Monetary Policy in a Small Open Economy with a Preference for Robustness

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March 17, 2006

Preliminary and incomplete

Abstract

We use robust control technques to study the effects of model uncertainty on monetary policy in an estimated, semi-structural, small open economy model. Compared to the closed economy, the addition of an exchange rate channel for monetary policy not only produces new trade-offs for monetary policy, but also introduces an additional source of specification errors. We find that exchange rate shocks are an important contributor to volatility in the open economy, especially when policy is set with discretion. The gains from commitment are therefore very large. The exchange rate equation is also particularly vulnerable to model misspecification, along with the equation for domestic inflation. A challenge for central banks in open economies is therefore to develop better empirical models for domestic inflation and the exchange rate.

Keywords: Robust monetary policy, model uncertainty, commitment versus discretion.

JEL Classification: E52, E61, F41.

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We are grateful for comments from Juha Kilponen and seminar participants at the Finnish National Economist Meeting and Sveriges Riksbank. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

1 Introduction

In this paper we use robust control theory to study how a central bank in a small open economy should set monetary policy in the face of model uncertainty. We assume that the central bank has doubts about the exact model specification, but is unwilling to specify a probability distribution over possible specification errors. Instead, the central bank designs policy for the worst-case outcome, allowing for specification errors that lie within a neighborhood of the preferred specification.[1] The model that we study is a semi-structural model of a small open economy estimated on U.K. data.

The small open economy structure differs mainly from the closed economy counterpart primarily in allowing the exchange rate channel to influence output and inflation. This exchange rate channel introduces not only additional shocks and trade-offs that the policymaker needs to consider when designing policy, but also introduces other sources of possible model misspecification.

We first show that the exchange rate is indeed a crucial variable for monetary policy, even when the central bank has complete confidence in the model. Exchange rate shocks account for a very large fraction of the volatility in all variables, especially when policy is set with discretion. The gains to commitment are therefore large in our estimated model. After introducing a preference for robustness, we find that the exchange rate equation is also particularly vulnerable to model misspecification. This vulnerability arises partly because the exchange rate equation is estimated with less precision than the other model equations, and partly because the exchange rate presents the policymaker with a challenging trade-off when responding to shocks. Due to this trade-off, the policymaker cannot offset the exchange rate distortion without adversely affecting other target variables.

Our results also suggest that the equation for domestic inflation (the open-economy Phillips curve) is prone to misspecification, just as it is in the closed economy. The other equations in the model, such as those for imported goods inflation and the output gap, are much less vulnerable to misspecification. A challenge for central banks in open economies, therefore, is to develop better empirical models for domestic inflation and the exchange rate.

Although model uncertainty—in particular uncertainty concerning exchange rate determination—is arguably very important for central banks in small open economies, surprisingly little formal research has explored the issue. Leitemo and Söderström (2005a) study the robustness of simple policy rules to uncertainty about exchange rate determination in a calibrated small open economy model. They conclude that a standard Taylor

[1] Leitemo and Söderström (2005b) study the qualitative aspects of robust policy in a similar small open economy theory model and are able to obtain analytical results.

rule that responds to CPI inflation and the output gap performs very well in the open economy. They also conclude that the Taylor rule is more robust to uncertainty about the formation of exchange rate expectations than are rules that respond also to the exchange rate. [To be completed: Cook (2001), Lees (2004), Batini, Justiniano, Levine, and Pearlman (2005), Leitemo and Söderström (2005b).]

The paper is organized as follows. In Section 2 we present the theoretical model and its empirical counterpart, estimated on U.K. data. In Section 3 we discuss the methods we use to construct the optimal policy when the policymaker has a preference for robustness. Section 4 applies these methods on the empirical model and discusses the results. Section 5 concludes.

2 The model: theory and empirics

We study the New Keynesian small open economy model developed by Monacelli (2003), which builds on Clarida, Galí and Gertler (2001, 2002). While Clarida, Galí and Gertler (2001, 2002) show that their model is isomorphic to the canonical New Keynesian closed economy model, in-so-much-as domestic inflation and the output gap are fully described by a two-equation system that is equivalent to a closed economy, but with a steeper IS curve, this isomorphism is lost if there is imperfect pass-through of exchange rate movements to import prices (Monacelli, 2005). The model that we study has incomplete pass-through because import price are subject to price stickiness. Because consumer price inflation is influenced by imported goods prices, in our model it is not possible to achieve full price stability by setting the output gap to zero. The interest rate policy required to generate a zero output gap destabilizes inflation through its influence on imported goods prices.

There is ample evidence supporting incomplete exchange rate pass-through,[2] so allowing for sticky imported goods prices seems reasonable, especially since it is likely to be important for the design of monetary policy. In the remainder of this Section we provide a brief description of the model, which is based on Monacelli (2005).

2.1 The model

Domestic firms operate in a monopolistically competitive environment, setting prices to maximize the expected discounted value of the firm. Following Calvo (1983), prices are set in a staggered manner and only domestic inputs are used in production. In this situation, the equation for domestic price inflation is given by a New Keynesian Phillips

[2] See, for instance, Campa and Goldberg (2005).

curve of the form

$$\pi_t^H = \beta \mathrm{E}_t \pi_{t+1}^H + \kappa_x x_t + \kappa_\psi \psi_t, \tag{1}$$

where $\pi_t^H \equiv p_t^H - p_{t-1}^H$ is the inflation rate for goods produced in the domestic economy, x_t is the output gap (the percent deviation of domestic output from its flexible-price level), and ψ_t is the deviation from the law of one price. This Law-of-One-Price (LOP) gap is the percent deviation of world market prices (measured in terms of domestic currency) from the domestic price of foreign goods:

$$\begin{aligned} \psi_t &\equiv e_t + p_t^* - p_t^F \\ &= q_t - (1-\gamma) s_t, \end{aligned} \tag{2}$$

where e_t is the nominal exchange rate, p_t^* is the world market price measured in foreign currency, p_t^F is the domestic price of imported foreign goods, $q_t \equiv e_t + p_t^* - p_t$ is the real exchange rate, and $s_t \equiv p_t^F - p_t^H$ is the terms of trade.

If import prices are flexible, then the law of one price holds, so $p_t^F = e_t + p_t^*$ and $\psi_t = 0$. However, with imported goods also subject to price stickiness, there is incomplete exchange rate pass-through, imported goods prices gradually adjust in response to movements in world market prices, and import price inflation obeys

$$\pi_t^F = \beta \mathrm{E}_t \pi_{t+1}^F + \lambda_\psi \psi_t. \tag{3}$$

Aggregate CPI inflation is a weighted average of domestic and imported goods inflation:

$$\pi_t = (1-\gamma)\pi_t^H + \gamma \pi_t^F,$$

where γ is the share of imports in domestic consumption.

On the demand side the economy is populated by infinitely-lived households that consume domestic and foreign goods and save in domestic and foreign one-period bonds. Output is demand-determined, and the optimal intertemporal consumption choice leads to an expression for the output gap of the form

$$x_t = \mathrm{E}_t x_{t+1} - \chi(r_t - \mathrm{E}_t \pi_{t+1}^H - \overline{rr}_t) + \zeta \mathrm{E}_t \Delta \psi_{t+1}, \tag{4}$$

where r_t is the one-period nominal interest rate and $\overline{rr}_t$ is the natural real interest rate, given by

$$\overline{rr}_t \equiv \phi \mathrm{E}_t \Delta y_{t+1}^* + \theta z_t, \tag{5}$$

where z_t is a domestic productivity shock and Δy_t^* is the growth rate of world output.

Finally, assuming perfect capital mobility, the optimal choice between domestic and foreign bonds implies that the nominal exchange rate is determined by the uncovered interest parity (UIP) condition

$$e_t = \mathrm{E}_t e_{t+1} - r_t + r_t^*, \tag{6}$$

where r_t^* is the foreign one-period nominal interest rate.

2.2 The empirical specification

The theoretical framework provides a simple description of private-sector behavior in an economy where goods prices are subject to stickiness. However, the framework abstracts from the information and decision lags that can give rise to gradual adjustments and inertial responses following shocks. Such inertial responses may be rationalized by firms using rule-of-thumb pricing (e.g., Galí and Gertler, 1999), and consumers being subject to habit formation (Fuhrer, 2000).

We adopt the empirical specification of the Monacelli (2005) model estimated by Leitemo (2006), who follows Rudebusch (2002a,b) in allowing data to influence the lead/lag structure of the economy. As in Rudebusch (2002a), Leitemo (2006) uses the expected annual inflation over the coming year to represent the forward-looking component of inflation in the Phillips curve. Furthermore, as in Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005), decisions are subject to a one-quarter implementation lag. The model is estimated on U.K. data[3] using GMM on quarterly data over the period 1980Q1 to 2001Q4.

The empirical model can be summarized as follows. Domestic inflation (the quarterly rate of change of the GDP deflator) is modeled as

$$\begin{aligned} \pi_t^H \; = \; & \underset{(0.081)}{0.58}\, \mathrm{E}_{t-1}\bar{\pi}_{t+3}^H + \underset{(-)}{0.42} \left(-\underset{(0.16)}{0.39}\pi_{t-1}^H + \underset{(0.056)}{0.22}\, \pi_{t-2}^H + \underset{(0.11)}{0.72}\pi_{t-3}^H + \underset{(-)}{0.45}\pi_{t-4}^H \right) \\ & + \underset{(0.13)}{0.28}\mathrm{E}_{t-1}x_t + \underset{(0.060)}{0.038}\mathrm{E}_{t-1}\psi_t + \varepsilon_t^H, \\ & \sigma = 0.021, \end{aligned} \tag{7}$$

where $\pi_t^H \equiv 4(p_t^H - p_{t-1}^H)$ quarterly rate of inflation measured at an annual rate, $\bar{\pi}_t \equiv (1/4)\sum_{j=0}^{3} \pi_{t-j}$ is the four-quarter inflation rate. Dynamic homogeneity is imposed the equations for both domestic and imported price inflation, implying that the coefficients on lagged inflation sum to unity. The output gap x_t is calculated by applying an HP filter

[3]The data are obtained from either the U.K. national accounts, the IMF, or the OECD.

to log real GDP. The LOP gap ψ_t is constructed from equation (2), using the detrended effective real exchange rate and terms of trade, where the share of imported goods in the consumer basket is set at $\gamma = 0.25$, as used by Batini and Haldane (1999) for the U.K. economy.

The equation for imported inflation is

$$\begin{aligned} \pi_t^F &= \underset{(0.047)}{0.78}\,\mathrm{E}_{t-1}\bar{\pi}_{t+3}^F + \underset{(-)}{0.22}\left(\underset{(0.19)}{1.11}\pi_{t-1}^F - \underset{(-)}{0.11}\pi_{t-4}^F\right) + \underset{(0.10)}{0.56}\mathrm{E}_{t-1}\psi_t + \varepsilon_t^F, \\ &\quad \sigma = 0.058, \end{aligned} \tag{8}$$

where $\pi_t^F \equiv 4(p_t^F - p_{t-1}^F)$ is the quarterly rate of change of imported goods prices measured at an annual rate, and $\bar{\pi}_t^F \equiv (1/4)\sum_{j=0}^{3}\pi_{t-j}^F$ is the four-quarter imported goods inflation rate.

The output equation is estimated to be

$$\begin{aligned} x_t &= \underset{(0.039)}{0.53}\,\mathrm{E}_{t-1}x_{t+1} + \underset{(-)}{0.47}\left(\underset{(0.076)}{1.36}\,x_{t-1} - \underset{(-)}{0.36}x_{t-2}\right) - \underset{(0.014)}{0.066}(r_{t-1} - \mathrm{E}_{t-1}\bar{\pi}_{t+2}^H) \\ &\quad + \underset{(0.012)}{0.11}\,\mathrm{E}_{t-1}\Delta\psi_t + \underset{(0.073)}{0.25}\,\mathrm{E}_{t-1}\Delta y_t^* + \varepsilon_t^x, \qquad (9) \\ &\quad \sigma = 0.0041, \end{aligned}$$

where r_t is the 3-month U.K. interest rate, and y_t^* is foreign output approximated by the OECD output gap.

The "risk-adjusted" uncovered interest parity condition was estimated as

$$q_t = \mathrm{E}_t q_{t+1} - (r_{q,t} - \mathrm{E}_t\pi_{q,t+1}) + rr_{q,t}^*, \tag{10}$$

$$\begin{aligned} rr_{q,t}^* &= \underset{(0.070)}{0.50}\,rr_{q,t-1}^* + \underset{(0.064)}{0.19}\,rr_{q,t-2}^* + \underset{(0.038)}{0.11}\,rr_{q,t-3}^* + \varepsilon_t^q, \\ &\quad \sigma = 0.037, \end{aligned} \tag{11}$$

where $r_{q,t} \equiv \frac{1}{4}r_t$, $\pi_{q,t} \equiv \frac{1}{4}\pi_t$ and $rr_{q,t}^* \equiv \frac{1}{4}rr_t^*$ are the U.K. 3-month interest rate, the quarterly CPI inflation rate, and the foreign (OECD) real interest rate, respectively, all expressed at quarterly rates.

Finally, the OECD output growth was modeled according to a first-order autoregressive process as

$$\begin{aligned} \Delta y_t^* &= \underset{(0.066)}{0.51}\,\Delta y_{t-1}^* + \varepsilon_t^{y*}, \\ &\quad \sigma = 0.0050. \end{aligned} \tag{12}$$

3 The robust control approach

The estimated model is taken as the central bank's "reference model," the model thought to best characterize the data-generating process. However, the central bank fears that this model is misspecified. To characterize monetary policy under such fears for misspecification, we use robust control theory. We deviate slightly from the standard robust control approach of Hansen and Sargent (2006) and others, and instead employ the structural-form solution methods developed by Dennis, Leitemo, and Söderström (2006), building on Dennis (2006) and Leitemo and Söderström (2004, 2005).

To apply these methods we begin by representing the reference model, and then distort the reference model through the inclusion of specification errors, which accommodate the central bank's concern for misspecification. This gives a "distorted model," of the form

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathrm{E}_t\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{A}_4\left(\mathbf{v}_t + \boldsymbol{\varepsilon}_t\right), \tag{13}$$

where $\mathbf{y}_t$ is the vector of endogenous variables, $\mathbf{u}_t$ is the vector of policy instrument(s), $\mathbf{v}_t$ is a vector of specification errors, $\boldsymbol{\varepsilon}_t$ is a vector of innovations, and $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, and $\mathbf{A}_4$ are matrices with dimensions conformable with $\mathbf{y}_t$, $\mathbf{u}_t$, and $\boldsymbol{\varepsilon}_t$ that contain the parameters of the model. The matrix $\mathbf{A}_0$ is assumed to be nonsingular and the elements of $\mathbf{A}_4$ are determined to ensure that the shocks are distributed according to $\boldsymbol{\varepsilon}_t \sim iid\,[\mathbf{0}, \mathbf{I}_s]$. The dating convention is such that any variable that enters $\mathbf{y}_{t-1}$ is predetermined, known by the beginning of period t. The specification errors, $\mathbf{v}_t$, are intertemporally constrained to satisfy the "budget constraint"

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbf{v}_t'\mathbf{v}_t \leq \eta, \tag{14}$$

where $\eta \in [0, \overline{\eta})$ represents the total budget for misspecification. When η equals zero, then equation (14) implies that $\mathbf{v}_t = 0$ for all t, in which case the distorted model, equation (13), collapses to the reference model.

The central bank's objective function is assumed to take the form

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{u}_t'\mathbf{Q}\mathbf{u}_t\right], \tag{15}$$

where $\mathbf{W}$ and $\mathbf{Q}$ are matrices containing policy weights and are assumed to be symmetric positive-semi-definite, and symmetric positive-definite, respectively.

The central bank sets policy so as to guard against the worst case misspecification, formulating policy subject to the distorted model with the view that the misspecification will be as damaging as possible. Private sector agents form expectations with the same view. The fear that the misspecification will be as damaging as possible is operationalized through the metaphor that the specification errors in $\mathbf{v}_t$ are chosen by an evil agent whose objectives are diametrically opposed to those of the policymaker. Hansen and Sargent (2006) show that the problem of minimizing equation (15) with respect to $\mathbf{u}_t$ and maximizing with respect to $\mathbf{v}_t$ subject to equations (13) and (14) can be replaced with an equivalent multiplier problem in which

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{u}_t'\mathbf{Q}\mathbf{u}_t - \theta\mathbf{v}_t'\mathbf{v}_t\right], \tag{16}$$

is minimized with respect to $\mathbf{u}_t$ and maximized with respect to $\mathbf{v}_t$, subject to equation (13). The multiplier $\theta \in [\underline{\theta}, \infty)$ is inversely related to the budget for misspecification, η, and represents the shadow price of a marginal relaxation in equation (14).

The solution to this problem returns decision rules for the policy instrument $\mathbf{u}_t$ and the specification errors $\mathbf{v}_t$ that are functions of the predetermined variables $\mathbf{y}_{t-1}$ and the shocks $\boldsymbol{\varepsilon}_t$. There are two distinct equilibria that are of interest. The first is the "worst-case" equilibrium, which is the equilibrium that pertains when the policymaker and private agents design policy and form expectations based on the worst-case misspecification and the worst-case misspecification is realized. The second is the "approximating" equilibrium that pertains when the policymaker and private agents design policy and form expectations based on the worst-case misspecification, but the reference model transpires to be specified correctly. Solving equation (13) with the optimal decision rules values for the instrument and the distortions produces the worst-case outcomes for $\mathbf{y}_t$, $\mathbf{u}_t$ and $\mathbf{v}_t$. To construct the approximating equilibrium, we set $\mathbf{v}_t = \mathbf{0}$, while retaining the equations for $\mathrm{E}_t\mathbf{y}_{t+1}$,and $\mathbf{u}_t$ generated by the worst case equilibrium, and substitute these into equation (13) to solve for $\mathbf{y}_t$. The solution procedures are described in detail in Dennis, Leitemo, and Söderström (2006), who allow for both commitment and discretion on the part of the central bank and the evil agent.

Until this point, the shadow price, θ, is taken as a free parameter. However, rather that setting θ arbitrarily, Anderson, Hansen, and Sargent (2003) describe the concept of a detection-error probability and introduce it as a tool of calibrating θ. Loosely speaking, a detection-error probability is the probability that an econometrician observing equilibrium outcomes would make an incorrect inference about whether the approximating equilibrium or the worst-case equilibrium generated the data. Smaller values for θ allow greater specification errors, which, for a given reference model, make it easier to statistically distinguish between the worst-case and approximating equilibria. In this study, we calibrate θ to generate a detection-error probability of 0.10, which allows the distortions to be of a reasonable magnitude, but not so large to make it inconceivable that they would not have previously been detected.

4 Robust monetary policy in the estimated model

In this section we apply our methodology to the estimated model. The central bank's objectives are assumed to be of a standard quadratic form, so monetary policy is directed toward stabilizing CPI inflation, the output gap and the interest rate around their long-run levels, which are normalized to zero, as per

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2 + \nu r_t^2], \tag{17}$$

where we set $\lambda = 1$, $\nu = 0.05$ and $\beta = 0.99$. We begin with the case where the central bank has complete confidence in its model, and thus no preferences for robustness, and characterize the equilibrium when policy is set with commitment and discretion. We then introduce a preference for robustness on the part of the central bank and discuss the worst-case and approximating equilibria. We set the robustness parameter θ to generate a detection error probability of 0.10, using $10,000$ draws of a sample of 200 observations.

4.1 The rational expectations equilibrium

The rational expectations equilibrium is characterized by both private agents and the central bank having full confidence in the model. Table 1 shows the unconditional variances of some key variables under commitment and discretion, along with the value of the loss function (17). The rational expectations equilibrium has fairly high volatility for all variables. This can be explained partly by the fact that the model is estimated over a period with relatively high volatility, as evidenced in the high regression standard errors. An alternative reason could be that the model is misspecified and fails to include all channels of adjustment in a correct manner, a possibility that motivates our analysis of robust policy below. In any case, the rational expectations equilibrium provides a natural baseline with which to compare the effects of robust policy.

Under both commitment and discretion, the variables specific to the open economy—imported goods inflation, the real exchange rate, and the law-of-one-price gap—are more volatile than the domestic rate of inflation and the output gap, suggesting that external shocks are an important driving force in the open economy. This impression is confirmed in Table 2, which shows the contribution of each shock to the unconditional variances and to loss reported in Table 1. It is clear that exchange rate shocks account for most of the

Table 1: Unconditional variances and loss in the rational expectations equilibrium

Variance in							Loss
π_t^H	π_t^F	π_t	x_t	Δq_t	ψ_t	r_t	
Commitment							
10.76	150.08	6.66	3.32	101.77	88.15	17.10	10.42
Discretion							
8.00	515.37	31.88	19.92	672.81	947.84	139.35	56.73

Note: The table shows the unconditional variances of key variables (in percent) and expected loss in the rational expectations equilibrium. The central bank loss function is given by equation (17), with $\beta = 0.99, \lambda = 1, \nu = 0.05$.

variability in almost all variables, especially when policy is set with discretion. To some extent this result arises because exchange rate shocks have a higher variance than the other shocks, but it also reflects the fact that these shocks give rise to a difficult trade-off for the central bank.

Table 1 also shows that the value of commitment is large in the model: central bank loss is more than 80% lower with commitment than with discretion.[4] These benefits to commitment arise mainly because commitment has a stabilizing effect on the exchange rate. As the exchange rate is a highly forward-looking variable, managing exchange rate expectations is particularly important in an open economy. Again, this is confirmed in Table 2, which reveals that exchange rate shocks account for a much smaller fraction of loss under commitment than under discretion. Additional results (available upon request) suggest that the benefit to commitment rises as imported goods prices become more flexible, because exchange rate volatility then has an even greater impact on prices in this situation. Furthermore, the gain from commitment is negligible in the closed-economy version of our model.[5]

Figures 1–10 show impulse responses to unit-sized shocks under commitment and discretion. (For now focus on the solid lines representing the rational expectations equilibrium.) These figures illustrate the difficult trade-off caused by exchange rate shocks, which have a stronger impact on the economy than other shocks, and therefore require a more forceful response from monetary policy. With commitment, the central bank is

[4]These results are also demonstrated for the theoretical model by Monacelli (2005).

[5]Dennis and Söderström (2005) examine the gains to commitment in a variety of estimated closed-economy models. They show that the gain depends not only on the degree of forward-looking behavior, but also on the existence of implementation and decision lags, which tend to reduce the gains to commitment. Our estimated open-economy model includes multi-period lags in all equations as well as one-period decision lags. Without these lags, the gains to commitment is likely to be even larger.

Table 2: Variance and loss decomposition in the rational expectations equilibrium

	Variance in							Loss
Shock	π_t^H	π_t^F	π_t	x_t	Δq_t	ψ_t	r_t	
Commitment								
ε_t^H	49.15	1.12	44.35	18.42	2.51	2.87	9.42	34.09
ε_t^F	0.02	23.54	32.98	0.22	0.58	0.92	0.83	20.98
ε_t^x	0.07	0.03	0.10	7.79	0.04	0.03	2.52	2.71
ε_t^q	50.73	75.22	22.48	72.88	96.85	96.15	84.61	41.74
ε_t^{y*}	0.03	0.09	0.09	0.69	0.03	0.03	2.62	0.48
Discretion								
ε_t^H	75.35	0.57	13.32	2.05	0.01	0.03	1.42	8.07
ε_t^F	0.86	7.73	7.46	1.21	0.14	0.96	0.80	4.64
ε_t^x	0.28	0.00	0.06	1.40	0.01	0.00	0.35	0.56
ε_t^q	23.12	91.69	79.07	95.04	99.82	98.99	96.90	86.52
ε_t^{y*}	0.38	0.01	0.09	0.31	0.03	0.02	0.54	0.22

Note: The table shows the percentage of the unconditional variances and central bank loss in the rational expectations equilibrium that is due to each shock.

able to manage expectations to better stabilize the economy and the initial shocks are often followed by reversals. Interestingly, the optimal policy makes the real exchange rate non-stationary. The real exchange rate only affects the economy through the LOP gap, which depends also on the terms of trade (see equation (2)). With imperfect pass-through, a real exchange rate that is non-stationary, but cointegrated with the terms of trade, allows the central bank to better stabilizes the LOP gap and the broader economy economy. But this requires the central bank to be able to commit to future policies: discretionary policy cannot manage the real exchange rate in this manner.[6] Due to the central bank's inability to manage expectations under discretion, the interest rate must respond more vigorously than under commitment, especially following external shocks.

4.2 The worst-case equilibrium

We now introduce a lack of confidence in the model by allowing an evil agent to choose specification errors, model distortions, to maximize central bank loss. The central bank chooses policy so as to minimize the impact of these distortions.

Table 3 shows the unconditional variances of the specification errors chosen in the

[6]The non-stationarity of the real exchange rate is not due to the specification of the empirical model, but is due to the presence of imperfect exchange rate pass-through. The same mechanism is also present in the theoretical specification, see Monacelli (2005).

Table 3: Unconditional variances of specification errors

θ	Specification error				
	v_t^H	v_t^F	v_t^x	v_t^q	v_t^{y*}
Commitment					
0.0145	201.70	13.97	8.05	132.99	0.61
Discretion					
0.0455	15.63	2.75	0.94	242.37	0.40

Note: The table shows the unconditional variances of the worst-case specification errors. The parameter θ is chosen so as to produce a detection error probability of 0.10.

worst-kind equilibrium when the central bank and the evil agent act under commitment and discretion. The distribution of the distortions tells us to which equation a distortion has the greatest impact on central bank loss. This will in general depend on whether we consider the equilibrium under commitment or discretion. Table 4 shows the unconditional variances of key variables under the worst-case and approximating equilibria, along with the value of the central bank loss function.

We first note that the variance of the distortions are of a larger magnitude in the commitment equilibrium. The ability of the evil agent to commit to future distortions allows him to have a greater impact on central bank loss. This is also illustrated by the impulse responses in Figures 1–10. Under commitment the distortions typically have a more persistent effect on the economy, introducing more volatility than under discretion. Thus, central bank loss in the worst-case equilibrium increases by more under commitment, where it is 75% larger than in the rational expectations equilibrium, while it is 25% larger under discretion.

A second important observation from Table 3 is the relative magnitudes of the distortions. Although under both commitment and discretion, the evil agent puts most emphasis on distorting the exchange rate and domestic inflation equations, there are important differences. Under discretion, the variance of the distortion to the exchange rate equation is 15 times greater than the second largest distortion, which is to the domestic inflation equation. Under commitment, on the other hand, the exchange rate distortions are less important relative to the other distortions, and are smaller than the distortions to the domestic inflation equation. This reflects the fact that the central bank under discretion is very vulnerable to exchange rate disturbances, as shown earlier for the rational expectations equilibrium.

What are the reasons why the evil agent distorts the domestic inflation and exchange rate equations? That the exchange rate equation is vulnerable to misspecification is

Table 4: Unconditional variances and loss under the robust policy

θ	Variable							Loss
	π_t^H	π_t^F	π_t	x_t	Δq_t	ψ_t	r_t	
Commitment								
Worst-case equilibrium								
0.0145	12.71	240.14	9.60	8.40	146.19	138.04	24.86	18.18
Approximating equilibrium								
0.0145	11.26	194.66	8.03	6.53	144.76	127.42	21.22	14.88
Discretion								
Worst-case equilibrium								
0.0455	8.72	609.13	37.64	27.93	823.85	1173.23	155.13	70.69
Approximating equilibrium								
0.0455	7.82	512.85	31.80	21.43	835.73	974.01	120.95	57.38

Note: The table shows the unconditional variances of key variables (in percent) in the worst-case and approximating equilibria. The parameter θ is chosen so as to produce a detection error probability of 0.10. The central bank loss function is given by equation (17), with $\beta = 0.99, \lambda = 1, \nu = 0.05$.

not a new insight. The views regarding the potential for exchange rate modeling and forecasting have not changed markedly since the pessimistic results reported by Meese and Rogoff (1983). But it is striking how strong these effects are, in particular under discretion. In the model, the exchange rate influences the LOP gap which directly influences all target variables. Firms set prices to reflect average future marginal costs, so domestic and imported goods inflation depend on the expected future sum of the LOP gap. Aggregate domestic demand on the other hand depends on the expected current LOP gap as consumers substitute consumption between foreign and domestic goods. The exchange rate is thus an important channel for the evil agent to increase volatility in all equations and increase central bank loss. Furthermore, exchange rate movements present difficult trade-offs for the central bank, making it even more attractive for the evil agent to introduce misspecification. Since the exchange rate channel has asymmetric effects on output and inflation, the exchange rate channel offer possibilities in which inflation and output can both be increased.

A third reason for why the UIP condition is distorted is the high variance of shocks to the (risk-premium corrected) foreign interest rate. As the robust control problem is formulated, it provides the evil agent a place to hide distortions behind high residual variance.

The foreign inflation equation is also subject to high residual variance and has a direct impact on the target variable. Nevertheless, the distortions to this equation are of a fairly small magnitude. This is because the distortions can be offset relatively easily through exchange rate movements induced by small interest rate adjustments. The exchange rate has a strong impact on this process (through the LOP gap) and the required exchange rate movements (and interest rate movement) induce only small changes to the other variables.

As for domestic inflation, it has been shown elsewhere (e.g., Leitemo and Söderström, 2004) that the Phillips curve in a closed economy is very vulnerable to specification errors, as such distortions create a more difficult trade-off for the central bank than other distortions. This result holds also in the open economy, although here the exchange rate equation is even more vulnerable to misspecification.

4.3 Robust policy and the approximating equilibrium

While the worst-case equilibrium reveals what specification errors are most damaging for the central bank, the approximating equilibrium provides information on the effects of the central bank's preference for robustness on monetary policy and the economy in the situation where there is no misspecification. Comparing the interest rate volatility for rational expectations in Table 1 and for the approximating equilibrium in Table 4, we see that the robust monetary policy is more volatile under commitment but less volatile under discretion than the non-robust policy. The insurance against model misspecification increases volatility in almost all variables, especially under commitment since the evil agent is able to do more damage.

Compared to the rational expectations equilibrium, loss is more than 40% larger in the approximating equilibrium with commitment, while it is essentially the same with discretion. The large increase in loss for the commitment case again reflects the greater damage caused by the evil agent when he can commit to future specification errors, thus necessitating stronger policy responses.

This impression is confirmed by Figures 1–10, where the impulse responses are more volatile in the approximating equilibrium than in the rational expectations equilibrium. Surprisingly, the smallest effect of robustness is in response to exchange rate shocks when policy is set with discretion (see Figure 8), where the effects are almost negligible.

5 Concluding remarks: What have we learned?

We set out to study the effects of model uncertainty on monetary policy in a small open economy. We first show that exchange rate shocks are an important source of volatility in the open economy also without taking model uncertainty into account. This is particularly the case when policy is set with discretion. Therefore the gain from commitment is very large in our estimated open-economy model.

When we introduce a preference for robustness, we find that monetary policy is primarily sensitive to distortions to the exchange rate and to domestic inflation, while distortions to the other equations are of minor importance. When policy is set with discretion, the exchange rate equation is most sensitive to misspecification, while with commitment, the domestic inflation equation is more vulnerable. Since especially exchange rate model uncertainty is perceived to be high also from an empirical modeling perspective, the sensitivity of the outcome to exchange rate model uncertainty poses a major challenge to monetary policy.

The policy implications of our results are obvious. To improve on monetary policy in the open economy, a better understanding of the models for the exchange rate and domestic inflation is crucial. Reducing the scope for misspecification in the other equations—for imported goods inflation and the output gap—seems to be of second-order importance. However, just as important, or even more important, is for central banks in open economies to increase their ability to commit to future policies. This may explain why open economies have been more willing to introduce formal targets for inflation, publish forecasts and improve on transparency.

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Figure 1: Impulse responses to a domestic inflation shock with commitment

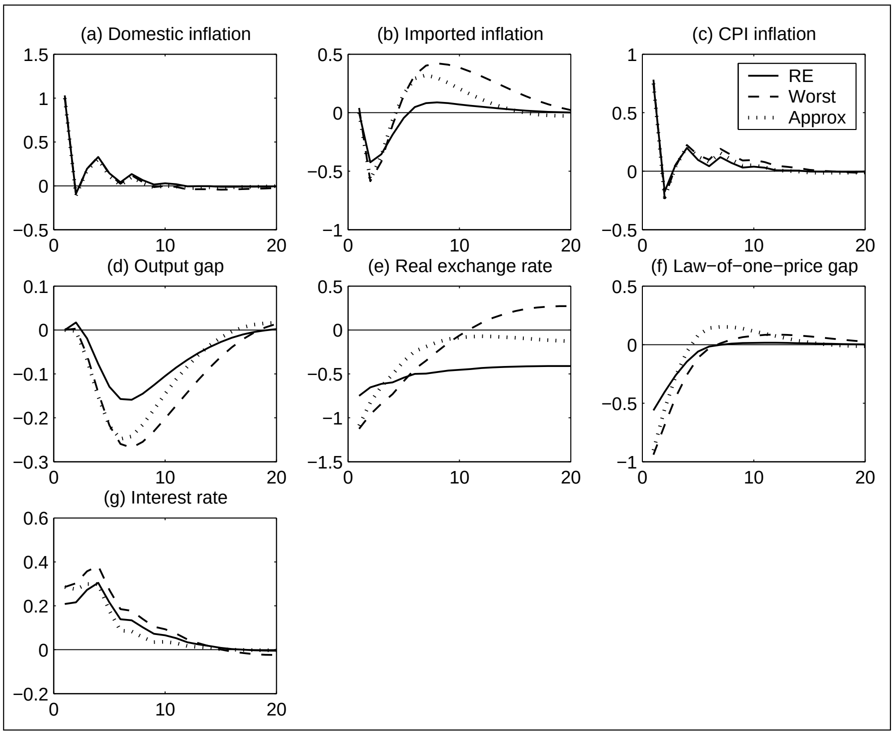

Figure 2: Impulse responses to a domestic inflation shock with discretion

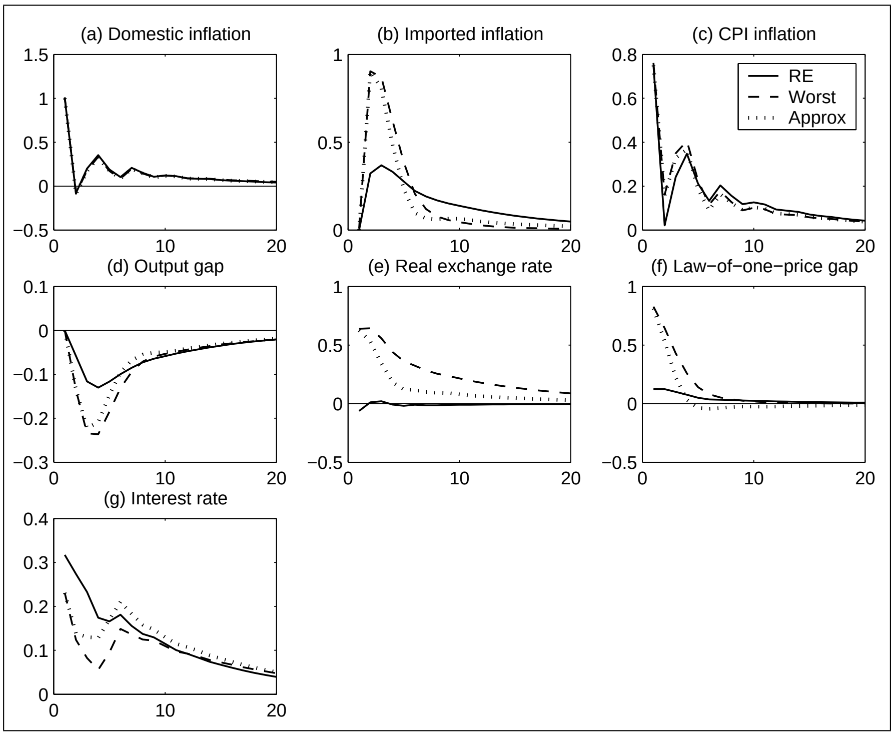

Figure 3: Impulse responses due to an imported goods inflation shock with commitment

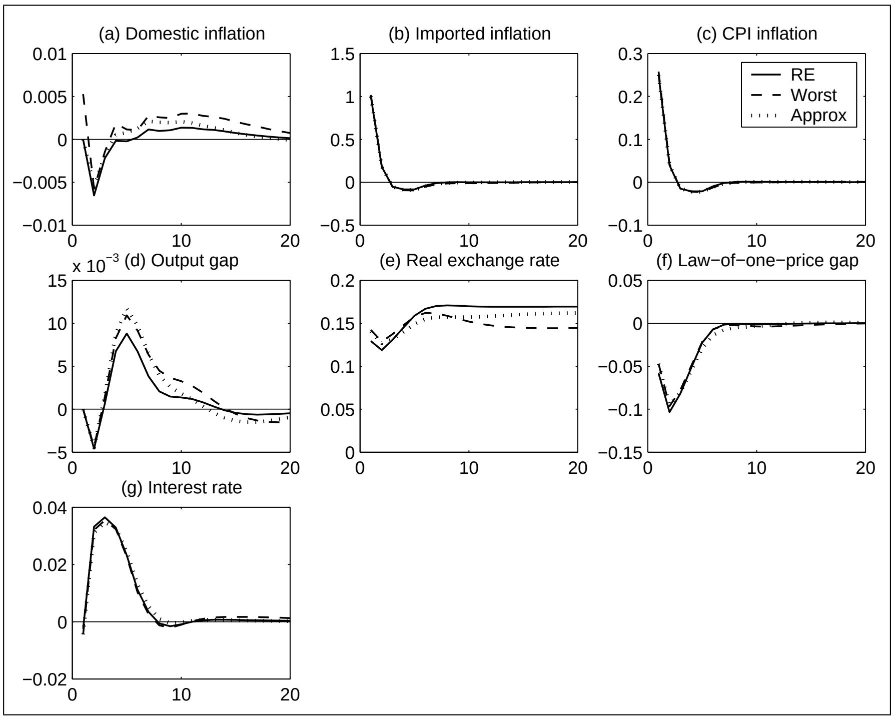

Figure 4: Impulse responses due to an imported goods inflation shock with discretion

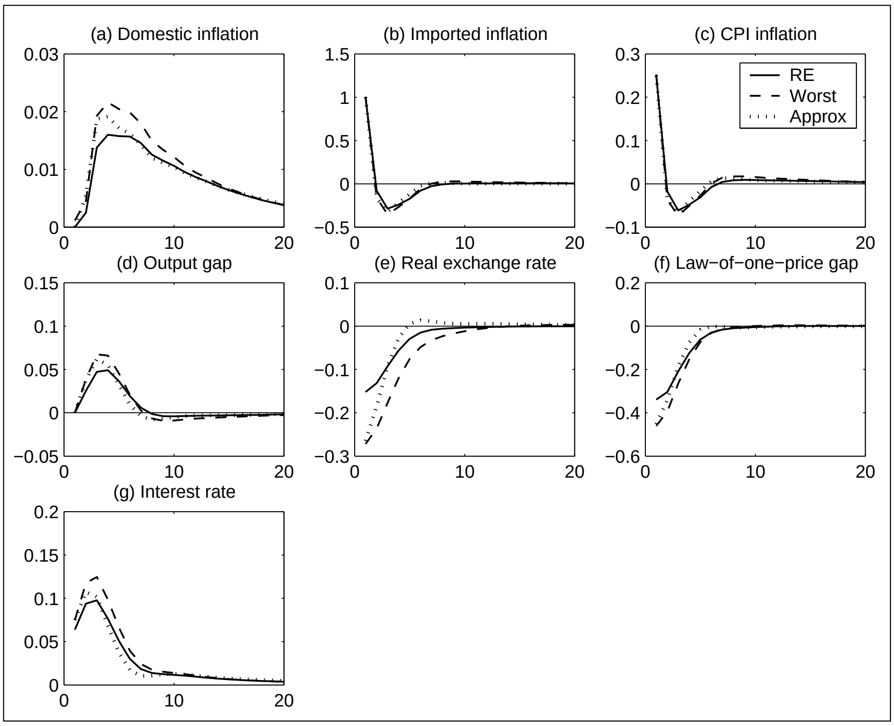

Figure 5: Impulse responses to a domestic demand shock with commitment

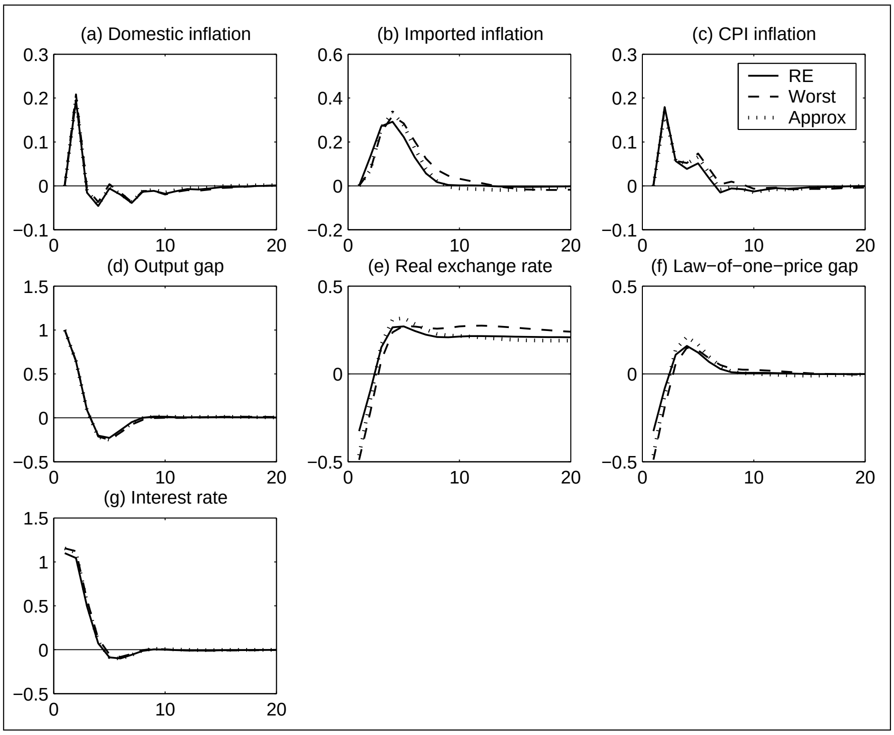

Figure 6: Impulse responses to a domestic demand shock with discretion

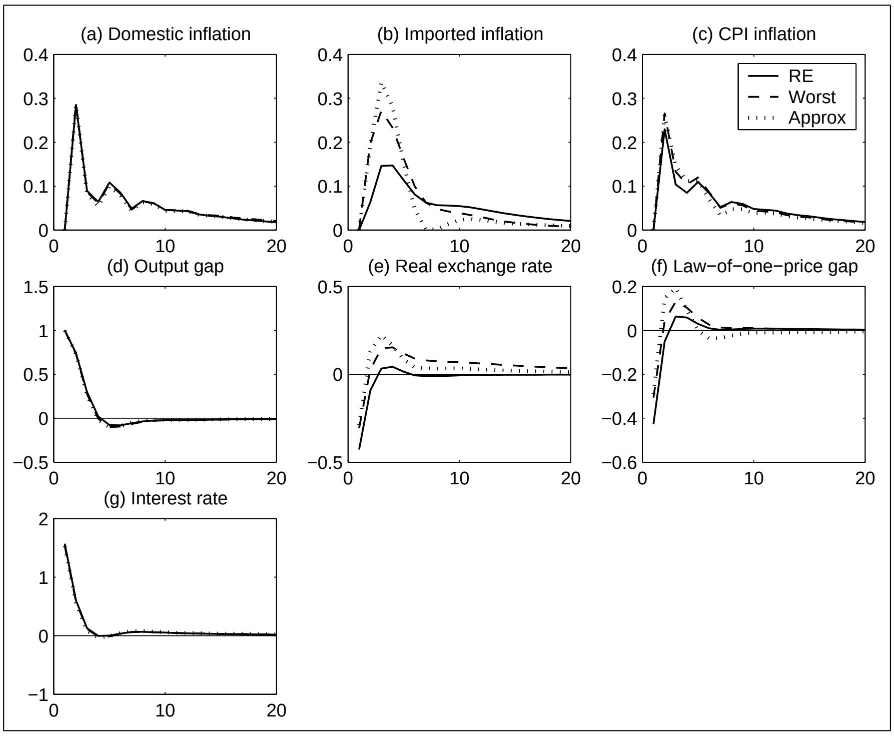

Figure 7: Impulse responses to an exchange rate shock with commitment

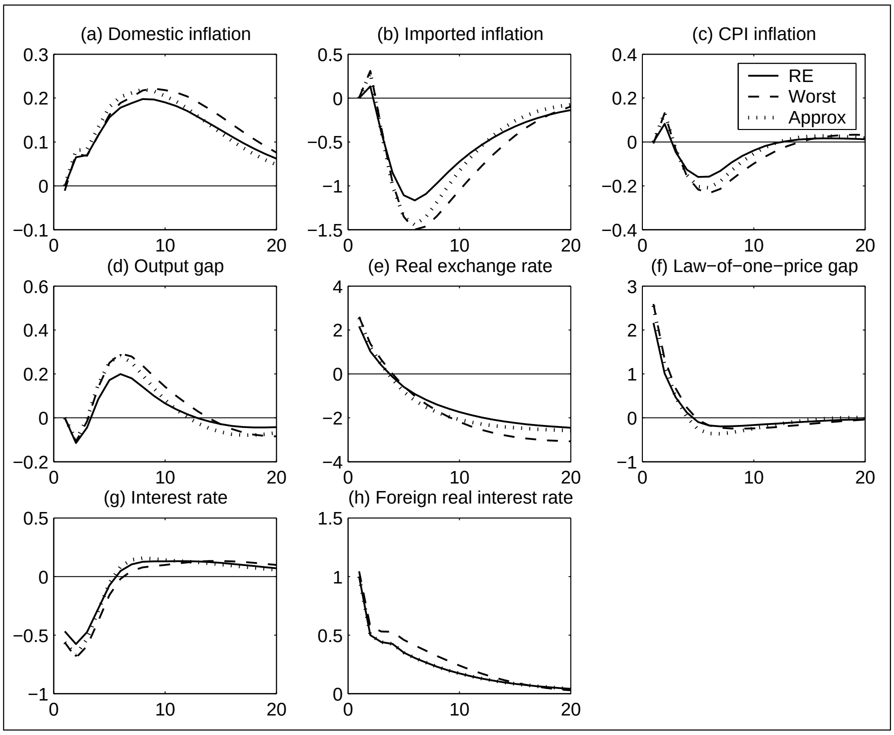

Figure 8: Impulse responses to an exchange rate shock with discretion

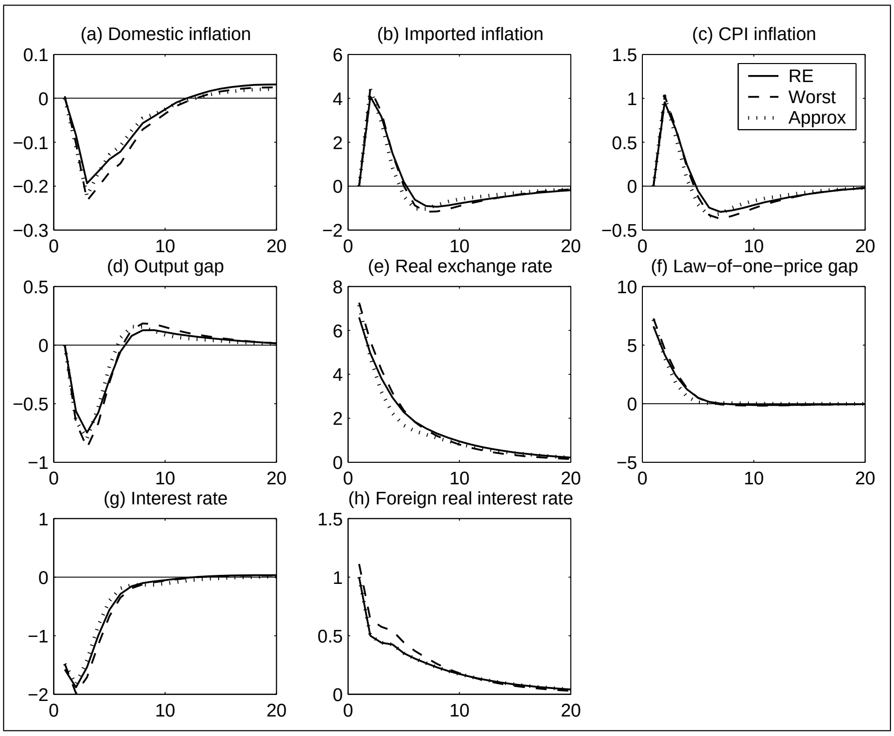

Figure 9: Impulse responses to a foreign output growth shock with commitment

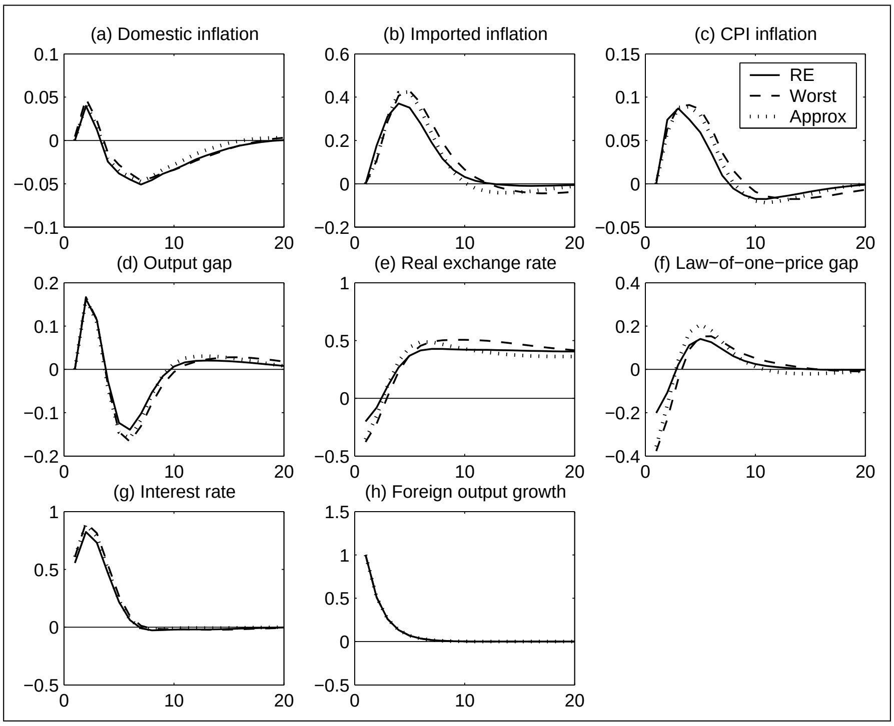

Figure 10: Impulse responses to a foreign output growth shock with discretion