Long Term Care: the State, the Market and the Family[*]

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Abstract

In this paper we study the optimal design of a long term care policy in a setting that includes three types of care to dependent parents: public nursing homes, financial assistance by children and assistance in time by children. The instruments are public nursing homes and subsidies to aiding children, both financed by a flat tax on earnings. The only source of heterogeneity is children's productivity. Parents can influence their children by leaving them gifts before they know whether or not they will need long term care, yet knowing the productivity of the children. We show that the quality of nursing homes and the level of tax-transfer depend on their effect on gifts, the distribution of wages and the various inequalities in consumption. We also consider the possibility of private insurance.

Keywords: long term care, altruism, bequests.
JEL classification: D64, H55, I118.

1   Introduction

The ongoing demographic ageing process represents a major challenge for the way our economies are organized both from a social, as well as from an economic point of view. Ageing can be felt across a large array of domains

touching all age groups, ranging from the very young to the oldest old. One often cited example is the provision of long-term care insurance to the oldest old, be it under the form of a private or a public system. Only a handful of countries or regions have set up such long-term care insurance systems which, incidentally, are also sometimes called dependency insurance. The relative scarcity of such systems, and the difficulties of organizing them, is linked to some conceptual problems intrinsically due to the issue at hand. First, a definition of who is a person in need of long-term care cannot always be stated objectively. However, this is not a sufficient reason to justify the lack of long-term care insurance programs around the world, since disability insurance systems are plagued by the same kind of problem but do exist. The second, and probably more fundamental reason, is that a lot of long-term care is not provided through a formal market mechanism, but rather through informal family arrangements. In this respect, the problem is similar to the child-care market, where family care is competing with market-provided care in private or public arrangements. From a social point of view, this duality of providers is an interesting one, as these two types of providers seem to function on a very different basis. While institutional care is essentially a provision of a contribution-based service by a public or private (for-profit or non-profit) provider, family care is at least partly motivated by some degree of altruism, which in turn implies that the caregiving family member also derives utility from this activity. Further, while institutional care usually implies some degree of public subsidization, and hence inter-family redistribution, this does not always hold true for family-care arrangements.

Yet the analogy between these two forms of care is limited. In contrast to the child-care market, the costs involved are much larger in long-term care insurance, as costs of medical and non-medical care are much more expensive at the end of the life-cycle than at the beginning because of the vastly different physical conditions of the people in question. Hence, the choice between family or institutional care has important budgetary implications that a government or a social planner cannot ignore. Another problem raised by cares such as child care or long term care provided out of altruism is that, depending on the opportunity cost of time, they can be provided directly by children in units of time, or obtained from the market through financial aid from children. One sees from this quick overview that the analysis of long term care is very complex, and that all aspects cannot be dealt with at the same time.

In this paper we study a society consisting of a number of pairs of parent-child. Parents are not altruistic, while children have a specific type of altruism: in that they are ready to help their parents if these lose their autonomy. In the absence of government policy dependent parents can be helped in two

ways: either children give them some financial aid or they provide them with assistance in time. Children have different productivities, and parents have a uniform endowment (wealth, pension). Market productivity varies, but productivity in terms of helping dependent parents is the same for all. As a consequence, children of dependent parents are divided into two groups. The low market productivity group helps their parents with time, and the other one provides financial assistance. Before knowing their own health status parents can give part of their endowment to their children in case of they lose so that in case of bad health they get better assistance.

We then introduce public policy consisting of three instruments: a uniform payroll tax, a subsidy for dependent parents receiving assistance (in kind or in cash) from their children, and institutionalized nursing assistance. Parents who receive this latter benefit cannot receive any help from their children. As it appears, children with middle level wages tend to have their dependent parents going to these nursing homes.

We are ultimately interested in the optimal policy chosen by a utilitarian government. But before doing that, we analyze the comparative statics of our model. In particular, we study the effect of policy variables and exogenous variables on the segmentation of our society into three groups.

Quite clearly such a model does not include all the aspects of long term care and it does rest on a number of assumptions. Some are pretty realistic; others are made to keep the analysis within reasonable limits. The only heterogeneity comes from differences in market productivity. The other characteristics such as altruism, initial endowment, productivity in assistance to dependent parents are equal for all.[1] The instruments are a payroll tax, lump-sum subsidy, and public nursing home. These restrictive policies are adopted for the sake of simplicity. Also, we consider the possibility of resorting to private long term care insurance, which is quite realistic.[2]

In an earlier paper [Pestieau and Sato, 2004], we only consider a tax-transfer policy. This paper thus extends this work in two different directions. We allow for the possibility of public nursing homes and also for the existence of private insurance. As it will be shown public nursing home and private insurance cater to parents with children of middle productivity. Low wages children prefer to help their parents with time; high wages children prefer to assist them with financial transfers.

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[1]In Jousten *et al.* (2003), the optimal long term care policy is analyed when the only source of heterogeneity is children's altruism.

[2]In a recent paper Finkelstein and McGarry (2004) underline two sources of heterogeneity in long term care insurance that are not observable: risk types and insurance preferences. They show that this double asymmetric information has negative efficiency consequences on the insurance market. We don't consider this issue here.

To avoid confusion, it is important to distinguish among the types of resources dependent parents can count on and among the types of provider of long term care. Assistance in time implies that the dependent parent stays home and is taken care of by his child. Assistance in cash or private insurance benefits allow the dependent parents to stay home and get some nursing service or to go to a private nursing home. Finally, the case of public nursing home is self-explanatory.

Among the scant evidence on upward intergenerational transfers from middle age children to their elderly parents, there is the study by Sloan *et al.* (2002) who use data from HRS. They show that a child with a high wage tends to transfer money rather than time and conversely for a child with a low wage. Ioannides and Kan (2000) using data from the PSID reach the same conclusion. Children's transfers (both money and time) are determined by their parents' needs and their own resources. High income children and children living far away tend to make transfers in money and not in time.[3]

The rest of the paper is organized as follows. The next section presents the basic model and some comparative statics results along with the *laissez-faire* solution with private insurance. The following section is devoted to the design of optimal tax transfer and nursing homes policy. A final section concludes.

2 The *laissez-faire*

2.1 The basic model

We consider a family consisting of a parent and his altruistic child. All families are *ex ante* identical except for the market productivity of children denoted w with density $f(w)$, distribution $F(w)$ and support (w_-, w_+). We assume that the parent chooses to leave a gift $\bar{G}$ to his child before knowing whether or not he needs long term care. When this is known, the child decides to help his dependent parent. Each parent faces a probability π of losing his autonomy which corresponds to a loss D. He has an initial endowment I and consumes d^D if dependent and d^N if autonomous.

His expected utility can be written as:

$$V = \pi \left[v\left(d^D\right) - D + H \right] + (1-\pi) v\left(d^N\right) = v(d) - \pi(D - H)$$

where $d^D = d^N = I - G$, I being his initial endowment. D is the utility loss implied by dependence and H is the help he gets from his child expressed in

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[3]See also Prouteau and Wolff (2003) for a study on French data reaching the same conclusion.

utility terms as well. Turning to the children, even though they are concerned by the consumption of their parents, dependent or not, they only help them in case of dependency. Denoting their utility by $u(\cdot)$ and their consumption by c^j $(j = D, N)$, we have

$$U^D = u\left(c^D\right) + \beta\left(v\left(d\right) + H - D\right)$$

and

$$U^N = u\left(c^N\right) + \beta v\left(d\right)$$

where $c^D = (1-h)w + G - s$, $c^N = w + G$ and $\beta \leqslant 1$ is a factor of altruism. Market labor supply is $(1-h)$ with h being the aid in time provided to dependent parents and s, is the amount of financial aid that allows children to purchase market services on behalf of their dependent parents. As we show h and s are mutually exclusive.

It is now time to define H. We assume that each child has one unit of time endowment. He can devote part of it to labor market in which case he earns w and he can devote another part of it to his parent. If he provides h to his parent given a constant productivity ω_0, this amount to a help of $\omega_0 h$. This child will also earn $(1-h)w$ as market earnings. Instead a child may want to help his dependent parent through financial aid, s, which is used to purchase market nursing services.

By assuming perfect substituability between these two forms of assistance, namely by positing:

$$H\left(\omega_0 h, s\right) = H\left(\omega_0 h + s\right)$$

with $H' > 0$ and $H'' < 0$, we know that children with $w \leqslant \omega_0$ will have $1 \geqslant h \geqslant 0$, $s = 0$ and those with $w > \omega_0$, $h = 0$ and $s \geqslant 0$.[4] For $I - G$ not too high, we expect interior solutions, namely, either $h > 0$ or $s > 0$. Formally, for $w \leqslant \omega_0$, h^* is the solution of

$$u'\left(c^D\right)w = \beta H'\left(\omega_0 h\right)\omega_0$$

and for $w > \omega_0$, s^* is the solution of

$$u'\left(c^D\right) = \beta H'\left(s\right).$$

If G is large enough we can expect that $h = 1$. The profile of $\omega_0 h + s$ is represented on Figure 1 below.

Given the expected behavior of his child, each parent can decide to leave him a certain fraction of his endowment. When making this choice, he does

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[4]In other words if $H\left(\omega_0 h, s\right)$ would allow for some complementarity between the two arguments, children could very well provide at the same time assistance in time and in cash.

not know yet whether or not he will need long term care but he knows his child's productivity. There is no parental altruism. The reason for such an early gift is insurance; it is also the only way to obtain care. With $\pi = 0$, there would not be such a gift. The optimal amount of gift will depend on w.

Up to now we distinguished two regimes depending on $w \gtrless \omega_0$. We will denote 1 the regime where children provide $\omega_0 h$ and 2 the regime where they provide s. We now consider the possibility of a third and intermediate regime defined by $w \in (\hat{w}_1, \hat{w}_2)$ with $\hat{w}_1 > \omega_0 > \hat{w}_2$. To obtain this regime, we introduce the possibility of a private insurance with compensation a and premium p, $p(a) = \pi a \theta$ where $\theta > 1$ reflects the fact that such an insurance cannot be actuarially fair for all sorts of reasons of informational and technological nature.

The parent instead of expecting assistance from his child at the cost G can thus buy that insurance. Parents with children of productivity $w < \omega_0$ have to compare the utility V_1 they get with aid h but cost G, and the utility V_3 they get with coverage a and cost $p(a)$. In other words, they choose private insurance if

$$v(I - G_1^*) + \pi H(\omega_0 h(w, G_1^*))$$
$$-v(I - \pi a^* \theta) + \pi H(a^*) < 0$$

where G_1^* and a^* are the optimal values chosen in regimes 1 and 3 respectively. It is not clear that this inequality can be verified. One can expect, e.g., that for an inefficient insurance market (large θ), no parent will ever buy private insurance. For the time being, we assume that there exists a value of w $(< \omega_0)$ for which the above inequality becomes an equality; we denote such a value $\hat{w}_1$.

Similarly we define the treshold value $\hat{w}_2$ as that for which

$$v(I - G_2^*) + \pi H(s(w + G_2^*)) = v(I - \pi a^* \theta) + \pi H(a^*).$$

We assume that private insurance and filial assistance are mutually exclusive. Thus, for w sufficient high children will prefer to help their parents than to let them rely on just private insurance whose coverage depends on both θ and I. Indeed, a^* is defined by

$$v'(I - \pi a^* \theta) \theta = H'(a^*).$$

To have a better grasp at this problem, we now turn to a simple illustration, using logarithmic utility functions.

2.2 The log-linear example

As parents move first and children second, we start by looking at the problem of each child. If his parent is healthy, he does not help him and benefit from the transfer G, if any. If his parent loses his autonomy, he helps him with h or s depending on his productivity.

2.2.1 Child's problem

Each child solves the following problem:

$$Max \quad \ln\left(w\left(1-h\right)+G-s\right)+\beta\ln\left(\omega_0 h+s\right)-\beta D+\beta\ln\left(I-G\right)$$

From the FOC, we obtain:

$$\text{For } w<\omega_0: \quad h^*=\frac{\beta}{1+\beta}\frac{w+G_1}{w} \quad \text{if} \quad G_1\leqslant\frac{w}{\beta}$$

$$=1 \quad \text{if } G_1>\frac{w}{\beta}$$

$$\text{For } w>\omega_0: \quad s^*=\frac{\beta}{1+\beta}\left(w+G\right).$$

2.2.2 Parents' problem without private insurance

$$Max \quad \ln\left(I-G\right)+\pi\ln\left(\omega_0 h+s\right)-\pi D.$$

Here, the FOC yields a supply function G_i^* that can be summarized by:

$$G_1^*=Max \quad \left[0, Min\left(\frac{\pi I-w}{1+\pi},\frac{w}{\beta}\right)\right]$$

$$G_2^*=Max \quad \left[0,\frac{\pi I-w}{1+\pi}\right].$$

We can now represent the values of $m\left(=h\omega_0 \text{ or } s\right)$ and G along the w-axis on Figure 1. On this axis $\underline{w}\equiv\dfrac{\pi\beta I}{1+\pi+\beta}$ and $\bar{w}\equiv\pi I$. Below $\underline{w}$, the dependent parent could count on a 100% assistance from his child and does not have to leave a lot. Above $\underline{w}$, children are so wealthy that a gift has no effect on the level of s. As we show in the appendix the profile we adopt here only holds for particular values of both I and ω_0.
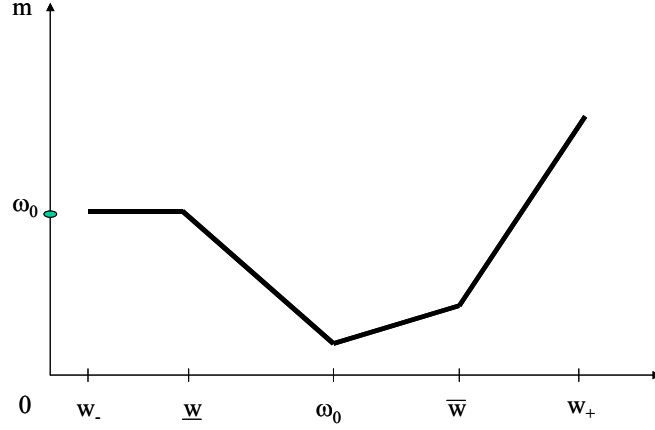
Figure 1: Child's assistance

Figure 2: Parent's gift
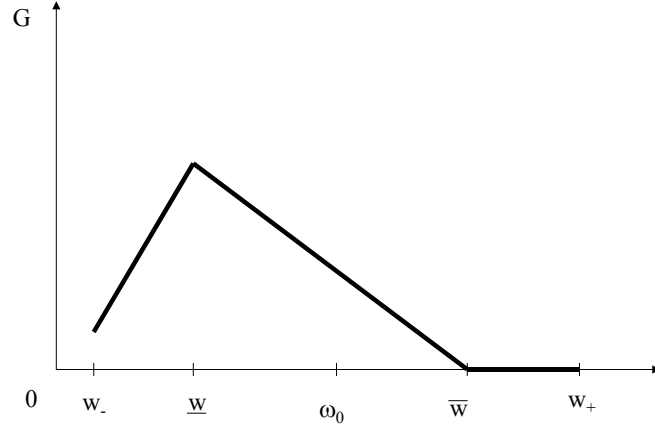
Substituting these values of G_i^* in the utility function of the parents we have the following expression that depend on the value of w.

$$w < \underline{w} : V_1 = \ln\left(I - \frac{w}{\beta}\right) + \pi \ln \omega_0$$

$$\underline{w} < w < \omega_0 : V_1 = \ln\left(1 + \pi\right)\ln\left(w + I\right) - \pi \ln w - (1 - \pi)\ln\left(1 + \pi\right) + \pi \ln \frac{\beta \pi \omega_0}{1 + \beta}$$

$$\omega_0 < w < \bar{w} : V_2 = \left(1 + \pi\right)\ln\left(w + I\right) - \left(1 + \pi\right)\ln\left(1 + \pi\right) + \pi \ln \frac{\beta \pi}{1 + \beta}$$

$$w > \bar{w} : V_2 = \pi \ln w + \ln I + \pi \ln \frac{\beta}{1 + \beta}$$

The profile of V_i is given on Figure 3.

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Figure 3: Parent's utility
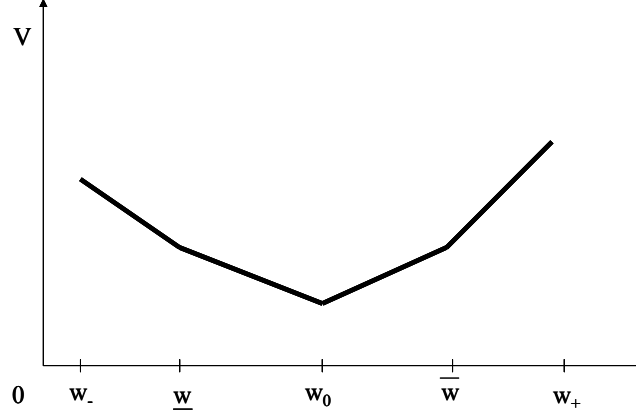
2.2.3 Parent's problem with private insurance

With the log-utilities, a^* is simply equal to $\dfrac{I}{\theta(1+\pi)}$ and the utility of the parents:

$$V_3 = (1+\pi)\ln I - (1+\pi)\ln(1+\pi) - \pi\ln\theta.$$

To obtain the values of $\hat{w}_1$ and $\hat{w}_2$ (assumed to exist), one respectively solves the following equations:

$$V_3 = V_1(w) \quad \text{and} \quad V_3 = V_2(w).$$

Explicitly, this gives

$$V_3 - V_1(\hat{w}_1) = (1+\pi)\left[\ln I - \ln(\hat{w}_1 + I)\right] + \pi\ln\frac{\hat{w}_1}{\omega_0} - \pi\ln\theta - \pi\ln\frac{\beta\pi}{1+\beta} = 0$$

and

$$V_3 - V_2(\hat{w}_2) = (1+\pi)\left[\ln I - \ln(\hat{w}_2 + I)\right] - \pi\ln\theta - \pi\ln\frac{\beta\pi}{1+\beta} = 0.$$

On Figure 4 we represent the value of V along the w-axis that is divided in three regimes: assistance in time, private insurance, assistance in cash. It is clear that for high values of θ (namely for very inefficient markets), the horizontal line V_3 could be below the minimum of V_1 and V_2.

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Figure 4: Parent's utility with private insurance
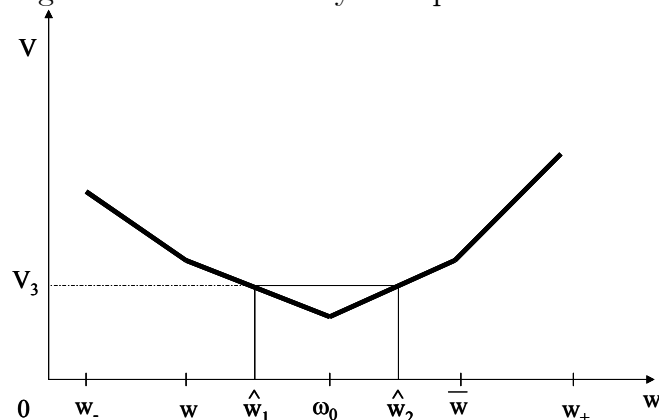
3 Public policy

We now introduce three policy instruments: an income tax t levied on children's earnings, a flat subsidy σ for children assisting their dependent parents and a public nursing home of quality g. The income tax is paid by all children; the uniform subsidy and the public nursing home are by assumption mutually exclusive. Parents who end up in a nursing home are not aided by their children.

As we will see in the absence of public nursing home and, for the time being, of private insurance the parents's utility has the U-shape of Figure 3. Thus it is pretty intuitive that if providing this facility is not too costly it can be attractive for families with children having productivity around ω_0. For families with very productive children, the quality of the public nursing home may be insufficient. For families with low productivity, the rational choice might be to rely on personal assistance given that children are so more productive at helping their dependent parents than working for an employer (at least in comparative terms).

For clarity sake, let us explicit the sequence of decision.

- Stage 1. The social planner chooses τ, σ and g.

- Stage 2. Each parent chooses whether or not he leaves some G and how much. If he anticipates that given (τ, σ, g) and in case of bad health he is better off in a nursing home, he does not leave anything. Otherwise, his child will help him through h or s, and he will *ex ante* leave him part of his wealth.

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- Stage 3. The child helps his unhealthy parent by comparing the alternatives: assistance in time or in cash.

We now look at each child's choice and thus at the parent's choice before turning to the determination of the optimal public policy.

3.1 Child's choice

A child with productivity w with a dependent parent chooses s or h to maximize:

$$u\left(\omega\left(1-h\right)+G+\sigma-s\right)+\beta\left[H\left(s+\omega_0 h\right)+v\left(I-G\right)-D\right]$$

where $\omega = w\left(1-t\right)$. As above we have to distinguish between two regimes to determine the optimal choice.

$$\omega \leqslant \omega_0: \quad s=0 \text{ and } 0<h^* \leqslant 1.$$
$$\omega_0\beta H'\left(\omega_0 h^*\right)\geqslant \omega u'\left(\omega\left(1-h\right)+G+\sigma\right)$$

$$\omega > \omega_0: \quad h=0 \text{ and } s^*>0.$$
$$\beta H'\left(s^*\right)=u\left(\omega+G+\sigma-s^*\right).$$

This yields the following supply functions:

$$h=h\left(\omega, G_1+\sigma\right)$$

or

$$s=s\left(\omega+G_2+\sigma\right).$$

Note that the subsidy and the gift have the same effect, but the subsidy is flat whereas the gift varies with w. We can also introduce the children's indirect utility functions (without the altruistic component):

$$u_1^D = u\left(\omega\left(1-h\left(\omega_1 G_1\sigma\right)\right)+G_1+\sigma\right) = \underset{+\ \ +\ \ +}{u_1^D\left(\omega, G_1, \sigma\right)}$$

and

$$u_2^D = u\left(\omega+G_2+\sigma-s\left(\omega_1 G_2, \sigma\right)\right) = \underset{+\ \ +\ \ +}{u_2^D\left(\omega, G_2, \sigma\right)}.$$

where the signs of the partial derivatives are given under each argument for well-behaved utility functions.

With a log-utility:

$$h^* = \frac{\beta}{1+\beta}\left(1+\frac{G+\sigma}{\omega}\right) \quad \text{if } G+\sigma\leqslant \omega/\beta$$
$$\text{if } G+\sigma > \omega/\beta$$

$$s^* = \frac{\beta}{1+\beta}\left(\omega+G+\sigma\right).$$

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We can also write the consumption of the child with a dependent parent:

$$c_i^D = \frac{\beta}{1+\beta}\left(\omega + G + \sigma\right).$$

It is the same for the two regimes. The consumption of the child when parent is healthy is trivial as it involves no choice:

$$c_i^N = \omega + G_i.$$

3.2   Parent's choice without nursing home

Given the above supply function $h^*\left(\omega, \sigma + G\right)$ and $s^*\left(\omega + \sigma + G\right)$ the parent of a child with productivity w maximizes

$$V_1 = v\left(I - G\right) + \pi\left[H\left(\omega_0 h^*\right) - D\right]$$

or

$$V_2 = v\left(I - G\right) + \pi\left[H\left(s^*\right) - D\right].$$

This yields two supply functions G_1^* and G_2^* depending on whether $\omega \lessgtr \omega_0$ and also two indirect utility functions:

$$V_1^* = \quad V_1^*\left(\omega, \sigma\right) \\ \qquad\qquad\;\; \text{-}\;\;\, + $$

and

$$V_2^* = \quad V_2^*\left(\omega, \sigma\right) \\ \qquad\qquad\;\; +\;\;\, + \quad .$$

In the log-utility case, these optimal gifts can be written as:

$$G_1^* = Max\left[0, Min\left(\frac{\pi I - \omega - \sigma}{1 + \pi}, \frac{\omega}{\beta} - \sigma\right)\right]$$

and

$$G_2^* = Max\left[0, \frac{\pi I - \omega - \sigma}{1 + \pi}\right].$$

The values of $m\left(= \omega_0 h \text{ or } s\right)$, G and V can be represented as above along the w-axis. There is only one difference which comes from the presence of a lump-sum transfer. There is a value of $w = \frac{\beta\sigma}{1 - t}$ below which the child devote all his time to his dependent parent and the latter does not find useful to make any gift because even with $G = 0$, $h = 1$.

In any case in this paper we assume that the range of (w_-, w_+) is such that the utility of the parent is first declining and than increasing as on Figure 5.

Figure 5

V

$\tilde{w}_1$ $\tilde{w}_2$

0   w_- $\dfrac{\beta\sigma}{1-t}$ $\underline{w}$ $\dfrac{\omega_0}{1-t}$ $\overline{w}$ w_+ w
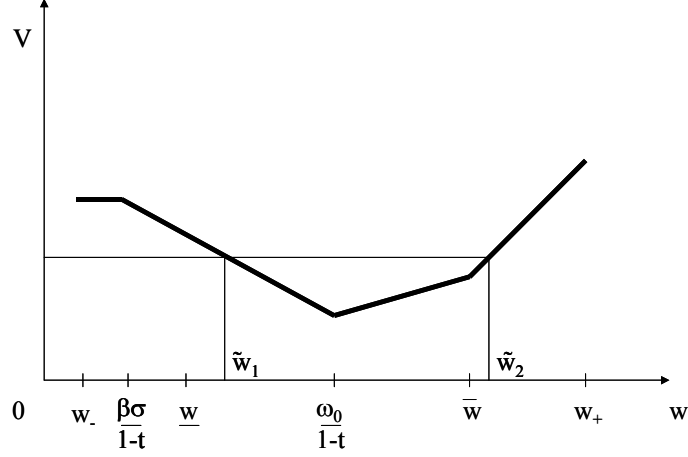
3.3 Parent's choice with public nursing home

Let us now introduce the possibility for the parent to go to the nursing home. This decision is taken in stage 1 and implies that he does not leave any gift, but also, that in case of bad health, he does not benefit from any filial assistance.

We denote by 4 the regime where children don't help their dependent parents and it is bounded by two levels of wage: $\tilde{w}_1$ and $\tilde{w}_2$.

The first one $\tilde{w}_1$ is determined by the equality between $V_1(\underset{-}{\omega},\underset{+}{\sigma})$ and $V_4 = V_4(g) = v(I) + \pi[H(g) - D]$. Similarly, $\tilde{w}_2$ is determinant by the equality between $V_2(\underset{+}{\omega},\underset{+}{\sigma})$ and $V_4(g)$. There are values of g that are so low that the parent would never choose to go to the public nursing home. This appears clearly on Figure 5.

From the equalities $V_1 = V_4(g)$ and $V_2 = V_4(g)$, we can write:

$$\tilde{w}_1 = \frac{1}{1-t}\varphi_1(\underset{+}{\sigma}, \underset{+}{g}) \quad \text{and} \quad \tilde{w}_2 = \frac{1}{1-t}\varphi_2(\underset{-}{\sigma}, \underset{+}{g}) > \tilde{w}_1.$$

We denote by n_4 the fraction of parents opting for the public nursing home.

$$n_4 = F(\tilde{w}_2) - F(\tilde{w}_1) = n_4(\underset{?}{t},\underset{-}{\sigma},\underset{+}{g}).$$

The effect of t in n_4 is ambiguous. Indeed

$$(1-t)^2 \frac{\partial n_4}{\partial t} = F'(\tilde{w}_2)\varphi_2 - F'(\tilde{w}_1)\varphi_1.$$

With a uniform density, given that $\varphi_2 > \varphi_1$, the number of dependent parents going to nursing homes increases with the tax rate, namely $\dfrac{\partial n_4}{\partial t} > 0.$

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3.4   The revenue constraint

The government collects a proportional payroll tax on children's earnings and use it to finance both subsidy and nursing homes. The labor supply of workers with productivity higher than $\tilde{w}_1$ is 1; that of workers with productivity below $\tilde{w}_1$ is $(1 - h^*)$ or 1 depending on whether or not their parents are dependent. Let us introduce the parameter q that reflects the cost of providing nursing home services. We expect that $q > 1$, which implies some inefficiency. The revenue constraint can be written as

$$\varphi\left(t, \sigma, g\right) = t\bar{y} - \pi\left(1 - n_4\right)\sigma - n_4 qg = 0$$

where

$$\bar{y} = \left(1 - \pi\right)\bar{\bar{w}} + \pi \int_{w_-}^{\tilde{w}_1} w\left(1 - h^*\right) dF\left(w\right) + \pi \int_{\tilde{w}_1}^{w_+} w dF\left(w\right).$$

In this expression $\bar{y}$ and $\bar{\bar{w}}$ are respectively average income and average wage; h^*, $\tilde{w}_1$, $\tilde{w}_2$ and n_4 are functions of policy tools.

For further use we can derive $\varphi\left(t, \sigma, g\right)$ with respect to its three arguments:

$$\varphi_t = -\pi[\int_{w_-}^{\tilde{w}_1} w\underset{-}{\frac{\partial h^*}{\partial t}} d\ F\left(w\right) + \tilde{w}_1 h^* \underset{+}{\frac{d\tilde{w}_1}{dt}}] - \pi\left(gq - \sigma\right)\underset{+}{\frac{\partial n_4}{\partial t}}$$

$$\varphi_\sigma = -\pi[\int_{w_-}^{\tilde{w}} w\underset{+}{\frac{\partial h^*}{\partial \sigma}} d\ F\left(w\right) + \tilde{w}_1 h^* \underset{+}{\frac{d\tilde{w}_1}{d\sigma}}] - \pi[\left(1 - n_3\right) + \left(gq - \sigma\right)\underset{-}{\frac{\partial n_4}{\partial \sigma}}]$$

$$\varphi_g = -\pi\ \tilde{w}_1\ h^* \underset{-}{\frac{d\tilde{w}_1}{dg}} - \pi[n_4\theta + \left(gq - \sigma\right)\underset{+}{\frac{\partial n_4}{\partial g}}].$$

The signs below the derivative hold for general utility functions. Only those pertaining to h^* rest on the logarithmic case.

4   Optimal policy

4.1   Unconstrained first-best

As a benchmark we first consider the resource allocation that a social planner would implement if he had perfect information and full control of the economy. The objective that we find appropriate is the sum of individual

utilities after having removed the altruistic component from the children's utility. In other words we consider a social welfare function:

$$SW = \int_{w_-}^{w_+} \{\pi \left[u\left(c^D\right) + v\left(d^D\right) - D + H\left(h\omega_0 + s + g\right)\right]$$
$$+ \left(1 - \pi\right)\left[u\left(c^N\right) + v\left(d^N\right)\right]\} d\, F\left(w\right).$$

This view is not properly utilitarist. Yet, if we were adding individual utilities this would amount to weight the welfare of the elderly people by $(1 + \beta)$ and not by 1.[5]

The first-best implies the equality of marginal utilities of consumption: $u'\left(c^D\right) = v'\left(d^D\right) = u'\left(c^N\right) = v'\left(d^N\right)$. It also implies that the best long term care technology is used; this involves using the contribution of children with $w < \omega_0$. Finally, we should have $u'\left(c^D\right) = H'\left(h\omega_0 + s + g\right)$, knowing that these three arguments are mutually exclusive.

4.2 Second-best optimality

We now turn to a second-best setting with imperfect information and restricted policy tools; namely linear taxation, lump-sum, but conditional subsidy and public nursing homes.

We write the problem of the government with the following Lagrangean expression.

$$\mathcal{L} = \int_{w_-}^{\hat{w}_1} \left(\bar{u}_1 + V_1\right) dF\left(w\right) + \int_{\hat{w}_1}^{\hat{w}_2} \left(\bar{u}_4 + \tilde{V}_4\right) dF\left(w\right) + \int_{\tilde{w}_2}^{w_-} \left(\bar{u}_2 + \check{V}_2\right) dF\left(w\right)$$
$$- \mu\left[\left(1 - n_4\right)\pi\sigma + n_4\pi qg - t\bar{y}\right].$$

where the $\bar{u}_i$ denotes the child's indirect utility net of the altruistic component.

$$\bar{u}_1 = \pi u\left(w\left(1 - t\right)\left(1 - h^*\left(w\left(1 - t\right), G_1^* + \sigma\right) + G^* + \sigma\right) + \left(1 - \pi\right)u\left(w\left(1 - t\right) + G_1^*\right)$$

$$\bar{u}_2 = \pi u\left(w\left(1 - t\right) + G_2^* + \sigma - s\left(w\left(1 - t\right), G_2^* + \sigma\right)\right) + \left(1 - \pi\right)u\left(w\left(1 - t\right) + G_2^*\right)$$

$$\bar{u}_4 = u\left(w\left(1 - t\right)\right).$$

We now derive the FOC:

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[5] See on this Hammond (1997), Cremer and Pestieau (2001).

$$\frac{\partial \mathcal{L}}{\partial t} = (\int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+}) \left[\pi \left(1 - \beta \right) H'\left(m \right) \frac{\partial m^*}{\partial t} + \left(\bar{u}'\left(c \right) - v'\left(d \right) \right) \frac{\partial G^*}{\partial t} \right]$$

$$dF\left(w \right) - \int_{w_-}^{w_+} \left[\pi u'\left(c^D \right) y^D + \left(1 - \pi \right) u'\left(c^N \right) y^N \right] dF\left(\omega \right) + \Delta_1 \frac{d\tilde{w}_1}{dt} + \Delta_2 \frac{d\tilde{w}_2}{dt}$$

$$+ \mu \left[\bar{y} + t \frac{\partial \bar{y}}{\partial t} - \pi \left(qg - \sigma \right) \frac{\partial n_4}{\partial t} \right] = 0. \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \sigma} = (\int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+}) \left[\pi \left(1 - \beta \right) H'\left(m \right) \frac{\partial m^*}{\partial \sigma} + \pi u'\left(c^D \right) + \left(\bar{u}'\left(c \right) - v'\left(d \right) \right) \frac{\partial G}{\partial \sigma} \right] dF\left(w \right)$$

$$+ \Delta_1 \frac{d\tilde{w}_1}{d\sigma} + \Delta_2 \frac{\partial \tilde{w}_2}{\partial \sigma} - \mu \left[t \frac{\partial \bar{y}}{\partial \sigma} - \pi \left(qg - \sigma \right) \frac{\partial n_4}{\partial \sigma} - \left(1 - n_4 \right) \pi \right] = 0. \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial g} = n_4 \, \pi \, H'\left(g \right) + \Delta_1 \frac{d\tilde{w}_1}{dg} + \Delta_2 \frac{\partial \tilde{w}_2}{\partial g}$$

$$- \mu \left[\pi q n_4 - \pi \left(qg - \sigma \right) \frac{\partial n_4}{\partial g} \right] = 0. \tag{3}$$

where $\Delta_1 = \bar{u}_1\left(\tilde{w}_1 \right) - u\left(\left(1 - t \right) \tilde{w}_1 \right)$ and $\Delta_2 = u\left(\left(1 - t \right) \tilde{w}_2 \right) - \bar{u}_1\left(\tilde{w}_2 \right)$ denote the difference utility for the child with productivity $\tilde{w}_i$ between helping his parent or not. As the choice is made by the parent, we cannot sign these two differences.

To interpret equations (1) and (2) we combine them as follows: $\frac{\partial \mathcal{L}^c}{\partial t} = \frac{\partial \mathcal{L}}{\partial t} + \frac{\bar{y}}{\pi \left(1 - n_4 \right)} \frac{\partial \mathcal{L}}{\partial \sigma}$.

$$\frac{\partial \mathcal{L}^c}{\partial t} = \overset{[1]}{\left(\int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+} \right) \left((1-\beta)\pi H' \frac{\partial m^c}{\partial t} \right.} + \overset{[2]}{\left. [\bar{u}' - v'] \frac{\partial G^c}{\partial t} \right) dF(w)}$$

$$\overset{[3]}{- \sum_{D,N} \pi_j cov \left(u'\left(c^j\right), y^j \right)} + \overset{[4]}{(1-\pi)\bar{y}^N E\left[u'\left(c^D\right) - u'\left(c^N\right) \right]}$$

$$\overbrace{- \bar{y}n_4 \left[\int_{\tilde{w}_1}^{\tilde{w}_2} u'\left(c^D\right) dF(w) - \left(\int_{w_-}^{\tilde{w}_1} + \int_{\tilde{w}_2}^{w_+} \right) u'\left(c^D\right) dF(w) \right]}^{[6]}$$

$$\overbrace{+ \Delta_1 \frac{d\tilde{w}_1^c}{dt} + \Delta_2 \frac{d\tilde{w}_2^c}{dt}}^{[7]}$$

$$\overset{[5]}{+ \mu \left[t \frac{\partial \bar{y}^c}{\partial t}} - \overset{[8]}{\pi (gq - \sigma) \frac{\partial n_4^c}{\partial t} \right]} = 0 \tag{4}$$

We now interpret the tax transfer formula (4) and the formula for g (3). But first we consider the case where g is not available.

4.2.1 The case with $g = 0$

Then $n_4 = 0$, and one can rewrite (4) as:

$$t = \frac{(1-\beta)\pi E H'(m)\frac{\partial m^c}{\partial t} + E[\bar{u}'(c) - v'(d)]\frac{\partial G^c}{\partial t} - \sum\limits_{j=D,N} \pi^j cov\left(u'\left(c^j\right), y^j \right) + (1-\pi)\bar{y}^N E\left[u'\left(c^D\right) - u'\left(c^N\right) \right]}{-\mu \frac{\partial \bar{y}^c}{\partial t}}.$$

$$\tag{5}$$

To interpret formula (5) we consider each of its components (defined in (4)).

The first term [1] in the numerator reflects the paternalistic action of the social planner. If $\beta = 1$, namely if the social planner and children have the same view on the parents' utility, this term vanishes. For $\beta < 1$, both t and σ are desirable if $\frac{\partial m^c}{\partial t} > 0$. In other words, if the tax-transfer policy encourages assistance and if the social planner puts more weight on the parents than the children, this policy should be encouraged. Yet one cannot excludes $\frac{\partial m^c}{\partial t} < 0$ in which case, a paternalistic government will choose a lower tax transfer than if it were not paternalistic. Using the logarithmic example, we see that $\frac{\partial m^c}{\partial t}$

is positive for $w < Max \left[\dfrac{\omega_0}{1-t}, \dfrac{\bar{y}}{\pi} \right]$. Roughly speaking, if the majority of children have a low productivity, namely a w below $\dfrac{\omega_0}{1-t}$ and $\dfrac{\bar{y}}{\pi}$, one can expect the tax-transfer policy to stimulate m. Then, the first term in the numerator of (5) is positive if $\beta < 1$.

The second term [2] reflects the effect of the tax-transfer on gifts, that expectedly narrow the difference between the marginal utilities of children and parents. If the tax-transfer package induces additional gift, then it will be higher for that reason. We know that $u'(c) > v'(c)$. With the log utility, $\dfrac{\partial G^c}{\partial t}$ is positive for $w > \bar{y}/\pi$. It is thus negative if the majority of children have a productivity below that threshold.

The third term [3] is made of covariances. It expresses the traditional equity consideration. The covariances are negative and they increase (in absolute value) with the concavity of $u(c)$ and the inequality of w. As it appears, there is a covariance for each state of nature. This is the traditional equity term that one finds in the literature on linear income tax.

The fourth term [4] in the numerator depends on the gap between children's consumption levels in the two state of nature. To the extent that $c^D < c^N$, this term pushes for relatively higher tax-transfers.

It is interesting to observe that we have here a number of sources of inequality: wage inequality, inequality between children with and without dependent parents, inequality among parents leaving different gifts. For the first two, we have some redistribution. Not for the last one.

Finally we come to the denominator [5]. It represents the traditional efficiency term. If the tax transfer policy has a low incidence on the tax base, it will be relatively high. Using the log linear illustration, it clearly appears that $\dfrac{\partial \bar{y}^c}{\partial t}$ is negative. This is because both the tax on earnings and the subsidy tend to foster h and thus to discourage market labor supply.

To sump up, assuming that the majority of children have a productivity below the average, the only term pushing for a low level of tax transfer is the second one. The tax then depresses gifts which contribute to the redistribution between parents and children. All the other terms push for a positive level of tax-transfer.

4.2.2 The case with $g > 0$

We now have to enlarge our tax formula. Using the notation of equation (4)

$$t = \frac{[1] \ + \ [2] \ - \ [3] \ + \ [4] \ - \ [6] \ + \ [7]}{- \ [5] \ + \ [8]}. \tag{6}$$

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We have to discuss the contribution of three additional terms $[6]$, $[7]$ and $[8]$ to the optimal tax transfer. Term $[6]$ gives the difference in children's marginal utility when they help their dependent parent or when they have them in a nursing home (in this case $c^D = c^N$). If this difference is positive (which is likely if the majority of children have an income below $\tilde{w}_1$), term $[6]$ has a positive effect on t.

Term $[7]$ gives the effect that the tax-transfer has on the bounds $\tilde{w}_1$ and $\tilde{w}_2$ each being weighted by the change in utility the child incurs going from one regime to another. We know that $\dfrac{d\hat{w}_1^c}{dt} > 0$; the sign of $\dfrac{d\hat{w}_2^c}{dt}$ is ambiguous. Suppose that it is positive as well. If Δ_1 is positive (the child is better off in regime 4 than in regime 1) and Δ_2 is also positive (the child is better off in regime 2 than in regime 4), then this term will have a positive influence on t.

The final term $[8]$ is the revenue cost of changing n_4. Increasing n_4 implies more spending on public nursing homes, but less in subsidies to children. Not surprisingly, if $gq > \sigma$, which is expected (particularly with a large q), and if $\dfrac{\partial n_4^c}{\partial t} > 0$, which is reasonable, this term is positive and it pushes for a lower tax.

Equation (3) gives the formula for the optimal determination of g. The marginal benefit (g for n_4 parents plus the effect on the two bounds) is equal to the marginal cost (the cost of providing g to the n_4 parents plus the effect of shifting from subsidy to nursing homes). The role of the efficiency parameter q is clearly important.

4.3   Private insurance versus public nursing home

Quite clearly, in this paper, private insurance and public nursing home are two ways for the parents to opt out of the family solidarity. The difference is that public nursing home is free whereas private insurance costs them a premium. Does it mean that in such a setting there is no earn for private insurance? Not really. If $q > 1$ is very high and θ relatively low, the optimal g may be lower than the optimal a.

5   Conclusion

The purpose of this paper was to design an optimal tax transfer policy for long term care. The setting is relatively simple. Each elderly person has an altruistic child who will help him in case of loss of autonomy. Help can be of two types: time for low productivity children, cash for high productivity

children. To foster help from their children parents can *ex ante* make a gift to their children. The government can subsidize children's assistance. But it can also directly provide the services of nursing homes. Middle productivity children tend to rely on nursing homes, but in that case they don't receive anything from their parents. Private insurance appears to be a substitute for public nursing homes, but not for children's assistance.

The case of public nursing homes is quite strong, particularly when private long term care insurance is inefficient. The case of subsidy for either type of assistance is not clear. For redistributive reason, a scheme of tax-subsidy is desirable as it narrows down some differences in consumption. At the same time, it can have undesirable effects on some type of assistance and on the level of *inter vivos* gifts. To clear this ambiguity one has to know more about the distribution of w, the level of I and the concavity of the utility function.

Two questions can be raised in conclusion. Is it realistic? Is it not too simplistic? The two questions are naturally related. As mentioned in the introduction, the problem of long term care is very complex. It is also relatively new. There is no much evidence on the socio-economic characteristics of people suffering from loss of autonomy and on those of their close relatives. If productivity was the only distinctive factor, the pattern discussed in this paper could be quite natural. There are however other characteristics. For example, altruism is not uniform across families. Some elderly don't even have children to care about them. Introducing differential altruism along with differential productivity could complicate the model. It is for example clear that in that case public nursing homes would cater not only to parents of middle productivity children with altruism, but also to all parents with non altruistic children. In this case we are faced with a moral hazard problem if altruism cannot be observed (see Jousten *et al.* (2003)). Another difficulty that we have assumed away is that loss of autonomy may not be observable. This leads to another moral hazard problem as it is tempting for healthy parents to mimick unhealthy parents. Again this would add an additional constraint to the design of an optimal tax-transfer scheme.

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Appendix

In the main text we have chosen values of I and ω_0 that cover the different regimes we were interested in. It is important to see that this is not necessarily the case.

With the log-linear utility function, we have the following values for the optimal gift:

$$G_1^* = Max\left(0, Min\left(\frac{\pi I - \omega - \sigma}{1 + \pi}, \frac{\omega}{\beta} - \sigma\right)\right) \qquad (A.1)$$

$$G_2^* = Max\left(0, \frac{\pi I - \omega - \sigma}{1 + \pi}\right) \qquad (A.2)$$

and we define two thresholds values of ω, $\underline{\omega}$ and $\bar{\omega}$.

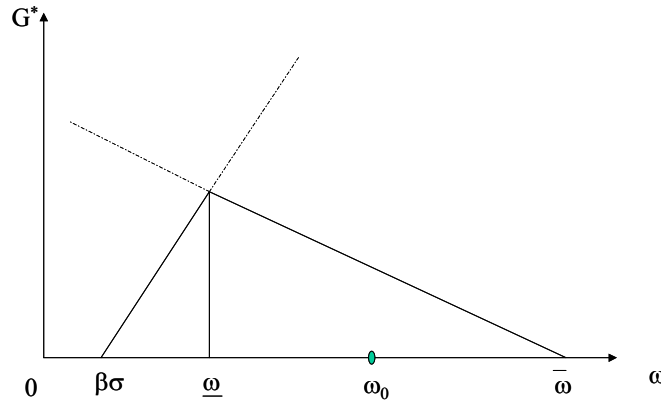$$\underline{\omega} = \frac{\beta\pi}{1 + \pi + \beta}(I + \sigma)$$

and

$$\bar{\omega} = \pi I - \sigma.$$

$\underline{\omega}$ is the value of ω at which the two terms in the Min expression of $(A.1)$ coincide and $\bar{\omega}$ is the value of ω above which $G^* = 0$.
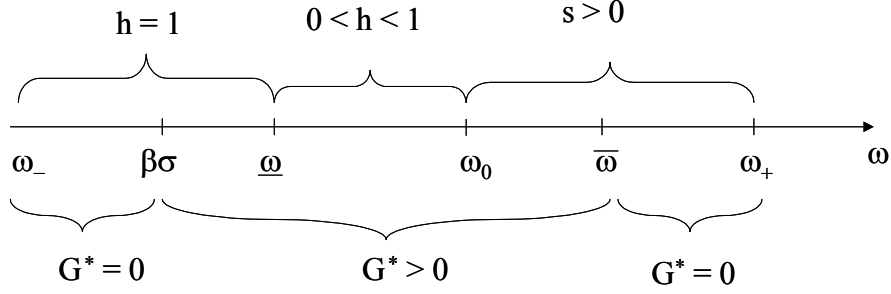
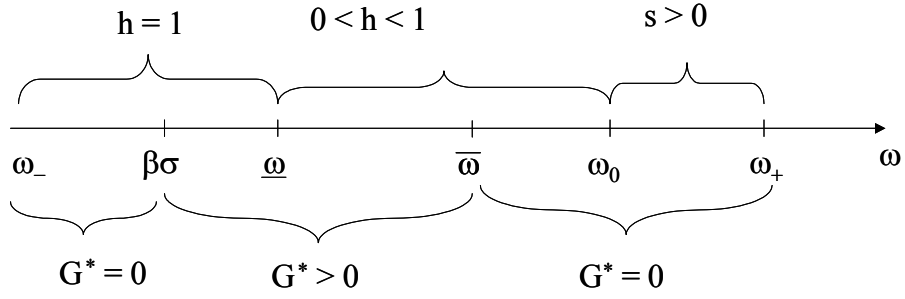Graphically we can represent G^* on Figure A.1.

Figure A.1

with ω_0 between $\underline{\omega}$ and $\bar{\omega}$.

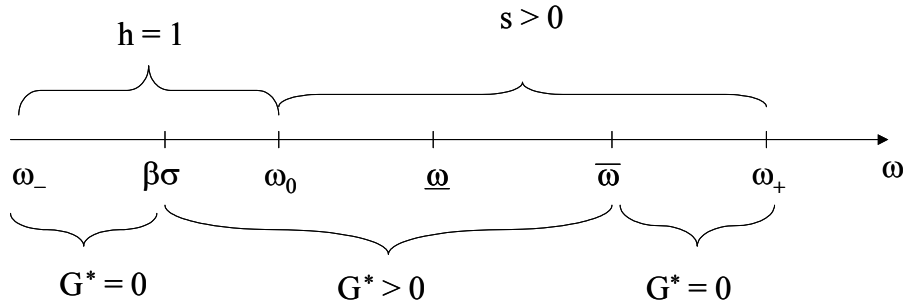In fact we can distinguish 3 cases depending on the respective values of I and ω_0.

1. $\omega_0 < \bar{\omega}$ or $I > \dfrac{\omega_0 + \sigma}{\pi}$

$h = 1$ $0 < h < 1$ $s > 0$

ω_- $\beta\sigma$ $\underline{\omega}$ ω_0 $\overline{\omega}$ ω_+ ω

$G^* = 0$ $G^* > 0$ $G^* = 0$

2. $\omega_0 > \bar{\omega}$ or $I > \dfrac{\omega_0 + \sigma}{\pi}$

$h = 1$ $0 < h < 1$ $s > 0$

ω_- $\beta\sigma$ $\underline{\omega}$ $\overline{\omega}$ ω_0 ω_+ ω

$G^* = 0$ $G^* > 0$ $G^* = 0$

3. $\underline{\omega} > \omega_0$ or $I > \dfrac{1 + \pi + \beta}{\beta\pi}\omega_0 - \sigma$

$h = 1$ $s > 0$

ω_- $\beta\sigma$ ω_0 $\underline{\omega}$ $\overline{\omega}$ ω_+ ω

$G^* = 0$ $G^* > 0$ $G^* = 0$

Note that we assume that $\beta\sigma < \omega_0$ or $\sigma < \dfrac{\omega_0}{\beta}$. We can represent these three cases in the plane (ω, I) on Figure A.2.

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Figure A.2

$s^* > 0$

$G^* = 0$

$G^* = 0$

$\bar{\omega} = \pi I - \sigma$

$G^* = \dfrac{\pi I - \omega - \sigma}{H\pi}$

$\omega = \dfrac{\beta \pi (I + \sigma)}{H\pi + \beta}$

ω_0

$G^* = \dfrac{\omega}{\beta} - \sigma$

and $h^* = 1$

βG

$h^* = 1$
and
$G^* = 0$

I_2

I_1

I_3

0

G/π

$I = \dfrac{H\beta}{\pi} G$

$\dfrac{\omega_0 + \sigma}{\pi}$

$\dfrac{1 + \pi + \beta}{\beta \pi}$

I
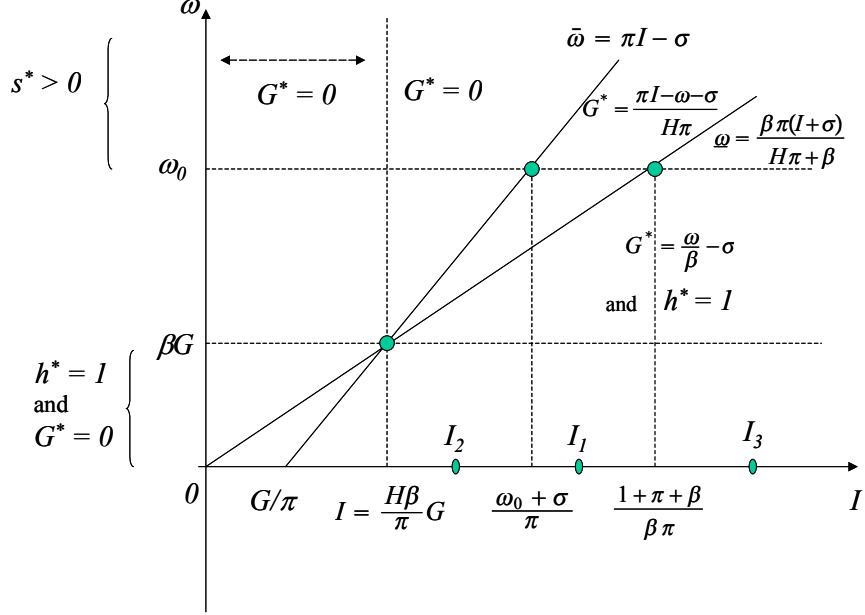
In this paper we assume that for a given value of ω_0, I is in the interval $\left(\dfrac{\omega_0 + \sigma}{\pi}, \dfrac{1 + \pi + \beta}{\beta \pi} \omega_0 - \sigma \right)$ which corresponds to case 1 or to a value of I given by I_1 on Figure A.2.

For a value I_2, we have case 2 and for a value I_3 we have case 3.