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Performance Pay, CEO Dismissal and the Dual Role of Takeovers
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Performance Pay, CEO Dismissal and the Dual Role of Takeovers

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PRELIMINARY AND INCOMPLETE

Abstract

We use a moral hazard framework with unknown agent ability to study the incentive implications of an active takeover market for incumbent managers. The takeover market mitigates agency conflicts by creating acquisition opportunities for successful managers, allowing shareholders to reduce monetary incentives ex-ante. Shareholders optimally allow some ex-post loss-making acquisitions. In addition, a more active takeover market always discourages board interference. In firms with strong boards, turnover and performance pay are non-monotonic in the intensity of the takeover threat. In firms with weak boards, turnover (performance pay) increases (decreases) with the intensity of the takeover threat. An externality between firms’ choices of governance arrangements arises. A more active board destroys acquisition opportunities for rival managers, thus forcing all firms to pay higher compensation.

JEL Classification: G34, J33, J63.

Keywords: Takeovers, board interference, compensation, CEO turnover.

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1 Introduction

This paper takes a new look at the incentive implications of an active takeover market for incumbent managers. The existing literature focuses on the threat of being taken over and its effects on management behavior. While Jensen (1986, 1989) and Scharfstein (1988) suggest that the takeover threat plays a disciplinary role which improves performance, Stein (1988) and Shleifer and Summers (1989) argue to the contrary that takeover pressure can have detrimental effects, leading to distorted investment decisions. Irrespective of their conclusions, these papers share the same view of takeovers as a threat to incumbent managers. A comprehensive analysis of the incentive implications of takeovers needs to consider that management behavior is shaped not only by the risk of being taken over but also by the chance of taking somebody else over. We propose that an active takeover market plays a dual role: it provides not only "sticks" but also "carrots" by creating acquisition opportunities for successful managers. In addition, we argue that a higher risk of being taken over may, under certain conditions, secure management's position in the firm. Greater takeover pressure may stifle the board's incentive to intervene to the extent that overall turnover risk decreases and agency conflicts between management and shareholders increase.

In a nutshell, this paper argues that an active takeover market mitigates moral hazard by offering acquisition opportunities for successful managers: the prospect of enjoying larger private benefits upon acquiring a poorly managed rival raises the manager's effort ex-ante. Since acquisition opportunities play a disciplinary role, a liquid takeover market with a sufficient supply of potential targets constitutes a public good. An externality in governance choices across firms arises. In equilibrium, board interference (which reduces the scope for value-enhancing acquisitions) is excessive and takeover activity is inefficiently low. At the same time takeover pressure stifles the board's incentive to discipline management, possibly to the extent that it aggravates agency conflicts in target firms.

According to the standard view, an active takeover market disciplines the manager by threatening him with dismissal upon poor performance if the board fails to intervene (Jensen 1986, 1989). In our model, following a takeover the acquiring manager can enjoy greater private benefits from running a larger firm. The prospect of taking over a rival in the future upon successful performance provides an implicit incentive to the manager to exert higher effort today in order to enjoy the larger private benefits associated with merged firms. Hence, acquisition opportunities mitigate agency conflicts and allow shareholders to reduce monetary incentives. As a consequence, an active takeover market may benefit shareholders even if it neither plays any disciplinary role vis-a-vis target firms nor creates any value directly (e.g. through the installation of a new management team).

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1In contrast to the existing literature on empire building, acquisitions are a remedy rather than a source of incentive problems in our model.
An externality between firms’ choices of governance standards arises. The externality operates through the takeover market and is based on the acquisition opportunity effect. In the present framework, board interference increases firm value by replacing incompetent managers, thereby reducing the scope for value enhancing takeovers. Hence, if boards are more effective, the takeover market is less liquid and there are fewer acquisition opportunities for successful managers. When designing the internal governance system, shareholders in each firm fail to internalize that the installation of a more vigilant board reduces acquisition opportunities for other managers which in turn necessitates higher compensation in rival firms. It turns out that there is excessive board interference in equilibrium.

In addition, this paper studies the interaction between board interference, takeovers, and performance pay. We consider a setting where an incumbent manager can be removed either by the board of directors or through a (hostile) takeover. The two governance mechanisms jointly determine the overall dismissal threat. In our model, firing a poorly performing manager unambiguously increases firm value for two reasons: it provides incentives for the manager to exert effort ex-ante and it increases firm value ex-post by replacing an unsuitable manager. Board intervention is costly and more intense takeover pressure always weakens the board’s incentive to intervene for two reasons. The prospect of a takeover reduces the cost of retaining an unsuitable manager and the takeover threat disciplines the manager, thus obviating the disciplinary role of the board.²

While the takeover threat and board interference are substitutes, a non-monotonic relation can emerge between the intensity of the takeover threat on the one hand and turnover and performance pay on the other hand. If the cost of board interference is low (i.e., if the board is potentially strong), the "crowding-out" effect (of board activity by takeovers) can be so strong that the manager’s position may be less secure when he is insulated from the takeover market. Supportive evidence is provided by Huang and Zhao (2009) who find that the sensitivity of CEO turnover to performance increases in firms with strong boards following the passage of antitakeover legislation. By weakening the firing threat, takeover pressure can aggravate agency conflicts in our model and force shareholders to raise performance pay. The negative relation between takeover pressure and turnover translates into a non-monotonic relation between takeover pressure and performance pay. In contrast, in firms with weak boards, i.e., with high costs of board interference, the negative impact of the takeover market on board interference is negligible. In this case, the relation between takeover threat and overall dismissal risk (performance pay) is always positive (negative).

A formalization of the above ideas requires an integrated model which includes both the takeover market and an internal governance system, i.e., performance pay and board interference. In order for acquisition opportunities and the firing threat to affect the

²See, for example, Kini, Kracaw, and Mian (2004) for supporting empirical evidence.
manager’s incentives we need a dynamic setting. Furthermore, one needs to consider explicitly the incentive problems in both target and acquiring firms in the takeover market. Along these lines, we develop a simple two-period moral hazard framework in which shareholders hire a manager of initially unknown ability. Subsequently, the manager can be dismissed in one of two ways: either the board hires a replacement or the firm is taken over. While the manager always wants to keep his job to enjoy future private benefits, firm value is maximized if he is fired whenever poor performance indicates that he is incompetent. As a consequence, the manager is exposed to career concerns in the sense that he has an implicit incentive to work in order to signal his suitability to shareholders. We allow for the possibility of an internal governance failure in which case the incumbent retains his position despite poor performance. Importantly, a takeover may correct the board’s failure and lead to removal of the incumbent.

Since the focus of the subsequent analysis is on the ex-ante incentive implications of the takeover market we abstract from any incentive or coordination problems pertaining to the takeover process itself. A firm’s role in the takeover market (bidder or target) is not a priori determined, but depends on interim-performance. Poorly performing firms are potential targets and well performing firms are potential acquirers.

The theoretical literature has not paid much attention to the ex-ante incentive implications of takeovers. Jensen (1989) and Scharfstein (1988) argue that the takeover threat plays a disciplinary role which improves performance, whereas Stein (1988) and Shleifer and Summers (1989) suggest that it can have detrimental effects, leading to distorted investment decisions. Irrespective of their conclusions, these papers share the same view of the takeover market as a threat to incumbent managers. We propose that, under certain circumstances, greater takeover pressure may secure the manager’s position in the firm because it weakens the board’s incentive to intervene. More generally, managerial incentives are not only affected by the risk of being taken over but also by the chance of taking over a rival. We argue that an active takeover market mitigates agency conflicts through the creation of acquisition opportunities.

Several recent papers study the interaction between firms’ choices of governance standards. Acharya and Volpin (2009) and Dicks (2009) uncover an externality in firms’ choices of governance that operates through the managerial labor market rather than the takeover market. The crucial assumption in Acharya and Volpin (2009) is competition for (or scarcity of) managerial talent. A firm may be forced to pay more compensation in order to prevent its manager from accepting more generous compensation in a poorly governed rival. A positive externality arises, whereby better governance in one firm allows competitors to reduce compensation. In contrast we argue that there is a negative relationship between a manager’s compensation and the quality of governance in rival firms due to diminished acquisition opportunities.

Finally, our paper is related to the literature on board of directors. Almazan and
Suarez (2003) show that the installation of a weak board (in which the manager can veto his own replacement) may be optimal when incentive provision through future control rents is cheaper than through incentive pay. In our model, a weak board mitigates agency conflicts in rival firms (by creating acquisition opportunities) but always reinforces the conflicts with one’s own manager.

LITERATURE REVIEW TO BE CONTINUED

The next section presents the setup. Section 3 analyses the effect of acquisition opportunities on managerial incentives in a simplified model and derives the firm’s optimal compensation scheme and liquidity policy. Section 4 explicitly models internal governance and derives the optimal combination of board interference and performance pay, as well as the optimal liquidity policy. Section 5 analyzes the relationship between turnover and performance pay on the one hand and the intensity of the takeover threat on the other hand. The interactions between the firms’ choices of governance arrangements and their optimal liquidity policies are the focus of section 6. Concluding remarks are in section 7.

2 Model

We consider a moral hazard problem with two periods of production. Shareholders hire a manager of unknown ability, denoted by $\theta$, to run their firm. The manager may be competent ($\theta = \bar{\theta}$) or incompetent ($\theta = \tilde{\theta}$). A priori, neither party knows the manager’s type, but both agree on the prior probability $p \in (0,1)$ that he is competent.\footnote{In defense of the assumption that the manager does not know his own type we follow Hermalin and Weisbach (2008) who argue that no one is born knowing whether he or she will be a competent CEO. Like the board, the manager only learns from actual performance whether he is suitable for the tasks demanded of him. Alternatively, $\theta$ may denote the quality of the strategy adopted by the manager. In this case, the assumption that the manager and the board share a common belief about the distribution of $\theta$ can be justified on the grounds that a strategy is only adopted if both parties are in sufficient agreement on its merits in the first place.} Everyone is risk-neutral and there is no discounting.

Initially, shareholders and the manager sign a contract (described in detail below) and the latter chooses a non-observable effort, denoted by $e \in \{e_l, e_h\}$, where $e_l < e_h$. He enjoys private benefits of $Z_1 > 0$ if he exerts low effort ($e = e_l$).

In a next step, the firm’s first-period cash flow, $X_1 \in \{0, X_1^H\}$, is realized, where $0 < X_1^H$. Cash-flows are contractible and depend on both managerial ability and effort. Let $q_l(\theta) = \Pr[X_1 = X_1^H | e_i, \theta]$ denote the probability of a high cash-flow given managerial abiltiy $\theta$ and effort $e_i$, and let $1 - q_l(\theta)$ denote the probability of a low cash-flow.

Assumption 1 $q_l(\bar{\theta}) = q_h(\bar{\theta}) = q_l(\tilde{\theta}) = 0$ and $q_h(\tilde{\theta}) = 1$. 

3
In words, a bad manager always produces a low cash-flow, irrespective of his effort, whereas a good manager can increase the probability of a high outcome through high effort. Shareholders receive the cash-flow $X_1$ net of any wage payments that they owe to the manager. Let $p(X_1)$ denote the posterior belief that the incumbent is competent following observation of $X_1$.

After the first period of production the manager can be dismissed by the board of directors. If the incumbent continues to run the firm in the second period, he produces a verifiable cash-flow $X_2 \in \{0, X^H_2\}$ that is only determined by his ability (to simplify the exposition): under competent management $X_2$ equals $X^H_2 > 0$. Otherwise, the cash-flow equals 0. In any case, the manager enjoys private benefits $Z_2 > 0$ from running the firm in the second period. Shareholders receive $X_2$ net of any wage payment to the manager. The expected second-period cash-flow under a new manager, hired randomly from the labor market, is $pX^H_2$.

A takeover market operates after a potential interference by the board of directors, but before a new manager is hired (if any) and before second-period production takes place. The firm may be either a bidder or a target in the takeover market. If the firm takes over a rival, it has to pay a price $P^a$ to the target and it incurs a takeover (or retooling) cost denoted by $c$. The takeover cost is drawn from a commonly known distribution function $F(c)$ on $[0, \tau]$ with $\tau > (1 - p)X^H_2$.

Denote by $X^a_2 \in \{0, X^H_2\}$ the gross return (before takeover cost and acquisition price) to shareholders from acquiring a rival. The return depends on the ability of the acquiring manager: if he is competent ($\theta = \bar{\theta}$), the post-acquisition value of the target is $X^a_2 = X^H_2$. If he is incompetent ($\theta = \bar{\theta}$), it is zero. In other words, the firm simply doubles its scale through a successful acquisition. The takeover cost $c$ realizes after the first period of production. Overall, the net return to shareholders from undertaking an acquisition is therefore $X^a_2 - c - P^a$. (The formation of the transaction price is described in the next section.) Finally, the manager of the acquiring firm enjoys additional private benefits $\Delta Z_2$ (with $\Delta > 0$) in the second period from running a larger firm following a successful takeover. Alternatively, the firm can be taken over by another company at the interim date. Denote by $P^t$ the sale price which shareholders receive if they decide to sell their firm. If the firm is taken over, it is run by the acquiring manager in the second period.

The contract between the manager and shareholders specifies payments to the manager contingent on the firm’s cash-flows. In addition, the contract fixes the level of liquidity available to the manager at the interim date in order to carry out acquisitions. Without loss of generality we can focus on contracts that include payments that are only contingent on first-period cash-flows, $(w(X^H_1), w(X^L_1))$. If the incumbent keeps his position after the first period (i.e., if he is neither sacked by the board nor removed through a takeover), he is in charge of the acquisition decision. Denote by $\hat{L}$ the funding level which the incumbent manager has at his disposal to carry out a takeover. The manager
is protected by limited liability.

To sum up, the timing of the game is as follows: (i) The parties sign a contract \((w(X_i^H), w(X_i^L), \hat{L})\) and the manager chooses an unobservable level of effort \(e \in \{e_l, e_h\}\). (ii) First-period cash-flow, \(X_1 \in \{0, X^H\}\) is realized and publicly observed. (iii) The board can dismiss the incumbent. (iv) The takeover market operates. (v) If the incumbent was dismissed in (iii) and the firm was not taken over in (iv), then the board hires a new manager from the labor market. (iv) Second-period cash-flow is realized.

Finally, we want to ensure that shareholders always find it optimal to induce high effort. The following condition is sufficient for high effort always being in the shareholders’ interest.

Assumption 2

\[ p[X_1^H + X_2^H - Z_1] \geq pX_2^H. \]

3 Acquisition Opportunities as an Incentive Device

This section analyzes the optimal compensation scheme and liquidity policy from shareholders’ perspective. We take a partial equilibrium view in the sense that we consider a single firm which faces an exogenous structure of the takeover market, i.e. it faces exogenous probabilities of finding a potential target or acquirer.

The game is solved by backward-induction. Consider first the case of poor first-period performance \((X_1 = 0)\). There is no doubt that the incumbent is incompetent \((p(X_1^L) = 0)\) and shareholders find it optimal to dismiss him. Hiring a new manager increases firm value from zero to \(pX_2^H\). Note also that it is never optimal to retain the incumbent in order to let him carry out an acquisition since \(X_2^a = 0 \leq c + P^a\). When deciding whether to sell their firm, shareholders anticipate that they can hire a new manager from the labor market if no transaction occurs. Their reservation price is \(pX_2^H\). Denote by \(\rho^l\) the exogenous probability that an acquirer shows up who offers a purchase price \(P^l \geq pX_2^H\). Consider in a next step the case of a high outcome in period one. First-period success perfectly reveals that the incumbent is competent \((p(X_1^H) = 1)\). He is retained by shareholders since \(X_2^H > pX_2^H\). In addition, the manager may launch an acquisition. Denote by \(\rho^a\) the probability that the successful manager finds some potential target company in the takeover market whose shareholders are willing to sell for a price \(P^a\). Given the availability of a potential target, the takeover cost \(c\) is observed and the manager goes through with the acquisition if he has the funding to do so, i.e., if \(\hat{L}\) exceeds the total acquisition cost \(c + P^a\). Hence, from an ex-ante perspective a successful manager faces a probability \(\rho^a F(\hat{L} - P^a)\) of taking over a rival. We assume that a firm cannot be taken over following a first-period high cash-flow. \(^4\)

\(^4\)The focus of the present paper is on the ex-ante incentive implications of takeovers. Mergers among
Finally, if a transaction occurs, the price equals the target’s outside option plus a takeover premium. Assuming that all firms in the takeover market possess the production technology described above, a target’s stand alone value is \( pX^H_2 \). Denoted by \( \nu \in [0, X^H_2(1-p)] \) the takeover premium obtained by shareholders in the target company, it corresponds to the fraction of the value increase from the transaction (gross of takeover cost) that is appropriated by target shareholders. For instance, a positive premium may be due to free-riding by target shareholders in a tender offer. An acquisition is ex-post efficient if \( X^a_2 - c \geq pX^H_2 \). The takeover cost has to be lower than the gross surplus, \( X^a_2 - pX^H_2 \), generated by a takeover. Note that the installation of a new manager by the board is (ex-post) more efficient than a takeover if \( pX^H_2 > X^H_2 - c \). An acquisition is loss-making, or value-decreasing, from the perspective of shareholders in the acquiring firm if the total acquisition cost exceeds the gross return, i.e., if \( X^a_2 \geq c + P^a \).

Shareholders maximize profits subject to incentive compatibility and limited liability constraints for the manager. (Remember that by Assumption 2 shareholders find it optimal to induce high effort):

\[
\max_{w(X_1^H), L} \left[ X_1^H - w(X_1^H) + X_2^H + \rho^a[(X^H_2 - P^a)F(\hat{L} - P^a) - \int_0^{\hat{L}-P^a} cf(c)dc] \right. \\
\left. + (1-p)[\rho^l P^l + (1-\rho^l)pX^H_2] \right]
\]

subject to the incentive compatibility constraint

\[
p \left[ w(X_1^H) + Z_2 + \rho^a F(\hat{L} - P^a)\Delta Z_2 \right] \geq Z_1
\]

and the limited liability constraint constraints \( w(X_1^H), w(X_1^L) \geq 0 \).

For example, with probability \( p \) the incumbent turns out to be competent and generates cash-flows \( X_1^H \) and \( X_2^H \). In addition, the firm can take over a rival with probability \( \rho^a F(\hat{L} - P^a) \). The integral corresponds to the expected takeover cost given a funding commitment \( \hat{L} \). If the incumbent is incompetent (which happens with probability \( (1-p) \)), an acquirer who offers a price \( P^l \geq pX^H_2 \) appears with probability \( \rho^l \). The left hand side of the incentive compatibility constraint gives the manager’s expected payoff if he exerts high effort, the right hand side if he exerts low effort. Remember that the manager does not know his own type when choosing his effort. If the manager works, first-period cash-flows are high with probability \( p \) in which case the manager receives an expected private benefit \( Z_2 + \rho^a F(\hat{L} - P^a)\Delta Z_2 \) in addition to his monetary reward. The term \( \rho^a F(\hat{L} - P^a)\Delta Z_2 \) is the expected benefit from running a larger firm following an successful firms are likely to be incentive-neutral. One way of formalizing this idea in our framework would be to postulate that, given a merger among two successful firms, each manager faces a probability of \( \frac{1}{2} \) to run the combined firm, implying a gain of \( \Delta Z_2 \), and a probability of \( \frac{1}{2} \) to be demoted to divisional manager, implying a reduction of private benefits of \( -\Delta Z_2 \).
acquisition. On the other hand, if $X_1 = X_1^L$ (which happens with probability $(1 - p)$), the manager is dismissed and thus loses his private benefits.

Since the objective function is decreasing in the wage $w(X_1^H)$, either the limited liability or the incentive compatibility constraint determines the wage. Rearranging the incentive compatibility constraint we find

$$w(X_1^H) \geq \frac{Z_1}{p} - \left[ 1 + \rho^a F(\bar{L}^* - P^a) \Delta \right] Z_2$$

The manager is exposed to implicit incentives. The private benefits $Z_2$ from running the firm in the second period provide an incentive to exert effort that complements the monetary reward $w(X_1^H)$. In particular, the takeover market relaxes the incentive compatibility constraint by offering additional private benefits with probability $\rho^a F(\bar{L}^* - P)$). The beneficial effect on incentives arises because the manager only stands a chance of launching an acquisition if he performed well in the first place. Note that if implicit incentives (as measured by $\left[ 1 + \rho^a F(\bar{L}^* - P^a) \Delta \right] Z_2$) are sufficiently large, the limited liability constraint binds ($w(X_1^H) = 0$) and the incentive compatibility constraint may be slack. In the following we focus on the case where the moral hazard problem is severe enough such that the incentive compatibility constraint for the manager is binding:

**Assumption 3** $Z_1 > (1 + \Delta)Z_2p$.

The private benefit from shirking is assumed to be sufficiently large such that additional monetary incentives are needed to ensure effort provision. We then obtain the following result:

**Proposition 1** (i) The optimal compensation scheme is

$$(w^*(X_1^H), w^*(X_1^L)) = \left( \frac{Z_1}{p} - \left[ 1 + \rho^a F(\bar{L}^* - P^a) \Delta \right] Z_2, 0 \right).$$

(ii) The optimal level of liquidity is

$$\bar{L}^* = X_2^H + \Delta Z_2.$$ 

Consider first the optimal payment scheme. While compensation is increasing in the private benefit from shirking, $Z_1$, it is decreasing in the future private benefit from running the firm, $Z_2$. Compensation is strictly decreasing in the acquisition opportunities offered by the takeover market ($\rho^a F(\bar{L}^* - P^a) \Delta Z_2$). The prospect of enjoying higher private benefits following a takeover raises the manager’s incentive to work ex-ante and thus allows shareholders to reduce compensation The higher the probability of a takeover, $\rho^a F(\bar{L}^* - P^a)$, the higher is the implicit incentive due to future acquisition. Hence, a liquid takeover market with a large supply of potential target firms ($\rho^a$ high) mitigates
moral hazard. In addition, compensation is decreasing in the level of liquidity available to the manager.

The optimal level of liquidity is strictly increasing in the gross profit from an acquisition ($X_2^H$) and in acquisition opportunities ($\Delta Z_2$). Note that if $\Delta$ was equal to zero, then $L^*$ would coincide with ex-post optimal level $X_2^H$. and there would only be value-enhancing acquisitions. In this case, the manager could only take over a rival if the gross return $X_2^H$ exceeded the total acquisition cost $c + P^a$. Due to the acquisition-opportunity-effect ($\Delta > 0$) investors grant the manager "excessive" funds ($L^* > X_2^H$) such that he can engage in loss-making acquisitions ex-post. The optimal liquidity policy trades off the ex-post marginal cost of a loss-making acquisition with the ex-ante marginal benefit of lower incentive pay (due to stronger implicit incentives). Taking a dynamic perspective of CEO incentives, the model thus provides a rationale for the occurrence of loss-making acquisitions. The above discussion is summarized in the following statement:

**Corollary 1** (i) Managerial compensation decreases with acquisition opportunities (as measured by $p^a F(\tilde{L}^* - P^a)$). (ii) Shareholders optimally allow some loss-making acquisitions.

In addition, Proposition 1 has the following implications. Both the level of compensation and its performance sensitivity (as measured by $w^*(X_1^H) - w^*(X_1^L)$) are decreasing in the amount of liquidity available to the manager. Given $L^*$, shareholders expected loss from a future acquisition is

$$l = p \rho^a \int_{X_2^H - P^a}^{L^* - P^a} cf(c) dc.$$

Firms with higher financial slack incur higher acquisition losses in expectation. Hence, the analysis suggests a negative relationship between the level of compensation and expected future acquisition losses. Ceteris paribus, if $\Delta Z_2$ increases, so does $l$ while $w^*(X_1^H)$ decreases. In addition, compensation should exhibit less performance sensitivity in firms where expected losses are high. Since $Z_2$ measures the private benefit from running the firm in the future, it is likely to be lower for a manager who is close to retirement.\(^5\) Hence, both the level of compensation and its performance sensitivity are likely to be lower for managers early in their careers. Conversely, the sensitivity of compensation to liquidity ($\frac{\delta w^*(X_1^H)}{\delta L^*}$) is likely to be larger for younger managers. Note that an acquisition may still be ex-post efficient ($c < (1 - p)X_2^H$) even though it implies a loss for the acquirer ($c + P^a > X_2^H$).

\(^5\)This interpretation of $Z_2$ suggests that explicit incentives should, ceteris paribus, be lower for younger managers which is supported by empirical evidence in Gibbons and Murphy (1992).
Shareholders must be able to commit in a credible way at \( t = 0 \) to a level of liquidity. Ex-post, they would never voluntarily provide funds in excess of \( X_2^H \) to finance an acquisition. The manager needs to be assured in a credible manner that he can carry out a loss-making takeover ex-post (as long as \( \hat{L}^* > c + P^a > X_2^H \)). Depending on the level of \( X_1^H \) two cases have to be distinguished.

Suppose that \( \hat{L}^* > X_1^H \). The intermediate income (or retained earnings) \( X_1^H \) are insufficient and the manager needs to be provided with additional funds. For example, the firm may obtain a non-revocable credit line at \( t = 0 \). A credit line \( \hat{L}^* - X_1^H \) in combination with a commitment to leave the intermediate income in the firm implements the optimal liquidity policy. Conversely, if the intermediate income is larger than the optimal level of liquidity (\( \hat{L}^* < X_1^H \)), funds need to be pumped out of the firm to prevent a successful manager from undertaking loss-making acquisitions ex-post. For instance, short-term debt of \( X_1^H - \hat{L}^* \) could be used to reduce the resources available to management.

To sum up, we show that the takeover market mitigates agency conflicts in bidder firms by providing acquisition opportunities for successful managers. According to the standard view, an active takeover market enhances efficiency in two ways. It increases firm value ex-post by replacing inferior managers and it disciplines managers ex-ante through the dismissal threat. The above discussion suggests a new channel that may operate independently from these two. Importantly, the reduction in agency costs also arises for non-transacting firms.

4 Internal Governance and the Dual Role of Takeovers

By assuming that the board can always freely dismiss a poorly performing manager the previous section ignored the disciplinary role played by the takeover market vis-a-vis target firms. There was no scope for takeovers to discipline poorly performing managers by threatening them with dismissal. This section extends the above setup by modeling explicitly the firm’s quality of internal governance. We allow for the possibility of internal governance failure whereby the incumbent is protected from dismissal by the board. Let \( \tau \in [0, 1] \) denote the ex-ante probability that the firm is well governed. The analysis in the previous section corresponds to the case \( \tau = 1 \).

The board of directors determines \( \tau \in [0, 1] \) in stage 0 at a cost \( \frac{1}{2} k \tau^2 \), with \( k \geq 0 \). We
assume that the board can ex-ante commit to a level of internal governance. Its choice is
publicly observed by the manager before he exerts effort. The firing decision is assumed
to be not contractible.

Two possible interpretations of the interference cost come to mind: in the absence
of any delegation problems between the board and shareholders, the cost \( \frac{1}{2}k\tau^2 \) can be
interpreted as the resources spent on assessing managerial performance, for instance, by
establishing a more transparent accounting system. When choosing \( \tau \), the benevolent
board maximizes firm value net of interference cost. Alternatively, the board may be
self-interested. In this case, the cost \( \frac{1}{2}k\tau^2 \) reflects the residual incentives of the board to
act against the interest of shareholders. Excessive leniency toward management may be
due, for example, to board members’ lack of independence, excessive workload, or simply
the desire to avoid conflicts.\(^6\) The above setup captures in a reduced form the notion that
there is scope for internal governance failure even if incentive mechanisms for the board
are designed optimally.\(^7\) If incentives are perfectly aligned (i.e. if \( k \) is equal to zero) then
\( \tau^* = 1.\(^8\)

The game is solved by backward induction. If the incumbent succeeded in the first
period the analysis coincides with that in the previous section. Similarly, with probability
\( \tau (1 - p) \) the firing decision is left to the board’s discretion upon poor performance \( (s = g) \)
and the analysis is unchanged: shareholders dismiss the incumbent and sell to an acquirer
if they can receive at least \( pX^H_t \). With probability \( (1 - \tau)(1 - p) \) internal control breaks
down and the incompetent incumbent cannot be fired. Now shareholders’ reservation
price when deciding whether to sell their firm is zero. We assume that a poorly performing
manager, who is not fired due to a breakdown of internal control, cannot access financial
resources \( \hat{L} \). Shareholders can obtain contingent liquidity provision by making access to
\( \hat{L} \) conditional on a contribution of funds by the incumbent. Since a poorly performing
manager disposes of no intermediate income at the interim date, he is thus unable to
take over a rival firm. Denote by \( \rho^t_b \) (respectively \( \rho^t_g \)) the probability the firm is taken
over upon poor performance given state \( s = b \) (respectively \( s = g \)) for a price \( P^t_b \geq 0 \)
(respectively \( P^t_g \geq pX^H_t \)). In anticipation of the general equilibrium results for these
probabilities, derived in section 6 below, this section proceeds on the assumption that \( \rho^t_b \)
exceeds \( \rho^t_g \). More precisely, \( \rho^t_g = \rho^t_b - \alpha \), with \( \alpha \in [0, \rho^t_b] \).

The board now has three instruments at its disposal to ensure that the manager exerts
high effort. It can offer a monetary reward for good performance in the first period. In
addition, it can threaten the incumbent with interference in the case of poor performance

\(^6\)For instance, one can interpret \( \tau \) as the monitoring effort exerted by the board in order to prevent
the manager from entrenching himself through a manager-specific investment à la Shleifer and Vishny
(1989). See also Almazan and Suarez (2003) who consider managerial compensation when the CEO can
veto his replacement.

\(^7\)Jensen (1993, p. 863) notes that "the available evidence does suggest that CEOs are removed after
poor performance, but the effect, ..., seems too late and too small to meet the obligations of the board".

\(^8\)Reference to Almazan and Suarez (2003).
through its choice of $\tau$. Finally, liquidity creates acquisition opportunities which relax the manager’s incentive compatibility constraint.

The board solves the following problem:

$$
\max_{w(X_1^H), w(X_2^H), \tilde{\omega}, \tau} \quad p \left[ X_1^H - w(X_1^H) + X_2^H + \rho^a \left[ (X_2^H - P^a) F(\tilde{\omega} - P^a) - \int_0^{\tilde{\omega} - P^a} cf(c)dc \right] \right] \\
+ (1 - p) \left[ \tau (\rho^i_g P^i_g + (1 - \rho^i_g) p X_2^H) + (1 - \tau) \rho^f_b P^f_b \right] - \frac{1}{2} k \tau^2
$$

subject to

$$
p \left[ w(X_1^H) + Z_2 + \rho^a F(\tilde{\omega} - P^a) \triangle Z_2 \right] + (1 - p)(1 - \tau)(1 - \rho^f_b) Z_2 \\
w(X_1^H), w(X_2^H) \geq 0 \quad \text{and} \quad \tau \in [0, 1]
$$

The objective function gives the expected firm value net of the cost of internal governance. For example, with probability $(1 - p) \tau$ first-stage cash-flows are low and the firm is well governed. Then shareholders sell their firm for a price $P^i_g$ with probability $\rho^i_g$ in stage 2. If no sale occurs, shareholders replace the incumbent and expect to receive a cash-flow $p X_2^H$. With probability $(1 - p)(1 - \tau)$ the manager is incompetent and the firm is poorly governed. Then shareholders find an acquirer who is willing to pay at least their reservation price of zero with probability $\rho^f_b$. If no acquirer shows up, the second-period cash-flow is zero. The left hand side of the incentive compatibility constraint gives the manager’s expected payoff if he exerts high effort. If $X_1 = X_1^L$ (which happens with probability $(1 - p)$), the manager may still receive the private benefit $Z_2$: with probability $(1 - \tau)(1 - \rho^f_b)$ both internal and external control mechanisms fail and the incumbent continues to run the firm.

Rearranging the incentive compatibility constraint yields:

$$
w(X_1^H) \geq \frac{Z_1}{p} - [\tau + (1 - \tau) \rho^f_b + \rho^a F(\tilde{\omega} - P^a) \triangle] Z_2
$$

The following assumption allows us to focus on interior solutions for $\tau$ in the subsequent analysis:

**Assumption 4** $k \geq p Z_2 + (1 - p) p X_2^H$.

The lower bound on $k$ ensures that the optimal level of internal governance is lower than one. Let $\tau^*$ denote the privately optimal level of internal governance. We then obtain the following result:

\footnote{In our framework, monitoring prevents entrenchment and thus relaxes the incentive compatibility constraint. To the contrary, other forms of monitoring by the board may inhibit effort. For example, direct observation of the manager’s intrinsic ability ex-post may weaken the firing threat ex-ante and thus make it more costly to induce effort (Crémer 1995).}
Proposition 2 The optimal internal governance system is

\[
(w^*(X_1^H), w^*(X_1^L)) = \left( \frac{Z_1}{p} - \left[ \tau^* + \rho_b^i(1 - \tau^*) + \rho^s F(\hat{L} - P^a) \Delta \right] Z_2, 0 \right)
\]

and

\[
\tau^* = \frac{1}{k} \{ p(1 - \rho_b^i) Z_2 + (1 - p) \left[ pX_2^H + \rho_b^i (P_g^i - pX_2^H) - \rho_b^i P_b^i \right] \} = .
\]

The optimal level of liquidity is

\[
\hat{L}^* = X_2^H + \Delta Z_2.
\]

The optimal level of liquidity is unchanged compared to the previous section. Consider the optimal compensation scheme. The term \( \left[ \tau + \rho_b^i (1 - \tau) + \rho^s F(\hat{L} - P^a) \Delta \right] Z_2 \) captures the effect of the two governance mechanisms on the wage. Governance provides an implicit incentive for the manager to work that allows shareholders to lower the explicit monetary reward which is needed to induce high effort. Both internal and external governance discipline the manager by threatening him with dismissal and the loss of private benefits in the case of poor performance. Given \( X_1 = X_1^L \), the probability of being fired is \( \tau + \rho_b^i (1 - \tau) \). Stronger internal control improves incentives (unless \( \rho_b^i = 1 \)). Conversely, the disciplinary role of the takeover market is redundant if internal governance is flawless (\( \tau = 1 \)) which was the case in the previous section. Thus, internal and external governance are substitutes. Note also that managerial turnover is independent of the probability of being taken over if \( \tau = 1 \).

The expression in braces on the RHS in (2) gives the marginal benefit from internal governance. Board interference increases firm value in two ways, by replacing incompetent managers (ex-post effect) and by providing stronger incentives to exert effort (ex-ante effect). The ex-ante effect is captured by the first summand in braces which is the reduction in the manager’s compensation due to the threat of being dismissed by the board upon poor performance. The second summand in braces reflects the ex-post effect, i.e. the change in future firm value due to the board’s ability to sack an incompetent manager: the first term, \( pX_2^H \), is the profit from interference in the absence of a takeover market. Note that the board’s ability to interfere also affects shareholders’ expected returns as a target in the takeover market: the summand, \( \rho_b^i (P_g^i - pX_2^H) \), is the expected takeover premium if the firm is well governed and the last term, \( \rho_b^i P_b^i \), is the expected premium if internal control breaks down.

With the possibility of an internal governance failure, an active takeover market plays a dual role. It offers acquisition opportunities to successful managers while posing a threat to poorly-performing ones. Note that the first effect is independent of the quality of internal governance. On the one hand, the takeover market thus substitutes for internal governance, on the other hand it provides additional incentives - as opposed to penalties.
- through the prospect of acquiring another firm. Furthermore, the decomposition in squared brackets in (1) shows that it is relatively unimportant whether $\rho_b^t$ or $\rho^s$ is larger (as they enter additively and both reduce managerial compensation). The only difference between the disciplining and the opportunity effects is that the former is a substitute to internal governance, while the latter is a complement.

We obtain the following comparative statics regarding the optimal level of $\tau$. A reduction in the cost $k$ improves internal governance. In the case of a benevolent board lower interference costs may result from the use of a more accurate accounting system which makes it easier to evaluate the manager’s performance. Alternatively, with a self-interested board a reduction in $k$ may be due to a more effective compensation scheme for board members or a regulatory requirement that the board meet without the CEO.

The ex-ante benefit of board interference (in terms of lower compensation) is decreasing in the takeover threat: even if internal control fails, the manager is removed through a takeover with probability $\rho_b^t$. The risk of being taken over disciplines the manager and thus reduces the need for the board to intervene. In the limit, as $\rho_b^t$ goes to one, the first summand goes to zero and the board does not play any disciplinary role. In terms of their ex-ante incentive implications board interference and takeovers are therefore substitutes. The higher is the manager’s future private benefit $Z_2$, the greater is the scope for providing implicit incentives through interference, i.e. the higher is $\tau^*$. Note that if the limited liability constraint binds this effect disappears.

With respect to the ex-post effect, one finds that $\tau^*$ is increasing in $X^H_2$. The higher the future cashflow, the more valuable is the board’s ability to interfere.

**Proposition 3** A more active takeover market discourages board interference ($\frac{\partial \tau^*}{\partial \rho_b^t} < 0$).

In other words, internal and external governance are substitutes in our framework. The intuition is straightforward. Board interference benefits shareholders in two ways, by replacing incompetent managers and by relaxing the incentive compatibility constraint ex-ante. The takeover market disciplines incumbent managers and thus weakens the board’s incentive to exert control. At the same time, the ex-post benefit of board interference is independent of the intensity of takeover pressure. The ex-post marginal benefit is always $pX^H_2$, irrespective of the level of $\rho_b^t$.

For future reference, let $\tau_{NT}^*$ denote the optimal level of internal governance in the absence of a takeover market, i.e. when $\rho_g^t = \rho_b^t = 0$. Then

$$\tau_{NT}^* = \frac{1}{c}[pZ_2 + (1 - p)pX^H_2].$$  \hspace{1cm} (4)
5 Managerial Turnover and Compensation

In a next step we consider the effect of an active takeover market on managerial turnover. In the absence of takeovers the probability that the manager is fired following poor performance is simply equal to $\tau_{NT}^*$ given in (4). With an active takeover market, managerial turnover conditional on $X_1 = X_1^T$ is

$$\Gamma^* = \tau^* (1 - \tau^*) \rho_b^I$$

where $\tau^*$ is given in (2) above. In this case, the incumbent is fired with probability $\rho_b^I$ even if the board is ineffective. Overall turnover is jointly determined by the intensity of the takeover threat and by the vigilance of the board.

Surprisingly, greater takeover pressure may lead to a decrease in managerial turnover. Two opposing effects of takeovers on the overall turnover probability have to be distinguished: on the one hand, Proposition 3 implies that an active takeover market increases the probability of an internal control failure ($\tau^* < \tau_{NT}^*$). On the other hand, the incumbent may be removed through a takeover. Even if the board is inactive, the incumbent may lose his position. Depending on which of the two effects dominates, overall turnover can increase or decrease compared to the no-takeover case. It is clear that the second effect always dominates as $\rho_b^I$ gets sufficiently large, implying that turnover increases ($\Gamma^* > \tau_{NT}^*$). In this case, the negative impact of a weaker board on overall turnover becomes negligible since the manager faces a high risk of being taken over whenever internal control breaks down. Hence, if the takeover market operates with little or no friction, the relation between takeover pressure and overall turnover is (weakly) positive. Conversely, one finds that if internal governance is very high in the absence of takeovers (i.e., if $\tau_{NT}^*$ is large) and if $\rho_b^I$ is sufficiently small, then the "crowding-out" effect of takeovers dominates and $\Gamma^* < \tau_{NT}^*$. Managerial turnover is higher in the absence of a takeover market.

Hence, we obtain the following result.

**Proposition 4** In firms with strong boards, turnover is non-monotonic in the intensity of the takeover threat ($\rho_b^I$).

Huang and Zhao (2009) study changes in the sensitivity of CEO turnover to performance following the adoption of antitakeover legislation. They find that the sensitivity indeed increases in firms with strong boards.

In a next step, we consider the relation between, on the one hand, compensation, and, on the other hand, board interference and the takeover market. In the absence of a takeover market, the optimal reward for the manager in case of first-period success is simply

$$w_{NT}^*(X_1^H) = \frac{Z_1}{p\Delta q} - \tau_{NT}^* Z_2.$$
With an active market for corporate control, the optimal reward is

\[ w^*(X_1^H) = \frac{Z_1}{P\Delta q} - [\Gamma^* + \rho^0 F(\hat{L}^* - P^\alpha)\Delta]Z_2. \]

The difference in compensation is

\[ w^*_{NT}(X_1^H) - w^*(X_1^H) = [\Gamma^* - \tau^*_{NT} + \rho^0 F(\hat{L}^* - P^\alpha)\Delta]Z_2. \]

In essence, compensation changes are the sum of changes in the aggregate dismissal probability and the acquisition opportunity. For instance, if the takeover market becomes more of a disciplining device (change in \( \rho^H_0 \) only), then compensation moves in opposite direction as managerial turnover, and the entire wage effect is due to the net change in \( \Gamma \).

**Proposition 5** In firms with strong boards, managerial compensation is non-monotone in the probability of a disciplinary takeover.

Furthermore, we obtain the following result.

**Proposition 6** In firms with strong boards, compensation levels (sensitivity) are (is) lower.

Pay-for-performance sensitivity is measured as the difference between \( w(X_1^H) - w(X_1^L) \) and since \( w(X_1^L) \) is normalized to zero, it is simply \( w(X_1^H) \). Ceteris paribus, compensation and pay-for-performance sensitivity is lower if the board is more effective, i.e., if the cost of interference is \( c \) is low. Supportive evidence is provided by Fahlenbrach (2009) who finds that high board quality is associated with lower CEO pay-for-performance sensitivity.

To sum up, an active takeover market offers not only "sticks" to poorly performing managers but also "carrots" to successful ones and thus mitigates agency conflicts.

### 6 Market Outcome and Externality

This section endogenizes \( \rho^H \) and \( \rho^F \). We show that the privately optimal level of internal governance exceeds the socially optimal one. When shareholders choose \( \tau \), they do not consider that better internal control reduces acquisition opportunities for rival managers, thus forcing rival firms to pay higher compensation.

In order to endogenize the takeover probabilities, suppose that there is a continuum of ex-ante identical firms with unit mass. Each firm is endowed with the technology described above. Initially, shareholders of each firm contract with a manager of unknown ability. Ability and noise are assumed to be distributed independently across firms. Because there is a continuum of firms there is no aggregate uncertainty (given that each
manager exerts high effort). By the law of large numbers the realized fraction of successful firms after period 1 is \( p \) and the realized fraction of poorly performing firms is \((1 - p)\). Hence, the set of potential acquirers (targets) in the economy after the first period equals \( p \) \((1 - p)\).

We consider the following matching technology: after being hired in stage 0, each manager prepares a bid for one rival firm (empire building). The preparation of a bid is costless and can be thought of as the formulation of a post-acquisition strategy for the combined entity. At this point, firms are indistinguishable and each manager randomly selects one potential target from the set of firms. If at all, a manager can only bid for the previously selected firm in stage 2 but not for any other one.

The above matching technology implies that

\[
\rho^g(\hat{L}_E) = pF(\hat{L}_E - P_g) \quad \text{and} \quad \rho^h(\hat{L}_E) = pF(\hat{L}_E - P_h)
\]

\[
\rho^a(\tau_E) = (1 - p)[\tau_E F(\hat{L} - P_g) + (1 - \tau_E)F(\hat{L} - P_h)]
\]

where \( \hat{L}_E \) and \( \tau_E \) denote the average levels of liquidity and internal governance in the economy. Importantly, the probability for a successful manager of acquiring another firm is decreasing in \( \tau_E \). If \( \tau_E \) increases, a successful manager is more likely to face a well-governed target. If \( P_g > P_h \), better internal governance leads to fewer takeovers. At the same time, the probability of being taken over is increasing in the average economy-wide level of liquidity.

Suppose that firms simultaneously and non-cooperatively choose their liquidity levels and governance arrangements ex-ante. It can easily be checked that there exists a unique symmetric equilibrium \((\hat{L}^e, \tau^e, w^e(X^H_1), w^e(X^H))\) which corresponds to the solution presented in Proposition 2 above, the only difference being that the takeover probabilities are now endogenous and given by (5) and (6) above. More precisely, the equilibrium level of liquidity is given by (3) in Proposition 2: \( \hat{L}^e = X^H_2 + \triangle Z_2 \). The equilibrium level of liquidity in turn determines \( \rho^g(\hat{L}^e) \) and \( \rho^h(\hat{L}^e) \) in (5) which can be substituted into (2) in Proposition 2. One thus obtains the equilibrium level of internal governance:

\[
\tau^e = \frac{1}{k} \left\{ p(1 - \rho^h(\hat{L}^e))Z_2 + (1 - p) \left[ pX^H_2 + \rho^g(\hat{L}^e)(P_g - pX^H_2) - \rho^h(\hat{L}^e)P_h \right] \right\}
\]

Together, \( \hat{L}^e \) and \( \tau^e \) fix the acquisition probability in (6). Finally, the equilibrium compensation scheme is found by substituting \( \rho^g(\hat{L}^e) \), \( \rho^h(\hat{L}^e) \), \( \tau^e(\hat{L}^e) \) and \( \rho^a(\tau^e) \) into (1). As a consequence, equilibrium compensation is increasing in the average level of internal governance in the economy. Better internal governance in other firms reduces acquisition opportunities and thus hardens a manager’s incentive compatibility constraint.

Denote by \((\hat{L}^o, \tau^o, w^o(X^H_1), w^o(X^H))\) the socially optimal governance arrangement and
the socially optimal liquidity policy which maximize joint profits for all firms. It is the solution to the following program:

$$\max_{w(X_1^H), \tau_E, \hat{L}_{E}} \mathcal{P} \left[ X_1^H - w(X_1^H) + X_2^H + (1 - p)\tau_E \left[ \int_0^{\hat{L}_{E} - P_g} (X_2^H - P_g - c) f(c) dc \right] \right] +$$

$$\mathcal{P} \left[ p (1 - p)(1 - \tau_E) \left[ \int_0^{\hat{L}_{E} - P_b} (X_2^H - P_b - c) f(c) dc \right] \right] +$$

$$(1 - p)\tau_E (\rho_g'(\hat{L}_{E}) P_g^e + (1 - \rho_b'(\hat{L}_{E})) p X_2^H) + (1 - \tau_E) \rho_b'(\hat{L}_{E}) P_b^e - \frac{1}{2} k \tau^2$$

subject to

$$w(X_1^H) \geq \frac{Z_1}{p} - \left[ \tau_E + (1 - \tau_E) \rho_b'(\hat{L}_{E}) + \Delta \rho'(\tau_E) \right] Z_2$$

$$w(X_1^H) \geq 0 \quad \text{and} \quad \tau_E \in [0, 1]$$

In the Appendix we derive the following result:

**Proposition 7** There is excessive board interference in equilibrium ($\tau^o < \tau^e$). The equilibrium level of liquidity is inefficiently low ($\hat{L}_{E} < \hat{L}^o$).

A comparison between the optimum and equilibrium shows that both the level of internal governance and the level of liquidity, that are chosen in equilibrium, deviate from the socially optimal levels. First, in equilibrium shareholders in each firm fail to internalize the negative impact of their governance effort on the acquisition opportunities of rival firms which hardens the incentive compatibility constraints for all other managers in the economy. Hence, $\tau^o < \tau^e$. Second, the privately optimal level of liquidity is lower than the socially optimal one. In equilibrium, each firm ignores that a higher level of $\hat{L}$ increases $\rho_b^e$ and thus disciplines the managers in all other firms.

Thus, profits of the corporate sector would increase if each firm deviated from the privately optimal governance arrangement and installed a weaker board. Weak boards lead to a more liquid takeover market by increasing the supply of potential target firms. However, the liquidity of the takeover market is a public good and the supply of target firms is too low in equilibrium.

In general, a firm’s privately optimal choice of $\tau$ entails an externality in our model if it affects the probability of making an acquisition or of being acquired (or both) of other firms in the economy. In these cases, privately optimal choices will not lead to a constrained efficient market outcome.

**7 Conclusion**

To be continued
Appendix

Proof of Proposition 3. Substituting $\rho_g^t = \lambda \rho_b^t$ in equation (3) gives

$$\tau^* = \frac{1}{c} \left\{ p(1 - \rho_b^t)Z_2 + (1 - p) \left[ pX_2^H + \rho_b^t [\lambda(P_g^t - pX_2^H) - P_b^t] \right] \right\}. \quad (1)$$

Then

$$\frac{\partial \tau^*}{\partial \rho_b^t} = \frac{1}{c} \left\{ -pZ_2 + (1 - p)[\lambda(P_g^t - pX_2^H) - P_b^t] \right\}. \quad (2)$$

Substituting $P_b^t = \psi pX_2^H + (1 - \psi) \overline{V}$ and $P_b^t = (1 - \psi)E[V]$ into the above equation yields

$$\frac{\partial \tau^*}{\partial \rho_b^t} = \frac{1}{c} \left\{ -pZ_2 + (1 - p)[\lambda \psi pX_2^H + (1 - \lambda)\overline{V}] \right\} \quad (3)$$

which is equivalent to

$$\frac{\partial \tau^*}{\partial \rho_b^t} = \frac{1}{c} \left\{ -pZ_2 + (1 - p)(1 - \psi)[\lambda \psi pX_2^H + (1 - \lambda)\overline{V}] \right\} \quad (4)$$

Hence, $\frac{\partial \tau^*}{\partial \rho_b^t} < 0$. ■

Proof of Proposition 4. $\Gamma^* = \tau^* + (1 - \tau^*)\rho_b^t$ with

$$\tau^* = \frac{1}{c} \left\{ p(1 - \rho_b^t)Z_2 + (1 - p) \left[ pX_2^H + \rho_b^t [\lambda(P_g^t - pX_2^H) - P_b^t] \right] \right\}. \quad (5)$$

Then

$$\frac{\partial \Gamma^*}{\partial \rho_b^t} = 1 - \tau^* + (1 - \rho_b^t) \frac{\partial \tau^*}{\partial \rho_b^t}$$

with

$$\frac{\partial \tau^*}{\partial \rho_b^t} = \frac{1}{c} \left\{ -pZ_2 + (1 - p)[\lambda(P_g^t - pX_2^H) - P_b^t] \right\} < 0. \quad (6)$$

Furthermore

$$\frac{\partial^2 \Gamma^*}{\partial (\rho_b^t)^2} = -2 \frac{\partial \tau^*}{\partial \rho_b^t} > 0.$$

Hence, $\Gamma^*$ is a strictly convex function of $\rho_b^t$. As $\rho_b^t \to 1$, $\Gamma^* \to 1$, and as $\rho_b^t \to 0$, $\Gamma \to \tau^*_{NT}$ where

$$\tau^*_{NT} = \frac{1}{c} [pZ_2 + (1 - p)pX_2^H].$$

As $\rho_b^t \to 1$, $\partial \Gamma^*/\partial \rho_T \to 1 - \tau^* \bigg|_{\rho_b^t=1}$, where

$$1 - \tau^* \bigg|_{\rho_b^t=1} = 1 - \frac{1}{c} (1 - p)((1 - \lambda)pX_2^H + \lambda P_g^t - P_b^t) > 0.$$
As $\rho_b^t \to 0$, $\partial \Gamma^*/\partial \rho_T \to 1 - \tau^*_N T + \partial \tau^*/\partial \rho_b^t$ where

$$1 - \tau^*_N T + \partial \tau^*/\partial \rho_b^t = 1 - \frac{1}{c} [pZ_2 + (1 - p)pX_2^H] + \frac{1}{c} \{ - pZ_2 + (1 - p)[\lambda (P^t_g - pX_2^H) - P_b^t] \}$$

$$= 1 - 2\tau^*_N T + \frac{1}{c} (1 - p)[\lambda (P^t_g - pX_2^H) + (pX_2^H - P_b^t)]$$
References


