

**FISCAL AND MONETARY POLICY INTERACTION:  
A SIMULATION-BASED ANALYSIS OF A TWO-COUNTRY  
NEW KEYNESIAN DSGE MODEL WITH HETEROGENEOUS HOUSEHOLDS**

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*This paper models a fiscal policy that pursues primary balance targets to stabilize the debt-to-GDP ratio in an open and heterogeneous economy where firms combine public and private capital to produce their goods. The model extends the European NAWM presented in Coenen et al. (2008) and Christoffel et al. (2008) by broadening the scope for fiscal policy implementation and allowing for heterogeneity in labor skills. The domestic economy is also assumed to follow a forward looking Taylor-rule consistent with an inflation targeting regime. We correct the NAWM specification of the final-goods price indices, the recursive representation of the wage setting rule, and the wage distortion index. We calibrate the model for Brazil to analyze some implications of monetary and fiscal policy interaction and explore some of the implications of fiscal policy in this class of DSGE models.*

## **1 Introduction**

DSGE models are now part of the core set of tools used by major central banks to assess the widespread effects of policy making. Building mostly on the recent New Keynesian literature (Monacelli, 2005, Galí and Monacelli, 2008, Smets and Wouters, 2003, Adolfson *et al.*, 2007, among others), these models have been further enriched in several aspects by the inclusion of alternative pricing assumptions, imperfect competition in distinct economic sectors, international financial linkages, and financial frictions. However, as Ratto *et al.* (2009) argue, “so far, not much work has been devoted towards exploring the role of fiscal policy in the (*DSGE*) New-Keynesian model”.<sup>1</sup>

DSGE models are a promising tool to understand the outcome of interactions between fiscal and monetary policies. The recent trend in modeling the fiscal sector in New Keynesian DSGE models is to include non-Ricardian agents and activist fiscal policies (Gunter and Coenen, 2005; Mourougane and Vogel, 2008; and Ratto *et al.*, 2009) mostly to assess the effects of shocks to government consumption on the aggregate economy, as well as the distributional effects of fiscal policies. However, the practice of fiscal policy usually goes beyond the decisions on consumption expenditures. The government often intervenes in the economy through public investment with important externalities upon private investment.

Ratto *et al.* (2009) are a recent attempt to account for the strategic role of public investment in policy decisions in a DSGE setup. They introduce a rule for public investment that responds to the business cycle and assume that public capital interferes in the productivity of private firms, but does not belong to factor decisions.

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<sup>1</sup> Rato, Röger and in 't Veld (p. 222). The italics are ours.

In this paper, we depart from the assumption that public investment is a type of externality. We assume that firms can rent capital services from a competitive market of private and public capital goods. The optimal composition of capital services will depend on the elasticity of substitution between both types of capital goods and on a parameter that captures the economy's "dependence" on public infrastructure. Households and the government have different investment agenda, and are faced with distinct efficiency in the transformation of investment to capital goods.

The reasoning for introducing public capital goods in this manner can be rationalized as follows. In our model, intermediate goods firms are the entities that actually use public capital. In the real world, there are both (mixed-capital) firms and government agencies utilizing capital owned by the government. By letting public capital enter firms' decisions, we believe we are approximating our model to the reality of a mixed-capital economy. The production technology distinguishes between the quality of each type of capital, and as such, the demand for public capital reacts to deviations of its rental rate to the calibrated value, which we assume to be subsidized in the steady state. In the real world, the government makes decisions on investment, and the efficiency with which such investment is transformed into capital goods can differ from the efficiency of the private sector's investment. In our model we empowered our government to decide on its public investment.

Our model builds on ECB's New Area Wide Model (NAWM) presented in Coenen *et al.* (2008) and Christoffel *et al.* (2008), hereinafter referred to as CMS and CCW respectively. However, there are important distinctions. First, we change the fiscal set-up. In the ECB NAWM, government consumption and transfers follow autoregressive rules. In our model, we introduce a fiscal policy rule that tracks primary surplus targets, that responds to deviations on the debt-to-GDP ratio and that also portrays an anti-cyclic response to economic conditions. In addition, we let fiscal transfers to be biased in favor of one of the household groups, and also introduce government investment through an autoregressive rule that also pursues an investment target. With a rule for the primary surplus, for government transfers and for public investment, government consumption thus becomes endogenous. This framework better approximates the theoretical setting of these models to the current practice of fiscal policy in a number of countries, including Brazil.

Second, we augment the labor market by introducing heterogeneity in labor skills. In Brazil, labor contracts are not usually flexible as to adjustments in daily hours worked. The most usual contracts set an 8-hour workday. Therefore, it seems reasonable to allow for the possibility that members of different social classes in average earn different wages for the same amount of hours worked.

Third, we correct some equations shown in CMS and CCW. The first refers to the specification of consumer and investment price indices, which we correct to guarantee that the producers of final consumption and investment goods operate under perfect competition. These modifications yield a representation of the economy's resource constraint that also differs from the one presented in CMS and CCW. We also correct the recursive representation of the wage setting rule and the wage distortion index.

Fourth, we introduce a deterministic spread between the interest rates of domestically and internationally traded bonds to account for the risk premium that can be significant in emerging economies.

Finally, monetary policy in the domestic economy is modeled with a forward looking rule to better approximate the conduct of policy to an inflation targeting framework.

We calibrate the structural parameters of our model for the Brazilian economy and the rest of the world (USA+EURO), leaving the monetary and fiscal policy rules of the rest of the world as specified in CMS and CCW. We assess the impulse responses to arbitrary magnitudes of the shocks

and analyze the implications of the interaction between fiscal and monetary policies. In particular, we assess the macroeconomic and distributional effects of shocks to government investment, primary surplus, transfers, and monetary policy, and analyze the effects of concomitant shocks to the fiscal and monetary policy rules. We proceed with a sensitivity analysis of the impact of varying degrees of rigor in the implementation of the fiscal rule, of fiscal commitment to a sustainable path of the public debt, and of the commitment of the monetary policy to the inflation target.

The adopted calibration of fiscal and monetary policy rules lies in a region of monetary activeness and fiscal passiveness. However, the model also shows stable equilibria under alternative calibrations where, in contrast, monetary policy is passive and fiscal policy is active. Apart from the specifications where the fiscal rule has a mute response to the public debt, active fiscal policies bring about strong cyclicalities in the impulse responses.

One of the important contributions of this paper is to show that an expansionist shock to the primary surplus is not equivalent to a shock to government consumption, as the former attains with a mix of cuts in both government consumption and investment. We also show that each one of the fiscal shocks – primary surplus, government investment and government transfers – has a distinct impact on the model dynamics.

Under the calibrated model, a shock that reduces the primary surplus has very short lived expansionist effects on output growth. A government investment shock, on the other hand, initially depresses output growth, since compliance with the fiscal rule requires government consumption to reduce. However, the government investment shock enables output growth expansion still within the first year after the shock. The inflationary effects of the shocks to the primary surplus and to government investment are mild, yet relatively long-lived. Shocks to government transfers have very short lived effects on economic growth. With the fiscal rule in place, an increase in government transfers induces some reduction in government consumption, which presses down production. Under our calibration, the distributional effects of all fiscal shocks end up being small, contrary to the findings of CMS and CCW likely due to the specification we adopted for labor heterogeneity.

We also experiment with different specifications of monetary and fiscal policy rules, and show that they have important effects on the models' dynamic responses and predicted moments.

Higher commitment to the stabilization of the public debt strengthens the contractionist impact of the monetary shock. The volatility of consumer price inflation increases, as does the correlation between inflation and output growth. Strongly (and negatively) correlated policy shocks also dampen the contractionist effect of the monetary policy shock.

We find a degree of fiscal rigor that jointly minimizes the influence of the primary surplus shock on inflation and of the monetary policy on GDP growth. As expected, a more rigorous implementation of the primary surplus rule implies lower variance of inflation and output growth, and significantly increases the influence of the monetary policy shock onto the variances of consumer price inflation and output growth.

Increasing the monetary policy commitment to the inflation target significantly reduces the volatility of inflation and its correlation with output growth. The variance of output growth poses a mild reduction. However, a higher commitment to the inflation target results in a higher stake of the variance of inflation being explained by the fiscal shock.

The model is also simulated under alternative monetary policy rules. Augmenting the rule to include an explicit reaction to the exchange rate variability or the output growth adds sluggishness to the reversal of inflation to the steady state after a monetary policy shock. However, the initial impact of the shock onto the economic activity is milder (yet more persistent). By activating the

policy shocks only, the response to the exchange rate volatility reduces the variance of inflation, output growth and the exchange rate. The monetary policy shock has a smaller effect on output variation and gains influence on the volatility of inflation.

On the other hand, a monetary policy rule that responds to output growth reduces output growth volatility, but increases the variance of consumer price inflation and the exchange rate. Under this policy rule, a shock to monetary policy loses influence over inflation variance, but also reduces its stake in the variance of output growth and the exchange rate.

The paper is organized as follows. Section 2 provides an overview of the model, focusing on the extensions proposed to the NAWM. Section 3 details the calibration strategy and the normalization to attain stationary representations of the aggregated variables. Section 4 analyses the impulse responses of the model and experiments with distinct types of policy orientation. The last section concludes the paper.

## 2 The model

In the model, there are two economies of different sizes that interact in both goods and financial markets. Except for monetary and fiscal policy rules, both economies are symmetric with respect to the structural equations that govern their dynamics, but the structural parameters are allowed to differ across countries.

Each economy is composed of households, firms, and the government. Households are distributed in two continuous sets that differ as to their access to capital and financial markets, and also to their labor skills. Families in the less specialized group, hereinafter referred to as group  $I = [1-\omega, 1]$ , can smooth consumption only through non-interest bearing money holdings, whilst the other group of households in group  $I = [0, 1-\omega]$ , with more specialized skills, has full access to capital, and to domestic and international financial markets. The differentiation in households' ability to smooth consumption over time, a feature adopted in CMS and CCW, allows for breaking the Ricardian Equivalence in this model. Within their groups, households supply labor in a competitive monopolistic labor market to produce intermediate goods. There are Calvo-type wage rigidities combined with hybrid wage indexation rules.

Firms are distributed in two sets. The first produces intermediate goods for both domestic and foreign markets, and operates under monopolistic competition with Calvo-type price rigidities combined with hybrid price indexation. The other set is composed of three firms, each one of them producing one single type of final good: private consumption, public consumption, or investment goods. Final goods firms are assumed to operate under perfect competition.

The government comprises a monetary authority that sets nominal interest rates and issues money, and a fiscal authority that levies taxes on most economic activities, and endogenously adjusts its consumption expenditures to comply with its investment, distributional transfers, and primary surplus rules.

A detailed derivation of the model is available in Appendix H. In the remaining of this section, we correct important equations in CMS and CCW and model a fiscal sector that is more in line with the current practice of fiscal policy in a wide number of countries. Public investment has spillover effects over private investment and affects the market for capital goods.

## 2.1 Wage setting

Household  $i \in I = [0, 1 - \omega]$  chooses consumption  $C_{i,t}$  and labor services  $N_{i,t}$  to maximize the separable intertemporal utility with external habit formation:

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[ \frac{1}{1-\sigma} (C_{i,t+k} - \kappa C_{i,t+k-1})^{1-\sigma} - \frac{1}{1+\zeta} (N_{i,t+k}^{1+\zeta}) \right] \right\} \quad (1)$$

subject to the budget constraint:

$$\begin{aligned} & (1 + \tau_t^C + \Gamma_v(v_{i,t})) P_{C,t} C_{i,t} + P_{I,t} I_{i,H,t} + R_t^{-1} B_{i,t+1} \\ & + \left( (1 - \Gamma_{B^F}(B_{I,t}^F)) r p \cdot R_{F,t} \right)^{-1} S_t B_{i,t+1}^F + M_{i,t} + \Xi_{i,t} + \Phi_{i,t} \\ = & (1 - \tau_t^N - \tau_t^{W_h}) W_{i,t} N_{i,t} + (1 - \tau_t^K) [u_{i,t} R_{K,H,t} - \Gamma_u(u_{i,t}) P_{I,t}] K_{i,H,t} + \tau_t^K \cdot \delta \cdot P_{I,t} \cdot K_{i,H,t} \\ & + (1 - \tau_t^D) D_{i,t} + TR_{i,t} - T_{i,t} + B_{i,t} + S_t B_{i,t}^F + M_{i,t-1} \end{aligned} \quad (2)$$

where  $W_{i,t}$  is the wage earned by the household for one unit of labor services,  $I_{i,H,t}$  is private investment in capital goods,  $B_{i,t+1}$  are domestic government bonds,  $M_{i,t}$  is money,  $B_{i,t+1}^F$  are foreign private bonds,  $S_t$  is the nominal exchange rate,  $R_{F,t}$  is the interest rate of the foreign bonds,  $r p$  is the steady state spread between interest rates of domestically and internationally traded bonds,  $\Gamma_{B^F}(B_{I,t}^F)$  is an extra risk premium when the external debt deviates from the steady state,  $\Gamma_v(v_{i,t})$  is a transaction cost on consumption,  $v_{i,t}$  is the money-velocity of consumption,  $D_{i,t}$  are dividends,  $K_{i,H,t}$  is the private capital stock,  $u_{i,t}$  is capital utilization,  $\Gamma_u(u_{i,t})$  is the cost of deviating from the steady state rate of capital utilization,  $R_{K,H,t}$  is the gross rate of the return on private capital,  $TR_{i,t}$  are transfers from the government,  $\Xi_{i,t}$  is a lump sum rebate on the risk premium introduced in the negotiation of international bonds, and  $\Phi_{i,t}$  is the stock of contingent securities negotiated within group  $I$ , which act as an insurance against risks on labor income. Taxes are  $\tau_t^C$  (consumption),  $\tau_t^N$  (labor income),  $\tau_t^{W_h}$  (social security),  $\tau_t^K$  (capital income),  $\tau_t^D$  (dividends) and  $T_{i,t}$  (lump sum, active only for the foreign economy). The parameter  $\kappa$  is the external habit persistence,  $\beta$  is the intertemporal discount factor,  $\frac{1}{\sigma}$  is the intertemporal elasticity of consumption substitution,  $\frac{1}{\zeta}$  is the elasticity of labor effort relative to the real wage, and  $\delta$  is the depreciation of capital. Price indices are  $P_{C,t}$  and  $P_{I,t}$ , the prices of final consumption and investment goods, respectively. Cost functions are detailed in Appendix A.

Households in group  $J$  maximize a utility function analogous to (1), but constrained on their investment choices, allowed to transfer wealth from one period to another only through non-interest bearing money holdings.

Within each group, households compete in a monopolistic competitive labor market. By setting wage  $W_{i,t}$ , household  $i$  commits to meeting any labor demand  $N_{i,t}$ . Wages are set à la Calvo, with a probability  $(1 - \xi_J)$  of optimizing each period. Households that do not optimize

readjust their wages based on a geometric average of realized and steady state inflation

$\bar{W}_{i,t} := \left( \frac{P_{C,t-1}}{P_{C,t-2}} \right)^{\chi_I} \pi_C^{1-\chi_I} W_{i,t-1}$ . Optimizing households in group  $I$  choose the same wage  $\tilde{W}_{i,t}$ , which we denote  $\tilde{W}_{I,t}$ .

Household  $i$ 's optimization with respect to the wage  $\tilde{W}_{i,t}$  yields the first order condition, which is the same for every optimizing household:

$$E_t \left\{ \sum_{k=0}^{\infty} \left( \xi_I \beta \right)^k N_{i,t+k} \left[ \begin{array}{l} \Lambda_{i,t+k} \left( 1 - \tau_{t+k}^N - \tau_{t+k}^{W_h} \right) \frac{\tilde{W}_{I,t}}{P_{C,t+k}} \left( \frac{P_{C,t+k-1}}{P_{C,t-1}} \right)^{\chi_I} \pi_C^{(1-\chi_I)k} \\ - \frac{\eta_I}{\eta_I - 1} \left( N_{i,t+k} \right)^\zeta \end{array} \right] \right\} = 0 \quad (3)$$

where  $\frac{\Lambda_{i,t}}{P_{C,t}}$  is the Lagrange multipliers for the budget constraint, and  $\eta_I / (\eta_I - 1)$  is the after-tax real wage markup, in the absence of wage rigidity (when  $\xi_I \rightarrow 0$ ), with respect to the marginal rate of substitution between consumption and leisure. The markup results from the worker's market power to set wages.

Equation (3) can be expressed in the following recursive form, which corrects the one presented in CMS after including the multiplicative constant  $(1 - \omega)^\zeta$  on the left hand side. This constant arises from the labor demand equation:

$$(1 - \omega)^\zeta \left( \frac{\tilde{W}_{I,t}}{P_{C,t}} \right)^{1 + \eta_I \cdot \zeta} = \frac{\eta_I}{\eta_I - 1} \cdot \frac{F_{I,t}}{G_{I,t}} \quad (4)$$

where:

$$F_{I,t} := \left( \left( \frac{W_{I,t}}{P_{C,t}} \right)^{\eta_I} N_t^I \right)^{1 + \zeta} + \xi_I \cdot \beta \cdot E_t \left\{ \left( \frac{\pi_{C,t+1}}{\pi_{C,t}^{\chi_I} \cdot \pi_C^{1-\chi_I}} \right)^{\eta_I (1 + \zeta)} \cdot F_{I,t+1} \right\}$$

$$G_{I,t} := \Lambda_{I,t} \left( 1 - \tau_t^N - \tau_t^{W_h} \right) \left( \frac{W_{I,t}}{P_{C,t}} \right)^{\eta_I} N_t^I + \xi_I \cdot \beta \cdot E_t \left\{ \left( \frac{\pi_{C,t+1}}{\pi_{C,t}^{\chi_I} \cdot \pi_C^{1-\chi_I}} \right)^{\eta_I - 1} \cdot G_{I,t+1} \right\}$$

and  $N_t^I$  is households group  $I$  aggregate labor demanded by firms, and  $W_{I,t}$  is household group  $I$ 's aggregate wage index. Superscripts in the labor variable represent demand. Subscripts represent supply.

The derivation of equation (4) is detailed in Appendix B.

## 2.2 Production

There are two types of firms in the model: producers of tradable intermediate goods and

producers of non-tradable final goods.

### 2.2.1 Intermediate goods firms

A continuum of firms, indexed by  $f \in [0,1]$ , produce tradable intermediate goods  $Y_{f,t}$  under monopolistic competition. We depart from the set-up in CMS by introducing mixed capital as an input to the production of these goods. We assume that firms competitively rent capital services from the government,  $K_{G,f,t}^S$ , and from households in group  $I$ ,  $K_{H,f,t}^S$ , and transform them into the total capital input  $K_{f,t}^S$  through the following CES technology:

$$K_{f,t}^S = \left[ (1 - \omega_g)^{1-\eta_g} \cdot (K_{H,f,t}^S)^{\frac{\eta_g-1}{\eta_g}} + (\omega_g)^{1-\eta_g} \cdot (K_{G,f,t}^S)^{\frac{\eta_g-1}{\eta_g}} \right]^{\frac{\eta_g}{\eta_g-1}} \quad (5)$$

where  $\omega_g$  is the economy's degree of dependence on government investment, and  $\eta_g$  stands for the elasticity of substitution between private and public goods, and also relates to the sensitivity of demand to the cost variation in each type of capital.

In addition to renting capital services, intermediate goods firms hire labor  $N_{f,t}^D$  from all groups of households to produce the intermediate good  $Y_t$  using the technology:

$$Y_{f,t} = z_t \cdot (K_{f,t}^S)^\alpha \cdot (zn_t \cdot N_{f,t}^D)^{1-\alpha} - \psi \cdot zn_t \quad (6)$$

where  $\psi \cdot zn_t$  is a cost, which in steady state is constant relative to the output. The constant  $\psi$  is chosen to ensure zero profit in the steady state, and  $z_t$  and  $zn_t$  are respectively (temporary) neutral and (permanent) labor-augmenting productivity shocks that follow the processes:

$$\ln(z_t) = (1 - \rho_z) \cdot \ln(z) + \rho_z \cdot \ln(z_{t-1}) + \varepsilon_{z,t} \quad (7)$$

and:

$$\frac{zn_t}{zn_{t-1}} = (1 - \rho_{zn}) \cdot gy + \rho_{zn} \cdot \frac{zn_{t-1}}{zn_{t-2}} + \varepsilon_{zn,t} \quad (8)$$

where  $z$  is the stationary level of total factor productivity,  $gy$  is the steady state growth rate of labor productivity,  $\rho_z$  and  $\rho_{zn}$  are parameters, and  $\varepsilon_{z,t}$  and  $\varepsilon_{zn,t}$  are exogenous white noise processes.

In equilibrium,  $K_{f,t}^S = u_{I,t} K_{f,t}$ , where  $K_{f,t}$  is the stock of capital used by firm  $f$ .

For a given total demand for capital services, the intermediate firm minimizes the total cost of private and public capital services, solving:

$$\min_{K_{H,f,t}^S, K_{G,f,t}^S} R_{K,t}^H K_{H,f,t}^S + R_{K,t}^G K_{G,f,t}^S \quad (9)$$

subject to (5).

The rental rate on private capital services results from the equilibrium conditions in the private capital market. The rental rate on government capital services also results from equilibrium conditions, this time in the market for government capital goods, but, in steady state, we calibrate  $\omega_g$  in order to have the rental rate of public capital goods exclusively covering expenses with capital depreciation, so as to portrait the idea that public capital is usually subsidized.

First order conditions to this problem yield the average rate of return on capital and the aggregate demand functions for each type of capital goods services:

$$R_{K,t} = \left( (1 - \omega_g) \cdot (R_{K,t}^H)^{1-\eta_g} + \omega_g \cdot (R_{K,t}^G)^{1-\eta_g} \right)^{\frac{1}{1-\eta_g}} \quad (10)$$

$$K_{G,t}^S = \omega_g \left( \frac{R_{G,t}}{R_{K,t}} \right)^{-\eta_g} K_t^S \quad (11)$$

$$K_{H,t}^S = (1 - \omega_g) \left( \frac{R_{H,t}}{R_{K,t}} \right)^{-\eta_g} K_t^S \quad (12)$$

All firms are identical since they solve the same optimization problem. The aggregate composition of capital services rented by intermediate goods firms can be restated by suppressing the subscript “ $f$ ” from (5), using (10), and aggregating the different types of capital services across firms:

$$K_t^S = \left( (1 - \omega_g)^{1/\eta_g} (K_{H,t}^S)^{\frac{\eta_g-1}{\eta_g}} + \omega_g^{1/\eta_g} (K_{G,t}^S)^{\frac{\eta_g-1}{\eta_g}} \right)^{\frac{\eta_g}{\eta_g-1}} \quad (15)$$

We also depart from CMS by introducing differentiated labor skills in the model. We reason that individuals with a lower degree of formal education are usually more constrained on their ability to analyze more sophisticated investment possibilities. In addition, it also seems reasonable to hypothesize that individuals with a lower degree of education will also have lower level of labor skills. Therefore, we make the assumption that the group of households that is investment-constrained in our model also has lower labor skills. This modeling strategy allows for a steady state where skillful workers can earn more yet working the same amount of hours as the less skilled. In addition to the labor differentiation arising from the assumption of monopolistic competition in the labor market, the non-homogeneity that we introduce here within household groups generates important differences in the impulse-responses of the model compared to CMS, as we show in Section 4.

The labor input used by firm  $f$  in the production of intermediate goods is a composite of labor demanded to both groups of households. In addition to the population-size adjustment ( $\omega$ ) that CMS add to the firm’s labor demand, we add the parameter  $v_\omega \in [0, 1/\omega]$  to introduce a bias in favor of more skilled workers. The resulting labor composite obtains from the following transformation technology:

$$N_{f,t}^D := \left( (1 - v_\omega \omega)^{1/\eta} (N_{f,t}^I)^{1-1/\eta} + (v_\omega \omega)^{1/\eta} (N_{f,t}^J)^{1-1/\eta} \right)^{\eta/(\eta-1)} \quad (14)$$

where:



$$N_{f,t}^I := \left[ \left( \frac{1}{1-\omega} \right)^{1/\eta_I} \int_0^{1-\omega} (N_{f,t}^i)^{1-1/\eta_I} di \right]^{\eta_I/(\eta_I-1)} \quad (15)$$

$$N_{f,t}^J := \left[ \left( \frac{1}{\omega} \right)^{1/\eta_J} \int_{1-\omega}^1 (N_{f,t}^j)^{1-1/\eta_J} dj \right]^{\eta_J/(\eta_J-1)} \quad (16)$$

and where  $\eta$  is the price-elasticity to demand for specific labor bundles,  $\eta_I$  and  $\eta_J$  are the price-elasticities for specific labor varieties. The special case when  $v_\omega = 1$  corresponds to the equally skilled workers assumption, as in CMS.

Taking average wages ( $W_{I,t}$  and  $W_{J,t}$ ) in both groups as given, firms choose how much to hire from both groups of households by minimizing total labor cost  $W_{I,t}N_{f,t}^I + W_{J,t}N_{f,t}^J$  subject to (14). It follows from first order conditions that the aggregate wage is:

$$W_t = \left[ (1-v_\omega \cdot \omega) \cdot W_{I,t}^{1-\eta} + v_\omega \cdot \omega \cdot W_{J,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (17)$$

and the aggregate demand functions for each group of households are:

$$N_t^I = (1-v_\omega \cdot \omega) \cdot \left( \frac{W_{I,t}}{W_t} \right)^{-\eta} \cdot N_t^D \quad (18)$$

$$N_t^J = v_\omega \cdot \omega \cdot \left( \frac{W_{J,t}}{W_t} \right)^{-\eta} \cdot N_t^D \quad (19)$$

### 2.2.2 Final goods firms

As in CMS, there are three firms producing non-tradable final goods. One specializes in the production of private consumption goods, another in public consumption goods, and the third in investment goods. Except for the firm that produces public consumption goods, all final goods producers combine domestic and imported intermediate goods in their production. The differentiation of public consumption goods stems from the evidence that usually the greatest share of government consumption is composed of services, which are heavily based on domestic human resources.

The existence of an adjustment cost to the share of imported goods in the production of final goods invalidates the standard result that the Lagrange multiplier of the technology constraint equals the price index of final goods. In this new context, we derive below the price index of private consumption goods and investment goods to ensure that final goods firms operate under perfect competition. The pricing of public consumption goods is exactly the same as in CMS.

#### 2.2.2.a Private consumption goods

To produce private consumption goods  $Q_t^C$ , the firm purchases bundles of domestic  $H_t^C$  and foreign  $IM_t^C$  intermediate goods. Whenever it adjusts its imported share of inputs, the firm

faces a cost,  $\Gamma_{IM^C}(IM_t^C / Q_t^C)$ , detailed in Appendix A. Letting  $\nu_C$  denote the bias towards domestic intermediate goods, the technology to produce private consumption goods is:

$$Q_t^C := \left\{ (\nu_C)^{1/\mu_C} [H_t^C]^{1-1/\mu_C} + (1-\nu_C)^{1/\mu_C} \left[ (1-\Gamma_{IM^C}(IM_t^C / Q_t^C)) IM_t^C \right]^{1-1/\mu_C} \right\}^{\mu_C / (\mu_C - 1)} \quad (20)$$

where:

$$H_t^C := \left( \int (H_{f,t}^C)^{1-1/\theta} df \right)^{\theta / (\theta - 1)}$$

$$IM_t^C := \left( \int_0^1 (IM_{f^*,t}^C)^{1-1/\theta^*} df^* \right)^{\theta^* / (\theta^* - 1)}$$

The firm minimizes total input costs:

$$\min_{H_t^C, IM_t^C} P_{H,t} \cdot H_t^C + P_{IM,t} \cdot IM_t^C \quad (21)$$

subject to the technology constraint (20) taking intermediate goods prices as given.

The price index that results from solving this problem is:<sup>2</sup>

$$P_{C,t} = (\Omega_t^C)^{1-\mu_C} (\lambda_t^C)^{\mu_C} \quad (22)$$

where:

$$\lambda_t^C = \left[ \nu_C P_{H,t}^{1-\mu_C} + (1-\nu_C) (P_{IM,t} / \Gamma_{IM^C}^S (IM_t^C / Q_t^C))^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \quad (23)$$

$$\Omega_t^C = \left\{ \nu_C (P_{H,t})^{1-\mu_C} + (1-\nu_C) \left( \frac{\Gamma_{IM^C}^S (IM_t^C / Q_t^C)}{(1-\Gamma_{IM^C}^C (IM_t^C / Q_t^C))} \right) \right\}^{\frac{1}{1-\mu_C}} \times (P_{IM,t} / \Gamma_{IM^C}^S (IM_t^C / Q_t^C))^{1-\mu_C} \quad (24)$$

In CMS, the multiplier  $\lambda_t^C$  is assumed to be the price index for one unit of the consumption good. However, this result is not compatible with their assumption that final goods firms operate with zero profits.

Notice that only when  $\Omega_t^C = \lambda_t^C$  do we obtain  $P_{C,t} = \lambda_t^C = \Omega_t^C$ . This requires

$$\left( \frac{\Gamma_{IM^C}^S (IM_t^C / Q_t^C)}{(1-\Gamma_{IM^C}^C (IM_t^C / Q_t^C))} \right) = 1, \text{ a very specific case.}$$

In general, when this equality does not hold, first order conditions and equation (22) can be combined to yield the following demand equations:

<sup>2</sup> Details of the derivation of (22) are shown in Appendix D.

$$H_t^C = v_C \left( \frac{P_{H,t}}{\Omega_t^C} \right)^{1-\mu_C} \left( \frac{P_{H,t}}{P_{C,t}} \right)^{-\mu_C} Q_t^C \quad (25)$$

$$IM_t^C = (1-v_C) \left( \frac{P_{C,t}}{\Omega_t^C} \right)^{1-\mu_C} \left( \frac{P_{IM,t} / \Gamma_{IM^C}^S (IM_t^C / Q_t^C)}{P_{C,t}} \right)^{-\mu_C} \frac{Q_t^C}{1-\Gamma_{IM^C} (IM_t^C / Q_t^C)} \quad (26)$$

These demand equations are different from the ones in CMS, and, as we show in subsequent sessions, they also result in important differences in the market clearing equations. In particular, the equation for the aggregate resource constraint of the economy now resembles the usual representation of national accounts.

### 2.2.2.b Investment goods

The pricing problem of investment goods is analogous to that of consumer goods. The investment goods price index, which also differs from CMS, is:

$$P_{I,t} = (\Omega_t^I)^{1-\mu_I} (\lambda_t^I)^{\mu_I} \quad (27)$$

where:

$$\Omega_t^I = \left\{ v_I (P_{H,t})^{1-\mu_I} + (1-v_I) \left( \frac{\Gamma_{IM^I}^S (IM_t^I / Q_t^I)}{(1-\Gamma_{IM^I} (IM_t^I / Q_t^I))} \right) \right\}^{\frac{1}{1-\mu_I}} \times \left( P_{IM,t} / \Gamma_{IM^I}^S (IM_t^I / Q_t^I) \right)^{1-\mu_I} \quad (28)$$

and:

$$\lambda_t^I = \left[ v_I P_{H,t}^{1-\mu_I} + (1-v_I) (P_{IM,t} / \Gamma_{IM^I}^S (IM_t^I / Q_t^I))^{1-\mu_I} \right]^{\frac{1}{1-\mu_I}} \quad (29)$$

## 2.3 Fiscal authorities

The domestic fiscal authority pursues a primary surplus target ( $sp$ ), levies taxes on consumption, labor, capital and dividends, makes biased transfers, and adjusts expenditures and budget financing accordingly.

The primary surplus  $SP_t$  is defined as:

$$\begin{aligned} SP_t = & \tau_t^C P_{C,t} C_t + (\tau_t^N + \tau_t^{W_h} + \tau_t^{W_f}) W_t \cdot N_t^D \\ & + \tau_t^K (R_{H,t} u_{I,t} - (\Gamma_u(u_{I,t}) + \delta) \cdot P_{I,t}) K_{H,t} + \tau_t^D \cdot D_t \\ & + u_{I,t} \cdot R_{G,t} \cdot K_{G,t} - P_{G,t} G_t - TR_t - P_{I,t} \cdot I_{G,t} \end{aligned} \quad (30)$$

where  $\tau_t^C$ ,  $\tau_t^N$ ,  $\tau_t^{W_h}$ ,  $\tau_t^{W_f}$ ,  $\tau_t^K$ , and  $\tau_t^D$  are rates of taxes levied on consumption, labor income, social security from workers, social security from firms, capital and dividends.  $P_{G,t} G_t$  stands for aggregate expenditures with government consumption,  $TR_t$  stands for government transfers, and  $P_{I,t} \cdot I_{G,t}$  stands for aggregate expenditures with government investment.

Table 1

## Empirical Estimate of the Primary Surplus Rule in Brazil

Dependent Variable: PRI_SUR_PIB_SA					
Method: Least Squares					
Sample (adjusted): 1996Q3 2009Q1					
Included observations: 51 after adjustments					
Convergence achieved after 1 iteration					
PRI_SUR_PIB_SA = C(2)*PRI_SUR_PIB_SA(-1) + C(4)*PRI_SUR_PIB_SA(-2)					
+ (1-C(2) - C(4))*(C(1) + C(3)*(DLSP_PIB_SA(-1)-2.1214))					
+ C(5)*(PIB_TRIM_SA(-1)/100 - 0.004962932)					
Variable		Coefficient	Std. Error	t-statistic	Prob.
C(2)	$\rho_1$	0.248161	0.094789	2.618042	0.0119
C(4)	$\rho_2$	0.167091	0.083178	2.008836	0.0504
C(1)	$sp$	0.041899	0.004038	10.37669	0.0000
C(3)	$\phi_b$	0.040928	0.012266	3.336770	0.0017
C(5)	$\phi_{gy}$	<b>0.269544</b>	0.107748	2.501619	<b>0.0160</b>
R-squared		0.710078			
Adjusted R-squared		0.684868			

The realization of the primary surplus is affected by deviations of the public debt and economic growth from their steady-states ( $B_y$  and  $g_y$ , respectively):

$$sp_t = \rho_{1,sp} \cdot sp_{t-1} + \rho_{2,sp} \cdot sp_{t-2} + (1 - \rho_{1,sp} - \rho_{2,sp}) \cdot \{sp + \phi_{B_y} (b_{Y,t} - b_Y)\} + \phi_{g_y} (g_{Y,t-1} - g_Y) + \varepsilon_{sp,t} \quad (31)$$

where  $sp_t = \frac{SP_t}{P_{Y,t} \cdot Y_t}$ ,  $b_{Y,t} = \frac{B_t}{P_{Y,t-1} Y_{t-1}}$ ,  $g_{Y,t} = \frac{Y_t}{Y_{t-1}}$ , the unindexed counterparts are steady-state ratios, and  $\varepsilon_{sp,t}$  is a white noise shock to the primary surplus.

For industrialized economies, Cecchetti *et al.* (2010) do not find evidence of a response of the primary balance to economic conditions. For Brazil, our empirical estimates for the primary balance rule show a significant anti-cyclic component (Table 1), which is also addressed, yet in a different manner, in Ratto *et al.* (2009). Estimations of the rule with only one lag in the primary balance do not show well-behaved residuals.

In our calibrations, the foreign economy is represented by the USA and the Euro area. Therefore, for the foreign economy, we adopt CMS's assumption that the fiscal authority does not follow a primary surplus target, and government expenditures with consumption,

$g_t = \left( \frac{P_{G,t}}{P_{Y,t}} \right) \left( \frac{G_t}{Y_t} \right)$ , follow an autoregressive process:

$$g_t = (1 - \rho_g) \cdot g + \rho_g \cdot g_{t-1} + \varepsilon_{g,t} \quad (32)$$

where  $g$  is the steady state value of government expenditures as a share of GDP and  $\varepsilon_{g,t}$  is a white noise shock to government expenditures. Specifically for the foreign economy, we assume that lump sum taxes exist and follow an autoregressive process of the type:

$$\left( \frac{T_t}{P_{Y,t} \cdot Y_t} \right) := \phi_{B_Y} \left( \left( \frac{R_t^{-1} \cdot B_{t+1}}{P_{Y,t} \cdot Y_t} \right) - B_Y \right) \quad (33)$$

where  $B_Y$  is the steady state value of government bonds.

For both economies, government transfers follow the autoregressive process:

$$\left( \frac{TR_t}{P_{Y,t} \cdot Y_t} \right) = (1 - \rho_{tr}) \cdot tr + \rho_{tr} \cdot \left( \frac{TR_t}{P_{Y,t} \cdot Y_t} \right) + \varepsilon_{tr,t} \quad (34)$$

where  $tr$  is the steady state value of government transfers, and  $\varepsilon_{tr,t}$  represents a white noise shock to government transfers.

Total transfers are distributed to each household group according to:

$$TR_{I,t} := \frac{(1 - \omega \cdot v_{tr})}{1 - \omega} TR_t \quad (35)$$

$$TR_{J,t} := v_{tr} \cdot TR_t \quad (36)$$

where  $v_{tr}$  is the bias in transfers towards group  $J$ .

Government investment follows an autoregressive rule of the form:

$$ig_t = (1 - \rho_{ig}) ig + \rho_{ig} \cdot ig_{t-1} + \varepsilon_{ig,t} \quad (37)$$

and public capital accumulation follows the rule:

$$K_{G,t+1} = (1 - \delta) \cdot K_{G,t} + \left( 1 - \Gamma_I \left( \frac{I_{G,t}}{I_{G,t-1}} \right) \right) I_{G,t} \quad (38)$$

The government budget constraint is thus:

$$\begin{aligned} & \tau_t^C P_{C,t} C_t + (\tau_t^N + \tau_t^{W_h} + \tau_t^{W_f}) \cdot W_t \cdot N_t^D \\ & + \tau_t^K (R_{K,t} \cdot u_{I,t} - (\Gamma_u(u_{I,t}) + \delta) \cdot P_{I,t}) \cdot K_t \\ & + \tau_t^D \cdot D_t + T_t + R_t^{-1} \cdot B_{t+1} + M_t + u_{I,t} \cdot R_{G,t} \cdot K_{G,t} \\ & - P_{G,t} G_t - TR_t - B_t - M_{t-1} - P_{I,t} \cdot I_{G,t} = 0 \end{aligned} \quad (39)$$

with  $T_t = 0$  for the domestic economy, which, using the primary surplus definition, can be stated as:

$$SP_t = (B_t - R_t^{-1} \cdot B_{t+1}) - (M_t - M_{t-1}) \quad (39)'$$

This equation makes clear that, in this model, money not only has an effective role in real decisions, but also matters for the adjustment of fiscal accounts. Increased money supply can alleviate the financial burden from public debt, a feature that approximates the theoretical model to the real conduct of economic policy.

## 2.4 Monetary authorities

The domestic monetary authority follows a forward-looking interest rate rule that is compatible with an inflation targeting regime:

$$R_t^4 = \phi_{R1} \cdot R_{t-1}^4 + \phi_{R2} \cdot R_{t-2}^4 + (1 - \phi_{R1} - \phi_{R2}) \cdot \left[ R^4 + \phi_{\Pi} \left( \frac{P_{C,t+3}}{P_{C,t-1}} - \Pi \right) \right] + \phi_{g_Y} (g_{Y,t-1} - g_Y) + \varepsilon_{R,t} \quad (40)$$

where  $\Pi$  is the annual inflation target,  $R^4$  is the annualized quarterly nominal equilibrium interest rate, which satisfies  $R^4 = \beta^{-4} \cdot \Pi$ ,  $g_Y$  is the steady state output growth rate, and  $\varepsilon_{R,t}$  is a white noise shock to the interest rate rule. Empirical evidence in Brazil suggests the presence of two lags in the policy instrument.<sup>3</sup>

For the foreign economy we adopt the representation in CMS:

$$R_t^4 = \phi_R \cdot R_{t-1}^4 + (1 - \phi_R) \cdot \left[ R^4 + \phi_{\Pi} \left( \frac{P_{C,t}}{P_{C,t-3}} - \Pi_t \right) \right] + \phi_{g_Y} \left( \frac{Y_t}{Y_{t-1}} - g_Y \right) + \varepsilon_{R,t} \quad (41)$$

## 2.5 Aggregation and market clearing

Any aggregated model variable  $Z_t$  denoted in per capita terms results from the aggregation

$$Z_t := \int_0^1 Z_{h,t} dh = (1 - \omega) Z_{I,t} + \omega Z_{J,t} \text{ where } Z_{I,t} \text{ and } Z_{J,t} \text{ are the respective per capita values of } Z_t \text{ for families } I \text{ and } J. \text{ Details on the aggregation that do not substantially differ from CMS are not shown.}$$

There are important distinctions in the aggregate relations that obtain from this model as compared to those in CMS. The first refers to the wage dispersion index, and the second to the economy's resource constraint, which are detailed below.

There are important distinctions in the aggregate relations that obtain from this model as compared to those in CMS. The first refers to the wage dispersion index, and the second to the economy's resource constraint, which are detailed below.

### 2.5.1 Wage dispersion

The equilibrium conditions between supply ( $N_{i,t}$ ) and demand ( $N_t^i$ ) for individual labor are:

$$N_{i,t} = N_t^i := \int_0^1 N_{f,t}^i df \quad (42)$$

$$N_{j,t} = N_t^j := \int_0^1 N_{f,t}^j df \quad (43)$$

Aggregating the demand of all firms for labor services yields:

<sup>3</sup> See Minella and Souza-Sobrinho (2009).

$$N_{i,t} = \frac{1}{1-\omega} \left( \frac{W_{i,t}}{W_{I,t}} \right)^{-\eta_I} N_t^I \quad (44)$$

$$N_{j,t} = \frac{1}{\omega} \left( \frac{W_{j,t}}{W_{J,t}} \right)^{-\eta_J} N_t^J \quad (45)$$

which can also be represented, using the group-wise aggregated labor demand equations, as a function of total demand for labor by the intermediate firms:

$$N_{i,t} = \frac{1-\nu_\omega \cdot \omega}{1-\omega} \left( \frac{W_{i,t}}{W_{I,t}} \right)^{-\eta_I} \left( \frac{W_{I,t}}{W_t} \right)^{-\eta} \cdot N_t^D \quad (46)$$

$$N_{j,t} = \nu_\omega \left( \frac{W_{j,t}}{W_{J,t}} \right)^{-\eta_J} \left( \frac{W_{J,t}}{W_t} \right)^{-\eta} \cdot N_t^D \quad (47)$$

The aggregate supply of labor from each household group,  $N_{i,t}$  and  $N_{j,t}$ , relates to the labor demand as:

$$N_{I,t} := \frac{1}{1-\omega} \int_0^{1-\omega} N_{i,t} di = \frac{\psi_{I,t}}{1-\omega} \cdot N_t^I \quad (48)$$

$$N_{J,t} := \frac{1}{\omega} \int_{1-\omega}^1 N_{j,t} dj = \frac{\psi_{J,t}}{\omega} \cdot N_t^J \quad (49)$$

where  $\psi_{I,t} := \int_0^{1-\omega} \frac{1}{1-\omega} \left( \frac{W_{i,t}}{W_{I,t}} \right)^{-\eta_I} di$  and  $\psi_{J,t} := \int_{1-\omega}^1 \frac{1}{\omega} \left( \frac{W_{j,t}}{W_{J,t}} \right)^{-\eta_J} dj$  are the dispersion indices.

We show in Appendix E that the wage dispersion indices  $\psi_{I,t}$  and  $\psi_{J,t}$  can be stated in a recursive formulation that differs from the working paper version of CMS as to the term of current consumer-price inflation that does not show in our equation:<sup>4</sup>

$$\psi_{I,t} := (1-\xi_I) \cdot \left( \frac{\tilde{W}_{I,t}}{W_{I,t}} \right)^{-\eta_I} + \xi_I \cdot \left( \frac{\pi_{C,t-1}^{\chi_I} \pi_C^{1-\chi_I}}{\pi_{W_{I,t}}} \right)^{-\eta_I} \cdot \psi_{I,t-1} \quad (50)$$

$$\psi_{J,t} := (1-\xi_J) \cdot \left( \left( \frac{\tilde{W}_{J,t}}{P_{Y,t} \cdot Y_t} \right) \left( \frac{W_{J,t}}{P_{Y,t} \cdot Y_t} \right)^{-1} \right)^{-\eta_J} + \xi_J \cdot \left( \frac{\pi_{C,t-1}^{\chi_J} \pi_C^{1-\chi_J}}{\pi_{W_{J,t}}} \right)^{-\eta_J} \cdot \psi_{J,t-1} \quad (51)$$

where  $\pi_{W_{I,t}}$  and  $\pi_{W_{J,t}}$  stand for household  $I$  and  $J$  wage inflation rates.

Aggregating the labor supply from household groups  $I$  and  $J$ , using equations (48) and (49), results in:

$$N_{S,t} := \psi_{I,t} \cdot N_t^I + \psi_{J,t} \cdot N_t^J$$

<sup>4</sup> Equation A.9, WPS 747/ECB.

which relates to the aggregate labor demand and the total wage dispersion index as:

$$N_{S,t} = \psi_t \cdot N_t^D \quad (52)$$

where total wage dispersion is  $\psi_t := \left\{ (1 - \omega) \cdot \left( \frac{W_{I,t}}{W_t} \right)^{-\eta} \psi_{I,t} + \omega \cdot \left( \frac{W_{J,t}}{W_t} \right)^{-\eta} \psi_{J,t} \right\}$ .

### 2.5.2 Aggregate resource constraint

The price indices derived in the previous sessions entail representations for the aggregate resource constraint of the economy that are importantly different from the ones presented in CMS and CCW. Aggregating household and government budget constraints, and substituting for the equations of external financing and optimality conditions of firms, we obtain the aggregate resource constraint of the economy:

$$P_{Y,t} \cdot Y_t = P_{C,t} \cdot Q_t^C + P_{I,t} \cdot Q_t^I + P_{G,t} \cdot Q_t^G + S_t \cdot P_{X,t} \cdot X_t - P_{IM,t} \cdot IM_t \quad (53)$$

which, using the price indices derived above, can also be restated as:

$$P_{Y,t} \cdot Y_t = P_{H,t} \cdot H_t^C + P_{H,t} \cdot H_t^I + P_{H,t} \cdot H_t^G + S_t \cdot P_{X,t} \cdot X_t \quad (54)$$

Despite the fact that these representations are standard for national accounts, they differ from the respective equations derived in CMS<sup>5</sup> and CCW, as we detail in Appendix F.

## 3 Model transformation and steady state calibration

In this section we describe the transformation of variables that render the model stationary, and detail the steady state calibration.

As we assume a technology shock that permanently shifts the productivity of labor, all real variables, with the exception of hours worked, share a common stochastic trend. Besides, as the monetary authority aims at stabilizing inflation, rather than the price level, all nominal variables share a nominal stochastic trend.

The strategy consists of three main types of transformation. Real variables are divided by aggregate output ( $Y_t$ ), nominal variables are divided by the price of aggregate output ( $P_{Y,t}$ ) and the variables expressed in monetary terms are divided by  $P_{Y,t} \cdot Y_t$ .

Although most transformations are straightforward, some are not trivial. Predetermined variables, such as capital, are scaled by dividing their lead values by  $Y_t$ ; wages, domestic bonds, and internationally traded bonds are scaled by  $P_{Y,t} \cdot Y_t$ . In addition, in order to make the Lagrange multipliers compatible with the adopted scaling strategy, we multiply them by  $Y_t^\sigma$ , resulting in  $Y_t^\sigma \cdot \Lambda_{I,t}$  and  $Y_t^\sigma \cdot \Lambda_{J,t}$  for households  $I$  and  $J$ , respectively.

<sup>5</sup> Equation (38) in CMS.



Table 2

## Steady State Ratios

Ratio	Value		Description
	Brazil	Rest of the World	
$TB/P_Y Y$	0.012	0.00	Trade balance
$X/Y$	0.128	0.00	Exports
$IM/Y$	0.122	0.00	Imports
$M/P_Y Y$	0.205	1.24	Money
$ROG/P_Y Y$	0.000	0.0	Government budget
$P_I I_G/P_Y Y$	0.019	0.02	Government investment
$T/P_Y Y$	0.000	0.00	Lump-sum taxes
$B/P_Y Y$	2.121	2.79	Public Debt
$SP/P_Y Y$	0.036	-0.005	Primary Surplus
$D/P_Y Y$	0.0	0.0	Dividends
$P_I I_H/P_Y Y$	0.162	0.25	Private Investment

The permanent technology shock,  $zn_t$ , should also be divided by the aggregate output. Re-scaling the production function for the intermediate goods results in:

$$\left(\frac{zn_t}{Y_t}\right)^{-1} = z_t \cdot \left(u_{I,t} \cdot \frac{K_t}{Y_{t-1}}\right)^\alpha \cdot (N_t^D)^{1-\alpha} \cdot \left(\frac{Y_t}{Y_{t-1}}\right)^{-\alpha} \cdot \left(\frac{zn_t}{Y_t}\right)^{-\alpha} - \psi$$

From the above, we can conclude that  $\frac{zn_t}{Y_t}$  is a stationary variable whenever the ratios  $\frac{K_t}{Y_{t-1}}$  and

$\frac{Y_t}{Y_{t-1}}$  are both stationary.

We now turn to the steady state calibration. For the domestic economy, we calibrate the model to reproduce historical averages of the Brazilian economy during the inflation targeting regime (Table 2). For parameters that are not directly derived from the historical averages in these series, we took the agnostic stance of using the same parameters adopted in the literature for Brazil, or, in its absence, we replicated the parameters in CMS.<sup>6</sup> The rest of the world is calibrated using an average of the values presented in CMS for the United States and the Euro Area.

Calibration and simulations are performed under the assumption of log-linear utility ( $\sigma = 1$ ). The steady state calibration starts by normalizing the stationary prices of intermediate goods at 1.

<sup>6</sup> An alternative strategy would be to calibrate the parameters to reproduce empirical moments of the endogenous series. We leave this for a companion paper with an estimated version of the model.

This normalization ensures that the steady state values of some variables are one, as is the case of final goods prices and Lagrange multipliers associated with the optimization problem of final goods firms. The steady state rate of capital utilization is also fixed at one for both economies. The remaining steady state ratios are calibrated accordingly, as shown in Table 3.

We calibrate the population size using LABORSTA<sup>7</sup> data on the economically active population in the world for the year 2007. The size of household's group  $J$  in the domestic economy was set to equal the share of households in Brazil that earn less than two minimum wages according to the PNAD 2007 survey. Also according to this survey, relative wages for household group  $I$  were set in our calibrations at 2.86.

The share of fixed costs in total production was set so as to guarantee zero profits in the steady state. The labor demand bias,  $v_\omega$ , was calibrated to ensure that households' groups  $I$  and  $J$  work the same amount of hours. For the stationary labor productivity growth rate, we set 2 per cent for Brazil and the rest of the world using data on GDP growth from the World Bank for the period 2000-07.

For Brazil, we calibrated the price elasticity  $\mu_C = 0.33$  according to Araújo *et al.* (2006). For the price elasticity  $\mu_I$ , we repeated the value set for  $\mu_C$ . The home biases  $v_C$  and  $v_I$  are obtained from the demand equations of imported goods using the steady state value for the supply of consumption and investment goods, and the import quantum.

The steady state primary surplus to output ratio,  $sp$ , was calibrated as the mean value of the primary surplus in the period 1999-2008. For the rest of the world, the value for  $sp$  was obtained implicitly from the NAWM calibration. The public debt ratio  $B_Y$  was set to be consistent with  $sp$ .

Government expenditures,  $g$ , for both Brazil and the rest of the world were set residually from the aggregate resource constraint. Government transfers,  $tr$ , for both Brazil and the rest of the world, were obtained so that household budget constraints close.

With the exception of consumption taxes,  $\tau^C$ , which were calibrated following Siqueira *et al.* (2001), Brazilian tax rates were calibrated based on the current tax law. The lump-sum tax bias,  $v_{ip}$ , which is active only for the foreign economy, was set to one, whilst the transfer bias,  $v_{tr}$ , was implicitly calculated from households  $I$  and  $J$  budget constraints.

We calibrated the price-elasticity to demand of government investment goods,  $\eta_g$ , to a value that is close to 1, arbitrarily approximating it to a Cobb-Douglas technology. This enabled us to calibrate  $v_g$  from the rental rate on government capital, which we assumed to be just enough to cover expenditures with depreciation.

The inflation target and the respective steady state nominal interest rate in the domestic economy were set according to historical Brazilian averages. The reaction coefficients in the monetary policy rule were calibrated according to Minella and Souza-Sobrinho (2009), where they show that the monetary policy in Brazil has in average shown an insignificant direct reaction to output.

The parameter  $\gamma_{v,2}$  that appears in the functional form of the consumption transaction for the domestic economy was set at the same value calibrated in CMS. The parameter  $\gamma_{v,1}$  follows from

<sup>7</sup> <http://laborsta.ilo.org/>

Table 3

## Calibrated Parameters and Steady State Variables

Parameter	Value		Description
	Brazil	Rest of the World	
<b>A) Households</b>			
$S$	0.00478	0.99522	Population size
$\beta$	0.98183	0.99756	Subjective discount factor
$\sigma$	1.00000	1.00000	Inverse of the intertemporal elasticity of substitution
$\kappa$	0.23280†	0.60000	Degree of habit persistence
$\zeta$	1.59000‡	2.00000	Inverse of the Frisch elasticity of labor supply
$\delta$	0.02500	0.02500	Depreciation rate
$\omega$	0.59260	0.25000	Size of household $J$
$\xi_I, \xi_J$	0.48660†	0.75000	Fraction of household members not setting wages optimally each quarter
$\chi_I, \chi_J$	0.75000	0.75000	Degree of wage indexation for household members
<b>B) Intermediate-good firms</b>			
$\alpha$	0.30000	0.30000	Share of capital income in value added
$\psi$	0.14909	0.41200	Share of fixed cost in production
$Z$	1.00000	1.00000	Stationary total productivity level
$\rho_z$	0.89000‡	0.90000	Productivity parameter
$\eta$	6.00000	6.00000	Price elasticity of demand for labor bundles
$\eta_I$	6.00000	6.00000	Price elasticity of demand for labor of household $I$
$\eta_J$	6.00000	6.00000	Price elasticity of demand for labor of household $J$
$\xi_H$	0.90000	0.90000	Fractions of firms not setting prices optimally each quarter
$\xi_X$	0.30000	0.30000	Fractions of firms not setting prices optimally each quarter
$\chi_H, \chi_X$	0.50000	0.50000	Degree of price indexation
$\theta_T$	1.00500	1.00500	Stationary labour productivity growth rate
$\rho_{zn}$	0.90000	0.90000	Labor productivity parameter
$v_\omega$	0.00438	1.00000	Labor demand bias

Table 3 (continued)

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## Calibrated Parameters and Steady State Variables

Parameter	Value		Description
	Brazil	Rest of the World	
<b>C) Final-good firms</b>			
$u_C$	0.87500	0.99650	Home bias in the production of consumption final goods
$u_I$	0.74999	1.00750	Home bias in the production of investment final goods
$\mu_C, \mu_I$	3.33000	1.50000	Price elasticity of demand for intermediate-goods
$\theta$	7.60000‡	6.00000	Price elasticity of demand for a specific intermediate-good variety
<b>D) Fiscal authority</b>			
$B_T$	2.12140	2.78840	Government debt as a share of quarterly GDP in the steady state
$\phi_{By}$	0.0409	0.10000	Primary surplus reaction to debt-to-output in the domestic economy and sensitivity of lump-sum taxes to debt-to-output ratio in the foreign economy
$\phi_{By}$	0.2695	n/a	Primary surplus reaction to output growth
$g$	0.1992	0.11099	Government consumption of public goods in the steady state
$\rho_g$	n/a	0.90000	Parameter governing public consumption
$tr$	0.1526	0.29231	Public transfers-to-GDP in steady state
$\rho_{tr}$	0.37717	0.90000	Parameter governing public transfers
$\tau^C$	0.16200	0.18300	Consumption tax rate
$\tau^D$	0.15000	0.00000	Dividend tax rate
$\tau^K$	0.15000	0.18400	Capital income tax rate
$\tau^N$	0.15000	0.14000	Labour income tax rate
$\tau^{W_h}$	0.11000	0.11800	Rate of social security contributions by households
$\tau^{W_f}$	0.20000	0.21900	Rate of social security contributions by firms
$sp$	0.03600	(0.00541)	Stationary primary surplus to output ratio
$\rho_{1,sp}$	0.2481	0.90000	Parameter of the first autoregressive term in the primary surplus rule
$\rho_{2,sp}$	0.1671	n/a	Parameter of the second autoregressive term in the primary surplus rule
$u_{tr}$	1.01300	0.42668	Household J transfers bias
$\eta_{tr}$	1.00000	1.00000	Household J lump-sum tax bias
$u_g$	0.05198	0.05590	Government investment bias
$\eta_g$	1.00100	1.00100	Elasticity of substitution between government and private investment goods
$ig$	0.01860	0.02000	Government investment-to-output ratio target
$\rho_{ig}$	0.90000	0.90000	Parameter governing government investment-to-output ratio

Table 3 (continued)

## Calibrated Parameters and Steady State Variables

Parameter	Value		Description
	Brazil	Rest of the World	
<b>E) Monetary Authority</b>			
$\Pi$	1.04500	1.02000	Inflation target
$\phi_{R1}$	1.13‡	0.95000	Degree of interest-rate inertia
$\phi_{R2}$	-0.51‡	0.00000	Degree of interest-rate inertia
$\phi_{\Pi}$	1.57000‡	2.00000	Interest-rate sensitivity to inflation gap
$\phi_{gY}$	0‡	0.10000	Interest-rate sensitivity to output-growth gap
$R$	1.03490	1.01240	Equilibrium nominal interest-rate
$\pi_H$	1.01110	1.00500	Steady state domestic prices inflation
$\pi_X$	1.00500	1.01110	Steady state export prices inflation
$\pi_C$	1.01110	1.00500	Steady state consumption prices inflation
<b>F) Adjustment and transaction costs</b>			
$\gamma_{WA}$	0.01545	0.47073	Parameter of transaction cost function
$\gamma_{WB}$	0.15000	0.15000	Parameter of transaction cost function
$\gamma_{WA}$	0.05271	0.03409	Parameter of capital utilization cost function
$\gamma_{WB}$	0.00700	0.00700	Parameter of capital utilization cost function
$\gamma_I$	3.00000	3.00000	Parameter of investment adjustment cost function
$\gamma_{IM^C}$	2.50000	2.50000	Parameter of import adjustment cost function
$\gamma_{IM^F}$	0.00000	0.00000	Parameter of import adjustment cost function
$\gamma_{BF}$	0.01000	0.01000	Parameter of intermediation cost function

Notes: Areosa, Areosa and Lago (2006): †, Minella and Souza-Sobrinho (2009): ‡

the equation that defines the consumption transaction cost, the calibrated values for money and consumption, and the equation that defines the money velocity. Finally, some autoregressive coefficients ( $\rho_{zn}, \rho_{sp}, \rho_{ig}$ ) were set at 0.9 following the NAWM calibration for  $\rho_z$ . For autoregressive coefficients referring to government consumption and transfers,  $\rho_g$  and  $\rho_{tr}$ , we used estimated coefficients obtained from isolated econometric regressions for Brazil.

#### 4 Simulations and policy analysis

In this session, we show impulse responses for shocks to: monetary policy, primary surplus, government transfers and investment.<sup>8</sup> The intention here is to understand how this model responds to shocks under the adopted calibration. We compare the model's predictions for alternative types of primary surplus and monetary policy rules. All simulations were done using the function "stoch\_simul" of DYNARE at MATLAB.

##### 4.1 Impulse responses of the calibrated model

Figure 1 shows the impulse responses of a 1 percentage point shock to the nominal interest rate. With this calibration, the shock affects inflation and output in the expected direction, but we do not obtain a hump-shaped response.<sup>9</sup> The trough in inflation and output growth occurs already in the first quarter. Inflation reverts back to the steady state in the third quarter, while the nominal interest rate remains above the steady state for about one year. Output levels return to the steady state in about 6 quarters.

Despite the fact that each policy rule responds to a different set of variables, in equilibrium the fiscal response intertwines with monetary conditions, the key linking element being the public debt. The interest rate hike puts pressure on the public debt, which rises above its steady trend and takes very long to revert to the steady state. Notwithstanding, the anti-cyclic component of the fiscal rule forces the primary surplus to initially react to the economic downturn, and the fiscal rule loosens through a reduction in the primary surplus of about 0.05 percentage points of GDP from its steady state. This reaction is enabled by an increase in government consumption that should also offset the reduction in expenditures with government investment. In the third quarter, public debt to GDP reaches a peak, and the output growth surpasses its stationary rate. This development puts pressure on the fiscal rule for a rise in the primary surplus of up to 0.10 percentage points of GDP, through a reduction in government consumption and levels of government investment below the steady state for longer than private investment. Consequently, the debt initiates a downward path, yet still above its steady state for a long time afterwards.

The economy decelerates in the aftermath of a monetary policy shock. Capital utilization is below the steady state and firms pay lower nominal wages to households. The amount of labor and consumption also drops. The impact on private investment and the stock of capital is almost negligible. The distributional effects, although very small, are less favorable to less specialized and more constrained households.

The dynamics of endogenous variables after the shock affects GDP composition. Although private consumption to GDP falls in the first quarter, it immediately bounces upwards after the second quarter mostly to replace investment and public consumption.

<sup>8</sup> The standard deviations of all shocks were arbitrarily set at 100bps. Their values are not meant to reflect their empirical counterpart.

<sup>9</sup> Minella (2003) and Silveira (2008) also report impulse responses of inflation and output after a monetary policy shock that lack the "hump shapeness" that is observed in other countries.

Figure 1

Impulse Responses to a Contractionist Shock to Monetary Policy

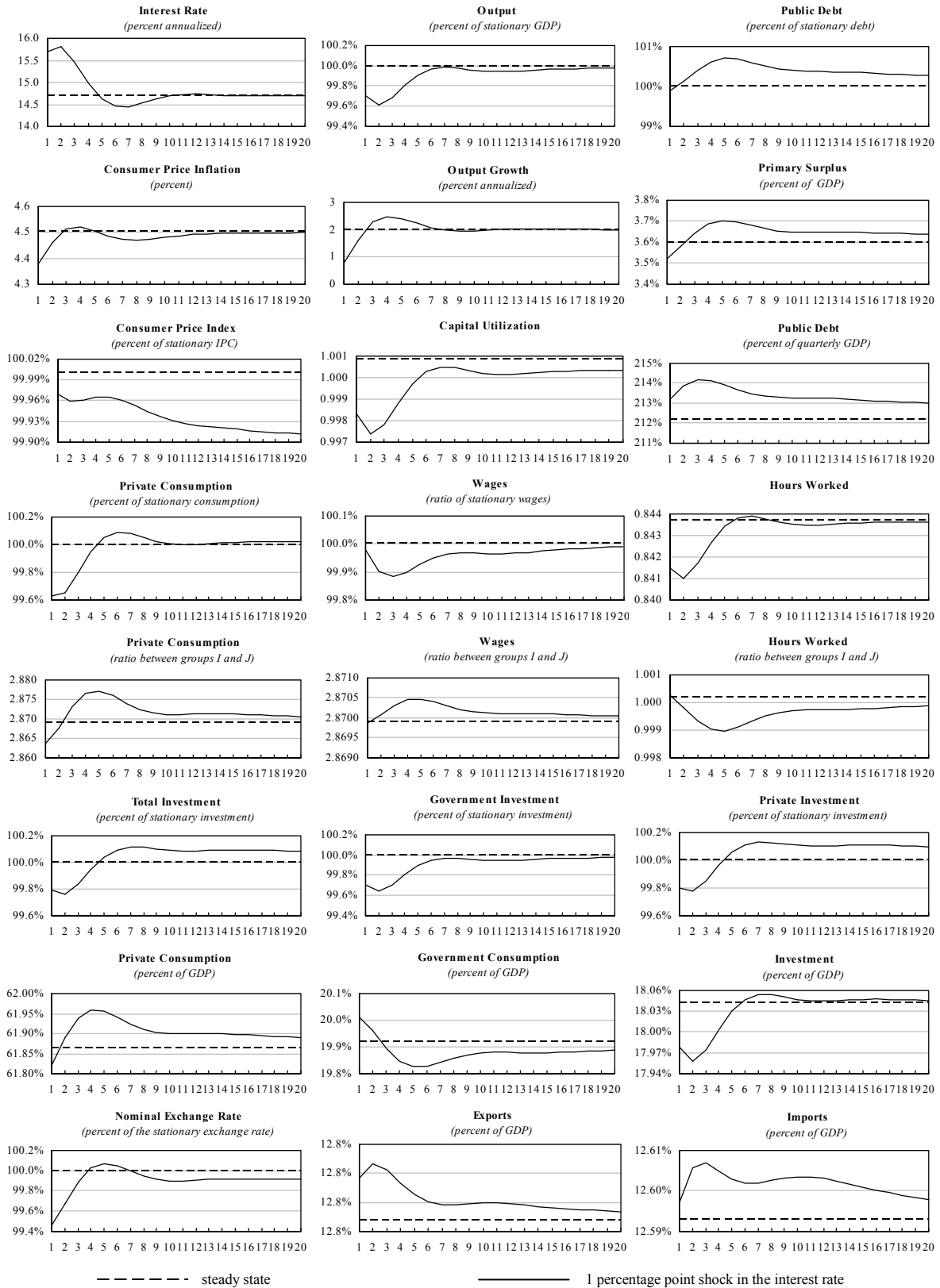


Figure 2 shows the impulse responses of a 1 percentage point reduction in the primary surplus. The shock initially increases government consumption by about 0.4 percentage points of GDP and raises public investment by 1 per cent from its steady state. Such expansionist effect initially boosts output growth to around 7 per cent per year, but in the second quarter, output growth falls to levels below steady state, where it reverts to afterwards. This shock has a smaller impact on the levels of private consumption and labor as compared to their steady state trends. The monetary effects of the fiscal shock comprise an increase of up to 0.2 percentage points in consumer price inflation, and, in spite of the contractionist stance of monetary policy, inflation remains above its steady state for a prolonged period.

The shape of the responses of inflation and public debt varies according to which shock is activated. For each shock, there is a distinct transmission mechanism. When the shock comes from the monetary policy, the response of the debt is more hump-shaped as the fiscal rule reacts to economic conditions. On the other hand, when the shock stems from the fiscal sector, the response of inflation becomes more hump-shaped, as the monetary policy rule reacts to the inflationary conditions imposed by the fiscal loosening.

To account for the fact that transfers are usually an instrument used for income distribution, the shock to government transfers (Figure 3) is biased towards less specialized and more constrained households. The hike in government transfers is enabled by a reduction in government consumption and public investment. These choices of cuts in government expenditures initially result in a significant downturn in economic activity. The fall in private consumption that could follow from depressed conditions stemming from the production side of the model does not occur possibly because of the direct injection of financial resources to households by the transfers (income effect) and also because monetary policy reacts to poor economic conditions and to the drop in inflation by keeping interest rates slightly below the steady state. Net public expenditures that result from the shock to transfers are not financed through debt issuance above steady state trends. In addition, the distributional effect of the shock vanishes after about 5 quarters.

A shock to government investment (Figure 4), of about 1 percentage point of GDP, crowds out private investment, as the rental rate of public capital is cheaper in the steady state. The rise in expenditures with public investment is financed through cuts in government consumption, driving the primary surplus down to levels below the steady state, and through debt issuance. Afterwards, the rise in public debt exerts a contractionist pressure on the fiscal rule, and the primary surplus rises after the third quarter. The initial inflationary spike results in a contractionist monetary policy reaction, and the final outcome is a drop in economic dynamism, with output below its steady state path for about 5 quarters. After the third quarter, the shock to government investment boosts output growth to above its steady state for a very prolonged time span. After the contractionist stance imposed by the fiscal and monetary adjustment unwinds, private consumption and wages rise a little above the steady state and remain there for a long time.

## 4.2 Policy analysis

To understand how the interaction of fiscal and monetary policy affects the model's predictions, we analyze impulse responses, variances and variance decompositions after policy shocks under a number of different specifications for the policy rules.

### 4.2.1 Sensitivity analysis

Figure 5 shows the impulse responses of a monetary policy shock with varying degrees of fiscal commitment with the stationary path of public debt. Greater commitment to the debt-to-GDP



Figure 2

Impulse Responses to an Expansionist Shock to the Primary Surplus

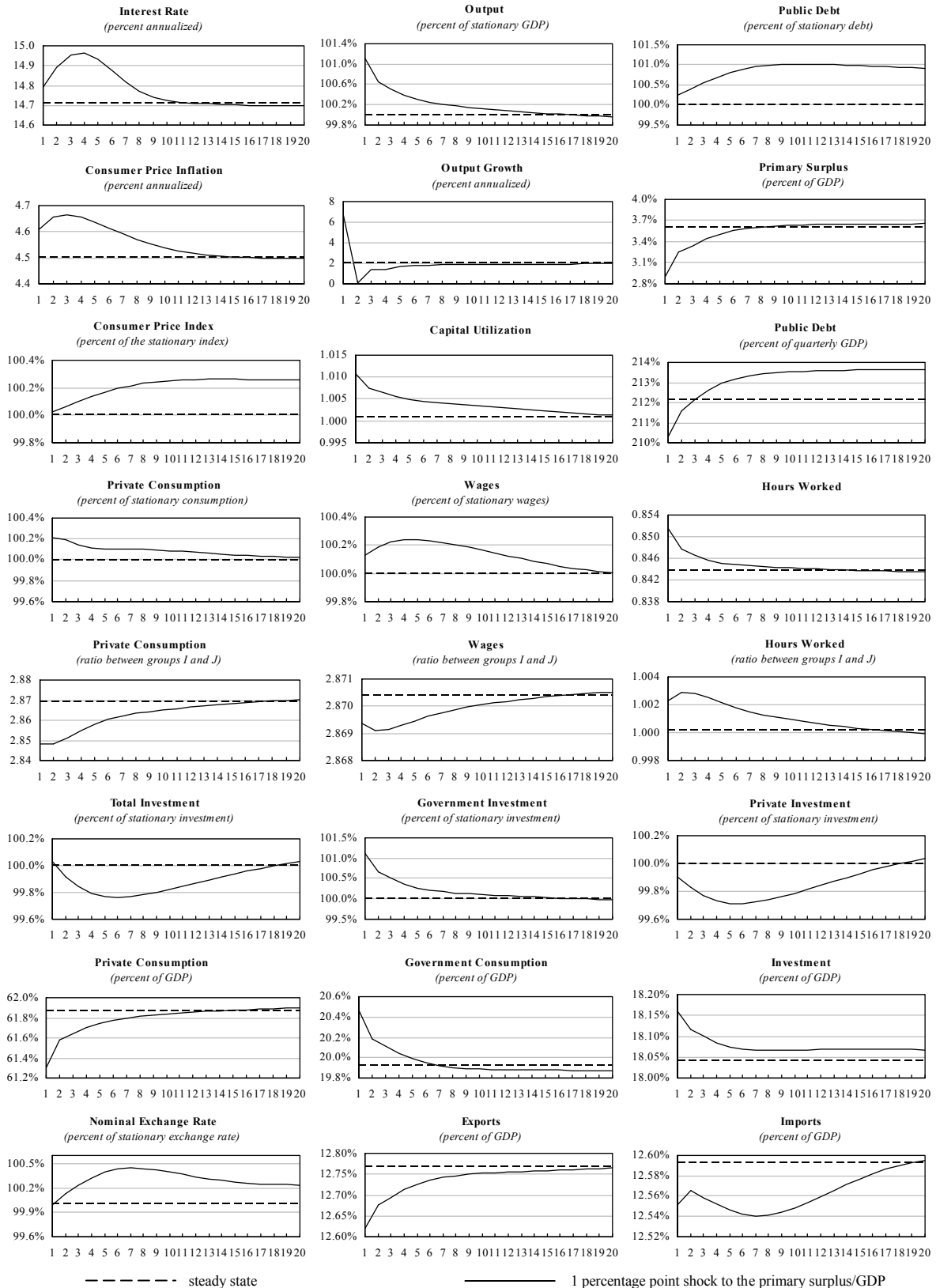


Figure 3

Impulse Responses to a Shock to Government Transfers

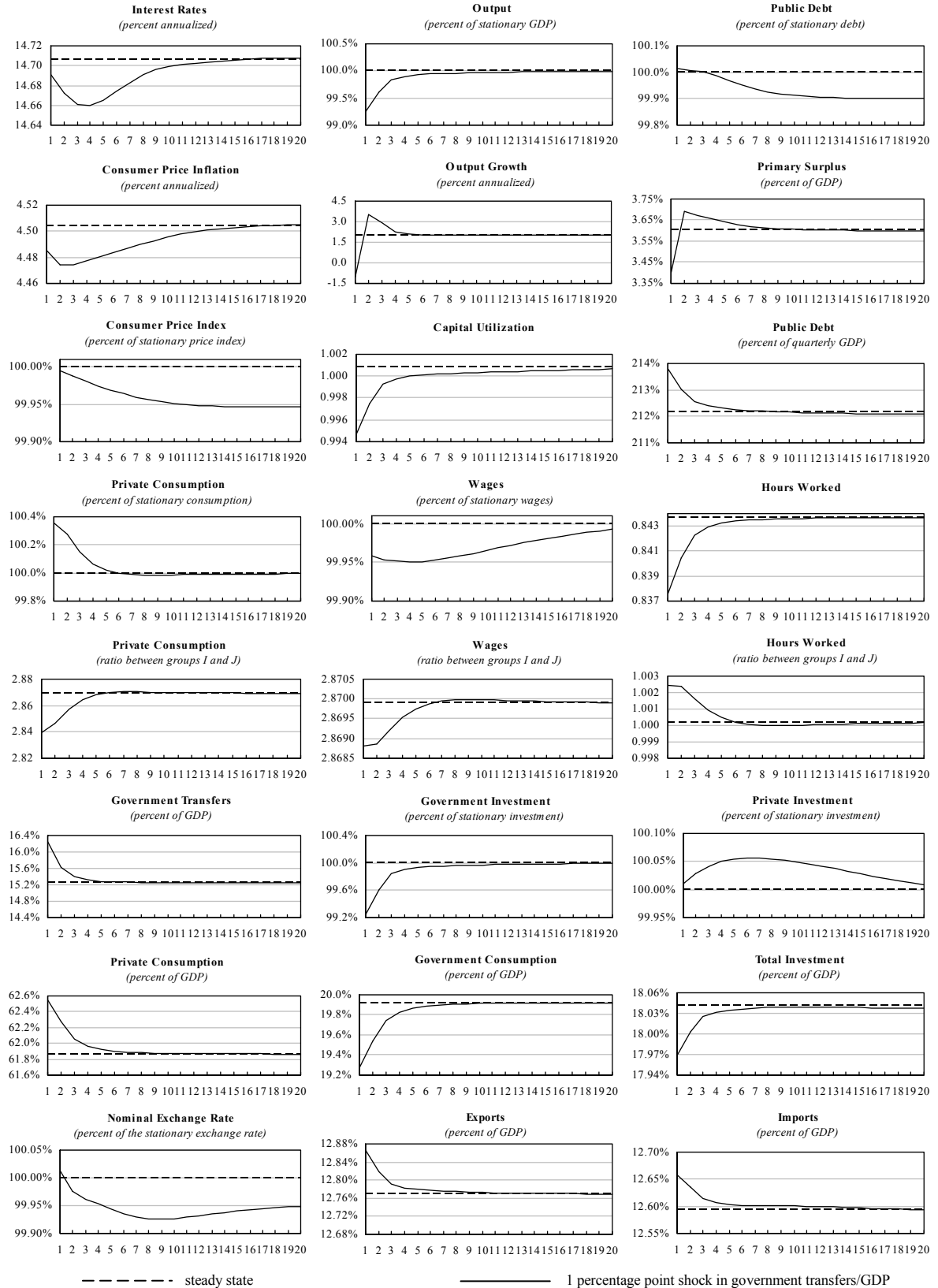


Figure 4

Impulse responses to a shock to government investment

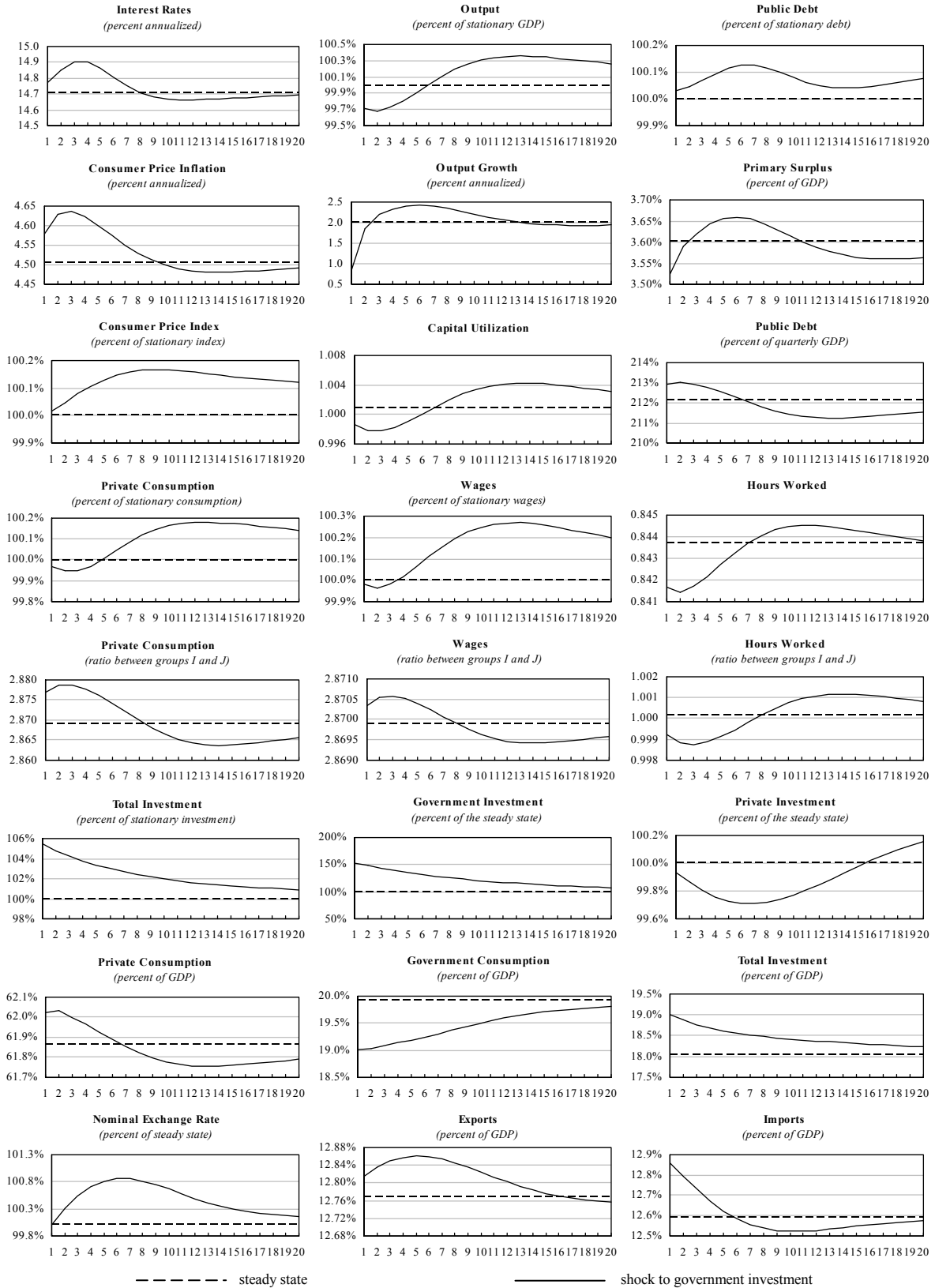


Figure 5

**Fiscal Commitment to the Steady State Level of the Public Debt:  
Impulse Responses of a Monetary Policy Shock**

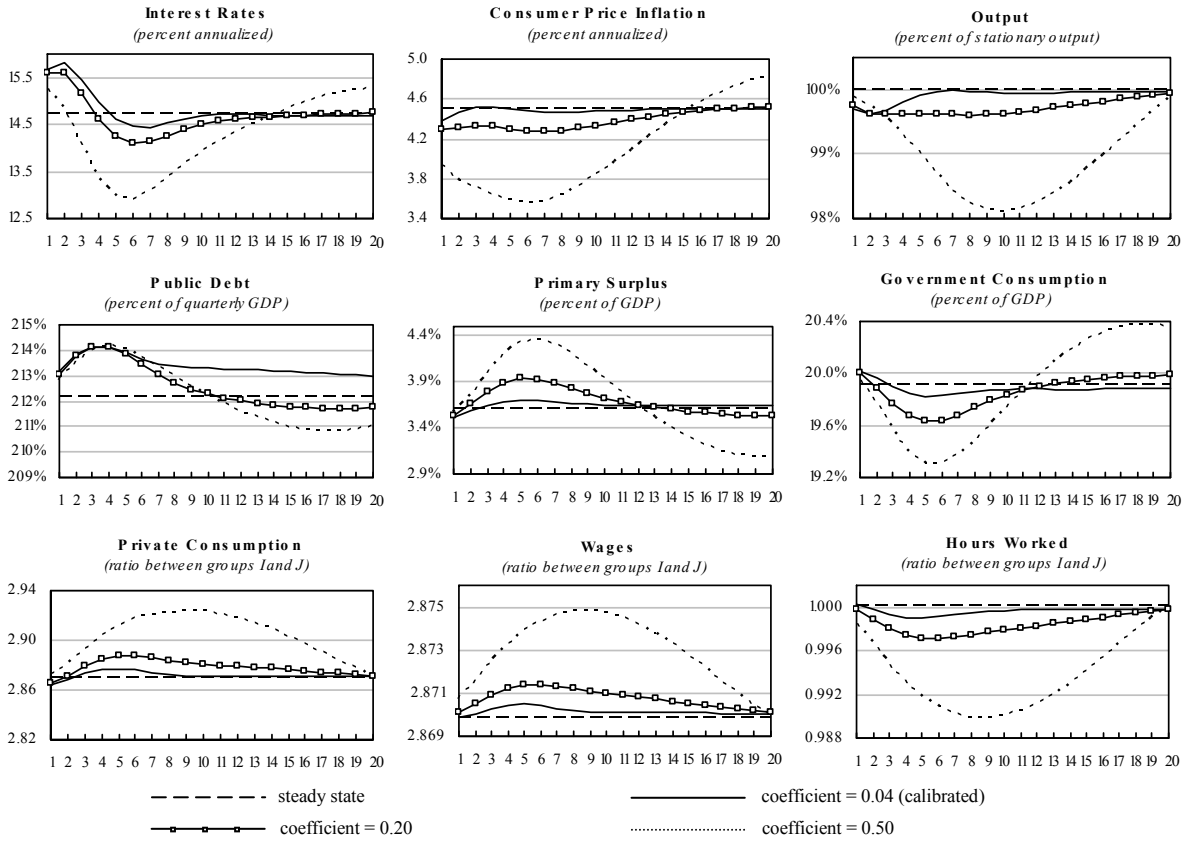
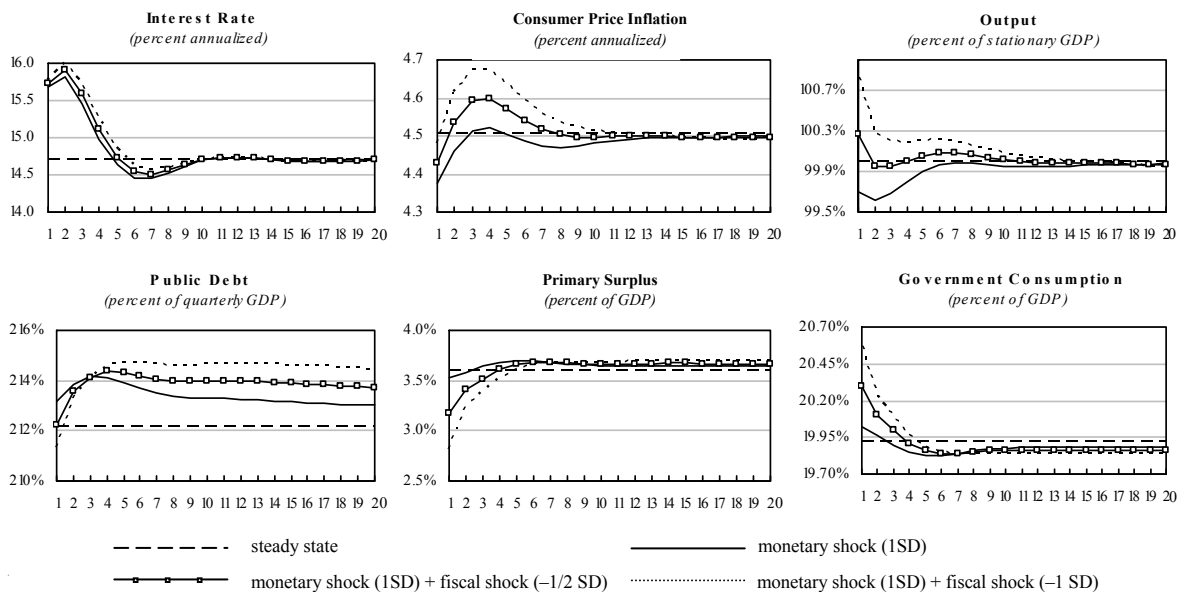


Figure 6

**Combination of Policy Shocks: Impulse Responses to a Monetary Policy Shock  
Varying the Rigor in the Implementation of the Fiscal Rule**



ratio implies that the government will post a stronger reaction to events that drive the public debt as a share of GDP away from its stationary trajectory. A contractionist monetary policy<sup>10</sup> increases interest rates and thus the service of the debt, which then triggers a reaction from the fiscal policy to stabilize the debt-to-GDP ratio. The stronger the reaction of the fiscal policy to the debt, the stronger the impact on output and inflation. The monetary policy rule then reacts to the effects on inflation from these economic conditions, lowering interest rates. The extreme case presented in the first plot, which corresponds to the case where the fiscal response to the debt is the greatest, illustrates that the initial increase in interest rates should be promptly reversed followed by an intense expansionist reaction in the medium-run to contain the excessive contractionist impact from the fiscal feedback. This calls for some sort of coordination between fiscal and monetary policy to attain the best policy combination to reduce the volatility that arises in inflation and output when both policies are in place. The plots also show that a stronger reaction to the debt-to-GDP ratio skews the distributive effects of the monetary policy shock a little more in favor of the group of more specialized households (group *I*) who also have more investment alternatives.

Table 4 shows variances and variance-decomposition when only the fiscal and monetary policy shocks are active. Under varying degrees of commitment to the stationary level of the debt, an increase in the coefficient of the fiscal rule associated with the deviation of the debt from its steady state increases the volatility of consumer price inflation and the correlation between inflation and output growth. As to the volatility of the output growth, the effects are non-linear. The shock decomposition shows that the influence of the monetary shock on output growth variance attains its least value with a coefficient of 0.18, a level that also grants the least variance of output growth.<sup>11</sup> On the other hand, the greatest influence of the monetary policy shock onto inflation variance obtains with a coefficient of 0.31.

Assuming that it is desirable to have the monetary policy affecting inflation more than the fiscal shock and conversely for the case of the output growth, we sought for a standard deviation of the fiscal shock that could jointly minimize the influence of the primary surplus shock on inflation and of the monetary policy shock on GDP growth. For a 1 percentage point standard deviation of the monetary policy shock and for a degree of fiscal commitment that minimized the unconditional volatility of output growth, the degree of fiscal rigor in the execution of the fiscal rule that implements this outcome is 0.47. The moments and variance decomposition that result are portrayed in Table 5. In the following figures and tables, the 0.47 standard deviation of the fiscal shock is used as benchmark. Figure 6 shows the impulse responses to a combination of a contractionist monetary policy shock and expansionist fiscal policy shocks, varying the rigor with which the fiscal rule is implemented. In the short run, the fiscal policy shock nullifies the impact of the monetary policy shock on inflation, and in the medium run, it actually generates some inflation, the more so the greater the rigor in the implementation of the fiscal rule. As to the public debt, as the fiscal policy shock increases in magnitude, there is additional pressure on the debt, and its initial increase gets steeper, accompanied by a higher persistence to revert back to the steady state.

Table 6 shows the effects on the variances, co-variances and variance decompositions of different degrees of correlation between policy shocks. In this exercise we start from one of the specifications of the fiscal rule shown in Table 4, corresponding to the one (coefficient of 0.18) where output growth attains its lowest volatility and is least impacted by a monetary policy shock. When a contractionist monetary policy jointly occurs with a loosening fiscal shock, which in the table is represented in the columns of negative correlations, the unconditional volatility of inflation

<sup>10</sup> Notice that in the benchmark calibration of the monetary policy rule, the direct reaction of the monetary policy to output is null. As a result, the exercises shown in the subsections that follow are conditional on the adopted parameterization.

<sup>11</sup> This could be suggestive of a region where optimal fiscal policy may lie on, but to be conclusive on this, we would need to conduct optimal policy analysis, which is beyond the scope of this paper.

Table 4

**Higher Commitment with the Stationary Path of the Public Debt in the Fiscal Rule**

Moments of the shocks ( <i>percent</i> )									
SD of the monetary policy shock <sup>(1)</sup> = 1.00									
SD of the fiscal shock = 1.00									
Corr. between shocks <sup>(1)</sup> = 0.00									
Fiscal commitment to the public debt									
Coefficient in the fiscal rule	0.04 <sup>(2)</sup>	<b>0.18</b>	<b>0.31</b>	0.50					
Moments of endogenous variables ( <i>percent</i> )									
SD of cons. price inflation	0.10	0.20	0.44	1.04					
SD of GDP growth	1.30	<b>1.28</b>	1.37	1.93					
Corr. between variables	4.78	9.68	29.41	58.85					
Variance decomposition ( <i>percent</i> )									
↓variance / → shock	MS <sup>(3)</sup>	FS <sup>(3)</sup>	MS	FS	MS	FS	MS	FS	
Consumer price inflation	15.63	84.37	47.98	52.02	58.48	<b>41.52</b>	45.16	54.84	
GDP growth	7.86	92.14	<b>5.22</b>	94.78	10.85	89.15	25.53	74.47	

<sup>(1)</sup> SD = standard deviation / Corr. = correlation.<sup>(2)</sup> Calibrated value.<sup>(3)</sup> MS = monetary shock / FS = fiscal shock (to the primary surplus).

Table 5

**Greater Rigor in Implementation of the Primary Surplus Rule**

Moments of the shocks ( <i>percent</i> )									
SD of the monetary policy shock <sup>(1)</sup> = 1.00									
SD of the fiscal shock = <b>0.47</b>									
Corr. between shocks <sup>(1)</sup> = 0.00									
Fiscal commitment to the public debt									
Coefficient in the fiscal rule	0.04 <sup>(2)</sup>	<b>0.18</b>	<b>0.31</b>	0.50					
Moments of endogenous variables ( <i>percent</i> )									
SD of cons. price inflation	0.06	0.16	0.36	0.79					
SD of GDP growth	0.69	<b>0.66</b>	0.76	1.25					
Corr. between variables	24.41	14.81	39.12	65.23					
Variance decomposition ( <i>percent</i> )									
↓variance / → shock	MS <sup>(3)</sup>	FS <sup>(3)</sup>	MS	FS	MS	FS	MS	FS	
Consumer price inflation	45.12	54.88	80.36	19.64	86.21	<b>13.79</b>	78.51	21.49	
GDP growth	27.45	72.55	<b>19.64</b>	80.36	35.06	64.94	60.34	39.66	

<sup>(1)</sup> SD = standard deviation / Corr. = correlation.<sup>(2)</sup> Calibrated value.<sup>(3)</sup> MS = monetary shock / FS = fiscal shock (to the primary surplus).

Table 6

## Varying the Correlation Between Monetary and Fiscal Policy (Primary Surplus) Shocks

Moments of the shocks ( <i>percent</i> )											
SD of the monetary policy shock <sup>(1)</sup> = 1.00											
SD between fiscal shocks = <b>0.47</b>											
Corr. between policy shocks	0.80	0.50	0.00	-0.50	-0.80						
Fiscal commitment to the public debt											
Coefficient in the fiscal rule = <b>0.18</b>											
Moments of the variables ( <i>percent</i> )											
SD of cons. price inflation	0.19	0.18	0.16	0.13	<b>0.11</b>						
SD of output growth	0.80	0.75	0.66	0.55	<b>0.47</b>						
Corr. between variables	18.44	17.40	14.81	9.95	<b>4.25</b>						
Variance decomposition ( <i>percent</i> ) – when the 1 <sup>st</sup> shock is in monetary policy											
↓variance / → shock	MS <sup>(3)</sup>	FS <sup>(3)</sup>	MS	FS	MS	FS	MS	FS	MS	FS	
Consumer price inflation	95.27	4.73	88.74	11.26	80.36	19.64	78.70	21.30	86.04	13.96	
GDP growth	80.49	19.51	53.70	46.30	19.64	80.36	<b>13.68</b>	86.32	44.07	55.93	
Variance decomposition ( <i>percent</i> ) – when the 1 <sup>st</sup> shock is in the fiscal rule											
↓variance / → shock	MS <sup>(3)</sup>	FS <sup>(3)</sup>	MS	FS	MS	FS	MS	FS	MS	FS	
Consumer price inflation	80.63	19.37	53.94	46.06	19.64	80.74	<b>12.83</b>	87.17	42.86	57.14	
GDP growth	95.23	4.77	88.68	11.32	80.36	19.64	78.90	21.10	86.33	13.67	

<sup>(1)</sup> SD = standard deviation / Corr. = correlation.

<sup>(2)</sup> Calibrated value.

<sup>(3)</sup> MS = monetary shock / FS = fiscal shock (to the primary surplus).

and output growth falls. This result was in line with what the previous discussion on Figure 6 implied. Economic stimuli from expansionist fiscal and monetary shocks add variance to both inflation and output, and also expand the correlation between these two variables.

Table 7 shows the impact of monetary policy rules that react more to deviations of expected inflation from the target. Notice that the coefficient of reaction to output growth is null under all monetary policy rules that we experiment with here. In this exercise, we used the same specification for the fiscal rule in Table 6. Under these assumptions, a more hawkish monetary policy enacts a reduction in the variances of inflation and output growth. It also reduces the correlation between these two variables. However, as monetary policy becomes more hawkish, the fiscal shock gains some power to explain the variance of consumer price inflation. When the coefficient attached to inflation targets is set at 2.44, the monetary policy shock has the smallest influence on the variance of the output growth.<sup>12</sup>

We find an specific combination of monetary and fiscal commitment that grants the lowest volatility in output growth, bearing in mind that the benchmark monetary policy rule does not react

<sup>12</sup> This result is not indicative of an optimal reaction of monetary policy to stabilize output, as it is conditioned on the fact that the calibrated monetary policy rule does not react directly to output growth, while the fiscal rule does.

Table 7

## Varying the Monetary Policy Commitment to the Inflation Target

Moments of the shocks (percent)								
SD of the monetary policy shock <sup>(1)</sup> = 1.00								
SD of the fiscal shock = <b>0.47</b>								
Corr. between shocks <sup>(1)</sup> = 0.00								
Fiscal commitment to the public debt								
Coefficient in the fiscal rule = <b>0.18</b>								
Monetary policy commitment to the inflation target								
Coefficient in the mon. policy rule	1.20	1.57 <sup>(2)</sup>	<b>2.44</b>	5.2				
Moments of endogenous variables (percent)								
SD of cons. price inflation	0.82	0.16	0.07	0.04				
SD of GDP growth	0.73	0.66	0.63	0.61				
Corr. between variables	25.52	14.81	8.40	0.00				
Variance decomposition (percent)								
↓variance / → shock	MS <sup>(3)</sup>	FS <sup>(3)</sup>	MS	FS	MS	FS	MS	FS
Consumer price inflation	93.01	6.99	80.36	19.64	64.72	35.28	60.37	39.63
GDP growth	29.57	70.43	19.64	80.36	<b>18.13</b>	81.87	22.08	77.92

<sup>(1)</sup> SD = standard deviation / Corr. = correlation.

<sup>(2)</sup> Calibrated value.

<sup>(3)</sup> MS = monetary shock / FS = fiscal shock (to the primary surplus).

directly to output conditions. Such combination is shown in the second column of Table 8. It increases the share of inflation variance that is attributed to the monetary policy shock, although the highest stake is still with the fiscal shock.

#### 4.2.2 Fiscal and monetary policy activeness

In Dynare, the model shows a unique solution for time paths of endogenous variables under two regions of policy activeness<sup>13</sup> (Figure 7), maintaining the remaining parameters as they were originally calibrated. Under active monetary policy ( $\phi_{\pi} > 1.1$ ), the equilibrium is unique if the response of the fiscal rule to deviations of the public debt to its steady state ratio ( $\phi_{B_T}$ ) remains in the positive interval of  $[0.03, \infty)$ , where the original calibrated parameter belongs, or in the interval  $(-\infty, -1.21)$ . In the former interval, the stronger the reaction of the fiscal rule to the debt-to-GDP ratio, the more cyclical are the responses of the output (Figure 8).

The model also shows a unique solution (in Dynare) in regions where monetary policy is passive (5<sup>th</sup> to 8<sup>th</sup> columns of Figure 8).<sup>14</sup> Again, the greater the magnitude of the reaction of the

<sup>13</sup> Active and passive policies are used here in the sense described in Schmidt-Grohé and Uribe (2006) and Leeper (1991). Woodford (2003) uses the term “locally Ricardian” for active policies.

<sup>14</sup> Schmidt-Grohé and Uribe (2006) also obtain regions of implementable policy with Taylor coefficients lower than 1.



Table 8

## Policy Rules That Minimize Output Volatility

Moments of the shocks ( <i>percent</i> )				
SD of the monetary policy shock <sup>(1)</sup> = 1.00				
SD of the fiscal policy shock = 1.00				
Corr. between shocks <sup>(1)</sup> = 0.00				
Fiscal commitment to the public debt				
Coefficient in the fiscal rule	0.04 <sup>(2)</sup>			<b>0.27</b>
Monetary policy commitment to the inflation target				
Coefficient in the mon. policy rule	1.57 <sup>(2)</sup>			<b>4.50</b>
Moments of endogenous variables ( <i>percent</i> )				
SD of cons. price inflation	<b>0.10</b>			<b>0.10</b>
SD of output growth	1.30			<b>1.17</b>
Corr. between variables	4.78			-15.58
Variance decomposition ( <i>percent</i> )				
↓variance / → shock	MS <sup>(3)</sup>	FS <sup>(3)</sup>	MS	FS
Consumer price inflation	15.63	84.37	25.31	<b>74.69</b>
GDP growth	7.86	92.14	<b>3.88</b>	96.12

<sup>(1)</sup> SD = standard deviation / Corr. = correlation.

<sup>(2)</sup> Calibrated value.

<sup>(3)</sup> MS = monetary shock / FS = fiscal shock (to the primary surplus).

fiscal rule to the debt-to-GDP ratio, the stronger the cyclicity of the responses. However, for practically null responsiveness of the fiscal rule to the debt and of the monetary policy rule to the inflation target, the model reestablishes lower cyclicity.

#### 4.2.3 Alternative types of monetary policy rules

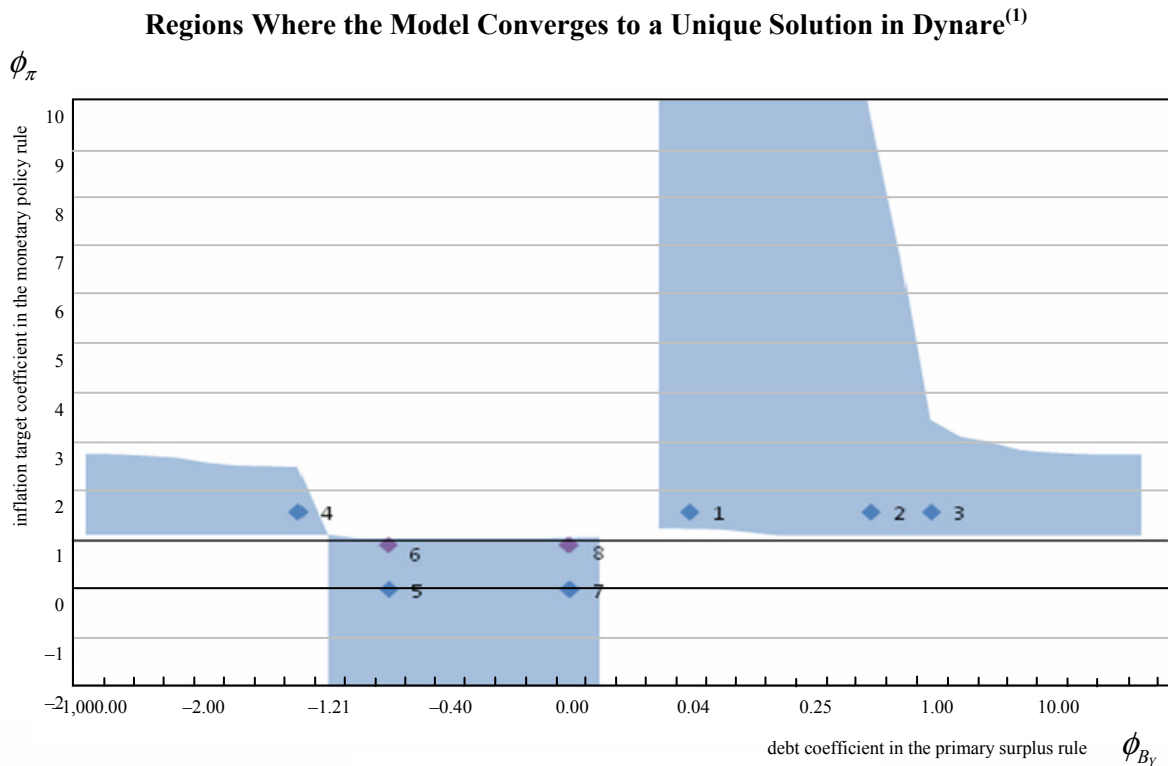
The model can also be used to analyze the effects of adopting a distinct monetary policy rule. Table 9 compares the moments and shows a variance decomposition of key endogenous variables under alternative types of monetary policy rules. If the monetary policy rule directly reacts to changes in the exchange rate,<sup>15</sup> the volatility of inflation and output growth reduces. The absolute magnitude of the correlation between economic growth and inflation drastically reduces.

If the monetary policy rule reacts to the gap in output growth,<sup>16</sup> the variance in output growth reduces, albeit with an increase in the variance of consumer price inflation and the exchange rate. The monetary policy shock also contributes less to the variances of inflation, output growth and the exchange rate.

<sup>15</sup> The coefficient of reaction to the deviation of changes in the exchange rate from its steady state was arbitrarily set at 1 in this exercise.

<sup>16</sup> The coefficient of reaction to the deviation of output growth from its steady state was arbitrarily set at 0.79 in this exercise.

Figure 7



<sup>(1)</sup> The regions of convergence were plotted only for the interval  $\phi_{B_y} \in (-1000, 100)$  and  $\phi_{\pi} \in (-2, 10)$ . The colored region continues in the area beyond the plotted limits.

The numbered dots represent the points selected to draw impulse responses in Figure 8.

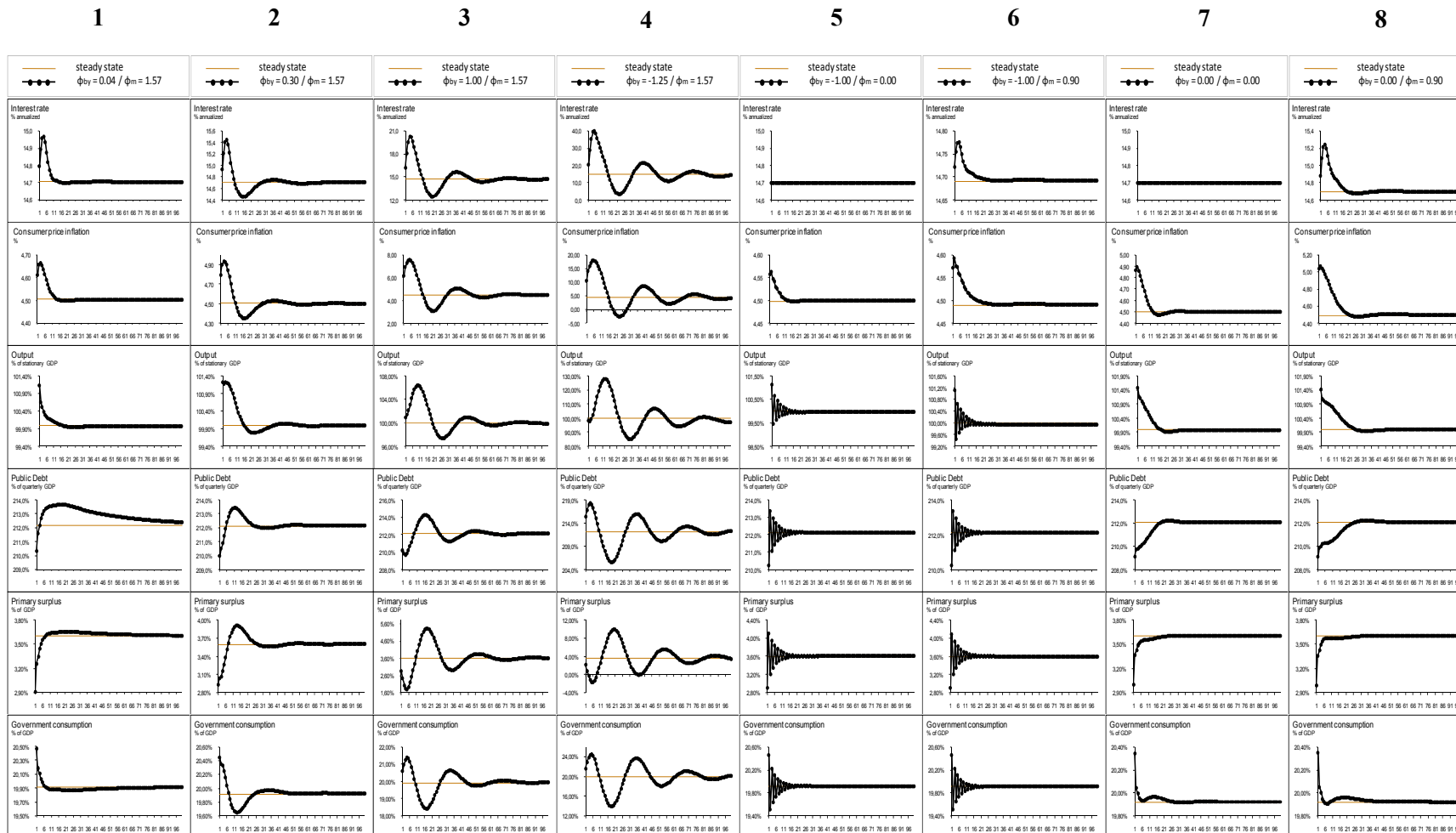
Impulse responses to different types of monetary rules have distinct shapes. Figure 9 shows that the introduction of an explicit reaction of the monetary policy to either output growth or to changes in the exchange rate brings about greater persistence to the drop in inflation. The initial impact on output growth is a little milder, yet the persistence is also more pronounced. Backward looking rules, on the other hand, do not substantially alter the dynamics of the main macroeconomic variables after a monetary policy shock.

## 5 Conclusion

In this paper we revised the work in CMS and CCW, correcting important equations relating to prices, wages and the aggregate resource constraint of the economy. In addition, in order to better approximate the modeled economy to the current practice of fiscal policy in a number of countries, including Brazil, we introduced a different modeling strategy of the fiscal sector. We let the government track a primary surplus and a debt-to-GDP target, using its instrument also as a response to economic conditions, and allowed the government to invest and the private sector to decide upon the utilization of public and private capital. We also extended the model to introduced labor specialization in order to allow for wage heterogeneity amongst households that supply the same amount of worked hours.

Figure 8

Some Plots of Impulse Responses to a Fiscal Policy Shock Under Distinct Combinations of Policy Parameters in the Regions Where the Model Converges to a Unique Solution in Dynare<sup>(1)</sup>



<sup>(1)</sup> The numbers in each column of graphs indicate the combinations of policy reactions plotted (and equally numbered) in Figure 7.

Table 9

## Alternative Monetary Policy Rules

Moments of the shocks ( <i>percent</i> )						
SD of the monetary policy shock <sup>(1)</sup> = 1.00						
SD of the fiscal policy shock = 1.00						
Corr. between shocks <sup>(1)</sup> = 0.00						
Monetary policy rules						
	calibrated model	calibrated rule + reaction to the exchange rate		calibrated rule + reaction to the output growth		
Moments of endogenous variables ( <i>percent</i> )						
SD of inflation	0.10	<b>0.04</b>			0.41	
SD of GDP growth	1.30	1.27			<b>0.85</b>	
SD of exchange rate variation	0.68	<b>0.22</b>			1.28	
Corr. between consumer price inflation and GDP growth	4.78	0.46			-7.51	
Corr. between consumer price inflation and exchange rate variation	48.84	40.25			46.36	
Corr. between GDP growth and exchange rate variation	8.58	-25.58			-78.61	
Variance decomposition ( <i>percent</i> )						
	MS <sup>(3)</sup>	FS <sup>(3)</sup>	MS	FS	MS	FS
Consumer price inflation	15.63	84.37	97.67	<b>2.33</b>	10.14	89.86
GDP growth	7.86	92.14	<b>1.75</b>	98.25	2.80	97.20
Exchange rate variation	89.4	10.6	86.16	13.84	<b>5.1</b>	94.9

<sup>(1)</sup> SD = standard deviation / Corr. = correlation.

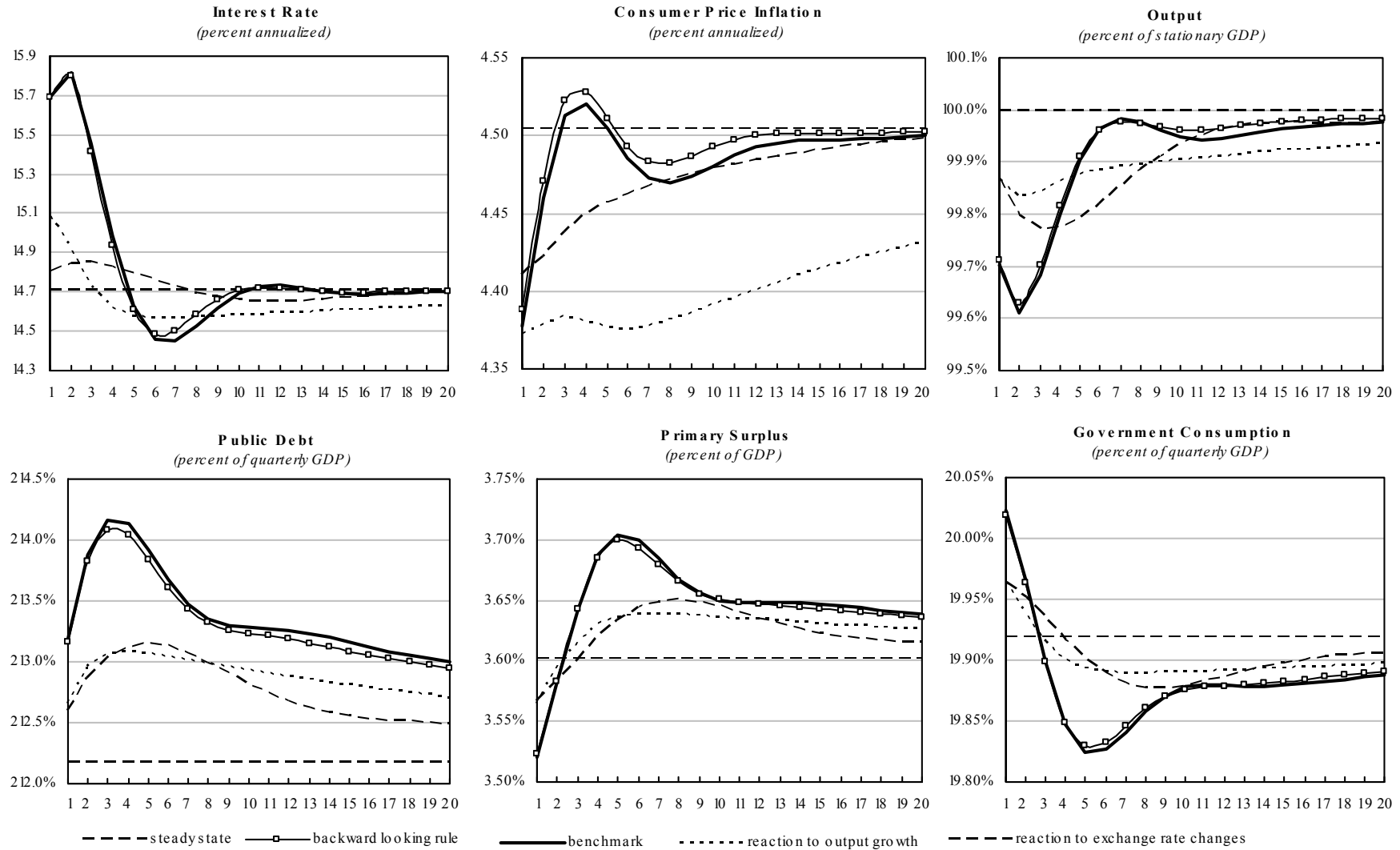
<sup>(2)</sup> Calibrated value.

<sup>(3)</sup> MS = monetary shock / FS = fiscal shock (to the primary surplus).

Under the adopted calibration, the model responses to monetary policy shocks are short-lived. The simulations show an important endogenous interaction of monetary policy conditions with fiscal policy responses, although policy rules are not directly responsive to one another. Expansionist primary surplus shocks can boost economic activity, yet with significant implications to inflation. Shocks to government investment also put pressure on inflation, and, although the immediate response of output growth is negative, it soon reverses to a prolonged economic expansion. On the other hand, the simulations show that fiscal transfer shocks, aimed at redistributing income, negatively affect general economic conditions as consequence of the fiscal rule.

**Figure 9**

**Impulse Responses to a 1 Percentage Point Monetary Policy Shock Under Alternative Monetary Policy Rules**



Different specifications for the policy rules significantly affect the results implied by the model. The simulations with different degrees of fiscal commitment to the stationary path of the public debt and with greater rigor in the implementation of the primary surplus rule make explicit that the strength of one policy affects the impact of the other on important variables such as output and inflation. Increasing fiscal commitment to the stationary debt-to-GDP ratio enhances the contractionist impact of a monetary policy shock upon inflation, albeit at the cost of a higher impact on output growth in the medium-run. The volatility of inflation and output growth increases, as does the correlation between them. On the other hand, a more rigorous implementation of the primary surplus rule implies, as expected, lower variance of inflation and output growth, but the correlation between them increases with the degree of rigor.

Simultaneous shocks to the primary surplus rule and to monetary policy make explicit the contrasting objectives of these policies. Primary surplus shocks dampen the contractionist effect of the monetary policy shock onto inflation and output, and also reduce the variance of inflation and output growth.

A higher commitment to the inflation target in the monetary policy rule reduces the variance of inflation and output growth, and their correlation, with the drawback that the fiscal shock gains importance in affecting the variance of inflation.

Different specifications of monetary policy rules also yield qualitatively distinct predictions. Rules that directly react to changes in the exchange rate or to the output gap reduce the variance of output growth. However, an explicit reaction to the output growth increases the variance of inflation. A monetary policy reaction to the exchange rate holds the following outcomes: the variance of inflation and the correlation between inflation and output growth reduce, and the monetary policy shock gains a much greater stake at the variance of inflation.

Our model finds stable equilibria in regions where the fiscal policy rule is active and the Taylor principle does not hold. Impulse responses with some combinations of policy reactions in the region of fiscal-activeness show that the responses can be either well-behaved or strongly cyclical. For these cases, the model reestablishes lower cyclicity for practically null responsiveness of the fiscal rule to the debt and of the monetary policy rule to the inflation target.

## **APPENDIX**

Please contact the authors to request a copy of the Appendix, or download a complete version of the working paper at <http://www.bcb.gov.br/pec/wps/ingl/wps204.pdf>

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