Why Have Interest Rates Fallen Far Below the Return on Capital

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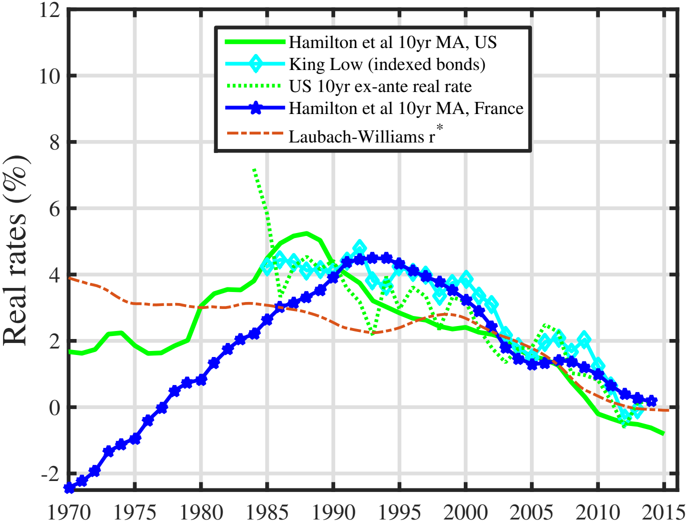
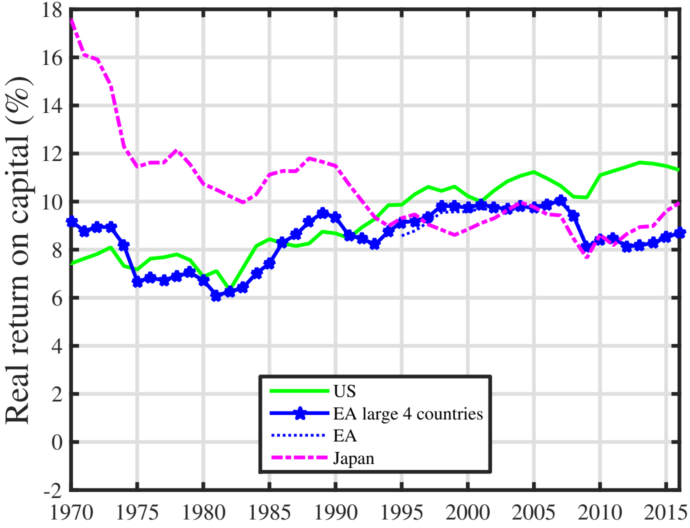
The decrease of real interest rates



The Deleveraging model of free fall r

(Eggertsson and Krugman)

Which does not reflect the evolution of capital return



The "higher risk aversion"/ "safe assets shortage" model of increased $r^K - r$

investors are "desperate" for liquid/safe assets (Caballero, Farhi and Gourinchas)

The usual suspects

- Low rates have been loosely tied to “secular stagnation”
- A number of potential explanations have been cited:
  - productivity slowdown
  - changing demographics (population slowdown, *increased longevity*)
  - change in the price of investment goods
  - tightening of borrowing constraint
  - *shortage of safe assets*
  - *rising inequality*

Our goal

- quantitative assessment of the various factors cited
- embed them in a single, tractable model
- explain both the evolution of capital return and risk-free rate
    - this means having risk, and attitudes toward risk, in the model

Related literature

- low rates: King and Low (2014); Hamilton et al. (2016); Holston et al. (2016); **?**
- safe assets: Coeurdacier et al. (2015); Caballero et al. (2008); Caballero and Farhi (2014)
- deleveraging: Eggertsson and Krugman (2012); Korinek and Simsek (2016); Farhi and Werning (2013)
- secular stagnation: Bean et al. (2015); Rachel and Smith (2015); **?**
- demographics: Carvalho et al. (2016); Gagnon et al. (2016)
- risk: Kozlowski et al. (2015); Hall (2016)
- return on capital: Caballero et al. (2017)

The Model

- add risk to Eggertsson and Mehrotra (2014) and Coeurdacier et al. (2015).
- time is discrete, infinite
- 3-period OLG structure (y, m, o)
  - population N_t, growth rate g_L
- recursive preferences with Epstein-Zin-Weil utility function
- capital and labor (supplied inelastically), age-specific productivities $(e^y, 1, 0)$
- output $Y = K^\alpha (AL)^{1-\alpha}$
  - productivity A: trend growth g_A + shock with variance σ (only source of risk)
  - growth in price of investment g_I

Preferences

Epstein and Zin (1989)–Weil (1990) recursive preferences:

$$V_t = U(c_t, E_t V_{t+1}) = \left(c_t^{1-\rho} + \beta \left(\left(E_t {V_{t+1}}^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

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CES functional form applied to

- time: $\left(c_t^{1-\rho} + \beta \left(\cdot_{t+1} \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}$
  - ρ: inverse of intertemporal elasticity of substitution
- risk: $\left(E_t V_{t+1}{}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$
  - γ: risk aversion
- when $\rho = \gamma$
  - standard time-additive preferences
  - tension between
    - high γ required to match asset pricing
    - low ρ required to match consumption growth with interest rates

Budget constraints

- young borrow from middle-aged up to a fraction θ of their $t+1$ labor income
  - we focus on equilibria where this binds
  - no other frictions (e.g., price stickiness)
- middle-aged lend to young, buy capital from old, invest
- old collect returns, sell depreciated capital

$$
\begin{aligned}
c_t^y &= b_{t+1}^y + w_t e_t^y \\
& \quad b_{t+1}^y \leq \theta_t E_t(w_{t+1}/R_{t+1}) \\
c_{t+1}^m - b_{t+2}^m + p_{t+1}^k k_{t+2}^m &= w_{t+1} - R_{t+1} b_{t+1}^y \\
c_{t+2}^o &= (p_{t+2}^k(1-\delta) + r_{t+2}^k) k_{t+2}^m - R_{t+2} b_{t+2}^m
\end{aligned}
$$

market-clearing:

$$
g_{L,t} b_{t+1}^y + b_{t+1}^m = 0
$$

Production

$$Y_t = (N_{t-2}k_t^m)^\alpha \left[A_t(e_t^y N_t + N_{t-1})\right]^{1-\alpha}$$

- $N_{t-2}k_t^m$: capital (chosen by current old in the previous period)
- $e_t^y N_t + N_{t-1}$: labor (of young and middle-aged)

Competitive factor markets:

$$\begin{aligned} w_t &= (1-\alpha)A_t^{1-\alpha}k_t^\alpha \\ r_t^k &= \alpha A_t^{1-\alpha}k_t^{\alpha-1} \end{aligned}$$

both written in terms of the capital/labor ratio k_t defined as

$$k_t \equiv \frac{N_{t-2}k_t^m}{e_t^y N_t + N_{t-1}} = \frac{k_t^m}{g_{L,t-1}(1+e_t^y g_{L,t})}.$$

Solution strategy

- only the middle-aged have an intertemporal problem
  - how much to save
  - in what form: bonds or capital
- write the middle-aged's Euler equation and substitute equilibrium quantities
  - quantity of bonds determined by young's constraint
  - Euler equation also relates risk-free rate R and return to capital R^k
- we derive a law of motion expressed in terms of R or equivalently k

Solution strategy (2)

Middle-aged FOCs:

$$
\begin{aligned}
(c_t^m)^{-\rho} &= \beta \left[E_t (c_{t+1}^o)^{1-\gamma} \right]^{\frac{\gamma-\rho}{1-\gamma}} E_t \left[(c_{t+1}^o)^{-\gamma} R_{t+1}^k \right] \\
(c_t^m)^{-\rho} &= \beta \left[E_t (c_{t+1}^o)^{1-\gamma} \right]^{\frac{\gamma-\rho}{1-\gamma}} E_t \left[(c_{t+1}^o)^{-\gamma} \right] R_{t+1}.
\end{aligned}
$$

Define $R_{t+1}^m = \alpha_t R_t^k + (1-\alpha_t) R_{t+1}$ and express budget constraints as

$$
\begin{aligned}
W_t &= Y_t - c_t^m \\
c_{t+1}^o &= R_{t+1}^m W_t.
\end{aligned}
$$

Portfolio choice: set α_t so that

$$
E_t ({R_{t+1}^m}^{-\gamma}) R_{t+1} = E_t \left({R_{t+1}^m}^{-\gamma} R_{t+1}^k \right)
$$

Saving decision:

$$
Y_t = \left(1 + (\beta \phi_t R_{t+1}^{1-\rho})^{-\frac{1}{\rho}} \right) W_t
$$

Then use market clearing to express Y_t, W_t, R_{t+1}^m in term of the aggregate capital stock

Law of motion

$$\underbrace{\left(1+(\beta\phi_t)^{-1/\rho}R_{t+1}^{1-1/\rho}\right)^{-1}}_{\text{saving rate}}\underbrace{(1-\frac{\theta_{t-1}}{\tilde{a}_t})(1-\alpha)(\frac{R^k_{t+1}}{g_I}-1+\delta)k_t}_{\text{income}}$$

$$=\underbrace{g_{L,t}\left[\underbrace{\alpha(1+e^y g_{L,t+1})}_{\text{capital}}+\underbrace{\left(\frac{1}{\xi_t}\right)(1-\alpha)\theta_t(1-g_I\frac{1-\delta}{R_{t+1}})}_{\text{bonds}}\right]k_{t+1}}_{\text{investments}}$$

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- overlapping generations
  - saving only done out of labor income
- borrowing constraint
  - disappears if $\theta=0$, $e^y=0$
- risk
  - ϕ_t: precautionary saving, acts like discount factor distortion ($\lesseqgtr 1$)
  - $1/\xi_t$: portfolio choice

Risk terms

The factors ϕ_t and ξ_t are

$$\begin{aligned} \xi_t &= \frac{\mathbb{E}_t({u_{t+1}}^{-\gamma}\tilde{a}_{t+1})}{\mathbb{E}_t({u_{t+1}}^{-\gamma})} \\ \phi_t &= \left[\mathbb{E}_t {u_{t+1}}^{1-\gamma}\right]^{(\gamma-\rho)/(1-\gamma)} \mathbb{E}_t {u_{t+1}}^{-\gamma} {v_t}^{\rho} \end{aligned}$$

with

$$\begin{aligned} u_{t+1} &\equiv \alpha(1+e^y g_{L,t+1})\tilde{a}_{t+1} + (1-\alpha)\theta_t \\ \tilde{a}_{t+1} &\equiv \frac{A_{t+1}^{1-\alpha}}{\mathbb{E}_t A_{t+1}^{1-\alpha}}. \end{aligned}$$

only functions of (moments of) the exogenous process A_{t+1}

- when $\delta \neq 1$, ϕ_t involves R_{t+1} as well

Risky steady state

to account for risk in a tractable way, we appeal to the concept of "risky steady state":

- exogenous trends as in the data
- productivity shock is assumed i.i.d.
- in the law of motion, $\tilde{a}_t$ set at its mean, $\tilde{a}_{t+1}$ is stochastic
- agents take into account the uncertainty

Risk and borrowing constraint

When $\delta = 1$, $\rho < 1$, $\theta = 0$, $e^y = 0$ (no young):

$$\phi_t \simeq 1 + \frac{1}{2}\gamma(1-\rho)\sigma^2$$

$$\frac{1}{\xi_t} \simeq 1$$

Risk and borrowing constraint

When $\delta = 1$, $\rho < 1$:

$$\phi_t \simeq 1 + \frac{1}{2}\gamma(1-\rho)\frac{\alpha^2(1+e^y g_{L,t+1})^2}{(\alpha(1+e^y g_{L,t+1})+(1-\alpha)\theta_t)^2}\sigma^2$$

$$\frac{1}{\xi_t} \simeq 1+\gamma\frac{\alpha(1+e^y g_{L,t+1})}{\alpha(1+e^y g_{L,t+1})+(1-\alpha)\theta_t}\sigma^2$$

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$$\frac{1}{\xi_t} \simeq 1 + \gamma\frac{\alpha(1+e^y g_{L,t+1})}{\alpha(1+e^y g_{L,t+1}) + (1-\alpha)\theta_t}\sigma^2$$

law of motion:

$$\left(1 + (\beta \underset{\substack{\theta,\sigma \\ -+}}{\phi_t})^{-1/\rho} R_{t+1}^{1-1/\rho}\right)^{-1} (1 - \frac{\theta_{t-1}}{\tilde{a}_t})(1-\alpha)\frac{R_{t+1}^k}{g_l} k_t$$

$$= g_{L,t}\left[\alpha(1+e^y g_{L,t+1}) + \underset{\substack{\theta,\sigma \\ -+}}{\left(\frac{1}{\xi_t}\right)}(1-\alpha)\theta_t\right] k_{t+1}$$

Long run determinants

of the bond interest rate r and the return on capital r^K

$\delta = 1$, $\rho = 1$:

- Observable factors
  - productivity growth g_A
  - evolution of working age population g_L
  - trend in investment price g_I
- Unobservable factors
  - borrowing constraint θ
  - variance of the shock on the trend of productivity σ.

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$$\begin{aligned} r &= \bar{r} + (g_L - 1) + (g_A - 1) - \frac{1}{2}(g_I - 1) + c\theta + \gamma u(\underset{+}{\theta|_\sigma}, \underset{-}{\sigma^2}) \\ r^K &= r + \gamma v(\underset{-}{\theta|_\sigma}, \underset{+}{\sigma^2}) \end{aligned}$$

The wedge between r and r^K is only affected by θ and σ

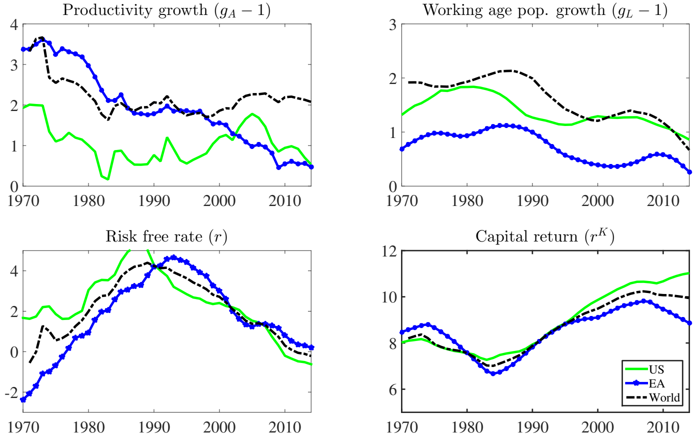
Empirical strategy

- our targets are the risk-free rate and the return on capital
- we segregate the usual suspects into
  - the observables: productivity, demographics, price of investment
  - the "less observables": borrowing constraint, productivity risk
- Three steps:
  1. input the observables, set θ and σ constant to match the levels of the targets
  2. input the observables, compute θ to match the risk-free rate, keep σ constant
  3. input the observables, compute σ to match the risk-free rate, keep θ constant
  4. input the observables, compute θ and σ to match both targets
- repeat for US and Euro area (and the world)
- then stare at the pictures...
- caveats
  - we interpret the generations loosely (10-year averages)
  - risk-free rates before the 1980s are less meaningful (financial repression etc), so we focus on 1990s to present

Model calibration and data sources

Parameters		
T	length of period (years)	10
β	discount factor	0.98^T
α	capital share	0.28
γ	risk aversion	100
ρ	inverse of IES	0.8
δ	capital depreciation rate	$0.1 * T$
e^y	relative productivity of young	0.3
Factors		
$g_{L,t}$	growth rate of population 20-64	US, EA (France), China, Japan: OECD
$g_{I,t}$	investment price growth	DiCecio (2009)
$g_{A,t}$	productivity growth	US: Fernald (2012), Euro: NAWM model
R_t	real interest rate	US: Hamilton et al. (2016), France
R_t^k	return on capital	US, EA: our calculations à la Gomme et al. (2015)
$\tilde{a}_t$	productivity shock	$\ln(\tilde{a})$ is a i.i.d. $N(-\sigma^2/2, \sigma^2)$
Free parameters		
θ	borrowing constraint on young	
σ^2	variance of $\tilde{a}_t$	

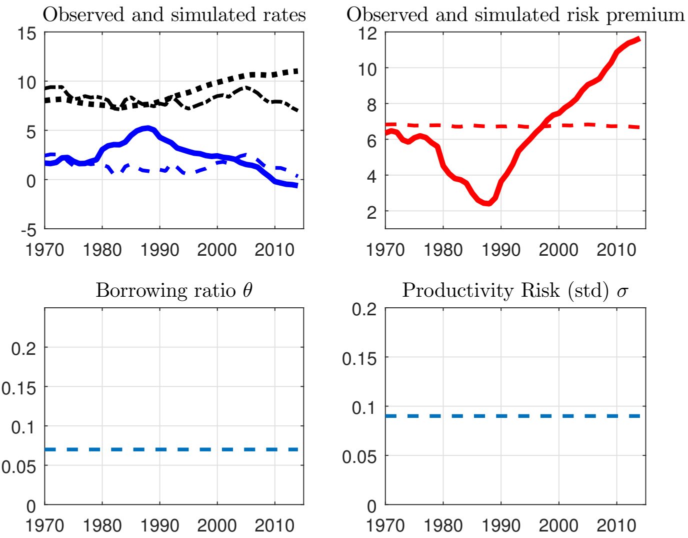
The inputs



Productivity growth ($g_A - 1$)
Working age pop. growth ($g_L - 1$)
Risk free rate (r)
Capital return (r^K)
US
EA
World
1970
1980
1990
2000
2010

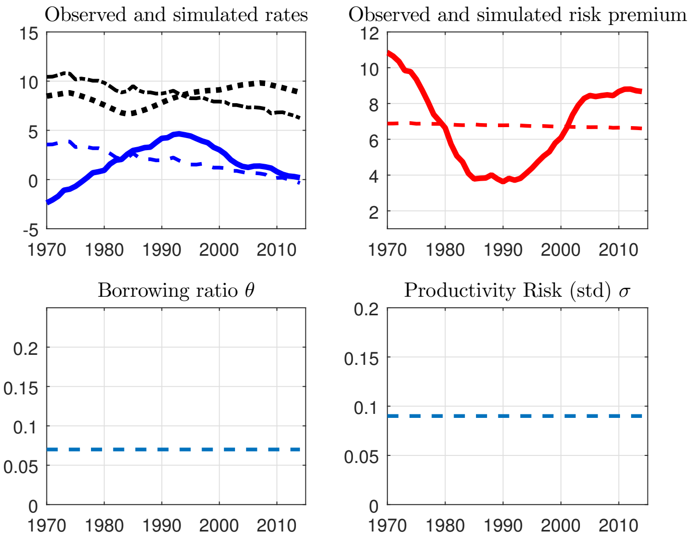
Impact of observable factors, in the US

Observable factors explain about 1.4% from 1992 to 2014



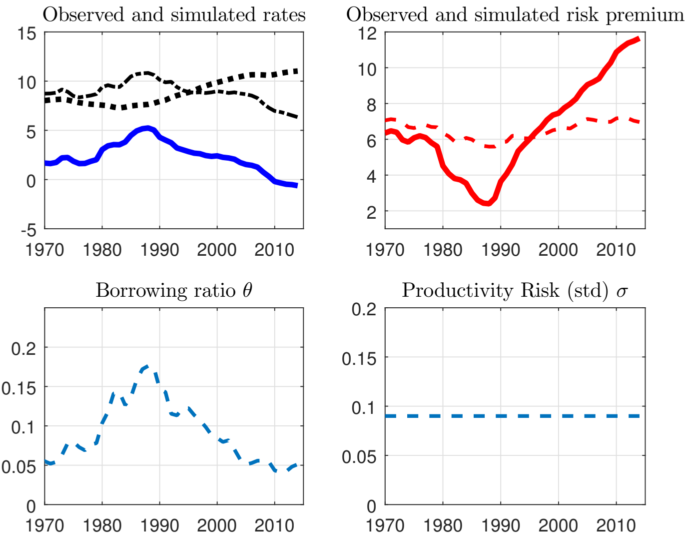
Impact of observable factors, in the EA

Observable factors explain about 1.8% from 1992 to 2014



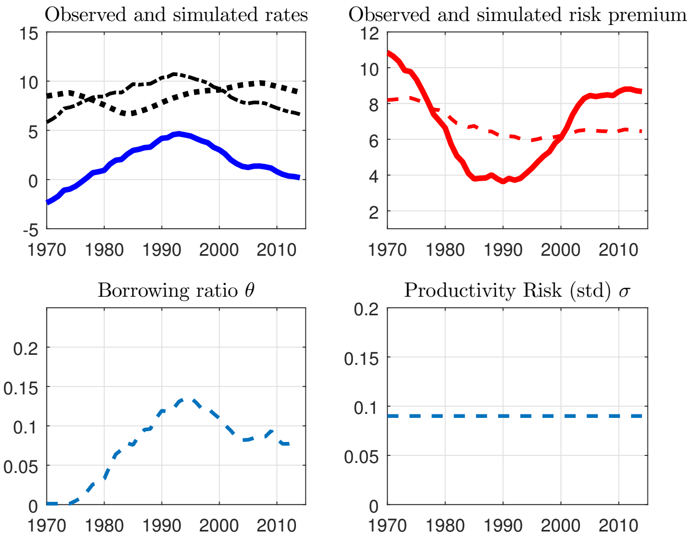
Impact of the borrowing constraint, in the US.

A tighter constraint can account for the fall in the risk-free rate and 0.8% increase of the risk premium



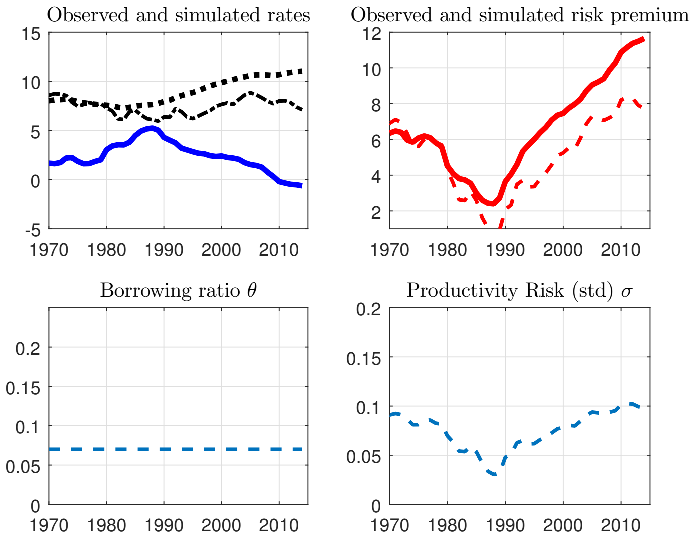
Impact of the borrowing constraint, in the EA.

A tighter constraint can account for the fall in the risk-free rate and 0.7% increase of the risk premium



Impact of risk, in the US.

A higher risk perception can account for the fall in the risk-free rate and the increase in the risk premium



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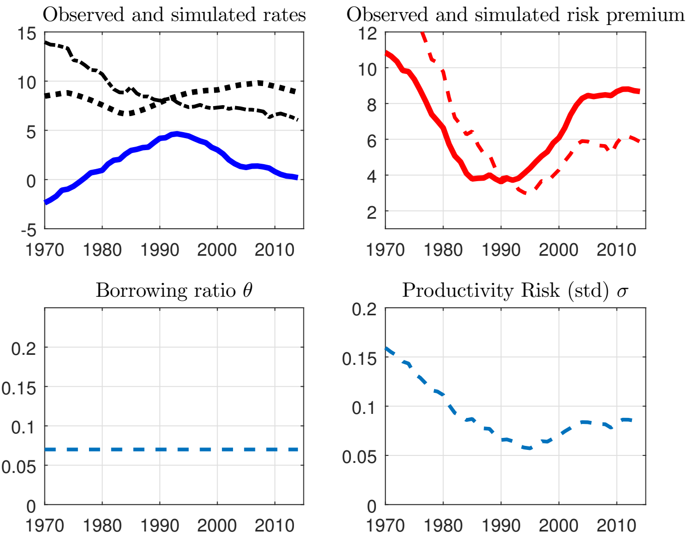
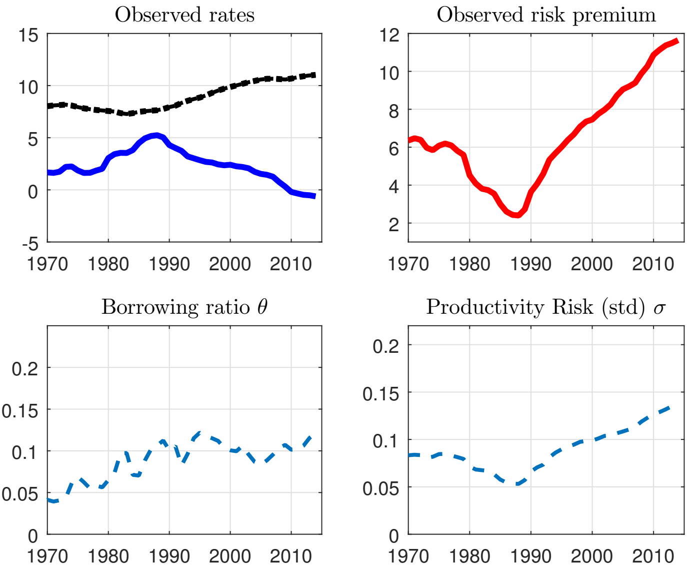


Impact of risk and the borrowing constraint, in the US.

With higher risk perception data are consistent with non decreasing debts



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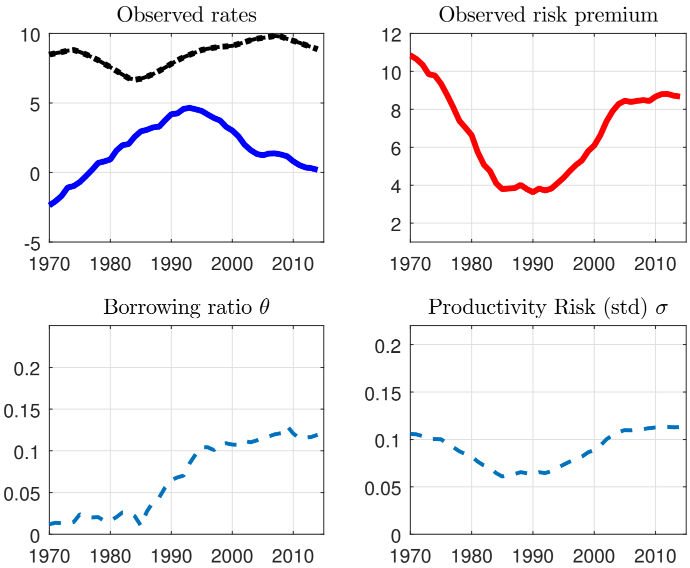
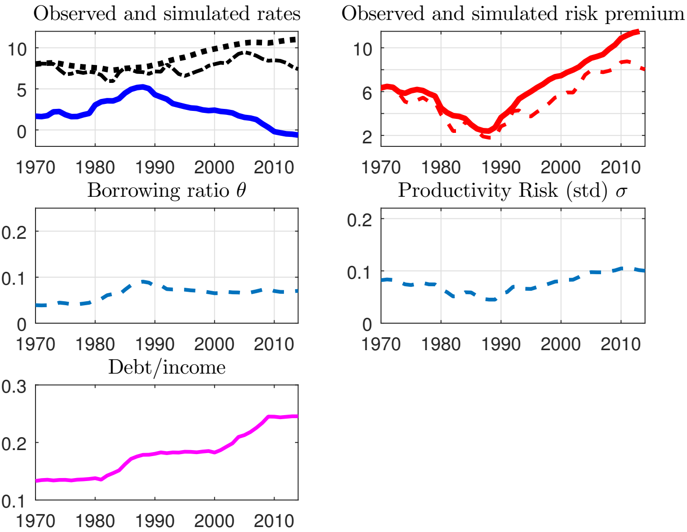
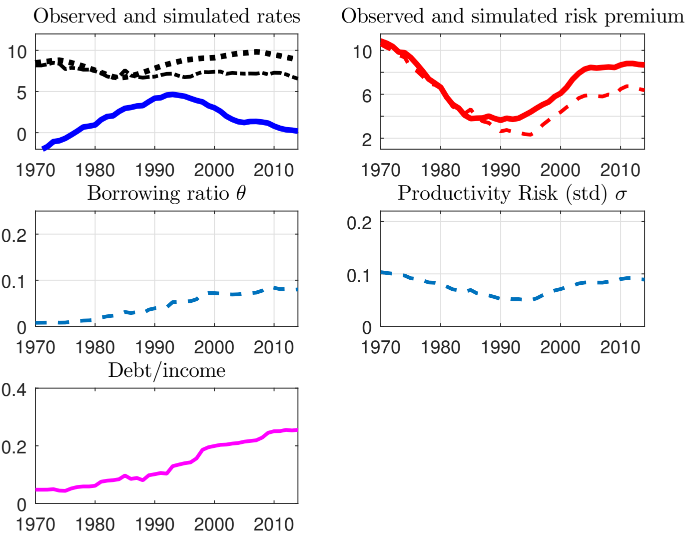


Borrowing constraint and risk, in the US.

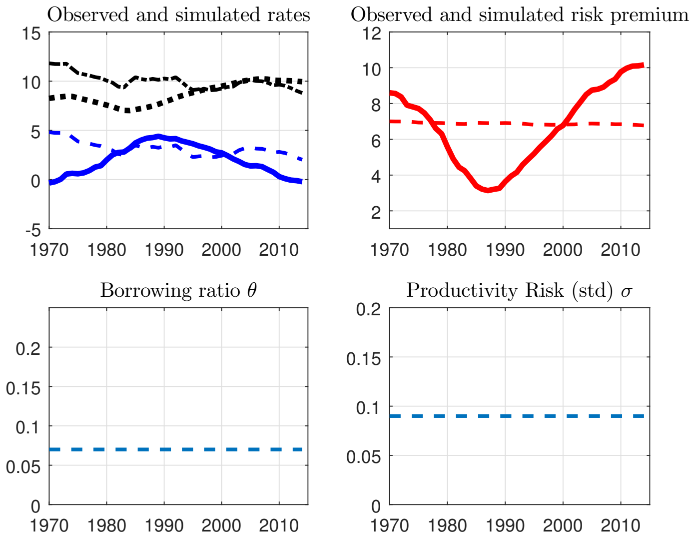


Borrowing constraint and risk, in the EA.



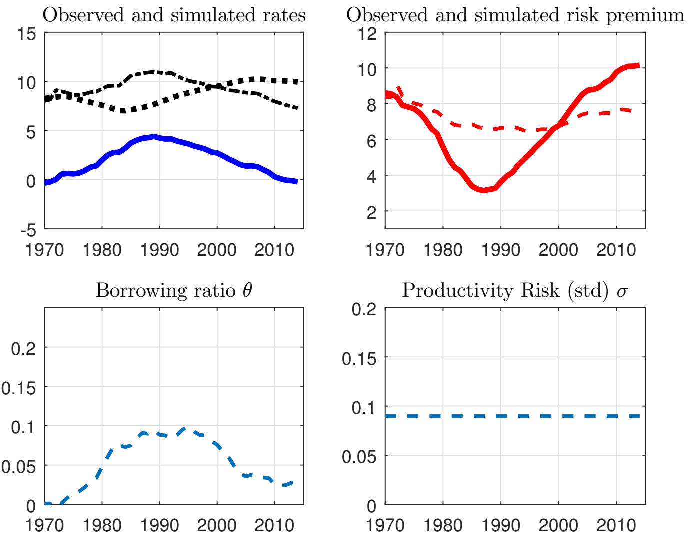
Global perspective

Impact of observable factors



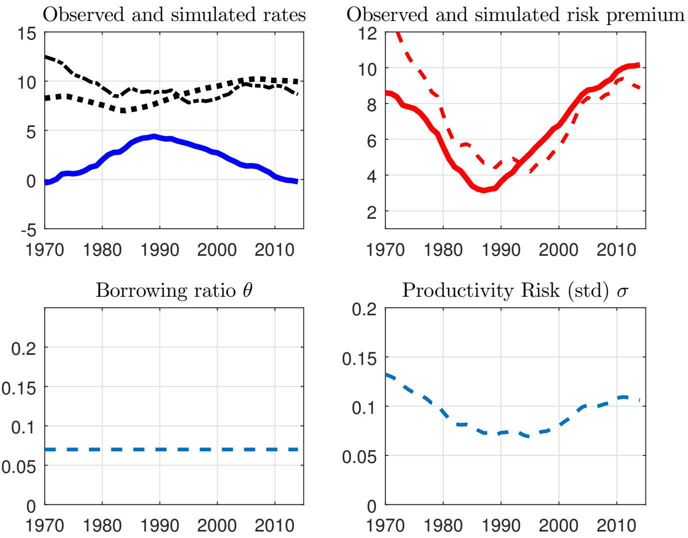
Global perspective

Impact of the borrowing constraint

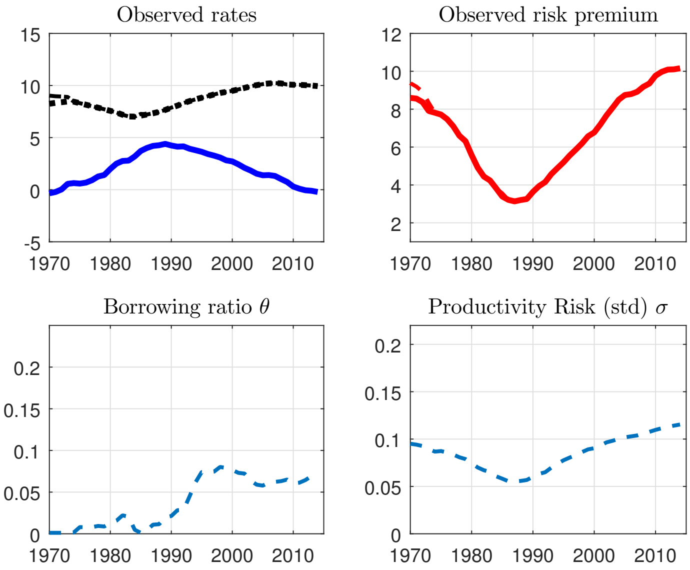


Global perspective

Impact of risk



Global perspective
Impact of risk and the borrowing constraint

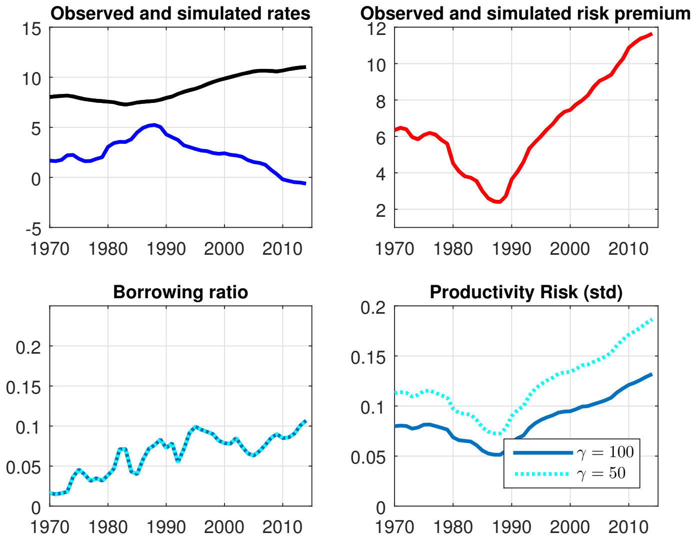


Conclusion

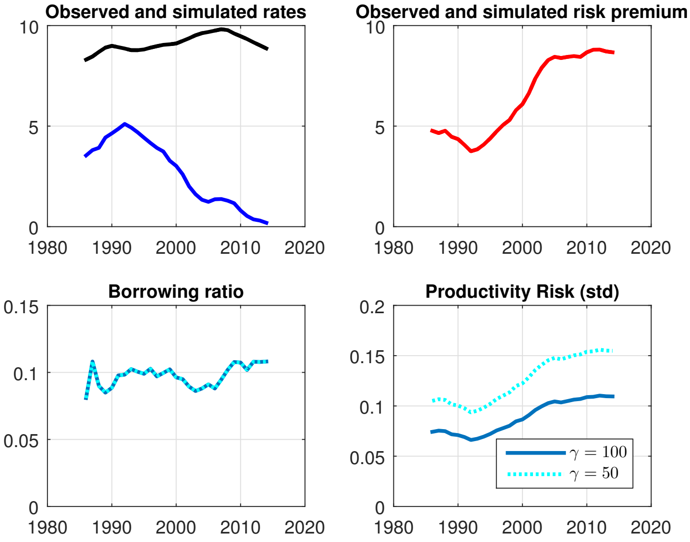
- usual suspects aren't enough
  - deleveraging story
- increased (perception of) risk can account for the patterns
  - but it's a residual
- more work to be done on
  - longevity
  - inequality (through a bequest motive)
  - exogenous supply of safe assets

Additional slides

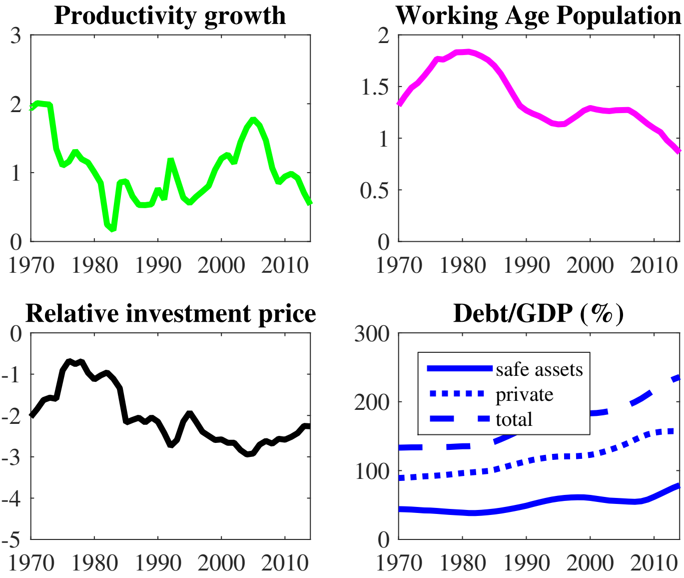
Sensitivity to γ (US)



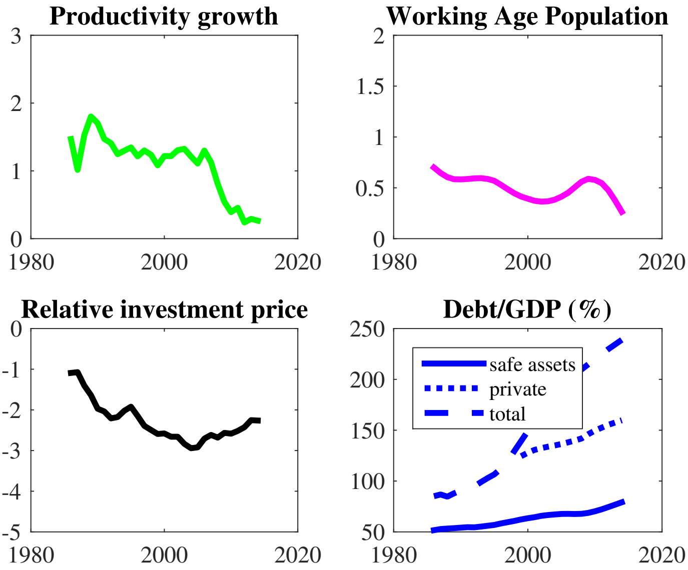
Sensitivity to γ (EA)



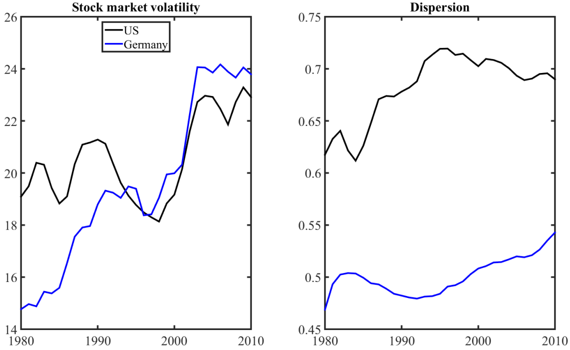
The inputs for the US



The inputs for the EA



Measures of uncertainty



Source : Bachmann et al. (2012), dispersion index is based on survey expectations data (disagreement and forecast errors).

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