Medium Run Redux

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Abstract: Differences in the medium-run performance of continental Europe and ÄAnglo-Saxon" economies have been the subject of much debate. This has been motivated by the persistently high levels of unemployment in continental Europe, the nature of its factor income shares and factor substitutability and its response to various supply and demand shocks from the early 1970s onwards. Our paper tries to confront explanations of this "medium-run" performance in a consistent, coherent optimizing framework. On the supply side, we estimate using a normalized CES function with time-varying factor-augmenting technical progress. Whilst, on the demand side, after including factor adjustment costs, we derive dynamic forward-looking factor demand equations compatible with this long-run system. We find that the elasticity of substitution lies below unity (around 0.6), that technical progress is such that the growth contribution of labor-augmenting technical progress is constant, while that of capital is continuously decreasing. Consequently, and consistent with our preferred "medium-run" concept, capital-augmenting technical progress plays an important but declining role in total factor progress. Furthermore, in an imperfectly-competitive framework, introducing changing sectoral shares as an explanation for a rising markup (and hence declining labor share), and accounting for differing technical progress in factor efficiencies, we can model the distribution of time-varying factor incomes shares in the euro area.

Keywords: Medium Run, Elasticity of Substitution, Supply-Side System, Factor Augmenting Technical Progress, Factor Income Distribution.

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1. Introduction

The stability of factor income shares has generally considered as a stylized fact that describes economic developments relatively well in most economies. However, during the last 30 years the development of income shares in many countries of continental Europe has not been compatible with this "fact". To illustrate, after increasing strongly in the 1970s, the GDP share of labor income in the euro area has continuously decreased during the subsequent two decades (Figure 1). In this respect development in the US, or more generally in "Anglo-Saxon" countries, has differed being broadly in line with the stylized fact of the stable labor income share.

Blanchard (1997) and Caballero and Hammour (1998) were among the first to pay serious attention to these differences. Through differences in the labor markets and wage formation in Europe and the US, they linked the differences in the development in income shares to another striking difference in the US and European development, i.e. the high and highly persistent unemployment rate in continental Europe as opposed to the largely stable US case. Additional elements included by them to explain the dynamics of European development, were the oil price shocks of 1970s coupled with the slow adjustment of capital, low short-run but high long-run substitutability between capital and labor.

At the outset, such considerations cast doubt on the suitability of the popular (unitary elasticity) Cobb-Douglas production function, for two principle reasons.

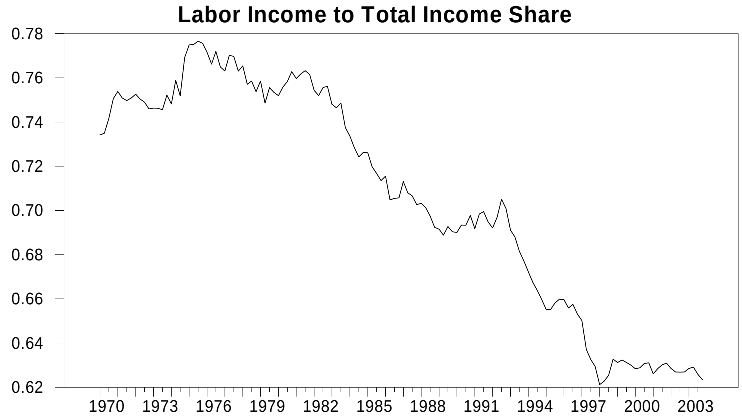
First, such a function implies that marginal labor costs are proportional to nominal unit labor costs. Coupled with a constant price elasticity of demand, this means that the output price should depend on nominal unit labor cost with a unit elasticity. This, in turn, implies, as a tautology, that real unit labor costs (or labor income share) should be stationary (or at least trendless). Hence, the humped-shaped (non-stationary) pattern of the labor-income share observed at the euro-area level contradicts that framework.

Second, Cobb-Douglas of course implies constant factor income shares. This property meets the essential condition for a steady state in neoclassical growth models and accords with the most prominent of the empirical stylized facts of long-term economic development: the relative stability of factor income distribution despite a secular rise in capital intensity and per-capita income. It also follows immediately that the direction of technical change is irrelevant for income distribution in the Cobb-Douglas world. It is thus impossible to determine empirically any bias in the direction of technical change. In contrast, pronounced trends or shifts in factor income distribution visible in many countries over what Blanchard (1997) called the "medium run" support the more general CES function and make possible biases of technical progress an important issue.

Many articles tried to explain this coincidence of rising unemployment and a hump-shaped behavior of factor income share in continental Europe with the help of models that incorporate particular assumptions about factor substitution and technological change. Caballero and Hammour (1998), Blanchard (1997) and Berthold *et al.*, (2002) assume a production technology with purely labor-augmenting technical progress and a relatively high (above unity) elasticity of substitution in the long run, with short-run putty-clay characteristics. Consequently, for example, a wage-push shock would lead at first to only a small decline in employment and an increase in the labor-income share. In the long run, however, labor is replaced over-proportionally by capital and, with rising capital intensity, the labor-share will fall again. Critics of this line of explanation argued that Europe also experienced a decline in capital formation since the 1970's. Declining capital intensity, however, can cause a decline in employment and a rise in the capital income share only if the elasticity of substitution does not exceed unity, Rowthorn (1999).[1] As an alternative explanation, Acemoglu (2002, 2003) suggests an elasticity of substitution below unity. Thus, oil crises, coupled with rigid labor markets may have induced persistent (albeit transient) capital-augmenting technical progress; attractive features of such a framework is that it coincidences asymptotically with the usual balanced growth condition of purely labor-augmenting technical progress.

[1] We can briefly mention some other important papers in this area. Bentolila and Saint-Paul (1998) introduces changes in the relative price of imported materials, in the skill mix, in union bargaining power of labour unions or in current and expected adjustment costs as possible factors affecting the development of the labour income share. Alcalá and Sancho (2000) find out quite similar time profiles of European inflation and the labour income share and suggest inflation for a proxy for uncertainty in explaining the markup. de Serres *et al.* (2000) studied the possible role of aggregation bias due to sectorally differentiated wage shares and conclude that in many countries aggregation bias explains at least part of the decline in labour income share. As a possible source for the observed decrease in the labour income share in 1980's and 1990's Blanchard (1997) mentions, albeit skeptically, also an increase in the markup. In our framework, however, an upward trend in the aggregate euro-area markup is an important element.

Figure 1



Note: Max 0.776 (1975:3), Min 0.621 (1998:1)

Indeed, values of the substitution elasticity above unity appear neither theoretically attractive[2] nor empirically robust. Unitary elasticities, on the other hand, though somewhat more regularly reported, precisely fall foul of the European experience (for the reasons given earlier).[3] To illustrate, Yuhn (1991) surveys many studies for the US economy and reported elasticities not exceeding 0.6. Chirinko *et al.*, (1999)'s cross-section analysis report elasticities ranging from 0.25-0.40. Studies for various euro area countries (e.g., Bolt and van Els [2000], Ripatti and Vilmunen [2001]) similarly suggest a value robustly below one (see also Klump *et al*. [2004, Table 2]).

In this paper, we introduce a framework that treats these various issues discussed in a consistent and coherent manner. This allows us to study and potentially discriminate between alternative "medium-run" views. As our main focus bears on the medium run our optimization framework captures besides the long run, also short-run developments. For instance, an interesting implication of our integrated, optimization framework is that adjustment costs associated with adjusting the labor input have also spillover effects on price setting. Having derived the first order conditions of optimization, we then show that, for empirical purposes, these conditions can be decomposed into non-stationary, long run and stationary, dynamic parts. The key elements of our approach are the following.

First, based on non-stationary conditions, we estimate a rich supply-side system. In our production function, for example, the elasticity of factor substitution (a key parameter in "medium-run" discussions) is not constrained to unity; we estimate using a "normalized" CES function (de La Grandville [1989], Klump and de La Grandville [2000]), the main advantage of which is the removal of the dependency of factor shares parameter from the substitution parameter. This markedly improves estimation and identification. Furthermore, we model technical progress as factor augmenting and time-varying. Our results suggest an elasticity value well below unity (around 0.6) and that the growth contribution of labor-augmenting technical progress is approximately constant. Capital-augmenting technical progress, however, in the medium run imparts a permanent though declining contribution to total factor progress.

Second, to account for stylized features of the euro area data, we allow (as in Willman [2002]) the aggregate-level markup to be time varying. Our theoretic framework contains a multi-sector model of imperfect competition, where output is produced by an otherwise common technology except for the sectorally-differentiated

[2] In the standard neoclassical growth model, Solow (1956), showed that a CES production function with an elasticity of substitution above unity generates perpetual growth since scarce labor can be completely substituted by capital. Thus, the marginal product of capital remains bounded above zero in the long run.

[3] See Rowthorn (1999) and Duffy and Papageorgiou (2000), amongst others, for critiques of the empirical relevance of the Cobb-Douglas function. Furthermore, Antràs (2004) suggests that the Cobb-Douglas finding in many older econometric investigations may be due to an omitted-variable bias caused by the assumption of Hicks-Neutral technical change.

scale and technical progress parameters of the production function. By allowing price and income elasticities to differ across sectors, the aggregation of the firm-level conditions of profit maximization implies that the aggregated-level markup may develop secularly though the markup in each sector remains constant. The development of the aggregated level markup reflects changes in the production shares of sectors with the high or low markup and/or with the fast or slow speed of technical progress. Further, the assumption of non-isoelastic demand curves implies time variant sectoral markups, as also competing foreign prices affect the pricing behavior. This offers an avenue for explaining the hump-shaped development of the labor income share in the euro area.

Finally, and underlying these previous points, we undertake a thorough analysis of aggregate euro-area data and its implied accounting framework. In doing so, we identify a puzzle regarding the relationship between labor-income share (whose declining trend we already noted), capital shares and the aggregate markup. The close, inverse mapping between labor and capital income that we should witness in the data is absent in the 1970s; there is no level shift in the labor-income share corresponding to that in the capital-to-labor ratio at the end of 1970s and in the early 1980s. Our capital income is an imputed concept and thus sensitive to variations in the measured user-cost-of-capital and real interest rate. After the non-stationary part (production function, time-varying aggregate markup parameters), we estimate the dynamic demand equations for factor inputs and the price equation (including the spillover effects from deviation of effective from normal working hours).

The paper proceeds as follows. Section 2 outlines the theoretical model. Section 3 introduces the Normalized Production with Time-Varying Factor Augmenting Technical Progress. Section 4 describes some key features of the euro area data that we seek to explain. This is followed by a discussion of the modeling of the dynamic adjustments in our model. Section 6 discuses the empirical results and, finally, we conclude.

2. The Theoretical Model

2.1 Maximization Problem of the Firm

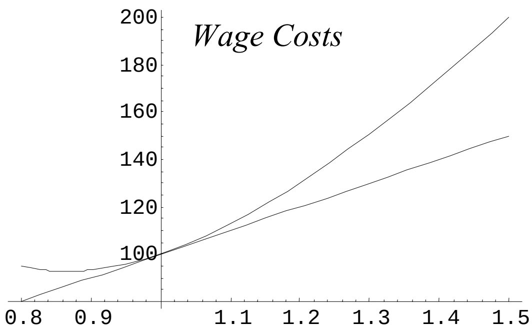
The output of a firm, *Y,* is defined by the production function $F(K_{t,} H_t)$, where *K* is the capital stock, and H_t is effective labor hours defined by identity $H_t = N_t h_t$, *N* the number of employees, and *h* effective working hours per employee. In the spirit of indivisible labor (e.g., Rogerson [1998]) we assume that labor contracts are made in terms of fixed (or normal) working hours per employee (normalized here to unity), $\overline{h} = 1$. If hours exceed normal hours, then they are associated with extra compensation. We further assume that employers have only limited possibilities to decrease paid hours when effective hours fall below normal hours. Hence, total wage costs can be presented as a convex function of the deviation of effective hours from normal hours. The following quadratic function gives a local approximation of this relation in the neighborhood of effective hours equaling normal hours,

$$W_t\left(H_t + a_h \frac{(H_t - N_t)^2}{H_t} \right) \qquad (1)$$

where W_t is the real wage rate of normal working hours which each firm takes as given.

Figure 2 gives a graphical presentation of this relation; the vertical axes presents total wage costs and horizontal axes effective hours per employee. To illustrate, with the number of employees set arbitrarily at 100 and W=1, the linear boundary presents the dependency of total wage costs if deviations of effective hours from normal hours were not associated with extra wage costs, i.e. parameter $a_h = 0$. Convex curvature in wage costs results when $a_h > 0$. It is easy to understand that the higher the curvature parameter a_h, the greater is the incentive to adjust total effective hours, *H*, by changing the number of employees (instead of changing working hours per employee). In fact, if changing the number of employees is costless, all adjustment is done via the number of employees and, independently from the size of a_h, effective hours *H* equals *N* for all periods. However, we believe that in the real world changes in the number of employees are associated with, for example, non-trivial recruitment, training and firing costs.

Figure 2
Wage Costs and Effective Hours



$h\ (\bar{h} = 1)$

Hence, accounting for adjustment cost associated with changes in the number of employees as well as the adjustment costs associated with the capital stock and investment, the real cash flow (*CF*) can be defined as,

$$CF_t = P_tY_t - W_t\left[H_t + a_h\frac{(H_t - N_t)^2}{H_t} + A_N(N_t, N_{t-1})\right] - I_t - A_K(K_t, K_{t-1}, K_{t-2},) \tag{2}$$

Where P_t denotes the real price of output of the firm in terms of aggregate price level (or investment deflator). $A_i(\cdot)$ represents adjustment costs associated with changes in factor *i* (exact functional forms will be defined later).[4] Note, that for capital, we assume that adjustment costs can additionally accommodate changes in the rate of capital stock accumulation; this extra costs essentially reflects time-to-build considerations in the formation of investment. If the firm pays out its net cash flow to its owners each period, then the rate of return in the end of period *t* (or equally at the beginning of period *t+1*) is,

$$1 + r_t^K = \frac{E_t(Q_{t+1})K_t + CF_t}{Q_tK_{t-1}} \tag{3}$$

where *Q* refers to the rental price of capital and E_t denotes the normal expectations operator.

Solving equation (3) recursively forward yields the expected value of the firm (V_t),

$$V_t = Q_tK_{t-1} = E_t\sum_{i=0}^{\infty}\prod_{j=0}^{i}\left(\frac{1 + r_{t-1}^K}{1 + r_{t+j-1}^K}\right)CF_{t+i} \tag{4}$$

We assume that firms make their investment, labor demand and pricing decisions maximizing the expected value of (4). Hence, after inserting (2) into (4) and utilizing the financial market equilibrium condition, $r_{t+i}^K - r_{t+i} = 0$, the firm's maximization problem becomes,

[4] Notice, unlike most other studies, we separate the adjustment costs associated with changes in the number of employees and the costs associated changes in the effective hours per worker. This is natural to introduce in our "medium-run" context since there is considerable evidence that labor-adjustment costs in the euro area are higher than in other comparable countries, e.g., McMorrow (1996).

$$Max\ V_t = E_t \sum_{i=0}^{\infty} \prod_{j=0}^{i} \left(\frac{1 + r_{t-1}}{1 + r_{t+j-1}} \right) \left\{ \begin{array}{l} P_{t+i} Y_t - W_{t+i} \left[H_{t+i} + \frac{a_h}{2} \frac{\left(H_{t+i} - N_{t+i}\right)^2}{H_{t+i}} + A_N\left(N_{t+i}, N_{t-1+i}\right) \right] \\ - I_{t+i} - A_K\left(K_{t+i}, K_{t-1+i}, K_{t-2+i}\right) \end{array} \right\} \tag{5}$$

subject to the constraints set by the production function (6), the downward-sloping demand function (7) and the law of motion for capital (8),

$$Y_t = F\left(K_t, H_t\right) \tag{6}$$

$$P_t = P\left(Y_t\right) \tag{7}$$

$$K_t = \left(1 - \delta\right) K_{t-1} + I_t \tag{8}$$

where $0 < \delta \leq 1$ is the depreciation rate. Taking the FOCs with respect to Output, Effective Hours, Labor, Capital and the Lagrangean multiplier Λ, we obtain,

$$P_t \left(1 + \frac{\partial P / \partial Y}{P/Y} \right) - \Lambda = 0 \tag{9}$$

$$-W_t \left[1 + a_h \frac{\left(H_t - N_t\right)}{H_t} - \frac{a_h}{2} \frac{\left(H_t - N_t\right)^2}{H_t^{\,2}} \right] + \Lambda \frac{\partial F\left(K_t, H_t\right)}{\partial H_t} = 0 \tag{10}$$

$$-W_t \left[-\frac{a_h\left(H_t - N_t\right)}{H_t} + \frac{\partial A_N\left(N_t, N_{t-1}\right)}{\partial N_t} \right] - \left(1 + r_t\right)^{-1} E_t W_{t+1} \frac{\partial A_N\left(N_{t+1}, N_t\right)}{\partial N_t} = 0 \tag{11}$$

$$\begin{aligned} P_t(1 + \frac{\partial P/\partial Y}{P/Y}) \frac{\partial F(K_t, H_t)}{\partial K_t} - \frac{\partial A_K(K_t, K_{t-1}, K_{t-2})}{\partial K_t} - 1 \\ + E_t\left(1 + r_t\right)^{-1} \left\{ -\frac{\partial A(K_{t+1}, K_t, K_{t-1})}{\partial K_t} + (1 - \delta) \right\} \\ + E_t\left[\left(1 + r_t\right)\left(1 + r_{t+1}\right)\right]^{-1} \left\{ -\frac{\partial A(K_{t+2}, K_{t+1}, K_t)}{\partial K_t} \right\} = 0 \end{aligned} \tag{12}$$

$$F\left(K_t, H_t\right) - Y_t = 0 \tag{13}$$

Conditional on the expected development of real wages and explicit functional forms for the demand function, the production function, the cost function associated to the deviation of total effective working hours from normal working hours and the adjustment cost functions, the system (9)-(13) determines optimal price setting (9 and 10), number of employees (11) and demand for capital inputs (12) and effective hours (13) – all conditional on demand-determined output.

2.2 Supply-Side System

However, for estimation purposes it is useful to separate from the entire supply system, that part determining the non-stationary long-run development. It can be easily seen that equation (11) reduces to $H_t = N_t$, (i.e., normal working hours N_t equals effective hours H_t) when neglecting (albeit temporarily) adjustment costs associated with changes of factor inputs, $A_i = 0$. This simplified system is by definition obtained by. Thus, the five-equation supply-side system (9)-(13) reduces to the following classical three-equation form:

$$\frac{\partial F}{\partial N_t} = (1+\mu)\frac{W_t}{P_t} \tag{14}$$

$$\frac{\partial F}{\partial K_t} = (1+\mu)\frac{r_t+\delta}{P_t(1+r_t)} \tag{15}$$

$$Y_t = F(K_t, N_t) \tag{16}$$

with the markup, $1+\mu = \left(1+\frac{\partial P/\partial Y}{P/Y}\right)^{-1}$.

The derivation of system (14)-(16) is based on the optimization of a single firm. Only under somewhat restrictive assumptions can it be used to represent the aggregate level supply-side system. Therefore, to be able better to track the stylized features of the aggregated euro-area data modify the system by allowing technology level of the production function as well as the demand curves faced by firms to deviate across sectors. Thereafter, we define explicit functional form of the production function (see Section 3). We assume it to be the normalized CES production function allowing for time-varying, factor-augmenting technical progress.

2.3 Sectoral Aggregation [5]

System (14-16), holds on aggregate level only if competition (or monopoly power) faced by all firms is the same for all firms (i.e., μ is the same) *or,* if μ is allowed to vary across firms, then the income elasticity of all goods produced by each firm must be equal (i.e., unity) – implying constant output shares for all firms. If this assumption does not hold, the aggregate level markup necessarily contains a trend (Willman [2002]). In estimating the system, this effect can be captured by adding an additional common trend to equations (9) and (10).

To illustrate, consider an economy with *m* production sectors. In each sector, technology is assumed identical except for the level of technology, which is allowed to be sector specifics. Hence, the linearly homogenous production function of the representative firm in sector i can be written as $A_i F(N_i, K_i) = A_i f(k_i) N_i$, A_i is the technical level parameter in sector i. Optimization implies the firm-level equivalent of system (14-16),

$$P_i = (1+\mu_i)W\left[\frac{\partial F}{\partial N_i}\right]^{-1} = (1+\mu_i)\frac{W}{[f(k_i)-k_i f'(k_i)]} \tag{14'}$$

$$\frac{c}{w} = \frac{f'(k_i)}{[f(k_i)-k_i f'(k_i)]} \tag{15'}$$

[5] For brevity, some steps in the following derivations have been compressed. All proofs are available in Willman (2002), McAdam and Willman (2004).

$$Y_i = A_i f(k_i) N_i \tag{16'}$$

Comparing (14'-16') with the earlier system, we have normalized (14'), for later convenience, on the aggregate price and re-expressed (15) in terms of the ratio of factor prices, i.e., (14) divided by (15).

From the point of view of aggregation, an implication of (15') is that the capital-labor ratio is common across sectors, i.e. $k_i = k \ \forall i$.[6] Let us further define $s_i(t) = Y_i(t) \Big/ \sum_{i=1}^{m} Y_i(t) = Y_i(t)/Y(t)$ as the output share of sector *i*, and the identity $pY = \sum_{i=1}^{m} p_i Y_i$.

Aggregating (14') and (16') results in an otherwise similar aggregate level relations except that the aggregate markup and the technology level contain a time-varying component depending on the developments of sectoral output shares. Further, it can be shown that changes in sectoral production shares can be reduced to a single composite trend variable if income elasticities of the demand for goods produced by each sector differ across those sectors.

Consequently after some linearization the aggregate-level correspondence of (14') and (16') can be presented in log form as,

$$\log P = \log\left(\frac{w}{f(k) - kf'(k)}\right) - \log A + \underbrace{\log(1+\mu_A) + \eta \cdot t}_{mark-up} + \Gamma \cdot t \tag{14''}$$

$$\log\left(\frac{Y}{N}\right) = \log\left(f(k)\right) + \log A + \Gamma \cdot t \tag{16''}$$

Where $A = \left(\sum \bar{s}_i A_i^{-1}\right)^{-1}$; $\mu_A = \sum_{i=1}^{m} AA_i^{-1}\bar{s}_i \mu_i$; $\Gamma \cdot t = \sum_{i=1}^{m} AA_i^{-1}\left(s_i(t) - \bar{s}_i\right)$; and $\eta \cdot t = \sum_{i=1}^{m} \frac{A(\mu_i - \mu_A)}{A_i(1+\mu_A)}\left(s_i(t) - \bar{s}_i\right)$.

Note, that the constancy (or stationarity) of sector-level mark-ups μ_i need not imply the constancy of the aggregate mark-up. The necessary and sufficient condition for constancy, is that the mark-up in each sector i is the same. If this is not true, then changes in output shares are transmitted into the aggregate mark-up.

Moreover, from equation (16''), it can be seen that if technology level differs across sectors, $A_i \neq A_j$ $(i \neq j)$, then the aggregated change of overall technical progress depends on the changes of sectoral production shares. It is straightforward to see that the growth of production shares of sectors i with $A_i > A$ ($A_i < A$) contributes positively (negatively) to the aggregated growth of productivity.

One additional remark. Let us relax the assumption that the price elasticities $\varepsilon^j \ \forall j$ (and hence markups) are constant. This is the case, if, in some sectors – assume for example in the foreign-competing sector (*j*=*x*) – firms face the following AIDS demand function[7]:

[6] To assume otherwise would preclude us deriving an aggregate production function resulting from sectorally-differentiated non-homogeneous labor (at least) across sectors and corresponding sectorally-differentiated wage rates.

[7] In terms of the AIDS expenditure system, the share of country *i* exports in world imports (at current prices) is: $\kappa_i = \frac{P_i^x \cdot X_i}{P_f \cdot D_f} = a_i - \theta_{ii} \cdot \log P_i^x + \sum_j \theta_{ij} \log P_{ij}^x$, where $\sum_i a_i = 1$, $\theta_{ii} = \sum_j \theta_{ij}$ and P_f is a weighted index of export prices P_j^x , i.e. the competing foreign price (see Deaton and Muellbauer, 1980).

$$\kappa = \frac{P^x \cdot Y^x}{P_f \cdot D_f} = a + \theta \cdot \log\left(\frac{P_f}{P^x}\right); a, \theta > 0 \quad (17)$$

where κ is the market share of exporting sector in nominal terms, D_f is world market demand and P_f is the competing foreign price level. Compared to the world market the size of the exporting firm is small, which allows us to treat both world market demand D_f and the price level P_f as exogenous.

We can show that this equation implies a time-varying price elasticity.[8] The markup in the export sector and the log of the markup can be presented as:

$$\log(1+\mu^x) \approx \log(1+\overline{\mu}^x) + \frac{1}{1+\overline{\mu}^x}\log\left(\frac{P_f}{P^x}\right); \overline{\mu}^x = \frac{a}{\theta} \quad (18)$$

The fact that the export-sector markup depends on the competitive pressure of foreign prices allows us to write the economy-wide markup in (14''') as:

$$\log(1+\mu_A) = \log(1+\overline{\mu}_A) + \phi \cdot \log\left(\frac{P_f}{P^x}\right) \quad (19)$$

where ϕ, reflecting the competitors price pressure in the markup is given by $\phi = \frac{s_0^x}{1+\overline{\mu}^x}$, $\overline{\mu}_A$ is the value of μ_A calculated in terms of $\overline{\mu}^x$ and s_0^x is production share of the export (or open) sector in the base (reference) period.

This connection between changing sectoral shares and rising markups, resonates relatively well for the euro-area. It is widely recognized, for example, that across the main industrialized countries, mark-ups measured for the service industries typically exceed those for manufacturing (e.g. Oliveira Martins *et al.* [1996]), given that, amongst other things, manufacturing is subject to more intense (international) competition. Moreover, the most noticeable sectoral shifts in the euro area have been away from manufacturing and towards services. For example, from 1970 to the end of the 1990s the US output share of services has barely changed (from 64.7% to 63.0) whilst that of the euro area moved, and moved in a very broad-based manner, from 51.5 to 61.3 (a 20% increase).

Interestingly, Blanchard (1997) suggests an increase in the aggregate markup as an explanation for declining labor share. He rejects this possibility on the basis that the completion of the Internal Market increased competition in the euro area. However, it should be borne in mind that such deregulation fell largely on sectors that were robustly open to intra- and extra-euro area competition and indeed that de-regulation of Services (as recommended by the European Commission [2004]) is still pending.[9] This outcome, i.e., growth skewed towards

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[8] Demand function (17) implies for the price elasticity, $\varepsilon^x = -1 - \frac{\theta}{\kappa} = -1 - \frac{\theta}{a + \theta \log(P_f / P^x)}$ and for the markup, $1 + \mu^x = \frac{\varepsilon^x}{\varepsilon^x + 1} = 1 + \frac{a}{\theta} + \log\left(\frac{P_f}{P^x}\right)$. To derive (18), we further linearized this later equation around the point $\log\left(\frac{P_f}{P^x}\right)$=0.

[9] Additionally, we could allow the pace of technical progress to differ across sectors, i.e., the production function of the representative firm in sector i would then be $A_i e^{\gamma_i t} f(k_i) N_i$, where γ_i is the pace of technical progress in sector i ($e^{\gamma_i t}$ can, for instance, be thought of as a general flag of firm-specific technical progress, of whatever kind, or a firm-specific Hicks Neutral component). This would introduce an additional trend into the markup equation (14'') as well as the production function itself (16''). However, we excluded this case for two reasons. First, it complicates the algebra with little additional benefit

high markup (essentially closed) services sectors and away from declining, potentially more competitive (open) manufacturing sectors, precisely strengthens our highlighted markup development since the size of the trend parameter, $\sum_{i=1}^{m} \frac{A(\mu_i - \mu_A)}{A_i(1+\mu_A)}$, in the aggregate markup (14') is effectively determined by the "markup gap" between these sectors.

3. Normalization Production with Time-Varying Factor Augmenting Technical Progress

3.1 Normalization of Production functions

In estimating the supply-side system (14-16), our technology assumption is the normalized CES production function allowing for time-varying factor-augmenting technical progress. The idea of normalizing CES functions was explicitly developed by de La Grandville (1989), Klump and de La Grandville (2000). It starts from the observation that a family of CES functions whose members are distinguished only by different elasticities of substitution needs a common fixed (or baseline) point. Since the elasticity of substitution is defined as a point elasticity, one needs to fix baseline values for per capita production, capital intensity and factor income shares (or the marginal rate of substitution). If technical progress is biased in the sense that factor income shares change over time the nature of this bias can only be classified with regard to the baseline values at the given fixed point. In merging the normalization method with the empirical systems approach, we follow Klump *et al.* (2004) and model technical progress with a flexible functional form, which allows the data to discriminate between the different forms of technical progress.

Since the focus of our analysis is on identifying possible biases in technical change, we concentrate on the following specification of the CES production function specification, David and van de Klundert (1965). This is a linear homogeneous CES function with technological change that is augmenting the efficiency of both factors of production:

$$Y_t = [(E_t^N \cdot N_t)^{-\rho} + (E_t^K \cdot K_t)^{-\rho}]^{-\frac{1}{\rho}} \tag{20}$$

where E_t^i represents the levels of efficiency of input factor *i* and $\rho = \frac{1-\sigma}{\sigma}$ is the substitution parameter (with σ the elasticity of substitution). The relationship between the CES production function (20) and the traditional Arrow *et al*., (1961) form which, instead of the two efficiency levels contains a distribution and a single efficiency parameter, has been explored by Klump and Preissler (2000). Both specifications can be regarded as two members of one family of normalized CES productions functions as long as they share the same baseline values of capital (K_0), labor (N_0), output (Y_0), and the marginal rate of substitution, $\frac{\partial Y_0 / \partial N_0}{\partial Y_0 / \partial K_0}$. This implies, that under imperfect competition, two members of one family also share the same fixed point for the distribution parameter $1-\pi_0 = (1+\bar{\mu}_A)\frac{W_0 N_0}{P_0 Y_0}$. [10]

---

since, when it comes to estimation, we would continue to treat such developments in terms of a composite trend (composing sectoral growth and technical level differences as in equation 9a) with a component in the pace of technical progress. Second, the evidence for slower technological change among service industries is more mixed. Though, on average, and in most countries, this would appear to hold true (e.g., Gouyette and Perelman [1997]).

[10] Under perfect competition, this is equal to the labor income share but, under imperfect competition with non-zero markup, it equals the share of labor income over total factor income: $\frac{P_0 Y_0}{1+\bar{\mu}_0}$.

The expression,

$$E_t^i = E_{t_0}^i e^{g_i(t)} \qquad ; \; g_i(t = t_0) = 0 \tag{21}$$

for $g_i(t),\ i = N, K$ defines the growth rates of factor-augmenting technical progress and $E_{t_0}^i$ represent the fixed points of the two efficiency levels, taken at the common baseline time $t = t_0$. Normalization of the CES function implies that members of the same CES family should all share the same fixed point and should in this point and at that time of reference only be characterized by different elasticities of substitution. To ensure that this property holds also in the presence of growing factor efficiencies, it follows that,

$$E_{t_0}^N = \frac{Y_0}{N_0}\left(\frac{1}{1-\pi_0}\right)^{\frac{1}{\rho}} ; E_{t_0}^K = \frac{Y_0}{K_0}\left(\frac{1}{\pi_0}\right)^{\frac{1}{\rho}} \tag{22}$$

The last expression ensures that at the common fixed point the factor shares are not biased by the growth of factor efficiencies but equal to the distribution parameters π_0 and $1-\pi_0$.[11]

Inserting assumptions (21) and (22) and the normalized values (4), (5) and (6) into (20) leads to a normalized CES function that can be rewritten in the form that resembles the Arrow *et al.*, (1961) variant,

$$Y_t = Y_0\left\{(1-\pi_0)N_0^{\rho}\left[N_t \cdot e^{g_N(t,t_0)}\right]^{-\rho} + \pi_0 K_0^{\rho}\left[K_t \cdot e^{g_K(t,t_0)}\right]^{-\rho}\right\}^{-\frac{1}{\rho}} \tag{23}$$

Thus, with factor augmenting technical progress the growth of efficiency levels is now measured by the expressions $N_0\, e^{\,g_N\,(t-t_0)}$ and $K_0\, e^{\,g_K\,(t-t_0)}$, respectively (implying, as in equation (21), $e^{g_i(t=t_0)} = 1$). As a test of consistent normalization, we see from (7) that for $t = t_0$ we retrieve $Y = Y_0$. Special cases of (23) are those used by Rowthorn (1999), Bentolila and Saint-Paul (2003) or Acemoglu (2003) where $N_0 = K_0 = Y_0 = 1$ is implicitly assumed, or by Antràs (2004) who sets $N_0 = K_0 = 1$. Caballero and Hammour (1997), Blanchard (1997) and Berthold *et al.*, (2002) work with a version of (7) where in addition to $N_0 = K_0 = 1$, also assume Harrod-Neutrality: $\frac{\partial g_N(t)}{\partial t} = \gamma_N > 0; \frac{\partial g_K(t)}{\partial t} = \gamma_K = 0$.

With treating sample averages as baseline values at the common point (and time) of reference and introducing an additional scaling parameter ζ so that $Y_0 = \zeta \cdot \overline{Y}$, $K_0 = \overline{K}$, $N_0 = \overline{N}$, $\pi_0 = \overline{\pi}$, $t_0 = \overline{t}$, and where $\overline{\pi}$ is the average capital income share. The scaling parameter ζ deviates from unity, when the estimated CES production function deviates from the log-linear Cobb-Douglas function with constant technical growth. Under perfect competition, the distribution parameter could be calculated directly, pre-recursively, from the data but, when associated with unobservable markup, it can be estimated jointly with the other parameters of the

[11] We follow Klump *et al.* (2004) who suggest that fixed points should be calculated on the basis of sample geometric averages, because over a longer period of time cyclical variations have netted out and even longer-term fluctuations have compensated. The choice of sample geometric average values can imply a problem of scaling, however, since the geometric average of each time series is calculated independently. Hence, fixed points calculated as the geometric averages of inputs correspond to the geometric average of output only if the production function is log-linear i.e. Cobb-Douglas with constant technical growth. Therefore, we capture and measure the possible emergence of such a problem by introducing an additional estimated parameter ζ whose role is to capture the effects of the deviation of the CES from the log-linear function on the fixed point output corresponding to the geometric averages of inputs.

model. Hence, we arrive at the final econometric specification of our normalized CES function with factor augmenting technological change. Per-capita output, as estimated, can be written in logarithmic form as:

$$\log\left(\frac{Y_t \;/\; \overline{Y}}{N_t \;/\; \overline{N}}\right) = -\frac{1}{\rho} \cdot \log\left[\overline{\pi} \cdot e^{\rho[g_N(t,t_0) - g_K(t,t_0)]}\left(\frac{K_t}{N_t}\frac{\overline{N}}{\overline{K}}\right)^{-\rho} + \left(1 - \overline{\pi}\right)\right] + g_N\left(t, t_0\right) + \log\zeta \qquad (24)$$

From the point of view of estimation, the advantage of normalized equation (24) over the un-normalized case, is that all parameter have clear economic interpretations with well-defined, plausible ranges. For comparison, we can re-write an un-normalized counterpart of (24):

$$Y_t = B\left\{b\left[e^{g_K(t,t_0)}K_t\right]^{-\rho} + (1-b)\left[e^{g_N(t,t_0)}N_t\right]^{-\rho}\right\}^{-\frac{1}{\rho}} \qquad (24')$$

where $b = \dfrac{\overline{\pi} \cdot \overline{K}^{\rho}}{\overline{\pi} \cdot \overline{K}^{\rho} + (1-\overline{\pi}) \cdot \overline{N}^{\rho}}$ and $B = \zeta \cdot \overline{Y}\left[b\overline{K}^{\rho} + (1-b)\overline{N}^{\rho}\right]^{\frac{1}{\rho}}$.

An important feature of the above un-normalized formulation is that the parameters *B* and *b* have no clear theoretic interpretation. They are composite parameters conditional on, besides the selected fixed points, other parameters, and the elasticity of substitution. Hence, varying sigma whilst keeping *B* and *b* constant, we find that each resulting CES function belongs to different families. Whereas for case (8), varying the elasticity parameter implies that each resulting CES function belongs to the *same* family. Hence, the main merit in using it, instead of the un-normalized form, is that all parameters have a clear empirical interpretation.

3.2 Technical Progress

Standard neo-classical growth theory suggests that, for balanced growth, technology should exhibit Harrod Neutrality. The intuition for this is as follows. Since labor is assumed the fixed factor, firms, in order to avoid an explosion of wage income (or the labor share), bias and concentrate technical improvements towards labor. Empirically, this would suggest that real labor costs grow in line with productivity, whilst user costs / real interest rates are stationary.

In the Cobb-Douglas environment, of course, the direction of technical change is irrelevant for income distribution since it is not possible to determine empirically any bias in the direction of technical change. In contrast, pronounced trends in factor-income distribution visible in many countries over the "medium run" (Blanchard [1997]) support the CES function and make possible biases of technical progress a key issue. In the CES environment, a steady state with constant factor income shares is only possible, if exogenous technical progress is purely labor augmenting. Acemoglu (2003) was able to derive this same result in a model with endogenous innovative activities but also demonstrates that, over quite significant periods of transition, growth of capital-augmenting progress can be expected resulting from endogenous changes in the direction of innovations.

In addition, earlier theoretical and empirical work on CES functions assumed exponential growth of both efficiency levels. However, following recent debates about biases in technical progress, it is not obvious that these growth rates should always be constant. Consequently, we apply the well-known Box-Cox transformation in defining factor-augmenting technical progress, $g_i(t,t_0)$:

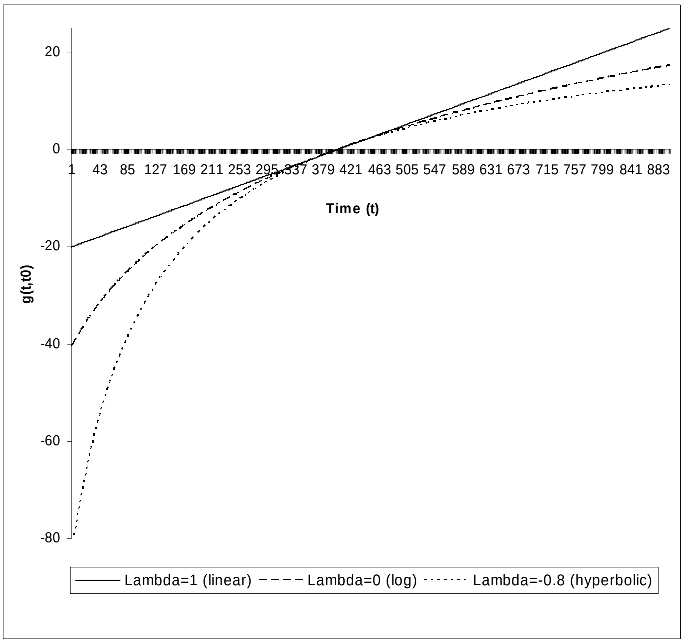
$$g_i(t,t_0) = \frac{\gamma_i}{\lambda_i}t_0\left(\left[\frac{t}{t_0}\right]^{\lambda_i} - 1\right) \; ; \quad \text{where } g_i(t = t_0) = 0 \qquad (25)$$

The size and sign of parameters, λ_i , define the functional form of technical progress; when λ_i =1 (=0) {<0}, technical progress functions, g_i , are linear (log-linear) {hyperbolic} functions in time. Thus, the level, $g_i(t,t_0)$, and growth rates, $\frac{\partial g_i(t,t_0)}{\partial t}$, of technical progress components for different λ_i have the following properties,

$$
\left.
\begin{array}{l}
g_i(t,\ t_0) \Rightarrow \begin{cases} \left.\begin{array}{l} \lim\limits_{t\to\infty} g_i(t,t_0) = \infty \\ \lim\limits_{t\to\infty} g_i(t,t_0) = \infty \end{array}\right\} & \text{if } \lambda_i \geq 0 \\ \lim\limits_{t\to\infty} g_i(t,t_0) = -\dfrac{\gamma_i t_0}{\lambda_i} > 0, & \text{if } \lambda_i < 0 \end{cases} \\
\dfrac{\partial g_i(t,t_0)}{\partial t} = \gamma_i \left(\dfrac{t}{t_0}\right)^{\lambda_i - 1} \Rightarrow \begin{cases} \dfrac{\partial g_i(t,t_0)}{\partial t} = \gamma_i, \forall t, & \text{if } \lambda_i = 1 \\ \lim\limits_{t\to\infty} \dfrac{\partial g_i(t,t_0)}{\partial t} = 0, & \text{if } 0 \leq \lambda_i < 1 \\ \lim\limits_{t\to\infty} \dfrac{\partial g_i(t,t_0)}{\partial t} = 0, & \text{if } \lambda_i < 0 \end{cases}
\end{array}
\right\} \quad (26)
$$

Thus, only in the case of hyperbolic curvature is there an upper bound of the contribution of factor *i* to the level of technical progress. The dynamic paths of technical progress, $g_i(t, t_0)$, are illustrated in Figure 3.

Figure 3
Functional Forms of Technical Progress



Furthermore, as discussed by Acemoglu (2002, 2003), there is no necessity to constraint technical progress to be solely Harrod-neutral; periods of capital-augmenting technical progress are also possible without being incompatible with a long-run balanced growth path. What is required is that technical progress is *asymptotically* Harrod neutral (implying a fading away of the capital component). Although, in any particular historical sample, it is not implausible that capital-augmenting technical progress, rather than strictly fading away, could retain a persistent role for aggregate TFP.

As should be clear, the benefit of our approach is that we can nest the above cases as limiting arguments. Consider, for example, the case,

$$\gamma_N > 0; \lambda_N = 1; \gamma_K = \lambda_K = 0 \tag{27}$$

This, coupled with the assumption of $\sigma >> 1$, corresponds to that used by Caballero and Hammour (1997), Blanchard (1997) and Berthold *et al.*, (2002) in explaining the decline in the labor share in continental Europe. However, assuming above-unity elasticity runs contrary to balanced growth. Furthermore, if the economy's growth rate is a positive function of this elasticity (e.g., Klump and Preissler [2000]), then we would expect, somewhat counterfactually, that the growth rate of continental Europe exceeds that of the US (given, as earlier noted, the bulk of studies claiming a below-unity elasticity for the US)

Another case, which we might speculatively call "Acemoglu Neutrality", can be nested as,

$$\gamma_N, \gamma_K > 0; \lambda_N = 1; \lambda_K < 1 \tag{28}$$

where, here, $\sigma < 1$ is natural.[12]

We might further identify two important cases within (28). Strong "Acemoglu Neutrality", $\lambda_K < 0$, implies that the contribution of capital to TFP is bounded and that its capital growth component returns rapidly to zero; whilst in the "weak" form, $0 < \lambda_K < 1$, capital imparts a continuing contribution to the level of TFP. Note, both forms are asymptotically Harrod-Neutral and thus consistent with a balanced-growth, steady state, where then the speed of labor-augmenting technical progress is constant (at γ_N). Two further cases with decreasing capital augmentation can be identified: $\lambda_N > 1 \left| \begin{array}{l} 0 < \lambda_K < 1 \\ \lambda_K < 0 \end{array} \right.$ and $\lambda_N < 1 \left| \begin{array}{l} 0 < \lambda_K < 1 \\ \lambda_K < 0 \end{array} \right.$. The first case implies explosive TFP growth, whilst the second to a case of permanent stagnation (i.e., TFP asymptotes to zero). Both cases, though seemingly implausible as long-run properties, never the less remain testable in a historical sample.

4. Some Stylized Facts of Euro Area Data [13]

Following Coenen and Wieland (2000), Fagan *et al*. (2001), Leith and Malley (2001), Galí *et al* (2001), Smets and Wouters (2003), etc, we model interactions at the aggregate euro-area level[14], using euro-area data from 1970q1-2003q4 from the AWM (Area Wide Model) database of Fagan *et al*. (2001).

Figures 4 (c) again shows the development of the share of labor income in GDP. Visually the series looks manifestly non-stationary; during the first half of the 1970s, the labor income share rose, but started to decline from

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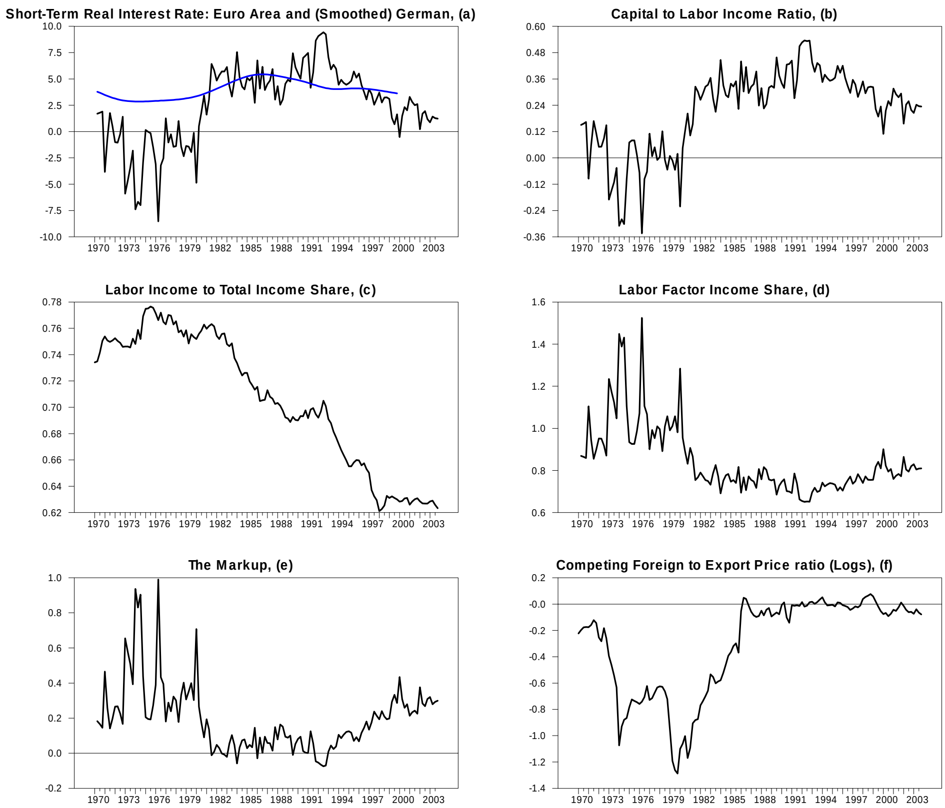
[12] The case $\lambda_K > 1$ is excluded from our present discussion since it is highly non-standard and, as our later results bear witness, not data coherent.

[13] The euro area data used, including all relevant transformations, is available on request; as are the (Rats programming) files to replicate all of our (later) results.

[14] Note that, though we use euro-area data, the data concerns and modeling strategy apply to many large constituent countries (e.g., Germany, France). Our analysis could, therefore, be mechanically performed at the level of individual euro-area country level; we leave this for possible future research.

the early 1980s onwards. Likewise, the capital-to-labor income ratio (Figure 4b) is non-stationary but its profile is, markedly different from that of the labor income share. [15] Essentially, we can observe two regimes in the capital-to-labor income share; a low level covering the most of 1970s and a shift in the late 1970s and early 1980s to a markedly higher level covering the rest of the rest of the sample period.

Figures 4
Some Stylized Features of Key Euro-Area Data



How might we explain these developments? We note that there should be a close, inverse mapping between the labor-income share and the capital-to-labor income ratio (assuming the mark-up is not excessively volatile). That is *not*, however, what we see in the data; there is no level shift in the labor-income share corresponding to that in the capital-to-labor ratio at the end of 1970s and in the early 1980s.

This suggests that there could be *other* explanations for the level shift in the capital-to-labor income ratio, linked e.g. to the way that capital income is constructed. Our capital income *QK* is an imputed concept. It is thus sensitive with respect to variations in the measured user-cost-of-capital (*Q*) and, hence, to variations in the real interest rate. Notably, as Figure 4a shows, the variation in the capital-to-labor income ratio, including the regime shift, essentially matches that of the real interest rate. Although there is of course nothing to prevent the ex-post real interest rate from being temporarily negative, a precarious feature in the observed development of the real interest rate is that it was negative for most of the 1970s. The use of a persistently negative real interest rate as an operational counterpart for the expected real interest rate used in the optimization framework seems contradictory, or at least worth investigating.

[15] The non-stationarity of both series is readily confirmed by ADF tests.

One possible explanation of the negative character of the measured real interest rate in the 1970s and of the upward level shift in the late 1970s and early 1980s might be that financial markets were highly regulated in Europe during the most of 1970s. In the late 1970s and early 1980s, this regulated system broke apart, at first perhaps partly due to leakage caused by financial innovations, and later due to the formal removal of regulations.[16] Under this explanation, measuring the regulated interest rate, does not measure the marginal cost of financing correctly. Figure 4a encourages this interpretation. It presents the development of ex post real interest rates (in terms of private consumption deflator) in the euro area and Germany (for graphical clarity, we present the latter as a HP-filtered smoothed series). We see that the real interest rate in the euro area was strongly negative throughout most of the 1970s, whilst the German rate was clearly positive through the whole sample (from the mid 1980s onwards, though, the two series are very similar). The German case is interesting since Germany took the lead in financial liberalization and all direct control had been removed before 1974, i.e. by the point of time at which real interest rates in other euro-area countries turned negative (e.g., Issing [1997]).[17]

To take into account the possibility that our data for the euro area real interest rate do not measure correctly the marginal cost of financing in the 1970s, a level shift dummy was constructed to correct the interest rate upwards in the 1970s.[18] The dummy-corrected interest rate could be interpreted as a shadow interest rate $\left(i^n\right)$, measuring the marginal cost of financing[19],

$$i^n = i + h \cdot DUM \tag{29}$$

Where *DUM* is presumed to be a smooth, level-shift dummy calibrated to be 1 in the early 1970s, and starting gradually to deviate from unity around 1976 and converging to zero at around 1983, after which i^n in practice equals to the observable interest rate i.[20] After replacing i by i^n we define capital income as,

$$QK = \frac{P_I\left[(i - 4 \cdot \pi^e) \cdot \Psi + \delta\right]}{400} \cdot K + h \cdot \Psi \cdot \frac{P_I K}{400} \cdot DUM \tag{30}$$

where Ψ is the ratio of the net to gross capital stock, π^e = inflation expectations (the *HP*-filter fit for one-period P_I inflation), P_I is the investment deflator, and where parameter *h* can be estimated jointly with the other parameters of the supply-side system (see also appendices 1 and 2).

The inclusion of the level shift dummy in the equation of the capital-to-labor income ratio allows the removal of the regime shift in the capital-to-labor income ratio, implying a rather stable evolution of the ratio.

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16 The level-shift in the real interest rate seems somewhat to precede the formal removal of financial regulations in many countries. It is quite possible, however, that financial innovations caused the regulated system to start to leak long before the formal removal of regulations.

17 The real interest rate in France and, especially, Italy was strongly negative throughout most of the 1970s. The real interest rate in France mimics the euro-area real interest rate relatively well; we have suppressed the corresponding figures but they can be found in Willman (2002).

18 This is in line with Coenen and Wieland (2000) who found a strong and significant negative dependence of euro-area aggregated demand on the German real interest rate, whilst the dependency of the weighted average of the euro area real interest rate was markedly weaker and statistically insignificant. Following Coenen and Wieland (2000), we also could have used the German real interest as a proxy for the real interest rate of the euro area. However, the drawback of this approach would be that we loose the information contained by the euro area real interest rate in the latter part of the sample, when we think that the euro area real interest rate measures reasonably well the real marginal cost of financing in the euro area. In addition, the size of the correction to the real interest rate implied by estimated parameter for the dummy may also serve as evidence or counter-evidence of our hypothesis.

19 This would, of course, presuppose the existence of a rather well functioning "grey" financial market. Then, when regulation is binding, the marginal cost of financing can be markedly above the average cost of financing, which the interest rate measures. After financial deregulation, under the *Modigliani-Miller* theorem, as our user cost definition assumes, the marginal and average costs of financing are equal.

20 The exact functional form used being, $DUM = \left(1 - \frac{1}{1 + \exp(2 - 0.3(t - 35))}\right)$.

Therefore, the non-stationarity of the share of labor income in GDP could be explained mainly by the evolution of the mark-up. Since the early 1980s, the downward trend can be explained by the observed increase in the output share of the services sector, coupled with a higher mark-up than in the manufacturing sector. Regarding the hump-shaped development of labor income share in the 1970s, we can resort to the widely-approved assessment of Bruno and Sachs (1985) (also shared by Blanchard [1997] and Caballero and Hammour [1998]). According to this assessment, countries in continental Europe were affected by large, adverse shifts in the "labor" supply in the 1970s. These shifts came from the failure of wages to adjust sufficiently to the productivity slowdown and from the adverse supply shocks of the 1970s, i.e. the sharp commodity (mainly oil) price increases. By contrast, the "Anglo-Saxon" countries appear to have been largely shielded from adverse labor-supply shifts in the 1970s.

However, as can be seen in Figure 3(f), these effects have been transmitted to our measure of the ratio of competing foreign prices to export prices, i.e. the price competitiveness of the open sector (i.e., parameter ϕ in equation (19)). Two oil price shocks and a weakening US dollar in the 1970s caused the losses of competitiveness in the euro area. However, as the Figure shows, there was a recovery in price competitiveness during the first half of 1980s, which reflects the appreciation of the US dollar and a gradual decrease in oil prices which ended up in the collapse of the OPEC cartel in 1986. Thereafter, until the end of our sample period, our measure of price competitiveness remained remarkably stable. (Appendix 1 further discuses some data transformations).

5 The Dynamics Adjustments of the Model.

We now return to the system (9)-(13) which determines the dynamic price setting, demand for labor and capital inputs and production of a profit maximizing firm. In the dynamic setting effective total working hours can be solved from the inverted production function implied by condition (13), i.e.

$$H_t = F^{-1}(K_t, Y_t) \tag{31}$$

Utilizing this result, conditions (9) and (10) yield the following rule for optimal pricing,

$$P_t = (1+\mu)\frac{W_t}{\partial F/\partial H_t}\left\{1 + a_h\left[1 - \left(\frac{N_t}{F^{-1}(K_t.Y_t)}\right)^2\right]\right\} \tag{32}$$

An interesting feature of (32) is that it introduces into the optimal price setting a cyclical (in the time series context, a stationary) component, namely the deviation of effective working hours from normal (cost-minimizing) working hours. By applying the following approximating:

$$\log\left(1 + \frac{a_h}{2}\left(1 - \left(\frac{N}{F^{-1}(K,Y)}\right)^2\right)\right) \approx \frac{a_h}{2}\left[1 - \left(\frac{N}{F^{-1}(K,Y)}\right)^2\right] \cong a_h \log\left(\frac{F^{-1}(K,Y)}{N}\right) \tag{33}$$

equation (32) can therefore be re-written as,

$$\log P_t = \log((1+\mu)\cdot W_t) - \log\left(\frac{\partial F}{\partial H_t}\right) + a_h \log\left(\frac{F^{-1}(K_t, Y_t)}{N_t}\right) \tag{34}$$

Hence, the deviation of effective working hours from normal hours has a spill-over effect to the pricing decision.

Note that (34) is not a dynamic equation as such: it defines the optimal price setting without menu costs or other frictional elements.

The dynamic labor demand rule is given directly by (11), which can be re-written as,

$$\frac{\partial A_N(N_t, N_{t-1})}{\partial N_t} + E_t \frac{W_{t+1}}{W_t(1 + r_t)} \frac{\partial A_N(N_{t+1}, N_t)}{\partial N_t} = a_h\left(1 - \frac{N_t}{F^{-1}(K_t, Y_t)}\right) \tag{35}$$

Whilst, condition (13) can be re-written as:

$$\frac{\partial A_K(K_t, K_{t-1}, K_{t-2})}{\partial K_t} + E_t(1 + r_t)^{-1}\left\{\frac{\partial A_K(K_{t+1}, K_t, K_{t-1})}{\partial K_t}\right\}$$
$$+ E_t[(1 + r_t)(1 + r_{t+1})]^{-1}\left\{\frac{\partial A_K(K_{t+2}, K_{t+1}, K_t)}{\partial K_t}\right\} = \frac{P_t}{(1 + \mu)} \frac{\partial F(K_t, H_t)}{\partial K_t} - \frac{r_t + \delta}{1 + r_t}, \tag{36}$$

where P_t is the relative price of output to the investment deflator. With the function F known effective total hours as well as the marginal product of capital are defined. However, to end up with estimable specifications we must define explicit functional forms of the adjustment cost functions A_N and A_K. As discussed in Willman *et al.* (1999), in an environment where relations between variables are multiplicative the translog adjustment costs result in particularly elegant results. The adjustment cost functions, and for comparison, their quadratic counterparts are:

$$A_N(N_t, N_{t-1}) = \frac{a_N}{2} \cdot \Delta N_t \Delta n_t \tag{37}$$

$$\approx \frac{a_N}{2} \cdot \frac{(\Delta N_t)^2}{N_{t-1}} \tag{38}$$

$$A(K_t, K_{t-1}, K_{t-2}) = \frac{a_K}{2} \cdot \Delta K_t \Delta k_t + \frac{a_K b_K^2}{2} \cdot \Delta K_{t-1} \Delta k_{t-1} - a_K b_K \cdot \Delta K_t \Delta k_{t-1} \tag{39}$$

$$\approx \frac{a_K}{2} \cdot \frac{(\Delta K_t - b_K \Delta K_{t-1})^2}{K_{t-1}} \tag{40}$$

where $n_t = \log(N_t)$, $k_t = \log(K_t)$ and $0 \le b_K \le 1$. Differentiating (37) and (39) with respect to N and K yields,

$$\left.\begin{aligned} \frac{\partial A_N(N_t, N_{t-1})}{\partial N_t} &= \frac{a_N}{2}\left(\Delta n_t + \frac{\Delta N_t}{N_t}\right) \cong a_N \Delta n_t \\ \frac{\partial A_N(N_{t+1}, N_t)}{\partial N_t} &= -\frac{a_N}{2}\left(\Delta n_t + \frac{\Delta N_t}{N_{t-1}}\right) \cong -a_N \Delta n \end{aligned}\right\} \tag{41}$$

$$\left.\begin{aligned}
\frac{\partial A_K\left(K_t, K_{t-1}, K_{t-2}\right)}{\partial K_t} &\cong a_K \Delta k_t - a_K b_K \Delta k_{t-1} \\
\frac{\partial A_K\left(K_{t+1}, K_t, K_{t-1}\right)}{\partial K_t} &\cong -a_K\left(1+b_K\right)\Delta k_{t+1} + a_K b_K\left(1+b_K\right)\Delta k_t \\
\frac{\partial A_K\left(K_{t+2}, K_{t+1}, K_t\right)}{\partial K_t} &\cong a_K b_K \Delta k_{t+2} - a_K b_K^2 \Delta k_{t+1}
\end{aligned}\right\} \quad (42)$$

Substituting these results into (35) and (36), we derive,

$$\Delta n_t - \frac{W_{t+1}}{\left(1 + r_t\right)} \Delta n_{t+1} = \frac{a_h}{a_N} \log\left(\frac{F^{-1}\left(K_t, Y_t\right)}{N_t}\right) \quad (43)$$

$$\begin{aligned}
&\frac{b_K}{\left(1+r_t\right)\left(1+r_{t+1}\right)}\Delta k_{t+2} - \left(\frac{b_K{}^2}{\left(1+r_t\right)\left(1+r_{t+1}\right)} + \frac{(1+b_K)}{\left(1+r_t\right)}\right)\Delta k_{t+1} + \left(\frac{b_K(1+b_K)}{\left(1+r_t\right)} + 1\right)\Delta k_t - b_K \Delta k_{t-1} \\
&\qquad = \frac{1}{a_K}\left(\frac{P_t}{(1+\mu)}\frac{\partial F(K_t, H_t)}{\partial K_t} - \frac{r_t + \delta}{1+r_t}\right)
\end{aligned} \quad (44)$$

Replacing r_t and r_{t+1} by their steady-state value, $\bar{r}$, equation (44) has one stable and two unstable roots: $b_K, \frac{1+\bar{r}}{b_K}$ and $1+\bar{r}$, thus, implying saddle-path stability. The dominant lead root thus equals the inverse of the average discount factor (where the discount factor is $\frac{1}{1+\bar{r}}$), being only slightly above unity, implies highly forward-looking behavior. The importance of the past, in turn, depends on the size of the adjustment parameter b_K. If $b_K = 0$, then the level of net investment could be changed without costs. Furthermore, it can be shown that equation (43) has one unstable and one stable root (see Table 1.4).

6 Estimation Results

6.1 Supply-Side

Tables 1.1-1.3 show the parameter estimates and their standard errors for various specifications of this system. (The exact form of our estimated supply-side system is presented in Appendix 2) For estimation, we use non-linear SUR which is the natural and most powerful estimator in this (cross-equation parameter) systems context. As can be seen, most parameters are significant at 1%, economically sensible, and relatively stable. The rows list the technical parameters ($\zeta, \gamma_N, \lambda_N, \gamma_K, \lambda_K, \sigma$), the parameters capturing the euro-area markup and its trend ($1 + \bar{\mu}_A$ and η respectively), the interest-rate financing dummy (*h*), and terms-of-trade effects (ϕ). Note that in estimation, we do not attempt to separate the aggregation contribution of technical progress (Γ), from the Box-Cox parameters.

Thereafter, we report Total factor progress (evaluated at the fixed point); and individual and joint (Wald) tests of key parameter restrictions. This is followed by residual stationarity tests and likelihood criteria. Table 1.1 uses real German interest rates (over the period 1970:1-1982:4) as a measure of real financing costs in the euro area, whilst Table 1.2 imposes a level shift in the euro-area real interest rate to respect the *average* of German real rates over that period. In Table 1.3, we allow this interest rate level shift to be freely estimated by the data. As regards technical progress forms we report an unrestricted form (column a), then impose linear labor-augmenting technical progress (b), hyperbolic capital-augmenting technical progress (c), and both forms jointly (d).

- Irrespective of the specification, many parameters appear quite stable. For example, the estimated elasticity of substitution (σ) appears robust and in almost all cases is around 0.6-0.65 (and significantly different from unity). Furthermore, Total factor progress growth comes out at around 0.3% per quarter (1.2% per annum). [21] Finally, the scale parameter (ζ) is, as expected, very close to but significantly different from unity.

- Similarly the sample-average markup ($1 + \bar{\mu}_A$) is found in a relatively tight range: around 12% (Tables 1.1-1.2) but around 5% thereafter (Table 1.3). These values are well in line with the implied markup components in the data when accounting for the corresponding level corrections in real interest rate in the 1970s. The intuition for such differences is straightforward: for a given output and labor share, a larger user-cost-of-capital increase the capital-income share and thus generates a lower aggregate markup. In Table 1.3, we impose a stronger (upward) financing correction for the euro-interest rates, and consequently derive down the markup.

- In most cases, within our sample period, capital-augmenting technical progress (evaluated at the middle point of the sample, t=t0) has been dominating, $\frac{\partial g_K}{\partial t}(t = t_0) = \gamma_K > \frac{\partial g_N}{\partial t}(t = t_0) = \gamma_N$.[22] The more common outcome in the literature tends to be the labor-saving variety. But in our framework, recall, technical progress is *time-varying*. In Figures 7-9, we plot (left panel) in log levels the paths for Total Factor Progress (TFP), Labor-Augmenting Factor Progress (LAUG) and Capital-Augmenting Factor Progress (KAUG), followed by (right panel) their first differences (prefixed by *D*) (i.e., their growth contributions), corresponding to each of the table cases. At the start of the sample the contribution of capital-augmenting technical progress to TFP dominates. However, the curvature effects (the $\lambda_K' s$) are such as to diminish that effect over time. As we know from our discussion in section 3.2. where we impose hyperbolic capital-augmenting technical progress (cases c and d in the tables), the growth (level) contribution decline to zero (non-zero constant). In other cases, i.e., $0 < \lambda_i < 1$ the level component is unbounded but its growth component approaches zero but only asymptotically (e.g. case a).

- Our results show that we cannot reject the restriction that $\lambda_N = 1$. This is in line with asymptotic Harrod Neutrality and balanced growth. However, crucially, we cannot reject a highly persistent (though declining) role for capital-augmenting technical progress. Specifically, to use our earlier taxonomy, our estimation results favor *weak* "Acemoglu Neutrality" since we reject hyperbolic capital curvature (i.e., *strong* "Acemoglu Neutrality"). Such a scenario may not correspond to a classical balanced-growth, steady state.

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[21] We calculate TFP using the well-known Kmeta (1967) approximation Applying this approximation around the fixed points $N_t = \bar{N}, K_t = \bar{K}$ and $t = \bar{t}$ to separate the total factor progress (TFP) term from the rest of the production function, we obtain,

$$\log\left(\frac{Y_t / \bar{N}}{N_t / \bar{Y}}\right) = \overbrace{\bar{\pi}\, g_K(t,\bar{t}) + (1-\bar{\pi})g_N(t,\bar{t}) - \left(\frac{1-\sigma}{\sigma}\right)\frac{\bar{\pi}(1-\bar{\pi})}{2}\left[g_N(t,\bar{t}) - g_K(t,\bar{t})\right]^2}^{TFP} + \log(\zeta) - \frac{\sigma}{1-\sigma}\log\left[\bar{\pi}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi})\right]$$

where $\bar{\pi} = 1-(1+\bar{\mu}_A)\cdot\alpha$, where α is the labor share ($\frac{W_0 N_0}{P_0 Y_0}$) evaluated at some fixed pint and where g_N and g_K refer to the estimated Box-Cox functions. Interestingly, when the elasticity of substitution does not equal unity, the factor augmentation introduces additional curvature into the estimated production function i.e., the squared-bracket term in power two above.

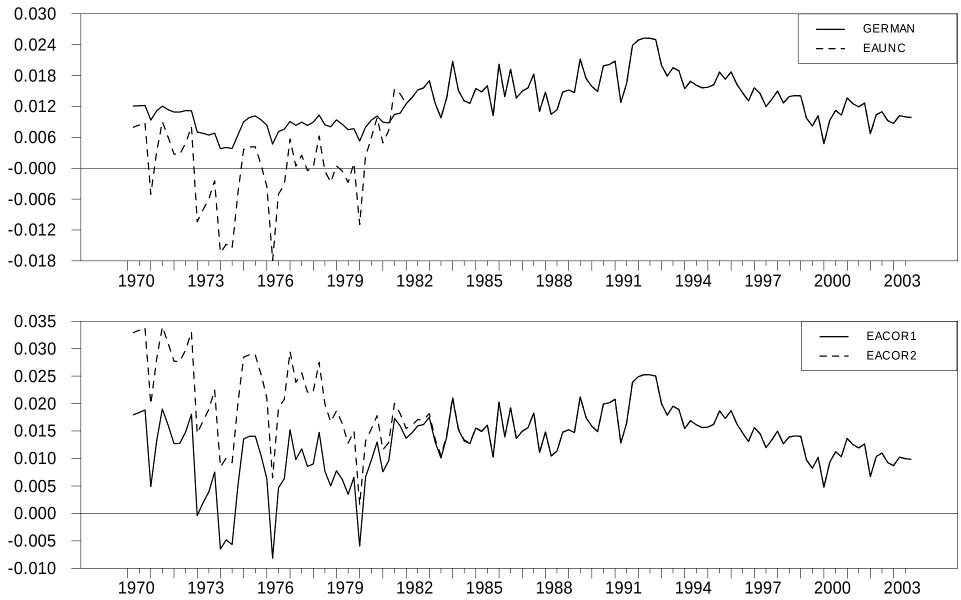
[22] Only in case 1.3e, is there (albeit marginally) labor-saving technology.

But nor does it represent an explosive, stagnating or unstable outcome. To return to the title of our paper, it may, however, be considered a “medium-run” outcome: where the effects of capital-augmenting technical progress are highly persistent and where labor, though asymptotically the scarce factor, in a sample characterized by high and persistent unemployment, cannot strictly be considered the constraining factor for growth. Furthermore, our results show that an above-unity elasticity of substitution is neither necessary nor sufficient to describe the declining factor shares in the euro area.

- In our supply-side system, note, in addition to the trends associated with factor-augmenting technical progress, that there are two trend-like variables: the markup, η, and level shift financing dummy, *h*. Given the difficulty in credibly disentangling freely-determined trends in technical progress and trends and shifts in these 2 dummies, some control may be reasonably required. This can appreciated from cases 1.3 a-d, where we allow the free estimation of all trend effects. This leads to an implausibly and atypically high value for the trending variables (see, for example, the jump in λ_K to 0.7 from the previous 0.3 values). For instance, constraining the trend in the markup to be consistent with the evidence of the other tables (e.g., 0.0025), yields cases 1.3 e and f. Since the constraint of linear labor-augmenting technical progress is not rejected by the data (the *p-value* being around 25%) we continue to the most parsimonious form (and our preferred specification) of 1.3 f.

- Figure 5 illustrates the consequences of using the different real interest rates which are used in the calculation of user cost. Looking at the top panel of Figure 5, we see that financing costs as measured by normal (i.e., uncorrected) euro area real interest rates, *EAUNC*, may well under-estimate true financing costs (benchmarking on the *German* rates). However what is the required correction, is not definitive on an a priori basis. In the lower panel of Figure 5, we present two plausible corrections: *EACOR1* applies a correction to respect the average of German real interest rates (i.e., corresponding to Table 1.2), and *EACOR2* were the correction is estimated freely (at around 0.3), roughly from Tables 1.3.

Figure 5
Alternative Measures of Real Interest Rates

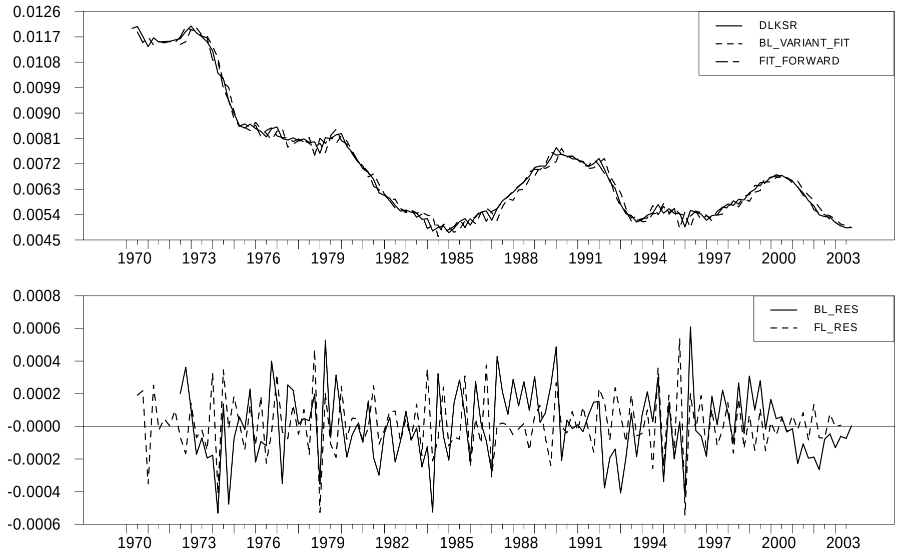


6.2 Estimation of Dynamic Equations

The estimates for the dynamic equivalents of labor demand (43), prices (34), investment (44) are given in Table 1.4. In all cases, the parameters are highly significant and the roots and over-identifying restrictions are acceptable. When looking at results, it is worth noting that, in the Investment equation, the adjust cost parameter, b_k, (the backward-looking root), is estimated to be quite high (0.65). This implies that in addition to costs associated with changing the level of the capital stock, it is not costless to change the level of investment either. Likewise, employment equation shows that also adjusting the number of employees are associated with significant costs (reflected by the roots, which are 0.89 and 1.14). Also, in line with our theoretical specification, staggered adjustment costs of the labor input has strong spillover effects on prices, reflected in $a_h = 0.69$. Thus, such spillover effects should be accounted for in specifying dynamic price setting equations, e.g., Calvo price-staggered framework. Indeed, as the DW statistics shows, the residual of the price equation is highly auto-correlated being in line with the view that there is considerable stickiness in price setting.

One further question is how well these equations track the data. This is not a straightforward question since the empirical performance of highly theoretical factor demand specifications (when compared say to ad-hoc equivalents) has long been an issue. Take the case of Investment, many authors have found that theory-based, forward-looking Investment functions perform relatively poorly compared to standard backward-looking variants (e.g., Chirinko [1993], Oliner *et al.* [1995]). Our results are not similar, although we also have a theoretically-founded forward-looking (FL) investment equation. As can be see from Figure 6, which graphs the fits in the upper panel and the implied residual in the lower panel, the fit of our forward-looking specification is not obviously inferior to a backward-looking (BL) one.[23] Both residuals, as can be readily confirmed, are stationary. Moreover, encompassing tests, would not lead to any particular specification being favored. (details available).

Figure 6
Dynamics of Alternative Investment Equations



7 Conclusions

This paper sought to contribute towards the "medium-run" debate as commonly applied to continental Europe. This debate has focused on key stylized facts: observing continental Europe's persistently high unemployment and, in turn, reflecting on the nature of its factor income shares, factor substitutability and response to various shocks from the early 1970s onwards.

We try to confront explanations of this "medium-run" performance in a consistent, coherent theoretical framework. Definitionally speaking, the medium run as such corresponds to the overlap of the long run (where supply factors govern events) and the short run (where demand presides). As regards the former, we have estimated a simultaneous supply side with a factor-augmenting normalized CES production function with time-varying technical progress. We argued that this specification allowed us to robustly identify and estimate important supply-side parameters. Another plus, was that, as estimated, the supply side plausibly nested a number of "medium-run" interpretations as limiting cases which could then be brought to the data. We found that the elasticity of substitution (a key parameter in "medium run" debates) was robustly estimated at significantly below unity (0.6) rather than above unity as several authors had argued. Further, though we could not reject Harrod Neutrality as an asymptotic property, we found a significant though declining role for capital-augmenting technical progress.

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[23] The BL variant used was,

$$\Delta k = \underset{(0.035)}{0.9970}\cdot \Delta k_{-1} + \underset{(0.1290)}{0.4680}\cdot \sum_{i=0}^{2} a_i \log\left(\frac{\partial F}{\partial K}\right)_{-i} - \underset{(0.0027)}{0.0050}\cdot \Delta\left(\Delta UC_{-4} - \Delta UC_{-7}\right)$$

$$- \underset{(0.0033)}{0.0063}\cdot \left(UC_{-6} - UC_{-9}\right) + \underset{(0.0031)}{0.0082}\cdot \left[\Delta \log(P_{-3}) + \Delta \log(P_{-5})\right] + \underset{(0.0045)}{0.0086}\cdot \left(\frac{\partial F}{\partial K} - UC\right)_{-2}$$

$$R^2 = 0.98;\ DW = 2.0764;\ a_0 = 1, a_1 = 2/3, a_2 = 1/3$$

Where *P* is the relative price of GDP to Investment Deflator.

Notwithstanding, we argued that this framework should be supplemented in order to fully track trends in factor incomes and in the aggregate markup. Accordingly, we first undertook a careful analysis of euro-area data and in particular draw attention to measurement problems associated with the real interest rate (and thus the user cost) and its implications for factor income shares (especially capital income) and, by definition, the identification of the markup. Second, in an imperfectly-competitive framework, we introduced changing sectoral output shares as an explanation for a rising markup (and hence declining labor share). Note, both elements (i.e., time-varying markup and CES technology) are required in our framework to explain factor developments in the euro area. The former, by itself, explains the non-stationary factor income shares but the Cobb-Douglas form implies that the ratio of factor incomes should be constant. Since it is not (recall Figures 4), we additionally require the CES form.

On the demand side, we sought to extend the conventional optimization framework to simultaneously capture adjustment costs associated with both labor and capital. Regarding adjustment costs associated with labor, a special feature of our framework was to decompose such costs into those associated with the deviation of effective hours from normal paid hours, on the one hand, and those associated with changes in the number of employees. This extension introduces deviation of effective hours from normal paid hours also into the conventional markup equation, which proves to be empirically highly significant. Regarding capital, besides adjustment costs in the level of the capital stock (as standard), we also introduce costs associated with changes in the rate of capital accumulation (i.e., the level of investment). Besides being theoretically well founded, all these dynamic specifications appear to have good data-congruent tracking properties (and comparable with corresponding conventional error-correction error correction functions).

To sum up: in our approach, we have modeled the main planks of the “medium-run” and euro area debate (i.e., declining factor incomes, non-stationary markups, robust identification of the elasticity of substitution, possible technical biases, marked sectoral shifts etc) in a consistent, coherent theoretical framework. Of course, our approach contains simplifying assumptions and therefore may not be able to capture all relevant factors or nuances for euro area, nor is our framework necessarily uncontroversial. Other promising extensions include endogenizing technical progress, adding wage formation dynamics and distinguishing between different skill varieties in the labor input. We leave these for possible future research.

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Table 1.1—Supply-Side Estimates (Using German Real Interest Rate 1970:1 - 1982:4)

	(a)	(b)	(c)	(d)
ζ	1.0143 (0.0014)	1.0139 (0.0014)	1.0135 (0.0014)	1.0114 (0.0013)
γ_N	0.0031 (0.0003)	0.0028 (0.0002)	0.0035 (0.0001)	0.0033 (0.0001)
λ_N	0.9261 (0.0726)	1.0000 (—)	0.8609 (0.0416)	1.0000 (—)
γ_K	0.0036 (0.0012)	0.0044 (0.0008)	0.0018 (0.0002)	0.0023 (0.0002)
λ_K	0.2356 (0.1177)	0.2804 (0.0861)	-0.0100 (—)	-0.0100 (—)
σ	0.6533 (0.0311)	0.6781 (0.0202)	0.6136 (0.0240)	0.6652 (0.0194)
$(1 + \bar{\mu}_A)$	1.1287 (0.0107)	1.1296 (0.0102)	1.1211 (0.0095)	1.1188 (0.0093)
η	0.0025 (0.0001)	0.0026 (0.0001)	0.0024 (0.0001)	0.0024 (0.0001)
h	..	..	..	..
ϕ	0.0227 (0.0029)	0.0237 (0.0028)	0.0209 (0.0029)	0.0234 (0.0028)
TFP	0.0032	0.0031	0.0031	0.0031
Parameter Restrictions				
$\lambda_N = 1$	1.0379 [0.3083]	..	11.1461 [0.0008]	..
$\lambda_K \approx 0$	4.3534 [0.0369]	11.3734 [0.0007]	..	..
$\lambda_N = 1, \lambda_K \approx 0$ (a)	10.6081 [0.0050]	..	..	..
Neutrality Assumption				
Harrod: $\gamma_K = \lambda_K = 0,$ $\lambda_N = 1$	76.0014 [0.0000]	..	..	..
Hicks $\gamma_N = \gamma_K,$ $\lambda_N = \lambda_K = 1$	549.6040 [0.0000]	..	..	..
Hicks Modified $\gamma_N = \gamma_K,$ $\lambda_N = \lambda_K$	73.5369 [0.0000]	..	..	..
Solow: $\gamma_N = \lambda_N = 0,$ $\lambda_K = 1$	1635.1717 [0.0000]	..	..	..
Stationarity				
ADF_p	-3.1507	-3.1702	-3.1191	-3.1821
$ADF_{ck/wn}$	-2.7103	-2.6883	-2.8164	-2.8208
$ADF_{Y/N}$	-2.7011	-2.6420	-2.9289	-3.0280
Likelihood (b)	-27.8284	-27.8237	-27.7972	-27.7583

Notes: Standard errors in parenthesis, p-values in squared brackets. “..” denotes non-applicable. TFP=Total Factor Progress. (a) we use a small negative value of -0.01. (b) We present the log determinant of the variance-covariance matrix which, except for additive constants, is proportional to the log likelihood (this is in line with the Rats output for estimation with non-linear SUR).

Table 1.2—Supply-Side Estimates
(Using *Average* Real Interest Rate As In Germany, 1970:1 - 1982:4, *h*=0.0128)

	(a)	(b)	(c)	(d)
ζ	1.0168 (0.0015)	1.0167 (0.0015)	1.0149 (0.0014)	1.0121 (0.0013)
γ_N	0.0029 (0.0003)	0.0026 (0.0002)	0.0036 (0.0001)	0.0033 (0.0001)
λ_N	0.8988 (0.0654)	1.0000 (—)	0.8228 (0.0345)	1.0000 (—)
γ_K	0.0046 (0.0013)	0.0062 (0.0010)	0.0017 (0.0002)	0.0024 (0.0002)
λ_K	0.3730 (0.1039)	0.4403 (0.0806)	-0.0100 (—)	-0.0100 (—)
σ	0.6200 (0.0293)	0.6589 (0.0185)	0.5509 (0.0216)	0.6366 (0.0179)
$(1+\bar{\mu}_A)$	1.1431 (0.0132)	1.1445 (0.0130)	1.1297 (0.0117)	1.1232 (0.0113)
η	0.0027 (0.0001)	0.0028 (0.0001)	0.0025 (0.0000)	0.0024 (0.0000)
h	0.0128 (—)	0.0128 (—)	0.0128 (—)	0.0128 (—)
ϕ	0.0224 (0.0029)	0.0240 (0.0027)	0.0182 (0.0027)	0.0224 (0.0027)
TFP	0.0033	0.0033	0.0032	0.0031
Parameter Restrictions				
$\lambda_N = 1$	2.3949 [0.1217]	..	26.3700 [0.0000]	..
$\lambda_K \approx 0$	13.5806 [0.0002]	31.2272 [0.0000]	..	..
$\lambda_N = 1, \lambda_K \approx 0$ (a)	23.9051 [0.0000]	..	..	..
Neutrality Assumption				
Harrod: $\gamma_K = \lambda_K = 0,$ $\lambda_N = 1$	52.1458 [0.0000]	..	..	..
Hicks: $\gamma_N = \gamma_K,$ $\lambda_N = \lambda_K = 1$	446.9961 [0.0000]	..	..	..
Hicks Modified $\gamma_N = \gamma_K,$ $\lambda_N = \lambda_K$	44.6910 [0.0000]	..	..	..
Solow: $\gamma_N = \lambda_N = 0,$ $\lambda_K = 1$	1427.6260 [0.0000]	..	..	..
Stationarity				
ADF_p	-3.1659	-3.1972	-3.1261	-3.2816
$ADF_{ck/wn}$	-3.6510	-3.6041	-3.8610	-3.8733
$ADF_{Y/N}$	-2.6644	-2.6197	-3.0581	-3.5528
Likelihood (b)	-27.4865	-27.4786	-27.4215	-27.3429

Note: See notes to Table 1.1.

Table 1.3—Supply-Side Estimates (Free estimate for *h*)

	(a)	(b)	(c)	(d)	(e)	(f)
ζ	1.0162 (0.0015)	1.0163 (0.0014)	1.0134 (0.0014)	1.0114 (0.0013)	1.0137 (0.0014)	1.0133 (0.0014)
γ_N	0.0014 (0.0006)	0.0019 (0.0003)	0.0035 (0.0001)	0.0033 (0.0001)	0.0030 (0.0002)	0.0028 (0.0002)
λ_N	1.2178 (0.3435)	1.0000 (—)	0.8517 (0.0367)	1.0000 (—)	0.9261 (0.0626)	1.0000 (—)
γ_K	0.0070 (0.0017)	0.0057 (0.0008)	0.0013 (0.0001)	0.0017 (0.0001)	0.0025 (0.0005)	0.0030 (0.0004)
λ_K	0.7019 (0.0932)	0.6602 (0.0860)	-0.0100 (—)	-0.0100 (—)	0.2025 (0.0654)	0.2319 (0.0579)
σ	0.6535 (0.0303)	0.6251 (0.0175)	0.5303 (0.0200)	0.5957 (0.0167)	0.6228 (0.0289)	0.6510 (0.0189)
$(1+\bar{\mu}_A)$	1.0399 (0.0101)	1.0408 (0.0100)	1.0491 (0.0093)	1.0494 (0.0092)	1.0491 (0.0092)	1.0487 (0.0092)
η	0.0033 (0.0002)	0.0032 (0.0002)	0.0025 (0.0001)	0.0025 (0.0000)	0.0025 (—)	0.0025 (—)
h	0.0382 (0.0022)	0.0375 (0.0020)	0.0320 (0.0015)	0.0318 (0.0015)	0.0321 (0.0015)	0.0322 (0.0015)
ϕ	0.0274 (0.0031)	0.0256 (0.0027)	0.0162 (0.0028)	0.0201 (0.0027)	0.0227 (0.0029)	0.0242 (0.0028)
TFP	0.0029	0.0029	0.0029	0.0029	0.0029	0.0029
Parameter Restrictions						
$\lambda_N = 1$	0.4023 [0.5259]	..	16.3666 [0.0001]	..	1.3952 [0.2375]	..
$\lambda_K \approx 0$	58.2927 [0.0000]	60.6812 [0.0000]	..	..	10.5596 [0.0011]	17.4491 [0.0000]
$\lambda_N = 1, \lambda_K \approx 0$ (a)	92.6344 [0.0000]	..	..	..	15.2409 [0.0005]	..
Neutrality Assumption						
Harrod: $\gamma_K = \lambda_K = 0,$ $\lambda_N = 1$	95.6612 [0.0000]	..	..	..	87.8115 [0.0000]	..
Hicks: $\gamma_N = \gamma_K,$ $\lambda_N = \lambda_K = 1$	264.1049 [0.0000]	..	..	..	588.6988 [0.0000]	..
Hicks Modified $\gamma_N = \gamma_K,$ $\lambda_N = \lambda_K$	5.8125 [0.0540	..	..	..	88.9183 [0.0000]	..
Solow: $\gamma_N = \lambda_N = 0,$ $\lambda_K = 1$	292.7132 [0.0000]	..	..	..	1436.3751 [0.0000]	..
Stationarity						
ADF_p	-3.6627	-3.6218	-3.4024	-3.5871	-3.4970	-3.5340
$ADF_{ck/wn}$	-6.0957	-6.1212	-6.2204	-6.2202	-6.2044	-6.2035
$ADF_{Y/N}$	-2.6247	-2.6338	-2.8613	-3.1504	-2.8124	-2.7862
Likelihood (b)	-28.4451	-28.4326	-28.1918	-28.1256	-28.2446	-28.2367

Note: See notes to Table 1.1.

Table 1.4—**Dynamic Equation Estimates**

Investment Function (Capital Demand) Estimates: Equation (44)	
a_K	0.0094 (0.0034)
b_K	0.6490 (0.0905)
Characteristic Roots	0.64901; 1.01533; 1.56442
J-Test	$\chi^2(15) =$ 7.8149 [0.9309]
Labor Equation Estimates: Equation (43)	
$a_h \ / \ a_N$	0.0154 (0.0058)
Characteristic Roots	1.1393; 0.8881
J-Test	$\chi^2(16) =$ 12.6484 [0.6983]

Notes: Standard errors in parenthesis, probability-values in squared brackets.
The equation is estimated predicated on a time-varying discount factor, β_t, whose sample average is 0.9849.

Table 1.5—**Price Equation Estimates,** Equation (34)

a_h	0.6901 (0.0646)
DW	0.176
R^2	0.442

Figures 7
Technical Progress Dynamics (Table 1.1)

Table 1.1 a

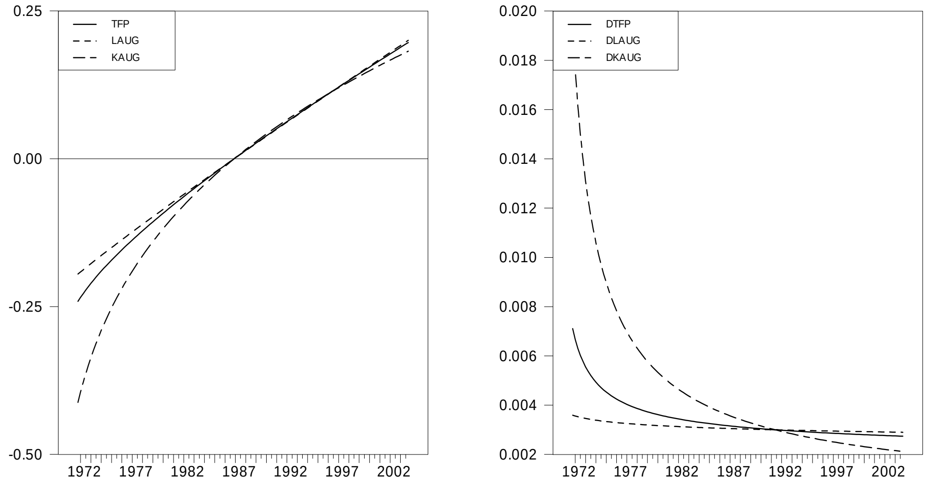
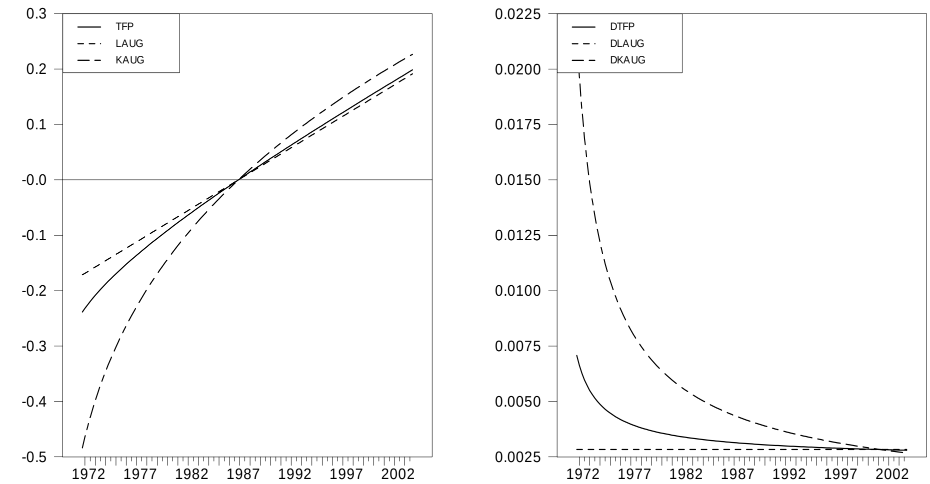


Table 1.1 b



Figures 7
Technical Progress Dynamics (Table 1.1. Cont.)

Table 1.1 c

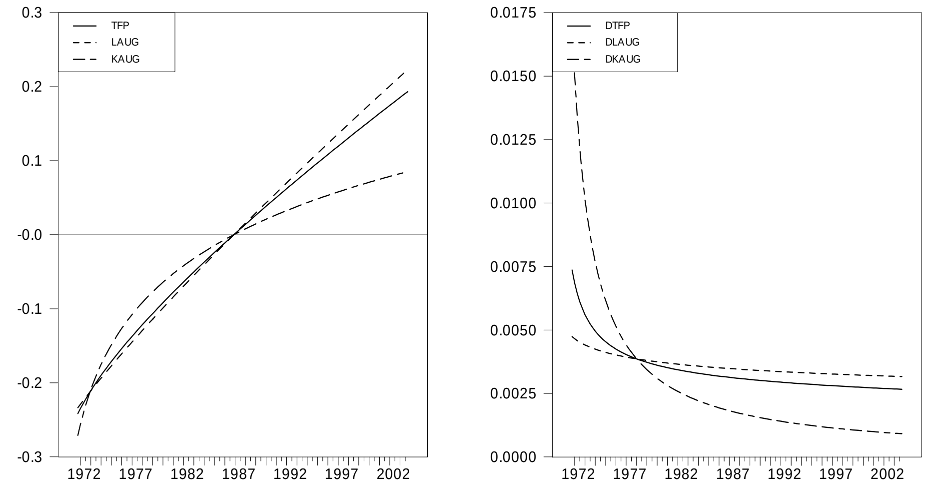
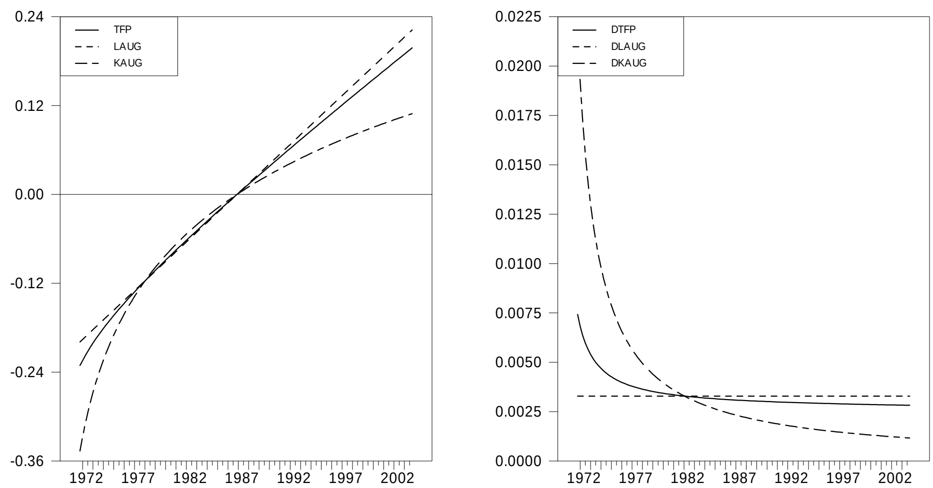


Table 1.1 d



Figures 8
Technical Progress Dynamics (Table 1.2)

Table 2.1 a

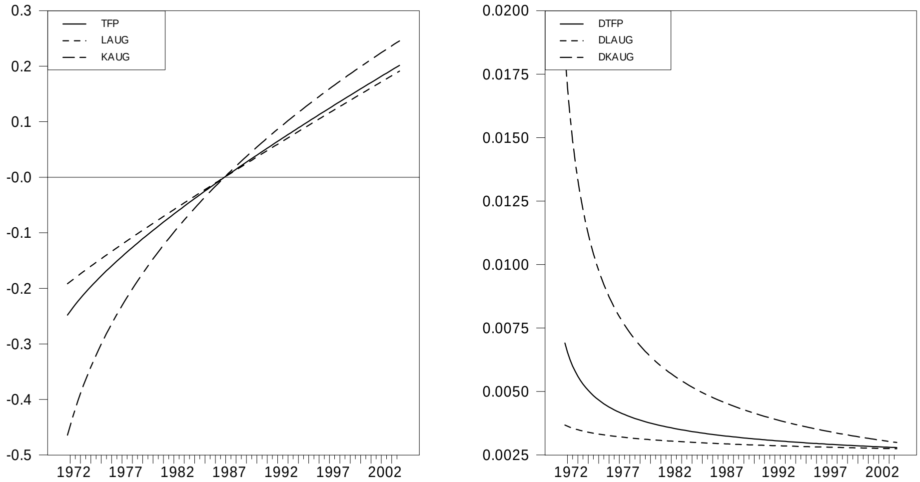
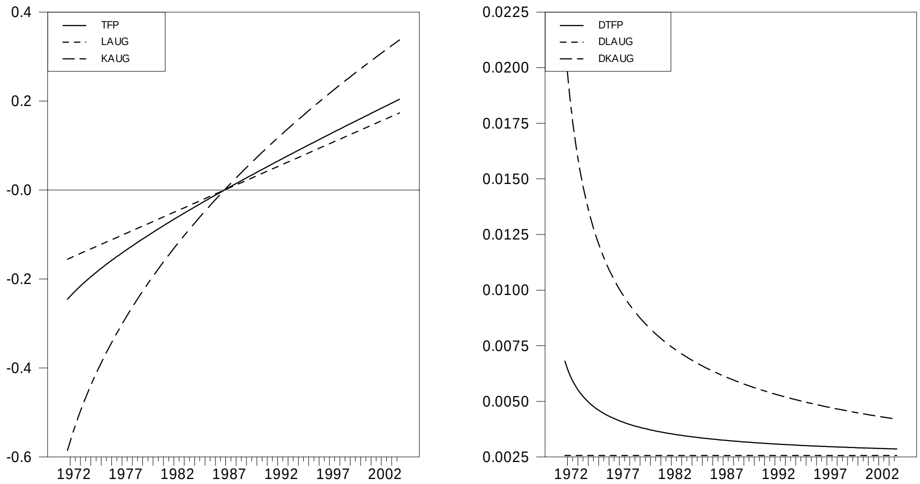


Table 2.1 b



Figures 8
Technical Progress Dynamics (Table 1.2, Cont.)

Table 2.1 c

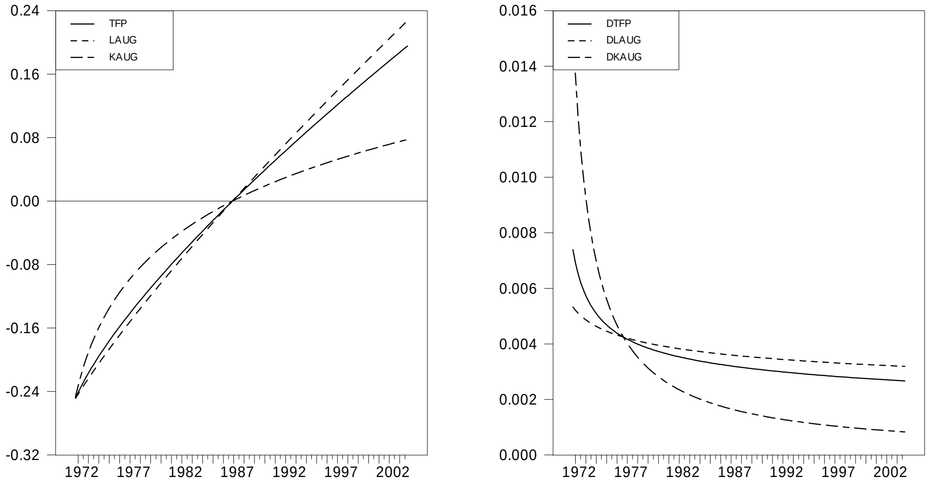
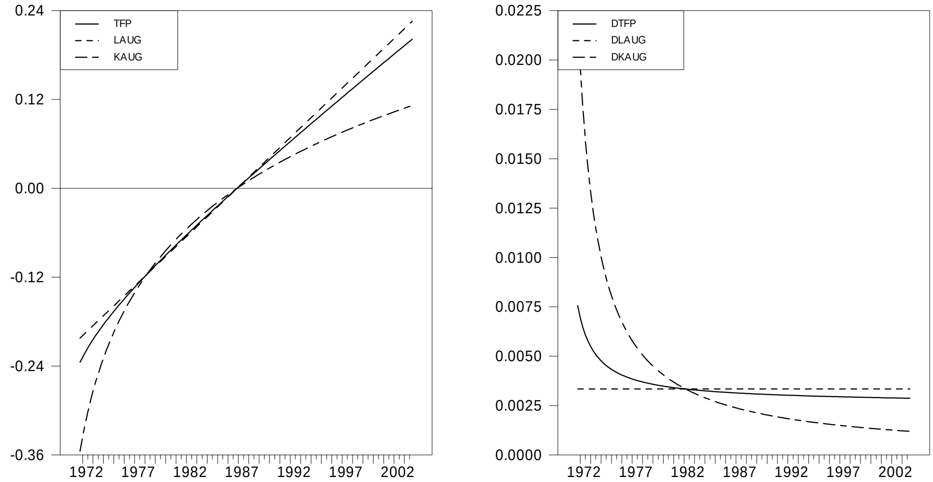


Table 2.1 d



Figures 9
Technical Progress Dynamics (Table 1.3)

Table 3.1 a

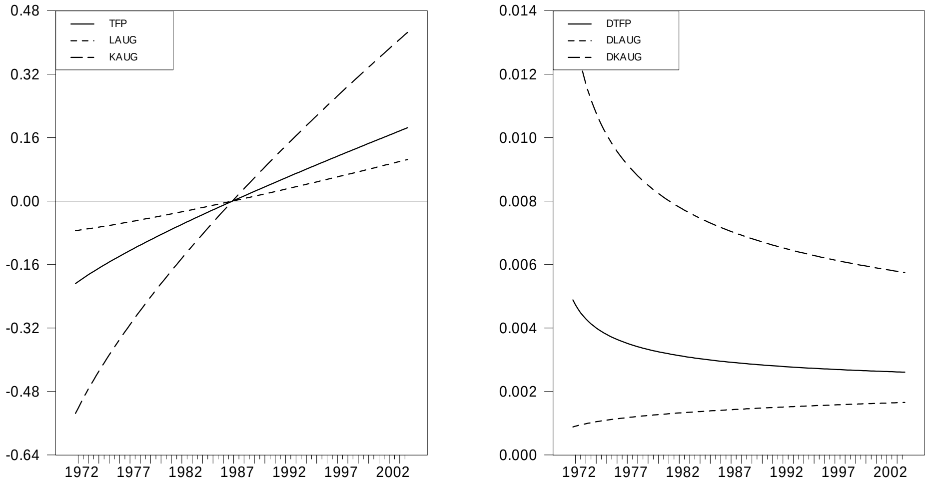
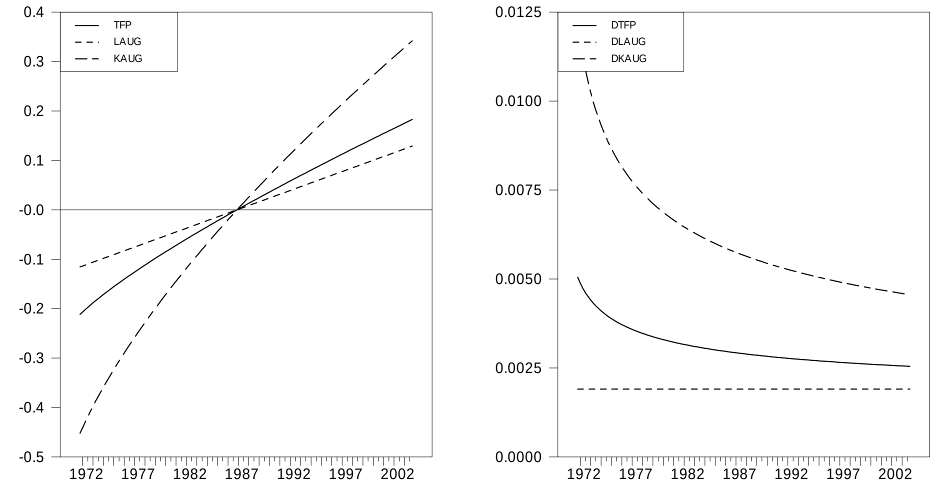


Table 3.1 b



Figures 9
Technical Progress Dynamics (Table 1.3, Cont.)

Table 3.1 c

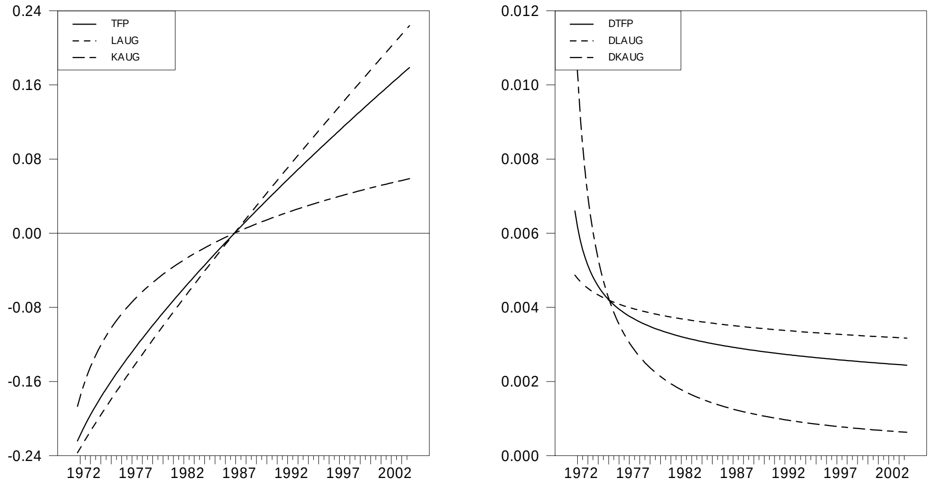
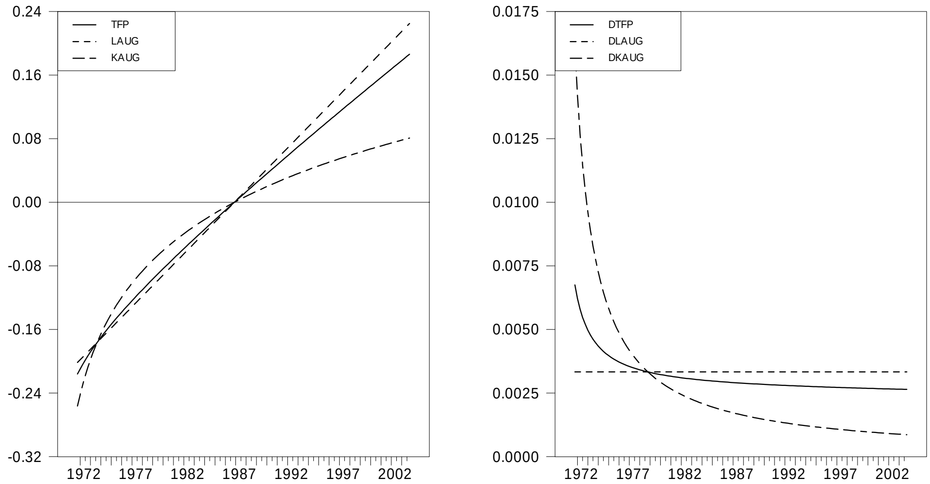


Table 3.1 d



Figures 9
Technical Progress Dynamics (Table 1.3, Cont.)

Table 3.1 e

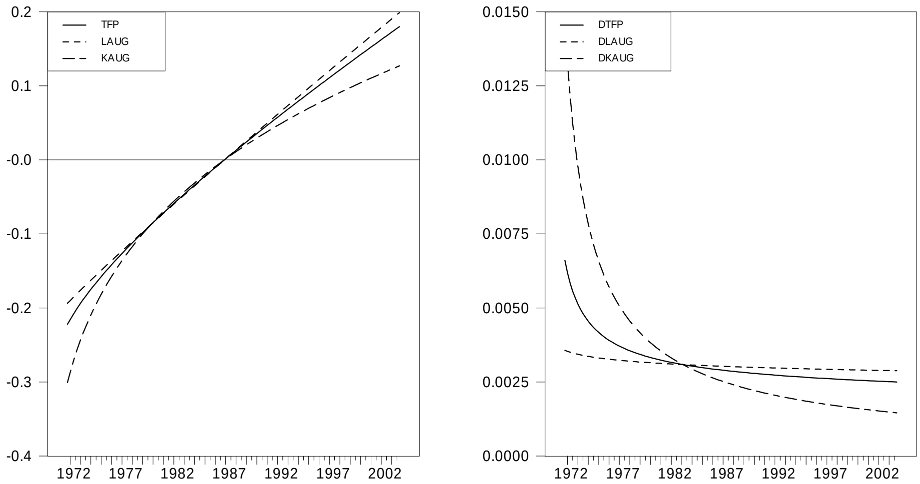
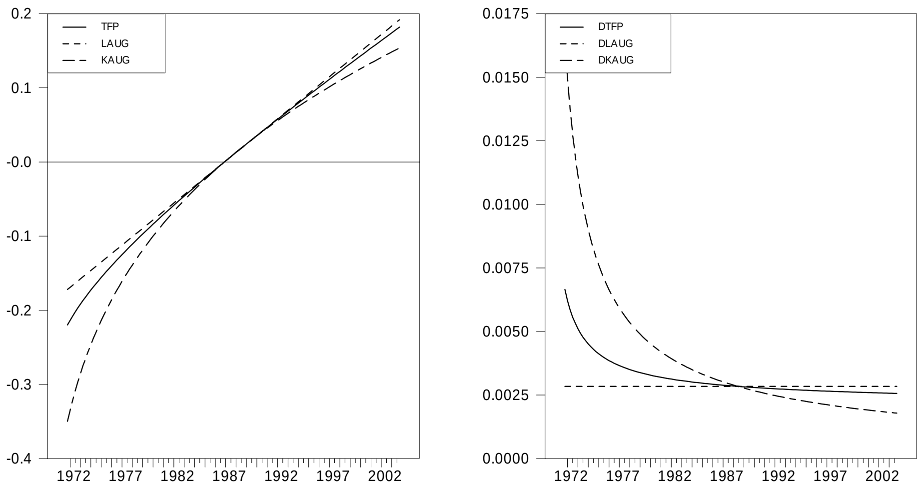


Table 3.1 f



Appendix One—Additional Data Transformations

Employment Income

As discussed by, for example, Gollin (2002), a problem in calculating labor-income is that it is unclear how the income of proprietors (self-employed) should be categorized in the labor-capital dichotomy. Some of the income earned by self-employed workers clearly represents labor income, while some represents a return on investment or economic profit. We follow a common practice to use compensation per employee as a shadow price of labor of self-employed workers.

Given a sample average value of 4% for unpaid employed labor (e.g., family members), we can calculate total labor income as:[24]

$$(1-0.04)\cdot(Compensation\,to\,Employees)\cdot\frac{Total\,Employment}{Employess}$$

Capital Stock Definitions

In calculating the capital income component we need, in addition to an operational counterpart for the user cost of capital *Q* (discussed in ?.?), the capital stock series *K*. However, regarding the capital stock, there are two different capital stock concepts available, i.e. gross and net capital stocks. The gross capital stock can be described as a capacity concept, i.e. it measures the potential volume of capital services which can be produced by the existing capital stock at a given point of time (e.g. Biorn and Olsen, 1989 and OECD, 1992). The net capital stock can be described as a wealth concept; capital has a value, which is derived from its ability to produce capital services today and in the future. Accordingly, the recommended practice in calculating the consumption of capital in national accounting statistics is to use the net capital stock.

The above argument supports the view that the net capital stock and the respective depreciation rate should be used in calculating the capital income component, while the gross capital stock should be used in the production function. To reconcile these views, we resort to the fact that, in practice, the ratio of net to gross capital stock is quite stable and, in the equilibrium growth path, this ratio should equal to the ratio of the respective depreciation rates.

Hence, on the basis of the steady-state condition $\frac{K_{net}}{K_{gross}}=\Psi=\frac{\delta_{gross}}{\delta_{net}}<1$ we can write,

$$K = P_I(r+\delta_{net})K_{net} = P_I(r\cdot\Psi+\delta_{gross})\cdot K_{gross}$$

where P_I is the investment deflator. The ratio, Ψ, calculated using Eurostat data is found to be 0.78.

With an annual depreciation rate of 4% in the euro-area (which is well in line with Eurostat data), the estimate for the capital income is defined as:

$$QK=\frac{P_I\left[(i-4\cdot\pi^e)\cdot\Psi+4\right]}{400}\cdot K$$

where i = interest rate, π^e = inflation expectations (the *HP*-filter fit for one-period P_I inflation)

Price Competitiveness

In constructing a series for the price competitiveness of open sector production, the deflator of euro area exports proxies the price of the total open sector production as constructed in the area-wide model data. As a measure for

[24] Four percent corresponds to the same average, although, note there is some mild trend in the ratio towards the end of the sample.

the competing foreign price of the open sector, we use the import price of non-primary goods. It is constructed utilizing the following quasi-identity:[25]

$$MTD = P_f^{1-m_t}\left(EEN * COMPR\right)^{m_t}$$

where *MTD* = deflator of the euro-area imports, *COMPR* = commodity prices (HWWA index), in US dollars, *EEN* = nominal effective exchange rate, m_t = elasticity estimate (the share of the primary goods imports of total euro area imports).

Appendix Two—Estimated Supply-Side System

For completeness, we now write down the full system as estimated. This shows the Box-Cox functional forms for the factor augmenting technical progress terms, the financing dummy, the terms a as estimated,

$$\log\left(\frac{w_t N_t}{p_t Y_t}\right) = \log\left(\frac{1-\bar{\pi}}{1+\bar{\mu}_A}\right) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log\zeta - \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right)\right] - \eta\cdot\left(t-\bar{t}\right) - \phi\cdot\log\left(\frac{P_f}{P^x}\right)$$

$$\left(\frac{Q_t}{P_t^I}\right) = \left(\frac{P_t^Y}{P_t^I}\right)\exp\left\{\log\left(\frac{\bar{\pi}}{1+\bar{\mu}_A}\right) + \frac{1}{\sigma}\log\left(\frac{Y_t}{K_t}\right) - \frac{1-\sigma}{\sigma}\left[\log\left(\frac{\zeta\cdot\bar{Y}}{\bar{K}}\right) + \frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K} - 1\right)\right]\right\}\cdot\exp\left[-\eta\left(t-\bar{t}\right) - \phi\cdot\log\left(\frac{P_f}{P^x}\right)\right]$$
$$+ h\cdot 0.78\cdot\frac{P_I K}{400}\cdot DUM$$

$$\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) = \log(\zeta) + \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right) - \frac{\sigma}{1-\sigma}\log\left[\bar{\pi}\cdot e^{\frac{1-\sigma}{\sigma}\left[\frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N}-1\right)-\frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K}-1\right)\right]}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\bar{\pi}\right)\right]$$

Where $1-\bar{\pi} = \left(1+\bar{\mu}_A\right)\frac{1}{T}\sum_{t=1}^{T}\frac{W_t N_t}{P_t Y_t}$, in other words, $1-\bar{\pi}$ is the average (or mid-sample) value of the labor income share. Thus, we define the distribution parameter solely in terms of labor income to isolate the financing dummy (*h*) from the specification of the rest of the estimated system.

[25] The area-wide model data contain the variables *MTD*, *EEN* and *COMPR*. The series for the import share of primary goods is calculated by the Directorate Statistics of the ECB on the basis of Eurostat data. This series covers the period from 1980/1981. In the 1970s, m_t is assumed to be constant, equaling to the value of 1980/1981.