Low frequency drivers of the real interest rate

A band-spectrum regression approach

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Banca d'Italia

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Overview of the paper

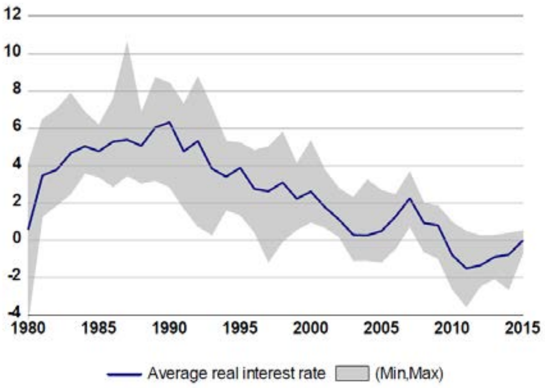

Figure 1: Real interest rate in advanced economies

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- Connections with the literature on estimation of the **natural rate of interest**; e.g. Laubach and Williams (2003), Lubik and Matthes (2015), Hamilton, Harris, Hatzius and West (2016), Holston, Laubach and Williams (2017) + structural macroeconomic models

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- Main finding: long-term movements of the real interest rate mainly reflect **productivity and demographic developments**.

Drivers of the real interest rate

(A) In a standard Solow growth model the **equilibrium** real rate is $r^* = \alpha \frac{n+g+\delta}{s}$

- Demographics (n, s)
- Technological change, human capital (g)
- Change in preferences (s)
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(B) Some have also emphasized a **disequilibrium** explanation.
Over-accomodating monetary policies may have induced a downwards bias in interest rates ...... e.g. Borio (2014), Juselius et al. (2016)

Some descriptive statistics

	Average		
	1980-1989	1990-1999	2000-2015
Working age population (% change)	0.88	0.63	0.36
Old age dependency ratio (%)	18.9	21.6	26.1
Total Factor Productivity (% change)	1.3	0.9	0.5
Human capital per person (% change)	0.74	0.62	0.46
Credit to GDP ratio	103.8	124.5	152.9
Income distribution (Gini index)	29.3	31.4	32.0

Table 1: Drivers of the real interest rate drivers: average values in advanced economies

Band-spectrum regression

- Hannan (1963), Engle (1974), Harvey (1978). Basic intuition: in a standard time series regression we explain the variability of y through the covariates X. The variance is just the integral of the spectrum of y (stationary). Can focus on a region of that integral.

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- How? **Transform time-domain data in the frequency-domain** using the finite Fourier transform. Perform **OLS regression on transformed data.** Specifically, for the linear model $y = X\beta + \varepsilon$ (with $\varepsilon \sim N\left(0, \sigma^2 I\right)$), pre-multiply the observations by an orthogonal, complex-valued $T \times T$ matrix W, with

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- **We select frequency bands by just deleting rows from $\widetilde{y}$ and $\widetilde{X}$**
- Serial correlation in ε is mapped into heteroschedasticity in $\widetilde{\varepsilon} \longrightarrow$ use robust standard errors.

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- $y = X\beta + \varepsilon$, where y is $NT \times 1$ and X is $NT \times k$. N countries stack one after. **Same coefficients β across countries.**

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- **We consider two low frequency bands, implying periodicities $P \geq 7$ and $P \geq 15$ years respectively.**
- Both pooled regressions and fixed-effects estimation are performed on annual data for the 1980-2014 period. Pooling/FE estimation is important because loss of degrees of freedoms

Results (benchmark specification)

- $y = \beta_1 \Delta tfp + \beta_2 \Delta adr + \beta_3 \Delta wpop + \beta_4 \Delta cy + \beta_5 IN.$

		Pooled		Fixed effects	
	Time domain	$P \geq 7$	$P \geq 15$	$P \geq 7$	$P \geq 15$
tfp	0.434***	0.828***	1.717***	0.984***	2.079***
age dependency	0.176	0.419*	0.997**	0.485*	1.268**
population 15-64	1.406***	2.010***	2.529**	1.453**	2.606**
credit-to-GDP	-0.016**	-0.015	-0.010	-0.013	-0.016
Gini index	-0.001	-0.042	-0.074	0.002	-0.095
R-square	0.13	0.15	0.41	0.22	0.51

Note: *=10%, **=5%, ***=1% significance

Results (business cycle frequencies)

	Pooled			Fixed effects		
	$P < 7$	$P \geq 7$	$P \geq 15$	$P < 7$	$P \geq 7$	$P \geq 15$
tfp	0.21**	0.83***	1.72***	0.20**	0.98***	2.08***
age dependency	-1.54***	0.42*	1.00**	-1.10**	0.49*	1.27**
population 15-64	0.01	2.01***	2.53**	-0.04	1.45**	2.606**
credit-to-GDP	0.05**	-0.02	-0.01	0.02	-0.01	-0.02
Gini index	0.02	-0.04	-0.07	-0.03	0.00	-0.095
R-square	0.03	0.15	0.41	0.05	0.22	0.51

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Contributions to the R-square

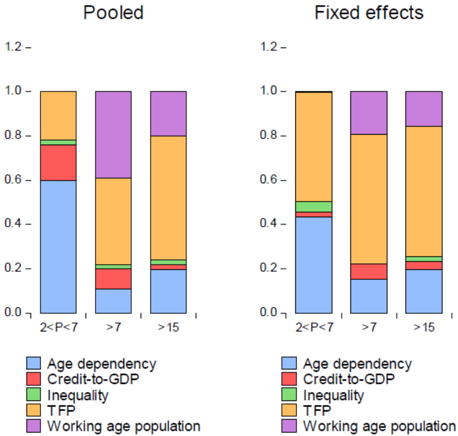


Figure 2: Specific contributions to total R^2

The natural rate of interest

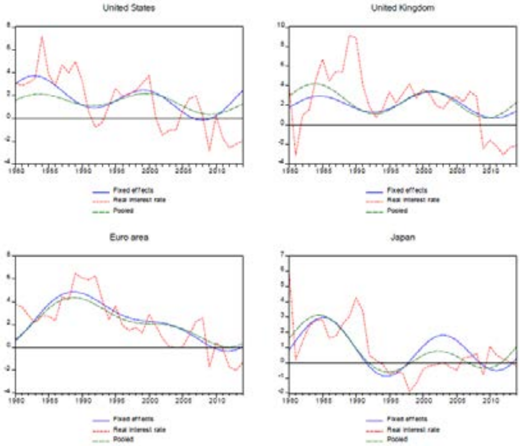
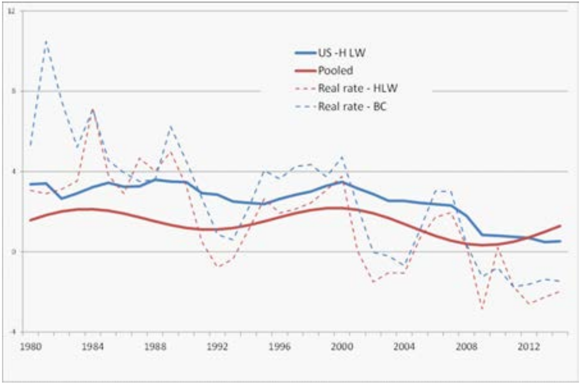


Figure 3: Fitted values - periodicity larger than 15 years

Comparison with Holston, Laubach and Williams (2017)



TFP or human capital?

	Pooled		Fixed effects	
	$P \geq 7$	$P \geq 15$	$P \geq 7$	$P \geq 15$
tfp	0.64**	1.23***	0.75**	1.66***
Human capital	7.01***	7.35***	6.58***	7.74***
Age dependency	0.59**	1.11***	0.59**	1.20***
Population 15-64	1.34**	1.90**	1.17*	1.97**
Credit-to-GDP	-0.01	-0.00	-0.01	-0.01
Gini index	-0.05	-0.07	-0.03	-0.12
R-square	0.33	0.56	0.37	0.67

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Concluding remarks

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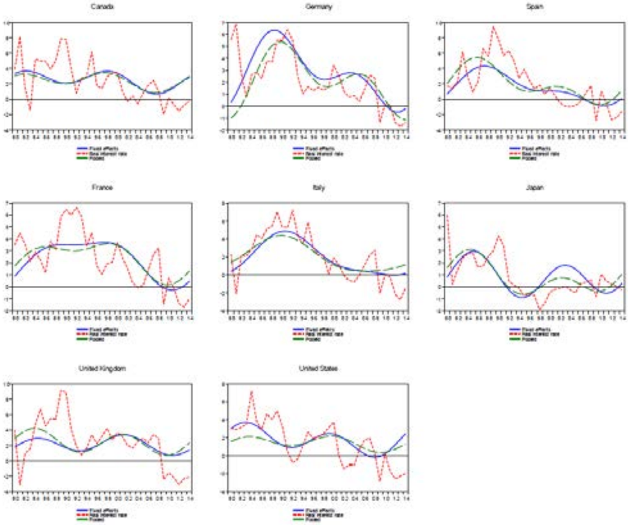
- Formal testing of different impacts across frequencies (business cycle vs longer run) can be performed through a Chow test.
- Our estimates of the natural rate show a much smaller decline in the US and UK than in the Euro area and Japan
- The model can be used to **project natural rates into the future** under plausible assumptions for demographic and TFP developments

Fitted data - periodicity > 15 years



Band-spectrum regressions

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$$z_{tj} = \begin{bmatrix} T^{-\frac{1}{2}} & \text{for } j = 1 \\ 2T^{-\frac{1}{2}} \cos\left[\frac{\pi j(t-1)}{T}\right] & \text{for } j > 1,\ j \text{ even} \\ 2T^{-\frac{1}{2}} \sin\left[\frac{\pi (j-1)(t-1)}{T}\right] & \text{for } j > 1,\ j \text{ odd} \\ T^{-\frac{1}{2}} (-1)^{t+1} & \text{for } j = T,\ T \text{ even} \end{bmatrix}$$

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- Frequency bands can be selected through an appropriate diagonal matrix A, filled with 1's on the diagonal entries corresponding to the included frequencies.

Robustness - detrending

		Pooled		Fixed effects	
	Time domain	$P \geq 7$	$P \geq 15$	$P \geq 7$	$P \geq 15$
tfp	0.48***	0.93***	1.95***	1.12***	2.30***
age dependency	0.12	0.38	1.03**	0.48*	1.37**
population 15-64	1.59***	2.24***	2.80**	1.55**	2.88**
credi-to-GDP	-0.020**	-0.018*	-0.012	-0.015	-0.017
Gini index	0.00	-0.05	-0.09	0.002	-0.10
R-square	0.16	0.14	0.42	0.24	0.51

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