Structural Vector Autoregressions with Markov Switching

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Motivation

- Identifying shocks is a central problem in SVAR models.
- Previous proposals:
  - Restrictions on instantaneous effects of shocks (Sims, 1980).
  - Restrictions on long-run effects of shocks (Blanchard/Quah, 1989; King/Plosser/Stock/Watson, 1991).
  - Sign restrictions (Canova/DeNicoló, 2002; Uhlig, 2005).
  - Bayesian methods (Koop, 1992).
  - Using statistical data properties
    - Heteroskedasticity (Rigobon, 2003; Lanne/Lütkepohl, 2008)
    - Nonnormal residual distribution (Lanne/Lütkepohl, 2009)
- This paper:
  Use Markov regime switching

Overview

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1. Model setup
2. Estimation
3. Inflation, Unemployment, Interest Rate
4. Conclusions

Reduced form VAR model

Variables of interest: $y_t = (y_{1t}, \dots, y_{Kt})'$

$$y_t = Dd_t + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t$$

- d_t – deterministic term.
- $u_t \sim (0, \Sigma_u)$.

Structural form VAR model

AB-model à la Amisano & Giannini (1997):

$$Ay_t = D^{(s)}d_t + A_1^{(s)}y_{t-1} + \cdots + A_p^{(s)}y_{t-p} + B\varepsilon_t$$

- A typically has ones on its main diagonal.
- A or B may be identity matrix.
- $\varepsilon_t \sim (0, \Sigma_\varepsilon)$.
- Σ_ε diagonal.
- $\Sigma_u = A^{-1}B\Sigma_\varepsilon B'A^{-1\prime}$.

Markov Switching

Markov process

$$s_t \in \{1, \dots, M\} \ (t = 0, \pm 1, \pm 2, \dots)$$

Transition probabilities

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i), \quad i, j = 1, \dots, M.$$

Conditional distribution of u_t

$$u_t | s_t \sim \mathcal{N}(0, \Sigma_{s_t}).$$

Identification of Shocks

Assumption: $M = 2$

A matrix decomposition result

$$\Sigma_1,\ \Sigma_2 \text{ positive definite}$$
$$\Rightarrow \exists \text{ a } (K \times K) \text{ matrix } B \text{ and } \Lambda = \text{diag}(\lambda_1, \dots, \lambda_K)$$
$$\text{such that } \Sigma_1 = BB' \text{ and } \Sigma_2 = B\Lambda B'$$

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B is (locally) unique if the λ_j's are distinct and ordered in a specific way (e.g., smallest to largest)

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Identification assumption

Instantaneous effects of shocks are the same across all states

More than two states I

Assumption: $M > 2$

Matrix decomposition

$$\Sigma_1 = BB', \quad \Sigma_i = B\Lambda_i B', \quad i = 2, \dots, M$$

imposes restrictions on covariance matrices which can be tested.

Test for state invariant B

LR test has asymptotic χ^2-distribution with $\frac{1}{2}MK(K+1) - K^2 - (M-1)K$ degrees of freedom.

More than two states II

Uniqueness of B

B is (locally) unique if for each pair of equal diagonal elements, for example, in $\Lambda_2 = \text{diag}(\lambda_{21}, \dots, \lambda_{2K})$ there is a corresponding pair of distinct diagonal elements in one of the other $\Lambda_i = \text{diag}(\lambda_{i1}, \dots, \lambda_{iK})$, $i = 3, \dots, M$.

Alternative decomposition

$$\Sigma_i = A^{-1} \Lambda_i^* A^{-1\prime}, \quad i = 1, \dots, M$$

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ML estimation

Use EM algorithm to optimize (pseudo) log likelihood.

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Research question and data

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Has US monetary policy changed or just the volatility of shocks?
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Data and variables

quarterly US data

Sample period: $1953Q1 - 2001Q3$

Variables:

- π_t – inflation rate based on chain weighted GDP price index
- u_t – unemployment rate
- r_t – yield on three-months treasury bills

Primiceri's identifying restrictions

$$A = \begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix} \qquad \begin{bmatrix} \pi_t \\ u_t \\ r_t \end{bmatrix}$$

Table: Estimates of Structural Parameters of MS Models for $(\pi_t, u_t, r_t)'$ with Lag Order $p = 2$ and Intercept Term (Sample Period: $1953Q1 - 2001Q3$)

	unrestricted model		restricted model	
parameters	estimates	std.dev.	estimates	std.dev.
λ_1	2.708	0.790	3.989	1.007
λ_2	8.230	2.381	7.956	1.979
λ_3	16.57	3.889	12.87	2.382
$\log L_T$	-148.12		-156.35	

Note: Standard errors are obtained from the inverse Hessian of the log likelihood function.

Table: Wald Tests for Equality of λ_i's for Unrestricted Model from Table 1

H_0	test value	p-value
$\lambda_1 = \lambda_2$	4.85	0.028
$\lambda_1 = \lambda_3$	11.70	0.001
$\lambda_2 = \lambda_3$	2.83	0.092

Test of Primiceri's identifying restrictions

Estimated B matrix

$$\hat{B} = \begin{bmatrix} 0.193\,(0.024) & -0.105\,(0.026) & 0.008\,(0.029) \\ -0.104\,(0.027) & -0.101\,(0.025) & -0.066\,(0.026) \\ 0.022\,(0.049) & -0.085\,(0.086) & 0.286\,(0.031) \end{bmatrix}$$

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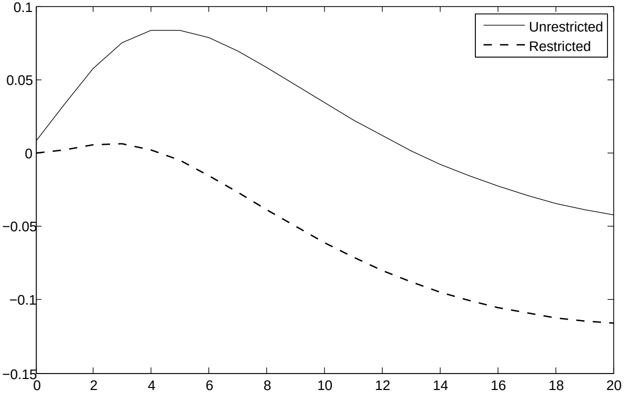
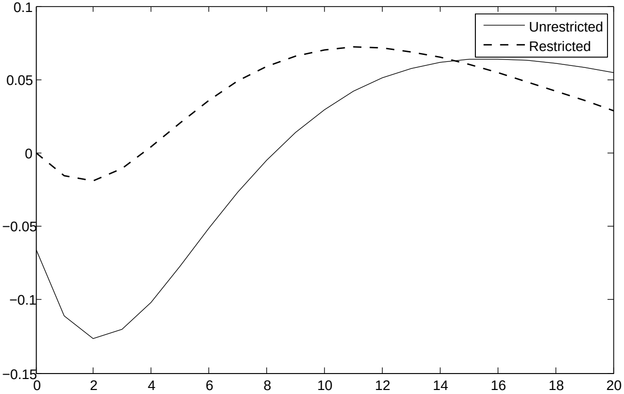
LR test of lower triangularity

16.47 (0.001)

Inflation responses to monetary policy shock



Unemployment responses to monetary policy shock

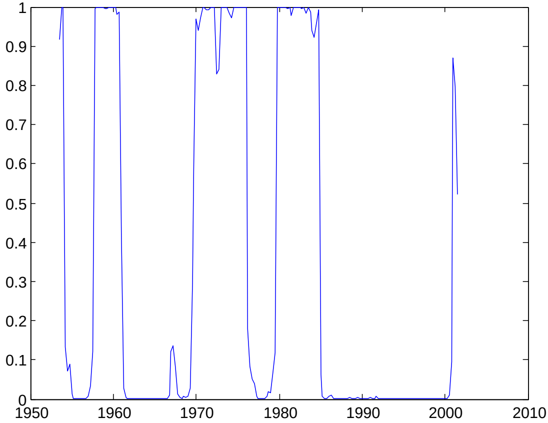


Interpretation of states

$$\hat{A} = \begin{bmatrix} 1 & 1.041 & 0.029 \\ -0.537 & 1 & -0.230 \\ 0.115 & 0.841 & 1 \end{bmatrix},$$

$$\hat{\Lambda}_1^* = \begin{bmatrix} 0.037 & 0 & 0 \\ 0 & 0.010 & 0 \\ 0 & 0 & 0.082 \end{bmatrix} \quad \text{and} \quad \hat{\Lambda}_2^* = \begin{bmatrix} 0.101 & 0 & 0 \\ 0 & 0.084 & 0 \\ 0 & 0 & 1.356 \end{bmatrix}.$$

Probabilities of State 2 ($\Pr(s_t = 2|Y_T)$) for the unrestricted model for $(\pi_t, u_t, r_t)'$.



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Conclusions

Identification assumptions:

- MS in the reduced form residuals.
- The impulse responses are invariant across regimes.

Advantages:

- Identifying assumptions in standard SVAR framework become overidentifying and can be tested.
- MS structure can be investigated with statistical methods.

Extensions

Remaining problems:

- Tests for number of states.
- Confidence intervals for impulse responses.
- Models with many variables and regimes are computationally difficult to handle because of difficult likelihood function.
- Theory for asymptotic inference for models with cointegrated variables is not complete.