OPTIMAL FISCAL POLICY IN THE POST-CRISIS WORLD

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To contrast the severe global recession of 2009, governments in most advanced countries implemented expansionary fiscal policies leading to a steep increase in public debt. As economies recover, a critical choice is whether to stabilize debt at post-crisis levels, or to bring it down to pre-crisis levels. On this issue, advices of international institutions and those coming from mainstream economic theory are at odds. While international institutions have called for a substantial and fast debt reduction, optimal fiscal policy literature calls for debt stabilization. The aim of this paper is to provide a formal theoretical rationale to the policy advices of international institutions in a DSGE model (the workhorse of mainstream optimal fiscal policy theory). In particular, we consider a model in which a benevolent government has to choose taxes and debt in order to finance an exogenous stream of public expenditure. We compare the optimal fiscal plan in two contexts. In the first one households are fully confident about government solvency. In the second, households believe that there is a positive default probability which is positively related to the level of debt. While in the first framework a temporary bad shock translates into a permanent increase in the debt level, in the second one the increase in government debt is only temporary.

"Only thing we have to fear is fear itself." F.D. Roosevelt

1 Introduction

To contrast the severe global recession of 2009, governments in most advanced countries implemented expansionary fiscal policies. These interventions have led to a steep increase in debt levels. According to the IMF, in the advanced economies of the G20 the debt-to-GDP ratio is projected to rise from 78 in 2007 to 118 per cent in 2014. While it is clear that ever-increasing debt-to-GDP ratios are inconsistent with government solvency and have to be avoided, a critical policy choice confronting policy-makers is whether to stabilize debt ratios at current levels, or bringing them down to pre-crisis levels. On this issue, advices of international institutions and those coming from mainstream economic theory are at odds.

On one side, international institutions have called for a substantial and fast debt reduction. For example, the December 2009 issue of ECB's *Monthly Bulletin* calls for adjustment measures which "succeed in putting debt ratios on a declining trajectory", to be implemented in 2011 at the latest; the ECOFIN Council (October 2009) agrees that "beyond the withdrawal of the stimulus measures, substantial fiscal consolidation is required in order to halt and eventually reverse the increase in debt"; the European Commission's *Communication from the Commission to the European Parliament and the Council: "Long-term Sustainability of Public Finances for a Recovering Economy"*, 2009, while recognizing that "a one-off increase in the stock of government debt need not put sustainability at risk", stresses that "while, prior to the crisis, the three prongs of the (Stockholm) strategy [*i.e.*, deficit and debt reduction, increases in employment rates and reforms of social protection systems] were options from which countries could choose, each of

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these pillars is now indispensable for most EU countries"; the IMF's *Strategies for Fiscal Consolidation in the Post-crisis World*, 2010, argues that "stabilizing debt ratios at post-crisis level would be insufficient".

On the other side, a surprisingly robust result in optimal fiscal policy theory is that public debt should on average be constant.[1] This has been demonstrated to be true both in a complete market framework (*i.e.*, in a framework in which the government has access to a full array of bonds for each maturity and for each contingency[2]) and in a more realistic incomplete market framework. In this latter setup, Ayagary, Marcet, Sargent and Seppälä (2002), "Optimal Fiscal Policy without State Contingent Debt", *Journal of Political Economy*, rigorously confirm the intuition of Barro (1979), "On the Determination of Public Debt", *Journal of Political Economy*, that negative shocks should have a permanent effect on public debt.[3] More precisely, the authors demonstrate that the optimal fiscal policy requires the debt to follow a random walk process, *i.e.*, its level tomorrow and in any future period is equal in expected terms to today's level. These results are also robust to the introduction of capital (see, e.g., Chari, Christiano and Kehoe (1994), "Optimal Fiscal Policy in a Business Cycles Model", *Journal of Political Economy*; Chari and Kehoe (1999), "Optimal Fiscal and Monetary Policy", in *Handbook of Macroeconomics*; and Scott (1999), "Does Tax Smoothing Imply Smooth Taxes", CEPR, Discussion Paper, No. 2172.

In summarizing this wide body of literature, Scott (2009), "Government Debt After the Crisis" concludes that economic theory suggests that "in the wake of large adverse shocks... the optimal response is to use debt as a buffer stock. Debt should show large and long term shifts and there is no presumption that governments need to reduce debt to pre-crisis levels". And that, in any case, "... fluctuations in government debt after such adverse shocks are long lasting... Debt stabilization occurs over decades not within a decade".

Is it possible to make sense of the policy advices of international institutions and practitioners in a model which shares features of the neoclassical dynamic general equilibrium models, which are the workhorse of standard optimal fiscal policy theory? The aim of this paper is to answer this question.

As in Ayagary *et al.* (2002), we consider a closed production economy with no capital and infinitely lived agents. Public spending follows an exogenous stochastic process. The problem of the representative household is to maximize its lifetime expected utility subject to the flow budget constraint. The government is benevolent: it chooses the level of debt and distortionary taxes on labor income to maximize households' expected utility subject to the feasibility constraint, households' beliefs and optimality conditions and debt sustainability. Moreover, the government acts under full commitment, *i.e.*, it always fulfils its promises. We believe that these two

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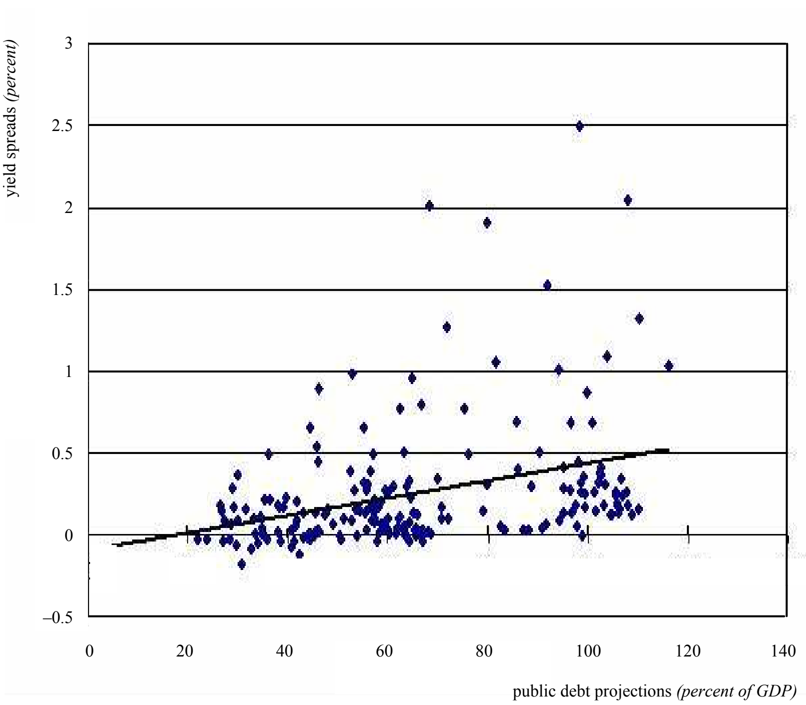
[1] The optimal taxation literature is immense and offering a comprehensive survey goes beyond the scope of this paper. See Barro, R.J. (1979), "On the Determination of Public Debt", *Journal of Political Economy*; Barro, R.J. (1989), "The Neoclassical Approach to Fiscal Policy", published in *Modern Business Cycle Theory*, Harvard University Press; Barro, R.J. (1995), "Optimal Debt Management", NBER, Working Paper, No. 5327; Barro, R.J. (1997), "Optimal Management of Indexed and Nominal Debt", NBER, Working Paper, No. 6197; Bohn, H. (1990), "Tax Smoothing with Financial Instruments", *American Economic Review*, No. 80; Kydland, F. and E.C. Prescott (1980), "Dynamic Optimal Taxation, Rational Expectations and Optimal Control", *Journal of Economic Dynamics and Control*; Lucas, R.E. Jr. and N.L. Stokey (1983), "Optimal Fiscal and Monetary Economy in an Economy Without Capital", *Journal of Monetary Economics*; Chari, V.V., L.J. Christiano and P.J. Kehoe (1994), "Optimal Fiscal Policy in a Business Cycles Model", *Journal of Political Economy*; Chari, V.V. and P.J. Kehoe (1999), "Optimal Fiscal and Monetary Policy", in *Handbook of Macroeconomics*; Ayagary, R., A. Marcet, T.J. Sargent and J. Seppälä (2002), "Optimal Fiscal Policy Without State Contingent Debt", *Journal of Political Economy*; Zhu, X. (1992), "Optimal Fiscal Policy in a Stochastic Growth Model", JET, among many others.

[2] Lucas, R.E. Jr. and N.L. Stokey (1983), "Optimal Fiscal and Monetary Economy in an Economy Without Capital", *Journal of Monetary Economics.*

[3] See also Marcet, A. and A. Scott (2010), "Debt and Deficit Fluctuations and the Structure of Bonds Markets", JET.

Figure 1

Debt Level and Yield Spread



assumptions are quite plausible if referred to advanced economies, in which the political cost of a default is likely to be prohibitive. Nevertheless, we also assume that households believe that with a positive probability the government could default on its own debt. This assumption captures the current situation, in which we observe financial markets assigning significant default probabilities even to the sovereign debt of advanced countries. For example, Figure 1 points to a positive relation between the amount of government debt and yield spread, a proxy for the sovereign risk premium, for 10 euro area countries in the period 2000-09. So we assume that households believe that there is a positive relation between the probability of default and the amount of outstanding debt. Over time they update their estimates of this relation as new data on government behavior become available.

We study the impact of expectations about government default on the optimal fiscal policy in two different set-ups. In the first one, when in the initial period the fiscal authority sets its plans agents are already sceptical about the government capability/willingness to honor its debt obligations. In the second one, agents are instead fully confident about debt repayment, but they may start fearing default if the government uses debt to absorb an adverse shock. These two cases are meant to capture two different situations. The first one refers to the post crisis situation, characterized by high debt levels and significant sovereign risk premia: here the government's problem is to design an optimal "exit strategy". The second one instead is meant to capture both the pre-crisis and the post-crisis period (crisis is modelled here as a very high decrease in productivity and output). The main problem here is to understand whether a "fiscal stimulus" in times of crisis, implying higher deficits and debts, is consistent with an optimal fiscal plan.

Our main findings are the following. First, when agents fear government default, a post-crisis fiscal consolidation becomes optimal. The intuition is that the interest rate on government debt is too high due to distorted expectations about government default. Therefore the marginal cost of higher distortionary taxes today is more than compensated by the expected future marginal benefits of lower distortionary taxes tomorrow. The incentive to reduce debt is stronger i) the more pessimistic agents are about government solvency and ii) for a given degree of pessimism, the higher the post-crisis debt level. Second, the state of agents' initial beliefs has an effect on the long-run mean value of the tax rate and debt. Third, while optimality still requires to increase debt to absorb the negative shock (as in Ayagary *et al.*, 2002), the possibility of a negative shock leads the government to run much higher primary surpluses before it materializes. *i.e.*, to create "fiscal room" in advance.

The paper proceeds as follows. Section 2 characterizes the optimal fiscal policy, and in Section 3 we solve it numerically. In Section 4 we characterize the fiscal plan in the case of an unexpected adverse shock. In Section 5 we compare the fiscal variables dynamics in two countries which differ for their initial debt level. Section 6 concludes.

2 The model

We consider an infinite horizon economy with an infinitely lived representative consumer and a benevolent fiscal authority. The government finances an exogenous stream of public consumption levying a proportional tax on labor income and issuing a one-period non state-contingent bond, which is the only financial asset in the economy. The government has a full commitment technology and always repays its debt. There are two sources of aggregate uncertainty, represented by a government expenditure shock and a technology shock. In Subsection 1 we briefly review optimal fiscal policy under the assumption that households are at any moment fully confident about government solvency, as in Ayagary *et al.* (2002). In Subsection 2 we modify this benchmark model assuming that households assign a positive probability to the event of government default. We show how the way in which households form their expectations change the constraints faced by the fiscal authority and consequently the optimal fiscal policy.

2.1 The rational expectations benchmark

Time is discrete and indexed by t=0,1,2.... At the beginning of each period there is a realization of a stochastic state $s=(g_t,\vartheta_t)\in S=G\times\Theta$. Let us define the history of events up to time t as $s^t=(g^t,\vartheta^t)$, where $g^t=(g_0,g_1,...,g_t),\vartheta^t=(\vartheta_0,\vartheta_1,...,\vartheta_t)$, and the conditional probability of s^r given s^t as $\pi(s^r|s^t)$; s_0 is non-stochastic.

2.1.1 The private sector

A representative household is endowed with one unit of time which can be used for leisure, l_t, or labor, n_t.

$$n_t(s^t)+l_t(s^t)=1 \qquad \forall t\geq 0,\ \ \forall s^t\in S^t \tag{1}$$

He chooses consumption $c_t(s^t)$, leisure and bond holdings $b_t(s^t)$ to maximize his lifetime discounted expected utility:

$$E_0\sum_{t=0}^{\infty}\beta^t u(c_t,l_t)=\sum_{t=0}^{\infty}\sum_{s^t}^{\infty}\beta^t u(c_t(s^t),l_t(s^t))\pi(s^t\mid s_0) \qquad (2)^4$$

subject to the period-by-period budget constraint:

$$b_{t-1}(s^{t-1})+(1-\tau_t(s^t))w_t(s^t)(1-l_t(s^t))=c_t(s^t)+p_t(s^t)b_t(s^t) \tag{3}$$

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4 The utility function satisfies the usual standard assumptions, *i.e.*, $u_{c,t}>0$, $u_{l,t}>0$, $u_{cc,t}<0$, $u_{ll,t}<0$.

where β is the discount factor, $\tau_t(s^t)$ is the state-contingent labor tax rate, $w_t(s^t)$ is the wage rate and $p_t(s^t)$ is the price of the one period bond.

The household's optimality conditions are:

$$\frac{u_{l,t}(s^t)}{u_{c,t}(s^t)} = w_t(s^t)(1-\tau_t(s^t)) \tag{4}$$

$$p_t(s^t) = \beta E_t \frac{u_{c,t+1}(s^{t+1})}{u_{c,t}(s^t)} \tag{5}$$

where, for notational simplicity, we denote from now on $u_{c,t}(s^t)$ and $u_{l,t}(s^t)$ as the marginal utility of labor and consumption in state s^t.

There is only one non-storable good, produced by a representative price-taker firm with a linear production technology given by:

$$y_t(s^t) = \vartheta_t(s^t) n_t(s^t)$$

Output, y_t, can be used either for private consumption or public consumption (g_t). Equilibrium in the good market and in the labor market requires:

$$y_t(s^t) = c_t(s^t) + g_t(s^t) \tag{6}$$

$$\vartheta_t(s^t) = w_t(s^t) \tag{7}$$

2.1.2 The government

The government finances the exogenous sequence of government expenditures levying taxes and issuing debt. Its policy $\tau_t(s^t), b_t(s^t)$ $\forall t \geq 0$ satisfies the period by period budget constraint:

$$b_{t-1}(s^{t-1}) + g_t(s^t) = \tau_t(s^t) w_t(s^t)(1-l_t(s^t)) + p_t(s^t) b_t(s^t)$$

The initial level of debt b_{-1} is exogenously given. Ayagary *et al.* (2002) show that the dynamic optimal taxation problem of the government is equivalent to the problem of maximizing:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{8}$$

under the following constraints:

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1-l_t)) = u_{c,0} b_{-1} \tag{9}$$

$$E_t \sum_{j=0}^{\infty} \beta^j (u_{c,t+j} c_{t+j} - u_{l,t+j}(1-l_{t+j})) = u_{c,t} b_{t-1}(s^{t-1}) \quad \forall t \geq 0, \quad \forall s^t \tag{10}$$

$$\underline{M} < \frac{E_t \sum_{j=0}^{\infty} \beta^t (u_{c,t+j} c_{t+j} - u_{l,t+j}(1-l_{t+j}))}{u_{c,t}(s^t)} < \overline{M} \qquad \forall t \geq 0, \ \forall s^t \tag{11}$$

$$\vartheta_t(1-l_t(s^t)) = c_t(s^t) + g_t(s^t) \tag{12}$$

Constraints (9) and (10) require that for any period and any state, the inherited level of debt is equal to the stream of expected future primary surpluses. They are equivalent to the intertemporal consumer budget constraint with both prices and taxes replaced using the households' optimality conditions, (4) and (5). If financial markets were complete, constraints (10) would be satisfied by choosing appropriately the vector of state-contingent bond, so they would not constrain the optimal choice of taxes. However, under incomplete markets, the government cannot adjust the inherited stock of debt in response to the current realization of the shock. Therefore, constraints (10) captures the idea that in any period the future path of taxes depends on the current state. Constraints (11) requires that debt limits be respected.

It can be shown that the solution to the government problem satisfies:

$$\tau_t = T(s_t, \psi_{t-1}, b_{t-1}) \qquad \forall t > 0 \tag{13}$$

$$b_t = D(s_t, \psi_{t-1}, b_{t-1}) \qquad \forall t > 0 \tag{14}$$

Equations (13) and (14) are the optimal policy rules for the labor tax rate and for bond holdings respectively. Both of them are time invariant functions of the current state s_t, the inherited bond holding b_{t-1} and the auxiliary state variable ψ_{t-1} which is equal to the sum of past lagrange multipliers, from period 0 till t–1, associated to the intertemporal budget constraints (10).[5]

Two observations are worth noting. First, by including the costate variable ψ_{t-1} in the vector of state variables the problem becomes recursive and standard solution techniques can be applied. Second, the presence of ψ_{t-1} and b_{t-1} makes the allocation and the cost of distortionary taxation state and history-dependent.

2.2 *Modeling fear of government default*

In the benchmark model of Subsection 2.1 households fully understand the government problem and therefore attach zero probability to the event of a government default, whatever the observed evolution of government debt. In particular, as households understand the risk-free nature of government bonds, they do not require to be compensated for any default risk. In this section we study what happens if agents abruptly – and wrongly – start to fear that the government might not fulfil the promise of always paying back its own obligations.

In particular, at time *t* the household believes that at time *t+1* debt will be honoured with probability $\hat{\pi}_t$ and will be instead repudiated with probability $(1-\hat{\pi}_t)$.

In this case, the optimality condition of the household is given by:

$$\begin{aligned} p_t(s^t,\delta^t)u_{c,t}(s^t,\delta^t) &= \beta \textstyle\sum_{s_{t+1}} u_{c,t+1}(s^{t+1},\delta_{t+1}=1,\delta^t)\tilde{\pi}(s^{t+1},\delta_{t+1}=1,\delta^t \mid s^t,\delta^t) = \\ &= \beta \textstyle\sum_{s_{t+1}} u_{c,t+1}(s^{t+1},\delta_{t+1}=1,\delta^t)\hat{\pi}(s^{t+1}\mid s^t,\delta^t)\hat{\pi}_t \end{aligned} \tag{15}$$

where $\delta_t \in \{0,1\}$ is equal to 1 if the government does not default on debt in period *t* and equal to 0 otherwise, and $\hat{\pi}_t$ is the probability that $\delta_{t+1}=1$ conditional on s^t and δ^t. The relevant expectations ($\tilde{\pi}$) are now with respect to s^t *and* the event of government default.

[5] This approach has been pioneered by Marcet and Marimon (2002).

We make two assumptions about how default expectations evolve. First, the higher the level of outstanding debt, the stronger the fear of government default, and in particular fear of default start to arise when the debt goes above some "psychological" threshold $\bar{b}$:[6]

$$\hat{\pi}_t = \frac{1}{1+\alpha_t \max(b_t - \bar{b})} \tag{16}$$

Second, we assume that agents revise their beliefs about the probability of a public default as new evidence about government behaviour becomes available. In the literature various ways have been proposed to model agents' learning.[7] We adopt the approach pioneered by Marcet and Sargent (1989). They study agents which are similar to an econometrician, *i.e.*, in each period they estimate recursively those parameters which are relevant for their decision, and whose values they ignore. In our model the only parameter that has to be estimated is α. Let α_t be the agents' estimate of α at time t. If agents use a constant gain algorithm with gain parameter equal to k, a special case of the algorithm studied by Marcet and Sargent (1989),[8] it can be shown that α_t is given by the following expression:

$$\alpha_t = \alpha_{t-1}(1 - kb_{t-1}{}^2)^{9} \tag{17}$$

Several observations are worth-noting. First, equation (16) nests the rational expectation case in which households understand that default cannot happen. In fact, when $\alpha_t = 0$, $\hat{\pi}_t = 1$. Second, under the condition that $|1 - kb_{t-1}{}^2| < 1$ equation (17) is such that α_t converges to its true value, 0.

It is important to stress the fact that the perceived default probability has no impact on the actual default probability, which is always equal to 0. We believe that these features of the model capture the challenges that advanced countries are facing in the aftermath of the huge fiscal stimulus packages put in place to contrast the recent crisis. More generally we aim to derive optimal strategies for policymakers which do not see default as a viable policy option but have to take into account the link between the design of fiscal policy, default expectations and macroeconomic variables.

Definition 1

Given b_{-1} and a stochastic process for the government expenditure g_t and the technology shock ϑ_t , a competitive equilibrium is an allocation $\{c_t, l_t, g_t\}_{t=0}^{\infty}$, state-contingent beliefs about government default probabilities $\{\hat{\pi}\}_{t=0}^{\infty}$,a price system $\{p_t, w_t\}_{t=0}^{\infty}$ and a government policy $\{\tau_t, b_t\}_{t=0}^{\infty}$ such that (a) given the price system, the beliefs and the government policy the

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6 In the remaining of the paper, we set $\bar{b} = 0$, without loss of generality.

7 For a comprehensive survey of learning models, see Evans and Honkapohja, (2001). Several papers have already used these models to explain real world phenomena. For example, Adam *et al.* (2006), Carceles and Giannitsarou (2007), and Cogley and Sargent (2008) introduce boundedly rational agents in a standard consumption based asset pricing model to fit some features of asset prices. Marcet and Nicolini (1998) and Adam *et al.* (2005) show how learning can be an explanation of hyperinflationary episodes. Kurz *et al.* (2005), Beaudry and Portier (2004, 2007), and Eusepi and Preston (2008) stress the importance of shifting expectations for business cycle fluctuations.

8 In any case, the economic intuition behind the result is robust to alternative learning scheme.

9 This formula is derived in the following way. Assume $b_t > 0$. Taking log of equation equation 17 we get $0 \approx -\alpha_t b_t$ where we use the fact that because of the assumption that government always honours its debt $\hat{\pi}_t$ tends to 1 and that $\log(1+x) \approx x$ for small x.

households' optimality conditions are satisfied; (b) given the allocation and the price system the government policy satisfies the sequence of government budget constraint (3); and (c) the goods and the bond markets clear.

Define:

$$A_t \equiv \prod_{k=0}^{t} \hat{\pi}_{k-1} \tag{18}$$

In the full credibility case A_t is constant and always equal to 1, while under learning it is not, unless the initial beliefs coincide with the rational expectations ones, *i.e.*, unless $\alpha_{-1} = 0$. Using households' optimality conditions to substitute out prices and taxes from the government budget constraint, Ayagary *et al.* (2002) show the constraints that a competitive equilibrium imposes on allocations. Using a similar argument, we show that under incomplete markets and bounded rationality the following result holds.

Proposition 1

Assume that for any competitive equilibrium $\beta^t A_t u_{c,t} \to 0$ almost surely. Given b_{-1} and α_{-1}, a feasible allocation $\{c_t, l_t, g_t\}_{t=0}^{\infty}$ is a competitive equilibrium if and only if the following constraints are satisfied:

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = A_0 u_{c,0} b_{-1} \tag{19}$$

$$E_t \sum_{j=0}^{\infty} \beta^j A_{t+j} (u_{c,t+j} c_{t+j} - u_{l,t+j}(1 - l_{+jt})) = A_t u_{c,t} b_{t-1} \tag{20}$$

$$\underline{M} < \frac{E_t \sum_{j=0}^{\infty} \beta^t A_{t+j} (u_{c,t+j} c_{t+j} - u_{l,t+j}(1 - l_{t+j}))}{A_t u_{c,t}(s^t)} < \overline{M} \tag{21}$$

with initial condition $A_{-1} = 1$.

Proof

We relegate the proof to the Appendix.

Equation (20) is the bounded rationality version of the intertemporal constraint on the allocation derived by Ayiagary *et al.* (2002) in a rational expectations framework, given in equation (20). The difference between equations (20) and (10) arises through the effect that government default expectations exert on bond prices. As expectations are not model-consistent, the primary surplus at time *t*, expressed in terms of marginal utility of consumption, is weighted by the product of one minus the expected default probabilities from period 0 till period *t*.

2.3 The government problem

Using the so-called primal approach to taxation, we can recast the problem of choosing taxes and bond holdings as a problem of directly choosing allocations of consumption and labor, under the constraint that they satisfy the conditions for a competitive equilibrium.

At this point a clarification is needed. When the households and the benevolent government share the same information, they maximize the same objective function. But when the way in which they form their expectations differ, as in this setup, their objective functions differ as well. In what follows we assume that the fiscal authority maximizes the representative consumer's welfare *as if* the latter were rational. Said differently, the government understands how agents behave and form their beliefs, and it understands that these beliefs are distorted.[10]

Definition 2

The government problem under learning is:

$$\max_{\{c_t, l_t, \alpha_t, A_{t+1}, b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to:

$$E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j} c_{t+j} - u_{l,t+j}(1-l_{t+j})) = A_t(s^t, \delta^t) u_{c,t}(s^t, \delta^t) b_{t-1}(s^{t-1}, \delta^{t-1}) \tag{22}$$

$$\underline{M} < \frac{E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j} c_{t+j} - u_{l,t+j}(1-l_{t+j}))}{A_t(s^t, \delta^t) u_{c,t}(s^t, \delta^t)} < \overline{M} \tag{23}$$

$$A_{t+1} = A_t \hat{\pi}_t(s^t, \delta^t) \tag{24}$$

$$\alpha_t(s^t, \delta^t) = \alpha_t(s^{t-1}, \delta^{t-1})(1 - k b_{t-1}{}^2(s^{t-1}, \delta^{t-1})) \tag{25}$$

$$c_t(s^t, \delta^t) + g_t = \vartheta_t(1 - l_t(s^t, \delta^t)) \tag{26}$$

for given b_{-1} and α_{-1}. Equations (22) and (21) constrain the allocation to be chosen among competitive equilibria. Equation (24) is the recursive formulation for A_t obtained directly from equation (18). Equation (25) gives the law of motion of beliefs. Equation (26) is the resource constraint. As in equations (22) and (21) appear expectations of future control variables, the problem is not recursive and standard solution techniques cannot be used.

The Lagrangian for the Ramsey problem can be represented as:

$$L = E_0 \sum_{t=0}^{\infty} \begin{matrix} \beta^t u(c_t, l_t) + \psi_t A_t(u_{c,t} c_t - u_{l,t}(1-l_t)) - \lambda_t b_{t-1} A_{t-1} u_{c,t} + \gamma_t(A_{t+1} - A_t \hat{\pi}_t) + \\ + \rho_t(\alpha_t - \alpha_{t-1}(1 - k b_{t-1}{}^2)) + \nu_t(\vartheta_t(1-l_t) - c_t - g_t) \end{matrix}$$

where $\psi_t = \psi_{t-1} + \lambda_t - \varepsilon_{1,t} + \varepsilon_{2,t}$, where $\beta^t \varepsilon_{1,t}$ and $\beta^t \varepsilon_{2,t}$ are the Lagrange multipliers attached to the upper and lower debt constraints respectively. Since A_t and α_t have a recursive structure, the problem becomes recursive adding A_t and α_{t-1} as endogenous state variables to the ones in the Ayagary *et al.* (2002) model, which are ψ_{t-1} and b_{t-1}.

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[10] The same assumption is made in Karantouniais *et al.* (2010) and Caprioli (2009).

First order necessary conditions $\forall t > 0$ are:[11]

∂c_t :

$$u_{c,t} + \psi_t A_t (u_{cc,t} c_t + u_{c,t}) - \lambda_{1,t} b_{t-1} u_{cc,t} A_t = \nu_t \tag{27}$$

∂l_t:

$$u_{l,t} + \psi_t A_t (u_{l,t} - u_{ll,t}(1 - l_t)) = \vartheta_t \nu_t \tag{28}$$

$\partial \alpha_{t+1}$:

$$\rho_t - \beta E_t \rho_{t+1}(1 - k b_t^{\,2}) + \gamma_t A_t \frac{b_t}{(1 + \alpha_t b_t)^2} = 0 \tag{29}$$

∂b_t :

$$-\beta E_t \lambda_{1,t+1} u_{c,t+1} A_{t+1} + \gamma_t A_t \frac{\alpha_t}{(1 + \alpha_t b_t)^2} + 2\beta E_t \rho_{t+1} \alpha_t k b_t = 0 \tag{30}$$

∂A_{t+1} :

$$\gamma_t - \beta E_t \gamma_{t+1} \frac{1}{(1 + \alpha_{t+1} b_{t+1})} - \beta E_t \lambda_{1,t+1} b_t u_{c,t+1} + E_t \psi_{t+1}(u_{c,t+1} c_{t+1} - u_{l,t+1}(1 - l_{t+1})) = 0 \tag{31}$$

3 Numerical solution

Together, the first order conditions and the constraints of the government program imply a stochastic non linear system of difference equations in the variables $c_t, l_t, \tau_t, b_t, \psi_t, A_{t+1}$ and α_t. We solve the system using standard collocation methods both in the case in which there are no doubts about debt repayment and in the case in which agents start to fear a government default. In both cases we consider a truncated AR(1) process for government expenditure and labor productivity:

$$g_t = \begin{cases} \underline{g} & if \quad (1-\rho_g) g^{ss} + \rho g_{t-1} + \varepsilon^g{}_t < \underline{g} \\ (1-\rho_g) g^{ss} + \rho g_{t-1} + \varepsilon^g{}_t & if \quad \underline{g} < (1-\rho_g) g^{ss} + \rho g_{t-1} + \varepsilon^g{}_t < \overline{g} \\ \overline{g} & if \quad \overline{g} < (1-\rho_g) g^{ss} + \rho g_{t-1} + \varepsilon^g{}_t \end{cases} \tag{32}$$

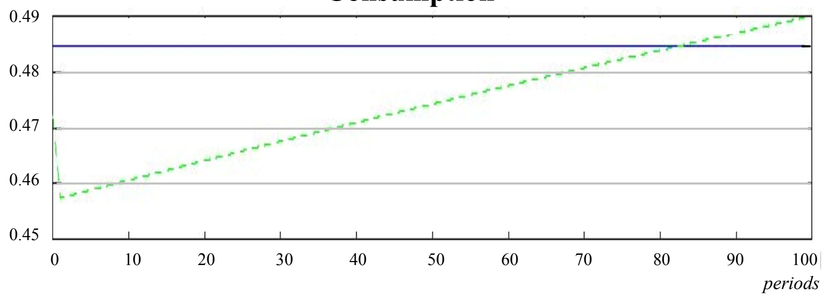
where $\varepsilon^g{}_t$ is assumed to be normally distributed with zero mean and σ^g standard deviation. Labor productivity has an analogous structure.

Figure 2 shows the path of consumption, primary surplus and government debt over GDP in two economies which are identical except for the fact that in the second one α starts at a value different from 0 (0.01). In both cases g_t and ϑ_t are constant and equal to their unconditional mean. Both economies start with the same positive level of debt (set

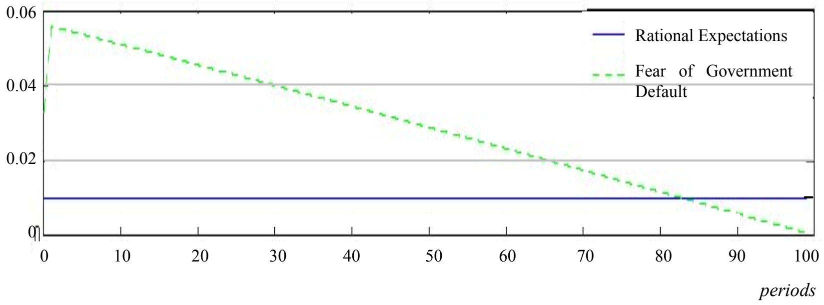
[11] As standard in the optimal fiscal policy literature, it is not easy to establish that the feasible set of the Ramsey problem is convex. To overcome this problem in our numerical calculations we check that the solution to the first-order necessary conditions of the Lagrangian is unique.

Figure 2

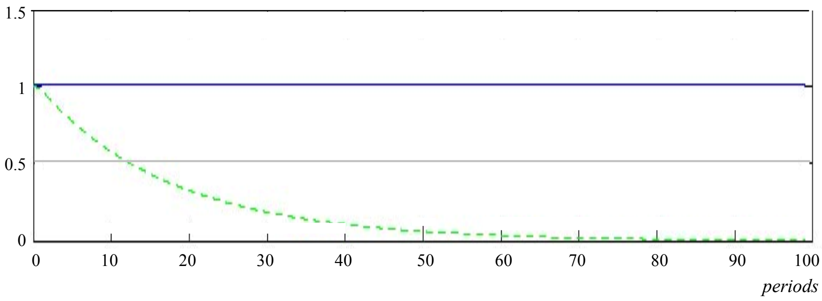
Rational Expectations Versus Fear of Default
Consumption



Primary Surplus
(percent of GDP)



Government Debt / GDP



equal to 100 per cent of GDP).[12] Given this parametrization, the initial default probability is equal to 5 per cent.

In the baseline case, government debt stays roughly constant at its initial value. This result is consistent with the main policy message coming out from the optimal fiscal policy literature. The intuition is that, as lump-sum taxes are not available, the only way to reduce debt is by increasing the distortionary tax rate today, which in turn would allow to reduce tax rates tomorrow. Under this path of taxes, households would initially enjoy less consumption and more leisure, whereas the contrary would be true later on (when the tax rate would be allowed to be lower, thanks to the reduction attained in the burden of debt). However, under standard assumptions on the utility function, households prefer to smooth consumption and leisure over time and states. Therefore a benevolent government keeps distortionary taxes as smooth as possible, and allows debt to fluctuate around the initial value. In other words, a policy of debt reduction is sub-optimal. This policy implication does not hold anymore in a context in which households fear government default. Instead, taxes are increased at the beginning and debt is correspondingly reduced. To get an intuition of this result, it is important to understand the trade-off now faced by the government. On one side, as in the baseline framework, taxes are distortionary and therefore the government would like to keep them as constant as possible. On the other side, the government is aware that the perceived probability of default is higher the higher the debt level. These expectations translate into

[12] Of course, changing the initial value does not affect the qualitative features of the result, as long as b_{-1} is above the threshold $\bar{b}$.

higher interest rates on government bonds and higher interest payments. Since agents are learning, the only way to manipulate distorted believes is by reducing debt. Fiscal consolidation becomes optimal because it is a way to correct distorted expectations.

Table 1

Average Allocation

	Full Credibility Model	**Partial Credibility Model**
Consumption	.31	.3
Leisure	.38	.39
Labor Tax Rate	.49	.51
Bond Holding	.2	.14
Primary Surplus	.0004	.01

Moving from a single realization to a fully-fledged simulation, Table 1 shows the average values for consumption and leisure and for fiscal variables (tax rate, government debt and primary surplus) in our two economies (averages are computed over 1000 simulated realizations of the shocks, for 20 time periods each). The qualitative results are confirmed. While in the rational expectation benchmark the mean value of bond holdings is equal to the initial one, in the economy with fear of default it is equal to 0.14, which means that fiscal consolidation is indeed optimal.

Correspondingly, in the second economy taxes and primary surpluses are on average higher (0.51 instead of 0.49 for taxes, 0.01 instead of 0.004 for the primary surplus). After 20 periods debt over GDP is equal to about 100 per cent in the case of a fully credible government, while it is equal to 35 per cent in the other scenario.

4 A step backward: are stimulus packages justified?

In Section 3 we studied a post-crisis situation, in which the debt has already reached the threshold above which scepticism about government commitment to debt repayment kicks in. In such a context, we showed that doubts about the capability/willingness of the government to pay back debt require a substantial, and possibly quite painful, fiscal consolidation. It is therefore natural to ask whether implementing a fiscal expansion in the event of a crisis can be justified, given that the stimulus might triggers fears of a government default.

To answer this question, in this section we do not focus on the post-crisis period only, but we aim at characterizing the optimal fiscal policy both before and after the crisis.

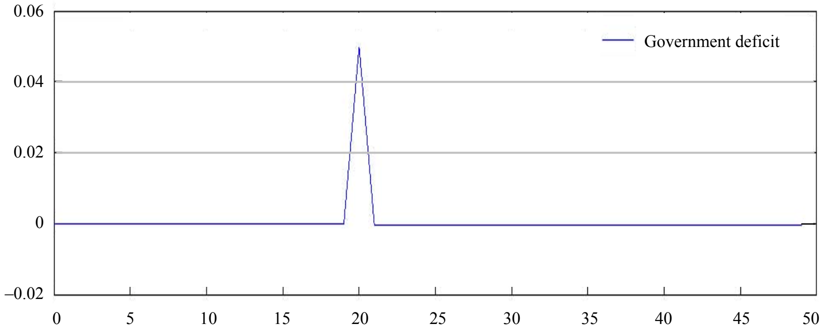
In particular, we assume that productivity ϑ_t is uncertain only at time $t = T$, when it can take two values, either ϑ_L or ϑ_H, with $\Pr ob(\vartheta = \vartheta_H) = \pi$ and $\Pr ob(\vartheta = \vartheta_L) = 1 - \pi$, but it is constant in all other periods: $\vartheta_0 = \vartheta_1 = ... = \vartheta_{T-1} = \vartheta_{T+j} = \vartheta_L(1-\pi) + \vartheta_H \pi \ \forall j \geq 1$.

Figure 3 shows the optimal way to react to a large decrease in the productivity under the rational expectation benchmark. Before period T the government sets a constant tax rate in all periods and runs a balanced budget in all periods. At T, conditional on the bad shock realization, the government runs a primary deficit and issues debt, which from that period onwards is rolled

Figure 3

Optimal Response to a Bad Shock Under Rational Expectations
(percent of GDP)

Government Deficit



Government Debt

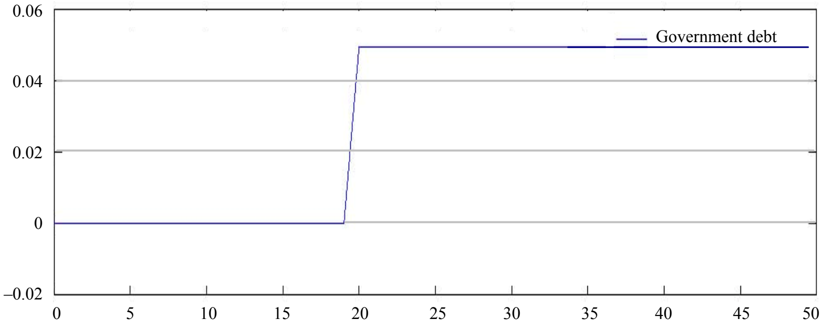
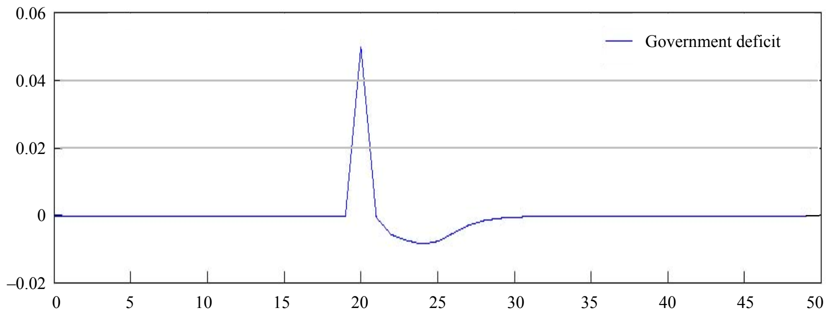


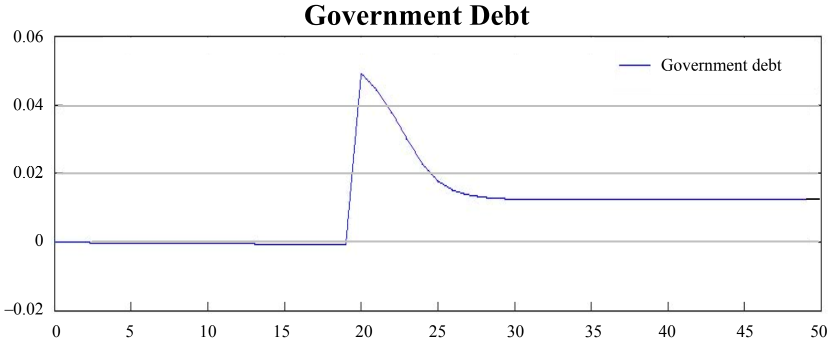
Figure 4

Optimal Response to a Bad Shock Under Fear of Default
(percent of GDP)

Government Deficit



Government Debt



over for ever. After the bad shock the tax rate is higher than before to pay for the higher debt services than before the crisis. But it is not optimal to bring debt to a lower levels.

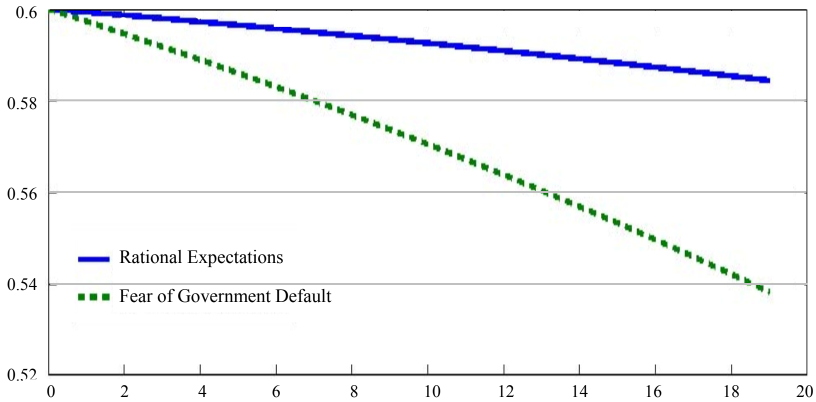
Things are different when agents fear government default. In particular consider an economy in which debt has been below the "psychological" threshold above which concerns for debt repayment start to appear. The government faces a trade-off concerning the way to cope with the crisis. If the government decides to react to the bad shock by issuing bonds, effects on consumption will be smoothed, but agents will start to fear default, which has costs because it suboptimally increases interest rates and interest payments.

What is the optimal way to respond to the shock in this case? Figure 4 offers a graphical answer to the question, for the case of $\pi = 0.5$, $\vartheta_H = 1.1$ and $\vartheta_L = 0.9$. As in the rational expectations benchmark, the optimal fiscal policy implies running a budget deficit in the event of a realization of a bad shock in T. So one could conclude that in adverse circumstances a fiscal stimulus is justified even if it induces fears concerning government debt.

However, this conclusion comes with several caveats. First, as we saw in the previous section, after the shock the government starts a fiscal consolidation aimed at reducing debt and increasing its credibility. Second, the jump in debt in T is lower with respect to the benchmark case. Third, the fact that agents may start fearing default at T influences the optimal fiscal policy even before period T. Figure 5 shows the dynamics of government debt before the realization of the shock both in the case of a fully credible government and in the case of a non fully credible government. It is apparent that, while starting from the same initial debt levels, the latter reduces debt much more than the former.[13] This provides a theoretical rationale to the policy prescription of building "fiscal space" in good times in order to be able to use fiscal policy as a counter-cyclical tool in bad times.

Figure 5

Optimal Front-Loading:
Rational Expectations Versus Fear of Default
Government Debt Before Period *T*
(percent of GDP)



5 Policy Implications for exit strategies: A tale of two countries

In the light of the model described above, how policy suggestions differ across different countries? First, the more investors are sceptical about the government willingness and/or ability to honor its debt, the more the fiscal authorities should pursue fiscal consolidation. Second, countries which are more indebted should act with more strength to reduce the debt burden. In both cases the consequences of distorted expectations are stronger, so more restrictive fiscal policies are required to restore trust in sovereign solvency.

We illustrate these insights using the German and the Italian cases. Both countries have been hardly hit by the economic crisis (in both GDP fell by about 5 per cent in 2009), but they have very different public finances (the debt-to-GDP ratio is at about 115 per cent in Italy and about 80 per cent in Germany). Moreover, perceived default risk as reflected in ratings, bond spreads and differences in the cost of credit default swap contracts, is significantly higher in the Italian case.

We calibrate the initial value for α to match the sovereign default expectations implicit in the prices of CDS contracts. We set the initial debt at the 2009 (post-crisis) level in the two countries. Figures 6 and 7 respectively show how primary deficit and debt/GDP should evolve in the two countries. The solid line refers to Germany, whereas the dashed line refers to Italy. The country facing a higher debt level and higher default premia runs higher primary surplus and reduces debt quicker than the other one.

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[13] The numerical example shown in Figure 5 has $\pi = 0$. In this scenario, debt is reduced between 0 and T–1 by about 3 per cent by a fully credible government and by about 11 per cent by a non fully credible government (in both economies the initial debt level has been set equal to 75 per cent of GDP).

Figure 6

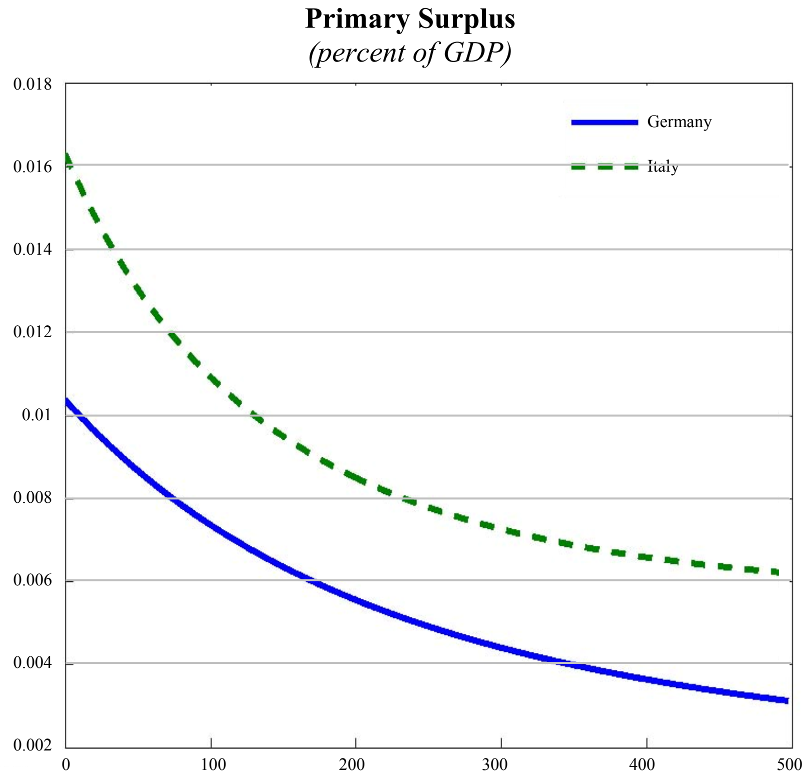
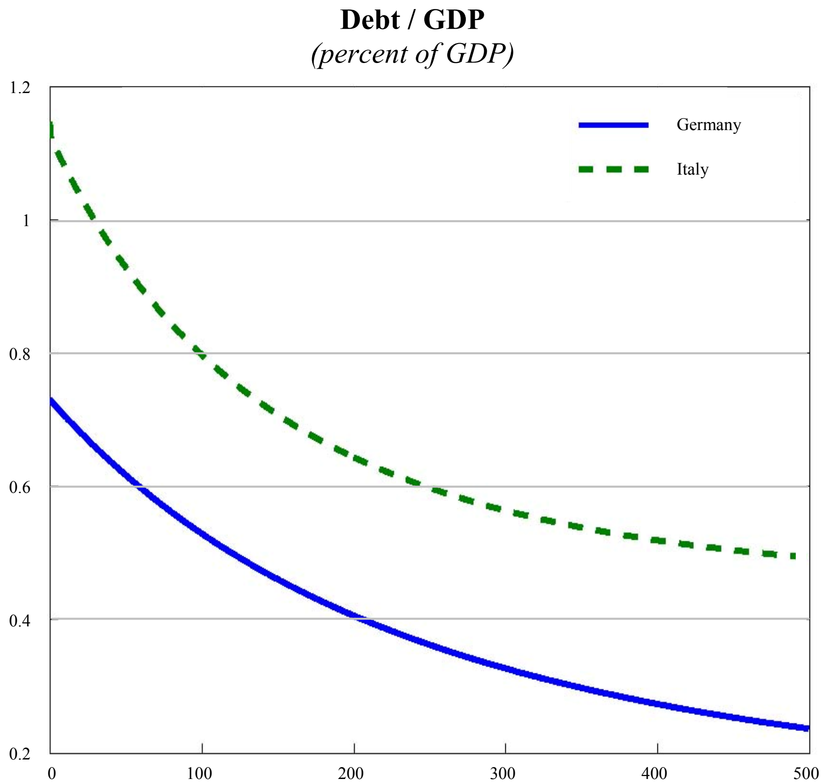


Figure 7



6 Conclusions and future research

To moderate the adverse consequences of the recent downturn, governments have intervened through expansionary fiscal policy. These interventions were justifiable but have led to a steep increase in public debts. As economies gradually recover from the recession, there is disagreement about whether to stabilize debt ratios at post-crisis levels, or to bring them down to pre-crisis levels.

This paper offers a first formal theoretical rationale, within the framework of standard optimal fiscal policy theory, for implementing a debt reduction policy after an economic crisis. Moreover, we derive the optimal size of consolidation as a function of the degree of government credibility and of the post-crisis level of debt.

If agents fully trusted the commitment of governments to always honor their debt obligations, no further fiscal consolidation would be required. But if agents fear government default and a frontloaded debt reduction reduces such fears (thereby reducing risk premia on sovereign bonds and interest rates) a quick fiscal consolidation path, such as the one advocated by several international organizations and observers, would be optimal.

The model can be extended in several possible dimensions. First, the assumption that default is not an equilibrium outcome should be relaxed. As our analysis refers to advanced countries, this assumption may be reasonable. Much less so for developing countries. Therefore one important extension would be to include a positive possibility of default in equilibrium. In this kind of model we conjecture that two possible equilibria can arise. When agents assign a low probability to the event of default, the low increase in the interest rate (with respect to the full credibility case) may be not enough to justify actual default. But when agents assign a very high probability of default, then the increase in the interest rate may support their believes because it may be optimal for the government to default. Because of the very high interest rate the cost of a transitory exclusion from the financial markets is lower than the distortionary cost of taxation to repay debt.

Another interesting extension would be to analyze fiscal and monetary coordination. In particular, it would be interesting to understand whether optimality requires that fiscal consolidation precedes or follows monetary tightening in the aftermath of a crisis, and whether a certain amount of inflation tax is an optimal way to pay the fiscal costs of the crisis.

Finally, in the paper we assumed that the government expenditure follows an exogenous stochastic process, as it is customary in the public finance literature. Because of this assumption, however, we cannot address the issue of the optimal composition of the post-crisis fiscal adjustment. In particular, should the fiscal authority reduce debt by higher taxes or by lower expenditure? Under standard assumptions on the utility and the production functions the optimal thing to do would probably be a mix of the two.

We leave all these extensions for future research.

APPENDIX

Proof of Proposition 1

First we show that constraints equation 3, equation 4 and equation 15 imply equation 20.

Consider the period-by-period budget constraint after substituting for the household optimality conditions:

$$b_{t-1} = \frac{u_{c,t} s_t}{u_{c,t}} + \beta E_t \frac{u_{c,t+1}}{u_{c,t}} \hat{\pi}_t b_t \tag{33}$$

where $s_t \equiv c_t - \frac{u_{l,t}}{u_{c,t}}(1-l_t)$, b_t is the amount of bond holdings and $\hat{\pi}_t$ is the perceived probability at time t about government default in $t+1$. Multiplying both sides of equation 33 by $u_{c,t}A_t$, where $A_t \equiv \prod_{k=0}^{t} \hat{\pi}_{k-1}$ we get:

$$b_{t-1} u_{c,t} A_t = (u_{c,t} c_t - u_{l,t}(1-l_t)) A_t + \beta E_t u_{c,t+1} A_t \hat{\pi}_t b_t \tag{34}$$

Notice that A_t has a recursive formulation given by:

$$A_t = A_{t-1} \hat{\pi}_{t-1} \tag{35}$$

Forwarding equation 35 one period we get:

$$A_{t+1} = A_t \hat{\pi}_t \tag{36}$$

Inserting equation 36 into equation 34 we get:

$$b_{t-1} u_{c,t} A_t = (u_{c,t} c_t - u_{l,t}(1-l_t)) A_t + \beta E_t u_{c,t+1} A_{t+1} b_t \tag{37}$$

Keeping iterating forward equation equation 37 and imposing the transversality condition $\lim_{t\to\infty} \beta^t A_t b_t u_{c,t} \to 0$, we get:

$$b_{t-1} u_{c,t} A_t = \sum_{j=0}^{\infty} \beta^j (u_{c,t+j} c_{t+j} - u_{l,t+j}(1-l_{t+j})) A_{t+j} \tag{38}$$

To prove the reverse implication, take any feasible allocation $\{c_{t+j}, l_{t+j}\}_{j=0}^{\infty}$ that satisfies equation 20.

Define:

$$b_{t-1} = E_t \sum_{j=0}^{\infty} \beta^j A_{t+j} u_{c,t+j} s_{t+j} \frac{1}{u_{c,t} A_t} \tag{39}$$

It follows that:

$$b_t = E_{t+1} \sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j} \frac{1}{u_{c,t+1} A_{t+1}} \tag{40}$$

$$\begin{aligned}
b_{t-1} &= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + E_t \sum_{j=1}^{\infty} \beta^j A_{t+j} u_{c,t+j} s_{t+j} \frac{1}{u_{c,t} A_t} = \\
&= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + \beta E_t \sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j} \frac{1}{u_{c,t} A_t} = \\
&= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + \frac{\beta}{u_{c,t} A_t} E_t u_{c,t+1} A_{t+1} E_{t+1} \sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j} \frac{1}{u_{c,t+1} A_{t+1}} = \\
&= \frac{A_t u_{c,t} s_t}{u_{c,t} A_t} + \frac{\beta}{u_{c,t} A_t} E_t u_{c,t+1} A_{t+1} b_t
\end{aligned} \tag{41}$$

Using equation 36 we get:

$$b_{t-1} = s_t + \frac{\beta}{u_{c,t}} E_t u_{c,t+1} \hat{\pi}_t b_t \tag{42}$$

Using the households' optimality conditions given by (4) and (15), equation (42) coincides with equation (3).

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