Firm and Worker Dynamics in an Aging Labor Market

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Banca d'Italia

Background

Figure I: Share of labor force aged 40+ & dynamism

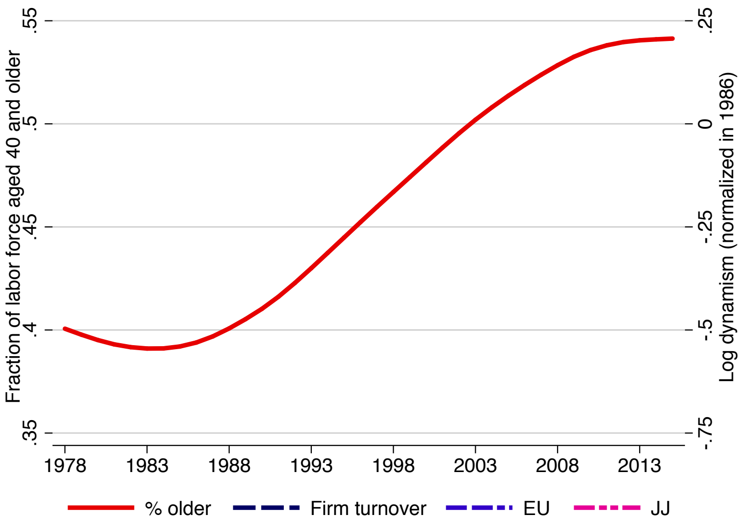
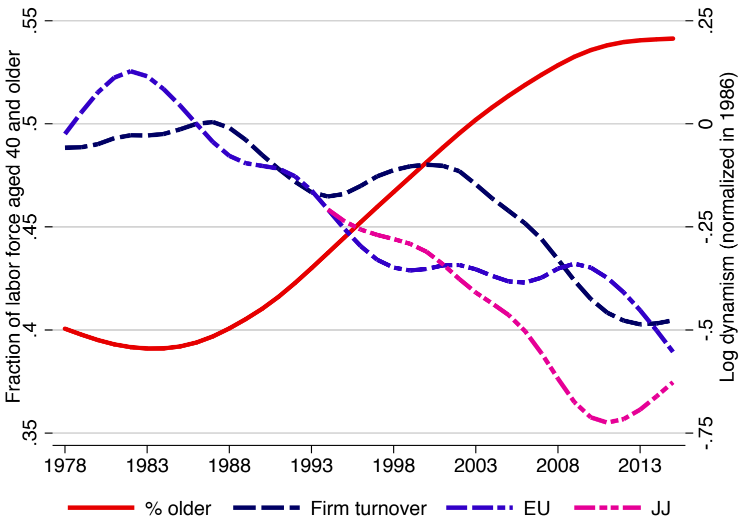


FIGURE I: SHARE OF LABOR FORCE AGED 40+ & DYNAMISM



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2. **Structural assessment**
    - Theory that links firm dyn., worker dyn. & growth to aging
    - **40–50% of declines** in firm & worker dynamism and **-$\frac{1}{4}$ percentage point** in annual economic growth
    - **Half due to equilibrium effects**

Related literature

Growth in frictional labor markets

- Bean & Pissarides ('93); Aghion & Howitt ('94); Mortensen & Pissarides ('98); Postel-Vinay ('98); Hornstein et al. ('07); Michau ('13)
- My contribution: Endogenous growth & on-the-job search

Declining dynamism, secular stagnation

- Gordon ('12); Hyatt & Spletzer ('13); Davis & Haltiwanger ('14); Fernald ('14); Malloy et al ('14); Karahan et al ('16); Hsieh & Klenow ('17)
- My contribution: Structural framework & quant. assessment of aging

Outline of talk

1. **Cross-state Evidence of the Impact of Aging**
2. **A Job Ladder with Creative Destruction**
3. **Structural Estimate of the Impact of Aging**
   - Life-cycle firm & worker dynamics
   - Impact of aging
   - Decomposing the mechanism

Cross-state Evidence on the Impact of Aging

State-fixed effect framework

- State-year data on dynamism & age composition 1978–2014

- Regress dynamism on share 40–64, controlling for state + year

$$\log(y_{st}) = \alpha \log\left(share_{st}^{40-64}\right) + \xi_s + \xi_t + \mathbf{X}_{st}\beta \qquad + \varepsilon_{st}$$

- Standard errors clustered at state and year

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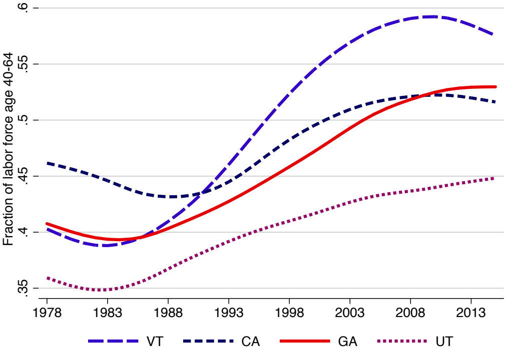
$$\log\left(y_{st}^{a}\right) = \alpha \log\left(share_{st}^{40-64}\right) + \xi_s + \xi_t + \mathbf{X}_{st}\beta + \xi_a + \varepsilon_{st}^{a}$$

- Standard errors clustered at state and year

- Variation in timing & magnitude of aging across states

Figure II: Fraction aged 40–64 in four selected states



Endogenous mobility

Issue: Mobility across states in response to dynamism

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   - Only effect on dynamism through current age composition
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Instrument current age composition with

1. 10-year lagged age composition
   - Only effect on dynamism through current age composition
   - Strong explanatory power on current age composition
2. Birth rates 40–64 years earlier
   - Only effect on dynamism through current age composition
   - Decent explanatory power on current age composition

Firm dynamism

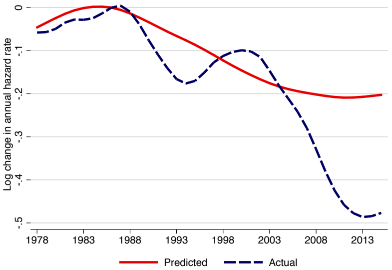
	(1)	(2)	(3)	(4)	(5)	(6)
	Labor force			Working age population		
	OLS	IV I	IV II	OLS	IV I	IV II
Panel A: Establishment dynamics						
Job reallocation	-0.448***	-0.527***	-1.183***	-0.518***	-0.539***	-0.978***
	(0.127)	(0.191)	(0.256)	(0.124)	(0.186)	(0.205)
Turnover	-0.630***	-0.961***	-1.573***	-0.774***	-0.984***	-1.300***
	(0.203)	(0.268)	(0.458)	(0.202)	(0.256)	(0.374)
Entry	-0.668***	-0.999***	-1.374***	-0.753***	-1.022***	-1.136***
	(0.189)	(0.247)	(0.498)	(0.188)	(0.245)	(0.409)
Exit	-0.600**	-0.940***	-1.753***	-0.809***	-0.962***	-1.449***
	(0.243)	(0.322)	(0.480)	(0.239)	(0.304)	(0.389)
Panel B: Firm dynamics						
Turnover	-0.764***	-1.266***	-1.680***	-0.923***	-1.296***	-1.411***
	(0.230)	(0.302)	(0.455)	(0.223)	(0.299)	(0.394)
Entry	-0.827***	-1.361***	-1.455***	-0.932***	-1.393***	-1.221***
	(0.199)	(0.278)	(0.506)	(0.195)	(0.291)	(0.440)
Exit	-0.712**	-1.203***	-1.795***	-0.921***	-1.231***	-1.484***
	(0.298)	(0.355)	(0.519)	(0.283)	(0.339)	(0.429)

Worker dynamism and growth

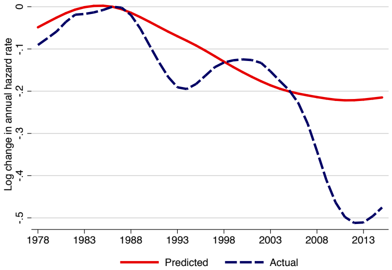
	(1)	(2)	(3)	(4)	(5)	(6)
	Labor force			Working age population		
	OLS	IV I	IV II	OLS	IV I	IV II
Panel C: Worker dynamics						
EU	-0.439***	-0.924**	-0.476	-0.494***	-0.939**	-0.425
	(0.145)	(0.375)	(0.582)	(0.159)	(0.406)	(0.506)
JJ	-0.477*	-0.113	-1.999*	-0.621***	-0.128	-3.165**
	(0.229)	(0.732)	(1.027)	(0.218)	(0.829)	(1.310)
UE	-0.088	-0.225	-0.744	-0.021	-0.228	-0.591
	(0.126)	(0.273)	(0.535)	(0.123)	(0.280)	(0.463)
Panel D: Growth in GDP per worker						
Growth	-0.066	-0.090**	-0.137**	-0.063	-0.092**	-0.115**
	(0.046)	(0.040)	(0.061)	(0.043)	(0.039)	(0.047)

Predicted impact of aging

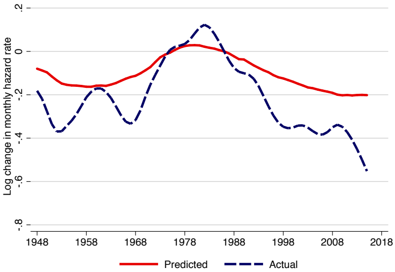
(A) Turnover rate



(B) Entry rate



(C) EU hazard



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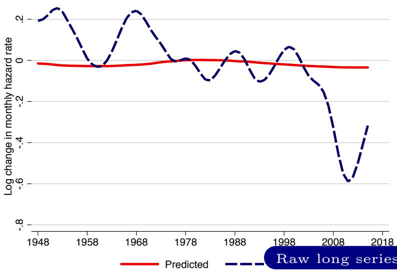

Raw long series

A Job Ladder Model with Creative Destruction

Key ingredients

- **Job ladder**: Ranking of firms that workers gradually climb
- **Entrepreneurial choice**
- **Creative destruction**: Entrants push out incumbents

Demographics & preferences

- **Agents**: Unit mass of individuals, $\mathbf{a} = \mathbf{1}, \ldots, \mathbf{A}$
  - Move to the next age at rate $\kappa(\mathbf{a})$
  - Oldest age group dies at rate $\kappa(\mathbf{A})$ and is replaced by offspring

- **Preferences**: Risk-neutral and altruistic w.r.t. offspring

$$\mathbb{E}_t \int_t^{\infty} \exp\left(-\tilde{\rho}(\tau - t)\right) \left[C\left(\tau\right) + \tilde{B}\left(\tau\right)\right] d\tau$$

where $\tilde{B}(\tau) = B(\tau)$ if unemployed; zero o.w.

Technology

- **Multiworker firms**: Idiosyncratic productivity $\tilde{\mathbf{z}}$

$$d\tilde{z}(t) = \mu_o dt + \sigma dW(t)$$

- **Production**: At match level, $\mathbf{y(z, x) = e^{\tilde{z}} \times x}$
  - x = quality of match; starts at $x = 1$
  - Jumps to x_b or x_g, $x_b < 1 < x_g$, with equal prob at rate ψ
  - Worker flows >> job flows

Two sources of growth:

1. Growth of incumbents at exogenous rate μ_o
2. Selection of firms at endogenous rate μ

$\Longrightarrow$ Total growth rate $\mu_e = \mu_o + \mu$

Transformation: $z = \tilde{z} - \underline{\tilde{z}}(t)$ etc.

- Incumbents fall behind at rate of obsolescence, $\mu = \mu_e - \mu_o$

When to switch employer & become entrepreneur

- Job finding rate λ from both U & E
- Entrepreneurship opportunities at rate γ
  - Entry cost $c \sim \Omega(a)$ and has to quit job (if employed)
  - Draws productivity from innovation distribution $\phi(z)$
  - Sells idea to MF and returns to labor market as unemployed
- Wage setting following Cahuc et al (2006)

Value of match & decision rules

$$\rho V(z, x_u, a) = y(z, x_u) - \underbrace{\mu \times \frac{\partial V(z, x_u, a)}{\partial z}}_{\text{obsolescence}} + \frac{\sigma^2}{2}\frac{\partial^2 V(z, x_u, a)}{\partial z^2} +$$

$$+ \underbrace{\kappa(a)\left[\tilde{V}(z, x_u, a+1) - V(z, x_u, a)\right]}_{\text{individual ages}} + \underbrace{\psi \sum_i \pi(x_i)\left[\tilde{V}(z, x_i, a) - V(z, x_u, a)\right]}_{\text{match quality is revealed}} +$$

$$+ \underbrace{\lambda\beta \int_0^\infty \left\{V(z', x_u, a) - V(z, x_u, a)\right\}^+ dF(z')}_{\text{new job offer}} + \underbrace{\gamma \int_{\underline{c}}^{\bar{c}} \{E + U(a) - V(z, x_u, a) - c\}^+ d\Omega(c; a)}_{\text{Entrepreneurship opportunity}}$$

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- **JJ mobility**: $V(\underline{z}(z, x, a), x_u, a) = V(z, x, a)$

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- **JJ mobility**: $V(\underline{z}(z, x, a), x_u, a) = V(z, x, a)$
- **Entrep. entry**: $\bar{c}(z, x, a) + V(z, x, a) = E + U(a)$

Firm's problem

Post vacancies v subject to cost $C(v) = r + c(v)$

- $c(v)$ is strictly convex flow cost per vacancy
- r is fixed cost associated with employing a unit of capital

$\Longrightarrow$ Stop paying => exit

$$\rho J(z) = \max_{v \geq 0} \left\{ v(1-\beta)q \left[\sum_a \left(\underbrace{u(a)\left\{V(z, x_u, a) - U(a)\right\}^+}_{\text{value from meeting unemployed individual}} \right) + \right.\right.$$

$$\left.\left. + \underbrace{(1-u)\int \left\{V(z, x_u, a) - V(z', x, a)\right\}^+ dG(z', x, a)}_{\text{value from meeting employed individual}} \right] - c(v) \right\} - \underbrace{r}_{\text{fixed cost}} - \underbrace{\mu J'(z)}_{\text{drift in } z} + \underbrace{\frac{\sigma^2}{2} J''(z)}_{\text{shocks to } z}$$

- **Vacancy policy**: $v(p)$ defined by FOC

Households own firms through mutual fund

- Avoids age of founder as state (Romer, 1990)
- Rents out K capital to firms in competitive market

$\Longrightarrow$ Factor in fixed supply => Creative destruction

Labor market: Cobb-Douglas matching function, $m = \chi V^{\alpha}$

Characterizing behavior & the equilibrium

Prop. 1 (Mismatch and worker dynamism)

(a) Better matched individuals are less likely to move

(b) A better matched labor market discourages vacancy creation

Prop. 2 (Mismatch and entrepreneurship)

(a) Better matched individuals are less entrepreneurial

(b) A better matched labor market discourages entrepreneurship

Prop. 3 (Amplification)

Rate of obsolescence increases in the aggregate entry rate, $\mu = \frac{e}{\zeta}$

Less entry => Lower rate of obsol. => Less mismatch => Less entry

Amplification

Aging

Amplification

Aging

Worker

$\uparrow$ **pairing**

with firms

Amplification

Aging ⇢ ↓ **Entry**

Aging ⇣

↑ **Worker pairing with firms**

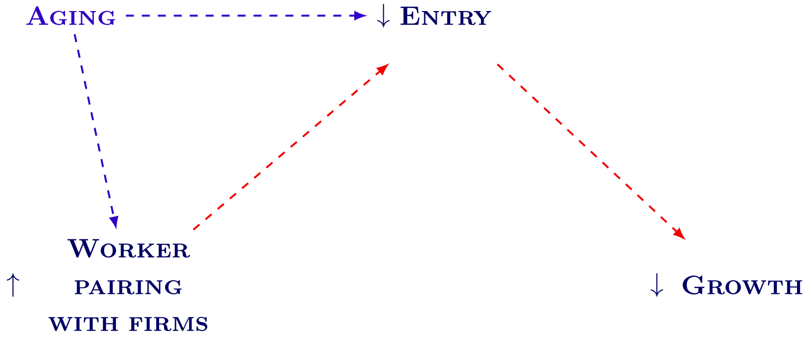
Amplification

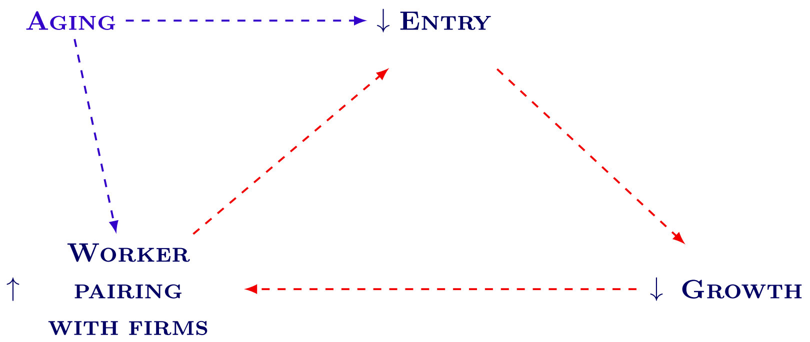
Aging

↓ Entry

↑ Worker pairing with firms

Amplification




Aging
↓ Entry
↑ Worker pairing with firms
↓ Growth

Structural Estimate of the Impact of Aging

Aging experiment within the model

- **Target**: salient features of aggregate firm & worker dynamism in BDS + SIPP in 2012–2014 Details Values

- **Validation**

  1. Life-cycle firm dynamics
  2. Life-cycle worker dynamics
  3. Link between worker and firm dynamics

- **Change age composition to 1986**
  - Reduce rate at which old individuals exit
  - Evaluate impact on dynamism holding everything else constant

The decline in dynamism

Table I: Firm dynamism

	(1) Data	(2) Model	(3) Share
Firm turnover	-0.026	-0.015	**56**
Job reallocation	-0.100	-0.039	**39**
Entry rate	-0.018	-0.012	**65**
Exit rate	-0.009	-0.003	**36**
Incumbent job reallocation	-0.046	-0.024	**53**

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More on UE/JJ | Graphs | Decker et al. (2017)'s empirical facts | Explanation of empirical facts

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Table II: Worker dynamism

	(1) Data	(2) Model	(3) Share
EU hazard	-0.003	-0.001	**36**
JJ hazard	-0.005	-0.002	**48**
UE hazard	-0.004	-0.001	**25**

The decline in dynamism

Table I: Firm dynamism

	(1) Data	(2) Model	(3) Share
Firm turnover	-0.026	-0.015	**56**
Job reallocation	-0.100	-0.039	**39**
Entry rate	-0.018	-0.012	**65**
Exit rate	-0.009	-0.003	**36**
Incumbent job reallocation	-0.046	-0.024	**53**

Table II: Worker dynamism

	(1) Data	(2) Model	(3) Share
EU hazard	-0.003	-0.001	**36**
JJ hazard	-0.005	-0.002	**48**
UE hazard	**-4%**	**-1%**	**25**

More on UE/JJ | Graphs | Decker et al. (2017)'s empirical facts | Explanation of empirical facts

The decline in dynamism

Table I: Firm dynamism

	(1) Data	(2) Model	(3) Share
Firm turnover	-0.026	-0.015	**56**
Job reallocation	-0.100	-0.039	**39**
Entry rate	-0.018	-0.012	**65**
Exit rate	-0.009	-0.003	**36**
Incumbent job reallocation	-0.046	-0.024	**53**

Table II: Worker dynamism

	(1) Data	(2) Model	(3) Share
EU hazard	-0.003	-0.001	**36**
JJ hazard	**-22%**	**-11%**	**48**
UE hazard	**-4%**	**-1%**	**25**

Aging has had negative growth effect

TABLE III: IMPACT OF AGING ON GROWTH & UNEMPLOYMENT

	(1) Data	(2) Model
Growth	-0.9	**-0.3**
Unemployment rate	-0.01	-0.01

Aging has had negative growth effect but positive level effect

Table III: Impact of aging on growth & unemployment

	(1) Data	(2) Model
Growth	-0.9	-0.3
Unemployment rate	-0.01	-0.01

Table IV: Log change in level of output, model

(1) Net output	(2) Discounted net output
0.06	-0.04

Aging has had negative growth effect but positive level effect

TABLE III: IMPACT OF AGING ON GROWTH & UNEMPLOYMENT

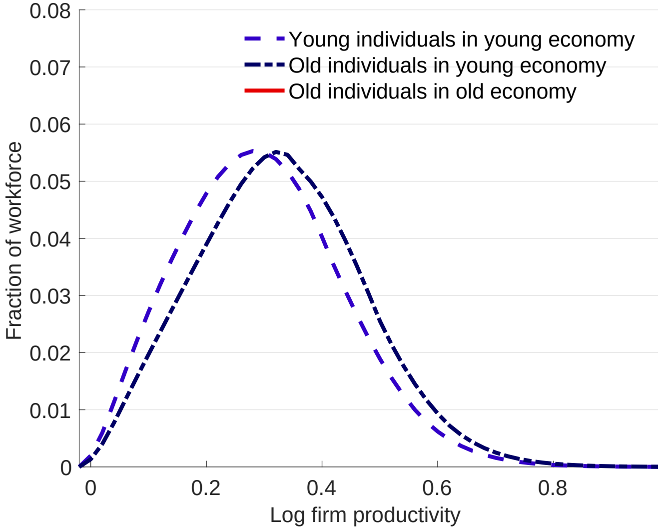
	(1) DATA	(2) MODEL
GROWTH	-0.9	-0.3
UNEMPLOYMENT RATE	-0.01	-0.01

TABLE IV: LOG CHANGE IN LEVEL OF OUTPUT, MODEL

(1) NET OUTPUT	(2) DISCOUNTED NET OUTPUT
0.06	**-0.04**

Direct & indirect effects

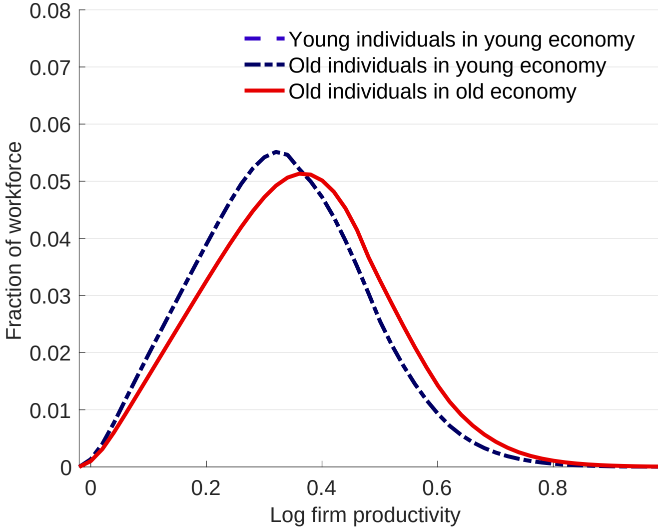
Figure IV: Employment distribution over firm productivity



Match quality

Direct & indirect effects

Figure IV: Employment distribution over firm productivity



Match quality

Equilibrium effects account for half of declines

$$\textbf{Hazard} = \sum_a \textbf{share}_a \underbrace{\int_{\mathbf{y}} Decision_a(\mathbf{y}) \times dEmployment_a(\mathbf{y})}_{\textbf{Age conditional rate}}$$

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$$\textbf{Hazard} = \sum_a \textbf{share}_a \underbrace{\int_{\mathbf{y}} Decision_a(\mathbf{y}) \times dEmployment_a(\mathbf{y})}_{\textbf{Age conditional rate}}$$

Table V: Decomposing the change in the JJ & entry hazard

	(1) Entry hazard	(2) JJ hazard
Direct effect	**10.5**	**7.0**

Details Shift share Life-cycle I Life-cycle II Emp. by size

Equilibrium effects account for half of declines

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Table V: Decomposing the change in the JJ & entry hazard

	(1) Entry hazard	(2) JJ hazard
Direct effect	**10.5**	**7.0**
Equilibrium effects	**11.7**	**6.1**
Decision rule	1.2	-17.3
Age cond. mismatch	10.4	23.3

Details | Shift share | Life-cycle I | Life-cycle II | Emp. by size

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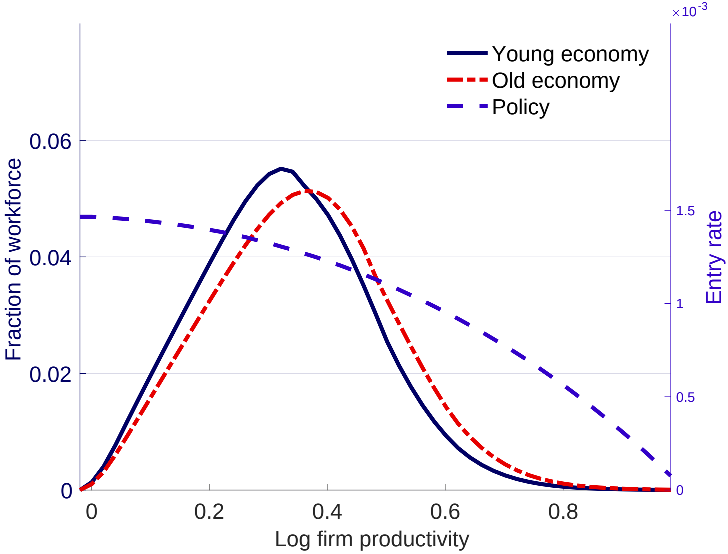
Decision rule	1.2	-17.3
Age cond. mismatch	10.4	23.3
Total effect	**22.2**	**13.1**

How much does entry fall with mismatch?

Figure V: Distribution of old individuals & entry policy



Additional exercises

1. Age-segregated labor markets
2. No aging of potential entrepreneurs
3. Approximate transition dynamics
4. Income dynamics

Fast-forward to conclusion

Conclusion

Aging explains

1. **40–50% of declines** in entry, exit, incumbent job reallocation, EU and JJ mobility; modest fall in UE
2. $-\frac{1}{4}$ **percentage points** decline in growth
3. **Half due to equilibrium effects**

Policy: Regulation/taxation or immigration?

Future research

1. **Aging typically accounts for at most half of declines**
   - Labor supply (Karahan et al, 2016)
   - Licensing (Kleiner and Krueger, 2013)
   - Training requirements (Cairo, 2013)
   - EPL (Autor et al., 2007)

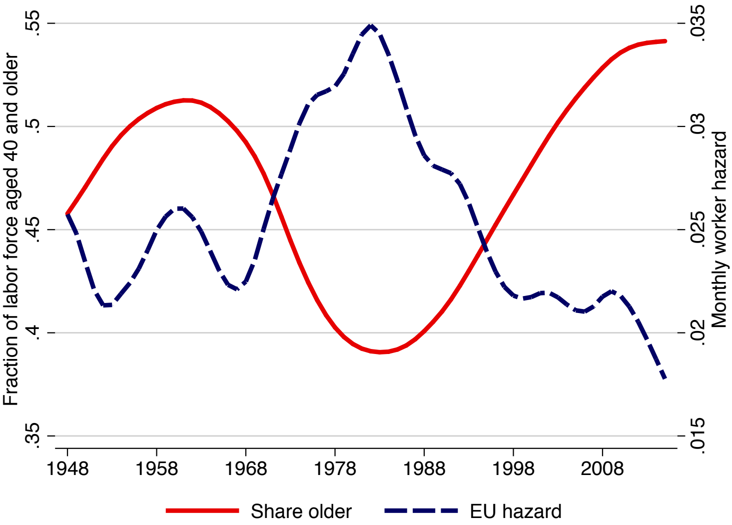
2. **Anecdotal evidence that aging has reduced dynamism & growth in other countries**
   - A rigorous cross-country analysis is missing

Thank you

Appendix A

A longer perspective

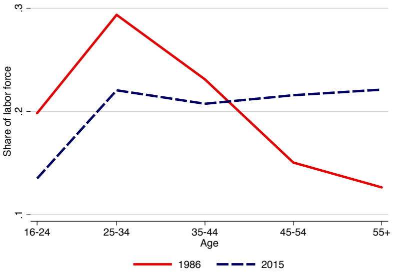
Figure VI: Share 40 and older and EU hazard



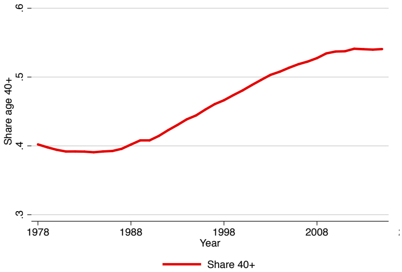
Back to motivation Back to conclusion

Age composition

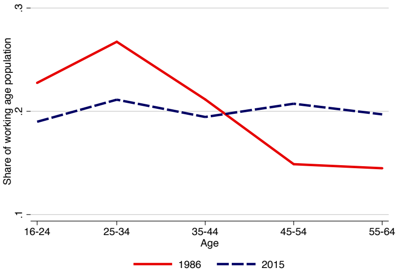
(A) Age distribution (labor force)



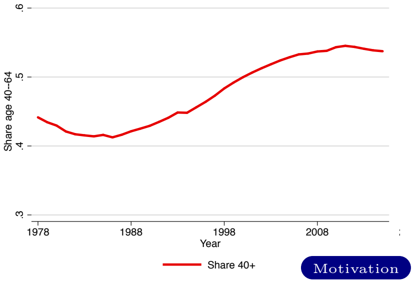
(B) Share 40+ (labor force)



(C) Age distribution (working age)



(D) Share 40+ (working age)



Motivation

Firm dynamics definitions

Data

- Business Dynamic Statistics (BDS) 1978–2015
- Annual data on firms and establishments covering private sector

Definitions

- Job creation: $JC_t = \sum_i \left(size_{it} - size_{it-1}\right)^+$
- Job destruction: $JD_t = \sum_i \left(-(size_{it} - size_{it-1})\right)^+$

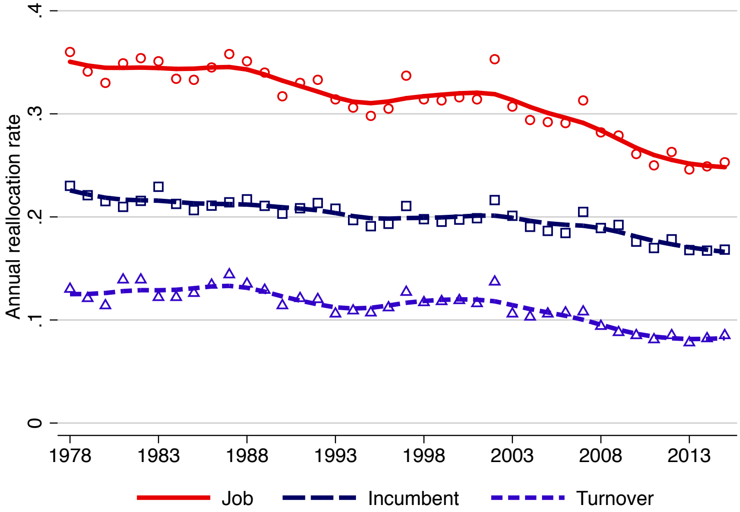
$$\underbrace{JC_t + JD_t} \quad = \quad \underbrace{JC_t^{inc} + JD_t^{inc}} \; + \; \underbrace{JC_t^{entry} + JD_t^{exit}}$$

Job reallocation$_t$ = Inc job reallocation$_t$ + Estabs. turnover$_t$

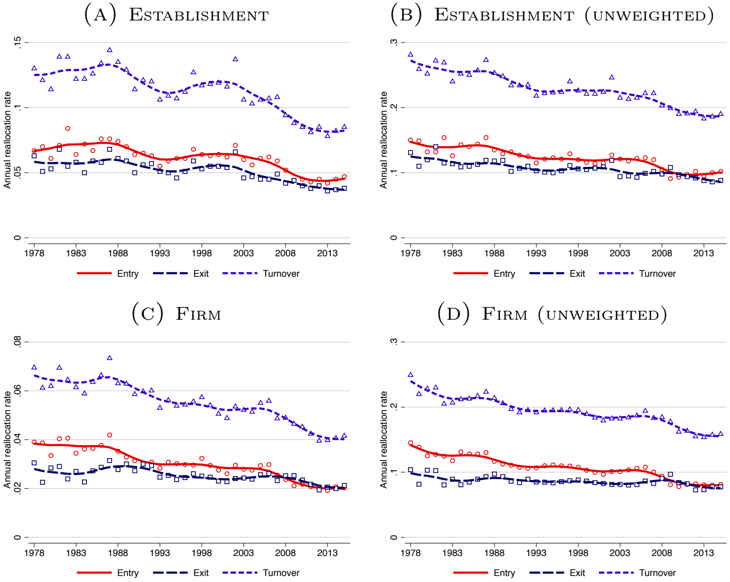
Motivation

Firm dynamics

Figure VIII: Establishment reallocation rates

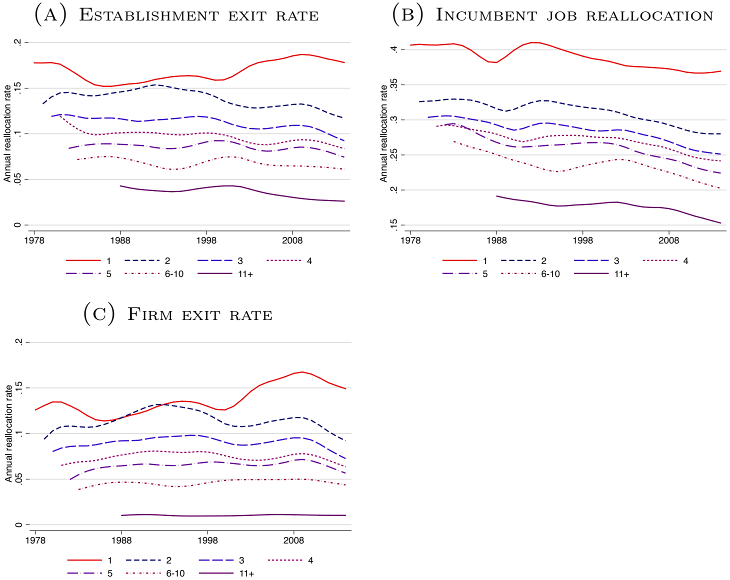


Entry and exit



Appendix Motivation

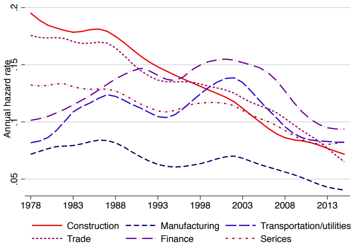
Dynamics by firm age



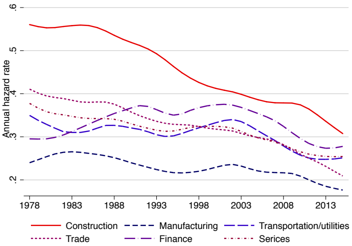
Appendix Karahan et al Motivation

Dynamics by industry

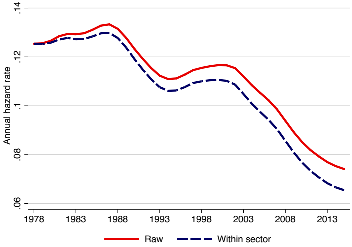
(A) Turnover



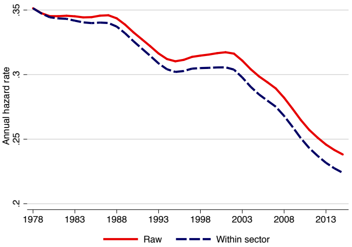
(B) Job reallocation



(C) Turnover



(D) Job reallocation



Appendix Motivation

Worker mobility definitions

Data

- SIPP (1984–2013)
- CPS (1978–2015)
- BLS (1948–2015)

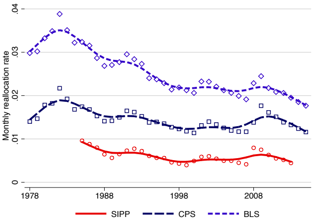
Definitions

- EU_{it} = employed in month t, unemployed in $t+1$
- UE_{it} = unemployed in t, employed in $t+1$
- JJ_{it} = employed in t, different main employer in $t+1$

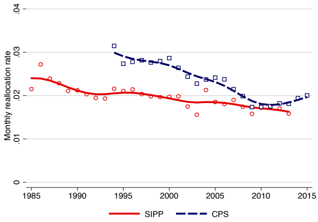
Worker dynamics

- Large fall in EU & JJ hazard
- Little evidence of secular decline in UE hazard

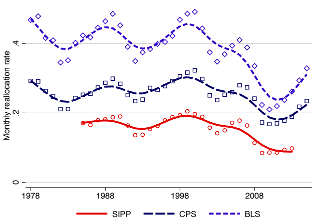
(A) EU hazard



(B) JJ hazard



(C) UE hazard

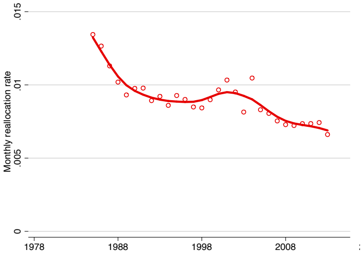


LN and NL Motivation

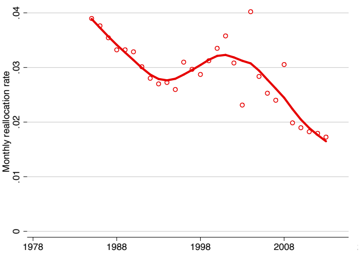
LN and NL flows

- Declines in the hazard of moving in and out of the labor force

(A) LN hazard



(B) NL hazard



Appendix Motivation

Growth

- Annual growth in real GDP per worker slowed from 2.6% in 1984–1988 to 1.7% in 2012–2016

Figure XIV: Annual HP-filtered growth rate

Motivation

Karahan et al. (2016)

Karahan, Pugsley and Sahin (2016)

- Labor supply growth explains $\frac{1}{4}$ of fall in start-up rate
- No change in incumbent life-cycle dynamics

Two key differences

1. "Quality"/composition of labor force rather than quantity
2. Partly different set of outcomes: Worker dynamics, incumbent dynamics and growth

Back

Shift-share

- Denote by $rate_a^{late}$ age-conditional mobility rates in 2012–2014
- Denote by $share_a^p$ the share of the labor force in age a in period p
- Direct effect = change due to shift in age composition under fixed age-conditional mobility rates

$$rate^{\text{direct}} = \sum_a rate_a^{late} \left[share_a^{early} - share_a^{late} \right]$$

Back

Shift-share

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Early		Late		% change		
	Raw	Direct	Raw	Direct	Raw	Direct	Share
Panel A: JJ mobility							
SIPP	0.024	0.019	0.017	0.017	45.4	12.3	27.1
Panel B: EU mobility							
SIPP	0.009	0.006	0.005	0.005	61.3	14.0	22.9
CPS	0.017	0.015	0.012	0.014	42.0	10.7	25.6
Panel C: UE mobility							
SIPP	0.175	0.101	0.090	0.093	94.5	8.6	9.1
CPS	0.251	0.199	0.221	0.196	13.7	1.9	13.9
Panel D: Entry to entrepreneurship							
Baseline	156.5	109.9	100	100	56.5	9.9	17.5
Opportunistic	156.5	110.0	100	100	56.5	10.0	17.7
Expect to grow	156.5	110.8	100	100	56.5	10.8	19.2

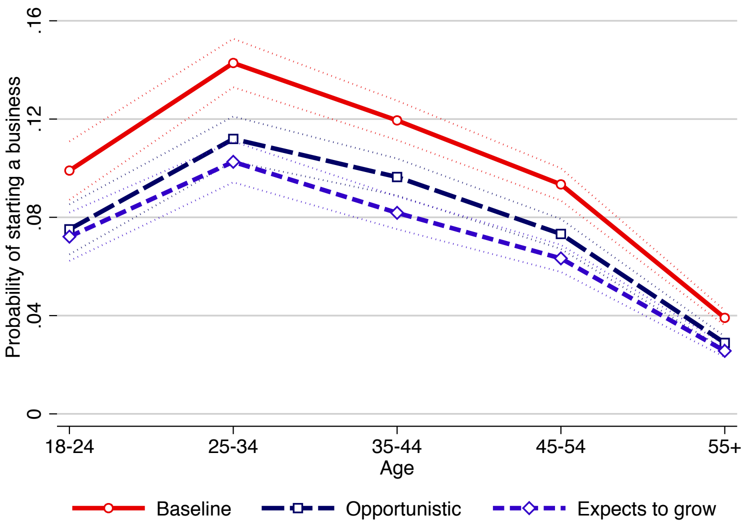
Back

Appendix B

Michael Bloomberg

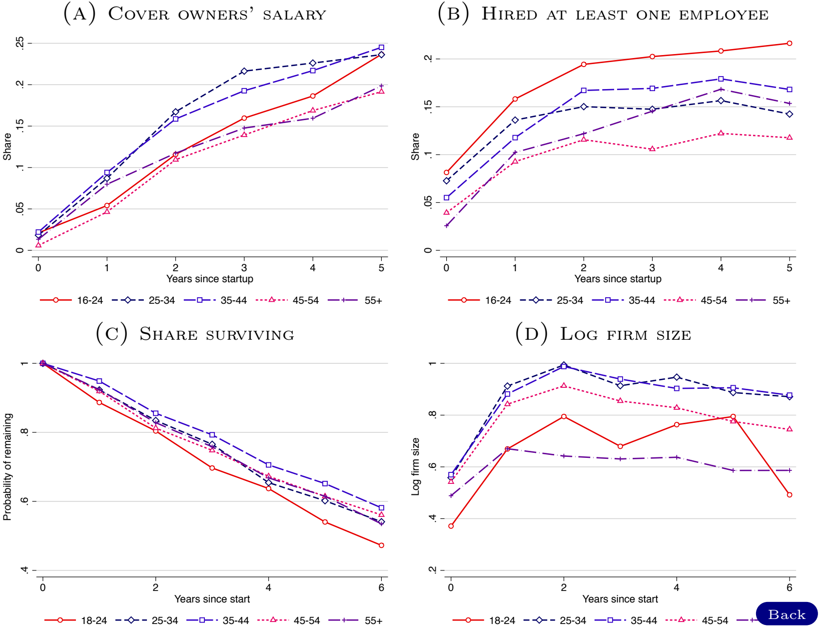
- Partner at Solomon Brothers, laid off in 1981 (at age 39)
- Started financial service company Bloomberg LP
- Current net worth: \$47.8bn
- **Would he have started Bloomberg if he had not been laid off?**
  - Walt Disney, JK Rowling, Thomas Edison, Mark Cuban, Oprah Winfrey, Sallie Krawcheck, Bernie Marcus and Arthur Blank...

Figure XV: Entrepreneurship entry by age



Model Calibration

Post entry performance by age of founder



Back

Search and matching

- Individuals may be either employed or unemployed

Back

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- Individuals may be either employed or unemployed

- Search with the same efficiency (normalized to one)

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- If firms post $\bar{v}$ vacancies, total number of matches equals $\chi\bar{v}^{\alpha}$

Back

Search and matching

- Individuals may be either employed or unemployed
- Search with the same efficiency (normalized to one)
- If firms post $\bar{v}$ vacancies, total number of matches equals $\chi \bar{v}^{\alpha}$
- Denote by λ rate at which individuals meet with open vacancies, q rate at which vacancy contacts individuals

$$\lambda = \chi \bar{v}^{\alpha}, \qquad q = \chi \bar{v}^{\alpha - 1}$$

Back

Appendix C

Wage setting

Offer matching framework of Cahuc et al (2006)

- Unemployed: Outside value plus β of surplus

Back

Wage setting

Offer matching framework of Cahuc et al (2006)

- Unemployed: Outside value plus β of surplus

- Employed (I): Poacher with lower valuation
  - Remain with current employer, (potentially) get updated value equal to poacher plus β of differential

Back

Wage setting

Offer matching framework of Cahuc et al (2006)

- Unemployed: Outside value plus β of surplus

- Employed (I): Poacher with lower valuation
  - Remain with current employer, (potentially) get updated value equal to poacher plus β of differential

- Employed (II): Poacher with higher valuation
  - Switch to poacher, get current match plus β of differential

Back

Wage setting

Offer matching framework of Cahuc et al (2006)

- Unemployed: Outside value plus β of surplus

- Employed (I): Poacher with lower valuation
  - Remain with current employer, (potentially) get updated value equal to poacher plus β of differential

- Employed (II): Poacher with higher valuation
  - Switch to poacher, get current match plus β of differential

$\Longrightarrow$ **Renegotiation when one party has credible threat**

Back

Balanced growth

- On the BGP, $\underline{Z}(t)$ and $\tilde{r}(t)$ grow at endogenous rate μ, while incumbent firm productivity in expectation does not change

Back

Balanced growth

- On the BGP, $\underline{Z}(t)$ and $\tilde{r}(t)$ grow at endogenous rate μ, while incumbent firm productivity in expectation does not change

- Study transformed economy in which $\underline{Z}(t)$ and $\tilde{r}(t)$ do not grow

Back

Balanced growth

- On the BGP, $\underline{Z}(t)$ and $\tilde{r}(t)$ grow at endogenous rate μ, while incumbent firm productivity in expectation does not change

- Study transformed economy in which $\underline{Z}(t)$ and $\tilde{r}(t)$ do not grow

- Normalize by $\underline{Z}(t)$ and denote by

Back

Balanced growth

- On the BGP, $\underline{Z}(t)$ and $\tilde{r}(t)$ grow at endogenous rate μ, while incumbent firm productivity in expectation does not change

- Study transformed economy in which $\underline{Z}(t)$ and $\tilde{r}(t)$ do not grow

- Normalize by $\underline{Z}(t)$ and denote by
  1. $z = \log(Z(t)/\underline{Z}(t))$ normalized log firm productivity

Back

Balanced growth

- On the BGP, $\underline{Z}(t)$ and $\tilde{r}(t)$ grow at endogenous rate μ, while incumbent firm productivity in expectation does not change

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- Normalize by $\underline{Z}(t)$ and denote by
  1. $z = \log(Z(t)/\underline{Z}(t))$ normalized log firm productivity
  2. r the normalized price of a marketing specialist

Back

Balanced growth

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- Study transformed economy in which $\underline{Z}(t)$ and $\tilde{r}(t)$ do not grow

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  2. r the normalized price of a marketing specialist
  3. $\phi(z)$ the normalized innovation distribution

Back

Balanced growth

- On the BGP, $\underline{Z}(t)$ and $\tilde{r}(t)$ grow at endogenous rate μ, while incumbent firm productivity in expectation does not change

- Study transformed economy in which $\underline{Z}(t)$ and $\tilde{r}(t)$ do not grow

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  1. $z = \log(Z(t)/\underline{Z}(t))$ normalized log firm productivity
  2. r the normalized price of a marketing specialist
  3. $\phi(z)$ the normalized innovation distribution
  4. $\rho = \tilde{\rho} - \mu$ the effective discount rate

Back

Balanced growth

- On the BGP, $\underline{Z}(t)$ and $\tilde{r}(t)$ grow at endogenous rate μ, while incumbent firm productivity in expectation does not change

- Study transformed economy in which $\underline{Z}(t)$ and $\tilde{r}(t)$ do not grow

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  1. $z = \log(Z(t)/\underline{Z}(t))$ normalized log firm productivity
  2. r the normalized price of a marketing specialist
  3. $\phi(z)$ the normalized innovation distribution
  4. $\rho = \tilde{\rho} - \mu$ the effective discount rate

$\Longrightarrow$ Incumbent firm productivity drifts at $-\mu$ while r is constant

Back

Value of unemployment

$$\rho U(a) = b + \underbrace{\kappa(a)\left[U(a+1) - U(a)\right]}_{\text{Aging}} +$$

Back

Value of unemployment

$$\rho U(a) = b + \underbrace{\kappa(a)\left[U(a+1) - U(a)\right]}_{\text{Aging}} + \underbrace{\lambda\beta \int_0^\infty \left\{V(z, x_u, a) - U(a)\right\}^+ dF(z)}_{\text{Job offer}}$$

- An individual meets firm with productivity z at rate $\lambda f(z)$

Back

Value of unemployment

$$\rho U(a) = b + \underbrace{\kappa(a)\left[U(a+1) - U(a)\right]}_{\text{Aging}} + \underbrace{\lambda\beta \int_0^\infty \left\{V(z, x_u, a) - U(a)\right\}^+ dF(z)}_{\text{Job offer}}$$

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  - Initial match productivity is unknown, $x = x_u$

Back

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  - Initial match productivity is unknown, $x = x_u$
  - Gets β of difference between value of match, $V(z, x_u, a)$, and $U(a)$

Back

Value of unemployment

$$\rho U(a) = b + \underbrace{\kappa(a)\left[U(a+1) - U(a)\right]}_{\text{Aging}} + \underbrace{\lambda\beta \int_0^\infty \left\{V(z, x_u, a) - U(a)\right\}^+ dF(z)}_{\text{Job offer}} + \underbrace{\gamma(a) \int_{\underline{c}}^{\bar{c}} \left\{E - c\right\}^+ d\Omega(c)}_{\text{Entrepreneurship opportunity}}$$

- An individual meets firm with productivity z at rate $\lambda f(z)$
  - Initial match productivity is unknown, $x = x_u$
  - Gets β of difference between value of match, $V(z, x_u, a)$, and $U(a)$
- Opportunity to start business at rate $\gamma(a)$

Back

Value of unemployment

$$\rho U(a) = b + \underbrace{\kappa(a)\left[U(a+1) - U(a)\right]}_{\text{Aging}} + \underbrace{\lambda\beta \int_0^{\infty} \left\{V(z, x_u, a) - U(a)\right\}^+ dF(z)}_{\text{Job offer}} + \underbrace{\gamma(a) \int_{\underline{c}}^{\bar{c}} \left\{E - c\right\}^+ d\Omega(c)}_{\text{Entrepreneurship opportunity}}$$

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- Opportunity to start business at rate $\gamma(a)$
  - Associated entry cost c drawn from Ω
  - E denotes expected value of entrepreneurship

Back

Value of unemployment

$$\rho U(a) = b + \underbrace{\kappa(a)\left[U(a+1) - U(a)\right]}_{\text{Aging}} + \underbrace{\lambda\beta \int_{0}^{\infty} \{V(z, x_u, a) - U(a)\}^{+} \, dF(z)}_{\text{Job offer}} + \underbrace{\gamma(a) \int_{\underline{c}}^{\bar{c}} \{E - c\}^{+} \, d\Omega(c)}_{\text{Entrepreneurship opportunity}}$$

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- Opportunity to start business at rate $\gamma(a)$
  - Associated entry cost c drawn from Ω
  - E denotes expected value of entrepreneurship
- **Decision rules:** $\mathbf{\underline{z}_u(x_u, a)}$

Back

Value of unemployment

$$\rho U(a) = b + \underbrace{\kappa(a)\left[U(a+1) - U(a)\right]}_{\text{Aging}} + \underbrace{\lambda\beta \int_0^\infty \left\{V(z, x_u, a) - U(a)\right\}^+ dF(z)}_{\text{Job offer}} + \underbrace{\gamma(a) \int_{\underline{c}}^{\bar{c}} \left\{E - c\right\}^+ d\Omega(c)}_{\text{Entrepreneurship opportunity}}$$

- An individual meets firm with productivity z at rate $\lambda f(z)$
  - Initial match productivity is unknown, $x = x_u$
  - Gets β of difference between value of match, $V(z, x_u, a)$, and $U(a)$
- Opportunity to start business at rate $\gamma(a)$
  - Associated entry cost c drawn from Ω
  - E denotes expected value of entrepreneurship
- **Decision rules:** $\underline{\mathbf{z}}_{\mathbf{u}}(\mathbf{x}_{\mathbf{u}}, \mathbf{a})$ and $\bar{\mathbf{c}}_{\mathbf{u}}$

Back

Value of match with known quality

$$
\begin{aligned}
\rho V(z,x,a) =& e^{z} - \underbrace{\mu \frac{\partial V(z,x,a)}{\partial z}}_{\text{drift in } z} + \underbrace{\frac{\sigma^2}{2} \frac{\partial^2 V(z,x,a)}{\partial z^2}}_{\text{shocks to } z} + \\
& + \underbrace{\kappa(a)\left[\max\left\{V(z,x,a+1), U(a+1)\right\} - V(z,x,a)\right]}_{\text{individual ages}} + \\
& + \underbrace{\lambda\beta \int_{0}^{\infty} \max\left\{V\left(z', x_u, a\right) - V(z,x,a), 0\right\} dF(z')}_{\text{new job offer}} + \\
& + \underbrace{\gamma(a) \int_{\underline{c}}^{\bar{c}} \max\left\{E - c - V(z,x,a) + U(a), 0\right\} d\Omega(c)}_{\text{entrepreneurship opportunity}}
\end{aligned}
$$

Value of entrepreneurship

- An individual who enters entrepreneurship draws an initial productivity z from Φ

Back to main

Value of entrepreneurship

- An individual who enters entrepreneurship draws an initial productivity z from Φ

- She gives the mutual fund a take-it-or-leave-it offer to purchase the business idea

Back to main

Value of entrepreneurship

- An individual who enters entrepreneurship draws an initial productivity z from Φ
- She gives the mutual fund a take-it-or-leave-it offer to purchase the business idea
- Hence the expected value of entry equals

$$E = \int_0^\infty J(z) d\Phi(z)$$

Back to main

Value of hiring to firm

- Denote by $J(z)$ the value of hiring to a firm

$$\rho J(z) = \max_{v \geq 0} \left\{ v(1-\beta) q \left[\sum_a \left(\underbrace{u(a) \left\{ V(z, x_u, a) - U(a) \right\}^+}_{\text{value from meeting unemployed individual}} \right) + \right.\right.$$

$$\left.\left. + \underbrace{(1-u) \int \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ dG(z', x, a)}_{\text{value from meeting employed individual}} \right] - c(v) \right\} - \underbrace{r}_{\text{fixed cost}} - \underbrace{\mu J'(z)}_{\text{drift in } z} + \underbrace{\frac{\sigma^2}{2} J''(z)}_{\text{shocks to } z}$$

Back to main

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$$\left.\left.+\underbrace{(1-u)\int\{V(z,x_u,a)-V(z',x,a)\}^+ dG(z',x,a)}_{\text{value from meeting employed individual}}\right]-c(v)\right\} - \underbrace{r}_{\text{fixed cost}} - \underbrace{\mu J'(z)}_{\text{drift in } z} + \underbrace{\frac{\sigma^2}{2}J''(z)}_{\text{shocks to } z}$$

- Post vacancies v subject to $\mathbf{c(v)}$

Back to main

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- Post vacancies v subject to $\mathbf{c(v)}$

- Has to pay fixed cost $\mathbf{r}$ to remain in hiring market

Back to main

Value of hiring to firm

- Denote by $J(z)$ the value of hiring to a firm

$$\rho J(z) = \max_{v\geq 0}\Bigg\{ v(1-\beta)q\Bigg[\sum_a\Bigg(\underbrace{u(a)\left\{V(z,x_u,a)-U(a)\right\}^+}_{\text{value from meeting unemployed individual}}\Bigg)+$$

$$+\underbrace{(1-u)\int\left\{V(z,x_u,a)-V(z',x,a)\right\}^+ dG(z',x,a)}_{\text{value from meeting employed individual}}\Bigg]-c(v)\Bigg\} - \underbrace{r}_{\text{fixed cost}} - \underbrace{\mu J'(z)}_{\text{drift in } z} + \underbrace{\frac{\sigma^2}{2}J''(z)}_{\text{shocks to } z}$$

- Post vacancies v subject to $\mathbf{c}(\mathbf{v})$
- Has to pay fixed cost $\mathbf{r}$ to remain in hiring market
- **Decision rules:** vacancy policy, $\mathbf{v}(\mathbf{z})$, and exit threshold, $\underline{\mathbf{z}}$

Back to main

Vacancy policy

$$v(z) = \left\{ \frac{(1-\beta)q}{c_v} \left[\underbrace{\sum_a u(a) \left\{ V(z, x_u, a) - U(a) \right\}^+}_{\text{Individual is unemployed}} + \underbrace{(1-u) \int \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ dG(z', x, a)}_{\text{Individual is employed}} \right] \right\}^{1/\eta}$$

Back to main

Vacancy policy

$$v(z) = \left\{ \frac{(1-\beta)q}{c_v} \left[\underbrace{\sum_a u(a) \left\{ V(z, x_u, a) - U(a) \right\}^+}_{\text{Individual is unemployed}} + \underbrace{(1-u) \int \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ dG(z', x, a)}_{\text{Individual is employed}} \right] \right\}^{1/\eta}$$

Less labor market mismatch $\implies$ less vacancy creation

1. Larger share of individuals are employed

Back to main

Vacancy policy

$$v(z) = \left\{ \frac{(1-\beta)q}{c_v} \left[\underbrace{\sum_a u(a) \left\{ V(z, x_u, a) - U(a) \right\}^+}_{\text{Individual is unemployed}} + \underbrace{(1-u) \int \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ dG(z', x, a)}_{\text{Individual is employed}} \right] \right\}^{1/\eta}$$

Less labor market mismatch $\implies$ less vacancy creation

1. Larger share of individuals are employed
2. Employed individuals are less mismatched

Back to main

Stationary equilibrium

Value functions $\{U, V, E, J\}$; policies $\{\bar{c}_u, \underline{z}_u(x,a), \underline{z}(z,x,a), \bar{c}(z,x,a)\}$; policies $\{\underline{z}, v(z)\}$; numbers $\{r, e, \mu, \bar{v}, \lambda, q\}$; and distributions $\{h(z), f(z), u(a), g(z,x,a)\}$; such that

1. Value and policy functions of unemployed, match and recruiting firm solve the respective problems
2. The aggregate entry rate e is consistent with individual behavior
3. The growth rate μ is consistent with the entry rate
4. Aggregate vacancies $\bar{v}$ are consistent with firm behavior and the finding rates are $\lambda = \chi \bar{v}^{\alpha}$, $q = \chi \bar{v}^{\alpha - 1}$
5. Distributions solve respective KFE and are stationary

Distributions and laws of motion Back

Four distributions

1. $h(z)$ denotes the pdf of recruiting firms
2. $f(z)$ denotes the *vacancy-weighted* pdf of recruiting firms
3. $u(a)$ denotes the mass of unemployed individuals of age a
4. $g(z, x, a)$ denotes the pdf of employed individuals

For all densities, upper case letters denote the corresponding cdf

Back

Distribution of recruiting firms

The distribution of recruiting firms, h, solves the KFE

$$0 = \mu h'(z) + \frac{\sigma^2}{2} h''(z) + e\zeta \exp(-\zeta z), \qquad z > 0 \tag{1}$$

subject to,

$$h(0) = 0, \qquad \int_0^\infty h(z) dz = 1, \qquad e = \frac{\sigma^2}{2} h'(0) \tag{2}$$

where e is the aggregate entry rate

- Last condition can be seen by integrating (1) from 0 to ∞, which gives $0 = -\mu h(0) - \sigma^2/2h'(0) + e$, and imposing $h(0) = 0$

Back

Entry and growth

(1) is a second-order ordinary differential equation with solution,

$$h(z) = \frac{e}{\mu - \frac{\sigma^2}{2}\zeta} \left[\exp(-\zeta z) - \exp\left(-\frac{2\mu}{\sigma^2} z \right) \right] \tag{3}$$

Back

Entry and growth

(1) is a second-order ordinary differential equation with solution,

$$h(z) = \frac{e}{\mu - \frac{\sigma^2}{2}\zeta}\left[\exp(-\zeta z) - \exp\left(-\frac{2\mu}{\sigma^2}z\right)\right] \tag{3}$$

where the growth rate of the economy is a function of the aggregate entry rate of entrepreneurs,

$$\mu = \frac{e}{\zeta} \tag{4}$$

Back

Offer distribution

The vacancy-weighted distribution of firms, $f(z)$, equals the density of recruiting firms at z times the amount of vacancies they post,

$$f(z) = \frac{v(z)h(z)}{\bar{v}} \tag{5}$$

where $v(z)$ is the firm's optimal vacancy policy and

$$\bar{v} = \int_0^\infty v(\tilde{z})\, dh(\tilde{z})$$

Back

Distribution of employment

On the BGP, $g(z,x,a)$ satisfies the KFE

$$
\begin{aligned}
0 =& \mu \frac{\partial g(z,x,a)}{\partial z} + \frac{\sigma^2}{2}\frac{\partial^2 g(z,x,a)}{\partial z^2} + \underbrace{\lambda \frac{u(a)}{1-u} f(z) \mathbb{1}\{x = x_u\} \mathbb{1}\left\{z > \underline{z}^u(x_u,a)\right\}}_{\text{inflow from unemployment}} + \\
&+ \underbrace{\kappa(a-1)\mathbb{1}\left\{z > \underline{z}^u(x_u,a)\right\} g(z,x,a-1)}_{\text{inflow from aging}} - \underbrace{\kappa(a) g(z,x,a)}_{\text{outflow from aging}} + \\
&+ \underbrace{\lambda f(z)\mathbb{1}\{x = x_u\} \int \mathbb{1}\left\{z > \underline{z}^e(z',x',a)\right\} G(dz',dx',a)}_{\text{inflow from lower rungs in job ladder}} - \underbrace{\lambda \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] g(z,x,a)}_{\text{outflow to higher rungs in job ladder}} + \\
&+ \underbrace{\psi \mathbb{1}\left\{z > \underline{z}^u(x,a)\right\} \pi(x) g(z,x_u,a)}_{\text{inflow from learning}} - \underbrace{\psi \mathbb{1}\{x = x_u\} g(z,x,a)}_{\text{outflow from learning}} - \underbrace{\gamma(a) g(z,x,a) \Omega\left(\bar{c}^e(z,x,a)\right)}_{\text{outflow to entrepreneurship}}
\end{aligned}
\tag{6}
$$

with $\pi(x_u) = 0$ and $g(z,x,0) \equiv 0, \forall z,x$, subject to workers exiting at the boundary so that the density is zero and the pdf integrates to one

Back

Unemployment

The mass of unemployed of each age group, $u(a)$, satisfies,

$$0 = -\underbrace{\lambda\left[1 - F\left(\underline{z}^{u}(x_u, a)\right)\right]u(a)}_{\text{outflow to employment}} + \underbrace{(1-u(a))\sum_x \frac{\sigma^2}{2}\frac{\partial g(\underline{z}^{u}(x,a),x,a)}{\partial z}}_{\text{individuals drifting below the threshold}} + \tag{7}$$

$$+ \underbrace{(1-u(a))\psi\pi(x_b)G\left(\underline{z}^{u}(x_b,a),x_u,a\right)}_{\text{individuals jumping below the threshold due to learning}} + \underbrace{\mathbb{1}\{a=1\}\kappa(A)}_{\text{newborn}} -$$

$$- \underbrace{\kappa(a)u(a)}_{\text{outflow from aging}} + \underbrace{\kappa(a-1)\left[u(a-1) + (1-u)\sum_x G\left(\underline{z}^{u}(x,a),x,a-1\right)\right]}_{\text{inflow from aging}} +$$

$$+ \underbrace{(1-u(a))\gamma(a)\int \Omega\left(\bar{c}^{e}(z,x,a)\right)G(dz,dx,a)}_{\text{entry to entrepreneurship}}$$

with the convention that $u(0) = 0$

Back

4 effects of aging on JJ hazard

$$JJ = \lambda \int \left[1 - F\left(\underline{z}^{e}(z,x,a)\right)\right] dG(z,x,a) \tag{8}$$

Back

4 EFFECTS OF AGING ON JJ HAZARD

$$
\begin{aligned}
JJ &= \lambda \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] dG(z,x,a) \\
&= \frac{1}{1-u} \sum_a m(a) \left(1 - \frac{u(a)}{m(a)}\right) \times \lambda \times \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] d\hat{G}(z,x|a) \qquad (8)
\end{aligned}
$$

Back

4 EFFECTS OF AGING ON JJ HAZARD

$$JJ = \lambda \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] dG(z,x,a)$$

$$= \frac{1}{1-u} \sum_a m(a) \left(1 - \frac{u(a)}{m(a)}\right) \times \lambda \times \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] d\hat{G}(z,x|a) \tag{8}$$

1. Changing $m(a)$ will affect the aggregate JJ hazard since older individuals typically are better matched

4 effects of aging on JJ hazard

$$JJ = \lambda \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] dG(z,x,a)$$

$$= \frac{1}{1-u} \sum_a m(a) \left(1 - \frac{u(a)}{m(a)}\right) \times \lambda \times \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] d\hat{G}(z,x|a) \qquad (8)$$

1. Changing $m(a)$ will affect the aggregate JJ hazard since older individuals typically are better matched

2. λ may change as firms respond to the changed economic environment by adjusting vacancy creation

Back

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$$

1. Changing $m(a)$ will affect the aggregate JJ hazard since older individuals typically are better matched

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3. F may change as firms change their vacancy posting decisions

Back

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\end{aligned}
$$

1. Changing $m(a)$ will affect the aggregate JJ hazard since older individuals typically are better matched
2. λ may change as firms respond to the changed economic environment by adjusting vacancy creation
3. F may change as firms change their vacancy posting decisions
4. Aging may give rise to changes in age-conditional labor market mismatch, $\hat{G}(z,x|a)$

Back

3 EFFECTS OF AGING ON ENTRY RATE

$$e = \frac{1}{M}\left\{(1-u)\int \Omega\left[\bar{c}^e(z,x,a)\right]\gamma(a)dG(z,x,a) + \Omega\left(\bar{c}^u\right)\sum_a u(a)\gamma(a)\right\} \tag{9}$$

Back

3 EFFECTS OF AGING ON ENTRY RATE

$$
\begin{aligned}
e &= \frac{1}{M}\left\{(1-u)\int \Omega\left[\bar{c}^{e}(z,x,a)\right]\gamma(a)dG(z,x,a) + \Omega\left(\bar{c}^{u}\right)\sum_{a} u(a)\gamma(a)\right\} \\
&= \sum_{a} m(a)\frac{\gamma(a)}{M}\left\{\left(1-\frac{u(a)}{m(a)}\right)\int \Omega\left[\bar{c}^{e}(z,x,a)\right] d\hat{G}(z,x|a) + \frac{u(a)}{m(a)}\Omega\left(\bar{c}^{u}\right)\right\} \qquad (9)
\end{aligned}
$$

Back

$$
\begin{aligned}
e &= \frac{1}{M}\left\{(1-u)\int \Omega\left[\bar{c}^{e}(z,x,a)\right]\gamma(a)dG(z,x,a) + \Omega\left(\bar{c}^{u}\right)\sum_{a} u(a)\gamma(a)\right\} \\
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\end{aligned}
$$

1. Changing $m(a)$ will affect the aggregate entry rate since age groups in general differ in their propensity to enter

Back

$$e = \frac{1}{M}\left\{(1-u)\int \Omega\left[\bar{c}^e(z,x,a)\right]\gamma(a)dG(z,x,a) + \Omega\left(\bar{c}^u\right)\sum_a u(a)\gamma(a)\right\}$$

$$= \sum_a m(a)\frac{\gamma(a)}{M}\left\{\left(1-\frac{u(a)}{m(a)}\right)\int \Omega\left[\bar{c}^e(z,x,a)\right]d\hat{G}(z,x|a) + \frac{u(a)}{m(a)}\Omega\left(\bar{c}^u\right)\right\} \quad (9)$$

1. Changing $m(a)$ will affect the aggregate entry rate since age groups in general differ in their propensity to enter

2. May affect $\bar{c}^e(z,x,a)$ ($\bar{c}^u$) as if for instance an older pool of hires discourages entry by driving up the effective cost of recruiting

Back

3 EFFECTS OF AGING ON ENTRY RATE

$$
\begin{aligned}
e &= \frac{1}{M}\left\{(1-u)\int \Omega\left[\bar{c}^{e}(z,x,a)\right]\gamma(a)dG(z,x,a) + \Omega\left(\bar{c}^{u}\right)\sum_{a} u(a)\gamma(a)\right\} \\
&= \sum_{a} m(a)\frac{\gamma(a)}{M}\left\{\left(1-\frac{u(a)}{m(a)}\right)\int \Omega\left[\bar{c}^{e}(z,x,a)\right] d\hat{G}(z,x|a) + \frac{u(a)}{m(a)}\Omega\left(\bar{c}^{u}\right)\right\} \qquad (9)
\end{aligned}
$$

1. Changing $m(a)$ will affect the aggregate entry rate since age groups in general differ in their propensity to enter

2. May affect $\bar{c}^{e}(z,x,a)$ $(\bar{c}^{u})$ as if for instance an older pool of hires discourages entry by driving up the effective cost of recruiting

3. Age conditional labor market mismatch—$\hat{G}(z,x|a)$ and $u(a)/m(a)$—may change through equilibrium effects

Back

Appendix D

Strategy

Target: Salient features of aggregate firm & worker dynamism in BDS + SIPP in 2012–2014

- Monthly frequency
- 3 age groups
- Pre-set a few parameters to standard values
- Remaining parameters internally

Back

Calibrated values

	Description	Target	Value
Panel A: Labor market mobility			
c_v	Cost of vacancy creation	Aggregate UE (2005–07)	$4.5 * 10^{-4}$
$\pi(x_b)$	P(match is low productive)	Aggregate EU	0.5
x_g	Productivity of high prod. match	Aggregate JJ	1.3
ψ	Rate of learning	Timing of decline in JJ with tenure	0.043
b	Flow value of unemployment	Indifference at margin	1.09
Panel B: Entrepreneurship			
ζ	Innovation distribution	Growth due to selection	20
$\gamma(a)$	Entrepreneurship opportunity	Entry rate and entry rate by age	$[4.2; 4.5; 2.1]*10^{-3}$
C	Dispersion in entry cost	Decline in entry with tenure	72
Panel C: Firms			
η	Curvature of vacancy creation	Size distribution of entrants	2
σ	Shocks to productivity	Size distribution	$7 * 10^{-3}$
d	Exit shock for firms	Average exit rate	$3.8 * 10^{-4}$
K	Capital	Average firm size	0.13

Calibrated values

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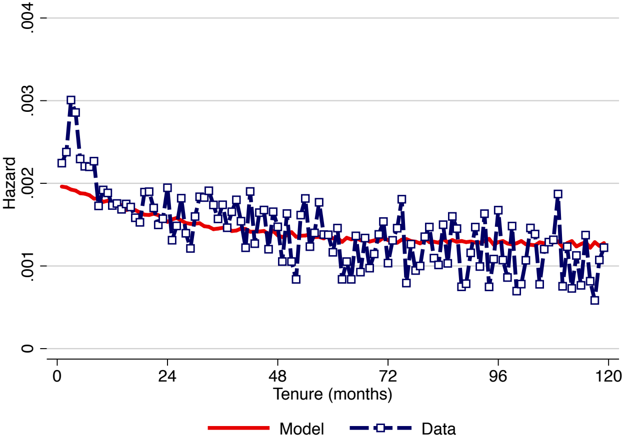
K	Capital	Average firm size	0.13

Calibrated values

- C => Elasticity of entry to net value

Figure XVII: Tenure profile of entrepreneurship entry hazard



Tenure distribution JJ by tenure Back

Calibrated values

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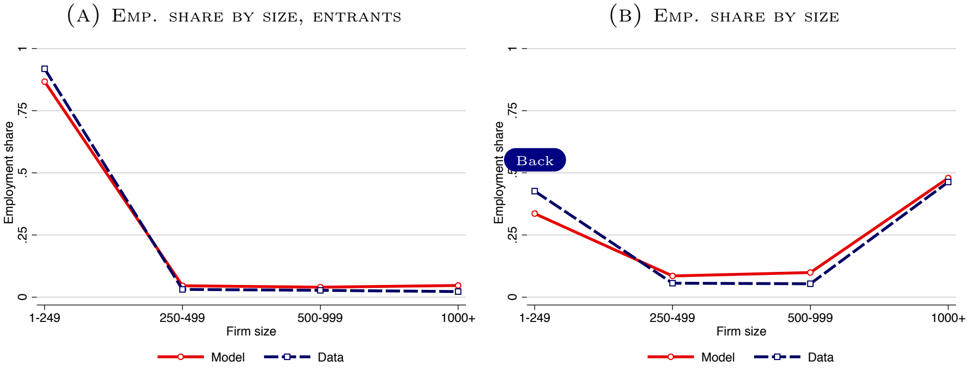
Calibrated values

- η => Elasticity of vacancy creation to net value



Calibrated values

	Description	Target	Value
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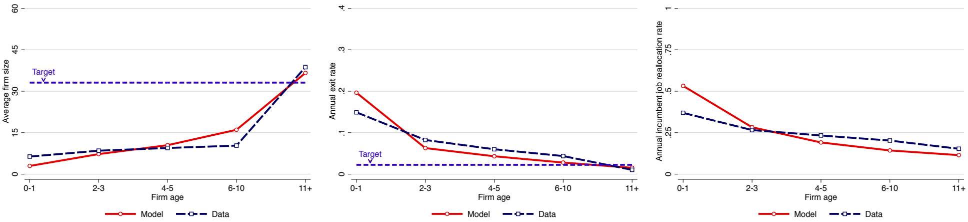
Labor market mobility Entrepreneurship Firms Back

Success I: Life cycle firm dynamics

- Calibration targets aggregate firm size and exit rate

$\Longrightarrow$ **Captures well life-cycle firm dynamics**

(A) Firm size (B) Exit rate (C) Incumbent job realloc.



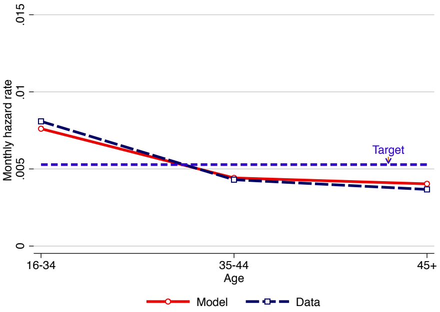
Employment shares by firm age | Wage by firm age | Exit by firm size | Wage by firm size | Back

Success II: Life cycle labor market mobility

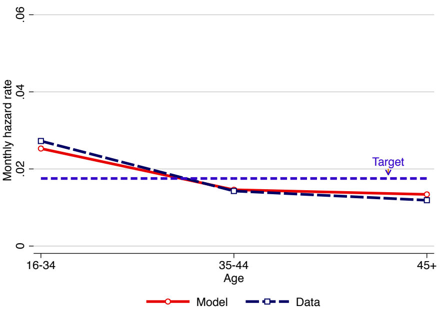
- Calibration targets aggregate JJ & EU hazard

$\Longrightarrow$ **Supports job ladder and learning mechanisms**

(A) EU



(B) JJ



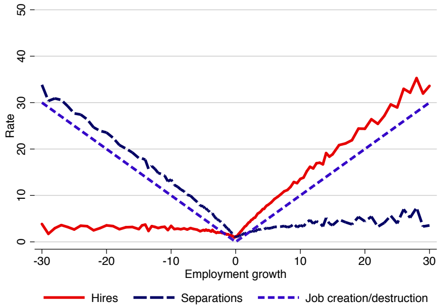
UE by age | Wage by tenure | Back

Success III: Linking firms and workers

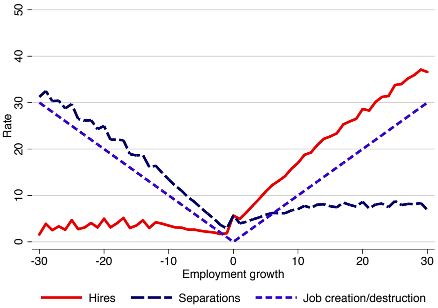
- Matches hire & separation rates as function of firm growth

$\Longrightarrow$ **Supports joint model of firm & worker dynamics**

(A) Data



(B) Model



Hires/separations by origin/destination by firm age | Average worker age by firm age | Back

Standard values

Table VI: Pre-set parameter values

	Description	Target	Value
ρ	Discount rate	Annual interest rate of 4%	0.0034
χ	Matching efficiency	Normalization	0.1
α	Elasticity of matching function	Petrongolo and Pissarides (2001)	0.7
β	Bargaining power	Bagger et al (2014)	0.3

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Calibration targets—individuals

	Description	Target	Value
c_v	Cost of vacancy creation	Aggregate UE (2005–07)	$4.5 * 10^{-4}$
$\pi(x_b)$	P(match is low productive)	Aggregate EU	0.5
x_g	Productivity of high prod. match	Aggregate JJ	1.3
ψ	Rate of learning	Timing of decline in JJ with tenure	0.043
b	Flow value of unemployment	Indifference at margin	1.09

Back

Calibration targets—individuals

- $\uparrow$ share of low-productive matches $\implies \uparrow$ EU hazard

	Description	Target	Value
c_v	Cost of vacancy creation	Aggregate UE (2005–07)	$4.5 * 10^{-4}$
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Calibration targets—individuals

- $\uparrow$ share of low-productive matches $\Longrightarrow \uparrow$ EU hazard
- $\uparrow x_g \Longrightarrow \uparrow$ opportunity cost of JJ mobility $\Longrightarrow \downarrow$ JJ hazard

	Description	Target	Value
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ψ	Rate of learning	Timing of decline in JJ with tenure	0.043
b	Flow value of unemployment	Indifference at margin	1.09

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Calibration targets—individuals

- $\uparrow$ share of low-productive matches $\Longrightarrow \uparrow$ EU hazard
- $\uparrow x_g \Longrightarrow \uparrow$ opportunity cost of JJ mobility $\Longrightarrow \downarrow$ JJ hazard
- $\uparrow \psi \Longrightarrow$ learning is faster $\Longrightarrow$ JJ falls quickly with tenure

	Description	Target	Value
c_v	Cost of vacancy creation	Aggregate UE (2005–07)	$4.5 * 10^{-4}$
$\pi(x_b)$	P(match is low productive)	Aggregate EU	0.5
x_g	Productivity of high prod. match	Aggregate JJ	1.3
ψ	Rate of learning	Timing of decline in JJ with tenure	0.043
b	Flow value of unemployment	Indifference at margin	1.09

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Calibration targets—entrepreneurs

	Description	Target	Value
$\gamma(a)$	Entrepreneurship opportunity	Entry rate by age	$[4.2; 4.5; 2.1] * 10^{-3}$
ζ	Innovation distribution	Growth due to selection	20
C	Dispersion in entry cost	Decline in entry with tenure	72

Entry by age Back

Calibration targets—entrepreneurs

	Description	Target	Value
$\gamma(a)$	Entrepreneurship opportunity	Entry rate by age	$[4.2; 4.5; 2.1] * 10^{-3}$
ζ	Innovation distribution	Growth due to selection	20
C	Dispersion in entry cost	Decline in entry with tenure	72

Entry by age Back

Calibration targets—entrepreneurs

- $\Omega \sim U(-C, C)$
- $\uparrow C \implies \downarrow$ change in entry for given change in value of entry
- Opportunity cost is positively correlated with tenure and hence decline in entry with tenure informs C

	Description	Target	Value
$\gamma(a)$	Entrepreneurship opportunity	Entry rate by age	$[4.2; 4.5; 2.1] * 10^{-3}$
ζ	Innovation distribution	Growth due to selection	20
C	Dispersion in entry cost	Decline in entry with tenure	72

Entry by age Back

Calibration targets—firms

- $\uparrow \eta \implies$ more costly to hire many workers $\implies$ less dispersion in initial firm size

	Description	Target	Value
η	Curvature of vacancy creation	Size distribution of entrants	2
d	Exit shock for firms	Average exit rate	$3.8 * 10^{-4}$
σ	Shocks to productivity	Size distribution	$7 * 10^{-3}$
K	Capital	Average firm size	0.13

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Calibration targets—firms

- $\uparrow \eta \implies$ more costly to hire many workers $\implies$ less dispersion in initial firm size

- Introduce small probability of firm death, d, that is independent of firm productivity

	Description	Target	Value
η	Curvature of vacancy creation	Size distribution of entrants	2
d	Exit shock for firms	Average exit rate	$3.8 * 10^{-4}$
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K	Capital	Average firm size	0.13

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Calibration targets—firms

- $\uparrow \eta \implies$ more costly to hire many workers $\implies$ less dispersion in initial firm size
- Introduce small probability of firm death, d, that is independent of firm productivity
- $\uparrow \sigma \implies \uparrow$ dispersion in steady-state firm productivity $\implies \uparrow$ dispersion in steady-state firm size

	Description	Target	Value
η	Curvature of vacancy creation	Size distribution of entrants	2
d	Exit shock for firms	Average exit rate	$3.8 * 10^{-4}$
σ	Shocks to productivity	Size distribution	$7 * 10^{-3}$
K	Capital	Average firm size	0.13

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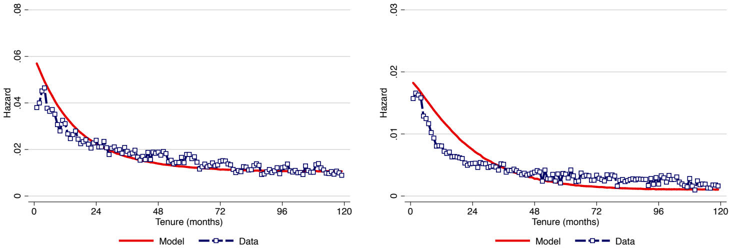
Worker mobility by tenure

Figure XX: Worker mobility by tenure

(a) JJ

(b) EU



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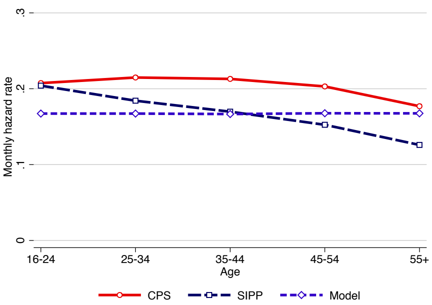
UE hazard with age

Figure XXII: UE hazard by age



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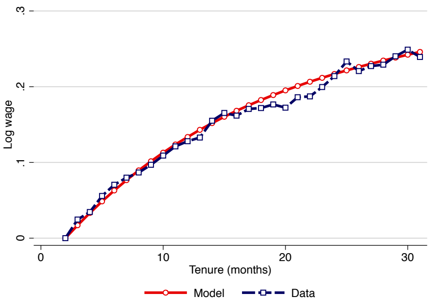
Average wage by tenure

- Model matches well average wages by tenure => confidence in β

Figure XXIII: Wage by tenure



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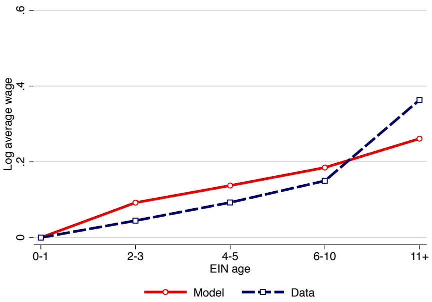
Average wage by firm age

Figure XXIV: Average wage by firm age



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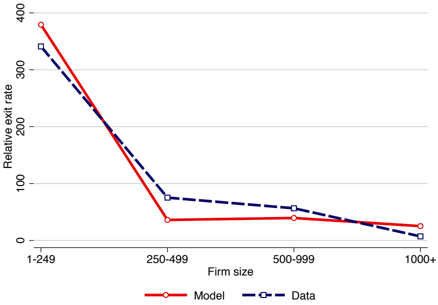
Exit rate by firm size

Figure XXV: Exit rate by firm size



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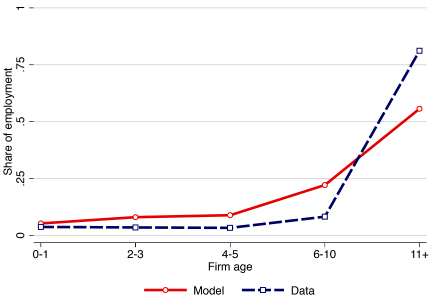
Employment share by firm age

Figure XXVI: Employment shares



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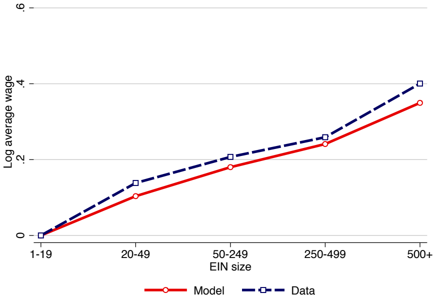
Average wage by firm size

Figure XXVII: Average wage by firm size



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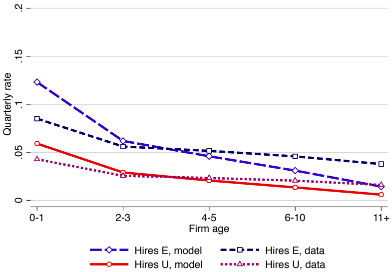
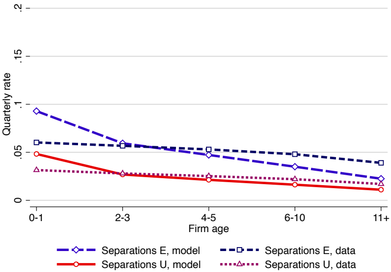
Hires and separations by origin and destination

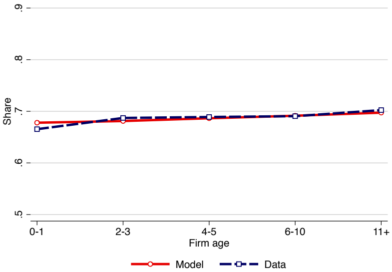
(A) Hires by origin



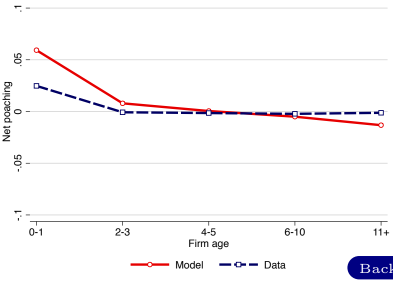
(B) Separations by destination



(C) Share of hires from other firms



(D) Net poaching



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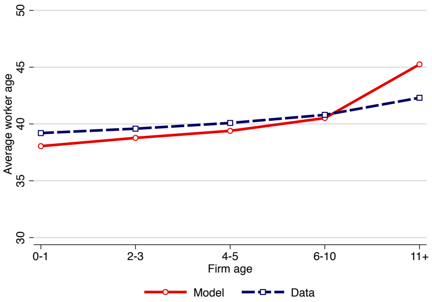
Average worker age by firm age

Figure XXIX: Average worker age by firm age



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Appendix E

Details on change in age composition

Change the age composition of the economy to 1986 and evaluate its impact on dynamism

- Increase the rate at which older individuals exit the market, $\kappa(3)$
  1. Increases the share of young people
  2. Shortens the time individuals expect to remain in the market

- The retirement age has not changed suggesting that individuals did not expect to spend less time in the market in the 1980s

$\Longrightarrow$ **Use original $\kappa(3)$ in value functions and new $\kappa(3)$ when computing individual transitions**

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Age composition

- Target change in share of older => Understates somewhat fall in the share of young

Table VII: Share of individuals in each age group by period

	(1)	(2)	(3)	(4)	(5)	(6)
	Early		Late		Change	
	Data	Model	Data	Model	Data	Model
Young	0.492	0.434	0.356	0.339	-0.136	-0.095
Middle aged	0.231	0.289	0.208	0.226	-0.023	-0.063
Older	0.277	0.277	0.436	0.436	0.159	0.158

Note: Empirical moments corresponds to the share of the labor force age 16–34 (young), 35–44 (middle aged) and 45+ (older) in 1986 and 2015 from the BLS.

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JJ versus UE

- Two opposing effects on vacancy creation

Back

JJ versus UE

- Two opposing effects on vacancy creation
    1. Firms post fewer vacancies conditional on productivity

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JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out

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JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out

$\Longrightarrow$ Only modest decline in λ

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JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out

  $\Longrightarrow$ Only modest decline in λ

- In contrast, the less dynamic economy implies that

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JJ versus UE

- Two opposing effects on vacancy creation
    1. Firms post fewer vacancies conditional on productivity
    2. Slower turnover rate shifts distribution of firms out

  $\Longrightarrow$ Only modest decline in λ

- In contrast, the less dynamic economy implies that
    1. Employment has shifted up the ranks of firms

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JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out

  $\Longrightarrow$ Only modest decline in λ

- In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity

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JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out

  $\Longrightarrow$ Only modest decline in λ

- In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity

  $\Longrightarrow$ Less likely individual accepts job offer

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JJ versus UE

- Two opposing effects on vacancy creation
  1. Firms post fewer vacancies conditional on productivity
  2. Slower turnover rate shifts distribution of firms out

  $\Longrightarrow$ Only modest decline in λ

- In contrast, the less dynamic economy implies that
  1. Employment has shifted up the ranks of firms
  2. A higher share of matches has learned its productivity

  $\Longrightarrow$ Less likely individual accepts job offer

$\Longrightarrow$ **JJ hazard falls over and above the decline in λ**

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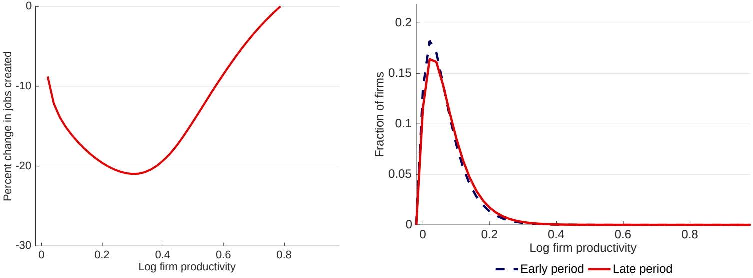
JJ versus UE (cont'd)

Figure XXX: Change in vacancy policy and firm distribution

(a) Vacancy creation

(b) Firm productivity distribution



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The decline in the passthrough

Decker et al. (2017)

1. The fall in job reallocation is not due to a more benign economic environment
2. Older firms adjust employment less in response to productivity shocks
3. Employment has shifted towards older firms, accounting for some of the decline in the passthrough
4. The response has fallen within firm age groups

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Explaining the empirical patterns

1. No change in variance of shocks

2. Lower passthrough of older firms as equilibrium outcome
    - Employment change to productivity shock is linked to #ranks
    - Log distance between ranks is larger further up the ladder
    - Shock moves firm fewer ranks at top => smaller employment response
    - Older, surviving firms are on average further up the ladder

3. Aging results in shift of employment towards older firms

4. Employment has also shifted up the ladder within age groups

Table Back

Passthrough in young and old economy

Table VIII: Passthrough from productivity to employment innovations

	(1) All firms	(2) Young firms	(3) Mature firms
Δ TFP	3.504***	5.604***	2.394***
Late period $\times \Delta$ TFP	-0.566***	-0.212***	-0.177***

Note: Young firms are $<$5 years, mature firms $\geq$ 5 years. Outcome variable is annual change in log firm size. Independent variable is annual change in log firm productivity. Weighted by employment.

$\Longrightarrow$ **Declines driven by weaker passthrough**

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Decomposition of level difference

Table IX: Decomposition of change in log output

(1) Age composition	(2) Firm productivity	(3) Match productivity	(4) Net output	(5) Discounted net output
0.014	0.044	0.004	0.055	-0.040

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Decomposition of level difference

Table IX: Decomposition of change in log output

(1) Age composition	(2) Firm productivity	(3) Match productivity	(4) Net output	(5) Discounted net output
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Decomposition of level difference

Table IX: Decomposition of change in log output

(1) Age composition	(2) Firm productivity	(3) Match productivity	(4) Net output	(5) Discounted net output
0.014	0.044	0.004	0.055	**-0.040**

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Decomposition

$$e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega \left[\bar{c}^e(z,x,a)\right] d\hat{G}(z,x|a) + \frac{u(a)}{m(a)} \Omega\left(\bar{c}^u\right) \right\}$$

$$JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1-u} \times \lambda \times \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] d\hat{G}(z,x|a)$$

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Decomposition

$$e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega \left[\bar{c}^e(z,x,a)\right] d\hat{G}(z,x|a) + \frac{u(a)}{m(a)} \Omega\left(\bar{c}^u\right) \right\}$$

$$JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1-u} \times \lambda \times \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] d\hat{G}(z,x|a)$$

Table X: Decomposing the change in the JJ and entry hazard

	(1)	(2)	(3)	(4)
	Entry hazard		JJ hazard	
	% change	% of total	% change	% of total
Direct: $m(a)$	**10.5**	47.5	7.0	53.6

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Decomposition

$$e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega \left[\bar{c}^e(z,x,a)\right] d\hat{G}(z,x|a) + \frac{u(a)}{m(a)} \Omega\left(\bar{c}^u\right) \right\}$$

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Policy: $\bar{c}^e(z,x,a)/\lambda \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right]$	**1.2**	5.4	-17.3	-133

Back

Decomposition

$$e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega\left[\bar{c}^e(z,x,a)\right] d\hat{G}(z,x|a) + \frac{u(a)}{m(a)} \Omega\left(\bar{c}^u\right) \right\}$$

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Back

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$$JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1-u} \times \lambda \times \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] d\hat{G}(z,x|a)$$

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Policy: $\bar{c}^e(z,x,a)/\lambda\left[1 - F\left(\underline{z}^e(z,x,a)\right)\right]$	1.2	5.4	-17.3	-133
Mismatch: $\hat{G}(z,x|a)$	**10.4**	47.2	23.3	179

Back

Decomposition

$$e = \sum_a m(a) \frac{\gamma(a)}{M} \left\{ \left(1 - \frac{u(a)}{m(a)}\right) \int \Omega\left[\bar{c}^e(z,x,a)\right] d\hat{G}(z,x|a) + \frac{u(a)}{m(a)} \Omega\left(\bar{c}^u\right) \right\}$$

$$JJ = \sum_a m(a) \frac{1 - \frac{u(a)}{m(a)}}{1-u} \times \lambda \times \int \left[1 - F\left(\underline{z}^e(z,x,a)\right)\right] d\hat{G}(z,x|a)$$

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Policy: $\bar{c}^e(z,x,a)/\lambda\left[1 - F\left(\underline{z}^e(z,x,a)\right)\right]$	1.2	5.4	-17.3	-133
Mismatch: $\hat{G}(z,x\|a)$	10.4	47.2	23.3	179
Total	22.2	100	13.1	100

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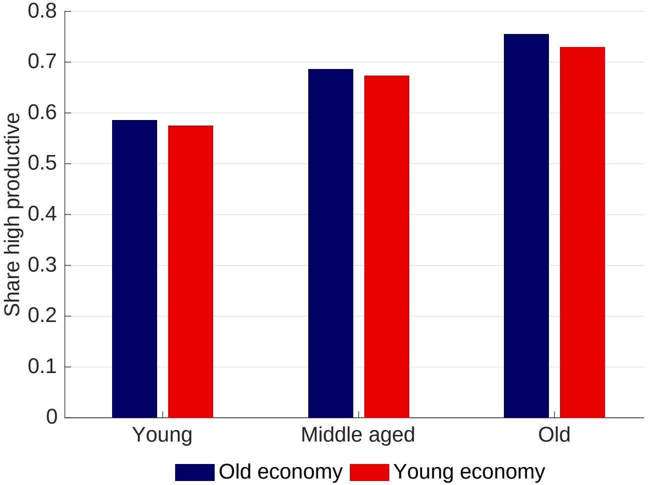
Shift in match productivity

Figure XXXII: Share with high match productivity



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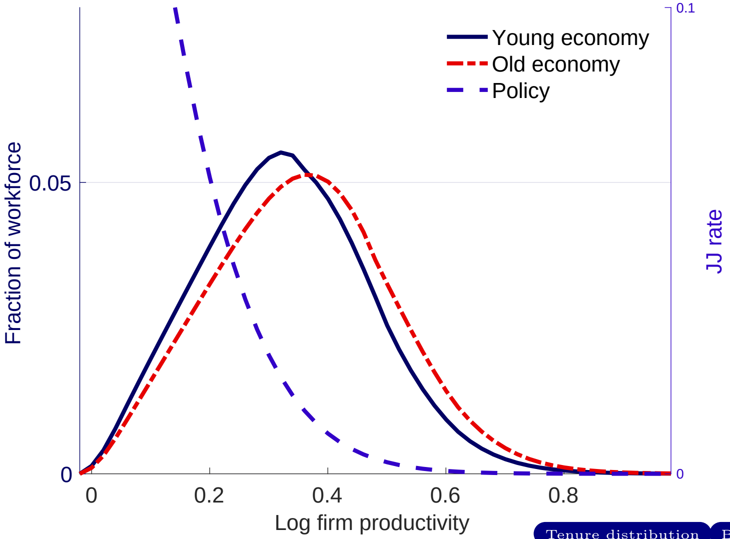
How much does JJ fall with mismatch?

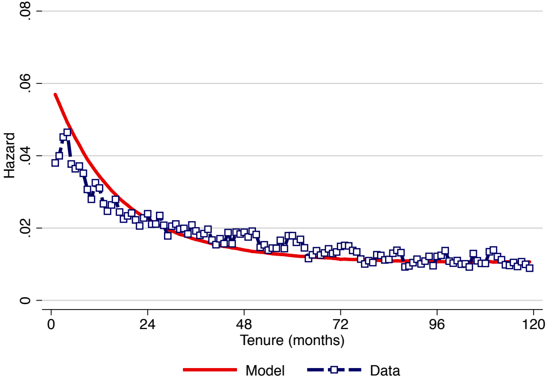
Figure XXXIII: Distribution of older individuals and JJ hazard



Tenure distribution Back

What moments of the data inform the effects

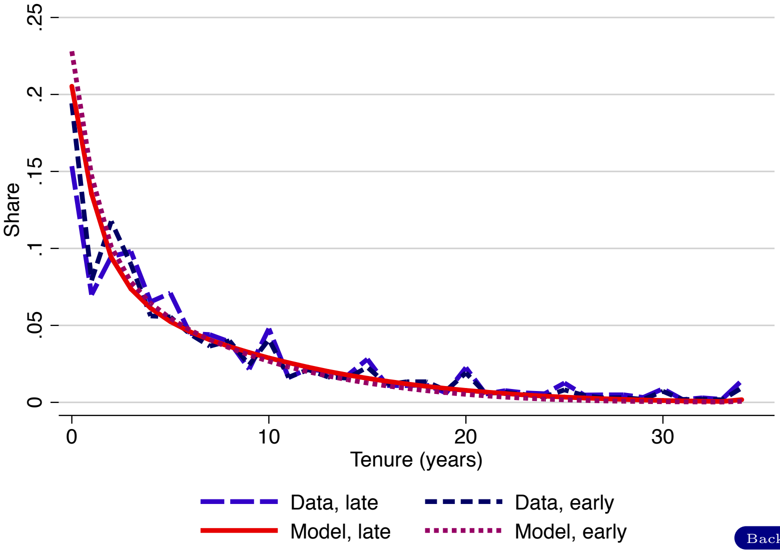
Figure XXXIV: Tenure profile of JJ mobility



$\Longrightarrow$ **Large equilibrium effects are not hardwired**

Tenure distribution Back

Figure XXXV: Tenure distribution



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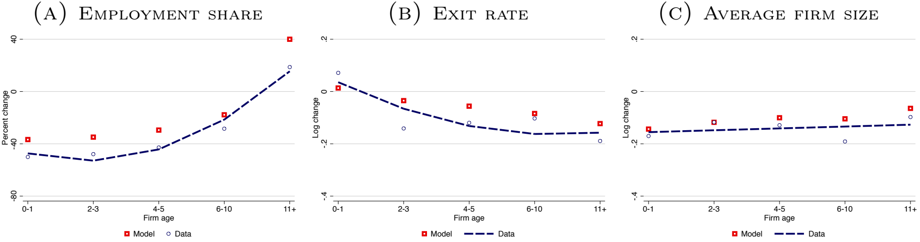
Change in life-cycle firm dynamics

Aging explains key changes in life-cycle firm dynamics

- Employment has shifted substantially towards older firms
- Exit has fallen the most for old firms
- Age conditional firm size has declined



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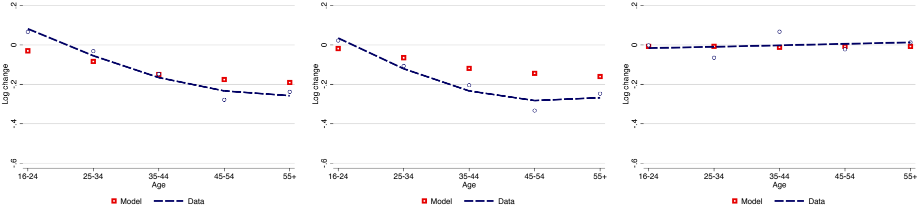
Change in life-cycle worker dynamics

- Aging in model replicates patterns across states
- Relatively larger effect on mobility rates late in careers

(A) EU (B) JJ (C) UE



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Shift-share analysis

Table XI: Shift share analysis with firm and worker age

	(1) Data	(2) Model
Panel A: Firm dynamics		
Exit	-0.008	-0.003
Direct effect	-0.008	-0.004
% of total	96.8	142.4
Incumbent	-0.045	-0.024
Direct effect	-0.010	-0.018
% of total	22.7	74.4
Panel B: Worker dynamics		
EU	-0.003	-0.001
Direct effect	-0.001	-0.000
% of total	20.7	34.4
JJ	-0.005	-0.002
Direct effect	-0.002	-0.001
% of total	40.8	51.7

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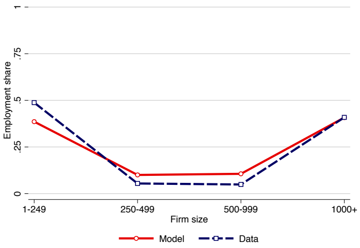
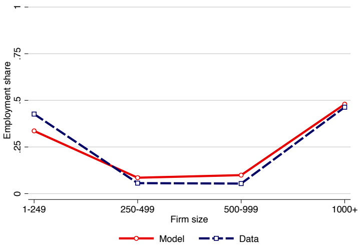
Employment shares by firm size

- Aging generates modest shift of employment to larger firms in line with the data over this period

(A) Early



(B) Late



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No aging of potential hires

$$\rho J(z) = \max_{v \geq 0} \left\{ v(1-\beta)q \left[\sum_a \left(\tilde{m}(a) \frac{u(a)}{m(a)} \left\{ V(z, x_u, a) - U(a) \right\}^+ \right) + \right.\right.$$

$$\left.\left. + \sum_a \left\{ \tilde{m}(a) \left(1 - \frac{u(a)}{m(a)} \right) \int \left\{ V(z, x_u, a) - V(z', x, a) \right\}^+ d\hat{G}(z', x|a) \right\} \right] - c(v) \right\} -$$

$$- r - \mu J'(z) + \frac{\sigma^2}{2} J''(z)$$

- Hold firms' expectations of age composition fixed at original age composition, $\tilde{m}(a)$

$\Longrightarrow$ No change in age-composition externality

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No aging of potential hires

Table XII: No direct congestion externality due to aging

	(1) Baseline	(2) No aging of hires	(3) Share
Panel A: Firm dynamics			
Entry rate	-0.012	-0.008	**72**
Job reallocation	-0.039	-0.031	**80**
Panel B: Worker dynamics			
EU hazard	-0.001	-0.001	**87**
JJ hazard	-0.002	-0.002	**72**
Panel C: Growth			
Growth per worker	-0.26	-0.18	**69**

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No aging of potential entrepreneurs

- Adjust $\gamma(a)$ to have no direct effect through aging entrepreneurs

Table XIII: No aging of potential entrepreneurs

	(1) Baseline	(2) No aging of entrep.	(3) Share
Panel A: Firm dynamics			
Entry rate	-0.012	-0.003	**27**
Job reallocation	-0.039	-0.009	**22**
Panel B: Worker dynamics			
EU hazard	-0.001	-0.001	**61**
JJ hazard	-0.002	-0.002	**65**
Panel C: Growth			
Growth per worker	-0.26	-0.11	**42**

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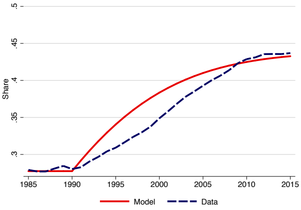
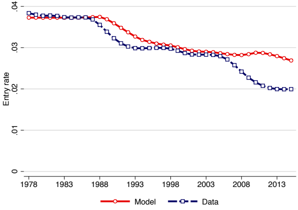
Transition dynamics

- Start with 1986 BGP
- Adjust $\kappa(3)$ and decision rules to 2014 BGP starting in 1990
- Relatively fast convergence of entry rate
- Level effect outweighs growth effect initially

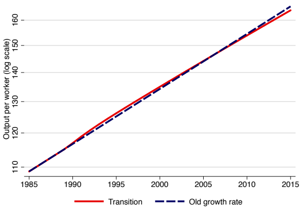
(A) Share older



(B) Entry rate



(C) Output per worker



Discussion Back

Discussion of transition

- Would want to eventually solve for full transition path
- Difficulty is that sequence of distributions $G(z, x, a; t)$ becomes a state
- Well known issue in search models—cannot boil down problem to shooting only an interest rate or average wage

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Inequality and income dynamics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Young		Old		Change		
	Data	Model	Data	Model	Data	Model	Share
Panel A: Inequality							
St.d of productivity	0.35	0.13	0.42	0.14	0.07	0.01	**14**
Variance of firm pay	0.40	0.45	0.48	0.46	0.08	0.02	**21**
Panel B: Annual income innovations							
St.d of innovations	0.55	0.54	0.51	0.52	-0.04	-0.02	**62**
Skewness	-0.21	-0.25	-0.31	-0.32	-0.10	-0.07	**71**

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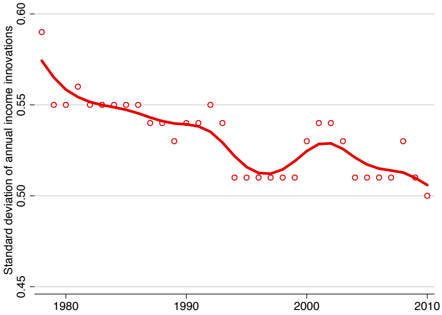
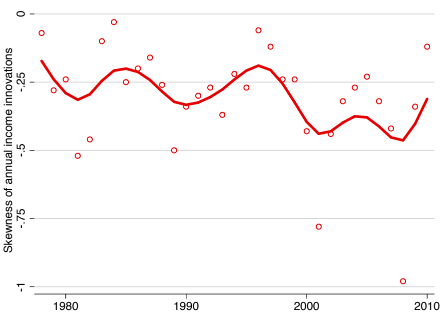
2nd and 3rd moments of income innovations

(A) Standard deviation



(B) Skewness



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Appendix F

Data

- Demographic data from the March CPS and Census Bureau's Intercensal Censi projections
- Establishment and firm dynamics from the BDS
- Merged CPS monthly files for worker mobility rates
- State real GDP per worker from state private sector GDP (BEA), regional CPIs (BLS), and private sector employment (BDS)

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Four other prominent changes

1. Increasing gender and racial diversity
   - Estimated coefficients on share female and non-white are in most cases not statistically significant
   - Typically predict a small *increase* in dynamism

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3. Increasing educational attainment
   - Share college is associated with higher dynamics
   - Hence also predicts an *increase* in dynamism

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Four other prominent changes

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    - Typically predict a small *increase* in dynamism

3. Increasing educational attainment
    - Share college is associated with higher dynamics
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4. Slowdown in labor supply growth
    - Confirming Karahan et al. (2016), labor supply growth is positively correlated with entry
    - But does not alter conclusion regarding the importance of the age composition

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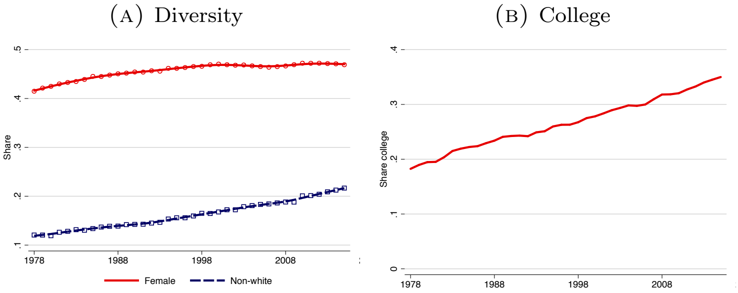
Increasing diversity and education

Figure XLI: Share female, non-white and with a college degree



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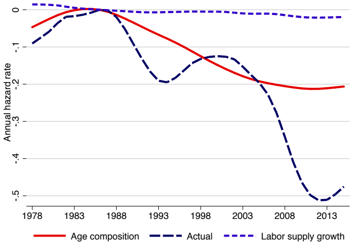
Labor supply growth

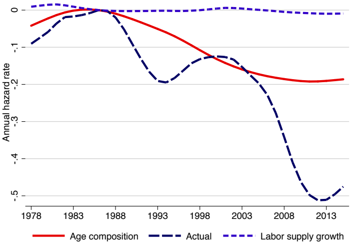
(A) Δ labor force

(B) Δ working age population

(C) Entry (lf)



(D) Entry (wp)



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