PRIVATIZING PENSIONS: MORE THAN AN INTERESTING THOUGHT?

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Privatization of public pension schemes, partial or complete, is on the political agenda in many countries. In the Netherlands, the discussion focuses on second-pillar pension schemes. Although these schemes are funded, they feature intergenerational risk sharing. This paper documents the consumption, labour supply and welfare effects of a privatization of these second-pillar schemes. It adopts a stochastic model of life-cycle planning that includes endogenous saving, investment and labour supply behaviour. The analysis offers a decomposition of the welfare effect of privatization in order to assess the effects of intergenerational risk sharing and of labour market distortions.

1 Introduction

A large literature has developed that assesses the welfare aspects of pension schemes. An important result is that defined-benefit (DB) pension schemes feature different types of intergenerational risk sharing that the market for whatever reason cannot provide. By this argument, DB pension schemes add to social welfare. Another result is that pension schemes generally aggravate already existing distortions on labour and capital markets, an aspect that decreases welfare. Some studies find that the risk sharing effects dominate (Nishiyama and Smetters, 2007; Fehr and Habermann, 2008), while others conclude that the distortions are dominant (Krueger and Kubler, 2006; Fuster *et al.*, 2007).

Almost without exception, the literature focuses on the case of PAYG-financed pension schemes. For the Netherlands, the case of funded schemes is more interesting. Moreover, the case of funded schemes differs from the PAYG case for two reasons. First, to the extent that the introduction of a funded scheme substitutes pension saving for private saving, the effect on aggregate saving may be minor. The case of the introduction of an unfunded scheme is known to be entirely different. Second, a funded scheme generally features a tight link between benefits and contributions. In contrast, in a PAYG scheme such a link either is weak or does not exist.[1]

This paper explores the effects of the privatization of a funded pension scheme. It therefore constructs an OLG model in which the rate of return on equity is stochastic and labour supply is endogenous. Unlike Teulings and de Vries (2006) and Bovenberg *et al.* (2007), households decide on the size and the portfolio composition of their private saving accounts. The idea that households do not save otherwise than through a pension fund is not only unrealistic, but would also in our case be misleading as households would be constrained from adjusting their private savings in order to compensate for a reduction of pension savings.

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[1] Lindbeck and Persson (2003) stress the usefulness of distinguishing carefully the concept of actuarial fairness from the financing concept (funded, unfunded).

Our model has a relation with a few models in the literature that combine capital income risk with aggregate labour income risk (Bodie *et al.*, 1985, Viceira, 2001, Cocco *et al.*, 2005 and Gomes *et al.*, 2008). Although our paper takes labour productivity and thus gross labour income as non-stochastic, labour income net of pension contributions is stochastic, as the rate of pension contributions in our model reflects shocks in the rate of return on equity. The paper that comes closest to our paper is that of Bodie *et al.* (1992) which includes labour income shocks that are perfectly correlated with stock price shocks.

Unlike Bodie *et al.* (1992) and Gomes *et al.* (2008), we adopt a specification in which labour supply is not driven by a wealth effect. Our motivation is that empirically, wealth effects are usually found to be small when compared with substitution effects (Lumsdaine and Mitchell, 1999). The implication is that labour supply is unresponsive to changes in financial wealth. Hence, labour flexibility cannot play a role in absorbing capital market shocks and the impact that the two above-mentioned studies find of labour flexibility on portfolio composition does not arise in our model.

Our approach is to analyze a hypothetical DB scheme. This allows us to give clear-cut answers on the question what is the role of typical elements of DB schemes, like the insurance against capital market uncertainty or lifetime uncertainty. The disadvantage is that real-world schemes are different, because of factors neglected in the simulation approach. See Samwick and Skinner (2004) and Poterba *et al.* (2007) for a comparison of the actual performance of DB and DC schemes.

Our analysis highlights four elements. The probably most well-known effect of DB schemes is intergenerational risk sharing. While the market does not allow trade with the unborn, DB schemes can. This type of risk sharing will be lost when the scheme is abandoned (Gordon and Varian, 1988; Bovenberg *et al.*, 2007; Gollier, 2008).

The second element is labour market distortions due to contingent transfers. If transfers among generations relate to labour income, they act as a wedge on labour supply. Hence, intergenerational risk sharing can result in effectively taxing or subsidizing labour supply. Both factors decrease social welfare and it is this welfare loss that will vanish when a DB scheme is abandoned.[2]

The third element also relates to labour market distortions, but now due to the fact that the contribution rate and the pension accumulation rate are uniform across generations. This element is common to DB plans (Bodie *et al.*, 1985) and is even legally prescribed in the Netherlands.[3] As the terminal value of a pension contribution is lower, the older the household, pension contributions are larger than the rights accumulated for young workers, whereas beyond a certain age (typically, about 45 years old) the opposite holds true. The uniformity of the pension contribution rate thus works as an incentive for young workers to take up leisure, whereas beyond a certain age, households are induced to increase their labour supply. The distortion of the labour supply decision of both groups of workers creates an additional welfare loss.

The fourth element is annuity markets. Pension schemes automatically convert the wealth upon retirement into an annuity, thereby insuring participants against lifetime uncertainty. This insurance could be achieved on the market as well, provided that annuity markets are perfect. In reality, annuity markets show large imperfections (Poterba, 2001). Our analysis takes this to the extreme and simply assumes (in the benchmark at least) that annuity markets do not exist.

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2 Taxes are levied also for other (non-pension) reasons, which increases the role of labour market distortions. Future work will take this into account.

3 Aarssen and Kuipers (2007) and Bonenkamp (2007) calculated the transfers between different age cohorts for the Netherlands that are due to the uniformity of the contribution rate and the build-up rate and found them to be quite large.

This paper will focus on the steady-state implications of the privatization of pension schemes. We will present four types of simulations: 1) privatization of the funded DB scheme (benchmark simulation); 2) the same as 1), but now assuming that perfect annuity markets exist; 3) a simulation that explores the role of labour supply endogeneity and 4) a simulation that explores the role of the uniform contribution rate. Together, these simulations indicate the overall contribution of the funded DB pension scheme to welfare and the contributions of various elements, among which insurance against lifetime uncertainty, uniform pricing and labour supply endogeneity.

Our analysis is not exhaustive. DB schemes offer additional advantages that our analysis does not capture. The obligatory nature of pensions prevents myopic households from saving too little. Moreover, pension funds may be better investors than individual households, able to achieve higher rates of return on average, less volatile rates of return or both. In addition, pension funds will be less subject to capital market constraints (*e.g.* borrowing constraints and short-selling constraints) than individual households.[4] These and other elements do not reduce the value of our results, but help to put them more in perspective.

The structure of our paper is as follows. The next section sets up our model. Then, we describe various aspects of the life-cycle behaviour of households in the baseline. Subsequently, we report the effects of the four simulations described above. We focus on the effects on consumption, labour supply and welfare. We end with some concluding remarks.

2 An OLG model with pensions

The model describes a small open economy for which factor prices are given. It consists of overlapping generations of households and a pension fund.

Households have a finite life with uncertain length. They enter the economy at the age of 20 and may work up to the age of 65. From that age onwards, they receive a pension until they die. The time of death is uncertain, but occurs at the age of 100 or before with certainty. We work with periods of five years, so we define the working phase of the life-cycle to consist of 9 periods, the retirement phase to consist of 7 periods and the life-cycle to consist of 16 periods. Households maximize a utility function by choosing their savings and their investment in risky assets at different ages in their lives. In the working phase of their life-cycle, they also choose optimally their consumption of leisure.

The pension fund in the model receives contributions from working generations and pays pensions to retired generations. Households are obliged to participate in this pension fund. This corresponds to the Dutch situation in which workers are obliged to participate in a pension scheme if they decide to sign a labour contract. The pension scheme is of the DB type: pension benefits relate to the individual's labour history, but are unrelated to both capital market rates of return and to the length of life. Shocks to pension wealth are absorbed by the contributions that the pension fund levies upon working cohorts.

This section develops the model that we use for our analysis. It starts by specifying the nature of the stochastic variables in the model. Subsequently, it specifies the model for households and that for the pension fund.

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4 See also Bovenberg *et al.* (2007) for an overview.

2.1 Stochastic assumptions

We focus on one major form of macroeconomic risks: equity return risk. The gross rate of return on equity follows a lognormal white noise process. The second asset in the economy is a bond, the return of which is riskless. The excess return on the risky asset is defined as:[5]

$$\tilde{e}_s(t) = \tilde{R}_s(t) - \tilde{R}_b \tag{1}$$

In equation (1), index b points to bonds and s to equity. The expected value of the excess return on equity is denoted as μ_s, whereas its variance is denoted as σ_s^2.

Our model distinguishes the case with and without a perfect life insurance market. In the former case, households receive an annuity return on their private savings that reflects their mortality risk (Yaari, 1965). As mortality rates are allowed to differ by age, the annuity return will be age-dependent. We abstract from macroeconomic longevity risk, so population growth at the level of generations is non-stochastic.

More precisely, in the simulations in which annuity markets are assumed to exist, the wealth of those who die at time t with age i, is transferred to the people of the same cohort who survive. This makes the effective rate of return on the two assets equal to $R_m(t,i) = \tilde{R}_m(t) / \zeta(t,i) \quad m = b, s$, where $\zeta(t,i)$ reflects the survival rate of cohort *i* in period *t*. Similarly, $e_s(t,i) = \tilde{e}_s(t) / \zeta(t,i)$. Hence, it is R_b and R_s (and e_s) that drive private savings if annuity markets are assumed perfect, rather than their equivalents $\tilde{R}_b$, $\tilde{R}_s$ (and $\tilde{e}_s$).

The literature offers a simple approach to price assets in complete markets in case of partial equilibrium modelling. Partial equilibrium models of small open economies assume exogenous given capital market developments. Equity income is the only source of uncertainty. Given these assumptions there is a unique stochastic discount factor which can be used to calculate the value of all assets and their derivatives. This unique stochastic discount factor reads as follows (see Cochrane, 2005, page 73):

$$\tilde{m}(t) = \frac{1}{\tilde{R}_b} - \frac{1}{\tilde{R}_b}\frac{\mu_s}{\sigma_s^2}\left(\tilde{e}_s(t) - \mu_s\right) \tag{2}$$

given the stochastic assumption made. This discount factor implies that non-stochastic income flows are discounted by the bond rate, because the last term disappears after taking expectations. However, stochastic income flows are discounted with a correction which depends on the covariance with the excess return.

Two examples may illustrate the working of the stochastic discount factor. Assume a bond price p_b that gives a pay out $d_b(+1)$ and a rest value $p_b(+1)$ in next year. According to asset valuation theory, it holds that:

$$p_b(t) = E\tilde{m}(t+1)\left(d_b(t+1) + p_b(t+1)\right) \tag{3}$$

This implies for the rate of return $\tilde{R}_b(t+1) = \left(d_b(t+1) + p_b(t+1)\right) / p_b(t)$

$$1 = E\tilde{m}(t+1)\tilde{R}_b(t+1) \tag{4}$$

[5] In this document we use suffixes as indicators for variables that refer to specific time periods or ages. For individual variables we use only the age suffix j, for intergenerational variables we use both the age suffix j and the time suffix t, for aggregated (macro) variables we use only time suffix t. At the individual level time and age are related on a one-to-one basis, so using the age indicator j is sufficient.

The same argument leads to:

$$1 = E\tilde{m}(t+1)\tilde{R}_s(t+1) \tag{5}$$

for the rate of return on shares. Subtract both equations to obtain for the excess return on shares:

$$0 = E\tilde{m}(t+1)\tilde{e}_s(t+1) \tag{6}$$

Equation (4) and (6) are easy to verify after substitution of the expression for the stochastic discount factor (equation (2)), taking expectations and using the definitions of the expected value and variance of the excess return on shares. All derivative assets can be valued using this stochastic discount factor, too. For instance, our model is characterized by stochastic net wages, due to stochastic pension premiums. This implies that human wealth, the discounted value of net wages, can be considered as a derivative asset of bonds and shares. The pay-out of human wealth (net wages) has to be valued with the stochastic discount factor $\tilde{m}$. In the household model we will use $m(t,i) = \zeta(t,i)\tilde{m}(t)$.

2.2 *The household decision problem*

An individual of age j maximizes his expected intertemporal utility, U, which is defined over his remaining lifetime:

$$U(j) = E_j \sum_{i=j}^{j_e} u(i)d_s(i) \text{ with} \tag{7}$$

$$d_s(i) = \prod_{l=j}^{i-1} \delta(l)^{-1}$$

Here, j_e (= 100 years) denotes the maximum attainable age.[6] The discount factor is defined as $\delta(l) = \tilde{\delta} / \zeta(l)$ with $\tilde{\delta}$ the time preference factor and $\zeta(j)$ the conditional (upon being alive at the start of year j) probability of living through the next period. E_j is the expectations operator, used to account for the uncertainty of utility derived from consumption.[7]

The felicity function, u, has as arguments the consumption of commodities, c, and the consumption of leisure, v:

$$u(i) = \frac{1}{1-\gamma}\left(\alpha_c c(i) + \alpha_v \frac{v(i)^{1-\beta}}{1-\beta}\right)^{1-\gamma} \text{ with} \tag{8}$$

$$\alpha_c, \alpha_v > 0,\ \beta > 1,\ \gamma > 0$$

$1/\gamma$ denotes the elasticity of intertemporal substitution and $1/\beta$ the price elasticity of leisure demand. We assume $\beta > 1$, ensuring that commodity consumption is always positive. α_c and α_v are utility weights of respectively the consumption of commodities and leisure.

The asset accumulation equation describes the development of household financial wealth, $w_f^h(i)$, through time:

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6 The consumption of children is attributed to their parents.

7 Note, we use as convention $\prod_{l=j}^{j-1} \delta(l)^{-1} = 1$.

$$w_f^h(i+1) = R_b(i+1)\left(w_f^h(i) + y(i) - c(i)\right) + e_s(i+1)w_s^h(i) \tag{9}$$

Equation (9) signals that households receive non-capital income $y(i)$, consume $c(i)$ and invest their savings in bonds and equity. Riskless bonds earn a yearly gross return R_b and equity earns an annual gross return R_s (with an excess return e_s). As explained in the previous section, the effective rates of return on the two assets depend on the household's mortality rate in case annuity markets are present. Hence, the effective rates of return are age-dependent. $w_s^h(i)$ denotes the household's investment in risky equity. Regarding the timing of transactions, we assume that all variables (transactions, demographic changes, stocks) are measured at the start of a period.

Non-capital income equals labour income $y_w(i)$ in the working ages, $i < j_r$, where j_r (= 65 years) denotes the maximum age in the working phase. Labour income depends on the working time, the wage rate $p_l(i)$ and the pension premium rate τ_p:

$$y(i) = y_w(i) = (1-\tau_p(i))\left(1-v(i)\right)p_l(i) \qquad \text{for } i < j_r \tag{10}$$

Working time is expressed as $1-v(i)$, indicating that we have normalized the time endowment to unity. Non-capital income equals pension income $y_p(i)$ in the retirement period ($i \geq j_r$). The pension level (replacement rate) at the start of the retirement period depends on the work effort over the past in an average-wage defined benefit (DB) system:

$$y_p(i+1) = y_p(i) + a\left(1-v(i)\right)p_l(i) \qquad \text{for } i < j_r - 1 \quad \text{and } y_p(5) = 0 \tag{11}$$

with a the accrual rate. Pension income is constant over time:

$$y(i) = y_p(i) = y_p(i-1) \qquad \text{for } i \geq j_r \tag{12}$$

The household's problem is to maximize expected intertemporal utility (7), subject to the asset accumulation equation (9), his initial amount of financial wealth, $w_f^h(i)$, and a Kuhn-Tucker condition that ensures that leisure does not exceed the time endowment of the household.

2.3 Household behaviour

In our model, households decide on their savings, on their investment in equity and on their leisure demand. We start to describe leisure demand. The equation that expresses leisure demand is as follows:

$$v(i) = \left(\frac{\alpha_c}{\alpha_v}\tilde{p}_v(i)\right)^{-\frac{1}{\beta}} \tag{13}$$

where the shadow price of leisure, $\tilde{p}_v$, is defined as the maximum of the actual price of leisure, p_v, and the ratio α_v / α_c. This ensures that leisure time does not exceed the time endowment of the household. In case the time constraint is binding, $\tilde{p}_v = \alpha_v / \alpha_c$; alternatively, $\tilde{p}_v$ equals p_v.

$$\tilde{p}_v(i) = \max\left\{\frac{\alpha_v}{\alpha_c}, p_v(i)\right\} \tag{14}$$

Two aspects of leisure demand deserve discussion. First, due to our felicity function, leisure demand does not depend on the household's financial or total wealth position. This accords with empirical evidence (Lumsdaine and Mitchell, 1999). Second, a Kuhn-Tucker condition ensures that leisure demand does not exceed unity. This holds true for retired workers, who will be assumed below to have zero labour productivity. It also applies to non-retired workers whose productivity falls below a certain level.[8] Our model thus captures the labour supply decision both at the intensive and the extensive margin.

The price of leisure consists of three components:

$$p_v(i) = (1-\tau_p(i))p_l(i) + p_r(i) \tag{15}$$

The first is the age-specific wage rate and the second the pension contributions which are proportional to the wage rate. The third component measures the discounted value of future pension income that can be attributed to the marginal hour of work, $p_r(i)$:

$$p_r(i) = ap_l(i)\sum_{h=j_r}^{j_e}\left(\prod_{l=i}^{h-1}R_b(l+1)^{-1}\right) \tag{16}$$

This component is also proportional to the wage rate.

Our specification of the felicity function implies that the consumption of commodities has a minimum that is strictly positive:[9]

$$c(i) > c_l(i) \equiv -\frac{\alpha_v}{\alpha_c}\frac{v(i)^{1-\beta}}{1-\beta} > 0 \tag{17}$$

Equation (17) demonstrates that this minimum amount of consumption is age-dependent and decreasing in leisure time. Because it relates to leisure time, we call this labour-induced consumption and denote it as c_l. This minimum amount of consumption plays an important role in our consumption equation, which reads as follows:

$$c(i) = c_l(i) + \left(\frac{1}{\alpha_c p_f(i)}\right)^{\frac{\gamma-1}{\gamma}}\left[w^h(i) - w_l^h(i)\right] \tag{18}$$

Here, w^h denotes total household wealth, which we will define below. w_l^h, the wealth that relates to current and future labour-induced consumption, is described by the following equation:[10]

$$w_l^h(i) = E_i\sum_{h=i}^{j_e}\left(\prod_{l=i}^{h-1}m(l+1)^{-1}\right)c_l(h) \tag{19}$$

The second term at the RHS of equation (18) reflects the basic feature of the standard life-cycle model, consumption being proportional with total household wealth. The first and third term however indicate that the life-cycle pattern of commodities consumption deviates from the pattern of this standard model, due to the interaction with leisure demand. In particular, the first and third term taken together establish that the household consumes more (fewer) commodities than prescribed by the standard model in years in which his labour supply is relatively high (low). Our

8 Actually, as long as labour productivity is below α_v / α_c, our model predicts zero labour supply. This indicates that retirement occurs in our model not only when labour productivity becomes sufficiently low, but also when the preference for leisure becomes sufficiently high.

9 Except if v would be zero, a case that we will not consider.

10 Note, households have expectations conditional on the state of the economy. These expectations depend on the state of the economy only and are time-invariant. We use the method of parameterized expectations (see Heer and Maussner (2005), chapter 3), *i.e.* we project $\sum_{h=i+1}^{j_e}\left(\prod_{l=i}^{h-1}m(l+1)^{-1}\right)c_l(h)$ on the state of the economy at time i using regression methods.

felicity specification thus brings about a positive correlation between consumption and labour supply and, given that labour supply is increasing with the wage rate, between consumption and current labour income. Hence, consumption and current income are more strongly correlated than in the standard life-cycle model, which may help to solve part of the excess sensitivity of consumption that is found in empirical research (Flavin, 1981).

Total wealth is defined as the sum of explicit assets (here, financial wealth) and implicit assets (here, human wealth, denoted w_h^h, and pension rights, denoted w_p^h):

$$w^h(i) = w_f^h(i) + w_h^h(i) + w_p^h(i) \tag{20}$$

Human wealth is defined as the discounted expected value of future after-tax labour income:[11]

$$w_h^h(i) = E_i \sum_{h=i}^{j_r} \left(\prod_{l=i}^{h-1} m(l+1) \right) (1 - v(h)) p_v(h) \tag{21}$$

Pension wealth is the accumulation of pension rights minus the pension benefits that have already been paid out, where $\delta_{j \geq j_r}$ equals one for the retirement years and is zero otherwise.

$$w_p^h(j+1) = R_b(j+1) \left[w_p^h(j) + (1 - v(j)) p_r(j) - \delta_{j > j_r} y_p(j) \right] \tag{22}$$

The price index of total wealth:

$$p_f(i) = \left[\sum_{h=i}^{j_e - 1} \alpha_c^{\frac{1-\gamma}{\gamma}} \prod_{l=i}^{h-1} \left(\frac{R_b(l+1)\phi(l+1)}{\delta(l+1)} \right)^{\frac{1}{\gamma}} \frac{1}{R_b(l+1)\phi(l+1)} \right]^{\frac{\gamma}{\gamma-1}} \tag{23}$$

is a composite of the constant utility weight α_c. As in the standard life-cycle model, the weighting factors refer to two effects. A rate of return higher than the rate of time preference increases savings on account of the substitution effect. The second element of the weighting factor describes the income effect of returns on investments. A high rate of return also adds to consumption possibilities, the income effect. If the intertemporal elasticity of substitution is below unity ($1/\gamma < 1$), the income effect dominates the substitution effect.

Different from the standard life-cycle model is the rate of return $R_b\phi$. This variable measures the certainty-equivalent rate of return. It differs from the risk-free rate of interest because the rates of return on equity and human wealth are stochastic and different from the risk-free rate of interest. The certainty-equivalent rate of return is age-specific. Indeed, pensioners do not own human capital and are therefore not subject to stochastic fluctuations in the rate of return on human wealth. In addition, workers of different age have different amounts of human capital and are therefore differentially affected by shocks in the rate of return on human wealth. The equation for the certainty-equivalent rate of return can be derived as follows:

$$\phi(l+1) = \left[E_i \left(1 + a_n(l) e_n(l+1) + a_s(l) e_s(l+1) \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{24}$$

Here, e_n and e_s are the excess rates of return on human capital and equity respectively. a_n and a_s measure the share of human wealth and equity in total household wealth respectively.

The third dimension of the household's decision problem is the allocation of wealth over bonds and equity. An age-dependent fraction of total wealth net of consumption, $a_s(i)$, is invested

[11] See footnote 11.

in the risky asset (where both total wealth and consumption are corrected for labour-induced consumption):

$$w_s^h(i) = a_s(i)R_b(i+1)\left[\left(w^h(i) - w_l^h(i)\right) - \left(c(i) - c_l(i)\right)\right] \tag{25}$$

The investment share in the risky asset, a_s is implicitly defined by the following two equations:

$$0 = E_i\left(1 + a_n(i)e_n(i+1) + a_s(i)e_s(i+1)\right)^{-\gamma} e_l(i+1) \text{ and } l \in \{n,s\} \tag{26}$$

with a_n the implicit portfolio share of human wealth and:[12]

$$e_n(i+1) = \frac{w_h^h(i+1) - w_l^h(i+1) + R_b(i+1)y_b(i)}{w_h^h(i) - w_l^h(i)} - R_b(i+1) \tag{27}$$

in which y_b is net broad labour income, *i.e.* labour income net of pension contributions but including pension rights and excluding labour-induced consumption: $y_b \equiv (1-v)p_v - c_l$.

The RHS of equation (26) can be approximated by a second-order Taylor expansion around zero. This leads to the following expression for the fraction of total household wealth that is invested in equity:

$$a_s(i) = \frac{\mu_s - \gamma a_n(i)\sigma_{ns}(i)}{\gamma\left[\sigma_s^2 - \mu_s^2\right]} \tag{28}$$

in which $\mu_s / (\gamma\left[\sigma_s^2 - \mu_s^2\right])$ is the tangency portfolio and $(a_n(i)\sigma_{ns}(i)) / \left[\sigma_s^2 - \mu_s^2\right]$ is the income hedge portfolio. $\sigma_{ns}(i)$ denotes the covariance between the excess return on equity and that on human capital. This covariance term is age-specific. It is positive for all working generations. The implicit portfolio share of human wealth can be approximated in the same way

$$a_n(i) = \frac{\mu_n - \gamma a_s(i)\sigma_{ns}(i)}{\gamma\left[\sigma_n^2 - \mu_n^2\right]} \tag{29}$$

The value function is defined as

$$V(i) = \frac{1}{1-\gamma} p_f(i)^{\gamma-1}\left[w^h(i) - w_l^h(i)\right]^{1-\gamma} \tag{30}$$

2.4 *The behaviour of retirees*

Retirees have zero labour productivity so that $p_v = \alpha_v / \alpha_c$ and $v = 1$. Pre-commitment consumption equals $c_l = (\alpha_v / \alpha_c)/(\beta - 1)$ and the equation for consumption adjusts correspondingly. Importantly, $e_n = 0$ for retirees, since they are not subject to labour income shocks. Hence, the investment share for pensioners does not contain a hedging component and is independent of age:

$$a_s(i) = \frac{\mu_s}{\gamma\left[\sigma_s^2 - \mu_s^2\right]} \tag{31}$$

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[12] The error terms e_n are calculated assuming perfect foresight. A future research step will be replacing this assumption with the rational expectation assumption.

which is comparable with Viceira (2001). This implies that $\phi(i)$ is age-independent for retired people.

2.5 The behaviour of workers

The leisure of workers is given by equation (13) and their consumption by equation (18). The investment in equity as a fraction of their total wealth is given in equation (28). A feature of the life-cycle model we employ is that human wealth as a fraction of total household wealth drops to zero at the age of 65. It can be derived that a_n displays a similar pattern. Labour income net of pension premiums is positively correlated with equity return shocks. Consequently, a_s increases over the life-cycle. As the hedging role of human capital diminishes over the life-cycle, households decide to invest an increasing fraction of their total wealth in equity. Although equation (28) is similar to that of Viceira (2001), the hedging demand for equity does not increase with age in that paper. The reason is that all workers in Viceira (2001) face the same probability to become retired, so that human wealth is actually independent of age. We consider our approach more realistic.

2.6 Pension sector

Pension funds start each period with a given amount of financial wealth w_f^p. They receive premium income $\tau_p y_{wg}$ from workers ($j < j_r$) and pay benefits y_p to retirees. The remainder is invested in bonds or equity. Assets have a return which is received at the start of next period. Assets evolve according to:[13]

$$w_f^p(t+1) = \tilde{R}_b(t+1)\left[w_f^p(t) + \tau_p(t) y_{wg}(t) - y_p(t)\right] + \tilde{e}_s(t+1) w_s^p(t) \tag{32}$$

in which the macro variables are obtained by aggregation over the age cohorts (for instance $y_p(t) = \sum_j n(t,j) y_p(t,j)$). In this equation τ_p is the pension premium rate and y_{wg} is gross wage income, *i.e.* income before premiums are paid ($y_{wg}(t) = \sum_j n(t,j)(1 - v(j) p_l(j))$). w_s^p denotes the amount that the pension scheme has invested in equity.

The pension benefits for ($j \geq j_r$) are given in pure DB: shocks are absorbed in the premium rate, while the built up remains time-independent. The representative pension fund uses a simple premium rule. It fixes the premium at a rate that gradually reduces deviations of financial wealth from the pension rights

$$\tau_p(t):\ E_t \Delta w_f^p(t+1) = \Delta w_p^h(t+1) - \mu\left(w_f^p(t) - w_p^h(t)\right) \tag{33}$$

The partial adjustment specification in equation (33) implies a gradual adjustment of financial wealth of the pension fund to its liabilities or, alternatively, a gradual convergence of the coverage ratio towards the level of unity. Hence, a deviation of the coverage ratio from one will generally not be eliminated in one period. This is essential, as it means that the pension scheme organizes risk sharing between non-overlapping generations of households, something that the private market is unable to organize. Unlike households, we do not let the pension fund optimize over the portfolio allocation of its financial wealth. We rather fix this portfolio allocation to the level that coincides with the portfolio allocation (for the case without pension funds) of the average household.

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13 See footnote 5 for notational conventions.

2.7 *Alternative model settings*

The benchmark version of our model abstracts from annuity markets, so private savings cannot be insured against longevity risk. An alternative model version assumes that annuity markets exist, so that there is full insurance against longevity risk at actuarially fair prices. As mortality rates are age-dependent, effective rates of return are also age-dependent in this case. Another model version assumes exogenous labour supply. In this model version, labour supply is not an instrument of the household optimization problem. A third alternative version assumes fair pension pricing. In this model version, the equation for the uniform pension contribution rate does not apply. Instead, each cohort faces a cohort-specific pension contribution rate, equal to the present value of the marginal pension right.

As the modifications that arise when implementing one of these alternative versions are pretty straightforward, we omit a detailed description with model equations.

3 Calibration and the numerical solution of the base run

The intertemporal substitution elasticity takes a value of 0.5 ($\gamma = 2$). The rate of time preference, $\tilde{\delta} - 1$, takes a value of 1.25 percent. The net risk-free rate, $\tilde{R}_b - 1$, equals 2 percent. The mean and the standard deviation of the excess rate of return on equity, μ_s and σ_s, are chosen to equal 1 and 10 percent respectively.[14]

Total available time a year is scaled to one and the annual gross wage rate to 2. The price elasticity of leisure equals –1/3 ($\beta = 3$). The parameters α_v and α_c are chosen such that annual leisure time and annual working time during working ages equal 0.5. This is achieved by taking values for α_v and α_c of 0.25 and 1 respectively.

The pattern of mortality rates is such that cohorts up to the age of 75 have a size of 10 and older cohorts have size equal to that of their predecessor minus 2, so that the last cohort in the model, aged 95-99, has size 2. In the simulations with a pension fund we assume an adjustment parameter $\mu = 0.5$ and an accrual rate $a = 0.0125$ a year.

We start in a world without pension funds and without insurance against longevity risk. These assumptions imply as only source of uncertainty equity income of households.

Figures 1 and 2 give more insight into the life-cycle behaviour of households in the model version without pension funds and without annuity markets. For convenience, we focus on the median case, *i.e.* we present results for the case in which the rate of return on equity equals its mean in all years: $\tilde{e}_s(t) = \mu_s$.

The left panel of Figure 1 portrays the development of financial wealth, human wealth and their sum, total household wealth, as a function of the age of the household. The household accumulates financial wealth during the working phase in order to finance consumption during retirement. Human wealth is highest when households enter the labour market and falls gradually to zero over the working phase.

The right panel of Figure 1 displays average consumption and income as a function of age. Consumption increases during the working ages due to the fact that the return on savings is larger

[14] The values taken for the mean and the standard deviation of the rate of return on equity are much lower than in the data. This is not so much of a problem, as this paper only *explores* the effects of pension reform. For a more thorough assessment of the issue, obviously more realistic values need to be included. We leave this for future research.

Figure 1

Age Profiles of Wealth (left panel) and of Consumption and Income (right panel)

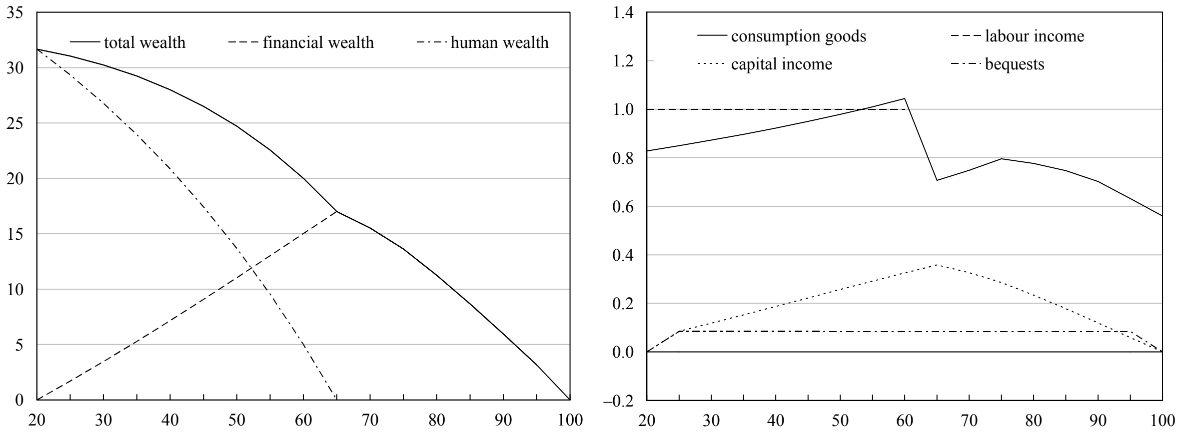


Figure 2

Equity as a Fraction of Total Wealth (left) and Financial Wealth (right)

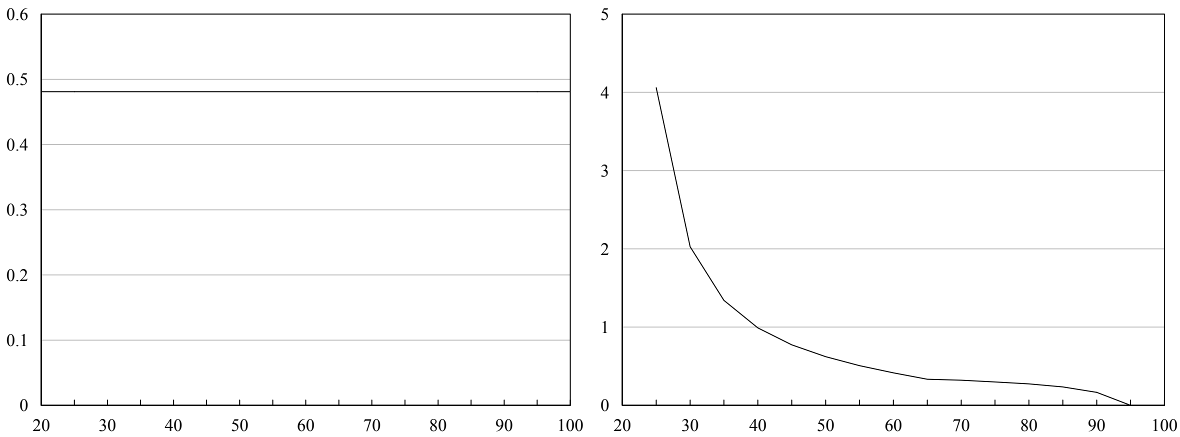
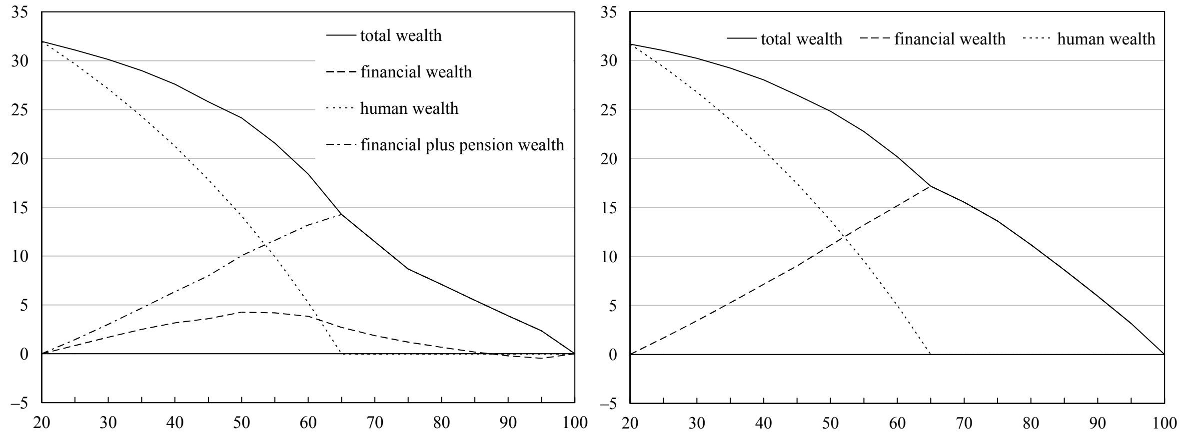


Figure 3

Wealth Profile with (left) and without Pension Funds (right)



than the rate of time preference. At retirement, consumption drops. This may look as a violation of the Euler condition that characterizes optimal consumption behaviour. It is not however. Retirement implies that the household is forced to reduce its labour supply to zero and to start consuming leisure at the maximum rate. In order to achieve marginal utility smoothing, the household then has to reduce the consumption of commodities upon retirement.

After the age of 65, consumption starts increasing again. The increase turns into a decline at later ages. This occurs in the years with a positive death probability. Without annuity markets households prefer to frontload consumption, *i.e.* the time preference increases relative to the return of savings in this period.

There are three sources of income: labour income, capital income and income from bequests. Labour income is generated during the working ages. Capital income develops in line with financial wealth. Bequests are constant over the life cycle. This is based on our assumption that in the absence of annuity markets aggregate wealth of those who die is distributed equally over all living households by the government.

The left panel of Figure 2 displays the fraction of households' total wealth invested in equity. Equity investment as a fraction of total household wealth is constant over the life-cycle, a well-known property of the CRRA function (Merton, 1969; Samuelson, 1969). Since financial wealth as a share of total wealth increases over the life-cycle, the ratio of equity investment over financial household wealth falls over the life cycle. Note that we have assumed perfect capital markets in which there are no short-selling constraints. Indeed, young cohorts start to invest about 4 times their stock of private savings into equity. Only at the age of 40, the share of financial wealth drops below unity and the household no longer needs to go short in riskless bonds.

4 Stochastic Simulations

4.1 Privatising pensions

We draw 100 different stochastic paths. For convenience, we only present the means. This section compares the case with pension funds (left panels) with that without pension funds (right panels).

The accumulation of private financial wealth is slower in the model with a pension fund for the obvious reason that pension savings and private savings are substitutes. It is not that obvious that the sum of private and pension savings in the model with a pension fund is also smaller than private savings in the model without a pension fund. The reason is that the insurance that the pension scheme provides against equity return and lifetime uncertainty reduces the need for precautionary saving, thereby decreasing the accumulation of financial wealth. Figure 4 shows the counterpart of this: the smaller savings in the model with a pension fund imply higher consumption during working ages, but lower consumption at higher ages.

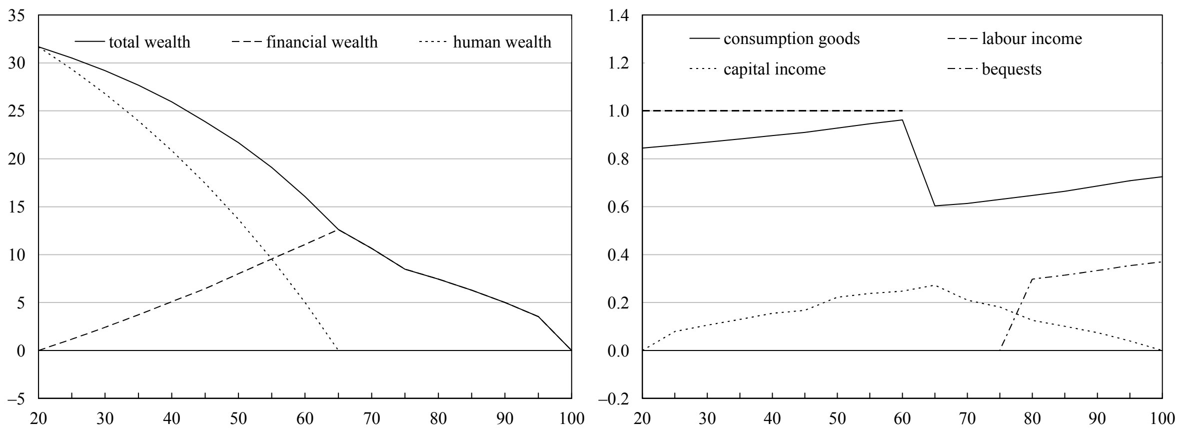
Privatization is calculated to imply a negative welfare effect. Although the pension scheme in our model distorts the labour market in two ways, the insurance that the pension scheme provides to the household against capital income risk and longevity risk obviously dominates. In particular, the welfare loss of privatization equals 13.3 per cent. To see how this effect can be decomposed, the next sections will calculate the effects of the same reform with alternative model versions.

Figure 4

Consumption and Income Profile with (left) and without (right) Pension Funds

Figure 5

Wealth Profile and Consumption with Annuity Market



4.2 *The contribution of annuity markets*

The previous section showed a welfare decline of 13.3 per cent for the steady state generations in case pensions are privatised. The absence of annuity markets in the model without a pension fund explains about 8 per cent of this overall welfare decrease. The reason is that without annuity markets, the household needs to hold additional savings to protect himself against under-consumption in case he lives longer as expected. Hence, with annuity markets, he can consume at a higher rate at more advanced ages (compare the right panels of Figure 4 and 5). In order to finance the higher consumption, consumption at lower ages is somewhat reduced.

4.3 *The contribution of diminished intergenerational risk sharing*

Coverage deficits are reflected in catching-up premiums, levied upon labour income. Similarly, higher than expected returns on equity imply surpluses in the pension scheme that translate into negative premiums on labour. The two cases have in common that they distort the labour supply decision of individual households. Privatization of the pension schemes removes this effect. This amounts to a 0.8 per cent welfare gain.

4.4 The influence of uniform premiums

The impact of uniform pension pricing when compared with actuarially fair pricing on welfare is about 2.5 per cent. Like catching-up premiums, uniform pension pricing leads to distortions on the labour market. Unlike catching-up premiums which in an average simulation will be close to zero, the implicit premiums that are due to uniform pension pricing are non-zero on an average simulation. They are positive for young workers and negative for old workers. This may help to explain our finding that the welfare gain that stems from the removal of the distortion due to uniform pricing is an order of magnitude larger than the welfare gain attached to the removal of catching-up premiums. Should we include taxes in our model, this conclusion may again be modified. This is beyond the scope of this paper, however.

5 Concluding remarks

Our analysis has shown that privatising a funded DB pension scheme is on net welfare-decreasing. The steady-state loss from privatization is 13.3 per cent. Of this, 8.4 per cent can be attributed to valuable intergenerational risk sharing between non-overlapping generations. This is lost when the scheme is privatized. Another 8 per cent is due to insurance against lifetime uncertainty. This is also lost upon privatization if annuity markets are assumed to be absent. Should we assume that well-functioning annuity markets exist, this part of the welfare loss can be avoided as households can switch to annuity markets to insure against lifetime uncertainty.

Pension schemes like the ones studied here are also known to distort labour markets. The fact that pension contributions are levied on labour income implies that the part of contributions that is used by the pension fund to restore the coverage rate acts as wedge on labour supply, similar to a labour income tax. The elimination of the labour market distortion that is due to the levying of (positive and negative) catching-up premiums produces a welfare gain, albeit quite meagre: 0.8 per cent.

Pension schemes distort the labour market for another reason as well. That is that the accumulation of pension rights and the pension contribution rate do not distinguish between generations. Since, the terminal value of pension contributions decreases with age, this means that young working generations pay more than what is actuarially fair; for older working generations, the opposite holds true. The labour market is distorted along two dimensions. Young generations supply too little labour and older generations too much. Privatization eliminates this inefficiency. The contribution to welfare is calculated to be another 2.5 per cent.

Overall, the welfare implications of labour market distortions are non-negligible, but small when compared to the welfare effects that are due to intergenerational risk sharing. This confirms earlier calculations, like those in Nishiyama and Smetters (2007) and Fehr and Habermann (2008).

Although these findings are interesting, our paper cannot be considered finalized. Future research will add a sensitivity analysis. It will also increase the number of stochastic simulations in order to get a more accurate estimation of the distributions of variables. It will also focus on the effects that will occur during the transition form a public to a private pension scheme.

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