Forecasting Evaluation and Combination

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Developments in time series econometrics and their uses
for macroeconomic forecasting in a policy environment

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Outline

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2. Models and data

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- $f_T(\mathbf{w})$ is a concave function.
- Given the evaluations p_{ti} from the alternative prediction models and a sample, finding $\mathbf{w}_T^* = \arg\max_{\mathbf{w}} f_T(\mathbf{w})$ is a straightforward convex programming problem.

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3. Output: growth rate in per capita real GDP
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6. Real wage: growth rate in real wage
7. Interest rate: Federal Funds Rate

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Log score of optimal pool: -974.9

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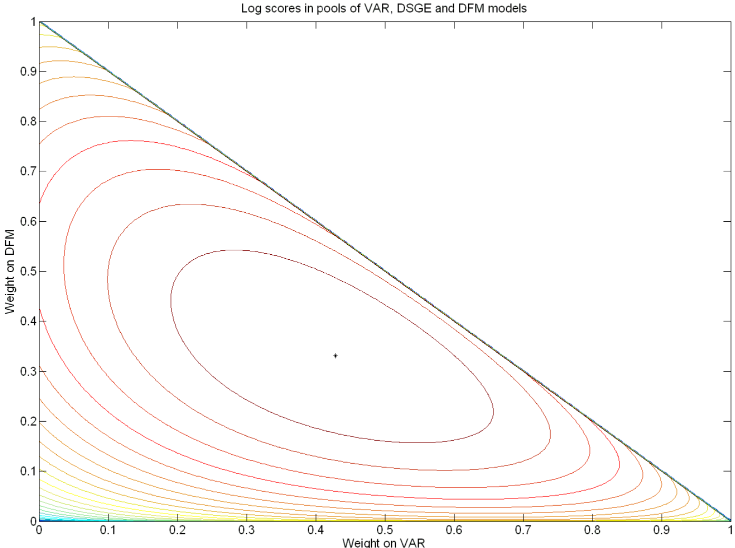
Log scores in pools of VAR, DSGE and DFM models
Weight on DFM
Weight on VAR
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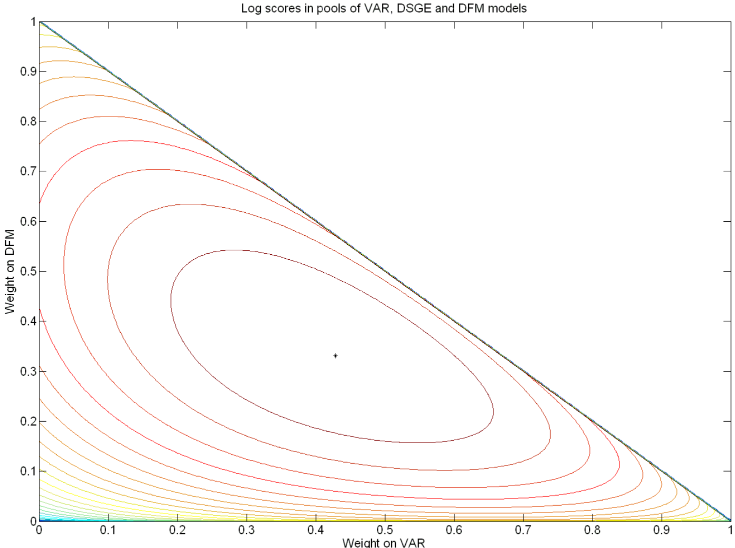
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1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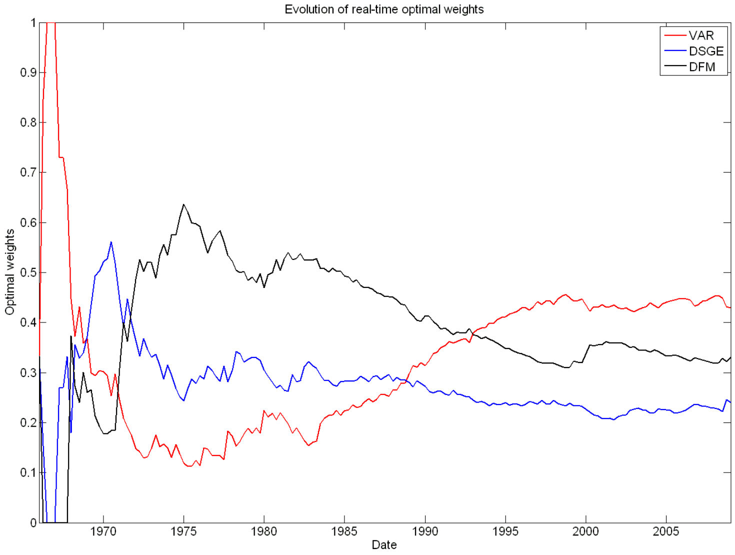
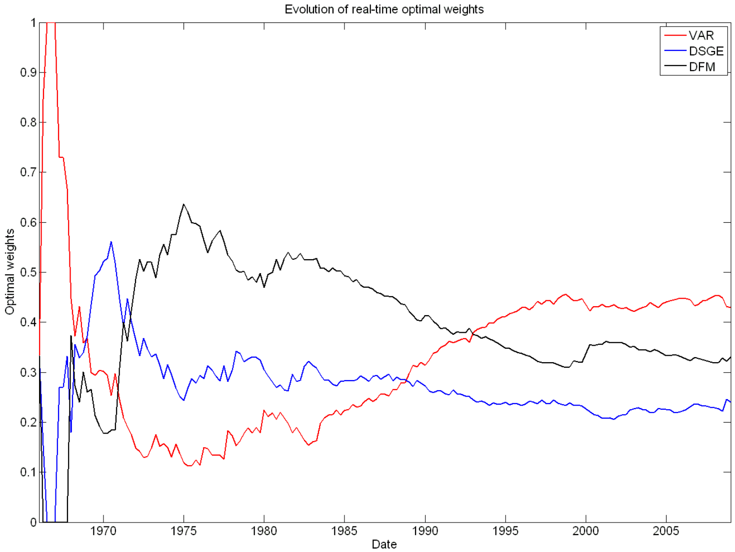

Evolution of real-time optimal weights
Optimal weights
Date
VAR
DSGE
DFM
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1970
1975
1980
1985
1990
1995
2000
2005


Evolution of real-time optimal weights
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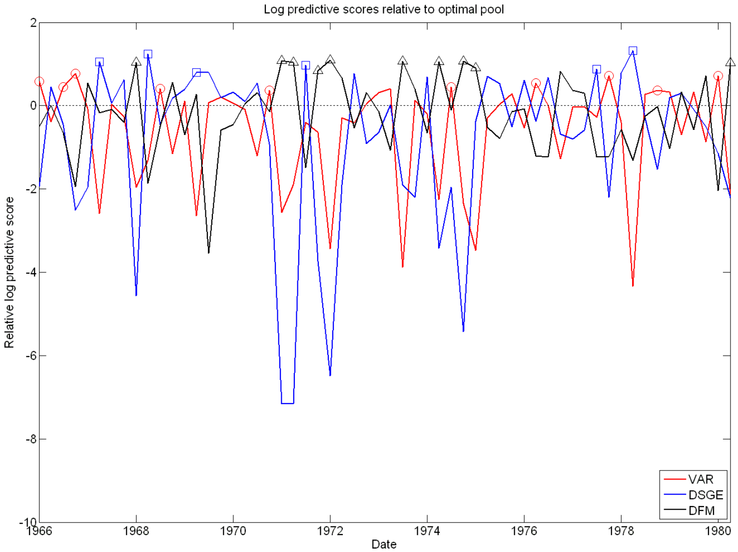
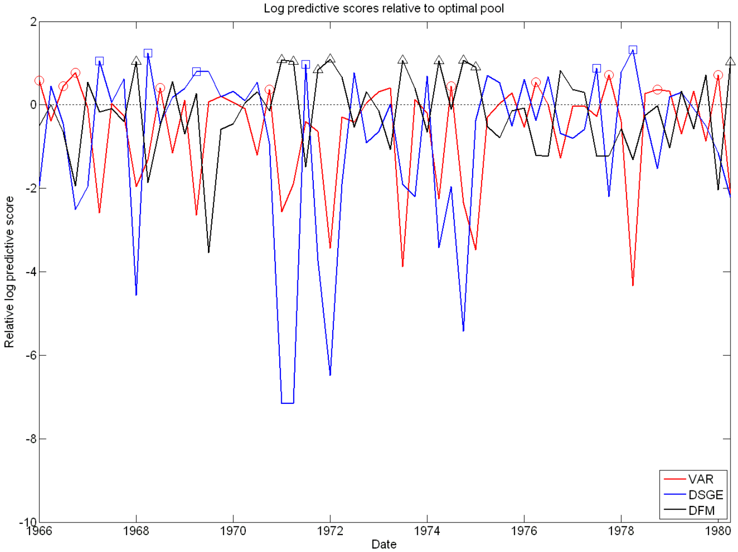

Log predictive scores relative to optimal pool
Relative log predictive score
Date
2
0
-2
-4
-6
-8
-10
1966
1968
1970
1972
1974
1976
1978
1980
VAR
DSGE
DFM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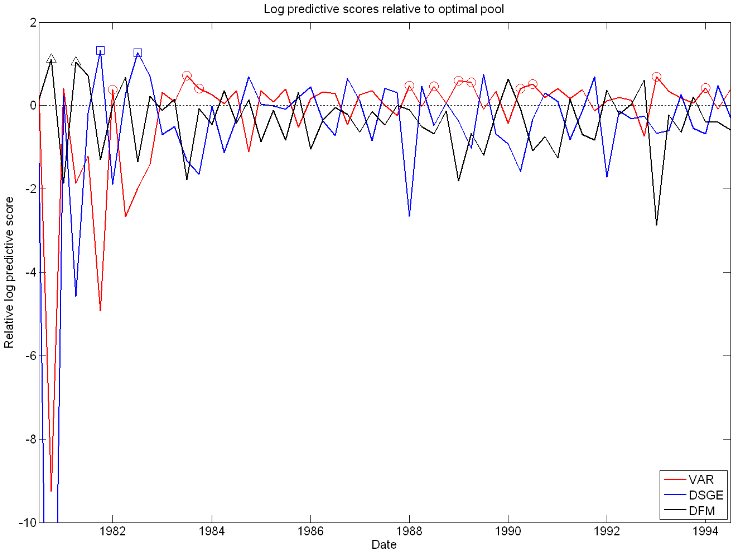
Log predictive scores relative to optimal pool
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2
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1982
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1988
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1992
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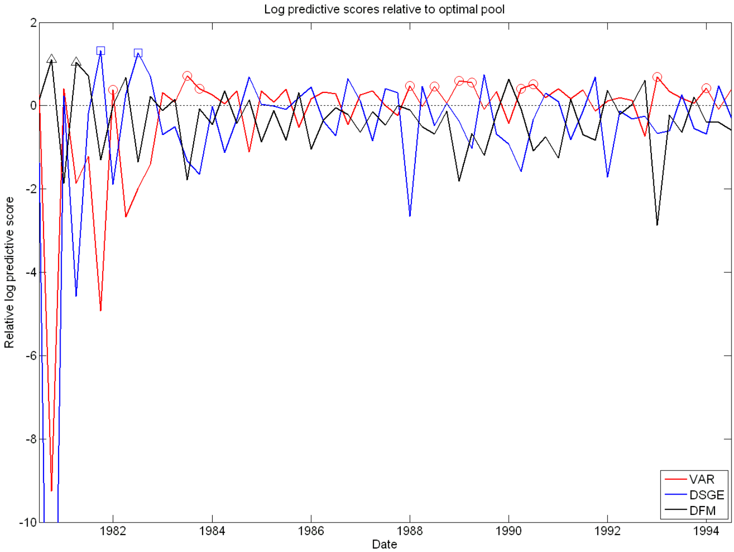
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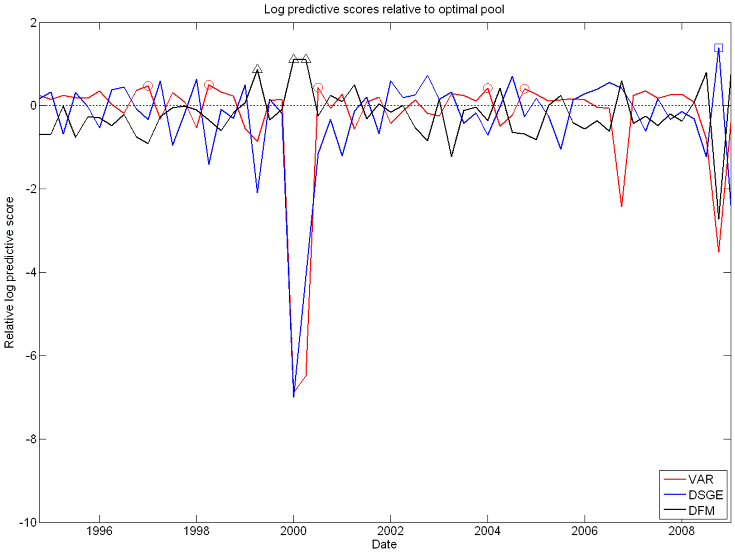
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2006
2008
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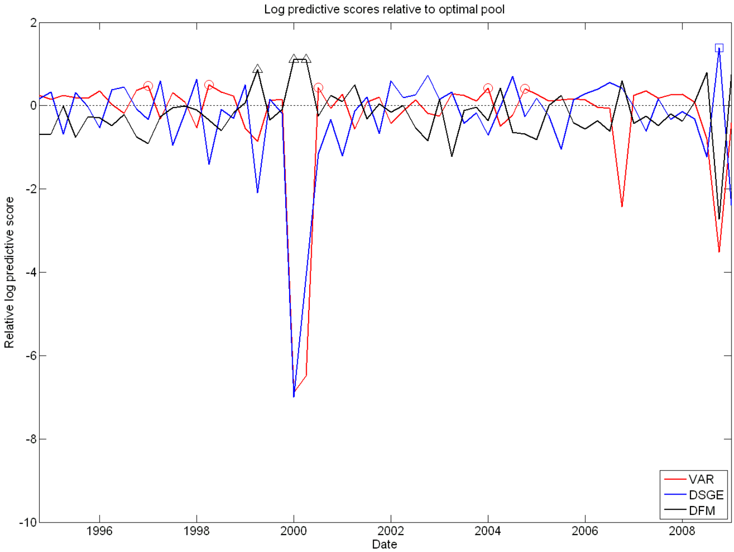
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