from the complete $T \times 1$ vector of realizations X_i that may contain observations not included in the dataset. Hence, $T^{\text{obs}} \leq T$. It is assumed that the relationship between observable and complete data is given by the linear relationship

$$X_i^{\text{obs}} = A_i X_i, \tag{5}$$

where A_i is a known $T^{\text{obs}} \times T$ selection or aggregator matrix that can tackle missing values or different frequencies of the data.[2]

Example 1: Missing monthly observations at the end of the sample For example, if X_i is the original vector of monthly observations of time series i and X_i^{obs} denotes the vector of available monthly observations. In case all observations are available, the matrix A_i is simply equal to the identity matrix I_T. However, if the last observation in

[2]See the general aggregator matrix in Angelini et al. (2006), section 2.

X_i^{obs} is not available, $T^{\text{obs}} = T - 1$ and the matrix A_i becomes

$$A_i = \begin{pmatrix} 1 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & 0 \end{pmatrix} = \begin{pmatrix} I_{T^{\text{obs}}} & 0_{T^{\text{obs}} \times 1} \end{pmatrix}_{T^{\text{obs}} \times T}, \tag{6}$$

where the last row of the identity matrix A_i corresponding to the missing value of X_i^{obs} has been removed. In our context, this transformation is particularly relevant, as at a certain point in time, monthly indicator observations are available differently. For example, financial time series are available immediately, whereas industrial production numbers are not available up to the current period.

Example 2: GDP as a monthly flow variable The key variable of interest, real GDP in natural logs, denoted as y_t^q, is a quarterly flow variable. Hence, the relationship between the observed quarterly GDP y_t^q and the unobserved monthly observation y_t^m can be written as

$$y_t^q = (1/3)(y_t^m + y_{t-1}^m + y_{t-2}^m), \tag{7}$$

where the time index t denotes a calendar month. Hence, equation (7) holds for $t = 3, 6, 9, ..., T$, assuming for simplicity that new quarterly observations are available in the last month of the quarter. More importantly, it ensures that the quarterly value is an average of monthly values in the same quarter. The factor models following the forecast literature typically explain the growth rate of GDP, as the forecast model discussed above in (1) and (3) requires stationary time series. Therefore, it may be useful to explain the observed quarterly growth rate $\Delta^q y_t^q = (1 - L^3) y_t^q$ by the unobserved monthly growth rate $\Delta y_t^m = (1 - L) y_t^m$ according to

$$\begin{aligned} \Delta^q y_t^q &= (1/3)(y_t^m + y_{t-1}^m + y_{t-2}^m) - (1/3)(y_{t-3}^m + y_{t-4}^m + y_{t-5}^m) && (8) \\ &= (1/3)(\Delta y_t^m + 2\Delta y_{t-1}^m + 3\Delta y_{t-2}^m + 2\Delta y_{t-3}^m + \Delta y_{t-4}^m), && (9) \end{aligned}$$

which is standard in the literature on GDP interpolation.[3] In matrix notation, stacking the time series observations of GDP yields the relationship $\Delta^q Y^q = A_y \Delta Y^m$, where

$$A_y = \frac{1}{3} \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & 0 & 1 & 2 & 3 & 2 & 1 & 0 & 0 & 0 \\ \dots & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 2 & 1 \end{pmatrix}, \tag{10}$$

[3] See, for example, Mariano and Murasawa (2004).

and $\Delta^q Y^q = (\dots, \Delta^q y^q_{T-6}, \Delta^q y^q_{T-3}, \Delta^q y^q_T)'$ and the corresponding monthly growth rates $\Delta Y^m = (\dots, \Delta y^m_{T-2}, \Delta y^m_{T-1}, \Delta y^m_T)'$. Of course, mixing frequencies together with missing values can also be considered for GDP in the present setup. This is particularly relevant in the real-time setting under discussion here, where often more timely monthly observations are available at the end of the sample than quarterly GDP observations. In the case of missing quarterly observations, the rows of matrix A_y corresponding to missing values of the quarterly GDP time series have to be removed. Assume, for example, that the final growth rate of GDP is missing at the end of the sample, i.e. $\Delta^q Y^q = (\dots, \Delta^q y^q_{T-6}, \Delta^q y^q_{T-3})$, but the monthly indicators are still available until month T. In this case, the bottom row of (10) has to be removed, and the final three columns of A_y become zero vectors, an analogous partitioning as in (6). This has consequences for the EM algorithm estimation as well as the forecasting methods to be discussed later.

The examples described above are the most relevant in our context of monthly GDP estimation. For other time series, the transformation matrices generally depend on the stock-flow nature of the time series and on the degree of stochastic integration.[4]

Steps of the EM algorithm Assume that we have a dataset comprised of quarterly time series including GDP and monthly indicators, and we want to use this dataset for factor estimation. Following Stock and Watson (2002a), the EM algorithm proceeds as follows:

1. Use an initial estimate of the missing data and an initial monthly estimates of quarterly data to obtain initial estimates of the factors and loadings. In our application, missing values and monthly estimates are simply set equal to the unconditional mean of the series. The initial estimates of the monthly observations for the quarterly data together with the observed monthly time series comprise a monthly dataset. Hence, monthly factors and loadings can be estimated as described above in the single-frequency case (4).

2. *E-step:* For iteration j, given factors and loadings from a previous iteration $j-1$ (or estimates of the first initial step 1), compute an updated estimate of the monthly or missing observations by the expectation of the $T \times 1$ vector X_i conditional on the observed data X_i^{obs} and the previous iteration factors and loadings for variable i according to

$$\widehat{X}_i^{(j)} = E\left(X_i | X_i^{\text{obs}}, \widehat{F}^{(j-1)}, \widehat{\Lambda}_i^{(j-1)}\right) \quad (11)$$

$$= \widehat{F}^{(j-1)}\widehat{\Lambda}_i^{(j-1)} + A_i'(A_i A_i')^{-1}\left(X_i^{\text{obs}} - A_i \widehat{F}^{(j-1)}\widehat{\Lambda}_i^{(j-1)}\right), \quad (12)$$

[4] See the different transformations in Stock and Watson (2002a), pp. 156-157. The I(0)-flow case is provided in Chow and Lin (1971), p. 373.

where $\widehat{F}^{(j-1)}$ is the $T \times r$ matrix of the factor estimates from the previous iteration, and $\widehat{\Lambda}_i^{(j-1)}$ is the $r \times 1$ vector of loadings for the i-th variable. The E-step estimate of the monthly data $\widehat{X}_i^{(j)}$ consists mainly of two components: the common component from the previous iteration, and the estimated idiosyncratic components $X_i^{\text{obs}} - A_i \widehat{F}^{(j-1)} \widehat{\Lambda}_i^{(j-1)}$ distributed with the matrix $A_i'(A_i A_i')^{-1}$.
If $\widehat{X}_i^{(j)}$ is quarterly GDP for example, the idiosyncratic component is a vector of quarterly values distributed among the corresponding months. Accordingly, the E-Step in (12) is repeated for all series in the sample that contain missing values or have to be transformed from quarterly to monthly frequency. For all monthly series without missing values, $A_i = I$ holds and (12) becomes simply $\widehat{X}_i = \widehat{F}\widehat{\Lambda}_i + \left(X_i^{\text{obs}} - \widehat{F}\widehat{\Lambda}_i\right) = X_i^{\text{obs}}$, which is in line with the single-frequency factor representation. Therefore, for time series with no data irregularities, no EM iterations are necessary.

3. *M-Step:* The estimated monthly observations for quarterly time series, the estimates for the missing observations of the monthly series together with the monthly time series without data irregularities are collected in the $N \times 1$ vector $\widehat{X}_t^{(j)}$. These monthly observations are used to re-estimate the factors $\widehat{F}_t^{(j)}$ and loadings $\widehat{\Lambda}^{(j)}$ by using principal components of the covariance matrix $\widehat{\Gamma}^{(j)} = T^{-1} \sum_{t=1}^{T} \widehat{X}_t^{(j)} \widehat{X}_t^{(j)\prime}$ according to (4). The estimates of the factors and loadings enter again step 2 above until some convergence criterion is reached. In our application below, the algorithm stops if the maximum percentage change of the variables' estimates is smaller than 10^{-4}.

The steps above provide monthly estimates of the quarterly variables as well as estimates for missing data. We denote the final monthly factor estimates as $\widehat{F}_t = \widehat{F}_t^{(J)}$ and final loadings as $\widehat{\Lambda} = \widehat{\Lambda}^{(J)}$ for $t = 1, \ldots, T$, where J is the final iteration of the EM algorithm run after convergence. Similarly, the final estimator of the monthly observations are denoted as $\widehat{X}_t = \widehat{X}_t^{(J)}$ for simplicity. The estimated monthly observations of GDP can be directly taken from the particular element of the estimated data vector $\widehat{X}_t$.

Some properties of the EM algorithm Note that within the sample, the EM algorithm provides monthly estimates that exactly fulfill the restriction (9). Multiplying (12) by A_i yields

$$A_i \widehat{X}_i^{(j)} = A_i \widehat{F}^{(j-1)} \widehat{\Lambda}_i^{(j-1)} + \left(X_i^{\text{obs}} - A_i \widehat{F}^{(j-1)} \widehat{\Lambda}_i^{(j-1)}\right) = X_i^{\text{obs}}. \qquad (13)$$

Hence, for all iterations $j = 1, \ldots, J$, the equivalence between the quarterly time series and the monthly estimates transformed to quarterly frequency is ensured. The restriction that the quarterly value of the estimated monthly series is equal to the observed quarterly value is an attractive feature of the EM algorithm discussed here. The EM algorithm shares this property with interpolation methods such as the widely used method proposed by Chow and Lin (1971).[5]

Note that the EM algorithm above can be used in different cases of data availability: it can be used for interpolation of the monthly in-sample values corresponding to the quarterly observations of GDP or other quarterly time series. Moreover, it can be used for extrapolation of monthly GDP in case monthly observations for indicators are available within a quarter for which the GDP observation is not available yet. Assume that we have T^{q} quarterly time series observations for GDP, but $(T^{\mathrm{q}} \times 3) + T^{\mathrm{miss}}$ observations for monthly indicators, in particular T^{miss} monthly observations at the end of the sample with no observations of GDP for the corresponding quarters. For example, if T^{miss} is equal to two, then we have observations of two months for monthly indicators available but no GDP observation for the corresponding quarter. This is a typical case in a real-time environment with special implications for the EM algorithm and the estimates of monthly GDP. In this case, the transformation matrix can be partitioned into

$$A_i = \begin{pmatrix} A_{i,1} & 0_{T^{\mathrm{q}} \times T^{\mathrm{miss}}} \end{pmatrix}, \tag{14}$$

where the partial matrix $A_{i,1}$ again depends on the stock-flow nature of the time series as in (6) and (10), and $0_{T^{\mathrm{q}} \times T^{\mathrm{miss}}}$ represents the missing observations of GDP at the end of the sample. This partitioned matrix implies for the distribution matrix $A_i'(A_i A_i')^{-1}$ in (12) that

$$A_i'(A_i A_i')^{-1} = \begin{pmatrix} A_{i,1}' \left(A_{i,1} A_{i,1}' \right)^{-1} \\ 0_{T^{\mathrm{miss}} \times T^{\mathrm{q}}} \end{pmatrix}. \tag{15}$$

Plugging this into (12) and partition $\widehat{X}_i^{(j)}$ in accordance with A_i above provides

$$\begin{pmatrix} \widehat{X}_i^{\mathrm{qava},(j)} \\ \widehat{X}_i^{\mathrm{qmiss},(j)} \end{pmatrix} = \widehat{F}^{(j-1)} \widehat{\Lambda}_i^{(j-1)} + \begin{pmatrix} A_{i,1}' \left(A_{i,1} A_{i,1}' \right)^{-1} \\ 0_{T^{\mathrm{miss}} \times T^{\mathrm{q}}} \end{pmatrix} \left(X_i^{\mathrm{obs}} - A_i \widehat{F}^{(j-1)} \widehat{\Lambda}_i^{(j-1)} \right), \tag{16}$$

where $\widehat{X}_i^{\mathrm{qava},(j)}$ contains the monthly estimated observations of variable i for which corresponding quarterly observations are available, and $\widehat{X}_i^{\mathrm{qmiss},(j)}$ with dimension $(T^{\mathrm{miss}} \times 1)$ contains the estimates for monthly periods where the corresponding quarterly observation is missing. The reformulated equation shows that in case no quarterly values are available at the end of the sample, the in-sample quarterly idiosyncratic components are not distributed among the final T^{miss} monthly observations, because the entries of the final T^{miss}

[5]See Chow and Lin (1971), p. 374, equation 17. For a comparison of methods, see Angelini et al. (2006).

rows of the distribution matrix $A_i'(A_iA_i')^{-1}$ are equal to zero. This simply reflects the fact that there is no quarterly idiosyncratic component at the end of the monthly sample. This result has also consequences for the estimates of $\widehat{X}_i^{(j)}$: the final T^{miss} entries $\widehat{X}_i^{\text{qmiss},(j)}$ are equal to corresponding entries the common component $\widehat{F}^{(j-1)}\widehat{\Lambda}_i^{(j-1)}$ only, as the quarterly idiosyncratic components do not affect these observations. Furthermore, this implies that the monthly idiosyncratic component of $\widehat{X}_i^{\text{qmiss},(j)}$ for the final T^{miss} observations is equal to zero. This result holds for all iterations $j = 1, \ldots, J$ and has implications for the forecasting methods to be discussed below.

2.3 Forecasting monthly GDP using the monthly factors

The EM algorithm as discussed above provides estimates of monthly observations for GDP. In more detail, it provides monthly observations for those quarters, where quarterly figures of GDP are available. Moreover, it can estimate monthly observations of GDP, when monthly indicators are available within a quarter without a corresponding GDP release available. In this sense, the EM algorithm already provides a forecast, although it might also be labelled as ‘extrapolation’ following the literature on time series interpolation such as Chow and Lin (1971). However, in many empirical macroeconomic datasets, monthly indicators are available in advance of GDP only up to a few months. If longer forecast horizons are required, we have to apply further forecasting techniques. In particular, we are interested in the conditional forecast of monthly GDP growth in month $T+h$, $\Delta y^m_{T+h|T}$, where T denotes the latest observation of monthly indicators available. As we cannot compare the monthly observations with data, the forecasts will be aggregated to quarterly figures by using (9) and then compared with the quarterly GDP releases. As highlighted by Boivin and Ng (2005), many different ways to forecast with approximate factor models can be found in literature, depending on the exact forecast design. Typically, factor forecasts are plug-in forecasts, where the factors estimated in a first step enter various types of dynamic forecast equations in the second step. Boivin and Ng (2005) discuss the differences of the methods for the single-frequency model in a unifying framework. These methods have to be modified, if quarterly and monthly data has to be used to forecast monthly data.

To motivate the different ways of forecasting with factor models, a discussion of the theoretical factor model (1) and (3) is useful. According to (1), forecasts $x_{i,T+h|T}$ can be written as

$$x_{i,T+h|T} = \Lambda_i' F_{T+h|T} + e_{i,T+h|T}, \tag{17}$$

Following (17), separate predictions of the factors and the idiosyncratic components together with information on the loadings Λ_i' can provide a forecast. The dynamic model of the factors (3) can help to express the forecast in terms of in-sample observations of the factors according to the Wiener-Kolmogorov prediction formula

$$F_{T+h|T} = \left[(I - A(L)L)^{-1}/L^h\right]_+ (I - A(L)L)F_T, \tag{18}$$

where $[\cdot]_+$ denotes the annihilation operator that sets negative powers of L to zero.[6] An obvious forecasting method is now to compute the factor forecast (18) in a first step and then plug it into (17) to obtain the final forecast $x_{i,T+h|T}$ in a second step. This provides an indirect factor-based forecast.

Regarding the forecast of the idiosyncratic component, one could also think of specifying a typically univariate model and plugging a resulting forecast into (17) as in Boivin and Ng (2005) for the single-frequency case. However, in the context of mixed-frequency data, we decided to neglect the idiosyncratic component. Note that for the case for GDP estimation with the EM algorithm, idiosyncratic components are available only up to month $T - T^{\text{miss}}$, as discussed below equation (16). As in many empirical datasets, including ours below, at least some of the monthly time series indicators are available more timely than GDP, $T - T^{\text{miss}} \geq 1$ holds, and we always have at least one month, where the idiosyncratic component is equal to zero as discussed in equation (16). Furthermore, as the factor model concept refers to the factors as main driving forces of the variables in the model, and the EM algorithm had to be modified for special types of serial correlations of the idiosyncratic components, we will for simplicity neglect the idiosyncratic component for the forecasting purposes in this paper.

Direct, indirect and unrestricted factor forecasts of GDP In empirical applications, there are different ways to specify the factor forecasts (18). As a first alternative, Boivin and Ng (2005) suggest to estimate a VAR according to (3) providing the lag polynomial $\widehat{A}(L)$. Given this polynomial, iterative multi-step forecasts (IMS), named following Chevillon and Hendry (2005), can be derived as

$$\widehat{F}^{\text{IMS}}_{T+h|T} = \left[(I - \widehat{A}(L)L)^{-1}/L^h\right]_+ (I - \widehat{A}(L)L)\widehat{F}_T, \tag{19}$$

again applying the Wiener-Kolmogorov prediction formula as in the theoretical case (18).[7] A more direct way to project the factors specifies a regression of $t + h$-dated variables to be predicted on t-dated regressors. The resulting forecasts are called direct multi-step (DMS) forecasts, again following Chevillon and Hendry (2005). In this case, the forecasts

[6] See Sargent (1987), p. 328.

[7] Another analytically equal way is to compute sequential one step-ahead forecasts that might be easier to implement computationally. See Lütkepohl (1993), p. 31, Boivin and Bg (2005), p. 125.

are

$$\widehat{F}^{\text{DMS}}_{T+h|T} = \widehat{\Pi}(L)\widehat{F}_T. \tag{20}$$

Regarding estimation, the lag orders of $\widehat{A}(L)$ from IMS and $\widehat{\Pi}(L)$ from DMS may differ in applications. Furthermore, whereas $\widehat{A}(L)$ is constant for different h, $\widehat{\Pi}(L)$ has to be estimated for different h. To specify the lag orders in the forecast models in our empirical application below, we employ the Bayesian information criterion. Irrespective of whether IMS or DMS is chosen, given the forecasts of the factors and the loading estimate $\widehat{\Lambda}'_{\Delta y}$, the forecast for GDP becomes simply

$$\Delta\widehat{y}^m_{T+h|T} = \widehat{\Lambda}'_{\Delta y}\widehat{F}_{T+h|T}. \tag{21}$$

Note that the differences between IMS and DMS arise only in empirical applications, where misspecifications might matter. For known lag polynomials $A(L)$ from (18) and given factors as in our theoretical setup, DMS and IMS forecasting are equivalent.[8] For empirical applications, an argument in favour of the DMS approach is that the potential impact of specification errors in the one-step ahead model and using the IMS approach can be reduced by using the same horizon for estimation as for forecasting. However, Monte Carlo simulation results by Marcellino et al. (2005) indicate that the DMS approach is not generally the best method to choose, although their comparison does not include factor forecasts. On the other hand, in the recent study by Boivin and Ng (2005), the DMS approach performs best (overall) in the forecast comparison of factor models. Moreover, Chevillon and Hendry (2005) favour the direct approach due to its robustness in case of misspecification. The evidence from those papers highlights the circumstances under which IMS or DMS forecasting might be preferable. However, in empirical applications, it is not clear whether the underlying conditions hold or not and, therefore, we consider both approaches in our application below.

A forecast that directly relates GDP to the estimated factors is the so-called unrestricted (U) forecast, as labelled by Boivin and Ng (2005) and introduced in the factor literature by Stock and Watson (1999). In the context of monthly GDP forecasting, we have

$$\Delta\widehat{y}^m_{T+h|T} = \widehat{\Xi}(L)\widehat{F}_T, \tag{22}$$

which is a function of the factors $\widehat{F}_T$ and their lags only. The estimate of the lag polynomial $\widehat{\Xi}(L)$ is provided by regressing $\Delta\widehat{y}^m_t$ onto $\widehat{F}_{t-h}$. Following (22), the unrestricted forecast is also a DMS forecast, as the left-hand-side projection is $t+h$-dated, whereas factors

[8] See the discussion in Clements and Hendry (1998), pp. 243-246 and Boivin and Ng (2005).

are t-dated. Compared with the DMS forecast, the in-sample estimate of loading matrix $\widehat{\Lambda}'_{\Delta y}$ is ignored. Furthermore, inserting (20) into (21) shows that the DMS which is based on the polynomial $\widehat{\Lambda}'_{\Delta y}\widehat{\Pi}(L)$, which combines the separately estimated loadings and lag polynomial of the factors. Hence, the U forecast is less restricted than the DMS forecast.

Summary of factor forecasts for monthly GDP Overall, we have three different forecasts for monthly GDP as summarised below in table 1. Note that the different

Table 1: Factor forecasts for monthly GDP

F-IMS:	$\Delta\widehat{y}^m_{T+h\|T} = \widehat{\Lambda}'_{\Delta y}\left[(I - \widehat{A}(L)L)^{-1}/L^h\right]_+ (I - \widehat{A}(L)L)\widehat{F}_T$
F-DMS:	$\Delta\widehat{y}^m_{T+h\|T} = \widehat{\Lambda}'_{\Delta y}\widehat{\Pi}(L)\widehat{F}_T$
F-U:	$\Delta\widehat{y}^m_{T+h\|T} = \widehat{\Xi}(L)\widehat{F}_T$

Note: F-IMS relies on the iterative multi-step (IMS) forecasts of the factors (19), that are used together with the in-sample loadings estimate for the GDP forecast (21). F-DMS is the factor forecast using direct multi-step (DMS) forecasts of factors from (20). F-U refers to unrestricted forecasts (22) using factors only.

approaches IMS, DMS and U are theoretically the same if the assumptions behind the theoretical factor model (1) and (3) were correct, as has been discussed by Boivin and Ng (2005). In real-world applications however, when the true factors and dynamics behind the model are unknown, it makes a difference whether IMS, DMS or U are used for forecasting as the estimators might have different small-sample properties. In some cases, the U forecasts work well in practice, see again Boivin and Ng (2005). Therefore, we follow the factor forecasting literature and try the different factor forecast methods above in the present application.

3 German real-time data

Having discussed different forecasting alternatives for monthly GDP, we now provide an empirical application using German real-time data.

Composition of time series Real-time data is characterised by two features: Firstly, due to revisions the actually available data in a particular month may differ substantially from the final values released later by a statistical office. Secondly, the dataset may suffer from the ragged-edge problem in econometrics, namely missing values for some of the variables at the end of the sample due to publication lags. To assess the importance of these issues in our context, we use a novel composite real-time dataset, which is an extension of the real-time dataset constructed by Gerberding et al. (2005a, 2005b). Our dataset includes quarterly demand components of GDP, industrial production and subcomponents as well as incoming orders by sector. All these series are subject to data revisions. In addition to these series, a variety of financial indicators, for example interest rates and spreads, stock price indices and exchange rates were added, which are taken as final values. Furthermore, survey results from the German *ifo institute* on business confidence and expectations were added, again assuming that the observations are not subject to revisions. Overall, the dataset consists of 52 time series, 39 monthly series and 13 quarterly series. Of the monthly series, 13 are subject to revisions, 26 are not. Compared with other applications, the dataset is somewhat smaller than the data sets of other applications using large factor models, but considerably larger than those used in applications of state-space or bridge-equation models.[9] To check whether the size of the dataset is conceptually appropriate for estimating monthly GDP using the EM algorithm discussed above, we have carried out some Monte Carlo simulations. The results show that the sample size of the dataset has only a minor effect on estimation, if the included data is informative enough for estimating the factors. This is in line with results obtained by Boivin and Ng (2006) for single-frequency data, where also relatively small datasets were found to be sufficient for forecast applications.[10] Details on these simulations can be found in appendix A.3.

The real-time data for the empirical application was taken from the Bundesbank Monthly Bulletin Supplement, Seasonal Adjusted Data. Hence, the releases of the dataset are the same as the releases of the Bulletin Supplement. The Bulletin is published by the mid of the month, for example, the Supplement April 2005 is released in the middle of April 2005. This Bulletin issue contains GDP data up to the fourth quarter 2004, and, hence, has a release delay of three months compared with the calendar. Production indices and incoming orders are available up to February 2005. CPI inflation is available until March 2005. The financial and survey time series are not taken from the Monthly Bulletin Supplement, and we assume that it is available up to March 2005. From one release to another, the availability of the data changes, but we can broadly distinguish three groups of data with respect to their timeliness: The GDP time series and the demand components have the lowest degree of timeliness and are not available for estimation up to a time lag of four months, followed by a group consisting of production and incoming

[9]In their large factor model for the USA, Bernanke and Boivin (2003), pp. 529-531, use about 80 time series, whereas Nunes (2005) estimates a six-variable state-space model for monthly GDP.

[10]See also the theoretical discussion in Heaton and Solo (2006), pp. 12-14.

orders, and the group of financial and survey data that have the highest degree of timeliness. The different release dates, which imply missing values at the end of the sample, are taken into account automatically by the EM algorithm as described in section 2. The releases used range from $1998M7$ to $2005M06$, providing 84 vintages that can be used for in-sample estimation and forecast comparison. More information about the time series can be found in the data appendix A.1.

Real-time data without timely monthly indicators In order to investigate the importance of the monthly observations that are available more timely compared with GDP and its demand components, another dataset is constructed, where observations of monthly data are available only up to the last month of the quarter for which GDP figures are available.[11] The quarterly time series vintages are the same as in the full real-time dataset. Hence, compared with the real-time dataset, timely information from the monthly data is missing, and we will investigate below, how the timeliness of the data affects the forecasting accuracy of monthly GDP.

Final vintage data To determine the importance of the real-time nature of the dataset, the forecast simulations were repeated using the final vintage dataset. This dataset consists of the same time series as the real-time dataset, except that data revisions known as of the final vintage of the dataset are incorporated. Note that, in this database, the timing and release conventions are consistent with the real-time database as in Bernanke and Boivin (2003, p. 530). Delays in publication are incorporated as in the real-time dataset, so the three-block structure of the real-time dataset is replicated. Therefore, the two datasets differ only with respect to revisions in the data. Overall, the number of time series subject to revisions is quite low in our dataset, although GDP is the key variable subject to revisions. Hence, their role in forecasting might be limited if their information content for the factors is low. However, this is an ultimately empirical question to be discussed below.

4 Real-time forecasting of German GDP

Below, we first show some in-sample results of estimating monthly GDP for Germany. Out-of-sample forecasting results follow.

Forecast simulation design The forecast simulations are carried out in a recursive way. With every new vintage, more time series information becomes available. The factor model is reestimated with the extended dataset, and forecasts are computed. Regarding

[11] For a similar construction of a dataset, see Baffigi et al. (2004).

the forecast horizon, monthly GDP estimates are provided for the current quarter and for the next quarter. Hence, we obtain GDP nowcasts and one-period-ahead forecasts. Due to the publication lags of GDP, however, the effective forecast horizon needed for estimation has to include an additional quarter. For example, for the data release October 2004 (2004M10), GDP is available only for the second quarter of 2004. For a forecast up to the first quarter 2005, we need a three-quarter-ahead forecast from the end of the GDP sample. As we work with monthly estimates of GDP, this corresponds to a maximum of nine-month forecast horizon for our model in order to compute the nowcast and the one-quarter-ahead forecast. Hence, from the 84 monthly data vintages, we can use $84 - 9 + 1 = 76$ vintages for forecast comparison using the maximum horizon of nine months.

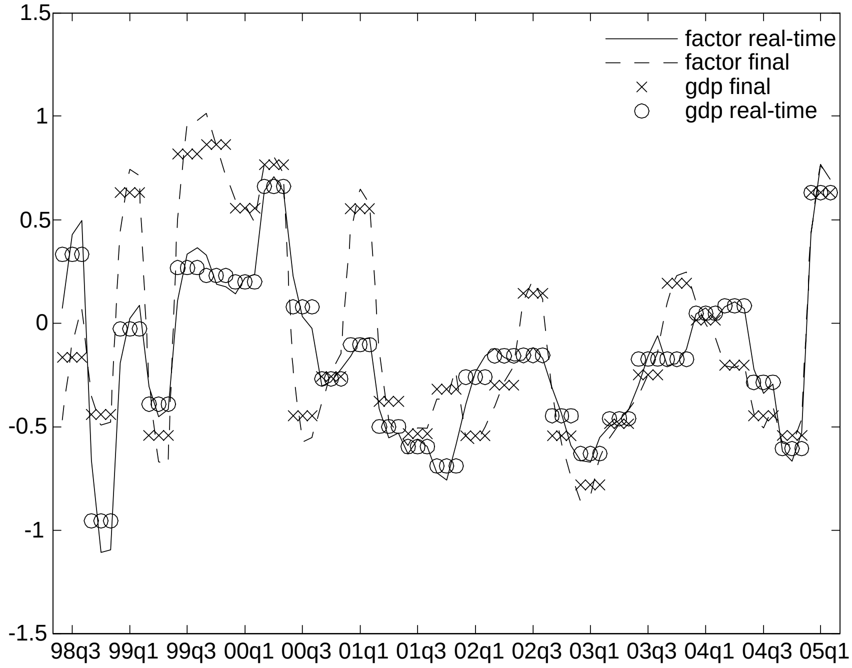
For forecasting using the factor model, the number of factors as well as the lag orders of the AR and VAR models described in table 1 have to be specified. Regarding the number of factors, there is considerable uncertainty about the appropriate way to choose in the empirical literature, since information criteria seem to give misleading results in some cases.[12] For example, Bernanke and Boivin (2003) use three factors for their real-time applications for the USA, whereas the Federal Reserve Bank of Chicago publishes a composite index based on a similar method, where only one factor of monthly data is chosen.[13] In our application, for both in-sample and forecasting results below, the number of factors was set equal to one. We also experimented with a larger numbers of factors, but the forecast performance was generally worse than in the one-factor model. Details can be found in the appendix A.2. The lag order of the AR and VAR models of the forecasting models was determined using the BIC. This fully specifies the forecast model at every forecast recursion.

In-sample estimation results To give an impression about the in-sample properties of the EM algorithm, below monthly GDP estimates are shown based on final data vintages and real-time data. The real-time estimates of monthly GDP are estimated immediately after a new release of GDP is available, so the results are first monthly estimates of GDP. Figure 1 shows the real-time and final monthly estimates of German GDP as well as the corresponding final and real-time quarterly data. Note that the figure contains results for the quarter-on-quarter growth rate, which was equally distributed among the respective months of the quarter. The monthly estimates from the factor model are, however, month-on-month growth rates Δy_t^m as shown in equation (9). To make these comparable to the quarterly GDP figures, they were transformed to monthly quarter-on-quarter growth rates $\Delta^q y_t^m = y_t^m - y_{t-3}^m$. Note that equation (9) is equivalent to the

[12] See, for example, Bernanke and Boivin (2003), footnote 7.

[13] See Evans et al. (2001) and Federal Reserve Bank of Chicago (2001)

Figure 1: Monthly GDP growth estimates, in-sample



Note: The figure shows quarter-on-quarter growth rates of final and real-time GDP data. The factor estimates are month-on-same-month-in-the-previous-quarter estimates. The GDP data is demeaned as is the data used to estimate the factors.

relationship $\Delta^q y_t^q = (1/3)(\Delta^q y_t^m + \Delta^q y_{t-1}^m + \Delta^q y_{t-2}^m)$. This implies that the quarterly average of the quarterly growth rates of the monthly estimates in figure 1 should be approximately equal to the observed quarterly growth rate of GDP. The in-sample results show that this is the case using the EM algorithm both for real-time data and final vintage data. Comparing real-time and final-vintage data results shows that in the first half of the sample, observations of GDP have been subject to notable revisions. Since the EM algorithm ensures that monthly and quarterly GDP are equal on average, revisions in quarterly GDP lead directly to deviations of the real-time monthly estimates from the monthly GDP estimates based on final data. This might overstate the role of data revisions in-sample, whereas out-of-sample, where GDP observations are not available, their role might be different, when the dynamics are more dependent on the more timely monthly data. In the second half of the sample, the revisions are small and monthly estimates deviate to a lesser extent, too.

Forecasting results In table 2, the out-of-sample forecasting results for German GDP are shown in terms of mean-squared forecast errors (MSE), where the forecasts are compared with GDP figures from the final release available in the dataset. Note that the forecast horizon is quarterly, because only the official quarterly figures released by the statistical office can be compared with the forecasts. The forecasts for horizon 1 can be

Table 2: Out-of-sample forecast results, MSE

row	estimation method	forecasting equation	dataset	forecast horizon 1	forecast horizon 2
1	EM factor	IMS	real-time	0.1867	0.2184
2	EM factor	DMS	real-time	0.1896	0.2269
3	EM factor	U	real-time	0.2135	0.3072
4	EM factor	IMS	real-time, no timely data	0.2148	0.2481
5	EM factor	IMS	final data	0.1857	0.2274
7	AR	IMS	real-time	0.2965	0.3120
8	AR	DMS	real-time	0.2572	0.3169
9	no change	—	real-time	0.3202	0.3683

Note: Numbers in the table show mean-squared forecast errors with respect to quarterly GDP. In the column 'forecasting equation', IMS denotes iterative multi-step forecasting, and DMS denotes direct forecasting. The AR model is an autoregressive model estimated with quarterly GDP data. The no change forecast is the forecast which is equal to the last observation available for GDP.

interpreted as nowcasts, as they are computed within current calendar quarters, whereas horizon 2 can be interpreted as a true one-quarter-ahead forecast. The monthly forecasts are obtained by using the forecast equations from table 1 and then transformed to quarterly numbers as described in the methodological section 2. In addition to the factor forecasts, also simple benchmark models are included in the forecast evaluation: 1) an autoregressive (AR) model which is estimated and forecast in real-time using iterative multi-step (IMS) forecasting, 2) an AR model using direct multi-step (DMS) forecasting, and 3) a no-change forecast which is equal to the last real-time observation of GDP. For comparison with the MSEs in the table, the variance of final GDP is 0.26. MSEs larger than this variance indicate uninformative forecasts. The forecasting results can be summarised as follows: Comparing direct versus iterative multi-step forecasting (rows 1 and 2) reveals advantages of the iterative approach, particularly at horizon two. The unrestricted forecast (row 3) performs worst. Removing timely monthly observations from the real-time sample leads to worse forecasting results (rows 1 and 4). Therefore, it seems to be beneficial to use the most timely information for forecasting monthly GDP. Data revisions have no clear impact on the forecasting accuracy (rows 1 and 5), as the use of final data leads to a lower MSE only for the nowcast, but not for the one-quarter-ahead forecast compared with the real-time forecasts.[14] However, the differences between the MSE results using these alternative datasets are only small. Hence, these results are in line with the conclusions of Bernanke and Boivin (2003), for example, where revisions had only a small impact on forecasting USA macroeconomic time series. Finally, both the AR model forecasts and the no-change forecasts are clearly outperformed, since all three benchmark forecasts have largest MSEs in the comparison (rows 7 to 9).

Forecast performance in subsamples Typically, a presentation of MSEs alone gives no information about the performance of the models' forecast performance over time, but decision-makers might also request models with a relatively stable forecast accuracy. In figure 1, we have seen different in-sample estimation results for monthly GDP as well as differences with respect to the size of revisions over time. Since this might also affect the forecast performance, we report the forecast results for the two consecutive subsamples of length three years at the end of the sample. The MSEs for these samples can be found in table 3. In the first subsample, the forecasts are hardly informative for future GDP. In the second subsample, however, the factor forecasts are informative. In both subsamples, the naive forecasts are outperformed. Using the final data improves the nowcast in the first subsample compared with using real-time data, whereas it is slightly worse in the second sample for both the now- and forecast. As the revisions of GDP releases are more relevant in the first subsample, see again figure 1, this seems to affect the forecasting accuracy whereas the effects of the minor revisions are small in the second sample.

Relative forecast performance over time In addition to the subsample results above, we now investigate the relative performance of the models for every observation. For this purpose, we present the forecasts, GDP growth, as well as squared forecast errors for every release in figure 2. A first impression of the forecast comparison is that the forecasts of the factor models are quite similar for most of the observations. Hence, in line with the MSE results, there are no big differences between factor estimates using real-time or final data or when removing the most timely monthly observations. The squared forecast errors shown in the second and fourth part of the figure decrease from the beginning to the end of the sample, again similar to the subsample forecast errors discussed in the previous section. Furthermore, in the first part of the sample, the simple AR benchmark model is more clearly outperformed by the factor models than in the second part, although the forecast performance of all models is quite poor in the first part of the sample. The same picture emerges from figure 3, where the squared forecast errors of the AR model divided by the squared forecast errors of the factor models are shown. A

[14] In Bernanke and Boivin (2003), table 1, p. 532, it is also often the case that forecasting using final data often does not lead to a reduction in MSE compared with forecasting using real-time data.

Table 3: Out-of-sample forecast results, MSE in subsamples

A. subsample 1999Q2 until 2002Q1

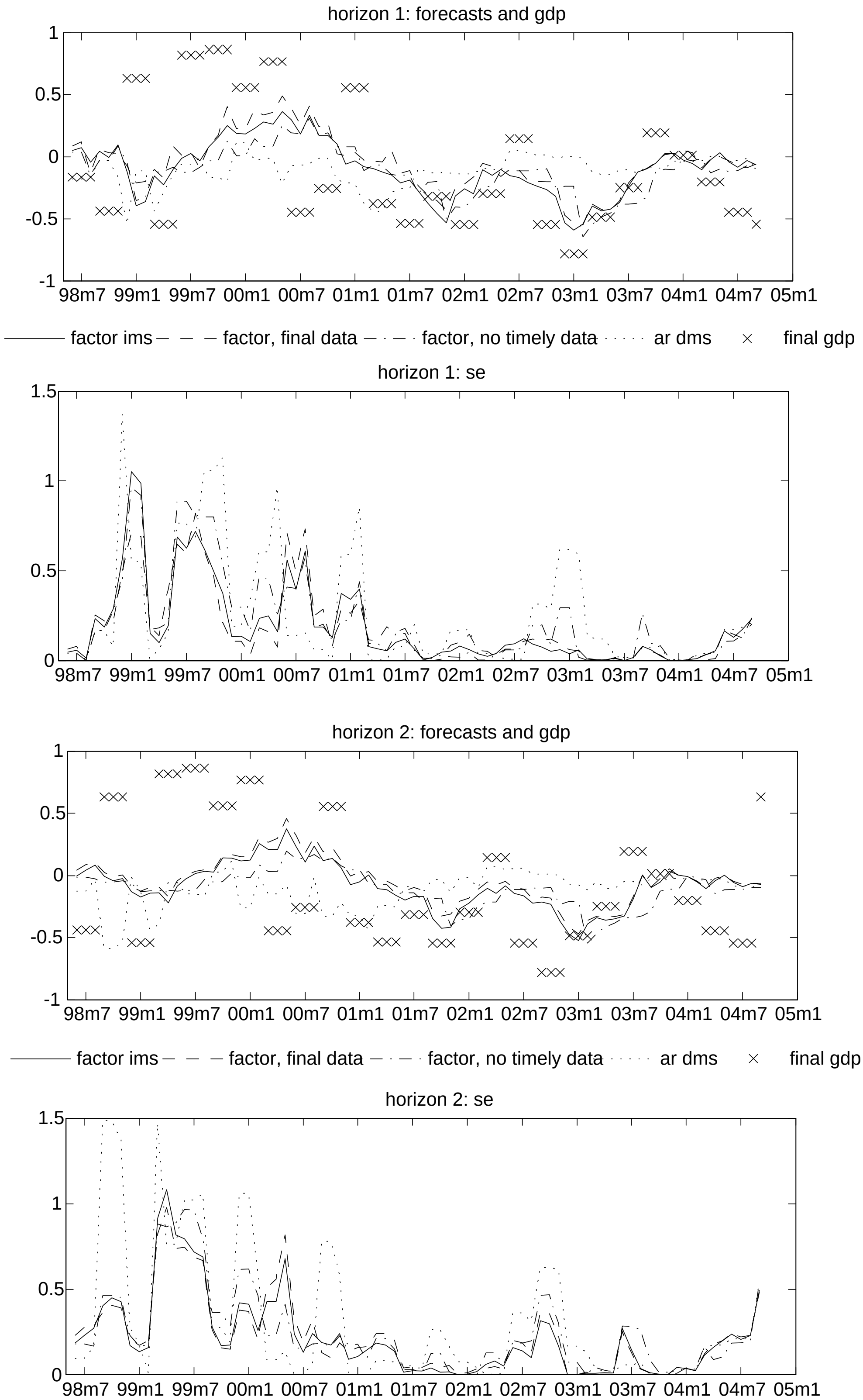
row	estimation method	forecasting equation	dataset	forecast horizon 1	forecast horizon 2
1	EM factor	IMS	real-time	0.3286	0.3553
2	EM factor	DMS	real-time	0.3331	0.3786
3	EM factor	U	real-time	0.3720	0.3786
4	EM factor	IMS	real-time, no timely data	0.3704	0.3771
5	EM factor	IMS	final data	0.3167	0.3627
7	AR	IMS	real-time	0.4967	0.5230
8	AR	DMS	real-time	0.3921	0.4877
9	no change	—	real-time	0.5348	0.5529

B. subsample 2002Q2 until 2005Q1

row	estimation method	forecasting equation	dataset	forecast horizon 1	forecast horizon 2
1	EM factor	IMS	real-time	0.0574	0.1029
2	EM factor	DMS	real-time	0.0599	0.0981
3	EM factor	U	real-time	0.0729	0.1581
4	EM factor	IMS	real-time, no timely data	0.0726	0.1408
5	EM factor	IMS	final data	0.0632	0.1117
7	AR	IMS	real-time	0.1143	0.1284
8	AR	DMS	real-time	0.1398	0.1698
9	no change	—	real-time	0.1299	0.2210

Note: The variance of GDP in the subsample of panel A is 0.340, wheras it is 0.149 in panel B. For the model descriptions and abbreviations, see table 2.

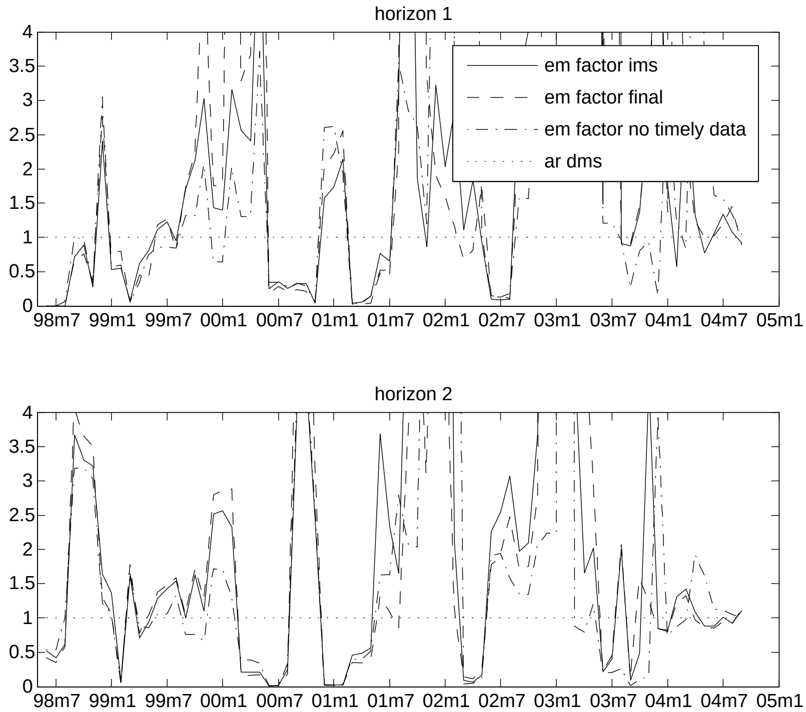
Figure 2: Forecasts and GDP, squared forecast errors over time



Note: The first and third figures show GDP growth rates and forecasts for the nowcasts and forecasts, respectively. The second and fourth figures show squared forecast errors.

value of the relative error below one indicates an inferior forecast performance during the respective period compared with the AR model. Again, the factor forecasts are similar. The figure also shows that there are some periods in time, where the EM factor forecasts perform worse than the AR model, as the relative squared errors are smaller than one. Hence, although the MSE of the factor models is substantially smaller according to table 2, there seems to be no insurance against temporary relative forecast failure of the factor models in some periods of time. This indicates that there is room for further improvements of the factor models used here. Overall, the only moderate forecast performance of all the models, particularly in the first part of the sample, indicates that forecasting German GDP in real-time is a difficult task. This is a finding in line with other work on the recent decline of forecast accuracy. D'Agostino et al. (2006), for example, find that only very few forecasts were informative for USA macroeconomic time series in the 90s, and only at short forecast horizons.

Figure 3: Relative squared errors of AR model to factor models



Note: The figure shows squared forecast errors of the AR model divided by the squared forecast errors of the factor models. A value of the relative error below one indicates an inferior forecast performance of the factor models.

Results for different compositions of the dataset Against the background of the forecast results obtained so far, we investigate now whether alternative compositions of the dataset might lead to improvements in forecast accuracy. Analysing different groups of data helps to identify the importance of indicator groups, in particular, which group of monthly and quarterly indicators are important for forecasting. Boivin and Ng (2006) find that the accuracy of the factor estimation can be affected by the information content of the data with respect to the factors.[15] Although no widely accepted formal procedure exists so far to identify relevant variables, a simple comparison of differently composed datasets might indicate directions for improving the forecast performance of the baseline model we have discussed above. We follow Forni et al. (2003) and Boivin and Ng (2006) and remove certain groups of data from the whole dataset, and compare the forecast results with the baseline model results. We distinguish four groupings of the data:

- A dataset without quarterly time series apart from GDP,
- A dataset without industry data,
- A dataset without the survey time series (business expectations, situation, consumer sentiment),
- A dataset without the financial time series (interest rates, exchange rates, stock market indices).[16]

The first group without quarterly time series other than GDP is motivated by the Monte Carlo simulation results from Angelini et al. (2006). As these time series have to be interpolated, are available only with delay and are subject to revisions in our sample, they might be a source of distortions for the factor estimation. The second group removes the industrial production indices and incoming orders. Among the monthly indicators these series are also subject to revisions and have the largest publication lag. The groups without survey data and financial time series remove the most timely observations from the data. Forecast results for these alternative datasets can be found in table 4. Without quarterly data and industrial data, the forecast accuracy is slightly worse than in the baseline model, as can be observed from panels A and B in the table. Therefore, there seems to be at least some information content in the quarterly data in addition to GDP, although removing them would not harm the forecast performance much. If survey data is removed from the dataset, see panel C, the forecasting accuracy deteriorates much more. For the financial data, the results are mixed. For the nowcast, the financial time series have a negative impact on the forecast accuracy, whereas they contribute to the forecast performance for the one-period-ahead forecast. Overall, the baseline composition of the model seems to be appropriate, as most of the data reductions lead to at least a slight deterioration in forecast performance.

[15] See also Heaton and Solo (2006) and the Monte Carlo results in appendix A.3.
[16] The individual variables can again be found in the data appendix A.1.

Table 4: Out-of-sample forecast results, MSE in subgroups

row	estimation method	forecasting equation	dataset	forecast horizon 1	forecast horizon 2
			0. Baseline model		
1	EM factor	IMS	real-time	0.1867	0.2184
2	EM factor	DMS	real-time	0.1896	0.2269
3	EM factor	U	real-time	0.2135	0.3072
4	EM factor	IMS	real-time, no timely data	0.2148	0.2481
5	EM factor	IMS	final data	0.1857	0.2274
			A. Model without quarterly data		
6	EM factor	IMS	real-time	0.1892	0.2185
7	EM factor	DMS	real-time	0.1922	0.2285
8	EM factor	U	real-time	0.2167	0.3103
9	EM factor	IMS	real-time, no timely data	0.2146	0.2458
10	EM factor	IMS	final data	0.1877	0.2267
			B. Model without industry data		
11	EM factor	IMS	real-time	0.1961	0.2201
12	EM factor	DMS	real-time	0.1997	0.2344
13	EM factor	U	real-time	0.2300	0.3255
14	EM factor	IMS	real-time, no timely data	0.2177	0.2410
15	EM factor	IMS	final data	0.1954	0.2272
			C. Model without survey data		
16	EM factor	IMS	real-time	0.5024	0.3367
17	EM factor	DMS	real-time	0.6686	0.3512
18	EM factor	U	real-time	0.6595	0.3262
19	EM factor	IMS	real-time, no timely data	0.2645	0.3018
20	EM factor	IMS	final data	0.5127	0.3117
			D. Model without financial data		
21	EM factor	IMS	real-time	0.1854	0.2399
22	EM factor	DMS	real-time	0.1809	0.2439
23	EM factor	U	real-time	0.2092	0.3258
24	EM factor	IMS	real-time, no timely data	0.2354	0.2898
25	EM factor	IMS	final data	0.1744	0.2297

Note: For the model descriptions and abbreviations, see table 2.

5 Conclusions

This paper discusses a factor model for estimating monthly GDP using mixed-frequency data. The EM algorithm is applied to cope with the problem of factor estimation with unbalanced real-time data. The EM algorithm provides monthly common components that represent estimates of the monthly observations for the time series data, in particular GDP. The empirical application to German GDP employs a novel, medium-sized real-time dataset. Monte Carlo simulations as well as forecast exercises, where some of the time series are removed from the full dataset, suggest that the size of the dataset is appropriate in the present context. The empirical forecasting results for German GDP show that data revisions have only a minor impact on the forecasting results. A much stronger effect is achieved by using monthly data that is available in advance of GDP. Including timely monthly observations in the forecasting model leads to substantial improvements in the forecast performance. Among the monthly indicators, the survey data has the biggest impact on the forecasting accuracy. Compared with simple benchmark models, the real-time factor forecasts lead to substantially smaller MSEs. However, a comparison of forecast errors over time reveals that there may be some time periods with a limited forecast performance relative to the benchmark models. This leaves room for improvements of the methods used here.

Since this is one of the first applications to this real-time dataset, a possible direction for future work could address a comparison with other methods for nowcasting and interpolating GDP at monthly intervals. Such an evaluation could provide an intuition for the relative performance of the method chosen here and, moreover, some sort of model averaging among competing short-term monthly GDP forecasting models may be attractive due to a higher degree of robustness against outliers.[17]

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A Appendix

A.1 Monthly and quarterly dataset

The composite real-time dataset in this paper is an extension of the real-time dataset constructed by Gerberding et al. (2005a, 2005b), from which mainly CPI inflation and GDP is suitable for our analysis. Overall, the dataset consists of 52 time series, 39 monthly series and 13 quarterly series described below. The time series are all taken from past releases of the Bundesbank Monthly Bulletin Supplement, Seasonal Adjusted Data, which is published mid month. The releases used here range from $1998M7$ to $2005M06$, providing 84 datasets that can be used for in-sample estimation and forecasting GDP.

Monthly time series The monthly series contain industrial production, incoming orders, and CPI inflation which are subject to revisions. In addition to these series, a variety of financial indicators (interest rates and spreads, stock price indices and exchange rates) as well as survey results were added, which were assumed to be known immediately and not to be revised.

- industrial production total manufacturing
- industrial production total excluding construction
- industrial production intermediate goods
- industrial production capital goods
- industrial production energy
- industrial production durable consumer goods
- industrial production non-durable consumer goods
- industrial production construction
- new orders received from the domestic economy: total
- new orders received from abroad: total
- new orders received from the domestic economy: capital goods
- new orders received from abroad: capital goods
- consumer price index total
- day-to-day money market rate
- 1 month money market rate
- 3 months money market rate
- government bond yield 1 to 2 years maturity
- government bond yield 9 to 10 years maturity
- yield spread: government bonds 1 to 2 years maturity minus three months rate
- yield spread: government bonds 9 to 10 years maturity minus three months rate
- CDAX share price index
- DAX German share index
- REX German bond index
- exchange rate US dollar/Deutsche Mark

- indicator of the German economy's price competitiveness against 19 industrial countries based on consumer prices
- ifo business situation: producers of capital goods
- ifo business situation: producers of durable consumer goods
- ifo business situation: producers of non-durable consumer goods
- ifo business situation: retail trade
- ifo business situation: wholesale trade
- ifo business expectations for the next six months: producers of capital goods
- ifo business expectations for the next six months: producers of durable consumer goods
- ifo business expectations for the next six months: producers of non-durable consumer goods
- ifo business expectations for the next six months: retail trade
- ifo business expectations for the next six months: wholesale trade
- ifo stocks of finished goods: producers of capital goods
- ifo stocks of finished goods: producers of durable consumer goods
- ifo stocks of finished goods: producers of non-durable consumer goods
- GfK consumer confidence index

Quarterly time series The time series included in the quarterly dataset are described below. The GDP expenditure, gross value added and income series are provided by the Federal German Statistical Office and seasonally adjusted by the Bundesbank as described in the Bundesbank Monthly Bulletin Supplement. All in all, 13 quarterly time series are used. In order to mix the quarterly frequency data with monthly frequency data using the EM algorithm as shown above, the type of time series has to be chosen appropriately. Here, all series are assumed to follow I(1) flow processes as GDP.

- gross domestic product
- private consumption expenditure
- government consumption expenditure
- gross fixed capital formation: machinery & equipment

- gross fixed capital formation: construction
- exports
- imports
- Gross value added: Production sector excl. construction
- Gross value added: Hotels, restaurants and transport
- Gross value added: Financial, real estate renting and business services
- Gross value added: Public and private services
- Gross wages and salaries
- Entrepreneurial and property income

Data transformation Natural logarithms were taken for all time series except interest rates. Stationarity was obtained by appropriately differencing the time series. Seasonal fluctuations in the releases of the Monthly Bulletin were eliminated using Census-X12 in real-time by Bundesbank staff. To eliminate scale effects, the series were standardised to have zero mean and the variance of GDP. Standardisation takes place every release of the data prior to the forecasting exercise.

A.2 Forecasting results for different numbers of factors

As discussed in the main text, the forecasts using the factor model are based on one factor. Below, we present forecasting results using up to four factors. In table 5, the out-of-sample forecasting results for German GDP are shown in terms of mean-squared forecast errors (MSE), where the forecasts are compared with GDP figures from the final release available in the dataset. We report results for IMS, DMS and U forecasting only with real-time data, as the results are the same with final data. As can be seen in the table, the forecast performance decreases, as the number of factors is increased from one to two (rows 5 to 9). If the number of factors is increased to three and four, the MSE increases sharply. In more detail, the MSE increases particularly for the nowcast (forecast horizon equal to one), but not for the one-quarter-ahead forecast and not for the forecast that uses only monthly data up to the quarter for which GDP is available (rows 12 and 16). This indicates that a too large number of factors worsens particularly the in-sample estimates, for which timely monthly observations are used to extrapolate monthly GDP. These results indicate that the EM algorithm fails to provide sensible estimates of monthly GDP is the number of factors is too large. This vulnerability is different to the single-frequency case, where the forecasting accuracy is often affected to a lesser extend

Table 5: Out-of-sample forecast results, MSE, different number of factors

row	estimation method	number factors	forecasting equation	dataset	forecast horizon 1	forecast horizon 2
1	EM factor	1	IMS	real-time	0.1867	0.2184
2	EM factor	1	DMS	real-time	0.1896	0.2269
3	EM factor	1	U	real-time	0.2135	0.3072
4	EM factor	1	IMS	real-time, no timely data	0.2148	0.2481
5	EM factor	2	IMS	real-time	0.2609	0.2784
6	EM factor	2	DMS	real-time	0.3069	0.3785
7	EM factor	2	U	real-time	0.3195	0.4607
8	EM factor	2	IMS	real-time, no timely data	0.2241	0.2585
9	EM factor	3	IMS	real-time	1.0239	0.3002
10	EM factor	3	DMS	real-time	1.6860	0.4377
11	EM factor	3	U	real-time	1.6142	0.3585
12	EM factor	3	IMS	real-time, no timely data	0.2903	0.3004
13	EM factor	4	IMS	real-time	1.1949	0.2935
14	EM factor	4	DMS	real-time	2.1272	0.4932
15	EM factor	4	U	real-time	2.1049	0.4999
16	EM factor	4	IMS	real-time, no timely data	0.2767	0.2827
17	AR	—	IMS	real-time	0.2965	0.3120
18	AR	—	DMS	real-time	0.2572	0.3169
19	no change	—	—	real-time	0.3202	0.3683

Note: Numbers in the table show mean-squared forecast errors with respect to quarterly GDP. In the column 'forecasting equation', IMS denotes iterative multi-step forecasting, and DMS denotes direct forecasting. The AR model is an autoregressive model estimated with quarterly GDP data. The no change forecast is the forecast which is equal to the last observation available for GDP.

by choosing a large number of factors.[18] As the results concerning the number of factors are robust to alternative specifications of the model, we provide results only for one factor in the main text.

A.3 Monte Carlo simulations

Since the mixed-frequency factor model discussed in the main text has not been discussed with respect to estimating monthly GDP in the literature before, we carry out some Monte Carlo simulations prior to an empirical application. Angelini et al. (2006) provide similar Monte Carlo simulations using the EM algorithm. Our results differ with respect to the use of estimating monthly I(1) flow variables as GDP, and the dimensions and composition of the dataset used. The simulation design used here aims at motivating the use of a medium-sized dataset for the empirical application to be discussed later. Since that real-world dataset is comparatively small in relation to other datasets used in the factor forecasting literature with single-frequency data, we discuss the estimation accuracy of the EM algorithm with artificial medium-scale datasets. The composition of the artificial datasets considers groups of monthly and quarterly variables, again motivating the use of empirical data later, where a number of quarterly indicators are used for estimating monthly GDP. Overall, the simulations carried out here are more closely related to Boivin and Ng (2006), where factor estimation in small samples of single-frequency data is investigated.

If the DGP based on the monthly frequency is known, we are able to assess the performance of the EM algorithm. We carried out two kinds of simulations: The first simulation design addresses the estimation of monthly observations from quarterly and monthly data for different variances of idiosyncratic noise. The second simulation setup considers missing values at the end of the sample due to publication lags of official data. In both cases, the EM algorithm provides estimates of factors and common components that fill the missing monthly observations for quarterly data and the missing observations at the end of the sample.

Mixed-frequency estimation The data-generating process for the factor model is in line with Stock and Watson (2002b) and Boivin and Ng (2006), extended for using mixed-frequency data. For simplicity, it is assumed that a single factor F_t drives the variables in the factor model. The common factor follows the AR(1) process

$$F_t = 0.5F_{t-1} + \sqrt{1-\alpha^2}\,\eta_t, \tag{23}$$

[18] See, for example, the forecasting results using a fixed number of factors in Stock and Watson (2002), tables 1-4.

for $t = 1, \ldots, T_m$, where $\eta_t \sim \mathcal{N}(0,1)$. Given the factors. The N_q quarterly time series are generated according to the factor model

$$x_{it}^{qm} = \sqrt{\omega_q} F_t + \sqrt{1-\omega_q}\, \epsilon_{it}^{qm}, \tag{24}$$

and $i = 1, \ldots, N_q$. The shock ϵ_{jt}^{qm} has a normal distribution with mean zero and unit variance. The quarterly time series are initially simulated at the monthly frequency, so $t = 1, \ldots, T_m$. To obtain a quarterly series, we assume that all x_{it}^{q} are I(1) flow variables observed at the last month of each quarter, so $x_{it}^{q} = (1/3)(x_{it}^{qm} + 2x_{it-1}^{qm} + 3x_{it-2}^{qm} + 2x_{it-3}^{qm} + x_{it-4}^{qm})$ for $t = 3, 6, 9, \ldots$. This was assumed to hold for GDP in equation (9). The variance of the variables is given by $\mathrm{var}\,(x_{it}^{qm}) = 1.0$ for the coefficient $0 \leq \omega_q \leq 1$, so ω_q and $(1-\omega_q)$ represent the variance contributions of the common and idiosyncratic components, respectively. The N_m monthly variables x_{it}^{m} are explained by the factor and an idiosyncratic component for $t = 1, \ldots, T_m$ according to

$$x_{jt}^{m} = \sqrt{\omega_m}\, F_t + \sqrt{1-\omega_m}\ \epsilon_{jt}^{m}, \tag{25}$$

where ϵ_{jt}^{m} is mutually and serially independently distributed with $\epsilon_{jt} \sim \mathcal{N}(0,1)$. The parameter ω_m measures the relative importance of the common components for the variance of the monthly variables. The smaller ω_q and ω_m are, the less informative is the dataset with respect to the factors and, therefore, the precision of the monthly estimates deteriorate. This allows us to investigate situations where the monthly or quarterly dataset is noisy in terms of large variance contributions of the idiosyncratic components (when ω_q and ω_m are small). Moreover, we can assess the relevance of the sample size for the estimation results for different N_q, N_m and T_m. In the Monte Carlo experiment, the monthly and quarterly observations are used to estimated the factors $\widehat{F}_t$, and the respective common components provide monthly estimates of quarterly variables. To compare the models based on different datasets, R replications of the DGP are computed. In order to assess the reliability of the factor estimates we follow Boivin and Ng (2006) and compute the statistic

$$S_{F,F_0} = \frac{1}{R} \sum_{r=1}^{R} \frac{tr(F_{0,r}{}' \widehat{F}_r (\widehat{F}_r' \widehat{F}_r)^{-1} \widehat{F}_r' F_{0,r})}{tr(F_{0,r}' F_{0,r})}. \tag{26}$$

A value close to one indicates a correspondence between the estimated factors $\widehat{F} = (\widehat{F}_1, \widehat{F}_2, \ldots, \widehat{F}_T)$ and the true factors $F_0 = (F_1, F_2, \ldots, F_T)$ simulated according to (23). To evaluate the estimation accuracy of the monthly observations for the quarterly data we compute the mean-squared error (MSE) as

$$MSE_q = \frac{1}{R}\sum_{r=1}^{R}\frac{1}{N_q}\sum_{i=1}^{N_q}\frac{1}{T_m}\sum_{t=1}^{T_m}(\widehat{x}_{it,r}^{qm} - x_{it,r}^{qm})^2, \tag{27}$$

where $\widehat{x}_{it,r}^{qm}$ is the estimated monthly observation for the variable i which is available only at quarterly frequencies for the estimation. The monthly estimate can be compared with the true monthly observation $x_{it,r}^{qm}$. The MSE is averaged over time, replications and all quarterly variables. The Monte Carlo simulations are carried out for $R = 500$ replications. The sample size of the monthly series T_m is set to 30, 60, 90 and 120, whereas the quarterly series have the respective sample sizes equal to $T_q = (1/3)T_m$. The number of monthly series N_m and quarterly series N_q are set to 20 and 40. The contribution of the factors to the overall variance of the quarterly variables is set to $\omega_q \in \{0.1, 0.5, 0.9\}$. The monthly series have either a common component variance contribution of $\omega_m = 0.9$ (very informative) or $\omega_m = 0.1$ (almost uninformative). In table 6, the results of the Monte Carlo experiment are shown. The results indicate a strong dependency of the estimation performance on the information content of the data. The less informative the time series are with respect to the factors, the less precisely are the factors and the monthly observations estimated. For example, for the case in row 6 ($T_m = 30$, $N_q = N_m = 20$, $\omega_q = \omega_m = 0.1$), the MSE of the estimated monthly observations is equal to 1.069, which implies that the estimates are almost uninformative, since the original series has variance equal to one by construction. If the number of time series observations are increased to $T_m = 60$, the MSE reduces to 0.649 (see row 12 of table 6). However, further increasing T_m improves the estimation only marginally. Similar results are obtained when the cross-section dimension is increased. However, if the dataset is informative about the factors ($\omega_q = \omega_m = 0.9$), increasing the sample size in both the cross-section and the time series dimension leads to minor improvements only. Thus, if the dataset is informative for estimating the factors, a small number of time series is sufficient to obtain reliable estimates. This is in line with simulation results by Boivin and Ng (2006), where it was found that small datasets already yield precise estimates of the factors as long as the time series are sufficiently informative for estimating common factors.[19]

Missing values due to publication lags Estimating monthly GDP in real-time requires not only to tackle the problem of mixed-frequency datasets but also the problem of missing observations at the end of the sample due to different publication lags of the official data. We therefore carried out additional Monte Carlo simulation, where the observations of the last period are randomly removed. For the forecast exercise, these

[19] There is also empirical evidence, that more data is not always better for factor analysis, see Inklaar et al. (2004), Den Reijer (2005), for example.

Table 6: Monte Carlo results for mixed-frequency estimation

row	T_m	N_m	ω_m	N_q	ω_q	S_{F,F_0}	MSE_q
1	30	20	0.900	20	0.900	0.991	0.092
2	30	20	0.900	20	0.500	0.990	0.366
3	30	20	0.900	20	0.100	0.991	0.641
4	30	20	0.100	20	0.900	0.624	1.384
5	30	20	0.100	20	0.500	0.616	1.280
6	30	20	0.100	20	0.100	0.565	1.069
7	60	20	0.900	20	0.900	0.993	0.081
8	60	20	0.900	20	0.500	0.992	0.349
9	60	20	0.900	20	0.100	0.992	0.618
10	60	20	0.100	20	0.900	0.712	0.494
11	60	20	0.100	20	0.500	0.705	0.592
12	60	20	0.100	20	0.100	0.661	0.695
13	120	20	0.900	20	0.900	0.994	0.075
14	120	20	0.900	20	0.500	0.993	0.341
15	120	20	0.900	20	0.100	0.993	0.609
16	120	20	0.100	20	0.900	0.745	0.373
17	120	20	0.100	20	0.500	0.737	0.509
18	120	20	0.100	20	0.100	0.698	0.649
19	30	40	0.900	40	0.900	0.993	0.092
20	30	40	0.900	40	0.500	0.994	0.365
21	30	40	0.900	40	0.100	0.996	0.637
22	30	40	0.100	40	0.900	0.779	0.478
23	30	40	0.100	40	0.500	0.776	0.561
24	30	40	0.100	40	0.100	0.741	0.779
25	60	40	0.900	40	0.900	0.995	0.078
26	60	40	0.900	40	0.500	0.995	0.346
27	60	40	0.900	40	0.100	0.996	0.616
28	60	40	0.100	40	0.900	0.829	0.260
29	60	40	0.100	40	0.500	0.825	0.450
30	60	40	0.100	40	0.100	0.798	0.643
31	120	40	0.900	40	0.900	0.997	0.073
32	120	40	0.900	40	0.500	0.996	0.339
33	120	40	0.900	40	0.100	0.997	0.607
34	120	40	0.100	40	0.900	0.853	0.223
35	120	40	0.100	40	0.500	0.848	0.424
36	120	40	0.100	40	0.100	0.824	0.626

Note: The statistics shown in the table can be found in (26) and (27).

missing observations are estimated using the EM algorithm based on the remaining data. To focus on this particular issue we assume a single-frequency DGP and, therefore, all other observations can be treated as given. According to the simulation design above, we employ equation (23) to simulate the common factors and equation (25) to simulate the time series x_{jt}^m for $j = 1, \ldots, N_m$ and $t = 1, \ldots, T_m$. Among the final observations at $t = T_m$, a fraction γ of the N_m observations is randomly deleted in each replication. The fraction of missing values is $\gamma = 0.1, 0.5, 0.9$. Thus, up to 90% of the data may be missing at the end of the sample. The sample size is given by $T_m = 50$ and $N_m = 50$. As above, we will focus on the statistic S_{F,F_0} that displays the correspondence between estimated and true factors and the MSE for the forecasts of the missing values, averaged over time series and replications. The results of the simulations can be found in table 7. As in the simulation carried out before, the results are driven by the coefficient ω_m that

Table 7: Monte Carlo results for estimating missing values at the end of the sample

row	T_m	N_m	γ	ω_m	S_{F,F_0}	MSE
1	50	50	0.100	0.900	0.998	0.104
2	50	50	0.500	0.900	0.998	0.104
3	50	50	0.900	0.900	0.997	0.122
4	50	50	0.100	0.500	0.980	0.505
5	50	50	0.500	0.500	0.979	0.530
6	50	50	0.900	0.500	0.976	0.626
7	50	50	0.100	0.100	0.816	0.936
8	50	50	0.500	0.100	0.812	0.982
9	50	50	0.900	0.100	0.778	1.280
10	100	100	0.100	0.900	0.999	0.100
11	100	100	0.500	0.900	0.999	0.102
12	100	100	0.900	0.900	0.999	0.111
13	100	100	0.100	0.500	0.990	0.505
14	100	100	0.500	0.500	0.990	0.510
15	100	100	0.900	0.500	0.989	0.547
16	100	100	0.100	0.100	0.909	0.933
17	100	100	0.500	0.100	0.907	0.943
18	100	100	0.900	0.100	0.902	1.012

Note: The statistics shown in the table can be found in (26) and (27).

operates as a signal to noise ratio of the common factors. The correspondence between factor estimates and the true factors is higher, the more informative the dataset is about the factors, whereas the effect of the number of missing values at the end of the sample is rather limited. Similarly, the MSE highly depends on the ω_m. Not surprisingly, the MSE increases, if more missing values occur at the end of the sample. On the other hand, if the data are only weakly correlated with the factors ($\omega_m = 0.1$), the MSE is close to one irrespective how many observations are missing at the end of the sample. The sample size has only a small impact on the size of the MSE, confirming the results obtained in the previous simulation.