# HOW COSTLY ARE THE PUBLIC SECTOR INEFFICIENCIES? AN INTEGRATED FRAMEWORK FOR ITS ASSESSMENT

Jorge Onrubia-Fernández<sup>\*</sup> and A. Jesús Sánchez-Fuentes<sup>\*</sup>

This paper provides a theoretical framework which integrates the conventional methodology for measuring the productive efficiency and the monetary assessment of social welfare changes associated with public sector performance. Two equivalent measures of social welfare changes generated by an improvement (or worsening) in productive efficiency are deduced using duality theory. The first one is obtained from the cost function, while the second one arises directly from the production function. Moreover, the paper induces the application of the theoretical framework proposed to empirical analysis.

## 1 Introduction

Nowadays, an essential issue to be analyzed in depth is the relationship between the productive efficiency of public sector and the potential budgetary savings associated with its improvement. Especially for advanced economies in which the current crisis effects are affecting the public finances in a more evident way. Quantifying these budgetary savings strongly constitute an alternative fiscal policy tool which goes beyond the traditional view of a fiscal consolidation (cut spending or tax hikes). This measure is not only helpful for short-term consolidation but also it is required to guarantee a sound long-term growth path.

Since the late eighties, the measurement of productive efficiency has received an increasing interest within the public economics area. This trend is even more evident for some specific sectors typically provided by the public sector: health, education, etc.. This growing literature has mainly focused on developing quantitative methodologies (usually grouped into parametric and non-parametric methods) from which we may achieve empirical measures of (technical, allocative or overall) efficiency with which a number of units – assumed to be homogeneous – have produced the public good(s) and service(s). Thus, all these measures usually provide us one scenario to compare their performance.

Without doubt these contributions measuring the productivity of public services are very useful to improve the management of public resources. However, there is lack of literature connecting these results with the potential budgetary gains that may arise from a reduction of public sector inefficiency.

In this vein, the OECD (2011) has recently highlighted the transcendence of implementing reforms addressed to increase the efficiency of public spending, specially for governments that are currently facing outstanding budgetary imbalances. In particular, the OECD refers to the need to improve the productivity of the public spending on education and health. In the first case, it is estimated that the gradual adoption of best practices in primary and secondary education could save resources around 0.5 per cent of GDP (with country range from 0.2 to 1.2 per cent), without

<sup>\*</sup> Universidad Complutense de Madrid.

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Corresponding author: Jorge Onrubia-Fernández. Department of Public Finance and Tax Systems. Universidad Complutense de Madrid – Campus de Somosaguas, Edificio 6, Desp. 6 – E-28223 Madrid (Spain). Phone: +34-91-3942442. Fax: +34-91-3942431. E-mail: jorge.onrubia@ccee.ucm.es

compromising the current educational targets. In the case of health, the resources released by improvements in productive efficiency could be even higher, around 2 per cent of GDP (range by country, between 0.4 and 4.8 per cent).

Moreover, the monetary gains are enormous in terms of social welfare. In this respect, it is important to account not only budgetary savings but also the monetary gains in terms of income and wealth derived from consuming a better education and health. Furthermore, from the marginal cost of public funds perspective, we should also consider the reduction in deadweight losses caused by distortionary taxes which provide these resources released.

The aim of this paper is to provide a theoretical framework which allows consistently integrate the conventional methodology for measuring the productive efficiency and the monetary assessment of social welfare changes associated with the public sector performance, defined in the basis of the output of any public activity. In particular, we deduce two measures of social welfare changes generated by an improvement (or worsening) in productive efficiency associated with the procurement of a public good. The first measure is obtained from the cost function, or in other words, from the supply side, while the second one arises directly from the production function. According to duality theory, both measures are equivalent and deducted from the same set of information.

The rest of the paper is organized as follows. In the second section, we introduce our theoretical framework, upon the basis of the conventional measures of efficiency (Farrell's radial approach). In the third section, we present our integrated approach which combines different dimensions typically involved in policy-makers decisions (welfare changes, measures of inefficiencies, etc.). Finally, the fourth section concludes.

## 2 The model

#### 2.1 Recent concerns on Public Sector Efficiency (PSE)

The monitoring of public sector activity and the potential derivation of measures of the Public Sector Efficiency (PSE) clearly justify the increasing interest observed on analyses related to the Public Sector Performance (PSP, hereinafter). This section briefly discusses the recent evolution of literature focused on the relevant concept, the Public Sector Efficiency (PSE, hereinafter), which refers to the efficient allocation and production of the public good and services. The existing literature comprises alternative approaches to measure - and to evaluate- the PSP and, consequently, the PSE. A non exhaustive description of how this literature has evolved is next. Firstly, a growing number of studies (Afonso *et al.*, 2005; Borge *et al.* 2008; and Clements, 2002, among others) translated the traditional approach used to analyze the productive efficiency of firms to the case of public sector units (countries, municipalities, schools, hospitals, etc.) with the aim of obtaining empirical measures of the PSE for a set of units and rank them. Secondly, some studies (Borge *et al.* 2008, among others) have also explored the identification of determinants of these empirical measures. An alternative perspective is considered by other authors (see Afonso *et al.*, 2010; and Casiraghi *et al.*, 2009, among others) in order to include the distributional concerns traditionally linked to the public sector activity into the efficiency analysis.

All in all, it can be observed that some caveats are still present. First, most of these analyses have focused on the productive efficiency or technical efficiency ( $\psi$ ). Thus, they have leaven out of the analysis issues related to the allocative efficiency ( $\gamma$ ), a relevant component of the overall efficiency ( $\eta$ ). This latter measure is our main interest in this paper. Second, the distributional concerns has not been yet fully incorporated to the analysis, although it is a component mostly involved in policy-makers decisions.

Our paper aims to fulfill all these caveats by combining the elements presented; (i) empirical measures of efficiency, (ii) welfare impact and distributional concerns, (iii) a monetary valuation of inefficiencies measured.

## 2.2 The public sector

This section introduces the notation used in subsequent sections and models the Public Sector Performance according to a framework which could be adapted to very different analysis.

Our model can be briefly described as follows. The public sector produces a vector of goods and services  $X = (x^1, ..., x^H)$  which we consider excludable unlike pure public goods.<sup>1</sup> Each  $x^h$  is produced by a public agency with the corresponding production function for the case of single output, such that:

$$x^{h} = f(Y) \tag{1}$$

where  $Y = (y_1, ..., y_n)$  is a vector of *n* inputs including fixed capital required for the activity and  $f \in S = \{(Y, X) : Y \text{ can produce } X\}$  with S the set of technologies.

The unitary price for each of these *n* inputs are included in the vector  $W = (w_1, ..., w_n)$ . Consequently, the total cost of producing  $x^h(c^h)$  is defined as:

$$c^{h}(x^{h}) = \sum y_{i} w_{i} \tag{2}$$

Assuming H = 1, for the sake of clarity in the presentation, this theoretical framework allows us to introduce the notation used in posterior sections by defining formally all the standard concepts of efficiency – mentioned above – from the inputs-oriented perspective.<sup>2</sup> First, given the minimum quantity of inputs needed for producing the level of output  $X(Y^*)$ , technical efficiency ( $\psi$ ) is defined as the ratio between Y and  $Y^*$ , such that:

$$\psi = \frac{\|Y^*\|}{\|Y\|} \tag{3}$$

Second, given the combination of inputs producing X at the minimum cost  $(Y^{**})$ , the allocative efficiency  $(\gamma)$  is defined as the following ratio:

$$\gamma = \frac{\left\|Y^{**}\right\|}{\left\|Y^{*}\right\|} \tag{4}$$

Third, the overall efficiency can be defined as the product of expressions (3) and (4):

$$\eta = \frac{\left\|Y^{**}\right\|}{\left\|Y\right\|} \tag{5}$$

Finally, we derive the corresponding expression for  $\eta$  in terms of production costs:<sup>3</sup>

<sup>&</sup>lt;sup>1</sup> Rivalry and excludability are assumed to consistently reflect changes in the demand observed for each public good.

<sup>&</sup>lt;sup>2</sup> Analogous definitions can be found in the literature according to the output-oriented measures (see Coelli, 2005) for a detailed comparison of both approaches). There are no divergences in the analyses carried out from both perspectives. Therefore, one of them can be excluded.

<sup>&</sup>lt;sup>3</sup> See Coelli (2005) for a detailed description.

$$\eta = \frac{c^{**}}{c} \tag{6}$$

where c and  $c^{**}$  are, respectively, the actual level of production costs and the production costs corresponding to  $Y^{**}$ , the efficient combination of inputs when producing X, from the technical and the allocative perspective.

### **3** PSE analysis: an integrated approach

#### 3.1 The "expenditure-efficiency" function

The framework described above can be observed from a different perspective, facing the dual version of the same problem. Under these circumstances, the production of public good (x) and its level of output  $(\hat{x})$  may be explained by the expenditure function assumed in production  $(\mathcal{C}(\hat{x}))$ , and the degree of overall efficiency  $(\eta(\hat{x}))$ . In other words, an "expenditure-efficiency" function  $(\Phi)$  which is implicit in the conventional production function of productive factors once the vector of input prices (W) is given:

$$x = f(Y)|_{W} \to x = \phi(c, \eta)|_{W}$$
<sup>(7)</sup>

First of all, from (6), we can express the budgetary cost of producing a quantity of public good from the vector of inputs  $(Y^{**})$  and the degree of overall efficiency reached in the productive process,  $\eta$ :

$$c(\hat{x}) = \eta^{-1} \sum_{i=1}^{n} y_i^{**} w_i$$
(8)

Secondly, by applying the inverse function theorem to the optimal technology  $f_{**}$  (that determining the overall efficiency condition,  $Y^{**}$ ), the optimal quantities of each input  $(Y_i^{**})$  to produce  $\hat{x}$  are obtained. Note that these values only depend on factor prices and technological parameters of the production function:

$$y_i^{**} = f_{**}^{-1}(\hat{x}, W), i \in \{1, 2, \dots, n\}$$
(9)

Next, by combining (8) and (9), and solving for  $\hat{x}$  we derive the expenditure-efficiency function,  $\Phi$ , as proposed:

$$\hat{x} = \phi(c(\hat{x}), \eta)|_{W}$$
(10)

To translate this general notation to our model, c(.) would be the amount of resources allocated for the provision of the public good, and  $\eta$  the degree of efficiency with which the public agency produces this good.

### 3.2 Changes in the PSE, welfare impact and monetary valuation

This section presents an integrated approach which allows us to integrate the different dimensions involved in the evaluation of the Public Sector Performance; (i) changes in the degree of efficiency, (ii) welfare impacts linked to public policies, and (iii) monetary valuation of effects. The latter may facilitate the understanding of the inefficiency costs. Moreover, an improvement in the degree of efficiency will help to provide the same public good or service but with a lower level of spending.

For the sake of clarification, we detail our assumptions. First, in the following analysis it is assumed that any change in the degree of efficiency is exogenous. However, as Gibbons (2005) discusses, the existence of internal disturbances in the organizations (misscoordination, lack of incentives, etc.) may be the source of inefficiencies. Second, the social welfare generated by consumption of public good (*x*) is measured in monetary value in the conventional way, that is, by computing the area under the curve of demand for the good and substracting the cost of the inputs used in its production.<sup>4</sup> Additionally, to obtain accurate measurements of changes in consumer welfare we assume the demand functions involved to be compensated.<sup>5</sup> All in all, this theoretical framework contributes to measure welfare impacts linked to changes (improvements/worsening) in the degree of efficiency ( $\eta$ ) with which the public good is produced. This analysis translates Myrick-Freeman and Harrington (1990) framework to our model.

Therefore, using our "expenditure-efficiency" function defined in (10), we have the following social welfare function:

$$\Omega = \Omega(Y, W, \eta) = \int_0^x p(u) du - \sum_{i=1}^n y_i w_i$$
<sup>(11)</sup>

where  $p(\cdot)$  is the compensated demand function specified in its inverse form.

From equation (11) one can derive the first order conditions with respect to each inputs used  $(y_i)$ , such that:

$$\frac{\partial\Omega}{\partial y_i} = p(x)\frac{\partial x}{\partial y_i} - w_i = 0, i = 1,...,n$$
(12)

which determine the input demand functions  $\mathcal{Y}_i^{**}(w_i, \eta)$  for all *i*. It should be noted here that these values are precisely those corresponding to the optimal vector of production factors,  $\mathcal{Y}^{**}$ . It allows us to compute the optimal output level of public good for a given level of productive efficiency:

$$x^{**}(\eta) = \varphi(y_i^{**}(w_i, \eta), \eta)$$
(13)

Likewise, we could define the social welfare function associated with the production of this public good by considering the overall productive efficiency ( $\eta$ ) as a main argument:

$$\Omega(\eta) = \overline{\sigma}(y_i^{**}(w_i, \eta), \eta)$$
(14)

Applying the envelope theorem to the algebraic analysis described above, we obtain the following proposition:

 $<sup>^4</sup>$  Note that, as we did in the previous sections, hereinafter the notation is simplified to a single public good x to highlight the underlying intuitions.

<sup>&</sup>lt;sup>5</sup> See Willig (1976) for a discussion on the accurate measurement of these areas.

Proposition 1: The net welfare gain is the value of the marginal contribution, in monetary terms, brought about by a reduction (or increase) of overall inefficiency in the production function, so that:

$$\frac{\partial\Omega(\cdot,\eta)}{\partial\eta} = p(x^{**})\frac{\partial x^{**}(\cdot,\eta)}{\partial\eta} - \sum_{i=1}^{n} w_i \frac{\partial y_i^{**}(\cdot,\eta)}{\partial\eta} = p(x^{**})\varphi_n\left(y_i^{**}(w_i,\eta),\eta\right)$$
(15)

Some interesting implications are next. First, this result defines a relationship between the production function and the changes in welfare computed in the light of modification of the degree of efficiency. Second, it can be observed that, under full productivity of all inputs, the value generated by an infinitesimal improvement in productive efficiency is explained by the increase in the output generated. Third, from a different perspective, this gain could be seen as an

approximation ( $\varphi_n$ ) to the optimal technology ( $\mathcal{Y}_i^{**}$ ).

Next, the dual version of this result is achieved. To do this, from (13) one can define the costs functions related to this production as a function of the optimal level of public good, the vector of inputs associated with the optimal technology and the degree of productive efficiency reached, so that:

$$c = c(x^{**}(\eta), \eta) \tag{16}$$

Accordingly, we can rewrite (11) as:

$$\Omega = \Omega(x^{**}, \eta) = \int_0^{x^{**}} p(u) du - c(x^{**}, \eta)$$
(17)

From this perspective, the social welfare, considered as the difference between consumer's surplus and producer's quasi-rents, is maximized for the level of optimal output determined by the equality between price and marginal cost:

$$p(x^{**}) = \frac{\partial c(x^{**}, \eta)}{\partial x}$$
(18)

Again, combining (17) and (18), the following proposition emerges:

Proposition 2: The net welfare gain (loss) is the value of the marginal contribution, in monetary terms, brought about by the reduction (increase) of production cost as a consequence of an improvement (worsening) of the degree of overall inefficiency:

$$\frac{\partial \Omega(x^{**}, \eta)}{\partial \eta} = -\frac{\partial c(x^{**}, \eta)}{\partial \eta}$$
(19)

Proof Given (17), we compute the total derivative with respect to the degree of efficiency ( $\eta$ ). That is:

$$\frac{d\Omega(x^{**},\eta)}{d\eta} = \frac{\partial\Omega(x^{**},\eta)}{\partial x}\frac{\partial x^{**}}{\partial \eta} + \frac{\partial\Omega(x^{**},\eta)}{\partial \eta}$$
(20)

where:

$$\frac{\partial \Omega(x^{**}, \eta)}{\partial x} = p(x^{**}) - \frac{\partial c(x^{**}, \eta)}{\partial x}$$
(21)

and:

$$\frac{\partial\Omega(x^{**},\eta)}{\partial\eta} = p(x^{**})\frac{\partial x^{**}}{\partial\eta} - \left(\frac{\partial c(x^{**},\eta)}{\partial x}\frac{\partial x^{**}}{\partial\eta} + \frac{\partial c(x^{**},\eta)}{\partial\eta}\right)$$
(22)

Firstly, as a consequence of (18), we could identify  $\frac{d\Omega(x^{**},\eta)}{d\eta}$  and  $\frac{\partial\Omega(x^{**},\eta)}{\partial\eta}$ 

Next, from (22), grouping conveniently and using again (18), we obtain the proposition.

Corollary: An improvement in the degree of overall inefficiency always involves an increase in social welfare.

Again, some interesting conclusions can be derived. First, this result defines a relationship between the costs function and the changes in welfare computed when the degree of efficiency is modified. Second, these results can be understood as follows. The infinitesimal improvements in productive efficiency obtained lead to a reduction in the cost of production and, consequently, they are welfare enhancing. Third, combining Propositions 1 and 2 we obtain that the two welfare measures proposed must coincide due to the duality in the relationship between the production function and the cost function, which is underlying in (equality).

To conclude with this subsection, some interesting lessons could be extracted regarding the application of this approach to empirical analyses. First, the final results would lead to monetary valuations of the changes in the overall efficiency, which becomes a very interesting tool from the policy-makers perspective. Second, our approach integrates elements related to efficiency and others related to the equity, which allows to explore this classical trade-off (next subsection will explore this point in depth). Third, this approach requires an estimate of the production function and the cost function as well, which may limit its application when information on the production procedure and/or the production costs is limited.

#### 3.3 Distributional issues

In this subsection, we analyze how the welfare gains from increased efficiency affect consumers of public goods and public sector itself as the producer. In this respect, we first identify the efficiency gains effects on consumer's welfare. Let  $\Omega^{C}$  be the measure of consumer surplus used (usually equivalent or compensatory variation), so that:

$$\Omega^{C} = \int_{0}^{x^{**}} p(u) du - p(x^{**}) x^{**}(\eta)$$
(23)

Then, the consumer's marginal gain is:

$$\frac{\partial \Omega^{C}}{\partial \eta} = -\frac{\partial p(x^{**})}{\partial x} \frac{\partial x^{**}(\eta)}{\partial \eta} x(\eta)$$
(24)

Alternatively, if we consider equation (13):

$$\frac{\partial p(x^{**})}{\partial x} = \frac{\partial p}{\partial x^{**}} \frac{\partial x^{**}}{\partial \eta}$$
(25)

Now, from the producer's perspective, we repeat a similar strategy. First, we define the producer's surplus in terms of  $\eta$ :

$$\Omega^{S} = p(x^{**})x^{**}(\eta) - \sum_{i=1}^{n} y_{i}^{**} w_{i}$$
(26)

where  $\mathcal{Y}_i^{**}$  is determined by the *n* input demand functions,  $\mathcal{Y}_i^{**}(w_i, \eta)$ . Again, the producer's marginal gain can be obtained by differentiating the previous expression:

$$\frac{\partial \Omega^{s}}{\partial \eta} = -\frac{\partial c(x^{**}, \eta)}{\partial \eta} + \frac{\partial p(x^{**})}{\partial x} \frac{\partial x^{**}(\eta)}{\partial \eta} x(\eta)$$
(27)

In the light of the previous expressions, the following proposition can be demonstrated:

Proposition 3: An improvement in the degree of overall inefficiency always lead to an increase in consumer's welfare. By contrast, this welfare gain is not guaranteed in the case of producers of public goods.

Proof: On the one hand, for consumers, this proof can be reduced to check the signs of the expressions mentioned above. As  $\frac{\partial p(x^{**})}{\partial x} \leq 0$  and  $x(\eta) > 0$ , depending on the sign of  $\frac{\partial x^{**}(\eta)}{\partial x}$ 

 $\partial \eta$  the consumer's net welfare gain will be positive or negative. The optimal vector of inputs (from the technological and the minimization of costs' perspective) is taken as given in (13). As a consequence, a reduction of inefficiency may, in principle, lead to a decreased level of output – in equilibrium. To clarify this latter statement, we differentiate the first order conditions mentioned above, in equation (18), to achieve the following expression:

$$\frac{\partial p(x^{**})}{\partial x}\frac{\partial x^{**}(\eta)}{\partial \eta} = \frac{\partial^2 c(x^{**},\eta)}{\partial x^2}\frac{\partial x^{**}(\eta)}{\partial \eta} + \frac{\partial^2 c(x^{**},\eta)}{\partial x\partial \eta}$$
(28)

Grouping conveniently:

$$\frac{\partial x^{**}(\eta)}{\partial \eta} = \frac{\frac{\partial^2 c(x^{**},\eta)}{\partial x \partial \eta}}{\frac{\partial p(x^{**})}{\partial x} - \frac{\partial^2 c(x^{**},\eta)}{\partial x^2}}$$

On the one hand, looking at the denominator, it is straightforward to establish that  $\frac{\partial p(x^{**})}{\partial x} - \frac{\partial^2 c(x^{**}, \eta)}{\partial x^2} < 0$ . On the other hand, any improvement in  $\eta$  lead to reductions in costs. Thus,  $\frac{\partial^2 c(x^{**}, \eta)}{\partial x \partial \eta} < 0$  and, consequently,  $\frac{\partial x^{**}(\eta)}{\partial \eta}$  is always positive.

All in all, we have proved that consumer's welfare increases can be derived from the response in the production costs to an improvement in overall efficiency.

On the other hand, for producers, using the price-elasticity of public good demand, defined

$$\mathcal{E} = \frac{p(x^{**})}{x \frac{\partial p(x^{**})}{\partial x}}, \text{ which is negative by definition, we can prove that } \frac{\partial \Omega^S}{\partial \eta} \text{ will only be}$$

as

$$\varepsilon \frac{\partial x^{**}(\eta)}{\partial \eta} > \varepsilon \frac{\frac{\partial c(x^{**},\eta)}{\partial \eta}}{p}$$

negative if and only if

That is, the difference between the social welfare change and the variation in the consumer surplus.

From Proposition 3, the distribution of welfare gains derived from an improvement in the degree of efficiency may be established. Our results indicate that the determinants are the optimal output response to this increase and the price-elasticity of demand. In short, three different possibilities are achieved:

(i) 
$$0 < \frac{\partial x^{**}(\eta)}{\partial \eta} < \varepsilon \frac{\frac{\partial c(x^{**},\eta)}{\partial \eta}}{p} \Leftrightarrow \frac{\partial \Omega^{C}}{\partial \eta} > 0, \frac{\partial \Omega^{S}}{\partial \eta} > 0$$
 (29)

(ii) 
$$\varepsilon \frac{\frac{\partial c(x^{**},\eta)}{\partial \eta}}{p} < \frac{\partial x^{**}(\eta)}{\partial \eta} \Leftrightarrow \frac{\partial \Omega^{C}}{\partial \eta} > 0, \frac{\partial \Omega^{S}}{\partial \eta} < 0$$
 (30)

In order to show a different perspective of the conclusions described so far, we consider now an example to illustrate (and reinforce) the underlying intuitions. Moreover, some implications for the empirical application of this approach are discussed.

We consider a scenario in which the overall efficiency to produce the public good x improves between two moments in time, from  $\eta_0$  to  $\eta_1$ . To quantify the value of social welfare generated by the change in the degree of efficiency, we may choose to integrate, alternatively, one of the two welfare change measures presented in Propositions 1 and 2, respectively, and use  $[\eta_0, \eta_1]$  as integration interval:

$$\Delta\Omega = \int_{\eta_0}^{\eta_1} p(x^{**}) \varphi_n(y_i^{**}(w_i,\eta),\eta) = -\int_{\eta_0}^{\eta_1} c(x^{**},\eta)$$
(31)

From the empirical point of view, the direct quantification of  $\Delta\Omega$  from any of the two alternatives shown in (31) requires to determine the changes in the equilibrium output and in the optimal combination of inputs caused by the change in the degree of productive efficiency. This informational requirement should be added to those previously mentioned when estimating the production and/or cost function.

On the contrary, this computation may be simplified when information on production levels of public good before and after to the change analysed is available. To do this, using (11), we simply need to calculate the difference between initial and final social welfare values:

$$\Delta\Omega = \int_0^{x_1} p(u) du - c(x_1, \eta_1) - \int_0^{x_0} p(u) du + c(x_0, \eta_0)$$
(32)

By using this quantification, it can be observed how the potential welfare gains resulting from improved efficiency come from the displacement of the supply curve (as there is a reduction in the cost function). In other words, marginal cost of producing public good goes from  $\partial x$ 

 $\partial c(x,\eta_1)$ 

to  $\partial x$ 

Following to Myrick-Freeman and Harrington (1990), we can obtain an alternative expression for (32) by incorporating the change experienced by the cost function.

To do this, we use the line integral of its gradient along any path between  $(x_0, \eta_0)$  and  $(x_1, \eta_1)$ , and integrate along the line connecting them, such that:<sup>6</sup>

$$\Delta\Omega = \int_{x_0}^{x_1} p(u) du - \int_{\eta_0}^{\eta_1} \frac{\partial c(x_0, \eta)}{\partial \eta} d\eta - \int_{x_0}^{x_1} \frac{\partial c(x_1, \eta)}{\partial x} dx$$
(33)

Figure 1 shows the net social welfare gain expressed in (33) (the shaded area marked  $\Delta\Omega$ ). For the sake of simplicity, we assume linearity for all the curves involved; both compensated public good demand, and marginal cost functions (pre- and post-).

According to the analysis presented above, we could additionally define welfare changes experienced by consumers and the public sector as public good supplier. On the one hand, consumers enhance their welfare by increasing the area under the compensated demand curve, as a consequence of the equilibrium price decrease, from  $p_0$  to  $p_1$ .

Net Social Welfare Gain

 $\partial c(x,\eta_0)$ 

Figure 1



Figure 2 shows the consumers' welfare gain, which is represented by the total upper shaded area. On the other hand, the net change in producer's welfare results from compensating for the decrease in their initial surplus due to the lower resulting price (the patterned upper shaded area) with the new surplus caused by the reduction of costs charted in the new marginal cost function (the lower shaded area marked  $\Delta \Omega^{S}$ ).

As a consequence, combining this graphical evidence with propositions presented above, we conclude that:

<sup>&</sup>lt;sup>6</sup> See Myrick-Freeman and Harrington (1990) for further details on the underlying method, which is out of the scope of this paper.

i) for any  $\eta > 0$ ,  $\Delta \Omega = \Delta \Omega^{C} + (\Delta \Omega^{S} - \nabla \Omega^{S}) > 0$ ;

ii) we have not any guarantee implying that  $(\Delta \Omega^S - \nabla \Omega^S) > 0$ .

### 4 Concluding remarks

In the light of the current economic situation, the near future points to intense (supra-/intra-) national social debates on the monitoring of public sector performance (health, education, etc.).

Particularly, advances economies are currently facing issues related to the reorganization of their welfare state. Within this framework, quantifying these budgetary savings strongly constitute an alternative fiscal policy tool which goes beyond the traditional view of a fiscal consolidation (cut spending or tax hikes). This measure is not only helpful for short-term consolidation but also it is required to guarantee a sound long-term growth path.



In this respect, important policy implications are derived from our results. First, this paper has presented an integrated approach which combines different dimensions involved in the usual policy-makers decisions (efficiency in the production of the public good, welfare impacts and monetary valuation). This proposal satisfies additional features in comparison to the usual methodologies extensively used so far. Mainly, our approach would allow to translate measures of (in)efficiencies into to a monetary value. Second, our proposal may be adapted to be used within a wide variety of empirical

applications monitoring and/or evaluating the public sector performance. In this respect, we have identified the information requirements. Finally, we have derived some analytical results which help to understand the underlying intuitions and their linkages.

Finally, this paper links and integrates two different fields growing in parallel so far. On the one hand, empirical analyses monitoring the public sector performance from the production side and, on the other hand, studies analyzing the welfare implications of public policy-makers. For instance, this approach may provide guidance to the design of fiscal consolidation programs, so that they are compatible with a more efficient use of public resources.

#### REFERENCES

- Afonso, A. and C. Scaglioni (2005), "Public Services Efficiency Provision in Italian Regions: A Non-parametric Analysis", Technical University of Lisbon, Department of Economics at the School of Economics and Management (ISEG), Working Paper, No. 2005/02.
- Afonso, A., L. Schuknecht and V. Tanzi (2005), "Public Sector Efficiency: An International Comparison", *Public Choice*, Vol. 123, No. 3, pp. 321-47.
  - (2010), "Income Distribution Determinants and Public Spending Efficiency", *Journal of Economic Inequality*, Vol. 8, No. 3, pp. 367-89.
- Borge, L.E., F. Torberg and P. Tovmo (2008), "Public Sector Efficiency: The Role of Political and Budgetary Institutions, Fiscal Capacity, and Democratic Participation", *Public Choice*, Vol. 136, pp. 475-95.
- Casiraghi, M., R. Giordano and P. Tommasino (2009), "Behind Public Sector Efficiency: The Role of Culture and Institutions" in S. Barrios, L. Pench and A. Schaechter (eds.), *The Quality of Public Finances and Economic Growth*, European Commission, Occasional Paper, No. 45, March.
- Charnes, A., W.W. Cooper and E. Rhodes (1978), "Measuring the Efficiency of Decision Making Units", *European Journal of Operational Research*, Vol. 2, No. 6, pp. 429-44.
- Clements, B. (2002), "How Efficient is Education Spending in Europe?", *European Review of Economics and Finance*, Vol. 1, No. 1, pp. 3-26.
- Coelli, T.J., D.S. Prasada Rao, C.J. O'Donnell and G.E. Battese (2005), An Introduction to Efficiency and Productivity Analysis, 2<sup>nd</sup> edition, Springer.
- Debreu, G. (1951), "The Coefficient of Resource Utilization", Econometrica, No. 19, pp. 273-92.
- Fakin, B. and A. de Crombrugghe (1997), "Fiscal Adjustment in Transition Economies: Social Transfers and the Efficiency of Public Spending: A Comparison with OECD Countries", The World Bank, Policy Research, Working Paper, No. 1803, Washington (D.C.).
- Farrell, M.J. (1957), "The Measurement of Productive Efficiency", Journal of the Royal Statistic Society, No. 120, pp. 253-81.
- Figlio, D.N. and L.W. Kenny (2009), "Public Sector Performance Measurement and Stakeholder Support", *Journal of Public Economics*, No. 93, pp. 1069-77.
- Gibbons, R. (2005), "Four Formal(izable) Theories of the Firm?", *Journal of Economic Behavior & Organization*, No. 58, pp. 200-45.
- Gupta, S. and M. Verhoeve (2001), "The Efficiency of Government Expenditure. Experiences from Africa", *Journal of Policy Modelling*, No. 23, pp. 433-67.
- Myrick Freeman III, A. and W. Harrington (1990), "Measuring Welfare Values of Productivity Changes", *Southern Economic Journal*, No. 56, pp. 892-904.
- OECD (2011), Economic Policy Reforms 2011. Going for Growth, OECD, Paris.
- Willig, R.D. (1976), "Consumer's Surplus Without Apology", American Economic Review, No. 66, pp. 589-97.