Risk Premium Shocks and the Zero Bound on Nominal Interest*†

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June 16, 2009

Abstract

There appears to be **a** disconnect between the importance of the zero bound on nominal interest rates in the real-world and predictions from quantitative DSGE models. Recent economic events have reinforced the relevance of the zero bound for monetary policy whereas quantitative models suggest that the zero bound does not constrain (optimal) monetary policy. This paper attempts to shed some light on this disconnect by studying a broader range of shocks within a standard DSGE model. Without denying the possibility of other factors, we find that risk premium shocks are key to building quantitative models where the zero bound is relevant for monetary policy design. The risk premium mechanism operates by increasing the spread between the rates of return on private capital and the risk-free government bonds rate. Other common shocks, such as aggregate productivity, investment specific productivity, government spending and money demand shocks, are unable to push nominal bond rates close to zero as the same risk premium spread mechanism is not at play.

1 Introduction

Recent economic events have highlighted the importance of the zero bound on nominal interest rates for monetary policy. Indeed, a number of central banks have lowered their policy interest rates to record lows. By the second quarter of 2009, policy interest rates will have fallen below one percent in Canada, England, the Euro Area, Japan, Sweden, Switzerland and the United States. From a theoretical perspective, Eggertsson and Woodford (2003) show, in the

*We thank Craig Burnside and Kevin Moran for their comments and suggestions. We are also grateful for useful discussions with our colleagues and seminar participants at the 2009 Bank of Canada / University of British Columbia workshop on Macroeconomics and Monetary Policy and the 2009 Canadian Economic Association meetings.

†The views expressed here are those of the authors, and not necessarily those of the Bank of Canada.

context of a two-equation macroeconomic model, that the zero bound has to be taken into account when formulating monetary policy because hitting the bound may, in principle, lead to large and protracted losses in output.[1]

In contrast to real-world events, quantitative DSGE models are often unable to find an important role for the zero nominal interest rate bound, even when the monetary authority follows a policy of targeting zero inflation. Christiano (2004) extends the analysis in Eggertsson and Woodford to include capital and government spending, and finds that the zero bound is not likely to bind. Schmitt-Grohe and Uribe (2005) study, inter alias, the zero bound problem in a medium-scale DSGE model with distortionary taxes and three shocks: aggregate productivity, investment-specific productivity, and government spending shocks. The model is calibrated to U.S. data and shows that under the optimal policy (which does not take zero-bound in account), the probability of the nominal interest rate approaching the zero bound is practically nil. This conclusion arises despite the fact that optimal average inflation rate in the model is slightly negative. Given the unsettled nature of this literature, Christiano (2004) argues that additional research allowing for a broader range of shocks may improve our understanding of the factors that occasionally force central banks to face the zero bound on nominal interest rates. This is the starting point for our paper.

In this paper, we construct a quantitative DSGE model that appears capable of capturing the relevance of the zero bound on nominal interest rates. Our model is a calibrated general-equilibrium model along the lines of Christiano (2004) and Schmitt-Grohe and Uribe (2005) but we consider a broader range of economic shocks.[2] Our results indicate that even under a zero inflation policy, historically-measured aggregate shocks - such as productivity, investment-specific productivity, government spending and money demand shocks - do not drive the nominal interest rate to its zero bound. The only shock in our analysis that forces the central bank to face the zero bound is a risk premium shock (perturbations that widen the spread between the rates of return on private capital and risk-free rate). Indeed, even conservatively measured risk-premium shocks (such as those reported in Campello, Chen and Zhang 2008) are capable of driving the risk-free nominal interest rate to zero. As such, our analysis focuses only on the exogenous component of the risk premium. We do so for two reasons. First, it greatly simplifies the solution and computation of our already non-linear model. Second, Huang and Huang (2002) have estimated that only a modest fraction (20 to 30 percent) of total risk premium can be explained by observable risk characteristics of individual firms.

Intuition for the "special" role of risk-premium shocks can be gained from the observation that these shocks change the spread between the expected rate of return on capital and the risk-free rate. This implies that either the expected

[1] Their results, however, are based on the application of a non-structural shock so it is not straightforward to isolate the source of the shock or its empirical magnitude.

[2] Our paper does not address questions about the real implications of the zero bound (see, e.g., Furher and Madigan 1997, Orphanides and Wieland 1998, and Wolman 2005) nor optimal monetary policy in the presence of zero nominal interest rate bound (see, e.g., Rotemberg and Woodford 1998, and Reifschneider and Williams 1999).

rate of return on capital must increase, or the risk-free rate must fall, or both, to accommodate the higher risk premium. For a wide range of plausible parameter configurations, much of the increase in the risk premium is accommodated by a fall in the risk-free rate, thus increasing the probability that the zero bound may bind. In contrast, the other aggregate shocks we examine do not move the rate of return on capital and the risk-free rate in opposite directions. Instead, both expected returns move in the same direction and by roughly the same proportion so the zero bound can only be reached with extreme realizations of these shocks.

Interestingly, our results are broadly consistent with past episodes where central banks hit or approached the zero nominal interest rate bound. Nominal policy interest rates in Japan since 1999, the United States and Switzerland in 2003-04 and many developed countries in 2008-09 hit or hovered above zero and these occurrences were preceded by significant turmoil in financial markets. More specifically, the collapse of an asset price "bubble" in the early 1990s, the rapid decline in the valuation of high technology related assets in 2000 and the breakdown of the sub-prime mortgage market in 2008 lead to bouts of zero or near zero policy rates as our model would predict.[3]

The remainder of the paper is organized as follows. Section 2 outlines the main features of our model and Section 3 describes its calibration. Section 4 presents our main result and Section 4 gives some sense of the robustness of the key result. Concluding remarks are provided in Section 5.

2 Model

The model is a standard real-business-cycle model extended to include sticky nominal prices, money and nominal government bonds. In the model, infinitely-lived households: (i) maximize an utility function which depends on consumption, money and leisure; (ii) decide on the amount of capital to accumulate given capital adjustment costs; and (iii) allocate the remaining wealth across fiat money and a risk-free government bond. Intermediate good firms produce differentiated goods by: (i) deciding on labour and capital inputs; and (ii) setting prices according to a Calvo (1983) specification. A representative final good producer combines intermediate goods into a final consumption good. The government finances exogenous government spending with lump sum taxes. And finally, a monetary authority sets the short-term interest rates, and lets the money supply to be determined by the demand for real balances. Lump-sum taxes are used to finance changes in the money stock. In the forthcoming formal description of the model, we focus on key relationships concerning investment, the capital stock, its marginal product, the risk-free nominal interest rate and

[3] Bank of Japan ex-Deputy Governor Ueda (2005), for instance, writes that many of the monetary policy measures adopted by Bank of Japan during its zero interest rate policy era were aimed at mitigating financial sector problems. Ueda goes on to say that the Bank of Japan was concerned about the rising risk premiums, and attempted to counteract them by lowering the "risk-free" nominal rate.

a risk premium term.

2.1 Households

The representative household maximizes expected utility

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\gamma}{\gamma - 1} \log \left(C_t^{\frac{\gamma-1}{\gamma}} + u_t^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) + \eta \log (1 - h_t) \right],$$

where C_t is consumption, M_t represents nominal balances, P_t is the price level, and h_t is hours worked. Total hours available to the household in each period is normalized to one. The parameters $\beta \in (0,1)$, γ, and η represent a discount factor, elasticity of substitution between consumption and real balances, and the weight on leisure in the utility function, respectively. The utility function also contains a money demand shock, u_t, of the form

$$\log (u_t) = (1 - \rho_u) \log \overline{u} + \rho_u \log (u_{t-1}) + \varepsilon_{ut}; \quad \rho_u \in (-1, 1) \text{ and } \varepsilon_{ut} \backsim iid(0, \sigma_u^2).$$

The budget constraint is given by

$$\begin{aligned} & C_t + I_t + CAC_t + \frac{B_t}{P_t} \frac{1}{R_t} + \frac{M_t}{P_t} \\ \leq \quad & W_t h_t + (q_t - \tau_{t-1}) \frac{P_t^k}{P_t} K_{t-1} + \frac{B_{t-1}}{P_{t-1}} \frac{1}{\pi_t} + \frac{M_{t-1}}{P_{t-1}} \frac{1}{\pi_t} + T_t. \end{aligned} \tag{1}$$

where I_t is investment, B_t represents one-period risk-free nominal bond, R_t is the gross nominal interest rate on the risk-free bond, W_t is the real wage rate, K_{t-1} is the capital stock from the previous period, π_t is the gross rate of inflation defined as P_t/P_{t-1}, P_t^k/P_t is the relative price of capital, and T_t is a composite term that contains profits, lump-sum taxes, and monetary injections $(M_t - M_{t-1})/P_t$. The term q_t is the marginal product of capital which includes the return to household and a risk premium denoted by τ_{t-1}, and CAC_t represents a capital adjustment cost which is specified as

$$CAC_t = \frac{\varphi}{2} \left(\frac{K_t}{K_{t-1}} - \gamma_k \right)^2 \frac{K_{t-1}}{X_t},$$

The parameter γ_k is the long-run average growth rate of the capital stock, φ is a positive parameter and X_t is investment-specific technology. Investment increases the household's stock of capital according to $K_t = (1-\delta)K_{t-1} + X_t I_t$ where $\delta \in (0,1)$ is the depreciation rate of capital.

As mentioned above, the gross return on capital, q_t, the risk-free interest, R_t, and the risk premium, τ_t, will be important components of the upcoming results so we focus specifically on two first-order conditions that may help us understand the role these variables play in our results:

$$\frac{\Lambda_t}{R_t} = \beta E_t \left(\frac{\Lambda_{t+1}}{\pi_{t+1}} \right), \tag{2}$$

$$\frac{\Lambda_t}{X_t}\left[1+\varphi\left(\frac{K_t}{K_{t-1}}-\gamma_k\right)\right]=\beta E_t\left\{\frac{\Lambda_{t+1}}{X_{t+1}}\left[1-\delta+q_{t+1}-\tau_t+\frac{\varphi}{2}\left(\left(\frac{K_{t+1}}{K_t}\right)^2-\gamma_k^2\right)\right]\right\}, \tag{3}$$

where Λ_t is the Lagrange multiplier associated with the period-t budget constraint.

It is useful to note that when we set $\varphi = 0$, assume full capital depreciation and ignore uncertainty, we can rewrite equation (3) as

$$\frac{R_t}{\pi_{t+1}} = \frac{X_t}{X_{t+1}}(q_{t+1} - \tau_t). \tag{4}$$

Given that X_t is a persistent technology shock, the ratio X_t/X_{t+1} is roughly constant and close to one. Equation (4) says that the risk premium τ_t is approximately equal to the spread between the marginal product of capital and the real interest rate. This implies that innovations in the risk premium will have a first-order effects on the real interest rate or the marginal product of capital, or both. Further, if the inflation rate is held constant then all the movements in the real interest rate will be reflected in one-to-one movements of the nominal risk-free rate.

2.2 Intermediate and final good producers

The final good Y_t is produced by combining a continuum of intermediate goods $Y_t(i)$ for $i \in [0, 1]$ that are imperfect substitutes according to a constant returns to scale technology given by

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where $\theta > 1$ is the elasticity of substitution between types of differentiated intermediate goods. The final goods sector is perfectly competitive so profit maximization leads to the following input-demand function for each intermediate good i

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t. \tag{6}$$

which specifies economy-wide demand for good i as a function of its relative price, $P_t(i)/P_t$, and aggregate output, Y_t.

Each intermediate good i firm produces $Y_t(i)$ units given the following production function

$$Y_t(i) = A_t K_t(i)^{1-\alpha} H_t(i)^{\alpha} \tag{7}$$

where $K_t(i)$ and $H_t(i)$ are capital input and labour hours input, and the aggregate productivity level A_t is given by

$$\log\left(\frac{A_t}{A_{t-1}}\right) = g_a + \varepsilon_{at}, \quad \varepsilon_{at} \sim N(0, \sigma_{\varepsilon_a}), \tag{8}$$

as in Fisher (2006).

In order to introduce nominal price stickiness into the model, producers of the intermediate goods are assumed to set prices according Calvo (1983) style contracts. Specifically, firms have a constant probability (d) that their price set in time t will still be in force at time $t+1$. When the ith intermediate good firm is allowed to re-optimize its price in period t, it sets its price to maximizes the discounted sum of its expected future profits.

2.3 Fiscal and Monetary Authorities

We assume government expenditures, G_t, are financed by lump-sum taxes[4] on households and that a fraction of government expenditures in GDP, $g_t = G_t/Y_t$, follows a stationary AR(1) process:[5]

$$g_t = \left(1-\rho_g\right)\bar{g} + \rho_g g_{t-1} + \varepsilon_{g,t}, \text{ where } \varepsilon_{g,t} \sim N(0, \sigma_{\varepsilon_g}).$$

For monetary policy, we follow Christiano (2004) in focusing on a very simple monetary policy which keeps net inflation precisely at zero in all periods, $\pi_t - 1 = 0 \; \forall \; t$. This policy has been quite prominent in the literature on optimal monetary policy with sticky nominal prices. King and Wolman (1999), for example, show in a sticky-price model that a monetary policy of keeping the price level perfectly constant in all periods is a close approximation to optimal monetary policy. The main reason for this finding is that a constant price-level effectively negates relative price distortion, and induces the economy to behave as a flexible-price economy. Khan et al. (2003) add a transaction demand for money to a sticky-price model, and find that optimal monetary policy can sometimes imply a very mild deflation with very small fluctuations of the price level around a declining trend. The mild deflation arises as optimal compromise between price stability, which minimizes relative price distortions, and the Friedman rule, which eliminates the cost of money holdings. Overall, Khan et al. suggest that eliminating price distortions is an important concern and the role of optimal monetary policy, to a first approximation, is to stabilize the price level. Goodfriend and King (2001) show that the near-optimality of price-level stabilization is likely robust across a wide variety of sticky-price models. Siu (2004) and Schmitt-Grohe and Uribe (2004) derive optimal fiscal and monetary policy under sticky prices and confirm that even small degrees of price rigidity imply very little volatility of optimal inflation. Finally, Schmitt-Grohe and Uribe (2005) reached a similar conclusion in a much larger model with various real and nominal frictions: the optimal inflation rate is nearly constant over time, albeit slightly negative as in Khan et al.[6]

[4] Time-varying capital income taxes can have a similar effect as risk-premium shocks analyzed in this paper. Such taxes would create a time-varying spread between the return on capital, which is taxed, and the risk-free rate, which is tax-exempt.

[5] With this process, the fraction is not constrained to lie between zero and one. It is, however, never a problem in the simulations and, thus, we retain this assumption for simplicity.

[6] In this paper we will not be characterizing optimal monetary policy with risk-premium shocks. We are planning to undertake that in future work.

2.4 Aggregation

We assume the presence of a rental market for capital that allows firms to rent their desired level of capital input. All firms have the same capital-to-labor ratio and real marginal cost $\psi_t(i)$. Also, firms that change their price in the same period choose the same price $P_t^*(i)$. As a result, we can drop the (i) argument for real marginal cost, ψ_t and newly chosen price, P_t^*. Integrating over the demand function (6) we obtain the following aggregate resource constraint:

$$Y_t^s = \left(C_t + \frac{K_t - (1-\delta)K_{t-1}}{X_t} + G_t + CAC_t \right) S_t \tag{9}$$

where

$$S_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di, \tag{10}$$

which under Calvo pricing has the law of motion:

$$S_t = (1-d) p_t^{*\,-\theta} + d{\pi_t}^{\theta} S_{t-1}. \tag{11}$$

Finally aggregate supply, Y_t^s, is given by

$$Y_t^s = A_t K_{t-1}^{1-\alpha} {h_t}^{\alpha} \tag{12}$$

where K_{t-1} and h_t are the aggregate capital stock and aggregate hours worked, respectively.

3 Calibration

We start this section with a brief overview of our calibration strategy. We measure aggregate shocks from the data, namely: (i) risk-premium shocks as measured by Campello, Chen and Zhang (2008) from micro data on corporate bond spreads of US corporations; (ii) aggregate productivity shocks derived from a TFP series obtained by fitting a Cobb-Douglas production function to aggregate capital, labour hours and real GDP; (iii) investment-specific shocks as estimated in Fisher (2006); (iv) money demand shocks, estimated from movements in the monetary base; and (v) government spending shocks calculated from the NIPA data.

The remaining parameters are calibrated by matching average values (first moments) of observable data, except for the capital adjustment cost parameter, φ. In order to calibrate φ, we need second moments from the model which in turn require us to specify a model of monetary policy that is congruent with the historical data. As such, we posit a forward-looking Taylor rules of the form

$$\log R_t = (1-\rho_R) \left(\log \bar{R} + \rho_\pi E_t \log \left(\frac{\pi_{t+1}}{\bar{\pi}} \right) + \rho_y \log \left(\frac{y_t}{\bar{y}} \right) \right) + \rho_R \log R_{t-1}, \tag{13}$$

which has been found by Taylor (1993), Clardia, Gali and Gertler (1999), Orphanides (2003) and others to capture well broad movements in Federal Reserve policy interest rates. The terms $\bar{R}$, $\bar{\pi}$, $\bar{y}$ are the steady-state values of R_t, π_t, and the de-trended output, y_t. The parameters, ρ_R, ρ_π, and ρ_y, that govern the response of the monetary authority to deviations from steady state, and the capital adjustment coefficient φ, are calibrated by stochastically simulating the model and matching a set of second moments from the data.

The following three subsections provide greater detail on our calibration exercise. Unless otherwise noted, we use the sample period 1974Q1 to 1998Q1 This data limitation is owing strictly to the availability of risk-premium data and our objective of maintaining a consistent sample period across the calibrations. More details on data sources and data transformations available in the appendix.

3.1 Calibrating aggregate shocks

We start by constructing a measure of risk-premium shocks using ex-ante equity risk premium data constructed by Campello, Chen and Zhang (2008) for the 1974Q1 to 1998Q1 sample period.[7] These authors exploit information on observable corporate bond spreads (relative to government bonds of the same maturity structure) to make inferences about the unobservable ex-ante risk premiums on common stock of the same corporation. In estimating those risk premia, the authors control for taxes and grade-specific default rates, as well as other observable determinants of the default risk such as leverage. We take the component remaining after accounting for these observable, firm-specific risk characteristics as our risk premium shock. In our model, an exogenous risk-premium shock drives a time varying wedge between the expected real return on capital and the expected real return on risk-free nominal bonds so the residual risk-premia component reported in Campello et al. appears to be a reasonable empirical counterpart.[8] Figure 1 plots two of the series constructed by Campello et al. The series are for BBB and AAA/AA corporations. It is clear from the figure, the two series are quite different especially at the beginning of the sample period, where the BBB series is much more volatile than AAA/AA series. We chose BBB series as our benchmark risk-premium shock, but we also report results for the AAA/AA series in the sensitivity analysis section.[9] To op-

[7] The dataset was downloaded from Lu Zhang's website in September 2008. The data is monthly so we converted them into quarterly data (to match the frequency of other variables) by simply taking the average over three months of each quarter. Since the risk-premiums are reported on the annualized basis, we also divided the values by four.

[8] A possible alternative model would combine both the exogenous risk premia and the endogenous risk premia of Bernanke, Gertler and Gilchrist (1999). The latter serves as compensation for expected default losses. We focus on the exogenous part of the risk premium for simplicity, and because, as Huang and Huang (2003) have estimated, only a smaller fraction (20-30 percents) of the total risk premium can be explained by observable risk characteristics of individual firms.

[9] The other three risk-premia series available from Lu Zhang's website are for A, BB, and B grade US corporations. We chose BBB grade as our benchmark because it was the median grade group. We also chose the least volatile AAA/AA series for our sensitivity analysis in order to stay on the conservative side with regard to the magnitude of the risk-premium

erationalize the benchmark shock, we estimate a simple AR(1) process from the equity premium series of the BBB grade corporations and obtain the following stochastic process:

$$\tau_t = (1 - 0.84) * 0.016 + 0.84 * \tau_{t-1} + \varepsilon_t^\tau, \quad \text{where } \varepsilon_t^\tau \sim N(0, 0.0079^2).$$

This stochastic process is the benchmark risk-premium shock in our model.

Next we calibrate the stochastic processes for the aggregate and investment-specific productivities. Following Fisher (2006) we assumed that both productivity shocks follow a similar process with stochastic trends:

$$\log(A_t/A_{t-1}) = g_a + \varepsilon_{at}, \quad \varepsilon_{at} \sim N(0, \sigma^2_{\varepsilon_a}) \tag{14}$$

$$\log(X_t/X_{t-1}) = g_x + \varepsilon_{xt}, \quad \varepsilon_{xt} \sim N(0, \sigma^2_{\varepsilon_x}) \tag{15}$$

We calibrate the drift terms g_a and g_x to match the growth rates of real per-capita GDP, and real per-capita capital stock in the data. Over the sample period, real GDP and real capital stock per working-age person grew at average rates of 0.43 and 0.72 percents per quarter, correspondingly. Inverting the derived growth rates of output $\gamma_y = \exp\left(\frac{g_a + (1-\alpha) g_x}{\alpha}\right)$ and capital $\gamma_k = \exp\left(\frac{g_a + g_x}{\alpha}\right)$, with the value of the labour share α set at 0.67, we obtain the implied average growth rates for TFP and investment-specific technological change:[10]

$$\begin{aligned} g_a &= \ln \gamma_y - (1-\alpha) \ln \gamma_k = 0.0043 - \frac{1}{3} 0.0072 = 0.0019, \\ g_x &= \alpha \ln \gamma_k - g_a = \ln \gamma_k - \ln \gamma_y = 0.0072 - 0.0043 = 0.0029. \end{aligned}$$

The standard deviation of the aggregate productivity shocks ($\sigma_{\varepsilon_a} = 0.0062$) is determined by fitting (14) to a TFP series generated from the Cobb-Douglas production function, $Y_t = A_t K_{t-1}^{1-\alpha} H_t^\alpha$, with aggregate real GDP, real capital and labour hours data. The standard deviation of the investment-specific productivity shocks, $\sigma_{\varepsilon_x} = 0.0055$ is set to be consistent with the results reported in Fisher (2006).[11]

In order to calibrate the money demand shock, we note that the first-order condition for real money balances is:

$$u_t = \frac{m_t}{c_t} \left(1 - \frac{1}{R_t}\right)^\gamma .$$

shocks.

[10] The obtained growth rate of the investment-specific technological change g_x is consistent with the average rate of decline in the price of investment goods relative to consumption goods, which was equal to 0.0028 over the 1974q1 - 1998q1 period. This relative price was computed by simply dividing the BEA "Gross private domestic investment" price index by the "PCE" price index.

[11] Fisher estimate $\widehat{\sigma}_x = 0.01158$ before 1980 and $\widehat{\sigma}_x = 0.00325$ after 1980. His empirical model imposes very few model-specific restrictions and encompasses a broad range of models, including ours. Since our sample 1974Q1 to 1998Q1, falls across both subperiods, we set $\sigma_x = 0.0055$, the weighted average of Fisher's estimates. We conduct the sensitivity analysis with this parameter later.

It follows that

$$\begin{aligned}\log u_t - \rho_u \log u_{t-1} &= (1-\rho_u)\log \bar{u} + \varepsilon_{ut} \\ &= \left(\log \frac{m_t}{c_t} + \gamma \log\left(1 - \frac{1}{R_t}\right)\right) - \rho_u \left(\log \frac{m_{t-1}}{c_{t-1}} + \gamma \log\left(1 - \frac{1}{R_{t-1}}\right)\right).\end{aligned}$$

Thus, we could, in principle, obtain values for $\log \bar{u}$, ρ_u, and σ_{ε_u} by estimating the equation

$$\log \frac{m_t}{c_t} + \gamma \log\left(1 - \frac{1}{R_t}\right) = \rho_u \left(\log \frac{m_{t-1}}{c_{t-1}} + \gamma \log\left(1 - \frac{1}{R_{t-1}}\right)\right) + (1-\rho_u)\log \bar{u} + \varepsilon_{ut}, \tag{16}$$

with m_t/c_t being the monetary base-to-consumption ratio, and R_t being the 90-day T-bill rate. Unfortunately, the parameters in (16) were not well identified empirically so we choose an alternative approach. We select a value for γ and then re-estimate the remaining parameters in equation (16) by OLS. We consider γs in a range identified in the literature, $[0, 0.2]$, (e.g., see Ball 2001) and then arrive at a final value, $\gamma = 0.06$, that maximizes the likelihood function. The resulting parameter values are: $\rho_u = 0.97$, $\bar{u} = 0.062$ and $\sigma_{\varepsilon_u} = 0.01$, which are similar to those estimated by maximum-likelihood methods in Dib and Christensen (2008).

Finally, we calibrate the stochastic process for the share of government consumption in GDP, g_t by fitting the AR(1) stochastic process to the observed share of government consumption in GDP. The result is

$$g_t = (1-0.98)\,0.162 + 0.98 g_{t-1} + \varepsilon_{g,t}, \text{ where } \varepsilon_{g,t} \sim N(0, 0.002^2),$$

which implies $\bar{g} = 0.162$, $\rho_g = 0.98$, and $\sigma_{\varepsilon_g} = 0.002$.[12]

3.2 Static calibration

Consistent with results reported in Clarida, Gali and Gertler (1999) over roughly the same period, we set the Federal Reserve's implicit inflation objective, $\bar{\pi}$, to be 3.6 percents. Estimates of the real interest rate are measured with substantial uncertainty, but they tend to lie between two and three percent over sample period under consideration (e.g. Laubach and Williams 2003, and Amato 2005). As such, our benchmark calibration for the real interest rate, $\bar{r}$, is 2.5, but we conduct sensitivity analysis over the two to three range. Given the benchmark value of $\bar{r}$ and the above growth rates of technology, the discount rate, $\beta = [\exp((g_a + (1-\alpha)\, g_x)/a)]/\bar{r} = 0.998$.

Further, we set the Calvo probability parameter $d = 2/3$, consistent with the micro literature on sticky nominal prices (e.g. Bils and Klenow, 2004).[13]

[12] We estimate this process over a longer 1974q1-2008q2 period. After a relatively stable period from 1974, the government consumption share in GDP declines steadily from 1991 to 1999 and then increases thereafter. As a result, the share appears nonstationary if one restricts attention to 1974q1 to 1998q1 period, which makes it difficult to fit a stationary process to the series.

[13] Variation in the value of d has little effect on the dynamics of the economy under the benchmark, zero-inflation policy.

The elasticity of the substitution between intermediate goods, θ, the preference weight on leisure, η, and the depreciate rate, δ, are jointly determined via a non-linear search algorithm which isolates values for these parameters by matching three data moments, namely: (i) the fraction of working hours ($h = 0.25$); (ii) the average private consumption to GDP ratio ($c/y = 0.65$); and (iii) the average labour income share 0.58 calculated from the NIPA data. The results are $\theta = 7.7$, $\eta = 2.7$ and $\delta = 0.026$.

3.3 Dynamic calibration

Finally, in order to calibrate the dynamic parameters, we log-linearize the model with a forward-looking Taylor rule (as discussed above) and solve for the predicted second moments of the model. Then we use a non-linear search algorithm to find the parameter values for the capital adjustment coefficient, φ, and for Taylor rule coefficients, ρ_R, ρ_π, ρ_y, so that to match the following four moments: (i) the standard deviation of the nominal investment (inclusive of net exports and government investment) to consumption ratio in the data (0.0326); (ii) the first-order autocorrelation coefficient of the 90-days nominal treasury bill rate (0.95); (iii) the standard deviation of the 90-days nominal treasury bill rate (0.0063) and (iv) the standard deviation of the nominal labour income share (0.0092). We focus on the first moment since its value in the model is influenced primarily by the investment adjustment costs. The two moments of the risk-free rate were chosen because of our focus on the behavior of the nominal interest rate relative to zero bound. With sticky nominal prices, the standard deviation of the labour income share is sensitive to the monetary policy rule, so we chose this moment to properly match the Taylor rule.[14] The calibrated values of the parameters are $\varphi = 18.6$, $\rho_R = 0.51$, $\rho_\pi = 1.29$, and $\rho_y = 0.034$.[15] Table 1 lists the calibrated benchmark parameter values.

Before concluding, we provide an indication of how well the model matches the data by comparing moments that were not directly used for calibration. Table 2 reports standard deviations (in percent) and first-order autocorrelation coefficients for working hours, inflation, nominal investment-to-GDP ratio, nominal investment-to-consumption ratio, labour share and the risk-free rate. The second and third columns of the Table report result based on US data, while

[14] Notice that none of our target second moments requires data de-trending.

[15] The calibrated values of the Taylor rule coefficients are broadly in line with the range of values estimated by other researchers. Clarida, Gali, Gertler (1999) estimated $\rho_R = 0.68$, $\rho_\pi = 0.83$, $\rho_y = 0.0675$ for 1960q1-1979q2, and $\rho_R = 0.79$, $\rho_\pi = 2.15$, $\rho_y = 0.23$ for 1979q3-1996q4 period. Orphanides (2003) reexamined the question, and found that, if one uses only the data available in real time, then the estimated values of Taylor rule coefficients, where more stable over these two periods: $\rho_R = 0.70$, $\rho_\pi = 1.64$, $\rho_y = 0.14$ in 1966q1-1979q2 and $\rho_R = 0.79$, $\rho_\pi = 1.80$, $\rho_y = 0.0675$ in 1979q3-1995q4. The difficulties with estimation of the forward looking Taylor rules are primarily due to unobservable nature of both the output gap, and the expected inflation. In any case, calibrated values of all the parameters in the model, other than the capital adjustment cost parameter φ, are completely independent of the Taylor rule coefficients. Moreover, the value of φ is determined primarily by the standard deviation of the investment-to-consumption ratio, which shows very little sensitivity to large variations in the Taylor rule coefficients.

fourth and fifth columns report model-generated moments. The numbers in bold are the moments that we targeted beforehand via calibration.

It is readily apparent that the model tends to underpredict the degree of persistence found in the data. Overall, however, the fit of the model seems satisfactory as measures of volatility are replicated quite closely. Interestingly, a variance decomposition shows that 86 percent of volatility in hours in the model is due to risk-premium shocks, suggesting that shocks emanating from financial markets have a powerful effect on the economy. This prediction of our model is quite similar, at least in spirt, to a number of recent papers. Nolan and Thoenissen (2009) work within a calibrated DSGE New Keynesian framework and find financial accelerator shocks to account for a large portion of the variance of output. Christiano, Motto and Rostagno (2007) augment a standard monetary DSGE model as developed by Christiano, Eichenbaum and Evans (2005) with financial markets to study, among other things, the role of financial shocks for business cycle fluctuations. The authors estimate their model on U.S. and Euro Area data and find financial market disturbances to be a key factor driving movements in important macroeconomic variables. Along the empirical margin, Gilchrist, Yankov and Zakrajsek (2009) carefully construct measures of credit market disruptions based on a broad range of credit spreads and estimate credit market shocks to be important for U.S. economic fluctuations.

4 Results

This section reports results generated from the non-linear model under a monetary policy of zero inflation. We focus on a zero ex-post inflation policy for three reasons. First, and perhaps most importantly, previous research within sticky price models has found zero inflation to be a good approximation to optimal monetary policy. Second, a zero inflation framework facilitates comparison with the results reported in Schmitt-Grohe and Uribe (2005) and Christiano (2004) who also consider the zero bound problem under a policy of zero inflation. Third, the non-linear model with non-zero inflation could not be solved as varying inflation leads to a larger state space that includes price dispersion, in addition to capital and exogenous shocks.[16] For computation of the model, we use the projection with endogenous-grid-points method developed in Carrol (2005).[17] The method allows us to handle a relatively large state-space problem complicated by non-linearities owing to the zero-bound constraint.

In contrast to the previous literature studying the zero bound within quantitative DSGE models, we find an important role for the zero bound on the nominal interest rate. Indeed, our quantitative model implies that the probability of approaching the effective zero bound (i.e. a risk-free rate less than 0.05

[16] Moreover, under Taylor rules with interest rate smoothing, one must also add lagged risk-free rate to the set of endogenous state variables.

[17] Details are in the Appendix.

percent) is about 1.7 percent.[18] In other words, the zero bound should bind, on average, once every 15 years. Further exploration indicates that the relevance of the zero bound is owing to the presence of a risk premium shock. More specifically, we shut down the risk-premium shocks by setting $\sigma_{\varepsilon_\tau} = 0$, while holding all other parameters at their benchmark valuers. We recompute the model assuming that the households know that the risk-premium will be constant over time. We then evaluate the probability of reaching the zero bound under the zero-inflation policy.[19] In this case, the probability of observing of risk-free interest rate less than 0.05 percent is virtually zero. To give a better sense of this result, we calculate a statistic that takes the lowest observed risk-free rate in the 10,000 quarters of simulation and then divides it by its standard deviation. This statistic, calculated to be 6.6, provides an indication of the distance between the lowest risk-free rate and the zero bound, normalized by the standard deviation of the risk-free rate.[20]

These results beg the question: What makes risk-premium different from the other shocks under consideration? Intuition for the "special" role of risk-premium shocks can be gained from the observation that these shocks are similar to time-varying taxes on capital in the sense that they drive a wedge between the (ex-ante) marginal rates of return on capital and savings. The higher risk premium leads to a widening of the spread between the expected rate of return on capital and the risk-free rate. This implies that either the expected rate of return on capital must increase, the risk-free rate must fall, or both rates must move apart to accommodate the higher risk premium. For a wide range of plausible parameter configurations, much of the increase in the risk premium is accommodated by a fall in the risk-free rate. This feature increases the probability that the zero bound may bind. In contrast, the other aggregate shocks we examine do not move the rate of return on capital and the risk-free rate in opposite directions. Instead, both expected returns move in the same direction and by roughly the same proportion so the zero bound can only be reached with extreme realizations of these shocks.

Overall, the results from the experiment without the risk-premium shocks are consistent with the findings reported in Schmitt-Grohe and Uribe (2005), which show that in a model with government spending, neutral productivity and investment-specific productivity shocks, optimal (near zero inflation) monetary policy is not constrained by the zero bound on nominal interest rates. In addition, the results with the risk-premium shocks support Christiano's (2004) conjecture that other shocks might make zero bound relevant for monetary policy design. Our results suggest that the presence of (at least) risk premium shocks may make a zero-inflation policy inconsistent with the objective of not

[18] Henceforth, we will refer to a risk-free rate falling below 0.05 percent as it being at the effective lower bound.

[19] Columns 2 and 3 of Table 5 show simulation results with and without risk-premium shocks, under a zero-inflation policy in the model with the benchmark parameter values.

[20] Alternatively we could report a ratio of the average risk-free rate and the standard deviation of the risk free rate (i.e. t-statistics). Given that the distribution of the risk-free rate is not symmetric, we feel such statistics would be less informative.

hitting zero-nominal bound.

5 Sensitivity analysis

In this section we examine the sensitivity of the model properties as well as our key result regarding the relative importance of risk-premium shocks for the zero bound on nominal interest rates. In particular, we report results from the following perturbations: (i) reduce the volatility of the risk-premium shocks; (ii) raise and lower the average real return on risk-free government bonds; and (iii) increase three-fold the magnitude of investment-specific technological shocks. We focus on these three cases since we find them to be quantitatively most important from a wide range of other sensitivity experiments conducted.

We conduct the sensitivity analysis in two steps. First, for each experiment, we recalibrate the model using the same procedure as in the benchmark case to maintain the same relative volatilities of the simulated macroeconomic variables as in the data. We then use the recalibrate model to generate data to compare with the benchmark model. This step gives us an indication of the sensitivity of the model properties to different experiments. In the second step, we use the recalibrated model, constrain monetary policy to follow a zero inflation policy, and study the zero bound problem as in the previous section.

5.1 Risk-premium shocks

The magnitude of the risk-premium shocks is clearly important for our results. As such, we examine the sensitivity of our results to a more conservative measure of risk-premium shocks. That is, we use Campello, Chen and Zhang's (2008) least volatile ex-ante equity premia series corresponding to the AAA- and AA-rated groups of U.S. corporations. This data produces a risk-premium shock series that is slightly more persistent ($\rho_\tau = 0.88$), but substantially less volatile ($\sigma_{\varepsilon_\tau} = 0.0029$) than the benchmark BBB risk premium shocks. Table 3 reports the recalibration results. The first column provides a list of model parameters, the second column reproduces the benchmark calibration results, and the third column gives the recalibrated parameter values based on the less volatile risk premium shock series. We see a number of small changes across the parameters but only the capital adjustment parameter (φ) displays a notable movement, specifically, a fall from 18.6 to 7.8. This decline allows the model to match the volatility of investment despite a lower variance of the risk premium shock.

The benchmark and recalibrated model moments are reported in Table 4. The first column of the table lists the variables under consideration. The second and third columns reproduce the standard deviations and the autocorrelations for the variables from the benchmark case. The following two columns report the same two moments for less volatile risk-premium experiment. Comparing the statistics listed in the four columns we see very little change across model moments, suggesting that the model is robust to changes in the volatility of the risk-premium shock.

Table 5 reports simulation results for the (re-calibrated) model under the zero-inflation policy. The variables under consideration are given in column one. The second and third columns reproduce statistics from the benchmark case. The second column shows results with risk-premium shocks while the third column displays corresponding results when the risk-premium shocks is shut down (by setting $\sigma_{\varepsilon_\tau} = 0$). The following two columns report the same statistics for the lower volatility risk-premium experiment. Looking across these rows, we see that the main message is unchanged: With the risk premium shocks there is a small but non-negligible probability of the risk-free rate being at the effective zero bound. In contrast, the version of the model without risk-premium shocks is less volatile and the probability of approaching the zero bound is extremely low. More specifically, a four standard deviation (of the risk-free rate) band separates the lowest (simulated) risk-free rate and zero. Overall, even with relatively more conservatively-measured risk-premium shocks, the main result is unchanged.

5.2 Real risk-free interest rate

The average real return on risk-free bonds determines the distance between the nominal risk-free rate and its zero bound, influencing the probability of approaching the bound. Moreover, the ex-ante real rate of return is not directly observable so there is some degree of uncertainty regarding its appropriate value. Given these two factors, we conduct a sensitivity analysis with two alternative rates of return, *viz.*, average rates of return of two and three percent annualized or $\bar{r} = 1.02^{0.25}$ and $\bar{r} = 1.03^{0.25}$, respectively. These two values cover a one percentage point range around our benchmark value of 2.5 percent.

Changes in the average real risk-free rate lead to a few changes in parameter calibrations. The fourth column of Table 3 contains the recalibrated parameter values under a lower real risk-free rate. While most of the parameters change only slightly, the capital adjustment parameter displays a relatively larger change, from 18.6 in the benchmark to 20.5 under this alternative scenario. A higher φ offsets an increase in investment volatility arising from the zero bound on the risk-free rate. When the average real risk-free rate is closer to zero, there is less room for the rate to adjust downward. When the spread between the rate of return on capital and the risk-free rate rises (because of an increase in the risk-premium), more of the adjustments is borne by the capital return, and hence by investment. The following column of Table 3 shows the re-calibrated parameter values with a higher average risk-free rate relative to the benchmark case. Again, the models parameters remain virtually the same except for the value of the capital adjustment coefficient which is now slightly lower than in the benchmark. The higher average risk-free rate gives more scope for the risk-free rate to adjust downward and, therefore, less adjustment by the returns to physical capital and investment is required.

Columns 6 to 9 of Table 4 show simulated moments from the model with the calibrated Taylor rule under two assumption for the real risk-free rate ($\bar{r} = 1.02^{0.25}$ and $\bar{r} = 1.03^{0.25}$). Again there is very little change relative to the moments generated from the benchmark calibration.

Table 5 reports simulation results for zero-inflation policy. As we can see from column 6, a lower average risk-free rates coupled with risk-premium shocks increases the probability of hitting the effective lower bound from 1.7 percent in the benchmark to 2.8 percent, or roughly once in 9 years. In the absence of risk-premium shocks (column 7), the variables are much less volatile and there is a wide buffer zone between the range of the simulated risk-free rates and the effective zero bound. Columns 8 and 9 report the risk premium and no risk premium shocks cases, respectively, under the assumption of a three percent average risk-free rate. These columns suggest that the qualitative results are similar under the two average risk-free rate cases. That is, although the probability of hitting the effective bound is lower, the conclusion regarding the relative importance of the risk premium shock stays intact.

Overall, we find that changing the value for the average risk-free interest rate does not affect the qualitative importance of risk premium shocks for hitting the effective zero bound on nominal interest rates.

5.3 Investment-specific shocks

The previous sections suggest that movements in investment may be a key component to our understanding of the zero bound problem in quantitative DSGE models. In this section, therefore, we consider the implications of an investment-specfic productivity shock that is threefold larger than its benchmark value (that is, $3 \cdot \sigma_{\varepsilon_x}$) for our main conclusion. The results are easily summarized. The last column of Table 3 presents the re-calibrated parameter values with the more volatile investment-specific shock series. Again, only the adjustment coefficient φ displays a meaningful change, from 18.6 to 19.9. The modestly higher value offsets an increase in the volatility of investment in an effort to match sample moments. The relative stability of the calibration results suggest the simulated moments from the recalibrated models should be quite similar to those from the benchmark model. This conjecture is confirmed by the statistics reported last two columns of Table 4. Finally, the latter two columns of Table 5 reports simulation results for zero-inflation policy with and without risk premium shocks. The quantitative results are similar to the benchmark and the qualitative results are unchanged. Overall, the results from this section indicate that the source of investment fluctuations is important for the zero bound issue. In particular, we find risk premium shocks only are capable of driving the risk-free rate to its effective lower bound. Other shocks such as investment-specific productivity shocks which also govern investment movements, even inflated threefold from their empirically measured values, do not induce the risk-free rate to reach its effective zero bound.

5.4 Discussion of the sensitivity analysis

The sensitivity analysis results suggest that, once the model is recalibrated to match data moments, the qualitative results are quite robust to a wide variation in the model's parameters. Under the zero-inflation policy benchmark, risk-premium shocks, even if conservatively measured, drive the risk-free rate to its effective zero bound. The other four shocks, even if grossly inflated, do not make zero-inflation policy inconsistent with the objective of staying away from the zero bound on the nominal interest rates.[21]

The robustness of the main finding stems from the fact that the risk-premium shocks have a first-order effect on the spread between the rates of return on capital and the risk-free rate. Owing to the fact that rapid changes in the rate of return on capital are costly, much of the adjustment to a rising spread is accommodated by the risk-free rate. The strength of the effect can be further illustrated by the impulse response functions of the risk-free rate after a one-standard deviation shock of each type. Figure 2 shows those impulse response functions computed in a model with the benchmark parameter values under the zero-inflation policy. The figure is quite striking. One standard deviation risk-premium shock makes the risk-free rate to fall by 17 basis points on impact. One standard deviation shocks of other four types make the risk free rate to change by at most one basis point. This relative strength of the effect of the risk-premium shocks on the risk-free rate shows up in our robustness tests.

6 Conclusions

Recent real world events have demonstrated the importance of the zero bound on nominal interest rates as consideration for monetary policy. Many quantitative DSGE models, however, finds that the zero bound is not a pressing constraint for monetary policy, even when the central bank follows a policy of zero inflation. In this paper, we attempt to resolve this apparent disconnect by studying a quantitative DSGE model with a broader range of shocks than examined in earlier work. We find that risk-premium shocks are the only shocks in our study that are capable of driving the risk-free rate to zero. The risk premium mechanism operates by increasing the spread between rates of return on private capital and the risk-free rate. Other common shocks, such as aggregate productivity, investment specific productivity, government spending and money demand shocks, are unable to push the risk-free rate close to zero since these shocks shift the risk-free rate and the expected return on capital in the same direction and roughly in the same proportions. These shocks, therefore, have weak implications for the zero bound problem and could only force nominal rates to zero following extreme realizations

In sum, our results suggest that careful consideration of risk premium shocks may improve our understanding of the zero bound on nominal interest rate problem within a quantitative DSGE framework. There are at least two avenues for future research. First, endogenizing the risk premium may lead to future insights on the zero bound problem, in particular, and monetary policy, in general. Second, it would be useful to derive optimal monetary policy in a DSGE model where the zero bound is important.

[21] These results are available from the authors upon request.

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A Detrending the model

There are two non-stationary processes for technology in the model A_t and X_t. As shown in Fisher (2006) we can detrend consumption, output, and other variables as follows:

$$y_t = \frac{Y_t}{\Omega_t}, \quad c_t = \frac{Y_t}{\Omega_t}, \quad m_t = \frac{M_t}{P_t \Omega_t}, \quad \lambda_t = \Lambda_t \Omega_t, \quad k_t = \frac{K_t}{\Omega_t X_t},$$

where

$$\Omega_t = A_t^{\frac{1}{\alpha}} X_t^{\frac{1-\alpha}{\alpha}}.$$

This implies that (non-detrended) consumption, output, and real money balances grow at the long run average rate $\gamma_y = \exp\left(\frac{g_a + (1-\alpha) g_x}{\alpha}\right)$, while capital stock grows at the long run average rate $\gamma_k = \exp\left(\frac{g_a + g_x}{\alpha}\right)$.

Defining $p_t^* = \frac{P_t^*}{P_t}$, restating all the household's first-order conditions and the market-clearing conditions with detrended variables and simplifying, we obtain the following system of equations:

$$\frac{c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + u_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t$$

$$\left(\frac{u_t c_t}{m_t}\right)^{\frac{1}{\gamma}} = 1 - \frac{1}{R_t}$$
$$\frac{\eta}{1-h_t} = \lambda_t w_t$$
$$\frac{\lambda_t}{R_t} = \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \frac{\Omega_t}{\Omega_{t+1}}\right)$$

$$\begin{aligned} & \lambda_t k_t \left[1 + \varphi\left(\frac{k_t}{\chi_t} - \gamma_k\right)\right] \\ = \quad & \beta E_t \left\{\lambda_{t+1}\chi_{t+1}\left[1 - \delta + q_{t+1} - \tau_t + \frac{\varphi}{2}\left(\left(\frac{k_{t+1}}{\chi_{t+1}}\right)^2 - \gamma_k^2\right)\right]\right\} \end{aligned}$$

$$1 = (1-d)\left(p_t^*\right)^{1-\theta} + d\pi_t^{\theta-1}$$
$$\Phi_t = \lambda_t \psi_t y_t + \beta d E_t \left[\pi_{t+1}^{\theta} \Phi_{t+1}\right]$$
$$\Gamma_t = \lambda_t y_t + \beta d E_t \left[\pi_{t+1}^{\theta-1} \Gamma_{t+1}\right]$$
$$p_t^* = \frac{\theta}{\theta-1} \frac{\Phi_t}{\Gamma_t}$$
$$q_t = (1-\alpha)\psi_t \left[\frac{\chi_t}{h_t}\right]^{-\alpha}$$
$$w_t = \alpha \psi_t \left[\frac{\chi_t}{h_t}\right]^{1-\alpha}$$
$$c_t + k_t - \left[1 - \delta - \frac{\varphi}{2}\left(\frac{k_t}{\chi_t} - \exp\left(\frac{g_a + g_x}{\alpha}\right)\right)^2\right]\chi_t = y_t(1 - g_t)$$
$$\chi_t^{1-\alpha} {h_t}^{\alpha} = y_t S_t$$
$$S_t = (1-d) {p_t^*}^{-\theta} + d {\pi_t}^{\theta} S_{t-1},$$
$$\chi_t = k_{t-1} \frac{\Omega_{t-1} X_{t-1}}{\Omega_t X_t}$$

plus a monetary policy equation

$$\pi_t = 1$$

under the zero-inflation policy, or

$$\log R_t = (1-\rho_R)\left(\log \bar{R} + \rho_\pi E_t \left[\log\left(\frac{\pi_{t+1}}{\bar{\pi}}\right)\right] + \rho_y \log\left(\frac{y_t}{\bar{y}}\right)\right) + \rho_R \log R_{t-1} + \varepsilon_{R,t}, \tag{17}$$

with a forward looking Taylor rule. In the Taylor rule $\bar{y}$ is simply the steady state value of the detrended output y_t.

B Computation

We use a combination of a parametrized-expectations approach with an endogenous grid method as in Carroll (2005) for non-linear computations of the model with zero-inflation policy.

1. Take Chebyshev grids over $(\ln k_t, \tau_t, g_t, \ln u_t)$, and over $(\ln \chi_t, \tau_t, g_t, \ln u_t)$, where $\chi_t = k_{t-1}\frac{\Omega_{t-1}X_{t-1}}{\Omega_t X_t}$.

2. Guess the expectation functions:

$$f_1^{(0)}(\ln k_t, \tau_t, g_t, \ln u_t) \equiv E_t\left(\beta\lambda_{t+1}\frac{\Omega_t}{\Omega_{t+1}}\right)$$

and

$$f_2^{(0)}(\ln k_t, \tau_t, g_t, \ln u_t) \equiv E_t\left\{\lambda_{t+1}\chi_{t+1}\left[1-\delta+q_{t+1}-\tau_t+\frac{\varphi}{2}\left(\left(\frac{k_{t+1}}{\chi_{t+1}}\right)^2-\gamma_k^2\right)\right]\right\}.$$

3. For each combination of the state variables $(\ln k_t, \tau_t, g_t, \ln u_t)$ from the grid, solve for χ_t, R_t, w_t, q_t, and $\widetilde{c}_t \equiv \frac{c_t}{\chi_t}$, $\widetilde{m}_t \equiv \frac{m_t}{\chi_t}$, $\widetilde{h}_t \equiv \frac{h_t}{\chi_t}$, $\widetilde{y}_t \equiv \frac{y_t}{\chi_t}$ $\widetilde{\lambda}_t \equiv \lambda_t\chi_t$ the following set of equations

$$\frac{\widetilde{c}_t^{-\frac{1}{\gamma}}}{\widetilde{c}_t^{\frac{\gamma-1}{\gamma}}+u_t^{\frac{1}{\gamma}}\widetilde{m}_t^{\frac{\gamma-1}{\gamma}}}=\widetilde{\lambda}_t$$

$$\left(\frac{u_t\widetilde{c}_t}{\widetilde{m}_t}\right)^{\frac{1}{\gamma}}=1-\frac{1}{R_t}$$

$$\frac{\eta}{\frac{1}{\chi_t}-\widetilde{h}_t}=\widetilde{\lambda}_t w_t,$$

$$\frac{\widetilde{\lambda}_t}{R_t\chi_t}=f_1^{(i)}(\ln k_t, \tau_t, g_t, \ln u_t)$$

$$\widetilde{\lambda}_t\frac{k_t}{\chi_t}\left[1+\varphi\left(\frac{k_t}{\chi_t}-\gamma_k\right)\right]=f_2^{(i)}(\ln k_t, \tau_t, g_t, \ln u_t)$$

$$q_t=(1-\alpha)\frac{\theta-1}{\theta}\left[\widetilde{h}_t\right]^{\alpha}$$

$$w_t=\alpha\frac{\theta-1}{\theta}\left[\widetilde{h}_t\right]^{\alpha-1}$$

$$\widetilde{c}_t+\frac{k_t}{\chi_t}-\left[1-\delta-\frac{\varphi}{2}\left(\frac{k_t}{\chi_t}-\gamma_k\right)^2\right]=\widetilde{y}_t(1-g_t)$$

$$\widetilde{h}_t^{\alpha}=\widetilde{y}_t$$

in which we take account of the fact that $\psi_t = \frac{\theta-1}{\theta}$, $S_t = 1$, and $p_t^* = 1$ under the zero-inflation policy, $\pi_t - 1 = 0$.

4. With the above variables computed, use the projection methods to approximate the following functions

$$g_1^{(i)}\left(\ln \chi_t, \tau_t, g_t, \ln u_t\right) \equiv \beta \widetilde{\lambda}_t$$

$$g_2^{(i)}\left(\ln \chi_t, \tau_t, g_t, \ln u_t\right) \equiv 1 - \delta + q_t + \frac{\varphi}{2}\left(\left(\frac{k_t}{\chi_t}\right)^2 - \gamma_k^2\right)$$

5. For each pair $(\ln k_t, \tau_t, g_t, \ln u_t)$ from the same grid as in step 1, use the Gauss-Legendre quadrature values of $(\varepsilon_a, \varepsilon_x, \varepsilon_\tau, \varepsilon_g, \varepsilon_u)$ together with their associated probabilities, and with the laws of motion

$$\ln \chi_{t+1} = \ln k_t - \frac{g_a + g_x}{\alpha} - \frac{\varepsilon_{a,t+1} + \varepsilon_{x,t+1}}{\alpha}$$

$$\tau_{t+1} = (1 - \rho_\tau)\,\bar{\tau} + \rho_\tau \tau_t + \varepsilon_{\tau,t+1}$$

$$g_{t+1} = \left(1 - \rho_g\right)\bar{g} + \rho_g g_t + \varepsilon_{g,t+1}$$

$$\ln u_{t+1} = (1 - \rho_u)\ln \bar{u} + \rho_u \ln u_t + \varepsilon_{u,t+1}$$

to compute the expectations

$$E_t\left(\beta \frac{\widetilde{\lambda}_{t+1}}{\chi_{t+1}} \frac{\Omega_t}{\Omega_{t+1}}\right) = E_t\left(\frac{g_1^{(i)}\left(\ln \chi_{t+1}, \tau_{t+1}, g_{t+1}, \ln u_{t+1}\right)}{\chi_{t+1}} \frac{\Omega_t}{\Omega_{t+1}}\right)$$

$$\begin{aligned} & E_t\left\{\beta \widetilde{\lambda}_{t+1}\left[1 - \delta + q_{t+1} - \tau_t + \frac{\varphi}{2}\left(\left(\frac{k_{t+1}}{\chi_{t+1}}\right)^2 - \gamma_k^2\right)\right]\right\} \\ = \; & E_t\left[g_1^{(i)}\left(\ln \chi_{t+1}, \tau_{t+1}, g_{t+1}, \ln u_{t+1}\right)\left\{g_2^{(i)}\left(\ln \chi_{t+1}, \tau_{t+1}, g_{t+1}, \ln u_{t+1}\right) - \tau_t\right\}\right]. \end{aligned}$$

6. Use the expectations computed above to update the approximated functions

$$f_1^{(i+1)}(\ln k_t, \tau_t, g_t, \ln u_t) \equiv E_t\left(\beta \frac{\widetilde{\lambda}_{t+1}}{\chi_{t+1}} \frac{\Omega_t}{\Omega_{t+1}}\right)$$

$$f_2^{(i+1)}(\ln k_t, \tau_t, g_t, \ln u_t) \equiv E_t\left\{\beta \widetilde{\lambda}_{t+1}\left[1 - \delta + q_{t+1} - \tau_t + \frac{\varphi}{2}\left(\left(\frac{k_{t+1}}{\chi_{t+1}}\right)^2 - \gamma_k^2\right)\right]\right\}$$

by fitting polynomial functions defined on the space of $(\ln k, \tau, g, \ln u)$.

7. Iterate on steps 3-6 until convergence of the expectations functions $f_1^{(i)}$, $f_2^{(i)}$.

C Data sources and data transformations

Monthly risk premium series were downloaded from Lu Zhang's website in September 2008. The data were converted into quarterly frequency by taking the average over three months of each quarter (or over two months in two quarters with missing BBB data points). Since the risk-premiums are reported on the annualized basis, we also divided the numbers by four.

Nominal labour-income share as well as the other five nominal ratios: 1) gross investment-to-GDP, 2) private consumption-to GDP, 3) government consumption-to-GDP, 4) gross investment-to-private consumption, and 5) monetary base-to-private consumption, were computed from the nominal, seasonally-adjusted, quarterly-frequency US NIPA data (taken from IFS-IMF dataset). The nominal investment series included government investment and net exports. The monetary base series came from BIS. The nominal labour-income share was computed by dividing the "Compensation of employees, received" series by the nominal GDP.

Seasonally-adjusted, quarterly-frequency real US GDP and real US capital stock series were taken from OECD Economic Outlook datasets. The aggregate hours worked index (total economy) was taken from Fransis & Ramey (2005) dataset downloaded from V. Ramey's website (June 10, 2008 version). The TFP series were constructed from the real GDP (Y), real capital (K) and aggregate hours worked (H) series as follows: $\ln A = \ln Y - \alpha \ln H - (1 - \alpha) \ln K$.

The annual-frequency US working-age population data also came from OECD Economic Outlook. These population data were converted to quarterly frequency by simple linear extrapolation.

The aggregate working hours-per-working age person *index* was taken from Fransis & Ramey dataset. We normalized this index to have its mean equal to 0.25, which is the average 1974q1-1998q1 fraction of the working hours in the dataset compiled by Cociuba, Prescott and Ueberfeldt (2009).

The risk-free rate series was taken to be the "3-Month Treasury Bill Rate: Auction Average" series available from the FRED database (TB3MA). We took the average of monthly rates in each quarter. We then divided the numbers by 4 to convert them from the annualized rates to quarterly rates.

Finally, the PCE inflation rate was computed from the BEA PCE (seasonally adjusted, quarterly-frequency) price index.

All the series, except the government consumption-to-GDP data, were taken over the sample period of 1974q1-1998q1, to be consistent with the ex-ante equity risk premia constructed by Campello, Chen and Zhang (2008). As was noted above, the nominal government consumption-to-GDP ratio was taken over a longer time period, 1974q1-2008q2, to avoid cutting the series off at the bottom of a 1991-1999 downward trend, which was largely reversed after that.

Table 1: Benchmark parameter values

Parameter	Description	Value
β	discount factor	0.998
γ	elast. of substitution between consump. and money	0.06
η	utility weight on leisure	2.7
φ	coefficient of capital adjustment	18.6
δ	capital depreciation rate	0.026
θ	elasticity of substitution for intermediate goods	7.7
α	coefficient on hours in Cobb-Douglas prod. function	0.67
d	Calvo probability of unchanged price next period	0.67
ρ_R	interest rate smoothing parameter in Taylor rule	0.51
ρ_π	Taylor rule coefficient on expected inflation	1.29
ρ_y	Taylor rule coefficient on output	0.034
$\bar{\pi}^4 - 1$	target inflation rate (annualized), percents	3.6
g_a	drift term for neutral productivity shock	0.0019
σ_{ε_a}	st. dev. of neutral productivity shock	0.0062
g_x	drift term for investment-specific productivity shock	0.0029
σ_{ε_x}	st. dev. of investment-specific productivity shock	0.0055
$\bar{g}$	average government consumption share in output	0.162
ρ_g	AR(1) coefficient for gov. consumption share shocks	0.98
σ_{ε_g}	st. dev. of government consumption share shock	0.002
$\bar{u}$	average value of money demand shock	0.062
ρ_u	AR(1) coefficient for money demand shocks	0.97
σ_{ε_u}	st. dev. of money demand shock	0.01
$\bar{\tau}$	average risk-premium	0.016
ρ_τ	AR(1) coefficient for risk-premium shocks	0.84
$\sigma_{\varepsilon_\tau}$	st. dev. of risk-premium shock	0.0079

Table 2: Calibration Results

	Data		Benchmark Model	
	st.dev.	ar(1)	st.dev.	ar(1)
Hours	0.80	0.99	0.80	0.78
Inflation	0.68	0.86	0.68	0.79
Invest./GDP ratio	1.60	0.93	1.64	0.82
Invest./Cons. ratio	**3.26**	0.93	**3.26**	0.82
Labour income share	**0.92**	0.87	**0.92**	0.65
Risk-free rate	**0.63**	**0.95**	**0.63**	**0.95**

Note: numbers in bold font are the calibration target moments. Standard deviations are in percentage points.

Table 3: Sensitivity analysis: calibrated parameter values

Parameter	Benchmark value	AAA/AA shock	real rate 2 %	real rate 3 %	σ_x= 1.65%
1	2	3	4	5	6
β	0.998		0.999	0.997	
η	2.7		2.7	2.7	
δ	0.026		0.024	0.028	
θ	7.7		7.7	7.7	
φ	18.6	7.8	20.5	17.0	19.9
ρ_R	0.51	0.51	0.51	0.51	0.53
ρ_π	1.29	1.27	1.27	1.30	1.38
ρ_y	0.034	0.028	0.031	0.038	0.001

Note: empty cells indicate parameters that would not be affected by the recalibration exercise.

Table 4: Sensitivity analysis: simulated moments with calibrated Taylor rules

Standard deviation of	Benchmark		AAA/AA		real rate=2%		real rate=3%		σ_x=1.65%	
	st.dev	ar(1)	st.dev	ar(1)	st.dev	ar(1)	st.dev	ar(1)	st.dev	ar(1)
1	2	3	4	5	6	7	8	9	10	11
Hours	0.80	0.78	0.80	0.77	0.80	0.78	0.81	0.78	0.80	0.78
Inflation	0.68	0.79	0.68	0.78	0.69	0.79	0.67	0.79	0.67	0.76
Invest./output	1.64	0.82	1.64	0.82	1.64	0.82	1.64	0.82	1.64	0.83
Invest./consump.	3.26	0.82	3.26	0.82	3.26	0.82	3.26	0.82	3.26	0.84
Labour income shr	0.92	0.65	0.92	0.61	0.92	0.65	0.92	0.65	0.92	0.63
Risk-free rate	0.63	0.95	0.63	0.95	0.63	0.95	0.63	0.95	0.63	0.95

Table 5: Sensitivity analysis: simulation results with and without risk-premium shocks under zero-inflation policy

Standard deviation of	Benchmark		AAA/AA		real rate=2%		real rate=3%		σ_x=1.65%	
	RP	no RP	RP	no RP	RP	no RP	RP	no RP	RP	no RP
1	2	3	4	5	6	7	8	9	10	11
Hours	0.54	0.29	0.50	0.31	0.55	0.29	0.53	0.29	0.53	0.31
Invest./output	1.58	0.29	1.45	0.50	1.59	0.30	1.58	0.27	1.58	0.53
Cons./output	1.85	0.99	1.73	1.09	1.85	0.99	1.84	1.00	1.82	1.08
Detr. output	1.55	0.97	0.97	0.79	1.62	1.02	1.51	0.92	2.15	1.77
Risk-free rate	0.31	0.06	0.26	0.08	0.30	0.06	0.32	0.06	0.32	0.12
p-val.of [0, 5bp]	1.70	0	0.58	0	2.76	0	0.48	0	1.75	0
min(R)/std(R)	-	6.6	-	4.1	-	4.9	-	8.1	-	2.1

Note: the last row shows the minimum risk-free rate divided by the standard deviation of the risk-free rate.

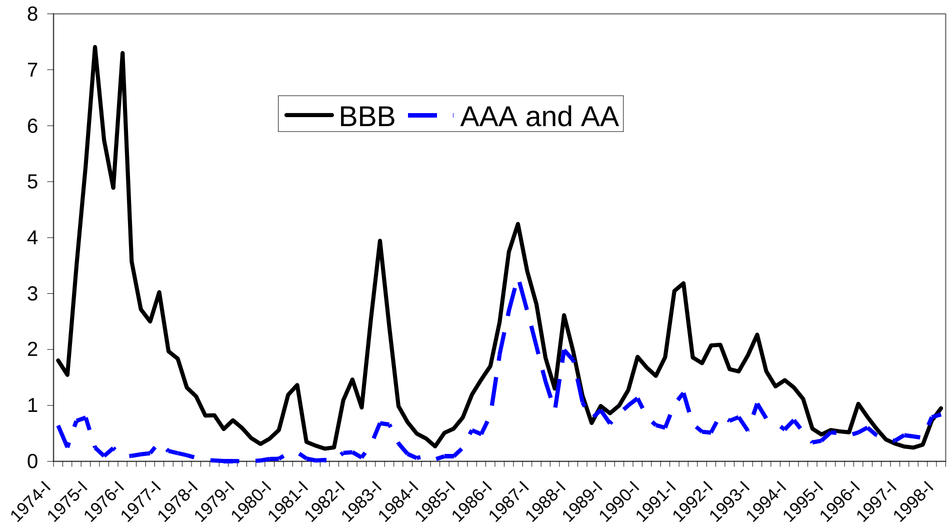

Figure 1: Ex-ante equity risk premia for BBB and AAA/AA US corporations. Source: Camplello, Chen and Zhang (2008). The original series were converted to the quarterly basis.

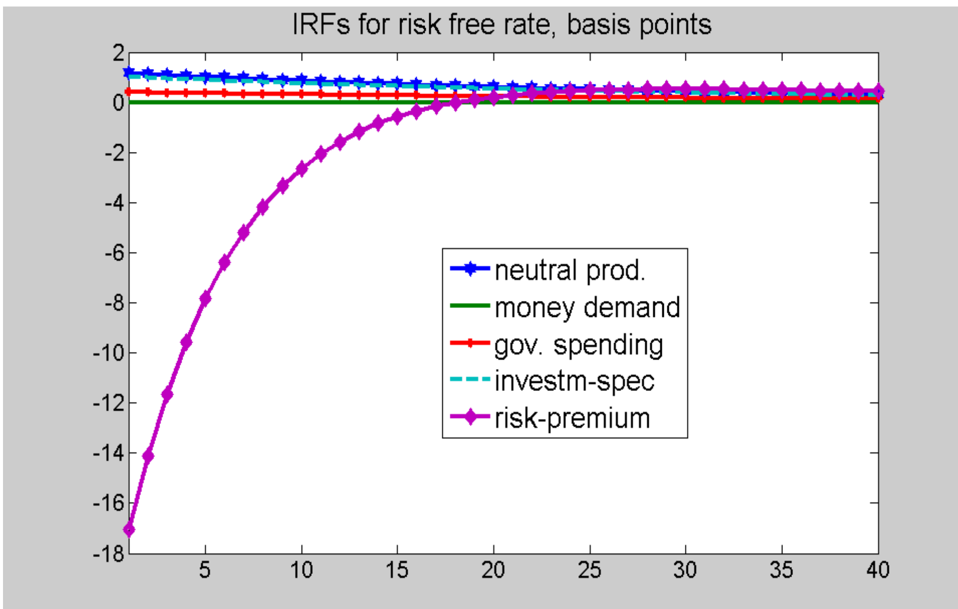

Figure 2: Impulse Response Functions of the risk-free rate to various shocks, under zero-inflation policy benchmark. Units: basis points in response to a one-standard-deviation shock.