Forecasting in the presence of recent and recurring structural change

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Abstract

Structural change is a major source of forecast failure. Immediately after a break, forecasting problems are particularly severe due to a lack of information about the new data generation process. Techniques exist for monitoring for structural change in real time, but the optimal post-break strategy is unexplored. We consider two approaches. First, monitoring for change and then combining forecasts from models that do and do not use data before the change; second, using methods robust to structural change. Robust methods include rolling regressions, forecast averaging over different windows and exponentially weighted moving average (EWMA) forecasting. We derive analytical results for the performance of robust methods relative to a full-sample recursive benchmark. For a model with stochastic breaks there is a ranking where the MSFE of rolling regression < forecast averaging < full sample regression < EWMA. Expressions are also derived for models involving deterministic breaks which may be more appropriate for structural change in small samples. We assess the methods with Monte Carlo experiments. Forecasting based on break monitoring improves performance at low cost; rolling regressions are effective for some cases including large breaks but may reduce performance in other cases; the EWMA is a still riskier strategy; and forecast averaging is a good compromise, improving performance in many cases. We apply the tests to a large number of UK and US macroeconomic series.

Key words: monitoring, structural change, forecast combination, robust forecasts.

JEL Classifications: C100, C590.

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1 Introduction

This paper addresses a pervasive problem that faces forecasters who need to generate projections in real time, faced by an environment that is subject to structural change.

Accounting for structural change is a critical empirical activity, for the obvious reason that if such changes are ignored then econometric relations are misspecified, from which numerous problems may flow. One area where it may be particularly important is forecasting. Clements and Hendry argue forcefully (in eg 1998a,b) that the main source of forecast error is structural change; Hendry (2000) argues that the dominant cause of these failures is the presence of deterministic shifts. Stock and Watson (1996) looked at many forecasting models of a large number of US time series, and found evidence for parameter instability in a significant proportion of the relations. Consequently there are large literatures on the identification of breaks, and methods that are robust to them.

But there is a relatively unexplored but critical question - how to forecast in the presence of recent and recurring breaks. Dealing with breaks in a forecast context has two important aspects which must be tackled differently, depending on whether the break was in the distant or immediate past. These relate first to the detection of the break, and second to the strategy adopted when adapting forecasts to the presence of the break. Both have received considerable attention for cases where the break occurred in the relatively distant past. The analysis of break detection has a long history - the seminal paper testing for a break at a known point was Chow (1960). Andrews (1993) introduced a methodology that allowed for unknown break-points: one influential paper is Bai and Perron (1998). But all these tests, by their nature, require some end-of-sample observations to perform the test. So timely real-time detection is simply impossible. And then when the break has been detected the question of how to modify the forecasting strategy arises. This in turn has been tackled by a number of authors. One significant recent contribution is by Pesaran and Timmermann (2007), who consider a number of alternative forecasting strategies in the presence of breaks. They conclude that forecast pooling using a variety of estimation windows provides a reasonably good and robust forecasting performance.

However, the joint problem of break detection and subsequent forecasting when the break occurs at the very end of the sample has received little attention. The mere definition of the end of the sample is itself potentially controversial. Perhaps the best way to define this is to note the circumstances under which the bulk of break detection procedures are not applicable. Between 5% and 15% of the sample size located at the end of the sample is usually necessarily assumed not to contain a break for the purposes of break detection. However, the real-time problem of break detection (where the hypothesis of interest is that there has been a recent break) has been tackled in the small literature on structural change 'monitoring'. In this situation, pioneered by Chu, Stinchcombe, and White (1996), break detection proceeds differently. As the forecaster monitors in real time for breaks, he or she carries out repeated tests. Clearly, this feature of the procedure implies the need for a different asymptotic framework. Since repeated tests have to be carried out, we need critical values that ensure rejection probabilities remain bounded by the significance level when breaks do not occur. This work has been refined by many others, including Zeileis, Leisch, Kleiber, and Hornik (2005), Leisch, Hornik, and Kuan (2000) and Kuan and Hornik (1995). Groen, Kapetanios, and Price (2008) extend the analysis to panel data sets.

So, oddly, this problem - which is the one actually facing practitioners - of how to adapt a forecasting strategy in the presence of recent breaks has not been discussed extensively in the literature.[1] Consequently it forms the main topic of the current paper. Two questions are crucial. The first is whether the forecaster attempts to detect and react to breaks, or adopts forecasting strategies that do not rely on break detection but are robust to them. The second relates to the environment faced by the forecaster: are breaks unique events, or by extension rare enough to be treated as unique for the purposes of forecasting or are they recurring and therefore need to be treated as such?

Following on from the work of Pesaran and Timmermann (2007), we consider the forecasting strategies they analyse in the context of recent breaks. Of all the strategies they consider, only forecast combination translates easily to the current framework. We provide and analyse one plausible translation which requires that breaks be detected in real time. Alternatively, we consider common forecasting strategies that do not detect breaks, but are robust to them.

One new strategy, we propose, involves monitoring and then combining full sample and post-break models. Clark and McCracken (2009a) write that 'it is possible that using a sample window based on break test estimates could yield better model

[1] We do not consider Bayesian approaches. One recent paper that in some ways is similar to our approach is Mahew and Gordon (2008). They use a Bayesian model combination approach that in principle allows them to exploit priors about when breaks may have occurred, and in which parameters.

estimates and forecasts. In practice, however, difficulties in identifying breaks and their timing may rule out such improvements (see, for example, the results in [Clark and McCracken (2009b)]'. One aspect of the current paper is to examine this in a systematic way.

The alternative robust models that we examine include the most popular robust methods for forecasting in the presence of past breaks: model averaging, rolling windows and exponentially weighted moving average (EWMA) models. In their discussion of some related empirical results, Clark and McCracken (2009a) write that in a forecast evaluation analysis, after 'aggregating across all models, horizons and variables being forecasted, it is clear that model averaging and Bayesian shrinkage methods consistently perform among the best methods. At the other extreme, the approaches of using a fixed rolling window of observations to estimate model parameters and discounted least squares estimation consistently rank among the worst.' By contrast, rolling regressions are advocated by Giacomini and White (2006). So another aspect of our current work is to re-examine the relative strengths of these methods in new data-sets, and also from theoretical and experimental perspectives, in the context of our novel setting of structural change occurring just prior to, or even during, the forecast period.

Our first contribution in this paper is to propose approaches for forecasting in the presence of recurring and recent breaks. The second is to provide some theoretical results for the performance of a number of robust forecasting strategies. Our third is to consider an extensive Monte Carlo study in which all the forecasting strategies are evaluated. We find that both of these sets of strategies are effective to some degree. But it emerges that robust strategies have, at least in the setup examined in our Monte Carlo study, the edge over forecast combination and monitoring (and that, as Clark and McCracken suggested, averaging may be broadly most useful). In many of the cases we consider they perform better, although monitoring does avoid bad forecast mistakes. A further advantage robust strategies enjoy is an ability to deal with recurring breaks. Our final contribution is to apply the methods we examined to a large number of US and UK macroeconomic time series, where we find results broadly consistent with the Monte Carlo study.

In the remainder of this paper, Section 2 provides the setup we explore and presents the forecasting strategies considered. Section 3 discusses some theoretical results. Section 4 reports on an extensive Monte Carlo study. Section 5 applies the methods to some macroeconomic time series. Finally, Section 6 concludes.

2 Forecasting strategies

We consider the multivariate model given by

$$y_{t+1} = \sum_{i=1}^{m} \mathcal{I}\left(\{T_{i-1} < t \leq T_i\}\right) \beta_i' x_t + u_{t+1}, \qquad t = 1, \ldots, T \tag{1}$$

$\mathcal{I}(A)$ is an indicator variable taking the value one if the event A occurs and zero otherwise. x_t is a $k \times 1$ vector of predetermined stochastic variables, β_i are $k \times 1$ vectors of parameters and u_t is a martingale difference sequence that is independent of x_t and has finite variance possibly changing at T_1. This model is a straightforward multiple structural break model that has been analysed extensively in the literature. The main point of departure from a standard analysis is to assume that some break dates are very close to the end of the sample at time T. In particular, we are interested in a setup where the forecaster is aware of the possibility of a break in real time and either actively looks for such a break or wishes to adopt a forecasting strategy that is robust to the occurrence of such a break. This is radically different to the standard situation of break detection because most break detection methods cannot detect breaks if $T_m/T \to 1$ as $T \to \infty$. It is well known that most tests for breaks assume that $T_m/T \to C$ $T \to \infty$, where $C \in (0,1)$.[2]

At this stage it is important to distinguish the cases where m is small and large. For illustrative purposes, it is convenient to restrict our small-m case to $m = 1$. If breaks are rare there is a realistic chance that their occurrence can be detected using some monitoring approach. In this case, a number of approaches to forecasting in the presence of breaks assumes that an estimate of the break date is available. We will consider these in detail below, but first we need to determine ways in which breaks can be detected when they occur at the very end of the sample. Thus the literature pioneered by Chu, Stinchcombe, and White (1996) is of use. Here it is assumed that the forecaster monitors in real time for breaks and therefore carries out repeated tests. This feature of the procedure implies the need for a different asymptotic framework since repeated tests have to be carried out using different critical values to ensure that break detection probabilities when breaks do not occur remain bounded by the significance level. This approach has been refined by many others, including Zeileis, Leisch, Kleiber, and Hornik (2005), Leisch, Hornik, and Kuan (2000), Kuan and

[2] One way to proceed is to disregard the model (1) and focus on a robust model such as a random walk or double-differenced model that may be biased but will be less affected by breaks, as Hendry (eg, 2000) has often suggested. We ignore this approach in the current paper, as we focus on the case where the forecaster has a specific view about the structure of the break.

Hornik (1995). Such an approach can provide an estimate of the break date which can be used as an input to a forecasting strategy.

If, on the other hand, breaks are frequent, monitoring for them becomes problematic. Monitoring inevitably entails delays in detection and if breaks are frequent they may not be easy to detect separately. In this framework forecasting strategies that account for breaks, not by monitoring them, but by being robust to their occurrence, seems to be the best course of action. Consequently, we now examine forecasting strategies both where the forecaster has estimated the break date(s) and where the forecaster has not, but wants to be robust to the presence of such breaks.

2.1 Forecasting strategies in the presence of a detected recent break

In this subsection, we discuss forecasting strategies that assume that recent breaks have been detected. It is important to note that detection has occurred using break monitoring and not some standard break detection procedure since, as we observed above, such procedures are invalid in this context. Our approach is related to the work of Pesaran and Timmermann (2007), who provide a detailed analysis of forecasting strategies when breaks occur in the more distant past. However, the problem for forecasting recent breaks is clearly much more difficult than that examined by Pesaran and Timmermann (2007) since post-break data are, by definition, in short supply. As a result the first four of the following strategies suggested by Pesaran and Timmermann (2007) are either not straightforwardly applicable or not attractive for obvious reasons.

1. Using model (1), estimated over post-window data.

2. Trading off the variance against the bias of the forecast by estimating the optimal size of the estimation window.

3. Estimating the optimal size of the estimation window using cross-validation.[3]

4. Combining forecasts from different estimation windows by using weights obtained through cross-validation as in 3.

5. Simple average forecast combination.

[3]Cross-validation holds back observations at the end of the sample for a post-sample exercise, in this case to establish a minimum MSFE estimation window.

For the final strategy, to use forecast combinations but not to estimate the weights, Pesaran and Timmermann (2007) suggest equal weights may work well.

Our proposed method builds on this last suggestion but is tailored to the specific problem. In particular, our strategy is specified as follows. The forecaster monitors for breaks. The forecasts are then produced using the model estimated over the whole sample as long as no breaks are detected.[4] Once the forecaster detects a break, it is assumed that the break has occurred at that point in time. Thus if $\hat{T}_1$ is the date the break is detected, it is also assumed to be the estimated date at which the break occurred. The fact that the monitoring procedure inevitably finds breaks with a delay, is not taken into account as it is very difficult to estimate this bias (see Groen, Kapetanios, and Price (2008) for evidence on its extent). Then, the forecaster makes two judgements, operationalised by the choice of two tuning parameters.

The first defines the time elapsed before the model can be reliably estimated post-break. This parameter is referred to as $\underline{\omega}$ in Pesaran and Timmermann (2007) and we retain this notation.

The second parameter is a window size that the forecaster deems acceptable for the post-break model to be the sole model used for future forecasting. We denote it by $\overline{f}$. That is then chosen to be the period over which the forecasts of the post-break and the no-break (or full-sample, ignoring the break) models will be combined. In other words, forecasts will be combined for the period $\hat{T}_1 + \underline{\omega}$ to $\hat{T}_1 + \underline{\omega} + \overline{f}$. We are assuming that there is a single break and that this is known to the forecaster. The forecasts after $\hat{T}_1 + \underline{\omega} + \overline{f}$ will therefore arise only from the post-break model.

The question of how the forecasts from the no-break (ie, forecasts using all currently available data and ignoring the break) and post-break (using only post-break data) model are combined has next to be addressed. Intuitively, the post-break model should receive increasingly more weight as new data come along. We have specified that the no-break model will be the sole model used prior to $\hat{T}_1 + \underline{\omega}$ and the post-break model will be the sole model used after $\hat{T}_1 + \underline{\omega} + \overline{f}$. A simple weighting scheme consistent with this choice is one where the weight for the post-break model increases linearly from zero prior to $\hat{T}_1 + \underline{\omega}$ to unity at $\hat{T}_1 + \underline{\omega} + \overline{f}$. More formally, the weight for the post-break model at time $\hat{T}_1 + \underline{\omega} + j - 1$ is $j/\left(\overline{f} + 2\right)$ whereas the weight for the no-break model is $1 - j/\left(\overline{f} + 2\right)$, where $j = 1, \ldots, \overline{f} + 1$. We consider this as our

[4]Thus we make the reasonable assumption that at the start of the monitoring period, the forecaster has considered the possibility of past breaks which have been accommodated by some unspecified method, if found present. We formally accommodate this in the Monte Carlo design by assuming there is at most one break, and that the forecaster knows this.

preferred forecasting strategy under the assumption that an estimate of the timing of the recent break is available, and is the one we focus on for our Monte Carlo work.

As stated above, in this exercise we assume that the forecaster knows that there is a single break. Obviously, in practice there would be a need for the forecaster to accommodate the possibility that further breaks occur. One solution would be to start monitoring for a new break as soon as the previous break has been detected by using only the post-break model. Then, the monitoring occurs at the same time as the forecast combining. We expect that the most relevant scenario is one where the forecaster stops combining forecasts before a new break is detected.[5]

Another possibility relates to the possibility of modifying the trade-off approach of Pesaran and Timmermann (2007) to obtain some estimate of the optimal delay one should apply before using the post-break model. This is problematic because no post-break data are available to be used to estimate post-break model parameters. Nevertheless, a grid of post-break model parameters could be used to produce a grid of possible optimal delays. This different delay parameters lead to different models whose forecasts can then be combined. But exploratory work indicated that this strategy was less effective than increasing the weight on the post-break model in the way described above.

2.2 Forecasting strategies that are robust to the presence of a recent break

The procedures involved in monitoring for breaks - as well as those involved in detecting distant past breaks - are not without problems. Unless the breaks are sufficiently large, they are difficult to detect or monitor for. Further, the estimators for the timing of the breaks have problems of their own. Evidence relating to this issue, at least, for monitoring, may be found in Groen, Kapetanios, and Price (2008). Therefore, it is perfectly reasonable to argue that one should not aim at detecting breaks; nevertheless, if the ultimate purpose is to provide good forecasts, then a forecasting strategy robust to the presence of recent breaks should be adopted. Such strategies should inevitably discount past data. There are a number of ideas in the literature about how to approach this.

One clearly relevant literature which has been growing rapidly over the recent

[5]It is reasonable to argue that if breaks occur more frequently than assumed here, the model itself must come under scrutiny. A clear path for addressing this is to endogenise the break process into the model following, eg, work by either Kapetanios and Tzavalis (2009) or Pesaran, Pettenuzzo, and Timmermann (2007). But an analysis of either course of action is beyond the scope of this paper.

past is on time varying coefficient models. Here the model (1) may be viewed as a measurement equation, augmented by a transition equation in terms of a vector of time varying parameters, β_t. Thus model (1) constitutes a state space model that can be analysed with widely available methods. In practice this might be regarded as a cumbersome apparatus. More pertinently, a number of specification issues are controversial and can affect the performance of the forecasts greatly. A more nuanced criticism can be centred on the fact that such a state space model is in fact a bilinear model which, rather than one of structural change, may represent a stationary process. Thus the time varying approach goes against the nature of the problem we try to address.

An alternative view of parameter instability is one where β_t is viewed as time dependent but deterministic. Such modelling attempts have a long pedigree in statistics starting with Priestley (1965). More recent examples include Dahlhaus (1996), Robinson (1989), Robinson (1991), Orbe, Ferreira, and Rodriguez-Poo (2005), Kapetanios (2006) and Kapetanios (2007). This line of work simply treats β_t as a deterministic process that can be estimated nonparametrically and considers standard nonparametric techniques for that such as kernel-based estimation. The easiest, most relevant and practical implementation of this idea is to estimate model (1) using a rolling window. The most important question then is to determine the size of the window. A number of considerations can be of use. A cross validation approach similar to that of Pesaran and Timmermann (2007) may be useful. Alternatively, this problem may be viewed as closely related to determining the bandwidth when estimating β_t by kernel methods. Since there are useful methods for this in nonparametric analysis, they can be used for this problem too.

We therefore consider rolling window estimation as an easy and powerful possibility for the problem we wish to address in this paper. There are two other widely available alternatives to rolling windows which share an interpretation as straightforward and easily implementable ways of modelling deterministically time varying coefficients. The first is based on estimating coefficients using exponentially weighted moving averages (EWMA). A detailed description may be found in Harvey (1989) but the idea is that, unlike rolling windows where only a subset of available observations receive a non-zero weight in estimation, all available observations receive some weight but older observations receive less weight. A parameter controls the rate of decline of weighting older observations, which plays a similar role to the rolling window size. A final alternative is to combine forecasts using different estimates of the coefficients where these estimates are obtained using all possible contiguous subsets of observations that include the latest available observation. This is advocated by Pesaran and Timmermann (2007) as a robust forecasting strategy when breaks occur in the past.

3 Some theoretical results

In this Section, we present some theoretical results for the robust forecasting strategies presented in the previous section, when multiple breaks occur. We concentrate on a simplified location model to make our analysis tractable. We consider two ways of modelling multiple structural breaks. The first is a novel one based partly on recent work by Koop and Potter (2007) and Kapetanios and Tzavalis (2009). It assumes that breaks are stochastic. The second is more traditional in assuming that breaks are exogenous deterministic shifts in parameters.

We start with the stochastic break model. Let the model be

$$y_t = \beta_t + \epsilon_t, \qquad t = 1, \ldots, T, \tag{2}$$

where

$$\beta_t = \sum_{i=1}^{t} \mathcal{I}\left(\nu_i = 1\right) u_i, \tag{3}$$

where ν_i is an i.i.d. sequence of Bernoulli random variables taking the value 1 with probability p and 0 otherwise. ϵ_t and u_i are also iid series independent of each other and ν_i with finite variance denoted by σ_ϵ^2 and σ_u^2 respectively. This is the simplest model that can accommodate multiple breaks. We are interested in the MSFE of a one step ahead forecast based on a model estimated over the whole period versus one that is estimated from a method that discounts early data. For simplicity at this stage we focus on a rolling regression. So we consider four alternative forecasts:

$$\hat{y}_{T+1|T} = \hat{\beta}_T, \qquad \text{where} \quad \hat{\beta}_T = \frac{\sum_{t=1}^{T} y_t}{T} \quad \text{(Full sample forecast)}\,, \tag{4}$$

$$\tilde{y}_{T+1|T} = \tilde{\beta}_T, \qquad \text{where} \quad \tilde{\beta}_T = \frac{\sum_{t=T-m+1}^{T} y_t}{m},\ m < T, \quad \text{(Rolling Forecast)}. \tag{5}$$

$$\bar{y}_{T+1|T} = \frac{1}{T}\sum_{i=1}^{T} \tilde{y}_{T+1|T}^{(i)}, \text{ (Forecast Averaging over Estimation periods)}$$

where we denote $\tilde{y}_{T+1|T}$ for a rolling window of size m by $\tilde{y}_{T+1|T}^{(m)}$, and finally

$$\breve{y}_{T+1|T} = \frac{\sum\limits_{t=1}^{T} \lambda\left(1-\lambda\right)^{T-t} y_t}{T}, \text{ (EWMA Forecast)}$$

for some $0 < \lambda < 1$. We wish to determine the mean square error of all these forecasts. under (2)-(3). We have the following theoretical results proven in the appendix:

Theorem 1 *Let the true model be given by* (2)-(3)*. Then,*

$$E\left(\hat{y}_{T+1|T} - y_{T+1}\right)^2 = \left(\tfrac{(T-1)(2T-1)}{6T} + 1\right) p\sigma_u^2 + \tfrac{(T+1)}{T}\sigma_\epsilon^2 = \frac{Tp\sigma_u^2}{3} + o(T)$$

Theorem 2 *Let the true model be given by* (2)-(3)*. Then,*

$$E\left(\tilde{y}_{T+1|T} - y_{T+1}\right)^2 = \left(\tfrac{(m-1)(2m-1)}{6m} + 1\right) p\sigma_u^2 + \tfrac{(m+1)}{m}\sigma_\epsilon^2 = \frac{mp\sigma_u^2}{3} + o(m)$$

Theorem 3 *Let the true model be given by* (2)-(3)*. Then,*

$$E\left(\bar{y}_{T+1|T} - y_{T+1}\right)^2 = 0.185T + o(T)$$

Theorem 4 *Let the true model be given by* (2)-(3)*. Then,*

$$E\left(\breve{y}_{T+1|T} - y_{T+1}\right)^2 = Tp\sigma_u^2 + o(T)$$

Obviously, given the underlying random walk nature of the stochastic breaks model, the MSFE for the full sample forecast is diverging at rate T. An obvious way this can be counteracted is to allow p to depend on T and specify it as $p_T = pT^{-1}$, thereby ensuring that breaks are rare enough not to induce random walk behaviour to the data. The specification of p does not affect the comparison of forecasts obtained via rolling or standard recursive regressions. In particular it is easy to see that for large T and m where $m/T \to 0$, we have that the leading term for recursive regressions is $T/3$ whereas for rolling regressions it is $m/3$ clearly implying that the recursive full sample regression has a larger unconditional MSFE. We next look at Pesaran's model averaging over all possible estimation periods. The result in Theorem 3 suggests that it has an MSFE of the same order but lower than the full sample forecast MSFE. This MSFE is higher than the MSFE of the rolling forecast. Finally, the EWMA forecast has the highest MSFE of all the other forecasts.

The alternative setup is the conventional approach where breaks are deterministic which may more accurately reflect small sample settings.

$$y_t = \begin{cases} \beta_1 + \epsilon_t & \text{if } \ t \le t_1 \\ \beta_2 + \epsilon_t & \text{if } \ t_1 < t \le t_2 \\ \quad\vdots & \quad\ \vdots \\ \beta_n + \epsilon_t & \text{if } \ t_{n-1} < t \le t_n \equiv T+1 \end{cases} \tag{6}$$

Define $t_i^* = t_i - t_{i-1}$ where $t_0 = 0$. Further, define $\beta_i^* = \beta_i - \beta_n$. Let $t_{n_m-1} < T - m < t_{n_m}$ for some $n_m \le n$. Also, define $\tilde{t}_{n_m} = t_{n_m} - T + m$, and $\tilde{t}_i = t_i$ for $i > n_m$. Then, it is straightforward to show that

$$E\left(\hat{y}_{T+1|T} - y_{T+1}\right)^2 = \left(\frac{\sum_{i=1}^{n-1} t_i^* \beta_i^*}{T}\right)^2 + \frac{(T+1)\,\sigma_\epsilon^2}{T} = B_1 + V_1$$

and

$$E\left(\tilde{y}_{T+1|T} - y_{T+1}\right)^2 = \left(\frac{\sum_{i=n_m}^{n-1} \tilde{t}_i \beta_i^*}{m}\right)^2 + \frac{(m+1)\,\sigma_\epsilon^2}{m} = B_2 + V_2$$

In this case it is clear that there is a tradeoff between the squared bias terms B_i $i = 1, 2$ and the variance terms V_i, $i = 1, 2$. Either method may dominate depending on the values of all parameters. Considering a simple case, where $n = n_m = 2$ and $t_1 = m = T/2$, we have that

$$E\left(\tilde{y}_{T+1|T} - y_{T+1}\right)^2 - E\left(\hat{y}_{T+1|T} - y_{T+1}\right)^2 < 0$$

if

$$\sigma_\epsilon^2 < \frac{m\left(\beta_2 - \beta_1\right)^2}{2}$$

which of course is satisfied for all $\beta_2 - \beta_1 \neq 0$ as long as $m \to \infty$ giving the standard result of work such as Pesaran (2007) in the simple case.

We next look at model averaging over all possible estimation periods. We have that

$$E\left(\bar{y}_{T+1|T} - y_{T+1}\right)^2 = \frac{1}{T^2}\sum_{i=1}^{T}\left\{\left(\frac{\sum_{j=n_i}^{n-1} \tilde{t}_j \bar{\beta}_j}{i}\right)^2 + \frac{(i+1)\,\sigma_\epsilon^2}{i}\right\}$$

Finally, we consider the EWMA estimator. This is given by

$$\breve{y}_{T+1|T} = \frac{\sum_{t=1}^{T} \lambda\left(1-\lambda\right)^{T-t} y_t}{T}$$

We wish to derive $E\left(\breve{y}_{T+1|T} - y_{T+1}\right)^2$. Again we do not consider stochastic breaks. It is straightforward to show that

$$E\left(\breve{y}_{T+1|T} - y_{T+1}\right)^2 = \left(\frac{\sum_{i=i}^{n}\sum_{j=t_{i-1}+1}^{t_i}\left(\lambda(1-\lambda)^{T-j}\beta_i - \beta_n\right)}{m}\right)^2 + \frac{\sigma_\epsilon^2 \sum_{t=1}^{T} \lambda(1-\lambda)^{T-t}}{T}$$

$$= \left(\frac{\sum_{i=i}^{n}\sum_{j=t_{i-1}+1}^{t_i}\left(\lambda(1-\lambda)^{T-j}\beta_i - \beta_n\right)}{m}\right)^2 - \frac{1}{T}\sigma_\epsilon^2 \frac{(-1)^{T+1} + (\lambda-1)^T}{(\lambda-1)^T}\left(-\lambda+1\right)^T$$

4 Monte Carlo analysis

In this section we consider the forecasting performance of the forecasting strategies discussed in Section 2. The Monte Carlo study contains three designs. The first simple design replicates one set of conditions used to derive the theoretical results of the previous section. The simple location with recurring breaks is used. For this set of experiments we have a clear expectation on the ranking of the various methods. However, the simple location model is not that interesting in practice. It was used in the theoretical section for its tractability. Our Monte Carlo study can be of use in providing indicative results for more complicated models. So, we consider an autoregressive model subject to structural change. For this model, we consider two different designs. In the first, we consider a single break which occurs during the forecast evaluation period. This case is designed to explore a situation where the forecaster believes that breaks are rare, and, so, in practise can be considered as unique events. As we argued in Section 2, it is reasonable then to monitor for breaks and react when breaks are detected by using our suggested forecast combination strategy. Of course, robust forecasting strategies are also applicable. The second allows frequent breaks to occur. Consequently, monitoring will not be a good strategy and only the robust forecasting strategies are considered.

4.1 Experiment design

For the simple location model we adopt the specification in (2)-(3). We specify that $p = 0.05, 0.1$ implying that breaks occur on average every 10 to 20 periods depending on these probabilities. We set $\epsilon_t \sim N(0,1)$. $u_t \sim i.i.d.U\left(u_l, u_u\right)$ where $\{u_l, u_u\} = \{-1,1\}, \{-0.9,0.9\}, \{-0.8,0.8\}, \{-0.7,0.7\}, \{-0.6,0.6\}$. The sample size is set to $T = 300$ and forecast evaluation starts at $t = 100$. 500 Monte Carlo replications are carried out. For this setup only robust forecasting strategies are considered. More details on the specification of these strategies are given below.

For the autoregressive experiment design, the generic model used is a dynamic autoregressive model

$$y_t = \alpha_t + \rho_t y_{t-1} + \epsilon_t, \qquad t = 1, \dots, T_0, \dots, T_1, \dots, T. \tag{7}$$

We begin with the specification of the single break case. Forecasting and break monitoring start at T_0 which we set to 100 and end at $T = 150$. 500 Monte Carlo replications are carried out. For the case of a unique break, the break occurs at T_1

which is set to 110. The break occurs either in the autoregressive parameter or the intercept. These parameters take the value ρ_1 or α_1 up to T_1 and ρ_2 or α_2 thereafter. More formally for the unique break case, the actual data generation process is

$$y_t = \begin{cases} \alpha_1 + \rho_1 y_{t-1} + \epsilon_t, & t = 1, \dots, T_1 - 1 \\ \alpha_2 + \rho_2 y_{t-1} + \epsilon_t, & t = T_1, \dots, T \end{cases} \tag{8}$$

If the intercept or the autoregressive parameter are assumed constant they take the values $\alpha_1 = \alpha_2 = 0$ and $\rho_1 = \rho_2 = 0$ respectively. We consider a large range of possible values for ρ_1, ρ_2, α_1 and α_2. So ρ_1 and ρ_2 take values from the set $\{-0.6, -0.4, -0.2, 0.2, 0.4, 0.6, 0.8\}$, while α_1 and α_2 take values from the set $\{-1.2, -0.4, 0.4, 0.8, 1.2, 1.6\}$. Monitoring is assumed to cease when a break is detected.[6] Forecasting (and our evaluation) stops at $T = 150$. Forecast evaluation is therefore over the period T_0 to T. The period over which model averaging occurs is $\hat{T}_1 + 5$ to $\hat{T}_1 + \overline{f}$ where $\hat{T}_1$ is the date at which the break is detected.[7] We also consider forecasting using robust strategies:

1. a rolling window where the size of the window is set to M,
2. forecast averaging of forecasts obtained using parameters estimated over all possible estimation windows,
3. exponential weighted moving average estimation of the parameters.

In particular the EWMA based least squares estimator of the regression $y_t = \beta' x_t + u_t$, $t = 1, \dots, T$, is $\hat{\beta}_{EWMA} = \left(\lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} x_t x_t'\right)^{-1} \lambda \sum_{t=1}^{T} (1-\lambda)^{T-t} x_t y_t$, where λ is a decay parameter. The choice of $0 < \lambda < 1$ is usually arbitrary, and to partially alleviate the effect of this we average forecasts obtained using different values of λ. Following suggestions in Harvey (1989) that λ should lie between 0.05 and 0.3, we consider $\lambda = 0.1, 0.2, 0.3$.

We now turn to the setup of the multiple break Monte Carlo study. In this case, we assume that breaks are stochastic. In particular, we specify that either the autoregressive parameter or the autoregressive model's intercept change as follows

$$\rho_t = \begin{cases} \rho_{t-1}, & \text{with probability} \quad 1-p \\ \eta_{\rho,t}, & \text{with probability} \quad p \end{cases}$$

$$\alpha_t = \begin{cases} \alpha_{t-1}, & \text{with probability} \quad 1-p \\ \eta_{\alpha,t}, & \text{with probability} \quad p \end{cases}$$

[6] Effectively, we are assuming the forecaster knows the structure of the model (in this as in other respects). In practice, it is plausible monitoring would recommence after some period.

[7] The choice of the delay in $\hat{T}_1 + 5$ is motivated on purely pragmatic grounds.

We specify that $p = 0.1, 0.05, 0.02, 0.01$ implying that breaks occur on average every 10 to 100 periods depending on these probabilities. $\eta_{i,t} \sim i.i.d.U\left(\eta_{il}, \eta_{iu}\right)$, $i = \rho, \alpha$, where

$$\{\eta_{\rho,l}, \eta_{\rho,u}\} = \{-0.8, 0.8\}, \{-0.6, 0.6\}, \{-0.4, 0.4\}, \{-0.2, 0.2\}$$

and

$$\{\eta_{\alpha,l}, \eta_{\alpha,u}\} = \{-2, 2\}, \{-1.6, 1.6\}, \{-1.2, 1.2\}, \{-0.8, 0.8\}, \{-0.4, 0.4\}.$$

When there are breaks in ρ, $\alpha = 0$, whereas, when there are breaks in α, $\rho = 0$. For this setup only robust forecasting strategies are considered, as is the case for the simple location model. The sample size is set to $T = 300$ and forecast evaluation starts at $t = 100$. Again, 500 Monte Carlo replications are carried out.

All forecasts are one-step ahead. The benchmark forecast disregards the possibility of a break and forecasts by estimating an AR(1) model over the whole available sample. We compare these in relative RMSFE (RRMSFE) terms to the benchmark forecasts.

4.2 Recurring breaks

We start our analysis of the results with the simple location model. We disregard the monitoring method here, which is obviously inappropriate where there are repeated breaks. Results are reported in Table 1. Given our analytical results, we have strong priors about the relative performance of the robust methods. In particular, we expect that rolling regressions with short windows will perform best, followed by rolling regressions with longer windows, forecast averaging and finally forecasting based on the full sample and EWMA. And this is roughly the ranking obtained from the Monte Carlo experiments, although there are configurations where any one of the methods outperforms the others.

Table 2 reports the results for the case where there are recurring breaks in persistence ρ, for constant α. For the largest shifts, a low order rolling window is the best performer. However, as the size of the shift declines, the small rolling window performance deteriorates, as the small data for estimation penalty dominates. The higher window rolling case is more robust, in the sense that it outperforms the full sample benchmark at all probabilities for large changes, and is close to the benchmark for small changes. In the best cases (large breaks), forecast averaging is comparable to the longer rolling window and generally no worse than the benchmark in the worst cases. So the cost in the worst cases is small, and this method could therefore be described as conservative. Moreover, the good performances are smaller than but comparable with the best cases. So forecast averaging emerges as a successful strategy. The EWMA never performs well, in all cases it performs worse than the alternative methods, and in many cases much worse.

Table 1: RRMSFE for forecasting strategies (Location Model); Recurring breaks

$p \backslash$ u_l / u_u	-1 / 1	-0.9 / 0.9	-0.8 / 0.8	-0.7 / 0.7	-0.6 / 0.6	-1 / 1	-0.9 / 0.9	-0.8 / 0.8	-0.7 / 0.7	-0.6 / 0.6
	Rolling Window ($M = 20$)					Rolling Window ($M = 60$)				
0.1	0.77	0.77	0.79	0.80	0.83	0.84	0.84	0.84	0.85	0.86
0.05	0.81	0.83	0.84	0.87	0.91	0.84	0.84	0.85	0.88	0.90
	Forecast Averaging					EWMA				
0.1	0.84	0.84	0.85	0.85	0.87	0.81	0.83	0.86	0.88	0.92
0.05	0.85	0.87	0.87	0.88	0.90	0.88	0.92	0.94	0.98	1.02

Table 2: RRMSFE for forecasting strategies (AR Model); Recurring breaks in ρ; $\alpha = 0$

$p \backslash$ $\eta_{\rho,l}$ / $\eta_{\rho,u}$	-0.8 / 0.8	-0.6 / 0.6	-0.4 / 0.4	-0.2 / 0.2	-0.8 / 0.8	-0.6 / 0.6	-0.4 / 0.4	-0.2 / 0.2
	Rolling Window ($M = 20$)				Rolling Window ($M = 60$)			
0.1	0.97	1.04	1.07	1.09	1.00	1.01	1.02	1.02
0.05	0.93	1.01	1.06	1.09	0.96	1.00	1.01	1.02
0.02	0.90	1.00	1.05	1.09	0.93	0.97	1.00	1.02
0.01	0.91	1.02	1.06	1.09	0.91	0.97	1.00	1.02
	Forecast Averaging				EWMA			
0.1	0.95	0.98	1.00	1.01	1.02	1.14	1.21	1.25
0.05	0.93	0.97	0.99	1.01	1.00	1.12	1.20	1.25
0.02	0.91	0.96	0.99	1.00	0.99	1.12	1.20	1.25
0.01	0.91	0.97	0.99	1.00	1.02	1.16	1.22	1.25

The results in Table 3 where the intercept shifts reveal less diversity. Overall, the best performer is again arguably the forecast average. In no cases is the EWMA best, and tends to be worst, often by wide margins.

We conclude that although no method is unambiguously superior, forecast averaging has the edge over rolling regressions, that for rolling regressions longer windows are more robust in the sense they avoid major errors, and that in many circumstances the EWMA is a poor forecast model.

Table 3: RRMSFE for forecasting strategies (AR Model); Recurring breaks in α; $\rho = 0$

$p\backslash$ $\eta_{\alpha,l}$ / $\eta_{\alpha,u}$	-2 / 2	-1.6 / 1.6	-1.2 / 1.2	-0.8 / 0.8	-0.4 / 0.4	-2 / 2	-1.6 / 1.6	-1.2 / 1.2	-0.8 / 0.8	-0.4 / 0.4
	Rolling Window ($M = 20$)					Rolling Window ($M = 60$)				
0.1	1.04	1.04	1.04	1.05	1.08	1.02	1.01	1.02	1.01	1.02
0.05	0.94	0.95	0.98	1.02	1.07	0.99	0.99	0.99	1.00	1.02
0.02	0.84	0.88	0.92	0.99	1.06	0.91	0.93	0.94	0.97	1.01
0.01	0.84	0.87	0.93	0.99	1.07	0.88	0.89	0.93	0.97	1.01
	Forecast Averaging					EWMA				
0.1	0.97	0.97	0.98	0.99	1.00	1.06	1.06	1.10	1.16	1.23
0.05	0.93	0.94	0.95	0.97	1.00	0.97	0.99	1.05	1.13	1.22
0.02	0.88	0.90	0.92	0.96	0.99	0.91	0.96	1.02	1.12	1.22
0.01	0.87	0.89	0.92	0.96	1.00	0.93	0.97	1.05	1.13	1.23

4.3 Single breaks

In our single break experiments we are able to consider our monitoring approach. Again, we consider breaks in either persistence or the mean. Table 4 reports the former for $\alpha = 0$. For monitoring, in some cases there are gains in forecast performance. However, in most cases the gains are more modest than with the other methods. But equally, there are no cases where monitoring leads to worse performance than the benchmark. The implication is that it is a conservative forecasting strategy, in the sense that it would tend to do somewhat better than the benchmark in some cases but will not lead to large forecast errors. In this setup, where there are gains, they tend to be greater for the shorter period. The rolling window methods perform much better for large breaks (again with the negative-positive asymmetry). Where they do well, a short post-break window improves the performance. But where they do worst, the opposite is the case. In general, a 60 period window offers a conservative strategy. However, the forecast averaging method outperforms the longer period rolling window in most cases and where it does worse than the benchmark, does not do so by a large margin. By contrast, although the EWMA does extremely well for some large changes, it does very badly for small changes or no structural change (along the diagonals). It is a risky strategy.

In Table 5 we consider a break in α. The results are qualitatively similar to those in Table 4.

4.4 Summary

Thus we can draw some tentative conclusions. A monitoring and combination strategy will improve forecast performance and is unlikely to lead to major forecast errors relative to the full sample benchmark: in that sense it is a conservative strategy. But forecast improvements are small. Where we are confident moderately large breaks are likely to occur or are occurring infrequently, rolling windows can be useful. But they may be susceptible to poor forecast performance, the more so the shorter the window. Longer windows flatten the performance profile at both ends. The EWMA can provide very large improvements for large shocks but in general is a risky strategy to adopt as it can lead to large errors. Overall, the forecast averaging method emerges as a good compromise between improved forecast performance in the face of large breaks and modest costs in other cases. And all this is broadly consistent with the analytical results.

5 Empirical application

In this section we examine how our methods would have fared when applied to a large range of UK and US quarterly data series.[8] In the UK, we use data on 94 series spanning 1977Q1 to 2008Q2, and examine two forecast evaluation sub-periods within this (1992Q1 to 1999Q4 and 200Q1 to 2008Q2). For the US, we have data on 97 series from 1960Q1 to 2008Q3, and examine three forecast evaluation sub-periods (1975Q1 to 1986Q2, 1986Q3 to 1997Q4, and 1998Q1 to 2008Q3). For each series, we compare RMSFEs to that from an AR(1) benchmark. The methods we report relate to those in the Monte Carlo study, and are monitoring using 40 and 60-period windows (M40 and M60), rolling-window forecasts using 40 and 60-period windows (R40 and R60), averaging across estimation periods (AV) and the exponentially weighted moving average (EWMA). The detailed results and variable descriptions are given in Appendices B and C for the UK and US respectively.

[8]We take no account or real-time data revisions.

Table 4: RRMSFE for alternative forecasting strategies; Single break in ρ; $\alpha = 0$

$\rho_1 \backslash \rho_2$	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
	Monitoring ($\overline{f} = 20$)								Monitoring ($\overline{f} = 60$)							
-0.6	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.92	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.95
-0.4	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.94	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
-0.2	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.95	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
0	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99
0.2	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
0.4	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Rolling Window ($M = 20$)								Rolling Window ($M = 60$)							
-0.6	1.09	1.06	1.00	0.94	0.84	0.74	0.61	0.48	1.01	1.01	0.99	0.96	0.93	0.88	0.82	0.74
-0.4	1.06	1.09	1.06	1.01	0.94	0.82	0.70	0.54	1.01	1.02	1.01	0.98	0.96	0.91	0.84	0.76
-0.2	0.99	1.07	1.09	1.06	1.01	0.93	0.81	0.64	0.98	1.01	1.02	1.01	0.99	0.95	0.89	0.79
0	0.90	1.01	1.07	1.09	1.08	1.02	0.90	0.74	0.93	0.98	1.01	1.02	1.01	0.98	0.94	0.84
0.2	0.80	0.91	1.01	1.07	1.09	1.08	1.00	0.87	0.88	0.94	0.99	1.01	1.02	1.01	0.98	0.90
0.4	0.70	0.84	0.94	1.02	1.08	1.09	1.08	0.97	0.85	0.91	0.95	0.99	1.01	1.02	1.01	0.95
0.6	0.61	0.74	0.84	0.94	1.02	1.08	1.11	1.07	0.81	0.88	0.92	0.96	0.99	1.01	1.02	1.00
0.8	0.53	0.66	0.76	0.86	0.95	1.01	1.08	1.12	0.81	0.86	0.91	0.94	0.96	0.99	1.01	1.02
	Forecast Averaging								EWMA							
-0.6	1.01	1.00	0.97	0.94	0.89	0.83	0.75	0.67	1.26	1.23	1.14	1.06	0.90	0.75	0.59	0.41
-0.4	1.00	1.01	1.00	0.97	0.94	0.87	0.80	0.70	1.22	1.26	1.23	1.14	1.05	0.87	0.71	0.51
-0.2	0.96	1.00	1.01	1.00	0.97	0.93	0.85	0.75	1.13	1.24	1.27	1.22	1.15	1.03	0.84	0.62
0	0.91	0.97	1.01	1.01	1.00	0.97	0.91	0.81	1.03	1.16	1.24	1.26	1.22	1.14	0.96	0.74
0.2	0.86	0.92	0.97	1.00	1.01	1.00	0.96	0.88	0.89	1.04	1.16	1.23	1.25	1.22	1.08	0.90
0.4	0.80	0.88	0.93	0.98	1.01	1.01	1.00	0.94	0.75	0.92	1.06	1.16	1.23	1.23	1.18	1.02
0.6	0.75	0.83	0.88	0.93	0.97	1.00	1.02	0.99	0.63	0.79	0.92	1.05	1.15	1.22	1.22	1.14
0.8	0.72	0.79	0.85	0.89	0.93	0.97	1.00	1.01	0.52	0.68	0.81	0.93	1.04	1.12	1.19	1.19

Table 5: RRMSFE for alternative forecasting strategies; Single break in α; $\rho = 0$

$\alpha_1 \backslash \alpha_2$	-1.2	-0.8	-0.4	0	0.4	0.8	1.2	1.6	-1.2	-0.8	-0.4	0	0.4	0.8	1.2	1.6
	Monitoring ($\overline{f} = 20$)								Monitoring ($\overline{f} = 60$)							
-1.2	1.00	1.00	0.99	0.95	0.90	0.86	0.83	0.82	1.00	1.00	0.99	0.97	0.95	0.93	0.92	0.91
-0.8	1.00	1.00	1.00	0.98	0.95	0.90	0.85	0.84	1.00	1.00	1.00	0.99	0.97	0.96	0.93	0.92
-0.4	0.98	1.00	1.00	1.00	0.99	0.95	0.90	0.86	0.99	1.00	1.00	1.00	0.99	0.97	0.95	0.93
0	0.95	0.99	1.00	1.00	1.00	0.99	0.95	0.91	0.98	0.99	1.00	1.00	1.00	0.99	0.98	0.95
0.4	0.90	0.95	0.99	1.00	1.00	1.00	0.98	0.95	0.95	0.98	0.99	1.00	1.00	1.00	0.99	0.98
0.8	0.86	0.90	0.95	0.98	1.00	1.00	1.00	0.99	0.93	0.95	0.98	0.99	1.00	1.00	1.00	0.99
1.2	0.83	0.87	0.90	0.95	0.98	1.00	1.00	1.00	0.92	0.93	0.95	0.97	0.99	1.00	1.00	1.00
1.6	0.81	0.83	0.86	0.89	0.94	0.99	1.00	1.00	0.91	0.92	0.93	0.95	0.97	0.99	1.00	1.00
	Rolling Window ($M = 20$)								Rolling Window ($M = 60$)							
-1.2	1.09	1.02	0.88	0.75	0.68	0.62	0.59	0.58	1.02	0.99	0.93	0.87	0.84	0.81	0.79	0.79
-0.8	1.02	1.09	1.03	0.88	0.75	0.67	0.61	0.59	0.99	1.01	0.99	0.93	0.88	0.83	0.81	0.79
-0.4	0.88	1.02	1.09	1.02	0.88	0.76	0.67	0.63	0.93	0.99	1.02	0.99	0.93	0.87	0.84	0.81
0	0.76	0.88	1.02	1.09	1.02	0.88	0.75	0.68	0.88	0.93	0.99	1.02	0.99	0.93	0.88	0.84
0.4	0.67	0.76	0.89	1.03	1.09	1.02	0.88	0.76	0.84	0.88	0.93	0.99	1.02	0.99	0.93	0.88
0.8	0.62	0.67	0.76	0.88	1.03	1.09	1.02	0.88	0.81	0.84	0.88	0.93	0.99	1.02	0.99	0.93
1.2	0.59	0.63	0.67	0.76	0.88	1.02	1.09	1.02	0.80	0.81	0.83	0.87	0.93	0.99	1.02	0.99
1.6	0.57	0.59	0.62	0.67	0.76	0.88	1.02	1.08	0.78	0.79	0.81	0.83	0.87	0.93	0.99	1.02
	Forecast Averaging								EWMA							
-1.2	1.01	0.98	0.90	0.83	0.79	0.76	0.73	0.73	1.25	1.16	0.97	0.81	0.71	0.63	0.59	0.58
-0.8	0.98	1.01	0.98	0.90	0.84	0.79	0.75	0.74	1.17	1.26	1.17	0.97	0.81	0.70	0.62	0.59
-0.4	0.90	0.98	1.01	0.98	0.90	0.84	0.79	0.76	0.97	1.16	1.26	1.16	0.97	0.81	0.69	0.64
0	0.84	0.91	0.98	1.01	0.98	0.90	0.84	0.79	0.80	0.98	1.17	1.25	1.16	0.97	0.80	0.71
0.4	0.79	0.84	0.91	0.98	1.01	0.98	0.90	0.84	0.70	0.81	0.98	1.17	1.26	1.17	0.97	0.82
0.8	0.76	0.79	0.84	0.90	0.98	1.01	0.97	0.90	0.64	0.70	0.82	0.97	1.16	1.26	1.16	0.98
1.2	0.74	0.76	0.79	0.84	0.90	0.98	1.01	0.98	0.60	0.64	0.69	0.82	0.98	1.16	1.25	1.16
1.6	0.73	0.73	0.75	0.79	0.83	0.91	0.98	1.01	0.57	0.59	0.62	0.70	0.81	0.98	1.17	1.26

5.1 UK results

An obvious prior question to ask is whether there is evidence of structural breaks in the series we examine. So we begin by performing Bai and Perron (1998) tests for structural breaks (mean shifts in an AR process), reported in Table 6. We identify 33 series containing breaks out of the total, so this suggests that structural change was indeed an important issue in the UK over this period. This test uses the full sample and this information would not be available in real time.

The full set of results is given in Appendix B for the two periods we examine. They are summarised in Table 7. We report the mean RRMSFE, the median (giving some indication of skewness), the minimum and maximum, the standard deviation and skewness. We also report the number of cases in which Diebold-Mariano tests reject equality of performance between the method and the full-sample null at 5% in favour of the robust method (DM(R)), while DM(FS) rejects against the robust method.

The theory for the stochastic case suggested that the EWMA RMSFE should exceed the average, which should exceed the rolling. On the mean and median RMSFE criteria in both periods the minima are delivered by the averaging method. The EWMA is not only the worst performer, but on average fails to beat the full sample AR, although in some cases it does extremely well (indicated by the very low values in the 'Minimum' rows). The monitoring method on average beats the benchmark, with a 40 period window outperforming 60 periods. The rolling window does better, however. Again, a shorter period is preferred. The rolling regressions also deliver low minima, especially for the shorter window. However, if the forecaster gives a high weight to avoiding extreme forecast errors, then using the monitoring method may be the best strategy. The maximum RRMSFE are close to unity in that case, and the variation in the RRMSFE also smallest. The EWMA, by contrast, is worst on this criterion. On the formal tests, in the first period the average ranks first, followed by the rolling and then the monitoring, while the EWMA is selected only slightly more often than would be expected by chance. The EWMA is significantly outperformed by the full-sample forecasts in more cases than it outperforms: by contrast, except for the rolling 60-period case there are only one or two rejections for the other methods. Similar results hold for the second period, although here the rolling 40-period is ranked first.

We conclude that over these periods averaging would have been a good strategy,

Table 6: Breaks identified: UK data (1992Q1 to 2008Q2)

Private sector output growth	1
GFK index score	1
Stock of net corporate debt	1
Nominal wages per worker	1
Sectoral M4	1
M4 liabilities to private non-financial corporations	1
Net lending to household sector	2
GDP	1
Gross National Income	1
Manufacturing	1
Manufacturing of textile & textile products	1
Manufacturing of leather & leather products	1
Manufacturing of wood & wood products	1
Manufacturing of non-metallic mineral products	1
Manufacturing of basic metals & fabricated prod	2
Manufacturing of electrical & optical equipment	1
Distribution, hotels & catering; repairs	1
Output Index: Total	1
Total adjustment to basic prices	1
GDP at market prices	1
Gross Value Added at factor cost	1
Money stock M4 (end period)	3
Notes & coins in circulation outside Bank of England	1
Total Government benefits paid to household sector	3
General Government: Final consumption expenditure	2
Household final consumption expenditure	2
Durable goods	1
Claimant count rate	1
Whole economy, inc bonus: % change 3 month average	1
Unemployed	3
Economically active	1
Total actual weekly hours worked	2
Imports: Total trade in goods and services excl MTIC fraud	1

Table 7: Summary Empirical Results for UK

	M40	M60	R40	R60	AV	EWMA
	First Period (1992Q1 - 1999Q4)					
Mean	0.972	0.980	0.925	0.959	0.903	1.029
Median	1.000	1.000	0.959	0.987	0.949	1.096
Minimum	0.619	0.737	0.006	0.005	0.047	0.005
Maximum	1.040	1.025	1.511	1.514	1.301	1.622
Std. Dev.	0.065	0.044	0.238	0.218	0.189	0.317
Skewness	-2.806	-2.819	-0.676	-0.636	-1.182	-0.525
DM(R)	12	12	16	16	22	8
DM(FS)	1	2	2	8	1	9
	Second Period (2000Q1 - 2008Q2)					
Mean	0.978	0.984	0.957	0.975	0.918	1.054
Median	1.000	1.000	0.974	0.984	0.951	1.056
Minimum	0.607	0.692	0.118	0.792	0.155	0.010
Maximum	1.050	1.031	1.525	1.235	1.265	2.228
Std. Dev.	0.058	0.043	0.170	0.085	0.157	0.301
Skewness	-3.783	-4.239	-0.725	0.383	-1.429	0.155
DM(R)	14	14	18	16	17	6
DM(FS)	2	2	4	4	1	8

Notes: The Table reports summary statistics on the set of Relative RMSFEs for alternative forecasting methods. M60: Monitoring using a 60-period window; M40: Monitoring using a 40-period window; R40: Rolling Forecast using a 40-period window; R60: Rolling Forecast using a 60-period window; AV: Averaging across estimation periods; EWMA: Exponentially Weighted Moving Average. DM(R) is the number of series for which the Diebold-Mariano test rejects in favour of the given robust method at the 5% significance level, while DM(FS) is the number of series for which the Diebold-Mariano test rejects against the given robust method at the 5% significance level.

although rolling regressions and monitoring would also have improved forecast performance. However, monitoring would have been a relatively conservative strategy, again in the sense that it would on average offer a small advantage over using the full sample and avoids making large forecast errors, while not offering large improvements in performance. This may well reflect the difficulty of detecting structural breaks.

5.2 US results

For the US, far fewer breaks are identified (Table 8). Consequently, there are fewer gains to using the methods (Table 9), although more so in the third period. The AV no longer emerges as the best average performer, but the EWMA remains both the worst on average and the most variable performer, with the best and worse individual forecasts in each period. The monitoring methods remain conservative in the sense we identified in the UK (small average gains and avoiding very poor performance). Based on the formal tests, there was little evidence that any model would have helped

Table 8: Breaks identified: US data

Industrial Production: Consumer Goods	1
Unemployment Rate: All Workers	1
Civilians Unemployed - 15 Weeks & Over	1
1-Year Treasury Constant Maturity Rate	1
Total Reserves of Depository Institutions	1
S&P 500 Finance Total return Index	1

forecast these series, with the exception of averaging in the third period, where there is some weak evidence in favour.

6 Conclusions

A common - perhaps the most common - source of forecast failure is the existence of structural breaks in the data generating process. It is often argued that such breaks are characterised by abrupt parameter shifts. In that context, it might be argued that a natural strategy for a forecaster operating in real time is to monitor for a break, and then to adopt a robust forecasting strategy. But there is a profound problem with this approach. The intrinsic difficulty is that by the nature of the exercise, there are few observations available either to estimate parameters or to evaluate forecasts. One method that has been advocated is to use combinations of differently specified models to forecast, and we assess this in a Monte Carlo experiment. Specifically, we use a simple combination of the no-break model using the full sample, and a model that uses only post-break data. As the sample in the latter case is inevitably short immediately post-break, we adopt a weighting scheme whereby for some period the weight on the post-break model linearly increases to 100%. But an equally widely advocated alternative to combination is to ignore the discreet nature of the hypothesised structural change and pursue some robust forecasting strategy that effectively allows for time variation in a simple but flexible manner. In general, robust methods weight recent observations more than distant. One simple approach is to use a rolling-window estimator. Others are to use combination methods or the exponentially weighted MA. This approach logically obviates the necessity to monitor for breaks. It has a cost - discarding data when there has not been a break - but also advantages: there is no delay in recognising a break has occurred, and it may be robust to varying forms of structural change.

We derive some theoretical results for the robust methods. In the stochastic break

Table 9: Summary Empirical Results for US

	M40	M60	R40	R60	AV	EWMA
	First Period (1975Q1 - 1986Q2)					
Mean	1.011	1.005	1.033	1.012	1.032	1.221
Median	1.000	1.000	1.033	1.007	1.034	1.212
Minimum	0.872	0.905	0.906	0.937	0.889	0.792
Maximum	1.171	1.106	1.135	1.355	1.291	2.594
Std. Dev.	0.032	0.020	0.042	0.042	0.057	0.212
Skewness	0.689	0.077	-0.266	5.515	0.455	2.695
DM(R)	2	2	0	0	0	0
DM(FS)	3	1	7	3	12	10
	Second Period (1986Q3 - 1997Q4)					
Mean	0.990	0.991	0.999	1.040	0.987	1.145
Median	1.000	1.000	0.999	1.029	1.008	1.161
Minimum	0.815	0.870	0.641	0.798	0.711	0.583
Maximum	1.092	1.054	1.284	1.414	1.113	1.732
Std. Dev.	0.032	0.024	0.100	0.101	0.070	0.227
Skewness	-2.027	-2.210	-0.400	0.978	-1.528	0.014
DM(R)	3	4	2	2	7	3
DM(FS)	4	2	1	14	2	3
	Third Period (1998Q1 - 2008Q3)					
Mean	0.998	0.991	1.002	0.977	0.952	1.307
Median	1.000	1.000	1.025	0.997	0.969	1.104
Minimum	0.842	0.877	0.311	0.324	0.513	0.333
Maximum	1.623	1.052	2.557	1.626	1.113	15.818
Std. Dev.	0.073	0.028	0.212	0.139	0.093	1.540
Skewness	6.153	-2.109	3.664	-0.259	-1.595	8.675
DM(R)	1	3	5	3	13	1
DM(FS)	0	0	6	5	0	6

Notes: The Table reports summary statistics on the set of Relative RMSFEs for alternative forecasting methods. M60: Monitoring using a 60-period window; M40: Monitoring using a 40-period window; R40: Rolling Forecast using a 40-period window; R60: Rolling Forecast using a 60-period window; AV: Averaging across estimation periods; EWMA: Exponentially Weighted Moving Average. DM(R) is the number of series for which the Diebold-Mariano test rejects in favour of the given robust method at the 5% significance level, while DM(FS) is the number of series for which the Diebold-Mariano test rejects against the given robust method at the 5% significance level.

case we examine, we obtain a clear ranking for the methods, with rolling regressions and forecast averaging being very good theoretical choices when robustness against structural change is important, and EWMA poor. We also derive expressions for MSFE in a deterministic case.

In our Monte Carlo exercise, the best methods can vary widely according to the particular break and parametrisation. Where we explore the monitoring method in the single break case we find the gains are small, although equally the costs (in poor forecasting cases) are also small. The results therefore make it hard to recommend a particular method, although the EWMA, consistent with the analytical results, should be avoided. Consequently, it is important to evaluate the methods using actual data. In an extensive exercise using US and UK data, we find that for both countries while the EWMA can occasionally do very well, on average it performs poorly and can perform very badly, consistent with the theoretical and Monte Carlo results. For the UK, where there are relatively many breaks identified in our full sample exercise, the best performing method is forecast averaging, although rolling regressions also beat the benchmark. Interestingly, for both countries monitoring brings only a small improvement in mean forecast performance, but is also a conservative strategy, in the sense that it can deliver improved forecast performance but is unlikely to lead to serious forecast failure relative to the benchmark. So while the Monte Carlo results suggest than monitoring offers little benefits, in practice it may be useful, and will avoid major forecast errors (relative to a full sample AR benchmark).

Appendix A Proofs

A.1 Proof of Theorem 1

We have that

$$\hat{y}_{T+1|T} - y_{T+1} = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{t}\mathcal{I}\left(\nu_i = 1\right)u_i + \frac{1}{T}\sum_{t=1}^{T}\epsilon_t -$$

$$\textstyle\sum_{i=1}^{T}\mathcal{I}\left(\nu_i = 1\right)u_i - \mathcal{I}\left(\nu_{T+1} = 1\right)u_{T+1} - \epsilon_{T+1} = \displaystyle\frac{1}{T}\sum_{t=1}^{T-1}\sum_{i=1}^{t}\mathcal{I}\left(\nu_i = 1\right)u_i -$$

$$\frac{1}{T}\left(T-1\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i = 1\right)u_i + \frac{1}{T}\sum_{t=1}^{T}\epsilon_t - \mathcal{I}\left(\nu_{T+1} = 1\right)u_{T+1} - \epsilon_{T+1}$$

Then,

$$\begin{aligned} E\left(\hat{y}_{T+1|T} - y_{T+1}\right)^2 &= \tfrac{(T-1)(2T-1)}{6T}p\sigma_u^2 + \tfrac{(T-1)^2}{T}p\sigma_u^2 - \tfrac{(T-1)^2}{T}p\sigma_u^2 + \tfrac{1}{T}\sigma_\epsilon^2 + p\sigma_u^2 + \sigma_\epsilon^2 \\ &= \left(\tfrac{(T-1)(2T-1)}{6T} + 1\right)p\sigma_u^2 + \tfrac{(T+1)}{T}\sigma_\epsilon^2 \end{aligned} \tag{A.1}$$

proving the result.

A.2 Proof of Theorem 2

Similarly to Theorem 1,

$$\tilde{y}_{T+1|T} - y_{T+1} = \frac{1}{m}\sum_{t=T-m+1}^{T}\sum_{i=1}^{t}\mathcal{I}\left(\nu_i = 1\right)u_i + \frac{1}{m}\sum_{t=T-m+1}^{T}\epsilon_t -$$

$$\textstyle\sum_{i=1}^{T}\mathcal{I}\left(\nu_i = 1\right)u_i - \mathcal{I}\left(\nu_{T+1} = 1\right)u_{T+1} - \epsilon_{T+1} = \displaystyle\frac{1}{m}\sum_{t=T-m+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}\left(\nu_i = 1\right)u_i$$

$$-\frac{1}{m}\left(m-1\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i = 1\right)u_i + \frac{1}{m}\sum_{t=T-m+1}^{T}\epsilon_t - \mathcal{I}\left(\nu_{T+1} = 1\right)u_{T+1} - \epsilon_{T+1}$$

giving

$$\begin{aligned} E\left(\tilde{y}_{T+1|T} - y_{T+1}\right)^2 &= \tfrac{(T-m)(m-1)^2 + 1/6m(2m-1)(m-1)}{m^2}p\sigma_u^2 + \tfrac{T(m-1)^2}{m^2}p\sigma_u^2 \\ &\quad - \tfrac{(2T-m)(m-1)^2}{m^2}p\sigma_u^2 + \frac{\sigma_\epsilon^2}{m} + p\sigma_u^2 + \sigma_\epsilon^2 \\ &= \left(\tfrac{(m-1)(2m-1)}{6m} + 1\right)p\sigma_u^2 + \tfrac{(m+1)}{m}\sigma_\epsilon^2 \end{aligned} \tag{A.2}$$

A.3 Proof of Theorem 3

We have

$$E\left(\bar{y}_{T+1|T}-y_{T+1}\right)^2 = E\left(\frac{1}{T}\sum_{i=1}^{T}\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)^2$$

$$=\frac{1}{T^2}\sum_{i=1}^{T}\sum_{j=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)\left(\tilde{y}^{(j)}_{T+1|T}-y_{T+1}\right)\right]$$

$$=\frac{1}{T^2}\sum_{i=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)^2\right]+\frac{1}{T^2}\sum_{i=1,i\neq j}^{T}\sum_{j=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)\left(\tilde{y}^{(j)}_{T+1|T}-y_{T+1}\right)\right]$$

By (A.2),

$$\frac{1}{T^2}\sum_{i=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)^2\right]=\frac{1}{T^2}\sum_{i=1}^{T}\left[\left(\tfrac{(i-1)(2i-1)}{6i}+1\right)p\sigma_u^2+\tfrac{(i+1)}{i}\sigma_\epsilon^2\right]=$$

$$p\sigma_u^2\left[1/T+\left(\frac{1}{T^2}\sum_{i=1}^{T}\left(\tfrac{(i-1)(2i-1)}{6i}\right)\right)\right]+\sigma_\epsilon^2\left[\frac{1}{T^2}\sum_{i=1}^{T}\tfrac{(i+1)}{i}\right]$$

But,

$$\frac{1}{T^2}\sum_{i=1}^{T}\left(\tfrac{(i-1)(2i-1)}{6i}\right)=1/6+o(1).$$

Further,

$$\frac{1}{T^2}\sum_{i=1}^{T}\tfrac{(i+1)}{i}=O(T^{-1}).$$

Next, we need to determine $E\left[\left(\tilde{y}^{(m_1)}_{T+1|T}-y_{T+1}\right)\left(\tilde{y}^{(m_2)}_{T+1|T}-y_{T+1}\right)\right]$ when $m_1\neq m_2$. Without loss of generality, we assume that $m_1>m_2$. We have

$$E\left[\left(\frac{1}{m_1}\sum_{t=T-m_1+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\left(\frac{1}{m_2}\sum_{t=T-m_2+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\right]=$$

$$\tfrac{(T-m_2)(m_2-1)^2+1/6m_2(2m_2-1)(m_2-1)}{m_1m_2}p\sigma_u^2+\frac{(T-m_1)(m_1-m_2)(m_2-1)}{m_1m_2}p\sigma_u^2+$$

$$\frac{(m_2-1)(m_1-m_2)(m_1-m_2+1)}{2m_1m_2}p\sigma_u^2=$$

$$\tfrac{(T-m_2)(m_2-1)^2+1/6m_2(2m_2-1)(m_2-1)}{m_1m_2}p\sigma_u^2+\tfrac{(2T-m_2-m_1+1)(m_1-m_2)(m_2-1)}{2m_1m_2}p\sigma_u^2$$

$$E\left[\left(\frac{1}{m_1}\left(m_1-1\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i\right)\left(\frac{1}{m_2}\left(m_2-1\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i\right)\right]=\tfrac{T(m_1-1)(m_2-1)}{m_1m_2}p\sigma_u^2$$

$$E\left[\left(\frac{1}{m_1}\sum_{t=T-m_1+1}^{T-1}\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\left(\frac{1}{m_2}\left(m_2-1\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i\right)\right]=\tfrac{(2T-m_1)(m_2-1)^2}{2m_1m_2}p\sigma_u^2$$

So, overall,

$$E\left[\left(\tilde{y}^{(m_1)}_{T+1|T}-y_{T+1}\right)\left(\tilde{y}^{(m_2)}_{T+1|T}-y_{T+1}\right)\right]=$$
$$\frac{m_2-6Tm_1^2-6Tm_2^2+3m_1m_2^2+3m_1^2+3m_2^2-m_2^3-9m_1m_2+12Tm_1m_2}{6m_1m_2}+$$
$$\frac{\sigma_\epsilon^2}{m_2}+p\sigma_u^2+\sigma_\epsilon^2$$

Thus,

$$\frac{1}{T^2}\sum_{i=1,i\neq j}^{T}\sum_{j=1}^{T}E\left[\left(\tilde{y}^{(i)}_{T+1|T}-y_{T+1}\right)\left(\tilde{y}^{(j)}_{T+1|T}-y_{T+1}\right)\right]= \tag{A.3}$$
$$\frac{2}{T^2}\sum_{i=2}^{T}\sum_{j=1}^{i-1}\frac{12Tij-6Ti^2-6Tj^2+3ij^2-j^3}{6ij}+o(T)$$

The first term in the RHS of (A.3) is a complicated sum involving reciprocals. As such it does not have a closed form expression involving solely elementary functions. However, we have simulated the limit and found that the first term of the RHS divided by $T/2$ converges to 0.0925.

A.4 Proof of Theorem 4

For Theorem 4, we consider the EWMA estimator. This is given by

$$\breve{y}_{T+1|T}=\frac{\sum\limits_{t=1}^{T}\lambda\left(1-\lambda\right)^{T-t}y_t}{T}$$

We have

$$\begin{aligned}\breve{y}_{T+1|T}-y_{T+1}&=\frac{\lambda}{T}\sum_{t=1}^{T}\left(1-\lambda\right)^{T-t}\left(\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)+\frac{\lambda}{T}\sum_{t=1}^{T}\left(1-\lambda\right)^{T-t}\epsilon_t\\&-\textstyle\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i-\mathcal{I}\left(\nu_{T+1}=1\right)u_{T+1}-\epsilon_{T+1}=\frac{\lambda}{T}\sum_{t=1}^{T-1}\left(1-\lambda\right)^{T-t}\left(\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\\&-\frac{1}{T}\left(T-\lambda\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i+\frac{\lambda}{T}\sum_{t=1}^{T}\left(1-\lambda\right)^{T-t}\epsilon_t-\mathcal{I}\left(\nu_{T+1}=1\right)u_{T+1}-\epsilon_{T+1}\end{aligned} \tag{A.4}$$

Then,

$$E\left[\frac{\lambda}{T}\sum_{t=1}^{T-1}\left(1-\lambda\right)^{T-t}\left(\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\right]^2=\frac{\lambda^2}{T^2}p\sigma_u^2\sum_{t=1}^{T-1}\left(\sum_{i=1}^{t}\left(1-\lambda\right)^i\right)^2$$

But,

$$\sum_{t=1}^{T-1}\left(\sum_{i=1}^{t}(1-\lambda)^{i}\right)^{2}=(1-\lambda)^{2}\sum_{t=1}^{T-1}\left(\sum_{i=0}^{t-1}(1-\lambda)^{i}\right)^{2}=(1-\lambda)^{2}\sum_{t=1}^{T-1}\left(\frac{1-(1-\lambda)^{t-1}}{\lambda}\right)^{2}$$
$$=\left(\frac{1-\lambda}{\lambda}\right)^{2}\sum_{t=1}^{T-1}\left(1-2(1-\lambda)^{t-1}+(1-\lambda)^{2(t-1)}\right)$$
$$=\left(\frac{1-\lambda}{\lambda}\right)^{2}\left(T-1-\frac{2-2(1-\lambda)^{T-1}}{\lambda}+\frac{1-(1-\lambda)^{2(T-1)}}{1-(1-\lambda)^{2}}\right)$$

So, overall

$$E\left[\frac{\lambda}{T}\sum_{t=1}^{T-1}(1-\lambda)^{T-t}\left(\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\right]^{2} \tag{A.5}$$
$$=\frac{(1-\lambda)^{2}p\sigma_u^2}{T^2}\left(T-1-\frac{2-2(1-\lambda)^{T-1}}{\lambda}+\frac{1-(1-\lambda)^{2(T-1)}}{1-(1-\lambda)^{2}}\right)=O\left(T^{-1}\right).$$

Next,

$$E\left[\frac{1}{T}\left(T-\lambda\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i\right]^{2}=\tfrac{(T-\lambda)^2}{T}p\sigma_u^2. \tag{A.6}$$

Next,

$$E\left(\frac{\lambda}{T}\sum_{t=1}^{T}(1-\lambda)^{T-t}\epsilon_t\right)^{2}=\left(\frac{\lambda}{T}\right)^{2}E\left(\sum_{t=1}^{T}(1-\lambda)^{T-t}\epsilon_t\right)^{2}=\left(\frac{\lambda}{T}\right)^{2}\sigma_\epsilon^2\sum_{t=1}^{T}(1-\lambda)^{2(T-t)} \tag{A.7}$$
$$=\left(\frac{\lambda}{T}\right)^{2}\sigma_\epsilon^2\left(\frac{(1-\lambda)^{2T}-1}{(1-\lambda)^{2}-1}\right)=O\left(T^{-2}\right).$$

Next,

$$E\left(\frac{1}{T}\left(T-\lambda\right)\sum_{i=1}^{T}\mathcal{I}\left(\nu_i=1\right)u_i\right)\left(\frac{\lambda}{T}\sum_{t=1}^{T-1}(1-\lambda)^{T-t}\left(\sum_{i=1}^{t}\mathcal{I}\left(\nu_i=1\right)u_i\right)\right)$$
$$=\frac{\lambda\left(T-\lambda\right)}{T^2}\sum_{t=1}^{T-1}t\left(1-\lambda\right)^{T-t}p\sigma_u^2$$

But,

$$\frac{\lambda\left(T-\lambda\right)}{T^2}\sum_{t=1}^{T-1}t\left(1-\lambda\right)^{T-t}=\frac{\left(T-\lambda\right)\left((1-\lambda)^{T+1}+\left(T\lambda-1\right)\left(1-\lambda\right)\right)}{T^2\lambda} \tag{A.8}$$
$$=1+O\left(T^{-1}\right)$$

Overall, combining the results in (A.5)-(A.8) implies that the term of the highest order when squaring and taking expectations in (A.4) is the term analysed in (A.6). This term is $O\left(T\right)$ whereas all the other terms are $o\left(T\right)$. Therefore, we conclude that

$$E\left(\breve{y}_{T+1|T}-y_{T+1}\right)^{2}=\tfrac{(T-\lambda)^2}{T}p\sigma_u^2+o(T).$$

Appendix B Detailed UK Results

Table B.1: Relative RMSFE Results for UK: First Period (1992Q1-1999Q4)

	M40	M60	R40	R60	AV	EWMA
Consumer Price Index	0.988	0.992	0.820	**0.969**	1.005	0.928
Private sector output annual growth	1.000	1.000	1.015	0.965	1.027	1.090
Private sector output quarterly growth	1.000	1.000	0.922	0.909	0.839	0.858
CBI survey: Employment intentions, next 3 months	1.000	1.000	0.865	0.963	0.881	0.981
UK FTSE All share dividend yield	1.000	1.000	1.019	0.998	1.024	1.198
Bank of England REPO rate	1.000	1.000	1.201	**1.208**	1.037	1.092
GFK index score	1.000	1.000	1.091	1.029	1.100	1.543
Nationwide House Price Index	1.000	1.000	**1.073**	0.997	1.017	1.068
Stock of net corporate debt	1.000	1.000	1.038	1.008	1.031	1.105
Constant market price imported consumption	1.000	1.000	1.030	1.043	1.012	1.036
Government procurement of goods and services (including investment)	1.000	1.000	0.990	1.009	1.034	1.240
Real post tax labour income, constant prices	1.000	1.000	1.101	1.006	1.000	1.151
Stock of notes and coins	0.768	0.834	0.930	0.845	0.783	0.781
Private sector productivity, hours-based measure	1.000	1.000	1.135	1.049	1.059	1.199
Long-term foreign nominal rate of interest.	1.000	1.000	1.036	1.012	1.024	1.371
Corporate bond real interest rate	1.000	1.000	1.158	**1.172**	1.001	1.070
Unit labour costs (private sector measure)	**0.919**	**0.944**	**0.814**	**0.867**	**0.728**	0.780
Nominal value of the firm	1.000	1.000	0.934	1.032	0.950	1.349
Tax revenue from corporation tax, current prices	1.000	1.000	1.003	0.896	0.738	0.940
Total tax payments of household sector	**1.000**	1.000	0.957	0.991	0.963	1.530
Real exchange rate	1.000	1.000	0.980	**1.061**	1.042	2.228
Average private sector weekly hours	1.000	1.000	0.804	0.856	0.840	1.088
Average whole economy average hours	1.000	1.000	0.777	0.836	0.795	0.883
Total private sector compensation spending	**0.942**	**0.960**	0.948	**0.947**	**0.866**	0.841
Total government compensation spending	0.954	**0.967**	**0.815**	0.863	0.832	0.936
Employment in heads	0.982	0.984	1.032	1.015	1.003	1.544
Private sector employment in heads	1.000	1.000	0.981	0.989	0.957	1.052
Nominal wages per worker	0.936	**0.954**	1.045	0.954	0.956	0.939
Nominal private sector wages per worker	**0.935**	**0.954**	0.961	0.940	0.908	0.860
Sectoral M4	1.000	1.000	0.896	1.015	0.812	0.815
Sectoral M4 Lending	1.000	1.000	0.960	0.972	0.871	0.934
M4 lending	0.924	0.941	0.744	0.930	**0.672**	0.694
Net lending to private non-financial corporations	1.000	1.000	1.099	1.036	0.917	0.828
M4 liabilities to private non-financial corporations	1.000	1.000	0.925	1.001	0.891	1.049
M4 liabilities to other financial corporations	1.000	1.000	0.962	0.957	0.909	1.192
Net lending to household sector	0.607	0.692	0.974	0.996	0.579	1.415
Gross Domestic Product	1.000	1.000	**0.617**	**0.792**	**0.597**	0.601
Gross National Income	0.904	0.934	0.830	0.866	0.763	0.914
Changes in inventories including alignment adjustment	1.000	1.000	1.005	1.010	1.029	**1.225**
IOP: All production industries	1.000	1.000	0.991	1.047	1.009	1.137
IOP: Mining & quarrying	1.000	1.000	1.193	0.990	1.097	1.162
IOP: Manufacturing	1.000	1.000	0.979	1.110	1.015	1.168
IOP: Manufacturing of food, drink & tobacco	1.000	1.000	1.042	1.009	**1.047**	**1.398**
IOP: Manufacturing of textile & textile products	1.000	1.000	0.977	1.042	1.029	1.158
IOP: Manufacturing of leather & leather products	1.000	1.000	**0.866**	0.929	0.882	0.905
IOP: Manufacturing of wood & wood products	1.000	1.000	1.003	1.003	1.010	1.255
IOP: Pulp/paper/printing/publishing industries	1.000	1.000	1.132	**1.105**	1.083	1.263

Notes: Bold entries indicate rejection for the Diebold-Mariano Test.

Table B.2: Relative RMSFE Results for UK: First Period (1992Q1-1999Q4)(cont.)

	M40	M60	R40	R60	AV	EWMA
IOP: Manufacturing coke/petroleum prod/nuclear fuels	1.000	**1.000**	1.024	1.029	1.034	1.331
IOP: Manufacturing of rubber & plastic products	0.972	0.980	0.975	1.117	1.004	1.114
IOP: Manufacturing of non-metallic mineral products	1.000	1.000	1.058	1.004	1.074	**1.411**
IOP: Manufacturing of basic metals & fabricated prod	1.000	1.000	0.908	0.915	0.953	1.373
IOP: Manufacturing of machinery & equipment	1.000	1.000	1.030	1.046	1.036	**1.408**
IOP: Manufacturing of electrical & optical equipment	1.000	1.000	0.909	0.932	0.929	0.989
IOP: Manufacturing of transport equipment	1.000	1.000	1.017	0.967	0.987	1.171
IOP: Extraction of oil & gas	0.982	0.988	1.278	0.996	1.144	1.177
UK Total Wages & Salaries	**0.940**	**0.956**	1.041	**0.934**	0.907	0.868
Business Investment (excl exceptional transfer)	1.000	1.000	0.906	0.950	0.947	1.054
CAPEX: Total excl exceptional transfer	**1.020**	**1.013**	0.982	0.989	0.970	1.223
Output Index: Distribution, hotels & catering; repairs	1.000	1.000	1.266	1.208	1.179	**1.591**
Output Index: Transport storage & communication	1.000	1.000	**0.740**	**0.806**	**0.716**	0.850
Output Index: Total	**1.000**	1.000	**0.714**	**0.804**	**0.678**	0.660
Total Gross Fixed Capital Formation	1.000	1.000	0.946	0.970	0.962	1.017
Total adjustment to basic prices (General Government + Rest of the World)	1.000	1.000	0.954	0.969	0.914	1.096
Gross Domestic Product at market prices	**0.893**	**0.928**	0.831	**0.883**	**0.701**	0.705
Gross Value Added at factor cost	1.000	1.000	0.728	0.793	**0.669**	0.669
Money stock M4 (end period)	**0.941**	**0.960**	0.855	1.008	**0.744**	0.783
Notes & coins in circulation outside Bank of England	0.992	0.991	0.998	1.013	1.008	**1.420**
Money Stock: Retail Deposits and Cash in M4	1.000	1.000	**0.118**	0.949	**0.155**	**0.010**
Total Government benefits paid to household sector	**0.901**	**0.932**	**0.777**	**0.856**	**0.789**	0.794
General Government: Final consumption expenditure	1.000	1.000	0.913	0.996	0.945	0.990
Household final consumption expenditure	1.000	1.000	**1.426**	0.979	1.169	1.302
Household final consumption expenditure, index	**0.929**	**0.953**	**0.729**	**0.904**	**0.755**	**0.640**
Durable goods	1.000	1.000	**1.525**	1.235	1.265	**1.627**
Semi-durable goods	1.000	1.000	**0.921**	0.941	0.955	1.130
Durable goods, index	0.947	0.963	**0.781**	**0.880**	**0.819**	0.928
Non-durable goods, index	**0.919**	**0.946**	**0.736**	**0.890**	**0.742**	**0.626**
Services, index	**0.903**	**0.934**	**0.797**	**0.882**	**0.801**	**0.741**
Semi-durable goods, index	**0.805**	**0.868**	**0.702**	**0.852**	**0.591**	**0.312**
Claimant count rate	1.000	1.000	0.968	1.001	1.032	1.159
Whole economy (incl bonus)	**0.866**	**0.907**	0.977	**0.902**	0.854	**0.637**
Whole economy (incl bonus): % change 3 month average	1.000	1.000	0.901	0.992	0.928	1.122
In employment, aged 16+	1.000	1.000	1.000	0.994	0.976	1.059
Unemployed, aged 16+	0.997	0.998	**0.962**	0.966	0.904	0.902
Economically active	1.000	1.000	1.016	0.996	0.932	0.982
Population aged 16+	1.050	1.031	**1.078**	1.035	1.016	1.022
Unemployment rate, aged 16+	1.000	1.000	**0.956**	0.974	0.924	0.964
Total actual weekly hours worked	1.000	1.000	0.902	0.953	0.772	0.606
PPI: Output of manufactured products	1.000	1.000	1.008	0.989	1.036	**1.359**
PPI: NSO :All Manufacturing excl duty	1.000	1.000	1.049	0.975	1.010	1.312
Imports: Total trade in goods and services excl MTIC fraud, curr prices	**1.000**	1.000	1.043	1.022	1.043	1.333
Exports: Total trade in goods and services excl MTIC fraud	1.000	1.000	**0.844**	0.934	**0.845**	0.866
Imports: Total trade in goods and services excl MTIC fraud, const prices	1.000	1.000	0.996	0.969	0.935	0.983
Balance of Payments: IM: Finished manufactures	0.984	0.989	**0.849**	**0.906**	0.877	0.902
Balance of Payments: Total Trade in Goods & Services	1.000	1.000	1.144	1.084	1.039	1.094

Table B.3: Relative RMSFE Results for UK: Second Period (2000Q1-2008Q2)

	M40	M60	R40	R60	AV	EWMA
Consumer Price Index	1.027	1.016	1.162	1.115	1.052	1.124
Private sector output annual growth	1.000	1.000	1.082	1.101	1.072	1.242
Private sector output quarterly growth	1.000	1.000	1.230	**1.458**	1.150	1.230
CBI survey: Employment intentions, next 3 months	1.000	1.000	1.129	1.129	1.074	1.398
UK FTSE All share dividend yield	1.000	1.000	1.031	0.973	0.993	1.145
Bank of England REPO rate	1.000	1.000	0.856	0.980	0.969	1.199
GFK index score	1.000	1.000	1.066	0.994	1.029	1.143
Nationwide House Price Index	1.040	1.025	1.088	1.048	1.027	0.845
Stock of net corporate debt	1.000	1.000	0.924	0.979	0.964	1.362
Constant market price imported consumption	1.000	1.000	0.991	1.036	0.976	1.295
Government procurement of goods and services (including investment)	1.000	1.000	1.001	0.975	0.968	1.326
Real post tax labour income, constant prices	1.000	1.000	0.934	0.868	0.940	1.160
Stock of notes and coins	0.983	0.988	1.212	1.025	0.991	1.054
Private sector productivity, hours-based measure	1.000	1.000	1.033	**1.120**	1.057	**1.355**
Long-term foreign nominal rate of interest.	1.000	**1.000**	0.930	0.995	1.001	1.556
Corporate bond real interest rate	1.000	1.000	0.796	0.915	0.911	1.096
Unit labour costs (private sector measure)	**0.880**	**0.918**	**0.553**	**0.613**	**0.634**	0.682
Nominal value of the firm	1.000	1.000	**0.754**	**0.788**	**0.780**	0.776
Tax revenue from corporation tax, current prices	1.000	1.000	**0.664**	0.750	**0.690**	0.705
Total tax payments of household sector	1.000	1.000	0.849	0.869	0.919	1.175
Real exchange rate	1.000	1.000	0.986	0.947	0.952	1.262
Average private sector weekly hours	1.000	1.000	0.889	0.881	0.905	1.213
Average whole economy average hours	1.000	1.000	0.847	0.839	**0.847**	1.080
Total private sector compensation spending	0.900	0.933	**0.733**	**0.702**	**0.755**	0.731
Total government compensation spending	**0.953**	**0.968**	1.063	0.897	0.921	1.223
Employment in heads	1.000	1.000	1.091	1.033	0.995	0.893
Private sector employment in heads	1.000	1.000	**0.607**	0.914	**0.658**	**0.464**
Nominal wages per worker	**0.748**	**0.827**	**0.571**	**0.639**	**0.625**	**0.535**
Nominal private sector wages per worker	0.778	0.846	**0.581**	**0.634**	**0.628**	0.700
Sectoral M4	1.000	1.000	1.028	1.053	0.950	1.095
Sectoral M4 Lending	1.000	1.000	1.133	**1.196**	1.029	1.202
M4 lending	0.927	0.949	0.907	1.022	0.892	0.961
Net lending to private non-financial corporations	1.000	1.000	1.091	1.187	1.030	1.553
M4 liabilities to private non-financial corporations	1.000	**1.000**	0.835	0.983	0.911	1.015
M4 liabilities to other financial corporations	1.000	1.000	0.970	1.091	0.925	0.898
Net lending to household sector	**0.897**	**0.931**	0.795	**0.690**	**0.765**	0.781
Gross Domestic Product	1.000	1.000	1.422	**1.512**	1.301	**1.622**
Gross National Income	**0.876**	**0.913**	**0.533**	**0.612**	**0.592**	0.683
Changes in inventories including alignment adjustment	1.000	1.000	1.054	1.000	1.027	**1.382**
IOP: All production industries	1.000	1.000	0.925	0.996	0.952	0.979
IOP: Mining & quarrying	1.000	1.000	0.986	0.992	0.908	1.029
IOP: Manufacturing	1.000	1.000	0.942	1.068	1.012	1.189
IOP: Manufacturing of food, drink & tobacco	1.000	1.000	1.017	0.997	0.990	1.125
IOP: Manufacturing of textile & textile products	1.000	1.000	1.028	**1.044**	1.019	1.142
IOP: Manufacturing of leather & leather products	1.000	1.000	1.118	1.042	1.038	1.169
IOP: Manufacturing of wood & wood products	1.000	1.000	0.922	0.943	0.966	1.139
IOP: Pulp/paper/printing/publishing industries	1.000	1.000	0.886	0.922	0.938	1.046

Table B.4: Relative RMSFE Results for UK: Second Period (2000Q1-2008Q2)(cont.)

	M40	M60	R40	R60	AV	EWMA
IOP: Manufacturing coke/petroleum prod/nuclear fuels	1.000	1.000	1.042	1.030	1.056	**1.531**
IOP: Manufacturing of rubber & plastic products	1.000	1.000	0.965	0.975	0.916	1.075
IOP: Manufacturing of non-metallic mineral products	1.000	1.000	0.952	1.085	1.011	1.285
IOP: Manufacturing of basic metals & fabricated prod	1.000	1.000	1.108	1.514	1.157	1.425
IOP: Manufacturing of machinery & equipment	1.000	1.000	0.916	0.903	**0.878**	0.813
IOP: Manufacturing of electrical & optical equipment	1.000	1.000	0.946	0.958	0.948	1.191
IOP: Manufacturing of transport equipment	1.000	1.000	1.031	1.087	1.059	**1.388**
IOP: Extraction of oil & gas	**0.881**	**0.918**	0.862	0.869	**0.766**	0.818
UK Total Wages & Salaries	0.869	0.904	0.831	0.813	0.822	0.852
Business Investment (excl exceptional transfer)	1.000	1.000	1.125	**1.185**	1.069	1.130
CAPEX: Total excl exceptional transfer	1.005	0.998	0.962	1.010	0.907	0.892
Output Index: Distribution, hotels & catering; repairs	1.000	1.000	1.008	1.039	0.986	1.065
Output Index: Transport storage & communication	1.000	0.999	**1.511**	**1.293**	**1.258**	1.317
Output Index: Total	1.010	1.005	1.416	1.307	1.165	1.530
Total Gross Fixed Capital Formation	1.000	1.000	1.094	1.061	1.060	**1.453**
Total adjustment to basic prices (General Government + Rest of the World)	**0.910**	**0.938**	**0.635**	**0.774**	**0.720**	0.892
Gross Domestic Product at market prices	**0.840**	**0.891**	**0.486**	**0.571**	**0.592**	0.618
Gross Value Added at factor cost	1.000	1.000	1.288	**1.384**	1.238	**1.603**
Money stock M4 (end period)	0.985	0.988	1.162	1.275	0.927	0.861
Notes & coins in circulation outside Bank of England	0.967	0.977	0.910	0.900	0.843	0.875
Money Stock: Retail Deposits and Cash in M4	1.000	1.000	**0.006**	**0.005**	**0.047**	**0.005**
Total Government benefits paid to household sector	0.900	0.931	0.814	0.873	0.842	1.126
General Government: Final consumption expenditure	1.000	1.000	0.684	**0.768**	**0.705**	0.713
Household final consumption expenditure	1.000	1.000	**1.280**	1.136	1.067	1.129
Household final consumption expenditure, index	**0.848**	**0.895**	**0.552**	**0.655**	**0.579**	**0.514**
Durable goods	1.000	1.000	1.105	1.087	0.986	1.109
Semi-durable goods	1.000	1.000	1.069	1.060	1.012	1.203
Durable goods, index	**0.905**	**0.936**	**0.648**	**0.766**	**0.699**	0.714
Non-durable goods, index	1.023	1.009	1.103	1.097	0.938	1.044
Services, index	**0.902**	**0.934**	**0.541**	**0.696**	**0.579**	**0.579**
Semi-durable goods, index	**0.944**	**0.961**	0.837	0.892	0.733	0.583
Claimant count rate	0.951	0.966	1.032	1.010	0.944	0.978
Whole economy (incl bonus)	0.619	0.737	0.447	0.513	0.540	0.319
Whole economy (incl bonus): % change 3 month average	1.000	1.000	0.626	0.646	0.633	0.466
In employment, aged 16+	1.000	1.000	**0.532**	**0.917**	**0.603**	**0.375**
Unemployed, aged 16+	0.921	0.943	0.805	0.887	0.805	0.731
Economically active	1.000	1.000	0.810	0.959	**0.797**	0.743
Population aged 16+	0.998	0.998	1.042	1.045	0.993	1.029
Unemployment rate, aged 16+	0.889	0.922	0.803	0.909	0.790	**0.641**
Total actual weekly hours worked	1.000	1.000	**0.467**	**0.650**	**0.566**	**0.507**
PPI: Output of manufactured products	**1.000**	1.000	0.996	0.995	0.994	1.164
PPI: NSO :All Manufacturing excl duty	1.000	1.000	0.956	0.966	0.962	1.162
Imports: Total trade in goods and services excl MTIC fraud, curr prices	1.000	1.000	1.106	1.046	1.019	1.180
Exports: Total trade in goods and services excl MTIC fraud	1.000	1.000	1.123	1.041	1.064	1.355
Imports: Total trade in goods and services excl MTIC fraud, const prices	1.000	1.000	1.164	1.052	1.071	**1.388**
Balance of Payments: IM: Finished manufactures	0.995	0.994	0.870	0.819	0.799	1.058
Balance of Payments: Total Trade in Goods & Services	1.000	1.000	1.051	1.006	1.007	**1.335**

Appendix C Detailed US results

Table C.1: Relative RMSFE Results for US: First Period (1975Q1-1986Q2)

	M40	M60	R40	R60	AV	EWMA
Industrial Production: Final Products (Market Group)	1.000	1.000	1.065	0.998	1.055	1.249
Industrial Production: Consumer Goods	1.000	**1.000**	1.000	0.978	1.031	1.414
Industrial Production: Durable Consumer Goods	1.000	1.000	1.022	0.987	1.029	1.478
Industrial Production: Nondurable Consumer Goods	1.000	1.000	0.953	0.972	0.965	1.276
Industrial Production: Business Equipment	1.000	1.000	**1.067**	0.998	1.047	1.135
Industrial Production: Materials	1.000	1.000	1.026	0.992	1.038	1.160
Industrial Production: Durable Materials	1.000	1.000	1.044	1.003	1.068	1.295
Industrial Production: Nondurable Materials	1.000	1.000	1.002	0.985	1.024	1.284
Industrial Production: Manufacturing (SIC)	1.000	1.000	1.040	0.997	1.043	1.207
Industrial Production Index	1.000	1.000	1.031	0.995	1.031	1.137
ISM Manufacturing: PMI Composite Index	1.000	1.000	1.078	1.030	1.116	1.536
Real Disposable Personal Income	1.000	1.000	1.008	1.008	0.943	0.988
Real Disposable Personal Income Less Transfer Payments	1.000	1.000	1.030	1.001	1.030	1.132
Civilian Labor Force: Employed, Total	1.020	1.012	1.011	0.998	1.028	1.349
Unemployment Rate: All Workers	**1.035**	**1.022**	1.124	0.978	1.152	1.197
Unemployment By Duration: Average Duration In Weeks	1.075	1.036	1.048	0.964	0.999	0.977
Civilians Unemployed - Less Than 5 Weeks	1.000	1.000	1.036	1.032	**1.071**	**1.391**
Civilian Unemployed for 5-14 Weeks	1.000	1.000	1.030	1.011	**1.078**	**1.441**
Civilians Unemployed - 15 Weeks & Over	1.000	1.000	1.057	0.999	1.130	**1.400**
Civilians Unemployed for 15-26 Weeks	1.000	1.000	1.029	1.004	**1.080**	**1.305**
Total nonfarm employment	1.000	1.000	1.040	1.011	**1.071**	1.212
Total private employment	1.027	1.017	**1.036**	1.007	**1.068**	1.273
Goods-producing employment	1.000	1.000	1.033	1.003	**1.068**	**1.257**
Natural resources and mining employment	1.171	1.106	1.113	1.057	1.291	2.594
Construction employment	**1.000**	1.000	1.071	1.001	1.109	1.293
Manufacturing employment	1.000	1.000	1.035	1.005	1.070	1.308
Durable goods manufacturing employment	**1.000**	1.000	1.027	1.005	1.071	1.314
Nondurable goods manufacturing employment	1.000	1.000	1.037	1.001	1.081	1.396
Service-providing employment	1.014	1.007	**1.059**	1.015	1.067	1.249
Trade, transportation and utilities employment	1.004	1.001	1.017	1.010	1.047	1.260
Retail trade employment	1.006	1.002	1.019	1.047	1.040	1.271
Wholesale trade employment	0.996	0.994	1.038	0.998	1.033	1.151
Financial activities employment	1.018	1.009	0.963	1.017	1.047	1.192
Private service-providing employment	1.010	1.006	1.033	1.009	**1.051**	**1.222**
Government employment	0.995	0.996	0.929	1.015	0.959	1.045
Manufacturing average weekly hours of production workers	1.000	1.000	1.040	1.012	**1.050**	1.279
Manufacturing average weekly overtime of production workers	1.000	1.000	1.061	1.008	1.081	1.467
Housing Starts: Total: New Privately Owned Housing Units Started	1.000	1.000	1.070	1.021	**1.057**	1.220
Housing Starts in Northeast Census Region	**1.000**	1.000	1.077	1.006	1.090	1.586
Housing Starts in Midwest Census Region	1.000	1.000	1.033	1.016	1.038	1.213
Housing Starts in South Census Region	1.000	1.000	1.050	1.018	**1.054**	**1.252**
Housing Starts in West Census Region	1.000	1.000	1.022	1.021	1.054	1.213
New Private Housing Units Authorized by Building Permit	1.000	1.000	1.047	1.007	1.024	1.369
New Orders, Consumer Goods & Materials	1.000	**1.000**	1.057	1.014	1.062	1.356
New Orders, Nondefense Capital Goods	1.000	1.000	1.050	0.982	1.027	1.314
Dow Jones Industrial Average	1.000	1.000	1.016	1.011	1.027	1.216
Japanese Yen-United States Dollar Exchange Rate	1.025	1.016	1.002	0.999	1.033	1.165
Canadian Dollar-United States Dollar Exchange Rate	0.963	0.973	0.989	1.015	0.985	1.135
Effective Federal Funds Rate	1.025	1.011	1.014	1.001	1.008	1.087

Table C.2: Relative RMSFE Results for US: First Period (1975Q1-1986Q2) (cont.)

	M40	M60	R40	R60	AV	EWMA
3-Month Treasury Bill: Secondary Market Rate	1.028	1.013	1.037	1.020	1.034	1.137
6-Month Treasury Bill: Secondary Market Rate	1.036	1.018	**1.048**	1.025	1.044	1.162
1-Year Treasury Constant Maturity Rate	1.037	1.019	**1.059**	1.025	1.051	1.171
5-Year Treasury Constant Maturity Rate	1.048	1.029	1.097	1.027	1.091	1.237
10-Year Treasury Constant Maturity Rate	1.043	1.026	1.085	1.016	1.090	1.246
Moody's Corporate AAA Yield	1.059	1.036	1.135	1.012	1.124	1.353
Moody's Corporate BAA Yield	1.029	1.017	1.067	1.010	1.038	1.082
Spread 3M-FF	1.090	1.049	0.984	0.977	1.081	1.451
Spread 6M-FF	1.054	1.028	0.999	0.984	1.050	1.334
Spread 1Y-FF	1.049	1.027	1.027	0.999	1.056	1.272
Spread 5Y-FF	1.039	1.021	1.023	1.001	1.021	1.109
Spread 10Y-FF	1.044	1.024	1.022	1.001	1.020	1.122
Spread AAA-FF	0.997	0.998	1.026	1.000	1.020	1.141
Spread BAA-FF	1.000	1.000	1.019	0.995	1.030	**1.205**
M1 Money Stock	0.956	0.965	0.906	0.955	0.897	1.002
M2 Money Stock	0.872	0.905	0.950	1.020	0.911	1.108
Money Supply - M2	0.997	0.998	0.961	1.027	0.909	0.923
Monetary base, adj for reserve requirement changes	0.952	0.963	0.933	0.948	0.889	1.104
Total Reserves of Depository Institutions	1.000	1.000	0.943	0.937	0.950	1.141
Non-Borrowed Reserves of Depository Institutions	1.042	1.025	1.022	1.019	1.044	1.203
Consumer Credit Outstanding - Nonrevolving	1.000	1.000	**1.061**	1.020	1.004	0.871
Commercial & Industrial Loans Outstanding	1.000	1.000	1.016	0.997	1.031	1.180
PPI: Finished Goods	1.040	1.024	1.131	1.077	0.993	0.904
PPI: Finished Consumer Goods	1.039	1.023	1.107	1.074	0.986	0.892
PPI: Intermediate Mat. Supplies & Components	1.032	1.019	1.085	**1.082**	1.001	1.039
PPI: Crude Materials	0.986	0.991	1.072	**1.080**	1.041	1.192
Consumer Price Index For All Urban Consumers: All Items	1.041	1.022	1.046	1.003	1.009	1.108
CPI-U: Apparel	1.012	1.003	0.996	1.002	0.949	1.047
CPI-U: Transportation	1.039	1.019	1.027	1.011	0.988	1.053
CPI-U: Medical care	1.020	1.005	1.120	1.000	0.936	1.186
CPI-U: Commodities	1.049	1.029	1.045	1.019	1.010	1.083
CPI-U: Durables	1.016	0.984	1.081	0.995	0.941	0.950
CPI-U: All Items Less Food	1.034	1.016	1.037	0.998	1.002	1.048
CPI-U: All Items Less Shelter	1.043	1.023	1.067	1.023	1.000	1.066
CPI-U: All Items Less Medical Care	1.043	1.023	1.045	1.007	1.005	1.088
Spot Market Price Index: BLS & CRB: all commodities	**1.000**	1.000	1.055	1.027	1.015	1.147
Construction: average hourly earnings of production workers	0.969	0.974	0.999	1.041	1.004	1.067
Manufacturing: average hourly earnings of production workers	0.982	0.985	1.046	**1.355**	0.925	0.792
U. of Michigan Index of Consumer Expectations	1.000	1.000	1.011	1.015	1.031	1.333
Dow Jones Industrials Total Return Index	1.000	1.000	0.995	1.012	1.023	1.212
S&P 500 Energy Total Return Index	1.000	1.000	1.051	1.034	1.054	1.524
S&P 500 Finance Total Return Index	1.000	1.000	1.029	0.994	**1.060**	**1.514**
S&P 500 Total Return Index	1.000	1.000	0.993	1.014	1.035	**1.266**
S&P 500 Transportation Total Return Index	1.000	1.000	1.002	1.006	1.030	1.317
S&P 500 Utilities Total Return Index	1.000	1.000	0.970	1.001	0.970	0.954
Dow Jones Corporate Bond Yield	1.000	1.000	**1.061**	0.997	1.079	1.363
USA Prime Rate	1.000	1.000	1.031	1.002	**1.067**	1.493
West Texas Intermediate Oil Price (US$/Barrel)	1.000	1.000	1.005	1.018	0.988	1.000

Table C.3: Relative RMSFE Results for US: Second Period (1986Q3-1997Q4)

	M40	M60	R40	R60	AV	EWMA
Industrial Production: Final Products (Market Group)	1.000	1.000	1.077	1.024	1.012	1.174
Industrial Production: Consumer Goods	1.000	1.000	1.034	1.029	1.036	1.385
Industrial Production: Durable Consumer Goods	1.000	1.000	1.060	1.046	1.037	1.235
Industrial Production: Nondurable Consumer Goods	1.000	1.000	0.854	0.886	0.911	1.173
Industrial Production: Business Equipment	1.000	1.000	1.055	0.996	1.011	1.101
Industrial Production: Materials	1.000	1.000	1.165	1.083	1.087	1.314
Industrial Production: Durable Materials	1.000	1.000	0.999	1.003	1.010	1.174
Industrial Production: Nondurable Materials	1.000	1.000	1.038	0.956	0.964	1.228
Industrial Production: Manufacturing (SIC)	1.000	1.000	1.107	1.027	1.047	1.290
Industrial Production Index	1.000	1.000	1.152	1.039	1.055	1.260
ISM Manufacturing: PMI Composite Index	1.000	1.000	1.090	1.034	1.039	1.398
Real Disposable Personal Income	1.000	1.000	0.960	0.888	**0.888**	1.064
Real Disposable Personal Income Less Transfer Payments	1.000	1.000	1.004	0.853	0.847	0.848
Civilian Labor Force: Employed, Total	1.000	1.000	1.002	1.026	1.042	1.334
Unemployment Rate: All Workers	0.989	0.987	0.985	**1.215**	1.004	0.770
Unemployment By Duration: Average Duration In Weeks	1.001	1.001	1.065	**1.067**	0.972	0.896
Civilians Unemployed - Less Than 5 Weeks	1.000	1.000	0.970	0.977	0.982	1.304
Civilian Unemployed for 5-14 Weeks	1.000	1.000	0.998	1.048	1.012	1.149
Civilians Unemployed - 15 Weeks & Over	**1.000**	1.000	1.007	1.056	1.011	1.059
Civilians Unemployed for 15-26 Weeks	1.000	1.000	0.875	1.012	**0.912**	0.810
Total nonfarm employment	1.000	1.000	0.999	1.055	1.012	1.162
Total private employment	**1.000**	1.000	0.974	1.079	1.002	1.171
Goods-producing employment	1.000	1.000	1.001	1.094	1.013	1.178
Natural resources and mining employment	1.000	1.000	0.740	1.031	**0.711**	0.760
Construction employment	1.000	1.000	0.903	**1.096**	0.982	1.380
Manufacturing employment	1.000	1.000	1.062	1.068	1.017	1.131
Durable goods manufacturing employment	**1.000**	1.000	1.013	1.055	0.995	1.121
Nondurable goods manufacturing employment	1.000	1.000	1.100	1.062	1.031	1.223
Service-providing employment	1.000	1.000	0.989	1.013	1.025	1.231
Trade, transportation and utilities employment	1.036	1.018	0.985	1.024	1.021	1.212
Retail trade employment	1.000	0.994	0.958	0.971	0.993	1.113
Wholesale trade employment	1.000	1.000	0.978	0.988	1.004	1.161
Financial activities employment	1.031	1.018	1.012	1.000	1.011	1.105
Private service-providing employment	**1.092**	**1.054**	0.973	1.042	1.044	1.392
Government employment	1.003	1.000	0.926	0.825	0.895	1.732
Manufacturing average weekly hours of production workers	1.000	1.000	1.055	1.053	1.034	1.565
Manufacturing average weekly overtime of production workers	1.000	1.000	0.996	0.991	0.958	1.103
Housing Starts: Total: New Privately Owned Housing Units Started	1.000	1.000	1.019	0.984	0.987	1.260
Housing Starts in Northeast Census Region	**1.000**	**1.000**	**1.098**	1.004	1.041	1.331
Housing Starts in Midwest Census Region	1.000	1.000	0.981	1.046	1.023	1.356
Housing Starts in South Census Region	1.000	1.000	0.995	1.009	0.999	1.167
Housing Starts in West Census Region	1.000	1.000	0.991	1.005	0.986	1.161
New Private Housing Units Authorized by Building Permit	1.000	1.000	1.016	1.001	0.990	1.346
New Orders, Consumer Goods & Materials	1.000	1.000	1.063	1.044	1.039	1.388
New Orders, Nondefense Capital Goods	1.000	1.000	1.067	1.126	1.079	1.569
Dow Jones Industrial Average	1.000	1.000	1.006	0.969	1.009	1.389
Japanese Yen-United States Dollar Exchange Rate	1.000	1.000	1.043	1.013	1.040	1.683
Canadian Dollar-United States Dollar Exchange Rate	1.000	1.000	1.086	1.029	1.018	1.021
Effective Federal Funds Rate	0.916	0.928	1.284	**1.322**	1.031	0.853

Table C.4: Relative RMSFE Results for US: Second Period (1986Q3-1997Q4) (cont.)

	M40	M60	R40	R60	AV	EWMA
3-Month Treasury Bill: Secondary Market Rate	0.933	0.941	1.235	**1.291**	1.019	0.897
6-Month Treasury Bill: Secondary Market Rate	0.959	0.961	1.163	**1.223**	1.018	0.963
1-Year Treasury Constant Maturity Rate	0.989	0.985	1.130	**1.169**	1.027	1.042
5-Year Treasury Constant Maturity Rate	1.004	1.001	1.012	1.054	1.027	**1.171**
10-Year Treasury Constant Maturity Rate	1.003	1.001	1.015	1.050	1.027	1.158
Moody's Corporate AAA Yield	0.999	0.998	1.040	1.059	1.027	1.114
Moody's Corporate BAA Yield	1.000	0.998	1.043	1.049	1.022	1.106
Spread 3M-FF	0.947	0.958	1.095	**1.414**	1.113	0.931
Spread 6M-FF	0.990	0.988	1.153	**1.369**	**1.109**	1.202
Spread 1Y-FF	1.000	1.000	0.969	1.126	**1.040**	1.196
Spread 5Y-FF	1.000	1.000	0.953	1.054	1.008	1.141
Spread 10Y-FF	1.000	1.000	0.959	1.076	0.979	0.979
Spread AAA-FF	1.000	1.000	0.960	1.067	0.964	0.888
Spread BAA-FF	0.994	0.996	0.958	1.120	0.985	0.960
M1 Money Stock	0.909	0.933	1.043	1.083	1.030	1.118
M2 Money Stock	0.999	0.999	1.167	**1.186**	1.000	0.855
Money Supply - M2	1.000	1.000	1.139	**1.154**	0.973	0.816
Monetary base, adj for reserve requirement changes	1.053	1.032	1.110	1.083	1.083	1.289
Total Reserves of Depository Institutions	0.939	0.955	0.896	0.923	0.939	1.116
Non-Borrowed Reserves of Depository Institutions	0.945	**0.960**	0.911	0.952	**0.918**	**0.583**
Consumer Credit Outstanding - Nonrevolving	1.000	1.000	0.888	**1.126**	0.933	0.982
Commercial & Industrial Loans Outstanding	1.000	1.000	1.064	1.065	1.000	1.156
PPI: Finished Goods	0.958	0.970	0.871	0.985	0.911	0.966
PPI: Finished Consumer Goods	0.966	0.975	0.883	0.991	0.929	1.023
PPI: Intermediate Mat. Supplies & Components	0.978	0.979	0.953	0.966	0.955	1.232
PPI: Crude Materials	1.000	1.000	1.055	1.031	1.059	1.697
Consumer Price Index For All Urban Consumers: All Items	0.971	0.978	0.888	1.004	0.909	0.925
CPI-U: Apparel	0.948	0.965	0.963	0.981	0.915	0.772
CPI-U: Transportation	0.960	0.971	0.891	0.957	0.937	1.308
CPI-U: Medical care	0.888	**0.911**	0.971	1.161	1.089	0.942
CPI-U: Commodities	**0.936**	**0.955**	0.802	**0.931**	**0.870**	0.910
CPI-U: Durables	0.948	0.961	1.103	**1.216**	0.991	0.868
CPI-U: All Items Less Food	0.977	0.983	0.877	1.012	0.919	0.991
CPI-U: All Items Less Shelter	0.959	0.970	0.868	0.960	0.892	0.905
CPI-U: All Items Less Medical Care	0.960	0.972	0.873	0.993	0.898	0.898
Spot Market Price Index: BLS & CRB: all commodities	1.000	1.000	1.019	1.027	1.034	**1.240**
Construction: average hourly earnings of production workers	0.998	0.999	**0.731**	**0.832**	**0.749**	**0.667**
Manufacturing: average hourly earnings of production workers	**0.815**	**0.870**	**0.641**	0.798	**0.737**	**0.637**
U. of Michigan Index of Consumer Expectations	1.000	1.000	0.978	0.949	0.943	1.114
Dow Jones Industrials Total Return Index	1.000	1.000	0.959	0.970	0.956	1.365
S&P 500 Energy Total Return Index	1.000	1.000	0.987	1.013	0.975	1.373
S&P 500 Finance Total Return Index	1.000	1.000	1.012	0.996	1.018	1.308
S&P 500 Total Return Index	1.000	1.000	0.953	0.992	0.950	1.280
S&P 500 Transportation Total Return Index	1.000	1.000	0.976	0.967	0.969	**1.375**
S&P 500 Utilities Total Return Index	1.000	1.000	1.059	1.099	1.043	1.260
Dow Jones Corporate Bond Yield	1.000	1.000	1.002	0.999	1.016	1.360
USA Prime Rate	1.000	1.000	0.935	**1.130**	1.013	1.223
West Texas Intermediate Oil Price (US$/Barrel)	1.000	1.000	0.907	0.934	0.959	1.488

Table C.5: Empirical Results for US: Third Period (1998Q1-2008Q3)

	M40	M60	R40	R60	AV	EWMA
Industrial Production: Final Products (Market Group)	1.000	1.000	1.052	0.958	0.968	1.194
Industrial Production: Consumer Goods	1.000	1.000	0.956	0.953	**0.920**	1.024
Industrial Production: Durable Consumer Goods	1.000	1.000	0.960	1.041	0.961	0.994
Industrial Production: Nondurable Consumer Goods	0.978	0.983	0.849	0.873	0.883	1.054
Industrial Production: Business Equipment	1.000	1.000	1.144	1.091	1.006	1.196
Industrial Production: Materials	1.000	1.000	1.039	0.982	0.952	1.133
Industrial Production: Durable Materials	1.000	1.000	1.107	1.010	0.965	1.194
Industrial Production: Nondurable Materials	0.964	0.976	0.919	0.925	0.944	1.049
Industrial Production: Manufacturing (SIC)	1.000	1.000	1.021	0.952	0.953	1.007
Industrial Production Index	1.000	1.000	1.055	0.966	0.957	1.082
ISM Manufacturing: PMI Composite Index	1.000	1.000	0.989	1.011	1.012	1.272
Real Disposable Personal Income	1.000	1.000	0.878	0.866	0.909	1.029
Real Disposable Personal Income Less Transfer Payments	1.000	1.000	**1.475**	1.110	1.049	1.282
Civilian Labor Force: Employed, Total	1.000	1.000	1.019	0.997	1.018	2.059
Unemployment Rate: All Workers	1.008	1.002	1.093	1.114	1.004	0.931
Unemployment By Duration: Average Duration In Weeks	0.948	0.957	**1.077**	0.992	0.991	0.952
Civilians Unemployed - Less Than 5 Weeks	1.000	1.000	1.074	1.053	1.029	1.265
Civilian Unemployed for 5-14 Weeks	1.000	1.000	1.163	1.076	1.047	1.275
Civilians Unemployed - 15 Weeks & Over	1.000	1.000	1.090	1.067	1.013	1.116
Civilians Unemployed for 15-26 Weeks	1.000	1.000	1.185	1.105	1.016	1.153
Total nonfarm employment	1.000	1.000	0.963	0.893	0.910	1.020
Total private employment	1.000	1.000	0.949	0.863	0.868	1.107
Goods-producing employment	1.000	1.000	0.950	0.861	0.833	1.083
Natural resources and mining employment	1.000	1.000	**0.311**	**0.324**	**0.513**	**0.333**
Construction employment	1.000	1.000	0.882	0.913	0.887	0.913
Manufacturing employment	1.000	1.000	0.923	0.835	0.799	1.175
Durable goods manufacturing employment	1.000	1.000	0.909	0.846	0.823	1.245
Nondurable goods manufacturing employment	0.958	**0.970**	0.792	0.759	**0.745**	0.796
Service-providing employment	1.000	1.000	0.956	0.915	0.939	1.104
Trade, transportation and utilities employment	1.000	1.000	0.845	0.856	**0.887**	0.949
Retail trade employment	1.000	1.000	**0.705**	**0.727**	**0.785**	0.989
Wholesale trade employment	1.000	1.000	0.901	0.920	0.932	1.074
Financial activities employment	1.007	1.002	0.963	0.974	0.968	1.043
Private service-providing employment	1.000	1.000	0.919	0.864	0.900	1.039
Government employment	1.052	1.022	0.803	0.857	0.995	2.691
Manufacturing average weekly hours of production workers	1.000	1.000	**1.108**	**1.071**	1.037	**1.306**
Manufacturing average weekly overtime of production workers	1.000	1.000	1.071	1.140	1.035	1.334
Housing Starts: Total: New Privately Owned Housing Units Started	1.000	1.000	1.019	1.169	1.012	1.219
Housing Starts in Northeast Census Region	1.000	1.000	1.055	1.048	1.017	**1.288**
Housing Starts in Midwest Census Region	1.000	1.000	0.995	1.058	0.984	1.045
Housing Starts in South Census Region	1.000	1.000	1.029	1.121	0.969	0.915
Housing Starts in West Census Region	1.000	1.000	1.044	1.150	0.979	0.937
New Private Housing Units Authorized by Building Permit	1.000	1.000	1.070	1.135	0.997	0.900
New Orders, Consumer Goods & Materials	1.000	1.000	0.872	0.929	**0.892**	0.817
New Orders, Nondefense Capital Goods	1.000	1.000	**1.195**	**1.078**	1.003	0.973
Dow Jones Industrial Average	1.000	1.000	1.079	1.066	1.048	1.423
Japanese Yen-United States Dollar Exchange Rate	1.000	1.000	0.990	1.013	1.005	**1.316**
Canadian Dollar-United States Dollar Exchange Rate	1.000	1.000	1.042	1.021	1.012	**1.320**
Effective Federal Funds Rate	0.970	0.971	0.910	0.897	**0.933**	0.839

Table C.6: Empirical Results for US: Third Period (1998Q1-2008Q3) (cont.)

	M40	M60	R40	R60	AV	EWMA
3-Month Treasury Bill: Secondary Market Rate	0.967	0.972	0.926	0.918	**0.946**	0.902
6-Month Treasury Bill: Secondary Market Rate	0.978	0.978	0.937	0.927	0.951	0.936
1-Year Treasury Constant Maturity Rate	0.981	0.979	0.946	0.926	0.963	0.984
5-Year Treasury Constant Maturity Rate	0.973	0.977	1.039	0.968	1.022	1.238
10-Year Treasury Constant Maturity Rate	0.973	0.979	1.024	0.961	1.017	1.154
Moody's Corporate AAA Yield	1.006	1.001	**1.095**	0.954	1.020	1.200
Moody's Corporate BAA Yield	1.062	1.037	1.102	0.964	1.032	1.248
Spread 3M-FF	1.073	1.040	1.045	1.018	1.020	1.248
Spread 6M-FF	1.064	1.038	1.074	1.022	1.041	1.339
Spread 1Y-FF	1.000	1.000	1.101	1.053	1.051	1.423
Spread 5Y-FF	1.000	1.000	1.030	1.032	1.034	1.179
Spread 10Y-FF	1.000	1.000	1.008	1.006	1.007	1.097
Spread AAA-FF	1.010	1.001	0.942	0.961	0.958	0.947
Spread BAA-FF	0.987	0.988	0.945	0.971	0.950	0.903
M1 Money Stock	1.000	1.000	1.057	1.037	1.025	1.511
M2 Money Stock	0.949	0.964	1.042	1.013	0.951	1.097
Money Supply - M2	0.953	0.966	1.055	1.030	0.958	1.121
Monetary base, adj for reserve requirement changes	1.083	1.052	1.072	1.054	1.070	1.970
Total Reserves of Depository Institutions	1.000	1.000	1.100	1.074	1.113	3.771
Non-Borrowed Reserves of Depository Institutions	1.623	0.917	2.557	1.626	0.718	15.818
Consumer Credit Outstanding - Nonrevolving	1.000	1.000	1.147	**1.116**	1.010	1.101
Commercial & Industrial Loans Outstanding	1.023	1.013	1.033	1.000	0.998	1.234
PPI: Finished Goods	1.000	1.000	1.065	1.033	0.972	1.214
PPI: Finished Consumer Goods	1.000	1.000	1.081	1.048	0.991	1.239
PPI: Intermediate Mat. Supplies & Components	1.000	1.000	1.096	1.054	1.018	1.275
PPI: Crude Materials	1.000	1.000	1.083	**1.045**	1.044	**1.591**
Consumer Price Index For All Urban Consumers: All Items	0.866	0.889	0.776	0.820	0.812	0.778
CPI-U: Apparel	**0.981**	**0.987**	**0.767**	0.869	**0.863**	0.877
CPI-U: Transportation	1.000	1.000	0.935	0.892	0.910	0.980
CPI-U: Medical care	0.892	0.901	0.851	0.905	**0.873**	0.944
CPI-U: Commodities	1.000	1.000	0.810	0.811	0.817	0.810
CPI-U: Durables	0.904	**0.927**	**0.815**	0.891	**0.899**	0.910
CPI-U: All Items Less Food	0.842	0.877	0.737	0.774	**0.802**	0.782
CPI-U: All Items Less Shelter	0.910	0.922	0.864	0.871	0.855	0.840
CPI-U: All Items Less Medical Care	0.879	0.897	0.798	0.827	0.820	0.806
Spot Market Price Index: BLS & CRB: all commodities	1.000	1.000	1.045	1.035	1.017	1.120
Construction: average hourly earnings of production workers	0.985	0.979	1.026	1.024	0.832	0.948
Manufacturing: average hourly earnings of production workers	1.000	1.000	**0.642**	**0.624**	**0.715**	0.696
U. of Michigan Index of Consumer Expectations	1.000	1.000	0.916	0.878	0.920	1.236
Dow Jones Industrials Total Return Index	1.000	1.000	1.042	1.068	1.023	1.111
S&P 500 Energy Total Return Index	1.000	1.000	0.921	0.943	0.940	0.958
S&P 500 Finance Total Return Index	1.000	1.000	**1.107**	1.043	1.004	1.012
S&P 500 Total Return Index	1.000	1.000	1.067	1.127	1.037	1.127
S&P 500 Transportation Total Return Index	1.000	1.000	1.025	1.047	1.021	**1.403**
S&P 500 Utilities Total Return Index	1.000	1.000	1.045	**1.063**	1.017	1.056
Dow Jones Corporate Bond Yield	1.000	1.000	1.056	1.059	1.031	1.298
USA Prime Rate	1.000	1.000	0.852	0.863	0.881	1.595
West Texas Intermediate Oil Price (US$/Barrel)	1.000	1.000	1.136	1.076	1.023	1.271

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