A NOTE ON OPTIMAL FISCAL RULE FOR TURKEY

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An optimal dynamic fiscal loss model for Turkey is presented in this note. The model is used as a benchmark to gauge the success of potential simple fiscal rules. Optimal linear and non-linear rules are shown to perform well.

1 Introduction

For many reasons, governments seek to have stable expenditure paths through time. That individuals have a preference for smooth consumption paths is a relatively well-understood and well-studied phenomenon. However, empirical evidence across countries show that governments' preference for smooth consumption may be even stronger than that of individuals. For most of the countries where governments can easily borrow to smooth their expenditures against shocks, standard deviation of government expenditures is significantly smaller than the standard deviation of consumption of private agents. Shocks to output, government expenditures, and financial sector are inevitable. Together with these shocks, governments' strong taste for smooth consumption make unexpected hikes in debt to output ratio quite common. However, governments can borrow to smooth consumption during bad times in a sustainable manner, only if they can achieve to reduce their debt levels during good times. Here lies an important time-inconsistency problem, and failing to solve this problem in a credible way may paralyze governments' ability to borrow in bad times, making them pay very high risk premia as a consequence. Amending fiscal rules into law in a credible manner can help solve this problem and may be used as the necessary commitment device. To serve as a successful commitment device a fiscal rule must be credible, simple, and transparent.

In an environment where there are no shocks coming to economy, the government's problem would be simple. The government would choose an ideal debt to output level and would balance its budget at this level. Through time, the debt to output ratio, and the government expenditure would both be smooth – except that the latter grows at the growth rate of output. In reality, however, there are significant shocks coming to the economic environment. Under the environment with shocks, it makes sense that the government determines an *ideal debt to output ratio* b^*, and a *desired smooth government expenditure path*, g_t^*, so that no matter what shocks come, it will stay close to this ratio and the path through time. There is a trade off between deviating from the ideal debt to output ratio, and deviating from the desired smooth government expenditure path. If the government chooses to stay very close to one of these, it will have to sacrifice from being close to the other. Therefore the government will have to balance out these two deviations according to its preferences, *i.e.*, how much these deviations matter for the government.

The problem of the government can be modeled as a *dynamic fiscal loss minimization problem* where given an initial debt, output level, and the government's expectations about future income path, it picks an optimal path of expenditures and debt for current and future periods. The government does that to minimize a measure of total sum of deviations from the ideal debt to output ratio, b^*, and the desired smooth government expenditure path, g_t^*, through time.

In this note, we model and solve a *dynamic fiscal loss minimization problem* for Turkey. We use the optimal solution to this problem as a benchmark to measure the success of potential simple

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fiscal rules. We calibrate the parameters and estimate the shock processes using Turkish data. Here we have two alternatives for modelling the income process. In the first alternative, a stochastic income process for Turkey can be estimated using historical data and a dynamic stochastic fiscal loss minimization problem can be solved. The second alternative is to use historical income data for future realizations of income in a dynamic fiscal loss minimization problem under the assumption of perfect foresight for government. Since estimating a reasonably accurate stochastic income process using historic data may be a problem, as a first step, we follow the second course in this note.

2 Economic environment

Consider a government which starts time zero with initial expenditure and debt levels, g_0 and b_0, respectively. Let the output in the economy at time zero be y_0. Assume that the economy grows at rate γ. Given the initial expenditure level, government desires to set its future expenditure levels through time according to $g_0^* = sy_0$ and $g_t^* = \gamma^t g_0^*$. Here s is the desired government expenditure to output ratio and given the initial desired government expenditure level g_0^* future expenditure levels smoothly grows at rate γ. This implies $g_t^* = s\gamma^t y_0$.

Given how the market's perceptions about the country's potential risk relates to its debt to output ratio, there is a desired level of debt to output ratio, denoted by b^*. The government seeks to keep its debt to output ratio $\frac{b_t}{y_t}$ as close to b^* as possible. Assume that government taxes income at the constant rate τ.

Consider a government seeking to minimize the dynamic loss function:

$$L = \min_{\{g_t, b_t\}_{t=1}^{\infty}} \left\{ \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^{t-1} \left[\underbrace{\alpha_g \left(\frac{g_t}{y_0 \gamma^t} - s \right)^2}_{\text{expenditure smoothing}} + \underbrace{\alpha_b \left(\frac{b_t}{y_t} - b^* \right)^2}_{\text{debt smoothing}} \right] \right\} \tag{1}$$

subject to:

$$g_t + (1+r) b_{t-1} = b_t + \tau y_t \tag{2}$$

$$b_0 \text{ given.}$$

In (1), the loss function has two components; expenditure smoothing term and debt smoothing term. Here g_t and b_t denote the government expenditure and government debt at time t, respectively. The political preference parameters α_g and α_b can be seen as conveying the relative importance of expenditure smoothing and debt smoothing for the government. The government is assumed to discount future loss at rate $\frac{1}{1+r}$. The time t budget constraint of the government is given by (2). Given the initial debt and output level, b_0 and y_0 the government decides about the expenditure and debt sequences $\{g_t, b_t\}_{t=1}^{\infty}$, that will satisfy the budget constraint and that will minimize the total fiscal loss. Notice that one of the two political preference parameters, α_g and α_b are redundant. We can normalize one of these parameters.

Since the variables γ_t, g_t and b_t all grow through time, transforming these variables into stationary ones will make the analysis more tractable. To this end, let $\widehat{y}_t = \frac{y_t}{\gamma^t}$, $\widehat{g}_t = \frac{g_t}{\gamma^t}$, and $\widehat{b}_t = \frac{b_t}{\gamma^t}$. The budget constraint can be transformed to:

$$\frac{g_t}{\gamma^t} + (1+r)\frac{b_{t-1}}{\gamma^t} = \frac{b_t}{\gamma^t} + \tau\frac{y_t}{\gamma^t}$$

which yields:

$$\widehat{g}_t = \tau\widehat{y}_t + \widehat{b}_t - (1+r)\frac{\widehat{b}_{t-1}}{\gamma}.$$

Notice that $\frac{b_t}{y_t} = \frac{b_t}{\gamma^t}\frac{\gamma^t}{y_t} = \frac{\widehat{b}_t}{\widehat{y}_t}$.

Let's restate the problem of the government. The government's problem in transformed variables reads:

$$L = \min_{\left\{\widehat{g}_t,\widehat{b}_t\right\}_{t=1}^{\infty}} \left\{\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t-1}\left[\alpha_g(\widehat{g}_t - g^*)^2 + \alpha_b\left(\frac{\widehat{b}_t}{\widehat{y}_t} - b^*\right)^2\right]\right\}$$

subject to:

$$\widehat{g}_t = \tau\widehat{y}_t + \widehat{b}_t - (1+r)\frac{\widehat{b}_{t-1}}{\gamma}$$

$$\widehat{b}_0 \text{ given.}$$

The Lagrange Equation corresponding to this problem reads:

$$L = \min_{\left\{\widehat{g}_t,\widehat{b}_t\right\}_{t=1}^{\infty}} \left\{\sum_{t=1}^{\infty}\left(\tfrac{1}{1+r}\right)^{t-1}\left(\left[\alpha_g(\widehat{g}_t - g^*)^2 + \alpha_b\left(\tfrac{\widehat{b}_t}{\widehat{y}_t} - b^*\right)^2\right]\right.\right.$$
$$\left.\left.\lambda_t\left[\widehat{g}_t - \tau\widehat{y}_t - \widehat{b}_t + (1+r)\tfrac{\widehat{b}_{t-1}}{\gamma}\right]\right)\right\}.$$

Efficiency conditions of this problem are:

$$\frac{dL}{d\widehat{b}_t} = \alpha_g(\widehat{g}_t - g^*)\frac{d\widehat{g}_t}{d\widehat{b}_t} + \alpha_b\left(\frac{\widehat{b}_t}{\widehat{y}_t} - b^*\right)\frac{1}{\widehat{y}_t} + \frac{1}{1+r}\alpha_g(\widehat{g}_{t+1} - g^*)\frac{d\widehat{g}_{t+1}}{d\widehat{b}_t} = 0,$$

and:

$$\widehat{g}_t - \tau\widehat{y}_t - \widehat{b}_t + (1+r)\frac{\widehat{b}_{t-1}}{\gamma} = 0.$$

Plugging this into the first condition yields:

$$\widehat{b}_t = \frac{\frac{\alpha_g}{\gamma}\left(\tau\widehat{y}_{t+1} + \widehat{b}_{t+1} - g^*\right) - \alpha_g\left(\tau\widehat{y}_t - \frac{1+r}{\gamma}\widehat{b}_{t-1} - g^*\right) + \alpha_b \frac{b^*}{\widehat{y}_t}}{\frac{\alpha_b}{\widehat{y}_t^2} + \frac{\alpha_g(1+r)}{\gamma^2} + \alpha_g}.$$

The first order condition given in (3) gives us optimal debt to output level through time, but unfortunately it is a relatively complex dynamic relationship. We can compute the optimal solution but it does not directly provide us a simple fiscal rule that we can practically use. However, we can use the optimal solution to gauge the relative success of potential simple rules. That is the route that we follow in the rest of our analysis.

3 Results

We pick parameter values that represent Turkish economy as close as possible. The tax rate τ is set to 0.3 so that government expenditures to output ratio is 0.3. Ideal debt to output ratio, b^* is picked as 0.3. We normalize α_b to one and we will consider a range of values for α_g. The average growth rate of real GDP in Turkey between 1970-2009 is used to calibrate γ, so that $\gamma = 4\%$. Initial output level y_0 is normalized to one and initial debt level b_0 is set to 0.45. Normalized real output growth numbers between 1970-2009 is used for future real output growth series.

The solution to the dynamic fiscal problem will be used as a benchmark to measure the success of potential simple linear rules. We will consider three potential fiscal rules:

i) sample fiscal rule considered:

$$d_t = d^* - 0.33(g_t - g^*) + 0.75(d_{-1} - d^*)$$

ii) optimized linear rule:

$$d_t = d^* - \alpha_g(g_t - g^*) + \alpha_d(d_{-1} - d^*)$$

iii) optimized non-linear rule:

$$d_t = d^* - \alpha_g(g_t - g^*)^{\phi_g} + \alpha_d(d_{-1} - d^*)^{\phi_d}$$

In order to grasp the optimal fiscal policy better, consider an environment with no output shocks where the economy grows at a constant rate, 4 per cent. Assume that the initial debt to output ratio is 45 per cent and desired level of debt to output ratio is 30 per cent. Figure 1 and 2 exhibits the transition of optimal debt to output and optimal government expenditure to output ratios during transition to the steady state of this economy for different values of alpha. Three values of alpha are used, $\alpha = 0.1, 10, 30$. For a low level of α – for $\alpha = 0.1$, for instance – debt smoothing is more important for the government compared to expenditure smoothing. As a result, at the expense of a more volatile expenditure path, the government chooses to have a debt to output ratio path close to the ideal level, 0.3. This is clearly seen in Figure 1.

For $\alpha = 0.1$, starting from a 45 per cent level, the transition to the steady state for debt to output ratio takes only one period. For larger α the transition takes longer as expected. For $\alpha = 30$, transition is slow; even after 20 years transition is not totally completed. Figure 2 exhibits government expenditure to output ratio during transition to the steady state for again $\alpha = 0.1, 10, 30$. For $\alpha = 0.1$, the transition is fast. It starts from a government expenditure to output ratio of 16 per cent, far below the desired level of 30 per cent. For a typical government this means

Figure 1

Debt-to-output Ratio During Transition to Steady State for Different α Values

(no output shocks)

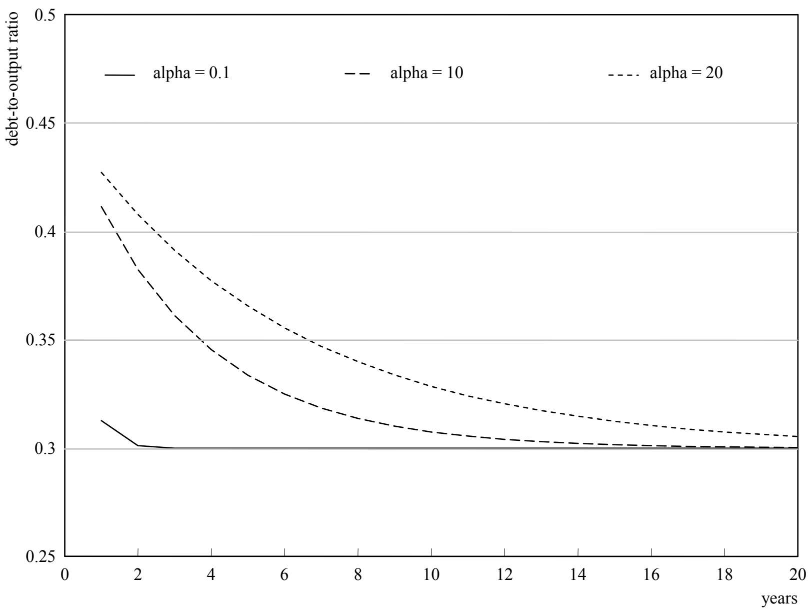
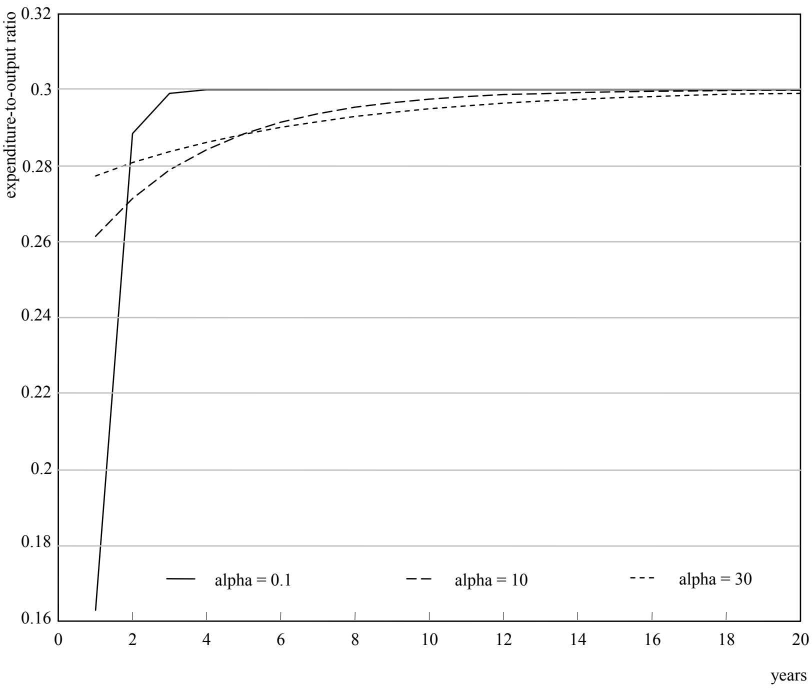


Figure 2

Expenditure-to-output Ratio During Transition to Steady State for Different α Values

(no output shocks)



a deadly tight fiscal policy on transition. Most of the governments would probably not be able stand that tight of a fiscal policy profile, showing us that $\alpha = 0.1$ does not represent a very realistic and credible preference parameter.

For higher level of α, however, the transition is more comfortable. For $\alpha = 30$, a two per cent cut in the expenditure to output ratio, initially during transition, does the job.

Using the historical output growth data for Turkey, Figure 3 plots the paths for optimal government expenditure-to-output ratio for $\alpha = 10$ and 30. Output shocks create fluctuations around the desired level of 30 per cent. Notice that the fluctuations are smaller for higher α. Similarly Figure 4 shows the optimal paths for debt to output ratio. As expected the transition is faster and fluctuations are smaller for lower α.

Figure 5 plots the paths for the expenditure to output ratio derived from the optimal solution and from the sample fiscal rule considered using $\alpha = 30$. The sample fiscal rule creates significant fluctuations in the ratio, around the ideal level, 0.3. Similarly, the next figure exhibits the

Figure 3

Government Expenditure-to-output Ratio During Transition with Actual Growth Numbers

Optimal Government Expenditure-to-output Ratio

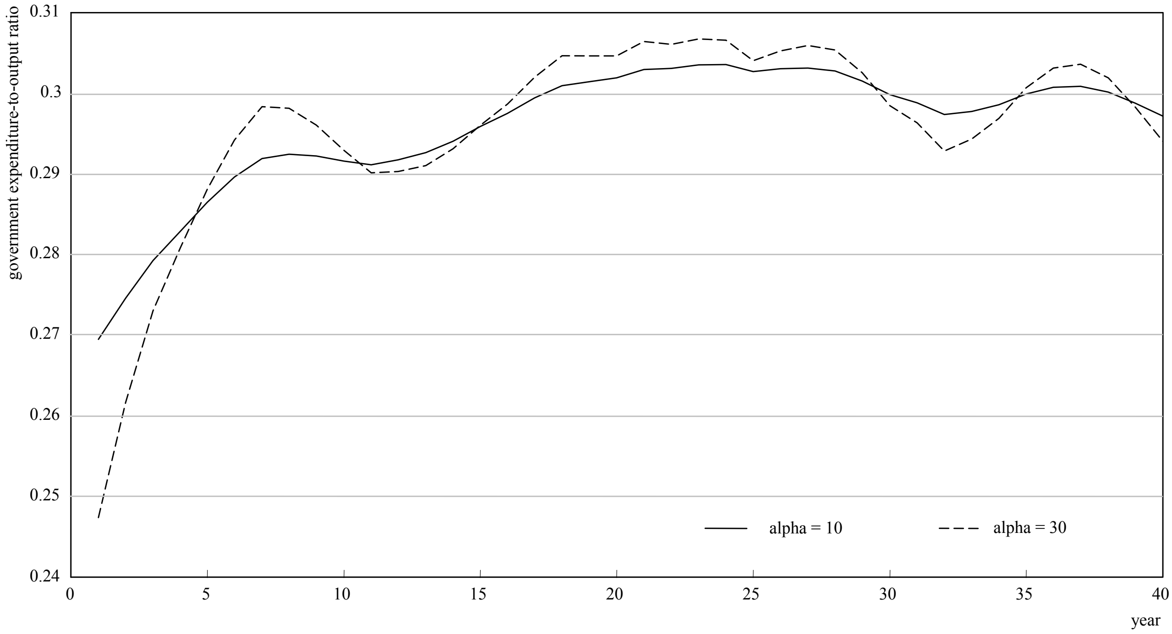


Figure 4

Government Debt-to-output Ratio During Transition with Actual Growth Numbers

Debt-to-output Ratio for Different Values of α

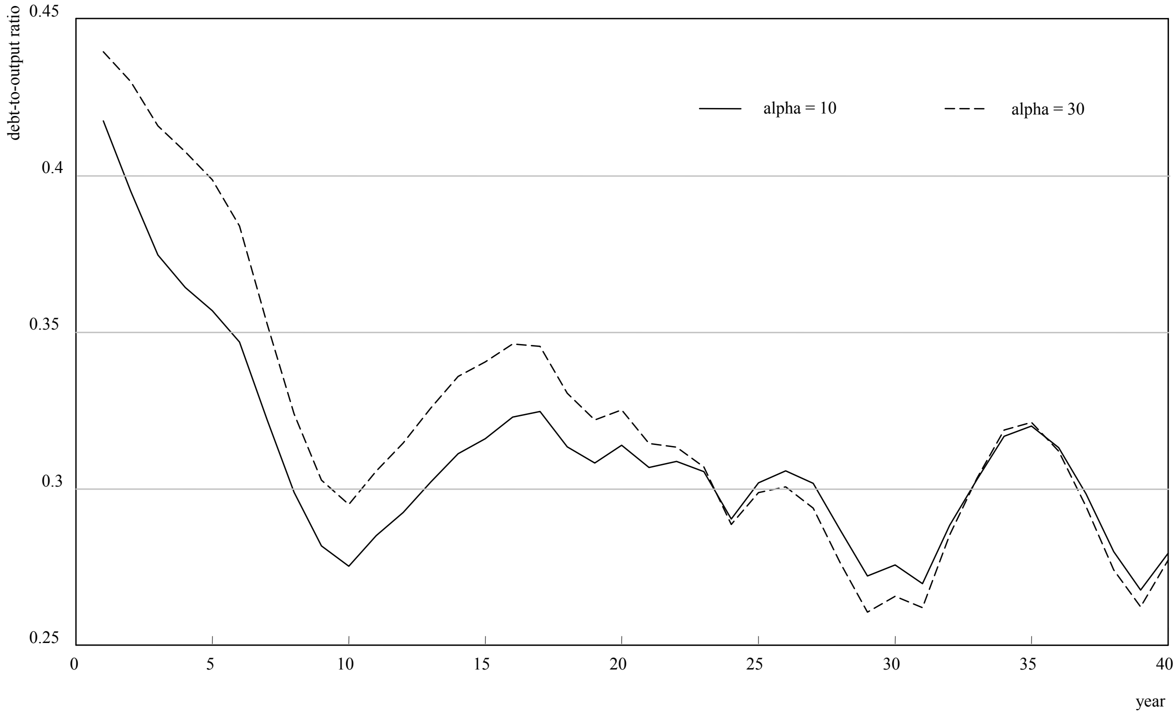


Figure 5

Expenditure-to-output Ratio Optimal Versus Sample Fiscal Rule Compared

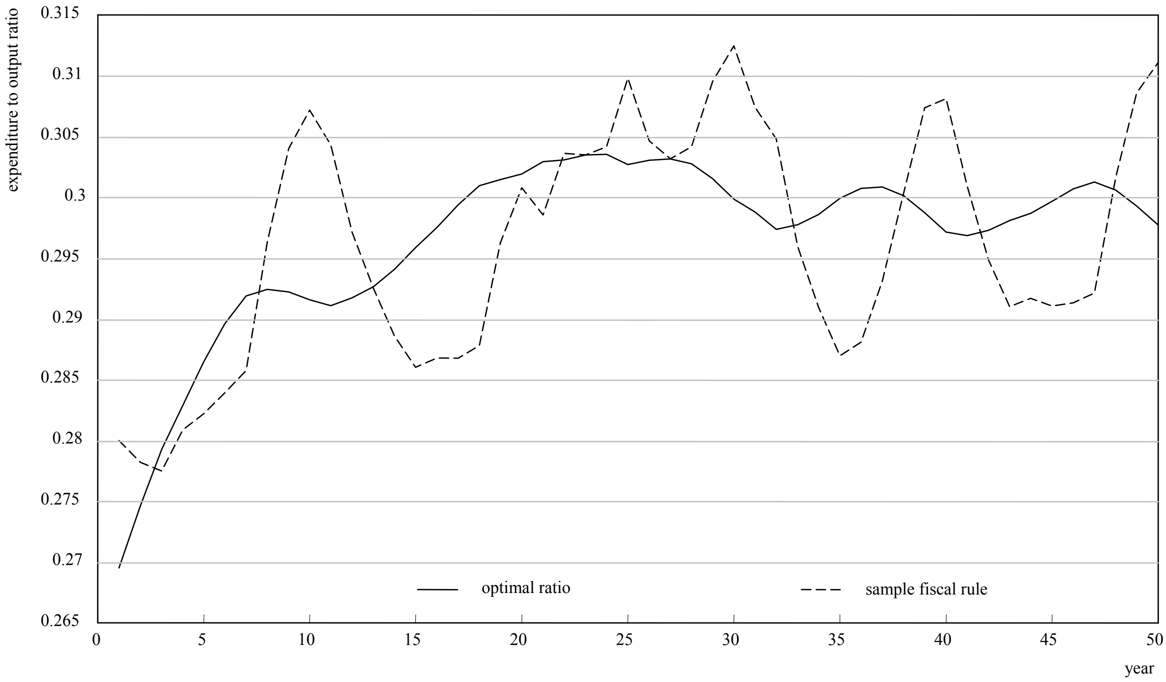


Figure 6

Optimal Debt-to-output Ratio Compared to the Sample Fiscal Rule

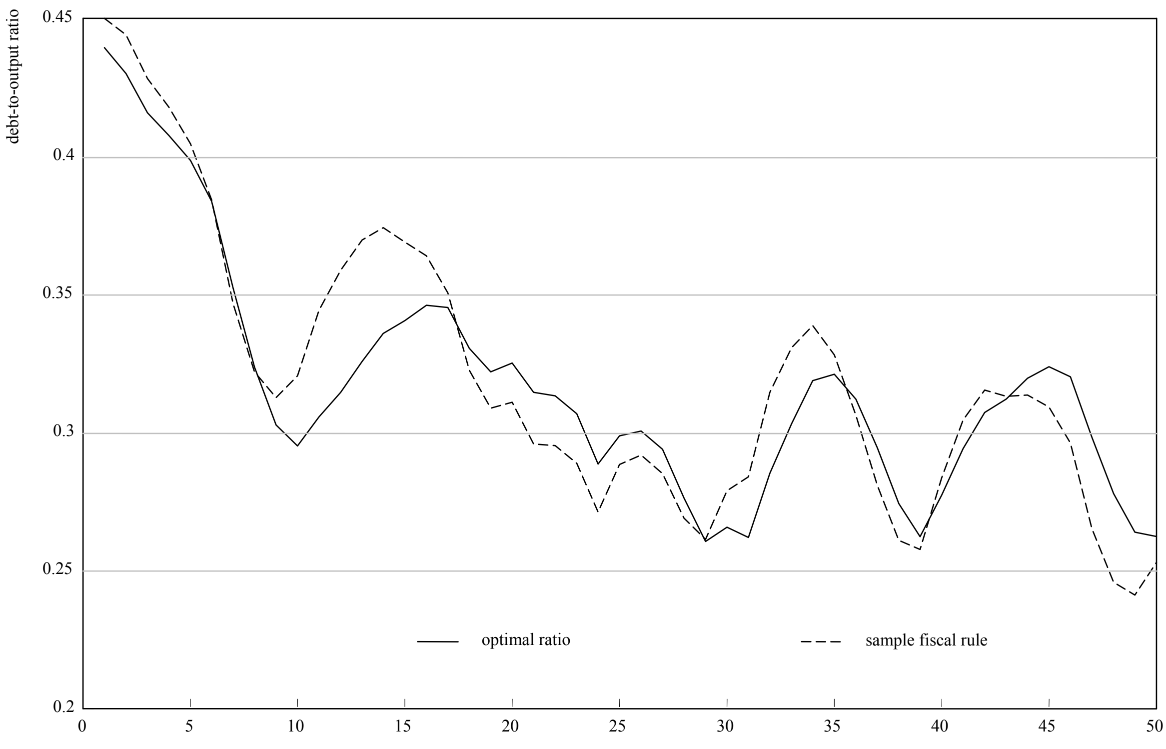


Table 1

	α_g	α_b	ϕ_g	ϕ_d
Sample fiscal rule	–0.33	0.75	1	1
Optimal linear rule	–0.24	0.86	1	1
Optimal non-linear rule	–0.21	0.98	1.13	0.88

Table 2

	Loss Function	Std. of Govt. Exp.	Std. of Govt. Debt
Optimal solution	0.1915	0.31	2.42
Sample fiscal rule	0.2494	0.80	3.17
Optimal linear rule	0.2217	0.55	2.84
Optimal non-linear rule	0.2115	0.39	2.79

paths for the debt to output ratio derived from the optimal solution and from the fiscal rule. The transition takes longer for the fiscal rule yet, there is not much difference in terms of volatility of the fluctuations between the optimal solution and the sample fiscal rule. Figure 5 and 6 show that in terms of debt to output ratio the sample fiscal rule performs quite similar to the optimal solution, however in terms of expenditure to output ratio its performance is not that satisfactory. The large fluctuations in the expenditure to output ratio may create significant burden on the governments trying to follow the considered sample rule, which may undermine the credibility of the program.

3.1 Optimal linear and non-linear rules

Consider the linear and non-linear rules of the following forms.

The linear rule:

$$d_t = d^* - \alpha_g(g_t - g^*) + \alpha_d(d_{-1} - d^*),$$

The non-linear rule:

$$d_t = d^* - \alpha_g(g_t - g^*)^{\phi_g} + \alpha_d(d_{-1} - d^*)^{\phi_d}$$

For the linear and the non-linear rules, optimal parameter values that jointly minimize the loss function are computed for $\alpha = 30$. Table 1 gives the parameter values for the optimal linear and non-linear rules. Notice that for the optimal non-linear rule the elasticity parameter of the growth term implies a convex (>1) where as the elasticity parameter of the debt term implies a concave (<1) relationship.

Table 2 gives the value of the loss function, standard deviation of government expenditures and the standard deviation of government debt for the optimal solution, the sample fiscal rule, optimal linear rule, and the optimal non-linear rule, respectively. Notice that the optimal linear rule improves the loss function significantly compared to the sample fiscal rule. Similarly, the volatility of the government expenditures is significantly reduced through optimal linear and non-linear rules. However, the volatility of government debt has not improved that significantly.

The paths of expenditure to output ratio are plotted for the optimal solution, the sample fiscal rule, the optimal linear and optimal non-linear rules respectively in Figure 7. The optimal non-linear rule notably reduces the magnitude of the fluctuations in the government expenditure to output ratio, making its application relatively comfortable for the government. Next, Figure 8 exhibits the paths of debt to output ratio for different rules. The paths do not differ from each other significantly.

The value of the loss function for different values of alpha in the range $\alpha \in [0.1, 60]$ is shown in Figure 9. By definition, the loss function is at minimum for all values of alpha for the optimal solution. The loss function is at maximum for the sample fiscal rule. Notice that for high values of α the loss function for the optimal non-linear rule approaches to the loss function of the optimal solution.

3.2 How robust are the parameter values to the value of alpha?

The optimal parameter values for the linear and non-linear rules are shown for different values of alpha in Figure 10. It is seen that optimal parameter values are quite robust to the political preference parameter α. The value range for α is [0.1,60] with increments of 0.1. This is a rather encouraging result, since the optimal fiscal rule seems to be almost independent of government's preference of α. Figure 11 plots the standard deviation of government expenditure derived from different rules for different α values. It is seen that for all values of alpha in the range the volatility of government spending is significantly lower for the optimal linear and non-linear fiscal rules.

Similarly, Figure 12 exhibits the standard deviation of debt to output ratio from different fiscal rules for the wide range of α. Notice that for reasonable values of α, *i.e.* $\alpha > 20$, in fact the optimal non-linear rule outperforms even the optimal solution in the dimension of debt volatility.

3.3 How robust are the results to the data starting point?

Since we are using actual growth data, the results may depend on the data starting point. Starting points have no significance for our study, therefore we need to show that the results are robust to different data starting points. To that end, we computed the parameters of the optimal non-linear fiscal rule for different starting points. In Figure 13, using each year in the 40 year growth data as the starting point, computed parameters are shown. It is seen that the parameters are relatively robust to the data starting point.

3.4 How robust are the results to other shocks?

In addition to shocks to output, other shocks like shocks to government expenditures and interest rate shocks may also be important. Here we add exogenous government expenditure shocks and interest rate shocks to the analysis. We use identically and independently distributed shocks with some persistence. Shocks are assumed to persist for two periods. We introduce these shocks in the following way so that the problem of the government now reads:

Figure 7

Government Expenditure-to-output Ratio for Different Rules

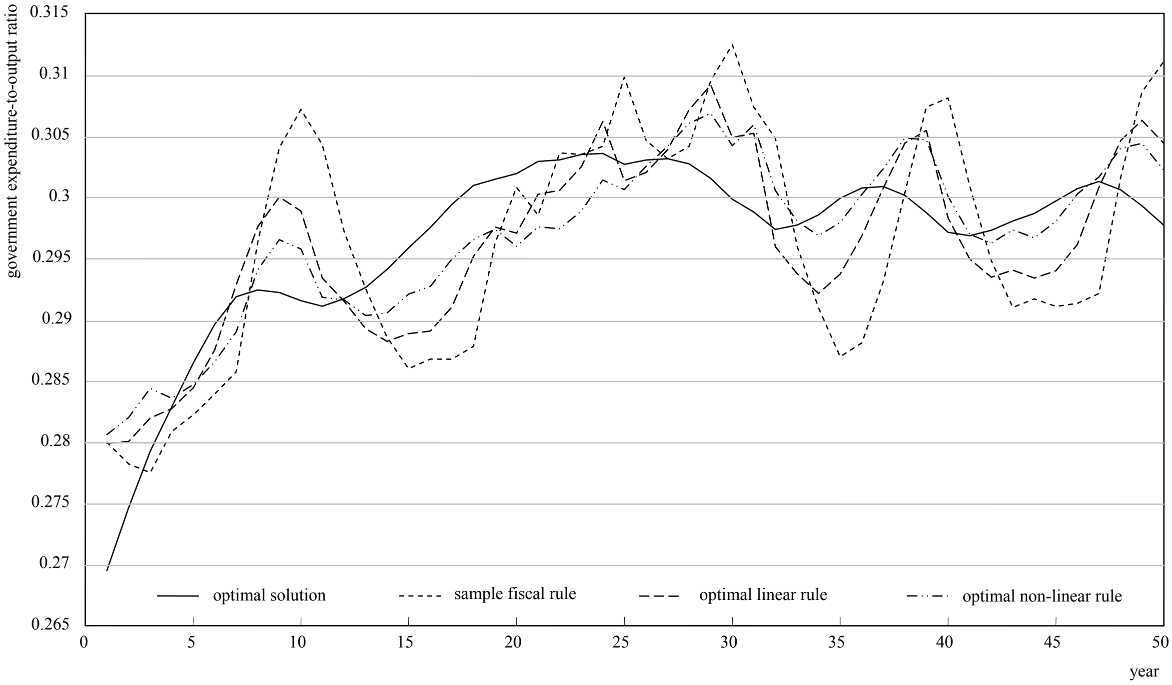


Figure 8

Debt-to-output Ratio, Different Fiscal Rules Compared

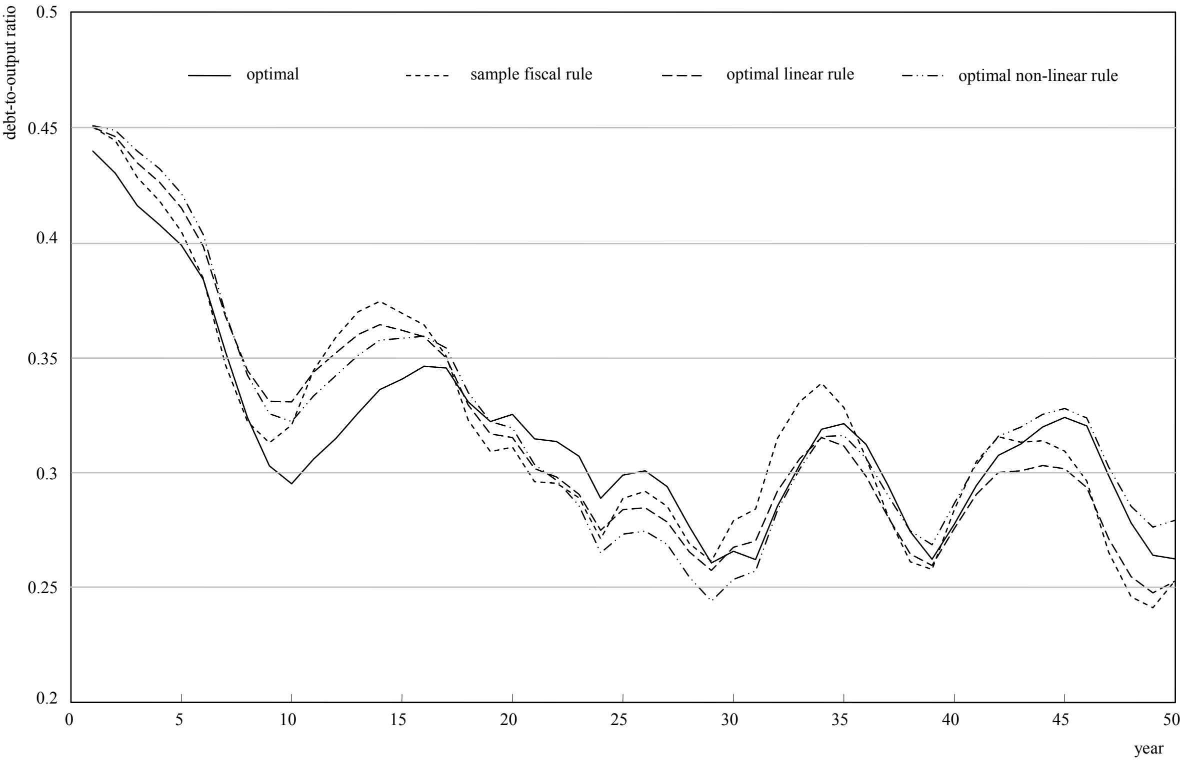


Figure 9

Value of the Loss Function for Different Rules for Different Levels of α

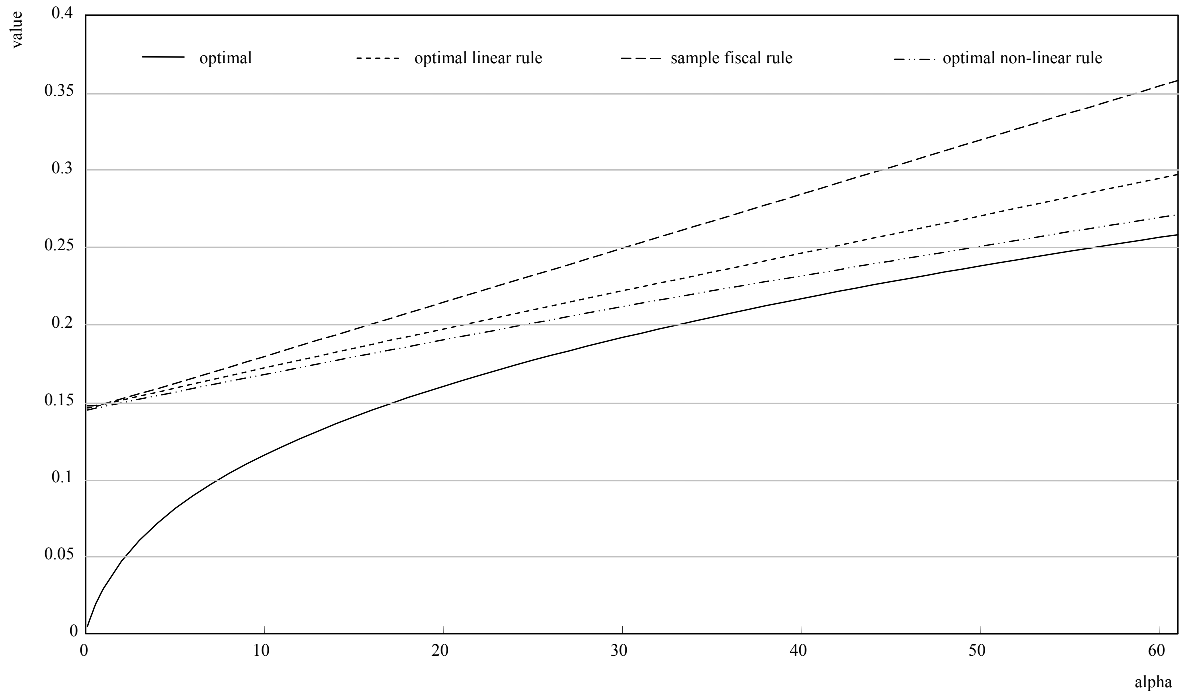


Figure 10

Optimal Parameter Values for Different α

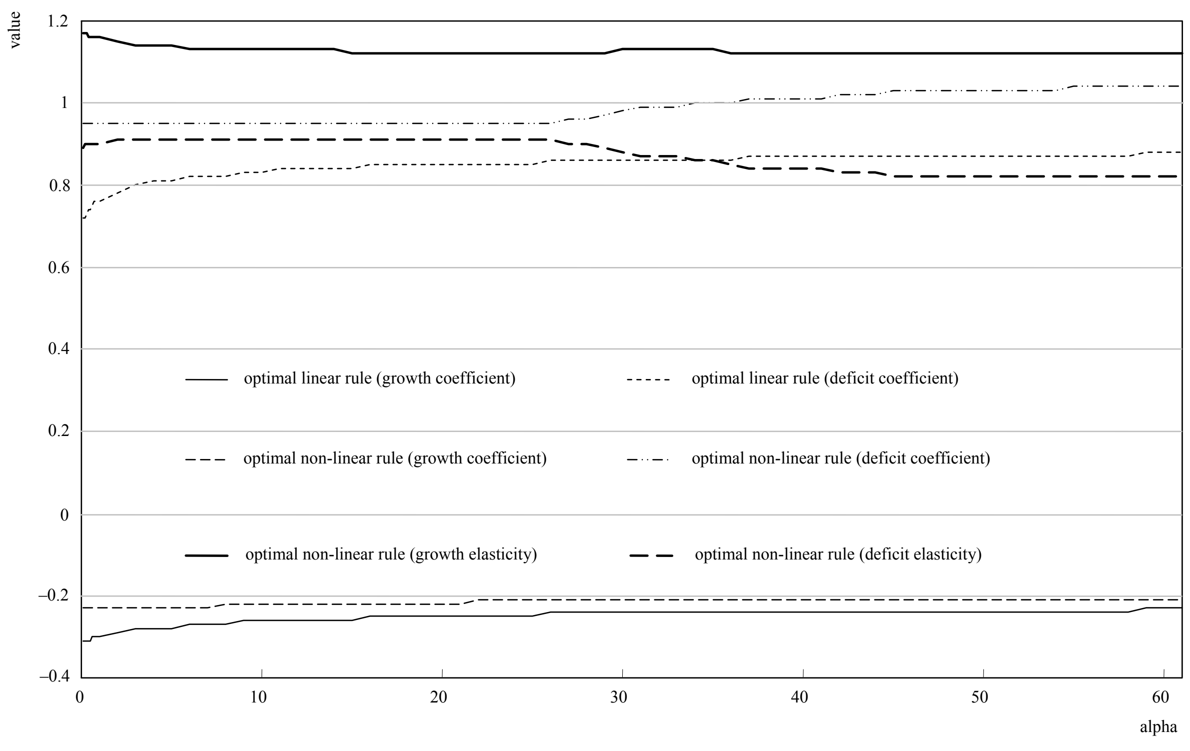


Figure 11

Standard Deviation of Government Expenditure for Different α

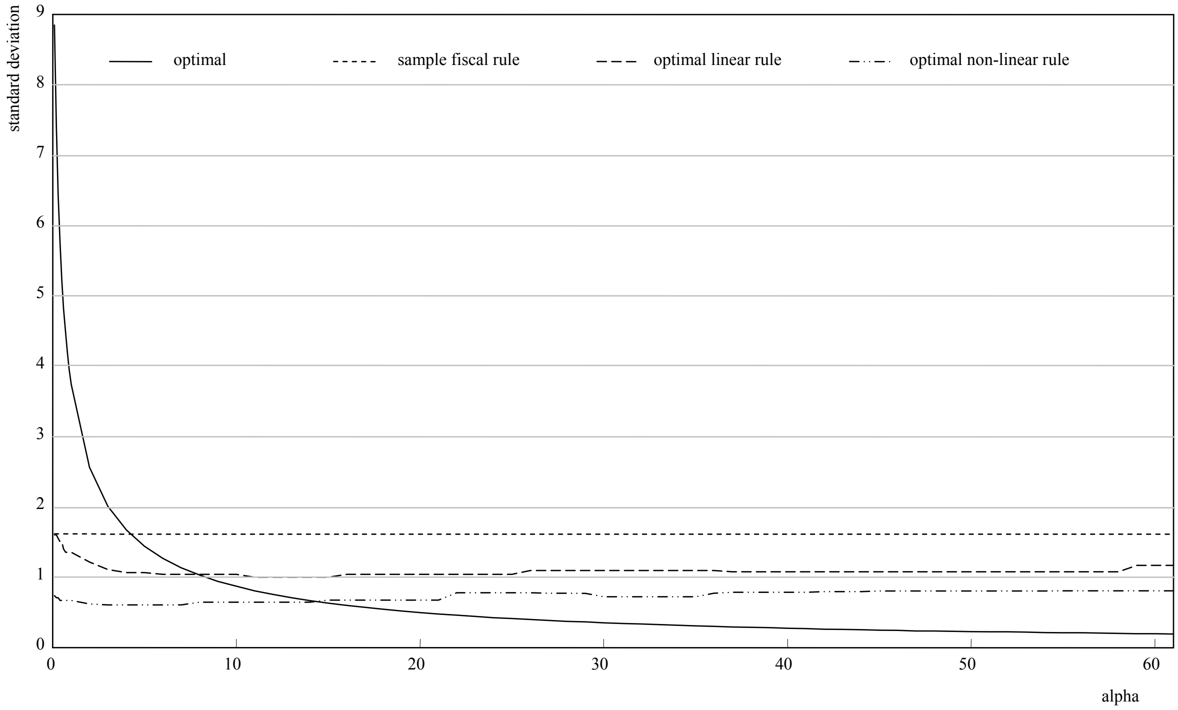


Figure 12

Standard Deviation of Debt-to-output Ratio for Different α

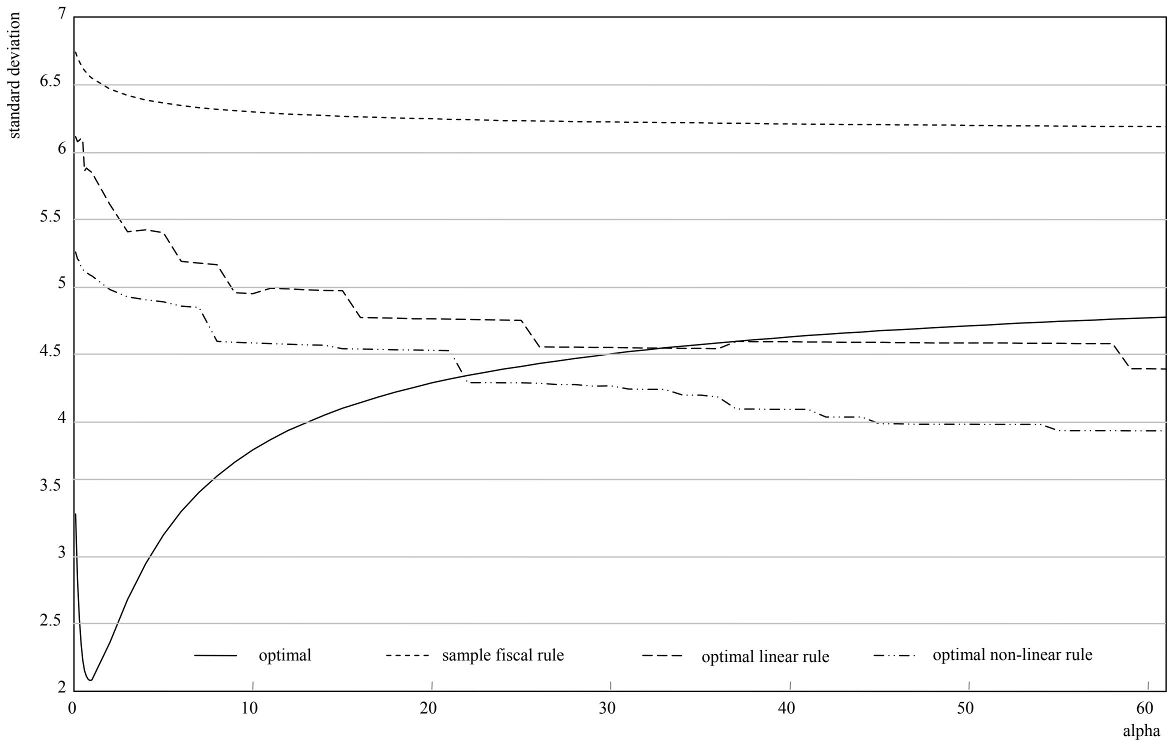
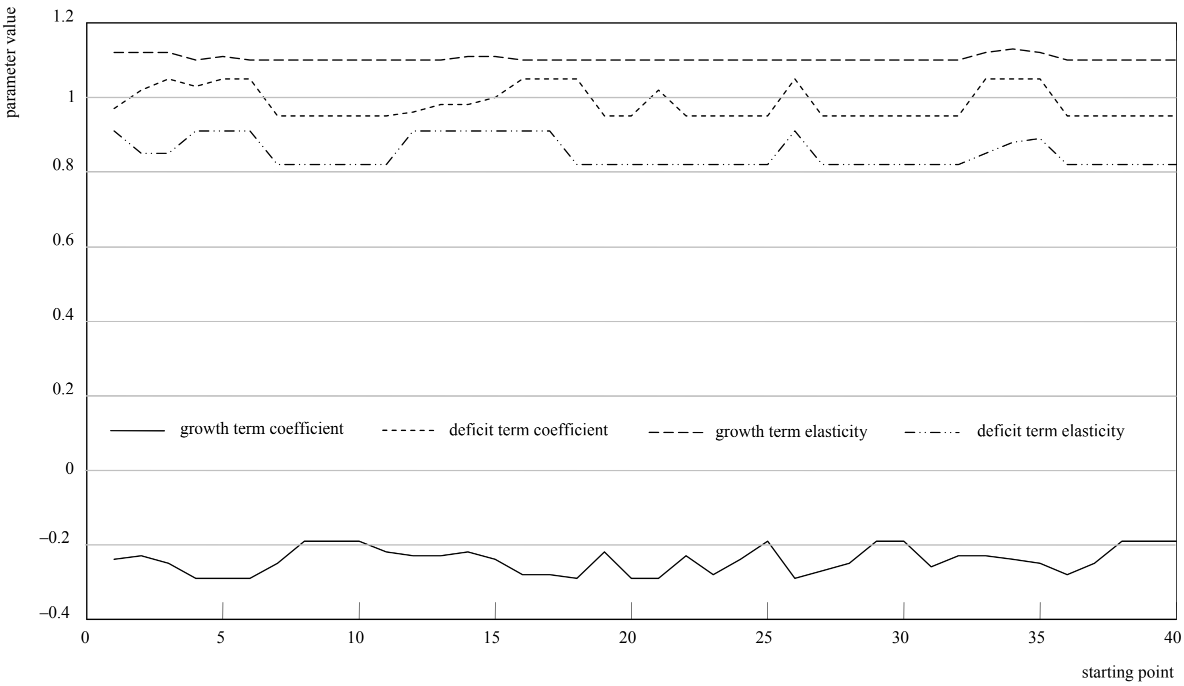


Figure 13

Sensitivity of Parameter Values for the Optimal Non-linear Rule



$$L = \min_{\{\hat{g}_t, \hat{b}_t\}_{t=1}^{\infty}} \left\{ \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} \left[\alpha_g \left(\hat{g}_t - \underbrace{z_t g^*}_{\text{expenditure shock}} \right)^2 + \alpha_b \left(\frac{\hat{b}_t}{\hat{y}_t} - b^* \right)^2 \right] \right\},$$

subject to:

$$\hat{g}_t = \tau \hat{y}_t + \hat{b}_t - (1 + r_t)\frac{\hat{b}_{t-1}}{\gamma},$$

z_t is iid with mean 1,

$$r_t = \underbrace{x_t r}_{\text{interest rate shock}}, \; x_t \text{ is iid with mean 1,}$$

$\hat{b}_0$ given.

Again, for the linear and the non-linear rule, optimal parameter values that jointly minimize the loss function are found for $\alpha = 30$. Table 3 gives the parameter values for the optimal linear and non-linear rules with government expenditure and interest rate shocks.

Notice that adding government expenditure and interest rate shocks does not change the values of optimal parameters for the linear and non-linear rules significantly (compare Table 1 with Table 3).

Table 3

	α_g	α_b	ϕ_g	ϕ_b
Sample fiscal rule	–0.33	0.75	1	1
Optimal linear rule	–0.26	0.86	1	1
Optimal non-linear rule	–0.23	1.00	1.14	0.89

4 Conclusions

- The form of the sample fiscal rule considered (including terms with deviations from potential growth and deviations from ideal deficit level) is successful, but the parameters can be significantly optimized using Turkish growth data.
- Optimized linear rule significantly improves the loss function compared to the sample fiscal rule. Volatility (standard deviation) of government expenditures is drastically reduced by the optimized rule – by more than 30 per cent (from 0.8 to 0.55 per cent), making the rule much easier to apply politically for the government, hence increases the credibility of the applicability of the rule. Volatility of government debt is reduced by around 10 per cent through the optimized linear rule. Hence much of the improvement comes through the smoother government expenditure profile achieved.
- An optimized non-linear rule can further improve performance significantly. Although the optimized elasticity parameters (powers) of the non-linear rule are close to one (close to linear), using optimized non-linear rule reduces the loss function significantly. Compared to the sample fiscal rule considered, using the optimal non-linear rule reduces the volatility (standard deviation) of the government expenditures by more than 50 per cent (from 0.8 to 0.39 per cent). The volatility of government debt is reduced by around 15 per cent. Therefore optimal non-linear rule can improve the performance of the fiscal rule very significantly.
- The optimal parameter values for the linear and non-linear rules do not depend on the value of the political preference parameter, α. This is a very encouraging result since it implies that our results are robust to government preferences. Hence we don't need to know the government's exact preference about α to come up with the optimal fiscal rule.
- Adding government expenditure and interest rate shocks to the environment does not change the optimal parameter values for the linear and non-linear rules either. Hence the results are robust to potential alternative sources of shocks too.
- The last two robustness results increase the applicability and credibility of the optimal rules.