EUROPEAN CENTRAL BANK

EUROSYSTEM

The flow of funds in a who-to-whom framework: balance-sheet interlinkages and propagation of quantity shocks

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How financial systems work: evidence from financial accounts

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European Central Bank

(**) Views should not be attributed to the ECB

What is this about…?

Using the w2w framework to study the propagation of shocks across the financial balance-sheets:

e.g. how are bank balance-sheets affected by sales of debt assets (to central banks)?

First (order) impact: balance-sheet reduction…

…but banks finance the sector that acquire the debt instruments sold: second order (positive) effect…

… and finance other sectors that also finance the acquiring sector: third order effect…

… infinite recursive propagation effects that can be decomposed analytically using w2w matrices

Outline

1 **Who-to-whom data and debt diffusion matrices**

2 An analytical decomposition of the propagation of shocks

3 Shock propagation and network centrality

Who-to-whom data

Columns break down a sector's liabilities by counterparty.

Government

Assets:	Liabilities :
Item 1 …	Item 1 …
Item 2 …	Item 2 …
	Debt issued : B+E+H

Rows break down its assets.

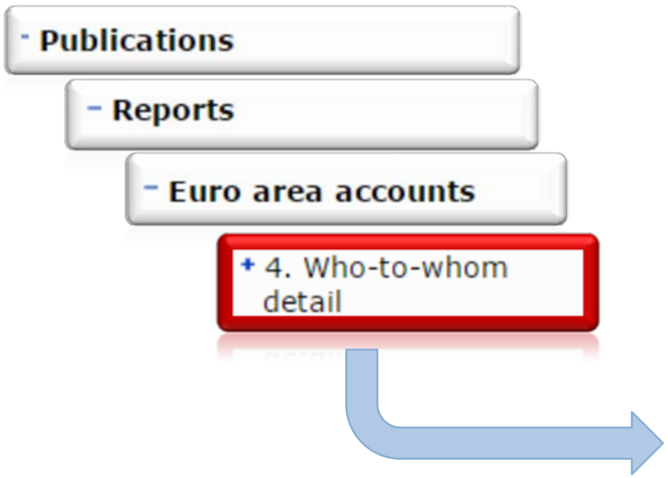
Banks

Assets:	Liabilities :
Item 1 …	Item 1 …
Item 2 …	Item 2 …
Debt held : A+B+C	

Creditor (holder)	Debtor (issuer)				
		Banks	Gov't	Corp.	**Total held**
	Banks	A	B	C	Banks: A+B+C
	Gov't	D	E	F	Gov't: D+E+F
	Corp.	G	H	I	Corp.: G+H+I
	Total issued	Banks: A+D+G	Gov't: B+E+H	Corp.: C+F+I	

The ECB provides euro area and country networks (with data from 13Q4) as data matrices…

- http://sdw.ecb.europa.eu/reports.do?node=1000005335



Who-to-whom detail

4.1.2 Short-term debt securities by counterpart sector
(EUR billions)

1. Transactions

2016 Q3		FINANCING: Total	Non-financial corporations	MFIs	Non-MMF investment funds	Other financial institutions	Insurance corporations and pension funds	General government	Households	Rest of the world
INVESTMENT	Total	42.6	-0.4	26.4	-3.6	6.3	0.0	3.8	0.0	10.2
	Non-financial corporations	0.6	0.8	3.2	0.0	-0.5	0.0	0.9	0.0	-3.8
	MFIs	20.1	0.6	26.5	0.0	9.0	0.0	-15.0	0.0	-1.1
	Non-MMF investment funds	12.3	1.0	3.4	0.0	-0.3	0.1	0.1	0.0	8.1
	Other financial institutions	7.8	-1.0	8.6	0.0	-6.5	-0.1	0.8	0.0	6.0
	Insurance corporations and pension funds	0.8	-0.4	0.5	0.0	-0.1	0.0	-0.4	0.0	1.2
	General government	-0.2	-1.3	0.0	0.0	0.1	0.0	1.0	0.0	-0.1
	Households	-0.2	-0.1	-0.7	0.0	0.1	0.0	0.5	0.0	-0.1
	Rest of the world	1.3	-0.2	-15.2	-3.6	4.5	0.0	15.8	0.0	

http://www.ecb.europa.eu/press/pr/stats/ffi/html/index.en.html

A simplified framework to study propagation (I)

	Debt on,				Other assets	TOTAL ASSETS
Assets of,	SN	S12K	S121	S13		
SN	$z_{1,1}$	$z_{1,2}$	$z_{1,3}$	$z_{1,4}$	n_1	t_1
S12K	$z_{2,1}$	$z_{2,2}$	$z_{2,3}$	$z_{2,4}$	n_2	t_2
S121	$z_{3,1}$	$z_{3,2}$	$z_{3,3}$	$z_{3,4}$	n_3	t_3
S13	$z_{4,1}$	$z_{4,2}$	$z_{4,3}$	$z_{4,4}$	n_4	t_4

Non-financial sectors (SN)

Banks (S12K)

Central bank (S121)

Government (S13)

A simplified framework … (II)

	Debt on,				Other assets	TOTAL ASSETS
Assets of,	SN	S12K	S121	S13		
SN	$z_{1,1}$	$z_{1,2}$	$z_{1,3}$	g_1	n_1	t_1
S12K	$z_{2,1}$	$z_{2,2}$	$z_{2,3}$	g_2	0	t_2
S121	0	$z_{3,2}$	0	g_3	0	t_3
S13	0	0	0	0	0	0

$\boldsymbol{t}$

$\boldsymbol{z}$ $\boldsymbol{g}$ $\boldsymbol{n}$

Diffusion matrix

$$\boldsymbol{t} = \boldsymbol{Z} * \boldsymbol{1} + (\boldsymbol{n} + \boldsymbol{g})$$

$$a_{i,j} = z_{i,j}/t_j$$

Financing per unit of investment

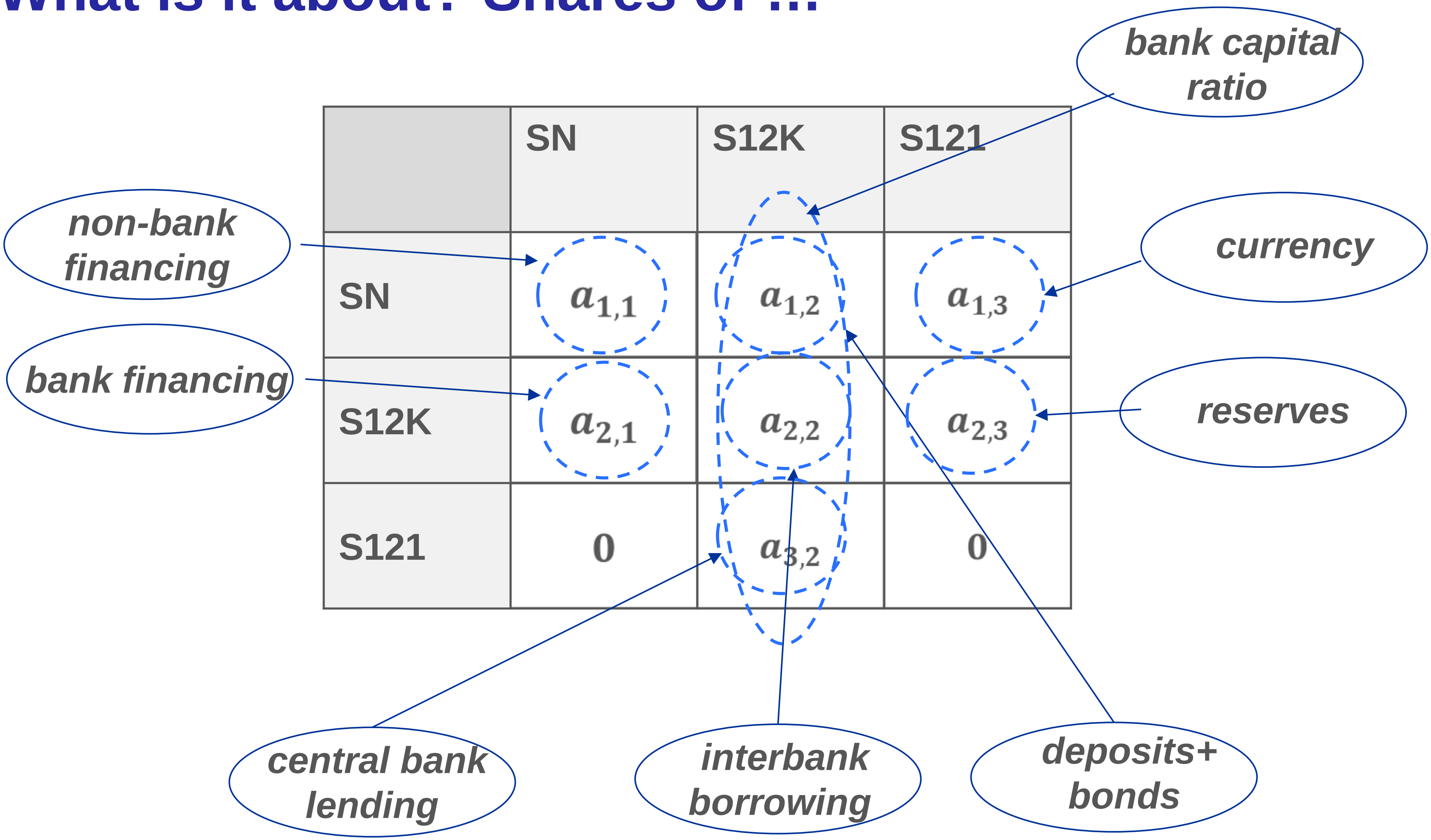
Diffusion matrix

$$\boldsymbol{t} = \boldsymbol{A} * \boldsymbol{t} + (\boldsymbol{n} + \boldsymbol{g})$$

$$\boldsymbol{t} = [\boldsymbol{I} - \boldsymbol{A}]^{-1} * (\boldsymbol{n} + \boldsymbol{g})$$

Leontief inverse

What is it about? Shares of …



An example …

	SN	S12K	S121
SN	0.1	0.6	0.3
S12K	0.5	0.25	0.7
S121	0	0.05	0

Back to initial example: how is sector investment (assets) affected by purchases by the central bank of bank assets?

$$\Delta \boldsymbol{g} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\Delta \boldsymbol{t} = [\boldsymbol{I} - \boldsymbol{A}]^{-1} * \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\Delta \boldsymbol{t} = \begin{bmatrix} 2.13 & 1.83 & 1.92 \\ 1.49 & 2.68 & 2.32 \\ 0.07 & 0.13 & 1.12 \end{bmatrix} * \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.09 \\ -0.36 \\ 0.98 \end{bmatrix}$$

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Continuing with the example, bank investment…

Reserves

Bank loans

$$-\mathbf{0.36} = -2.68 + 2.32 = \frac{(1 - a_{1,1}) + (a_{2,3}(1 - a_{1,1}) + a_{1,3}a_{2,1})}{\det(I - A)}$$

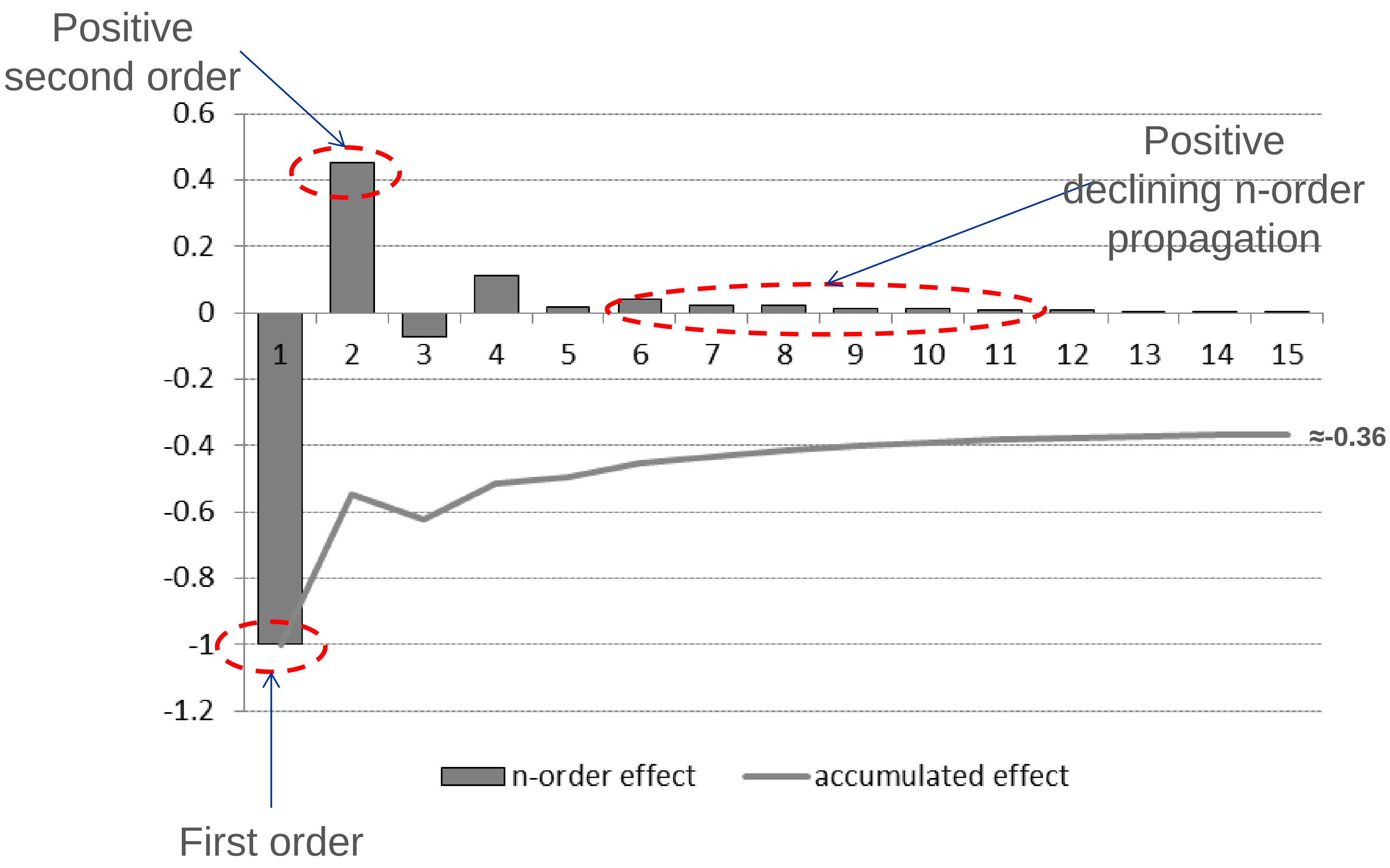
Diffusion:

$$\Delta \boldsymbol{t} = [\boldsymbol{I} - \boldsymbol{A}]^{-1} * \Delta \boldsymbol{g} = \Delta \boldsymbol{g} + \boldsymbol{A}\Delta \boldsymbol{g} + \boldsymbol{A}^2\Delta \boldsymbol{g} + \boldsymbol{A}^3\Delta \boldsymbol{g} + \cdots + \boldsymbol{A}^n\Delta \boldsymbol{g} + \cdots$$

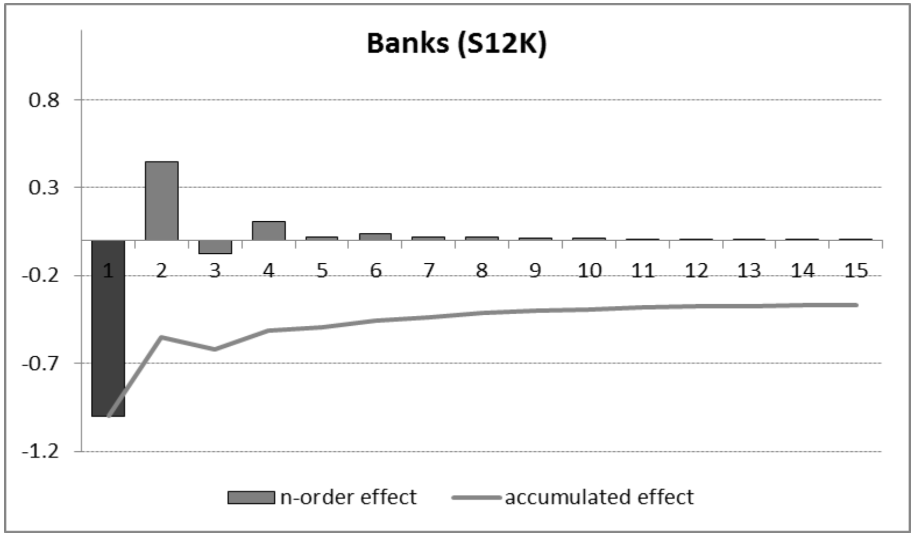
Purchases, sellings, first order effect

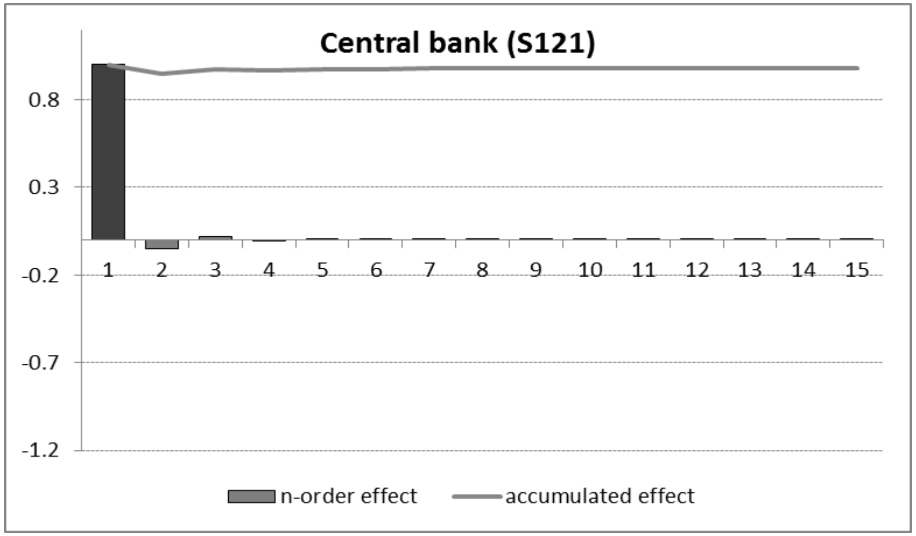
(n+1) order effects: **propagation effects**

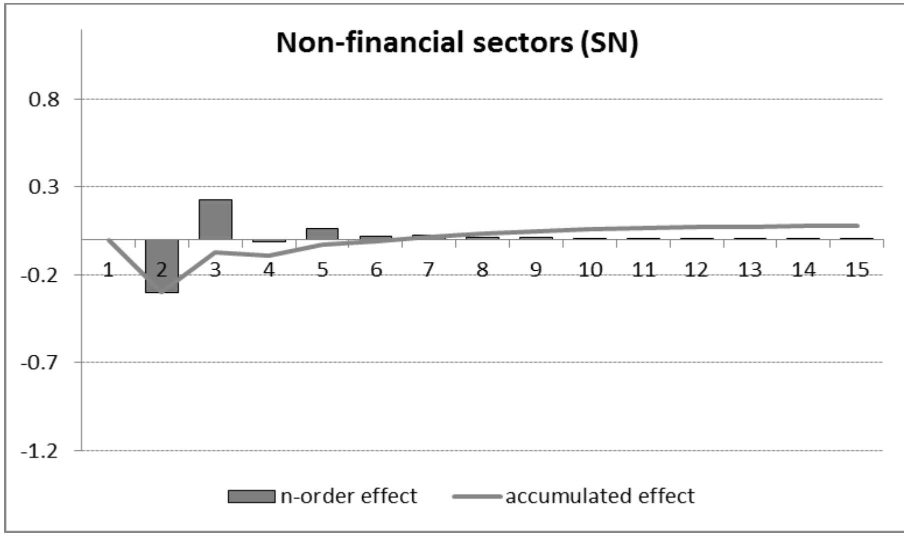
n-order effects on bank investment $(\Delta g = [0\ -1\ 1]')$



n-order effects on sectors' investment $(\Delta g = [0\ -1\ 1]')$







Second and higher order propagation effects are relevant for banks (and non-financial sectors), but not for the central bank… why?

Eigenbase representation (*A non-defective*):

n-order effect: "linear operation" $\boldsymbol{A}$ applied $n-1$ times on $\boldsymbol{g}$

$$\boldsymbol{A}^{n-1}\boldsymbol{g}$$

Decomposing the action of $\boldsymbol{A}$:

$$\boldsymbol{A}\boldsymbol{g} = \boldsymbol{V}\boldsymbol{E}\boldsymbol{V}^{-1}\boldsymbol{g}$$

Matrix of eigenvectors

Diagonal matrix of eigenvalues

i.e.:

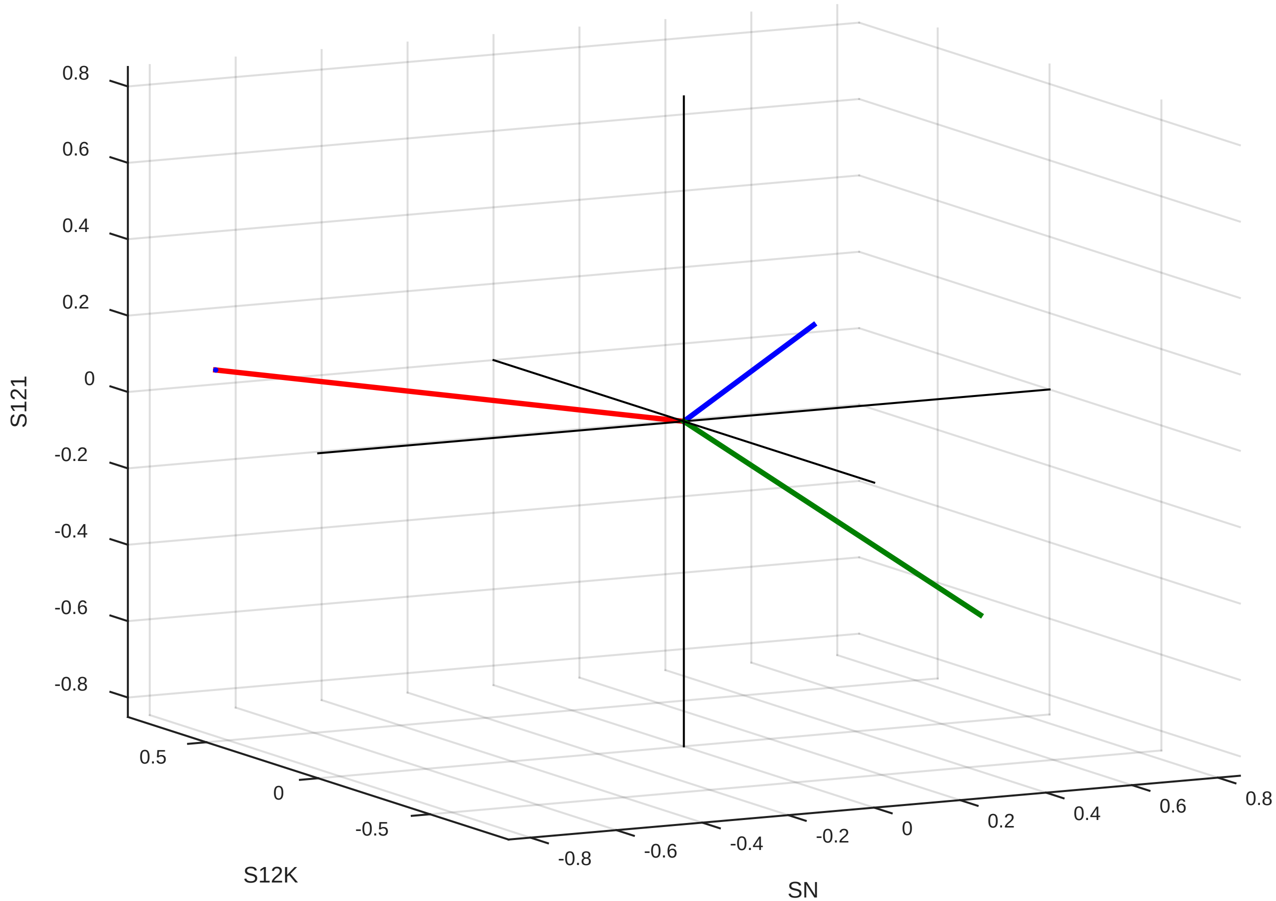
Second component of vector $\boldsymbol{V}^{-1}\boldsymbol{g}$ (component of $\boldsymbol{g}$ in the eigenbase)

$$\boldsymbol{A}^n\boldsymbol{g} = {\rho_1}^n c_1\boldsymbol{v_1} + {\rho_2}^n c_2\boldsymbol{v_2} + {\rho_3}^n c_3\boldsymbol{v_3}$$

Second eigenvalue

Second eigenvector, second column of $\boldsymbol{V}$

Eigenvectors in our example



$$\boldsymbol{v_1} = \begin{bmatrix} 0.68 \\ 0.73 \\ 0.05 \end{bmatrix}$$

$$\rho_1 = 0.76$$

$$\boldsymbol{v_2} = \begin{bmatrix} 0.75 \\ -0.66 \\ 0.08 \end{bmatrix}$$

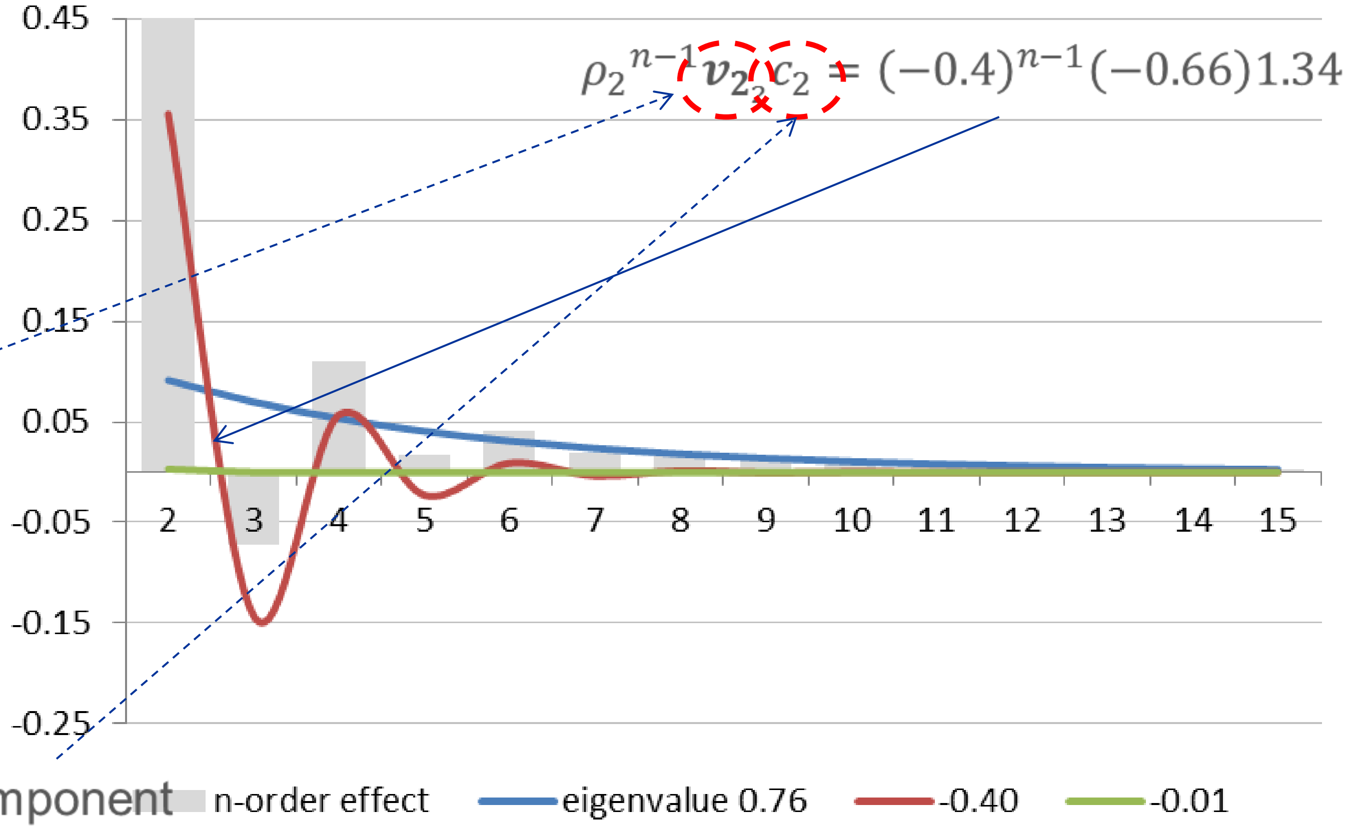
$$\rho_2 = -0.40$$

$$\boldsymbol{v_3} = \begin{bmatrix} 0.77 \\ 0.16 \\ -0.61 \end{bmatrix}$$

$$\rho_3 = -0.01$$

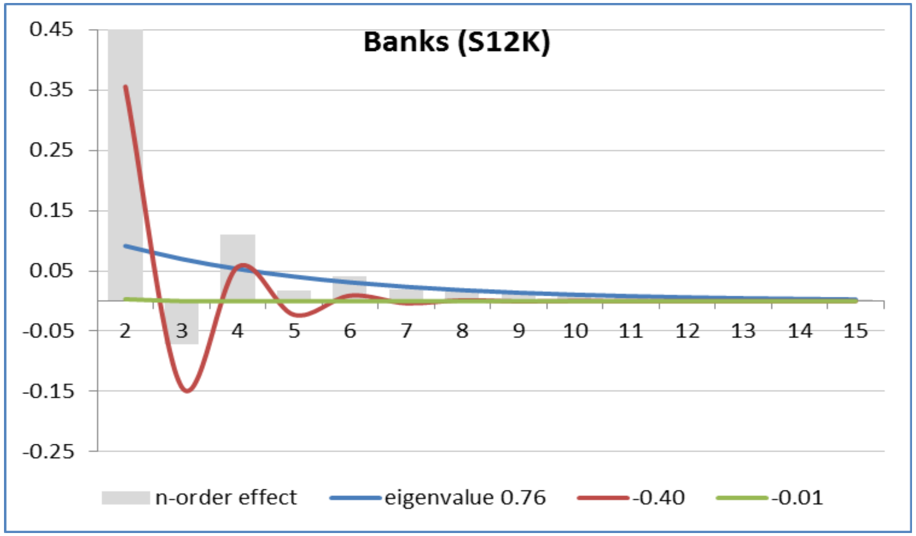
Eigenbase decomposition of (n>1)-order effects on bank investment ($\Delta g = [0\ -1\ 1]$)

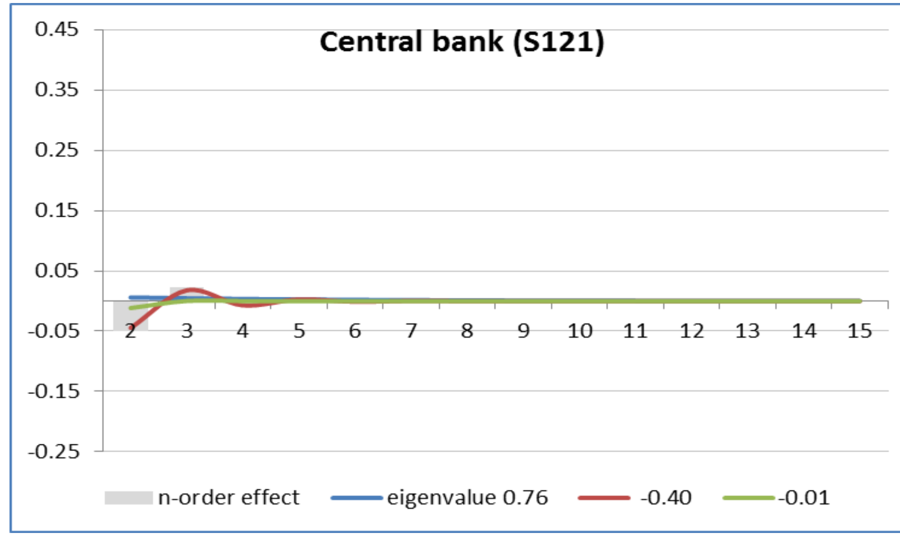
Second component (for sector S12K) of second eigenvector



Second component of g in the eigenbase (second component of $V^{-1}\Delta g$)

Eigenbase decomposition of (n>1)-order effects on sectors' investment ($\Delta g = [0\ -1\ 1]$)



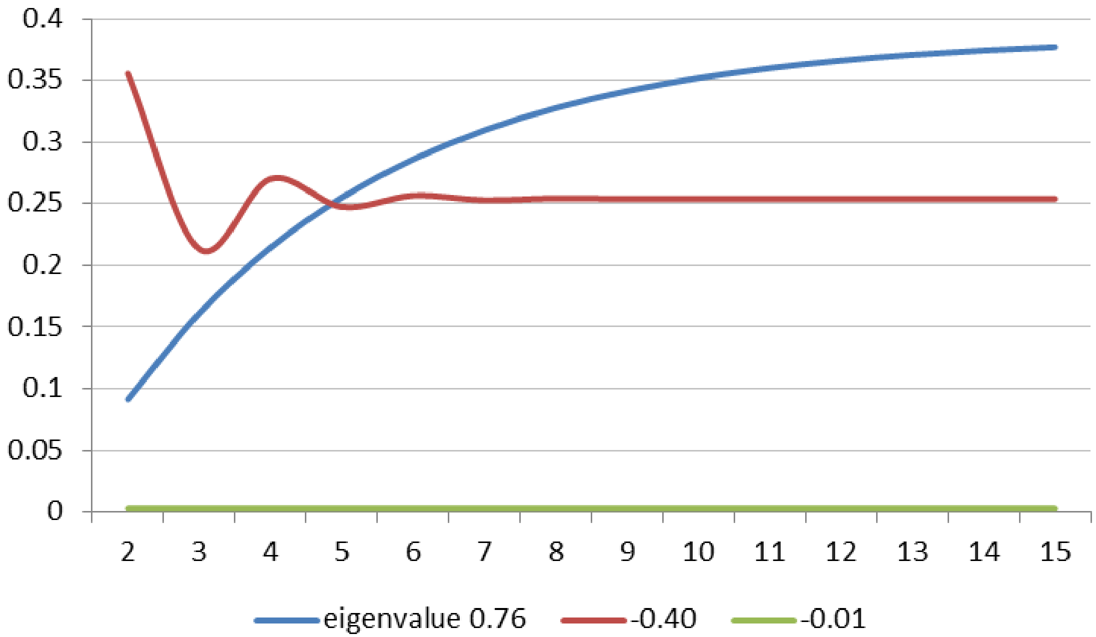


Why central bank propagation effects are small?: **the mathematical answer**

		Eigenvalues (abs)		
		0.76	***0.40***	***0.01***
components of eigenvectors (abs)	**S11**	0.68	0.75	0.77
	S12K	0.73	0.66	0.16
	S121	***0.05***	***0.08***	***0.61***

Low components in the large eigenvalues, high component in the small eigenvalue

Accumulated eigenbase decomposition of (n>1)-order effects on bank investment ($\Delta g = [0 \ -1 \ 1]$)



- ✓ Third eigenvector can be ignored: **dimensionality reduction**. For larger matrices, two, three eigenvectors are enough to describe (n>1)-order propagation effects (depending on “eigenvalue jumps”)
- ✓ After a few initial propagation effects, **propagation is dominated by** the first eigenvector associated to the **largest eigenvalue.**

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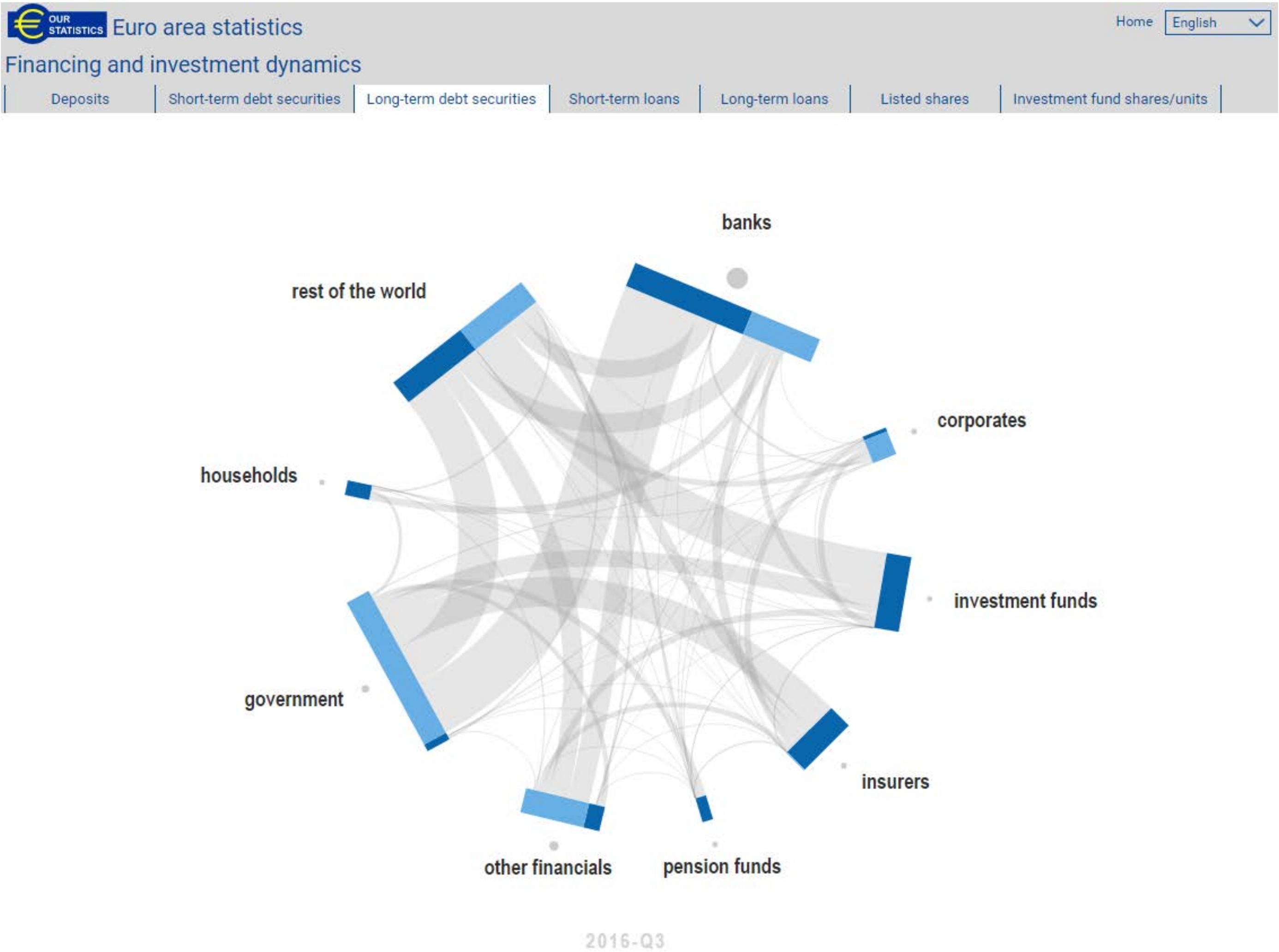
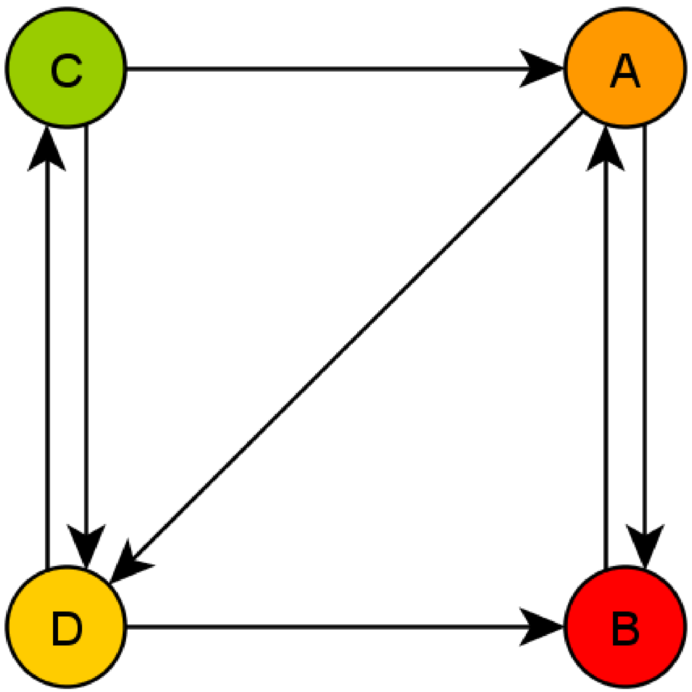
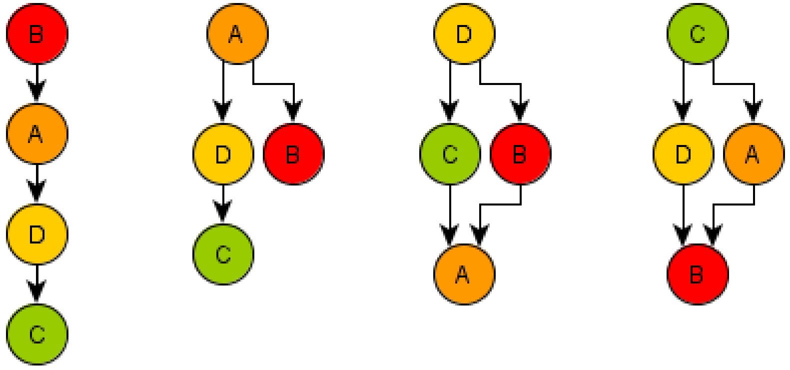
Who-to-whom as a network



ECB website for journalists: www.euro-area-statistics.org

Network centrality





✓interconnectedness ranking: **C-D-A-B**

✓For more complex networks (in particular weighted networks), the solution is not trivial

✓**Eigenvector centrality** provides "interconnectedness" scores/ rankings on the basis of the matrix representation of the network: **Perron eigenvector** (principal vector of Perron eigenvalue)

✓ here **(0.50 0.29 0.61 0.54)**

"Eigenvector centrality" is nothing but the eigenvector associated to **the largest eigenvalue of the eigenbase decomposition** (Perron eigenvalue) of the network matrix

The lack of propagation effects for the Central bank (S121) in the exercise above is a manifestation of its (relative) lack of **centrality.** On the contrary banks (S12K) are very central and show large (n>1)-order effects

Perron-Frobenius *theorem guarantees the existence of* ρ, v *for irreducible matrices*

Eigenvector centrality in financial data

- ✓ indicates sector interconnectedness via direct (first order) investment and financing links, but also indirect (second and higher order) links via financial intermediation

- ✓ Recursive interpretation: “the more a sector is linked to sectors with high score, the higher the score of the sector is”

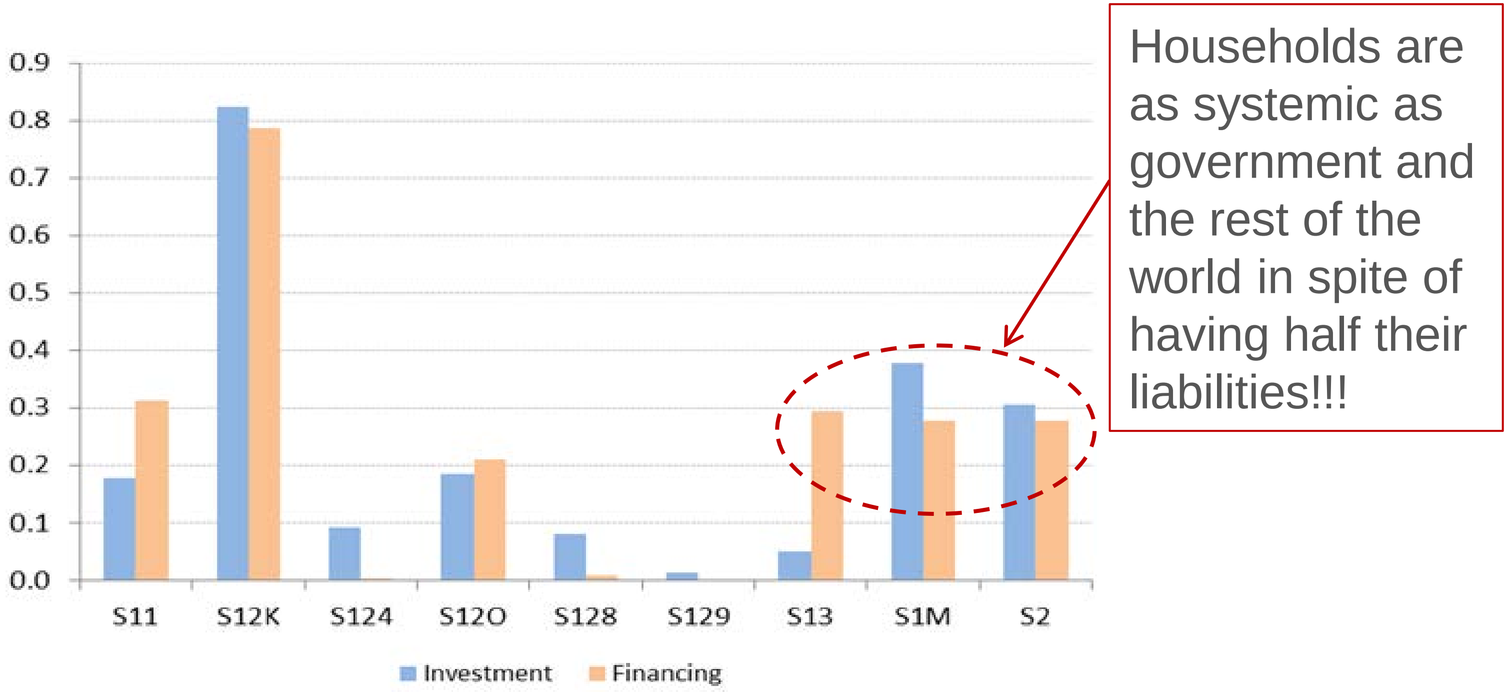
- ✓ Perron’s vector, when calculated on networks …

  - …showing **creditor-debtor** links, provides rankings of interconnectedness via investment: **vulnerability indicator**

  - …showing **debtor-creditor** links (represented by the transposed matrix of a creditor-debtor network), provides rankings of interconnectedness via financing: **systemic risk indicator**

Scores take into account indirect investment- financing links…

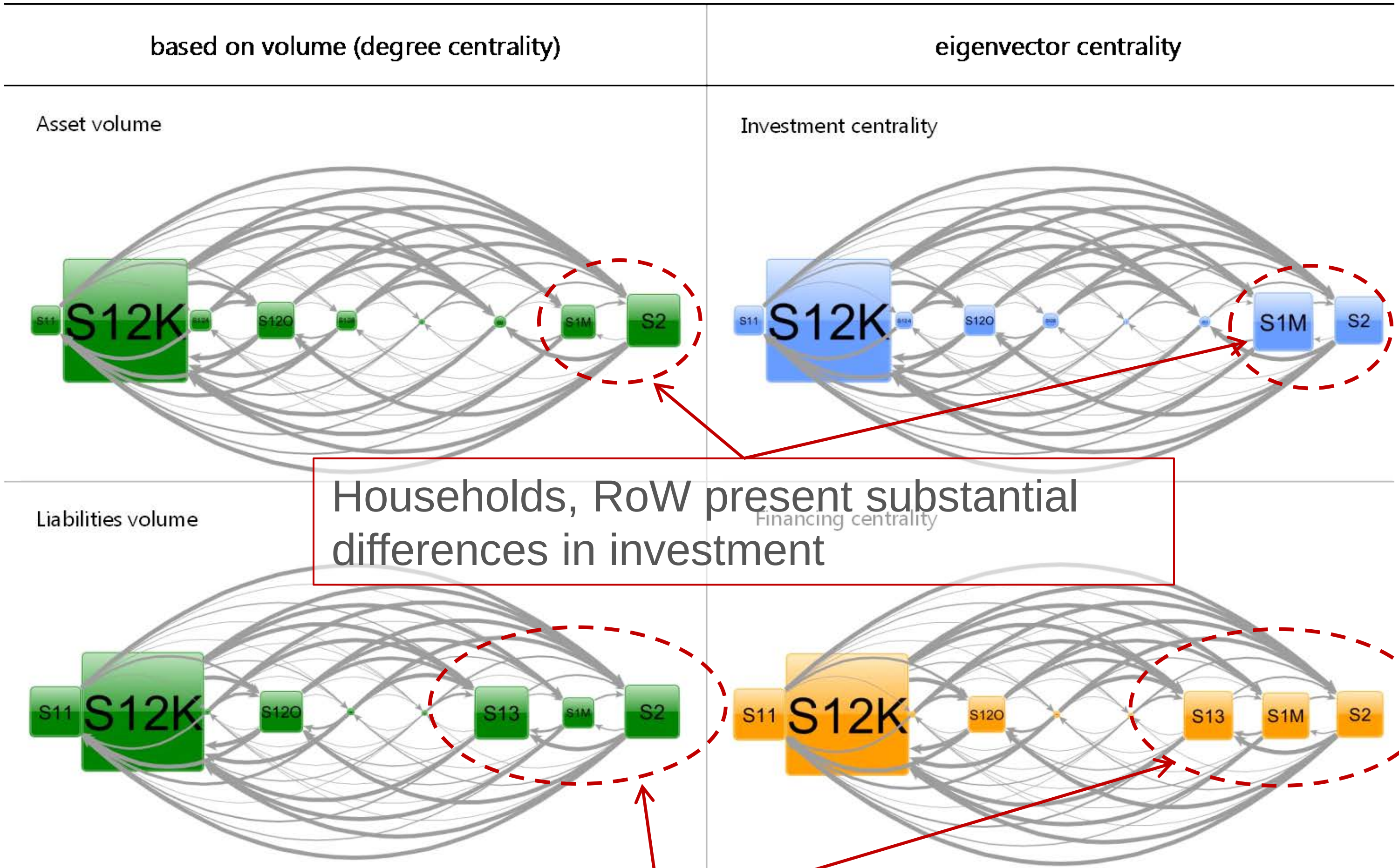
for debt in the euro area …



Notes:

- *Units: components of normalized Perron eigenvectors; network of debt (debt securities, loans and deposits); 16Q4*
- *S11: non-financial corporations; S12K: MFIs (S121+S122+S123); S124: investment funds; S12O: OFIs (S125+S126+S127); S128: insurance corporations; S129: pension funds; S13: general government; S1M: households and NPISHs (S14+S15); S12: rest of the world*

Indirect links matter. Rankings in the euro area …

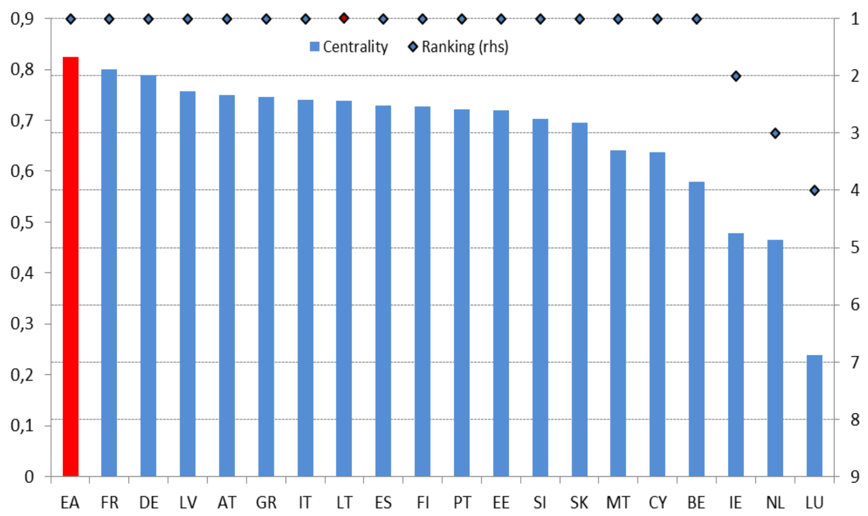


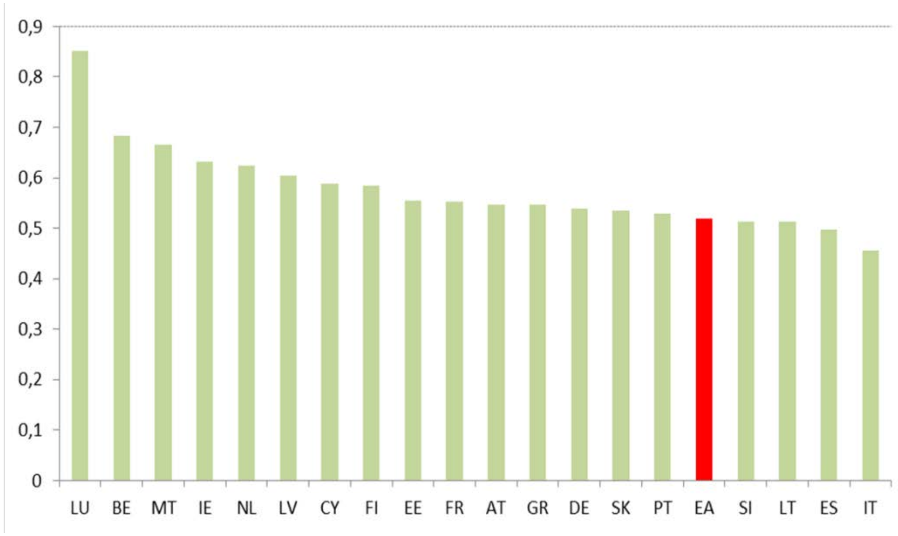
Households, government, RoW present substantial differences in financing

Notes:

- *S11: non-financial corporations; S12K: MFIs (S121+S122+S123); S124: investment funds; S12O: OFIs (S125+S126+S127); S128: insurance corporations; S129: pension funds; S13: general government; S1M: households and NPISHs (S14+S15); S12: rest of the world*

Eigenvector centrality in the euro area. Debt network. MFIs. Investment





Units : left panel:, S12K component in normalized Perron's eigenvector and ranking position; right panel: Perron's eigenvalue; ***debt network/matrix*** *; 16Q4*

- ✓ High centrality of MFIs in all euro area countries: high persistence of n-order propagation effects in quantity shocks
- ✓ Exceptions are IE, NL, LU, with lower centrality: OFIs to present high n-order effects, particularly persistent in LU (high Perron eigenvalue)
- ✓ **IT**, ES : MFI central and relatively higher n-order propagation effects, but less persistent

Summary and conclusions

- ✓ A **Leontief framework and eigenbase representation** applied to who-to-whom matrices allow for analyzing second and higher order indirect, propagation effects of sectors' asset acquisitions and disposals
- ✓ The eigenbase representation also enables **reduced dimensionality**: propagation n-order effects can be approximated as a linear combination or 2, 3 vectors
- ✓ Perron eigenvector, a standard measure of **network centrality, is dominating the propagation effects.** For large, sparse matrices, the Perron eigenvector is sufficient to characterize propagation
- ✓ A first examination of propagation based on eigenvector centrality for MFIs in the euro area (debt network) show that they are affected by large indirect effects: **e.g. disposals of assets by MFI result in less than proportional decreases in leverage**

References

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Thank you for your attention !

Reserve slides

Perturbing diffusion

$$\Delta t = \Delta g + A\Delta g + A^2\Delta g + A^3\Delta g + \cdots + A^n\Delta g + \cdots$$

$$\Delta t = \Delta g + A_0\Delta g + A_1A_0\Delta g + A_2A_1A_0\Delta g + \cdots + A_nA_{n-1} \ldots A_0\Delta g + \ldots$$

$$A_n = \delta\,(A_{n-1})$$

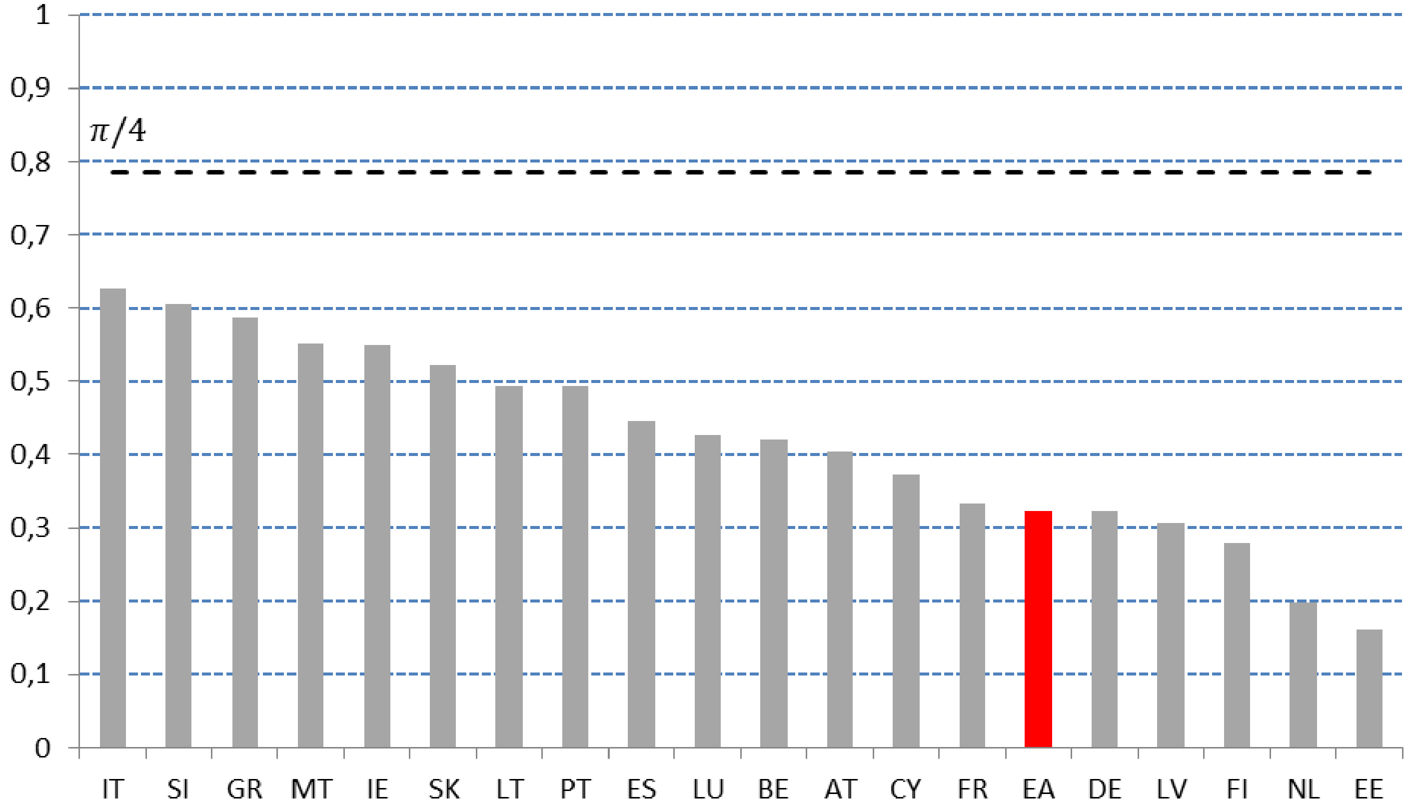
$$\delta\,(\rho) = \frac{w'\delta\,(A_{n-1})v}{w'v}$$

Left eigenvector

$$|\delta(\rho)| \leq \frac{1}{\cos\theta}\|\delta(A_{n-1})\|$$

Angle between right, left eigenvectors

Perron’s eigenvector angle



Note:

- *Units: angle between left and right eigenvectors of Perron’s eigenvalue (**debt network/matrix**); radians; 16Q4*