Analyzing Business and Financial Cycles Using Multi-Level Factor Models

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Paper's Contribution

- two estimation procedures for multi-level factor models are proposed
  - sequential least squares estimator
  - CCA
- two-level and three-level factor models are considered
- alternative estimation procedures for multi-level factor models are compared:
  - Two-stage PC estimators: bottom-up and top-down estimators
  - Two-step quasi ML estimator consider in Banbura et al. (2010)
- three empirical applications for the sequential least-squares estimator are presented

Theoretical Set-up

Baseline model is a two-level factor model:

- level-1 factors affect all economic sector
- level-2 factors affect only a specific economic sector

Feature of the model:

- lots of zero restrictions are imposed on factor loadings
- the number of factors grows with the number of sectors: the number of factors is allowed to grow without bounds as the number of sectors increases to infinity. when the number of sector is large the IC in Bai and NG (2002) cannot be used to consistently estimate the total number of factors.

Alternative Estimation techniques

- Two-step PC estimators
  - Top-down PC estimator

$$y_{r,it} = \gamma'_{r,i} G_t + e_{r,it}$$

$$e_{r,it} = \lambda'_{r,i} F_{r,t} + u_{r,it}$$

    problems arises because of the strong correlation among regional clusters of idiosyncratic components. Requirement is n_r fixed and $R \to \infty$

  - Bottom-up PC estimator PCA on region specific covariance matrix to extract

$$\tilde{F}_{r,t} = m_0 + m_r$$

    Second-step PCA on estimated factors $\tilde{F}$:

$$\tilde{G}_t$$

    problems arises in the second step because in general small number of regions.

- Two-step QMLE
  - Are the sector-specific factors allowed to be correlated one to the other?

Alternative Estimation techniques

- Iterative Principal Component Estimator of Wang (2010)
  1. Initial estimates for G and corresponding factor loadings Λ
  2. PCA on
  $$x_{r,t} = y_{r,t} - \widehat{\Lambda}_r \widehat{G}_t = \Gamma_r F_{r,t} + e_{r,t}$$
  to obtain $\widehat{\Gamma}_r$ and $\widehat{F}_{r,t}$
  3. PCA on
  $$z_{r,t} = y_{r,t} - \widehat{\Gamma}_r \widehat{F}_{r,t} = \Lambda G_t + u_t$$
  to obtain new $\widehat{G}$ and $\widehat{\Lambda}$
  4. iterate between 2 and 3 until convergence

Wang (2010) provides asymptotic theory for both the Factors (Global and Sector) and the loadings.

Breitung Eickmeier Sequential least square estimator

1. Initial estimates for G, $\hat{G}_0$.
2. Obtain an initial estimation for F, $\hat{F}_0$ by a PCA on

$$\hat{\Sigma}_r = \frac{1}{T}\sum_{t=1}^{T} Y_t M_{\hat{G}} Y_t^{'}$$

3. estimate the associated loading coefficients from $\sum_{r=1}^{N} n_r$ time series regressions

$$y_{r,it} = \lambda'_{r,i}\hat{G}_t + \gamma_{r,i}\hat{F}_{r,t} + u_{r,it} \quad (1)$$

to obtain the full set of factor loadings

$$\hat{\Gamma}_{(0)}* = (\hat{\Lambda}_0, \hat{\Gamma}_0)$$

4. a new estimator for the vector of factors is obtained from

$$\hat{F}^*_{t,1} = (\hat{\Gamma}^{*'}_{(0)}\hat{\Gamma}^*_{(0)})^{-1}\hat{\Gamma}^{*'}_{(0)} y_t$$

5. iterate between 3 and 4 until convergence

Why estimating (1) instead of applying the PCA?
How do you take into account the impact of the estimation errors of $\hat{G}_t$ and $\hat{F}_{r,t}$?
The efficiency of the estimates depends on T. T in general is not so large (In the second empirical application $T = 17$.)

Breitung-Eickmeier CCA estimator

Canonical Correlation Estimator

Consider the PCA estimation of the Global and Sector factors for two different sectors:

$$F^{+}_{r,t} = (G^{'}_{t}, F^{'}_{r,t}) \; F^{+}_{s,t} = (G^{'}_{t}, F^{'}_{s,t})$$

If G_t was observed, the CCA between $F^{+}_{r,t}$ and $F^{+}_{s,t}$ identifies the linear combination of $F^{+}_{r,t}$ that represent the Global Factors. However, as Breitung and Pigorsch (2013) show if $F^{+}_{r,t}$ is consistent estimator for the factor space $(G^{'}_{t}, F^{'}_{t})$, the linear combination $F^{+}_{r,t}$ converges in probability to HG_t.

- Very powerful tool
- when used as estimator (No iteration at all!!) small sample results very close to sequential least squares estimator
- Breitung and Pigorsch make use of this analysis to estimate the number of dynamic factors in a dynamic factor model. Maybe the same idea can be used here to estimate the number of common factors.

Monte Carlo

- Still in progress. The results of the simulations may change when more replications are added (especially for QMLE)
- add the results of Wang (2010) estimator for comparing the performance among the different estimators
- add more evaluation criteria to judge the goodness of Factor estimation (only average correlation)
  - true and estimated percentage of variance explained by the factors
  - report not only the mean but also the percentiles of the empirical distribution of the criteria over the replication
- impact of the estimation of the number of factors on the performance of the estimator
- what about the distribution of $\hat{F}$ and $\hat{G}$?
- why do you need to set the standard deviation of $F_r = 5$?