The Effects of Foreign Shocks when U.S. Interest Rates are at Zero*

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Abstract

This paper uses a two country DSGE model to show that the effects of an adverse foreign demand shock on the United States are greatly amplified if U.S. monetary policy is constrained from reducing nominal interest rates for a protracted period. In particular, a shock that causes foreign GDP to decline 1 percent causes U.S. output to fall about 0.8 percent in our benchmark specification in which the United States is mired in a liquidity trap, compared with the 0.3 percent fall that occurs in the normal situation in which the United States can freely adjust policy rates. This amplification in the presence of the binding zero bound constraint occurs because U.S. domestic demand contracts in response to higher real interest rates rise, in sharp contrast to the "textbook" implications of a negative external demand shock when policy rates can be freely adjusted.

Keywords: trade, DSGE models, zero lower bound

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1 Introduction

In the wake of the intensification of the financial crisis last fall, a broad array of U.S. trading partners – many of which had performed reasonably well through the first three quarters of 2008 – experienced sharp and coincident decelerations in activity that were virtually without precedent in the postwar period. An obvious question of interest to policymakers is how this downturn in foreign demand is likely to affect the United States.

A wide literature assessing the transmission of shocks across countries suggests that even a pronounced slowdown abroad would probably only exert a modest contractionary impact on the United States. Notwithstanding substantial growth in trade over the past two decades, U.S. exports of goods and services comprise only around 15 percent of national output. The correlation between U.S. growth and that of major U.S. trading partners is quite low, and has shown little tendency to rise with increased globalization Doyle and Faust (2005). Dynamic factor models that attempt to decompose output variation into country-specific and global factors typically find a relatively small role for the latter. Consistent with these results, large-scale structural models that estimate the impact of foreign shocks on the United States – such as the FRB/Global model used at the Federal Reserve Board – usually imply small, even if non-negligible, effects. Erceg, Guerrieri, and Gust (2006), for example, report that an aggregate demand shock that raises the real GDP of all major U.S. trading partners by 1 percent has a peak effect on the United States of about 0.25 percent.

Although this evidence supports the hypothesis that foreign shocks typically have a relatively small effect on the United States, a key limitation is that it is based on a historical period in U.S. monetary policy had latitude to offset shocks by adjusting policy rates. It is plausible that foreign shocks would have exerted considerably larger effects if U.S. policy had been constrained by a liquidity trap. Thus, an important practical issue is to assess how the transmission of foreign shocks might

change in the current environment in which the United States may face a persistent liquidity trap.

We investigate this question using a two country DSGE model that imposes an explicit zero bound constraint on short-term interest rate in each country, our proxies for the policy rates. Aside from the zero bound constraint, our model is quite similar to that developed in previous work by Erceg, Guerrieri, and Gust (2006). It includes many of the nominal and real frictions that have been identified as empirically important in the closed economy models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), including habit persistence in consumption and adjustment costs in investment. Moreover, it incorporates analogous frictions that are relevant in an open economy framework, including both local currency pricing (e.g. Betts and Devereux (1996)) and costs of adjusting trade flows.

We find that the impact of an adverse foreign demand shock on the United States is greatly amplified if the shock occurs against the backdrop of a liquidity trap in the United States (where the latter is interpreted as a situation in which unfavorable economic conditions preclude additional cuts in policy rates even prior to the arrival of the foreign shock). Thus, while a foreign output decline of 1 percent would induce U.S. GDP to fall by only around 0.3 percent in normal circumstances in which U.S. interest rates could decline, U.S. output falls nearly 0.8 persistent in our benchmark specification in which the United States is mired in a liquidity trap. The larger output contraction in the latter case is mainly attributable to rising real interest rates, as short-term nominal rates remain frozen while expected inflation falls. As a result, the contraction in net exports associated with weaker foreign demand is reinforced by a sharp contraction in private domestic demand. This contrasts with the familiar case in which policy rates can freely adjust, in which case lower real interest rates would cause private domestic demand to expand, and hence cushion the impact on U.S. output.

Our benchmark simulations assume that the liquidity trap is generated by an

adverse demand shock in the United States, and that agents expect that policy rates will remain at zero for two years prior to the negative foreign demand shock. While this seems a reasonable benchmark for the duration of the liquidity trap, we show through sensitivity analysis that the effects of the foreign demand shock on the United States are quite sensitive to the nature of the shock(s) that generates the liquidity trap; thus, in contrast to a standard log-linearized DSGE model framework, the effects of the foreign shock hinge on initial conditions about the baseline path of inflation, the output gap, and policy interest rates (including the number of periods that the policy rate is expected to remain at the zero bound). For an alternative baseline in which the liquidity trap is expected to prevail for just two quarters *prior* to the foreign demand shock, the foreign shock has only a slightly larger impact on U.S. GDP than in the case in which U.S. interest rates can respond in an unconstrained manner. The expectation that real interest rates will fall in the near-term, even if not immediately, suffices to boost domestic demand, and hence offset much of the impact of the shock. By contrast, the effects of the foreign demand shock are amplified relative to the benchmark if initial conditions are even bleaker, so that agents expect a deeper and more protracted recession prior to the arrival of the foreign shock.

Our simulation results are also sensitive to structural features that affect the path of expected inflation, and that influence the interest-sensitivity of domestic demand.

This paper is structured as follows: Section 2 describes our model, and Section 3 discusses the calibration and solution procedure. The results for our benchmark specification are presented in Section 4. Sensitivity analysis is provided in Section 5, and Section 6 concludes.

2 The Model

This section provides a brief description of a two country version of the model. Apart from the explicit treatment of the zero-lower bound on policy rates, the setup is closely related to Erceg, Guerrieri, and Gust (2006) and Erceg, Guerrieri, and Gust (2008). We focus on describing the home country, as the setup for the foreign country is analogous.

2.1 Firms and Price Setting

Production of Domestic Intermediate Goods. There is a continuum of differentiated intermediate goods (indexed by $i \in [0,1]$) in the home country, each of which is produced by a single monopolistically competitive firm. Firms charge different prices at home and abroad, i.e., they practice pricing to market. In the home market, firm i faces a demand function that varies inversely with its output price $P_{Dt}(i)$ and directly with aggregate demand at home Y_{Dt} :

$$Y_{Dt}(i) = \left[\frac{P_{Dt}(i)}{P_{Dt}}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_{Dt}, \tag{1}$$

where $\theta_p > 0$, and P_{Dt} is an aggregate price index defined below. Similarly, in the foreign market, firm i faces the demand function:

$$X_t(i) = \left[\frac{P^*_{Mt}(i)}{P^*_{Mt}}\right]^{\frac{-(1+\theta_p)}{\theta_p}} M^*_t, \tag{2}$$

where $X_t(i)$ denotes the foreign quantity demanded of home good i, $P^*_{Mt}(i)$ denotes the price that firm i sets in the foreign market (denominated in foreign currency), P^*_{Mt} is the foreign import price index, and M^*_t is aggregate foreign imports.

Each producer utilizes capital services $K_t(i)$ and a labor index $L_t(i)$ (defined below) to produce its respective output good. The production function is assumed to have a constant-elasticity of substitution (CES) form:

$$Y_t(i) = \left(\omega_K^{\frac{\rho}{1+\rho}} K_t(i)^{\frac{1}{1+\rho}} + \omega_L{}^{\frac{\rho}{1+\rho}} (z_t L_t(i))^{\frac{1}{1+\rho}}\right)^{1+\rho}, \tag{3}$$

where z_t is a country-specific shock to the level of technology. Firms face perfectly competitive factor markets for hiring capital and labor.

Prices of the intermediate goods are determined by Calvo-style staggered contracts, see Calvo (1983). Each period, a firm faces a constant probability, $1-\xi_p$, to reoptimize its price at home $P_{Dt}(i)$ and probability of $1-\xi_{p,x}$ of reoptimizing the price that it sets in the foreign country of $P^*_{Mt}(i)$. These probabilities are independent across firms, time, and countries.

Production of the Domestic Output Index. A representative aggregator combines the differentiated intermediate products into a composite home-produced good Y_{Dt} according to

$$Y_{Dt} = \left[\int_0^1 Y_{Dt}(i)^{\frac{1}{1+\theta_p}} di\right]^{1+\theta_p}. \tag{4}$$

The optimal bundle of goods minimizes the cost of producing Y_{Dt} taking the price of each intermediate good as given. A unit of the sectoral output index sells at the price P_{Dt}:

$$P_{Dt} = \left[\int_0^1 P_{Dt}(i)^{\frac{-1}{\theta_p}} di\right]^{-\theta_p}. \tag{5}$$

Similarly, a representative aggregator in the foreign economy combines the differentiated home products $X_t(i)$ into a single index for foreign imports:

$$M^*_t = \left[\int_0^1 X_t(i)^{\frac{1}{1+\theta_p}} di\right]^{1+\theta_p}, \tag{6}$$

and sells M^*_t at price P^*_{Mt}:

$$P^*_{Mt} = \left[\int_0^1 P^*_{Mt}(i)^{\frac{-1}{\theta_p}} di\right]^{-\theta_p}. \tag{7}$$

Production of Consumption and Investment Goods. For simplicity, we assume that the import content of consumption and investment is equal. Accordingly there is effectively only one final good A_t that is used for consumption or investment, (i.e., $A_t \equiv C_t + I_t$, noting that A_t can be interpreted as private absorption). A

single distributor combines its purchases of the domestically-produced goods with imported goods to produce final goods A_t according to

$$A_t = \left(\omega_A^{\frac{\rho_A}{1+\rho_A}} A_{Dt}^{\frac{1}{1+\rho_A}} + (1-\omega_A)^{\frac{\rho_A}{1+\rho_A}} (\varphi_{At} M_t)^{\frac{1}{1+\rho_A}} \right)^{1+\rho_A}, \tag{8}$$

where A_{Dt} denotes the distributor's demand for the domestically-produced good and M_t denotes the distributor's demand for imports. The quasi-share parameter ω_A determines the degree of home bias in private absorption, and ρ_A determines the elasticity of substitution between home and foreign goods in the long run. In the short run, this elasticity is lower, because we allow for adjustment costs φ_{At}:[1]

$$\varphi_{At} = \left[1 - \frac{\varphi_{M_A} \omega_A}{2} \left(\frac{\frac{M_t}{A_{Dt}}}{\frac{M_{t-1}^A}{A_{Dt-1}^A}} - 1 \right)^2 \right]. \tag{9}$$

Each representative distributor chooses a plan for A_{Dt} and M_t to minimize its discounted expected costs of producing the final good A_t:

$$\min_{A_{Dt}, M_t} \widetilde{\mathbb{E}}_t \sum_{k=0}^{\infty} \psi_{t,t+k} \left(P_{Dt+k} A_{Dt+k} + P_{Mt+k} M_{t+k} \right) \tag{10}$$

$$+ P_t \left[A_t - \left(\omega^{\frac{\rho}{1+\rho}} A_{Dt}^{\frac{1}{1+\rho}} + (1-\omega)^{\frac{\rho}{1+\rho}} (\varphi_t M_t)^{\frac{1}{1+\rho}} \right)^{1+\rho} \right]. \tag{11}$$

The distributor sells A_t to households at a price P_t. Accordingly, the prices of consumption and investment are equalized.

2.2 Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the intermediate goods-producing sector. For simplicity, a representative labor aggregator is

[1] Hooper, Johnson, and Marquez (2000) find that the short-run trade price elasticity is significantly smaller than the long-run elasticity in their study using aggregate data. This is qualitatively consistent with the results of industry studies as surveyed by McDaniel and Balistreri (2003).

assumed to combine households' labor hours in the same proportions as firms would choose. This labor index L_t has the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 N_t(h)^{\frac{1}{1+\theta_w}}\, dh\right]^{1+\theta_w}, \tag{12}$$

where $\theta_w > 0$ and $N_t(h)$ is hours worked by a typical member of household h. The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $W_t(h)$ as given. One unit of the labor index sells at the unit cost W_t:

$$W_t = \left[\int_0^1 W_t(h)^{\frac{-1}{\theta_w}}\, dh\right]^{-\theta_w}. \tag{13}$$

where W_t can be interpreted as the aggregate wage index. The aggregator's demand for the labor services of household h satisfies

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\frac{1+\theta_w}{\theta_w}} L_t. \tag{14}$$

The utility functional of a representative household h is

$$\widetilde{\mathbb{E}}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\sigma}\left(C_{t+j}(h) - \varkappa C_{t+j-1} - \nu_{ct}\right)^{1-\sigma} + \right.$$
$$\left. \frac{\chi_0}{1-\chi}(1 - N_{t+j}(h))^{1-\chi} + \frac{\mu_0}{1-\mu}\left(\frac{MB_{t+j+1}(h)}{P_{t+j}}\right)^{1-\mu} \right\}, \tag{15}$$

where the discount factor β satisfies $0 < \beta < 1$. As in Smets and Wouters (2003), we allow for the possibility of external habits. At date t household h cares about consumption relative to lagged per capita consumption, C_{t-1}. The household's period utility function depends on the current leisure $1 - N_t(h)$, the end-of-period real money balances, $\frac{MB_{t+1}(h)}{P_t}$, and a preference shock, ν_{ct}. The preference shock follows an exogenous first order process with a persistence parameter of ρ_ν. The budget constraint of each household is given by

$$P_t C_t(h) + P_t I_t(h) + MB_{t+1}(h) - MB_t(h) + \frac{e_t P^*_{Bt} B_{Ft+1}(h)}{\phi_{bt}} - e_t B_{Ft}(h)$$
$$= W_t(h) N_t(h) + \Gamma_t(h) - T_t(h) + R_{Kt}(1-\tau_{Kt}) K_t(h) - P_{Dt}\phi_{It}(h). \tag{16}$$

Final consumption and investment goods are purchased at a price P_t. Investment in physical capital augments the per capita capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}(h) = (1-\delta)K_t(h) + I_t(h), \tag{17}$$

where δ is the depreciation rate of capital. The term $R_{Kt}(1-\tau_{Kt})K_t(h)$ in the budget constraint represents the proceeds to the household from renting capital to firms net of capital taxes.

Financial asset accumulation consists of increases in nominal money holdings $MB_{t+1}(h) - MB_t(h)$ and the net acquisition of international bonds. Trade in international assets is restricted to a non-state contingent nominal bond. $B_{Ft+1}(h)$ represents the quantity of the international bond purchased by household h at time t that pays one unit of foreign currency in the subsequent period. P^*_{Bt} is the foreign currency price of the bond, and e_t is the nominal exchange rate expressed in units of home currency per unit of foreign currency. Following Turnovsky (1985) households pay an intermediation fee ϕ_{bt}.[2] The intermediation fee depends on the ratio of economy-wide holdings of net foreign assets to nominal output according to:

$$\phi_{bt} = \exp\left(-\phi_b\left(\frac{e_t B_{Ft+1}}{P_{Dt}Y_t}\right)\right). \tag{18}$$

If the home economy has an overall net lender position internationally, then a household will earn a lower return on any holdings of foreign bonds. By contrast, if the economy has a net debtor position, a household will pay a higher return on any foreign debt.

Households earn labor income, $W_t(h)N_t(h)$, lease capital to firms at the rental rate R_{Kt}, and receive an aliquot share $\Gamma_t(h)$ of the profits of all firms. Furthermore

[2]The assumption of an intermediation fee is adopted in order to ensure that the evolution of net foreign assets is stationary (assuming, as in this paper, that that the underlying shocks are stationary). See Schmitt-Grohe and Uribe and Bodenstein for a discussion. The intermediation cost is asymmetric, as foreign households do not face these costs. Rather, they collect profits on the monopoly rents associated with these intermediation costs.

they pay a lump-sum tax $T_t(h)$. We follow Christiano, Eichenbaum, and Evans (2005) in assuming that households bear a cost of changing the level of gross investment from the previous period, so that the acceleration in the capital stock is penalized:

$$\phi_{It}(h) = \frac{1}{2}\phi_I \frac{\left(I_t(h) - I_{t-1}(h)\right)^2}{I_{t-1}(h)}. \tag{19}$$

Households maximizes the utility functional (15) with respect to consumption, investment, (end-of-period) capital stock, money balances, and holdings of foreign bonds, subject to the labor demand function (14), budget constraint (16), and transition equation for capital (17). They also set nominal wages in staggered contracts that are analogous to the price contracts described above. In particular, with probability $1 - \xi_w$, each member of a household is allowed to reoptimize its wage contract.

2.3 Monetary and Fiscal Policy

We assume that the central bank follows an interest rate reaction function similar in form to the historical rule estimated by Orphanides and Wieland (1998). Thus, the short-term nominal interest rate is adjusted so that the *ex post* real interest rate rises when inflation exceeds its constant target value, or when output *growth* rises above some target value:

$$i_t = \max(0, \gamma_i(i_{t-1} - \bar{r} - \bar{\pi}) + \bar{r} + \bar{\pi}_t + \gamma_\pi(\pi_t - \bar{\pi}) + \gamma_y(y_t - y_{t-1} - g_y)). \tag{20}$$

In the above, i_t is the quarterly nominal interest rate, $\pi_t = log(P_{Dt}) - log(P_{Dt-1})$, $\bar{r}$ and $\bar{\pi}$ are the steady-state real interest rate and the central bank's constant inflation target. Also, $y_t - y_{t-1}$ is the quarterly growth rate of output, and g_y is its corresponding steady state value. The foreign central bank's interest rate reaction function is also assumed to be of the same form, but given the shocks we consider in this paper, the zero bound constraint is never binding on the foreign economy.

Some of the domestically-produced good is purchased by the government, but government purchases make no direct contribution to household utility. Government purchases are assumed to be a constant fraction of output $\bar{g}$. These purchases are financed through lump-sum taxes and capital taxes. Lump-sum taxes are adjusted so that the government's budget is balanced every period.

2.4 Resource Constraints

The home economy's aggregate resource constraint can be written as:

$$Y_{Dt} = C_{Dt} + I_{Dt} + G_t + \phi_{It}, \tag{21}$$

Thus, the composite domestically-produced good Y_{Dt} (net of investment adjustment costs ϕ_{It}) can be used as an input into final consumption or investment goods ($A_{Dt} = C_{Dt} + I_{Dt}$), or can be used directly to satisfy government demand. Moreover, since each individual intermediate goods producer can sell its output either at home or abroad (which is in turn "bundled" by the respective aggregator), there are also a continuum of resource constraints that apply at the firm level.

2.5 Calibration of Parameters

The model is calibrated at a quarterly frequency. The values of key parameters are presented in Table 1. We choose $\omega_A = 0.15$ to be consistent with an import share of output of 15%. We choose the domestic and foreign population levels, respectively ζ and ζ^*, so that the home country constitutes 25 percent of world output. Balanced trade in steady state implies an import (or export) share of output of the foreign country of 5 percent. Because the foreign country is assumed identical to the home country except in its size, $\omega_A^* = 0.05$. We set $\rho_A = 10$, so that the long-run price elasticity of import demand is 1.1. Non-zero values of the parameter ϕ_{M_A} allow us to create a wedge between the short- and long-run import price elasticities, but we relegate this to sensitivity analysis and set $\phi_{M_A} = 0$ in our benchmark calibration.

Monetary policy follows a simple Taylor-type rule with interest rate smoothing. The smoothing parameter γ_i is set at 0.9. The parameter γ_π governing the rule's response to inflation's deviation from the target rate is 0.5. The parameter γ_y that governs the rule's response to the output gap is 0.125 for the quarterly policy interest rate, equivalent to a coefficient equal to 0.5 when expressing the rule in terms of annualized policy rates. The economy's steady state real interest rate is set to 2% per year ($\beta = 0.995$). Given an inflation target of zero, the implied steady state nominal interest rate is two percent.

The values of remaining parameters are fairly standard in the literature, and are summarized in Table 1.

3 Solution Method and Baseline Path

With the exception of the monetary policy rule in equation (20), the equilibrium conditions of the model are linearized around the model's non-stochastic steady state. Thus, the only source of nonlinearity in the model is the lower bound constraint on the short-term nominal interest rates, the nonlinearity of principal interest to us.

We solve the model using an algorithm first proposed by Laffargue (1990) and extended by Boucekkine (1995) and Juillard (1996), which in turn builds on earlier work by Fair and Taylor (1983). This algorithm stacks all equations through time, which is equivalent to collapsing the Type I and II iterations in the Fair-Taylor shooting algorithm into one step. The size of the first-derivative used to implement a Newton-type recursion is kept manageable by exploiting the sparsity of the stacked system.

Following Anderson (1999), instead of using the steady state values as end point for the shooting algorithm, we restrict the end point following the solution computed using standard algorithms for linear models.[3] As shown by Anderson (1999), this

[3]For example, see Anderson and Moore (1985).

alternative restriction leads to an improvement in the numerical approximation to the solution of non-linear models reflected in a shorter length of the horizon needed to achieve the desired level of accuracy for the values at the beginning of the simulation.

An appropriately long simulation horizon makes the solution produced by our algorithm numerically-equivalent to that obtained following the method described by Eggertsson and Woodford (2003) and Jung, Teranishi, and Watanabe (2005) for simulations in which a shock immediately takes the model to the zero lower bound. Relative to the procedure suggested by Eggertsson and Woodford (2003), our method deals easily with shocks whose effects build up over time and only eventually lead the economy to the zero lower bound. Moreover our algorithm extends naturally to deal with both economies in the model constrained by the zero lower bound on nominal interest rates.

Our principal goal is to compare the *marginal* impact of foreign shocks on the home country when it faces a liquidity trap with the effects that occur in the more typical case in which policy rates can be freely adjusted. In the former case, the marginal impact of a foreign shock turns out to depend heavily on the economic conditions that precipitated the liquidity trap. Intuitively, the effects of an adverse foreign shock against the backdrop of a recession-induced liquidity trap in the home country should depend on the expected severity of the recession, and perceived duration of the liquidity trap. If the recession was expected to be shallow prior to the foreign shock, so that monetary policy was expected to have the latitude to cut interest rates in the near future, it is reasonable to expect that the effects of the foreign shock would not differ substantially from the usual case in which rates could be cut immediately.[4] By contrast, the effects of the foreign shock on the home country might be amplified substantially if it occurred against the backdrop of a steep recession in which policy rates were expected to be constrained from falling

[4]In the case of a linear model, the marginal effects of a shock are unrelated to the initial conditions.

for a protracted period.

We use the term "initial baseline path" to describe the evolution of the economy that would prevail in the absence of the foreign shock. Given agents' full knowledge of the model, the initial baseline path depends on the underlying shocks assumed to push the economy into a liquidity trap, including their magnitude and persistence, as these features play an important role in determining agents' perceptions about the duration of the liquidity trap.

Although we devote considerable attention to investigating the implications of different initial baselines, much of our analysis focuses on the effects of foreign shocks against the backdrop of an initial baseline path intended to capture a severe recession in the home country. This "severe recession" baseline is depicted in Figure 1. It is generated by a large and persistent preference shock ν_{ct} that reduces the home country's marginal utility of consumption. Given that the shock is assumed to occur exclusively in the home country, the foreign economy has latitude to offset much of the contractionary impact of the shock by reducing its policy rate.

Figure 1 shows the impulse responses of key home country to the taste shock, which we interpret as the model's initial baseline forecast path under the severe recession scenario. Real variables are reported in percentage deviation from their steady state values, and nominal variables as percentage point deviations. Policy rates immediately fall 2 percentage points below their steady state value – the maximum decline possible given the zero bound constraint – and remain frozen at this level for eight quarters. Given that the shock drives inflation persistently below baseline and that nominal interest rates are constrained from falling by the zero bound, real rates increase substantially in the near -term. This increase in real interest rates in the face of weakening aggregate demand helps account for a substantial output decline, which peaks in magnitude at about 15 percent below baseline. Real interest rates decline in the longer-run, which helps the economy recover. From the standpoint of external variables, lower long-term real interest rates cause the

home currency to depreciate in real terms, and an associated expansion of real net exports helps mitigate the effects of the shock on domestic output. However, this improvement in real net exports is somewhat delayed to the zero bound constraint, since higher real interest rates limit the size of depreciation of the home currency in the near-term.

For purposes of comparison, the figure also shows the effects of the same shocks in the case in which the home country's policy rates can be adjusted freely, i.e., ignoring the zero bound constraint. Clearly, the home nominal interest rate falls much more sharply. This allows real interest rates to decline in the near-term, and helps account for a much smaller fall in home output than in the benchmark framework in which the zero bound constraint is binding. The home output contraction is also mitigated by an improvement in real net exports due to a depreciation of the home country's exchange rate. Given that real interest rates fall very quickly, the real depreciation is considerably larger and more front-loaded than when the zero lower bound constraint is imposed, contributing to a more rapid improvement in real net exports.

As the shock used to construct the initial baseline forecast implies that the home policy rate reaches the zero-lower bound immediately (while the foreign policy rate is still well above zero) and is expected to eventually lift above zero, one can use the solution algorithm in Eggertsson and Woodford (2003) to show analytically that as long as any additional shock does not vary the number of periods for which policy rates are expected to remain at zero, the marginal effects are linear in the size of the shock's innovation. Similarly, one can also show that for any additional shock, there is a size of the shock's innovation above which the expected permanence of policy rates at the zero lower bound is extended. These propositions limit the arbitrary nature of the initial conditions substantially, but we relegate their formalization and proof to Appendix A.

4 International Transmission at the Zero Bound

We now turn to assessing the marginal impact of a negative foreign consumption preference shock ν^*_{ct} on the home economy when it faces a liquidity trap. As a prelude, it is helpful to first examine the effects of the foreign demand shock in the "usual case" in which policy rates can be freely adjusted. In this case, the responses of our (log-linearized) model are additive in the shocks, so that the marginal impact of the foreign shock does not depend on other shocks affecting the economy.

The dashed lines in Figure 2 show the effects of a persistent foreign consumption preference shock when the zero bound constraint is not imposed on home policy interest rates. We refer to this as the "NOZBI" ("no zero bound imposed") case below. Real variables are reported in the form of percentage deviations from their steady state values, and nominal variables as percentage point deviations. The foreign shock is scaled to induce a 1 percent reduction in foreign output relative to its steady state level. The foreign shock has a depressing effect on the home country's real net exports through both an activity channel (as the foreign shock causes foreign absorption to decline), and because the home real exchange rate appreciates (as foreign interest rates fall relative to home interest rates). This fall in external demand causes home output to decline by about 0.3 percent at peak impact, and also induces a persistent drop in inflation. However, the adverse impact on output is mitigated by rise in domestic absorption, which is "crowded in" because lower policy rates serve to reduce real interest rates.

The solid lines show the effects of same foreign consumption shock when the zero bound constraint is imposed on home policy rates, which we refer to as the "ZBI" (zero bound imposed) case. As discussed above, initial conditions as summarized by the baseline path are consequential for the marginal impact of shocks: the figure shows the marginal impact of the foreign shock when it occurs the backdrop of the severe recession scenario associated with Figure 1. To be specific, the marginal impacts are impulse responses derived from a simulation that adds both the adverse

domestic taste shock from Figure 1 and the foreign taste shock from Figure 2, and then subtracts the impulse response functions associated with the domestic taste shock alone. Thus, all variables are measured as deviations from the baseline path shown in Figure 1. Agents in the model are interpreted as expecting to be mired in a liquidity trap for two years prior to the arrival of the foreign shock.

The effects of the foreign demand shock in the ZBI case are strikingly different from the NOZBI case from both a quantitative and qualitative perspective. Although the foreign shock has nearly the same effect on foreign output across the two cases – reflecting the low degree of openness of the foreign economy, and that it has latitude to adjust interest rates to help offset the shock – the effects on home output are several times larger when the zero bound constraint is imposed. Home real net exports contract in the ZBI case for the same reasons as when the zero bound is not imposed, i.e., foreign activity declines, and the home real exchange rate appreciates. The amplification that occurs in the ZBI case is attributable to to a pronounced rise in the real interest rate, as the zero bound constraint keeps nominal rates from declining in the face of lower expected inflation. As a result, with both domestic absorption and net exports falling, output falls by nearly as much in the home country as abroad. Home net exports also decline more in the ZBI case, as rising domestic interest rates induce a larger appreciation of the domestic currency.

4.1 Alternative Initial Conditions

In Figure 3, we consider the marginal impact of the same foreign consumption shock ν^*_{ct} under different initial baselines. The solid lines show the impact of the foreign shock under our initial baseline above in which the zero bound is expected to last for eight quarters prior to the shock. The dashed line shows the effects of the foreign consumption shock when the initial baseline path is consistent with an even deeper and more protracted recession at home; in this case, the initial baseline implies that

the zero bound constraint is expected to bind for twelve quarters. The dotted lines show the marginal impact of the foreign consumption shock for an initial baseline in which the domestic recession is much shallower, and the liquidity trap more transient.

The impact of the foreign shock are largest for the initial baseline in which the zero bound binds for twelve quarters: U.S. output falls by almost one percent, the same magnitude as the decline in foreign output. The long duration of the zero bound contributes to a relatively large rise in real interest rates, and to a comparatively large appreciation of the real exchange rate – both of which serve to reduce the demand for home goods. By contrast, as seen by the dotted lines, the effects of the zero bound constraint are less pronounced when the zero bound constraint is more transient. The expectation that policy rates can be cut in the near future, even if not immediately, allows for a much more rapid rise in domestic absorption, and cushions the size and duration of the effects on the home country.

4.2 Positive and Negative Foreign Consumption Shocks

Figure 4 illustrates that when additional shocks change the expected number of periods, for which monetary policy is expected to be constrained by the zero lower bound, their marginal effects cease to be linear in the size of the shock's innovations.[5] The solid and and dotted lines in the figure show respectively the responses to contractionary and expansionary consumption shocks abroad but their magnitudes are now much larger than that of the shock whose effects are illustrated in Figure 2. The expansionary shock leads to such an increase in home exports that the marginal cost of production and home inflation jump up. The increase in inflation is large enough to lift the home economy from the zero lower bound on policy rates. The increase in policy rates reduces the fall in the real interest rate stemming from the inflation spurt. By contrast, with the contractionary shock abroad, the home

[5]The results in Figure 4 are in accordance with the propositions proved in Appendix A.

economy remains constrained by the zero lower bound. In that case, the policy rate cannot counter the rise in the real interest rate associated with deflation. This source of asymmetry for the real interest rate renders all other responses for the home country asymmetric.

By contrast, the responses for the Foreign country remain essentially symmetric, as the second round of spillovers from the home country back to the foreign country is quantitatively unimportant given modest trade ties in the calibration and the much larger size of the Foreign block relative to the home block. Finally, the smaller contraction in foreign demand, whose responses are denoted by the dashed lines in the figure, shows that linearity is maintained when shocks imply the same number of periods at the zero lower bound (as is the case for the responses denoted by the solid lines).

4.3 The Zero Lower Bound Binds in Both Countries

The discussion so far has focused on the marginal effects of a foreign disturbance on the home economy. However, our model also has implications for the spillover effects on the foreign country of shocks originating from the home country. Figure 5 illustrates the marginal effects of a home consumption shock. In the figure, the three lines shown refer to the case in which the zero lower bound binds in both countries (the solid lines), binds at home only (the dashed lines), and does not bind in either country (the dotted lines). In each case, the magnitude of the shock was kept unchanged. As is the case for spillovers from the foreign country to the home country, when the constraint on policy rates becomes binding, the spillover effects of home shocks to the foreign country are magnified. However, when the ZLB binds in both countries shock's reverberations back to the country of origin remain small. When the foreign economy is also at the ZLB, the greater contraction in foreign activity is accompanied by a greater fall in the relative price of foreign imports. Accordingly, home exports and GDP are little changed whether the foreign

economy is at the zero lower bound or not.

5 Sensitivity Analysis

5.1 Structural Parameters

We next examine how varying structural parameters affects the transmission of the same foreign demand shock, with particular attention to the effects of changes in the price-setting behavior (as captured by the degree of indexation in the Phillips Curve), and in the interest-sensitivity of domestic demand. Unfortunately, such parameter changes not only affect the transmission of a given foreign demand shock, but they also impact the constructed baseline. To maintain comparability across parameterizations, we adjust the baseline so that the expected duration of the economy at the zero lower bound remains unchanged.[6]

5.1.1 Inflation Dynamics

Figure 5 shows sensitivity of the domestic economy's reaction to a foreign demand shock when varying the degree of indexation to lagged inflation in the Calvo-type contracts for domestic prices. Following Christiano, Eichenbaum, and Evans (2005), the figure's right column considers price contracts such that firms that do not get to reoptimize their prices update them according to a partial indexation scheme based on the previous quarter's aggregate inflation rate. For ease of comparison,

[6]As described in section 3 the construction of the baseline assumes that in period 1 the home country experiences a negative consumption shock of given size and persistence such the home economy is at the zero lower bound for 8 quarters. If a structural parameter changes, the new baseline is constructed by changing the size of the negative consumption shock so that the economy is again expected to remain at the zero lower bound for 8 quarters. This adjustment, however, does not imply that the dynamics of variables are identical between old (old parametrization) and new (new parametrization) baseline. Nevertheless, as the only non-linear feature of the model stems from the zero lower bound on interest rates, we think that these differences across baselines are of minor importance for the key conclusions of our analysis.

the figure's left column reports again the results presented in Figure 2, turning off the indexation scheme. With indexation to lagged inflation, the response of domestic inflation to the foreign consumption shock is more persistent. Once firms start cutting prices, the deflation gains momentum and acquires a hump shape. The persistence of the inflation movement is increased by the indexation scheme. With a fixed nominal policy interest rate, constrained by the zero lower bound, real rates rise more persistently, leading to larger and more persistent drop in domestic private absorption. Accordingly, with indexation, the spillover effect of the negative foreign consumption shock on domestic GDP are larger and more persistent.

5.1.2 Interest-Sensitivity of Domestic Demand

The economy's intertemporal substitution elasticity has great influence over the cross-country spillover effects. Keeping other factors constant, greater interest sensitivity ought to deepen the spillover effects of foreign demand contractions, as domestic real rates rise. However, away from the zero lower bound, greater interest sensitivity accords the same interest rate reaction function describing monetary policy greater effectiveness in stabilizing the economy. Figure 7 allows a quick comparison of the spillover effects of foreign disturbances when the intertemporal substitution elasticity is varied from $\frac{1}{2}$ to 1. When the intertemporal substitution elasticity equals 1, greater effectiveness of monetary policy away from the ZLB, given forward looking agents, translate in reduced spillover effects even the policy is constrained by the ZLB.

5.2 Effects of Other Shocks

The magnification of foreign spillover effects is not peculiar to foreign preference shocks. Figure 8 shows the impulse responses for the case of a contraction in foreign government spending. Figure 9 shows the impulse responses for the case of a increase in the foreign capital tax rate. In both cases the cross-country spillover effects are

magnified at least twofold as measured by the reaction of home GDP relative to the same-size movement in foreign GDP.

6 Conclusions

When monetary policy is unconstrained by the zero lower bound on nominal interest rates foreign disturbances have limited spillover effects onto the U.S. economy as monetary policy can stimulate domestic demand by reducing interest rates. By contrast, at the zero lower bound the spillover effects of foreign shocks can be amplified greatly assuming that monetary policy does not resort to various unconventional forms of stimulus. In future work we plan to compare alternative policies that implement quantitative easing when the zero lower bounds becomes binding.

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Table 1: Calibration*

Parameter	Determines:	Parameter	Determines:
$\beta = 0.995$	s.s. real interest rate = 2% per year	$\delta = 0.025$	depreciation rate = 10% per year
χ_0	leisure's share of time = 1/2	$\chi = 10$	labor supply elasticity = 1/10
$\sigma = 2$	intertemporal substitution elast. 1/2	$\phi_b = 0.001$	interest elasticity of foreign assets
$\rho = -2$	capital-labor substitution elast. = 1/2	$\rho_A = 10$	long-run import price elasticity = 1.1
$\omega_A = 0.15$	import share of output = 15%	$\omega_A^* = 0.05$	foreign import share of output = 5%
$\zeta = 1$	population size	$\zeta^* = 3$	foreign population size
$\kappa = 0.8$	consumption habits	$\phi_I = 3$	investment adjustment costs
$\theta_w = 0.1$	wage markup = 10%	$\theta_p = 0.1$	domestic/export price markup = 10%
$\xi_p = 0.75$	price contract expected duration = 4 quarters	$\xi_w 0.75$	wage contract expected duration = 4 quarters
$\xi_{px} = 0.5$	export price contract expected duration = 2 quarters		
$\gamma_i = 0.9$	monetary policy's weight on lagged interest rate	$\gamma_\pi = 0.5$	monetary policy's weight on inflation
$\gamma_y = 0.125$	monetary policy's weight on output gap		

* Parameter values for the foreign country are chosen identical to their home country counterparts except for the population size ζ^* and the import share ω_A^*.

Figure 1: Severe Domestic Recession Scenario (Initial Baseline Path)

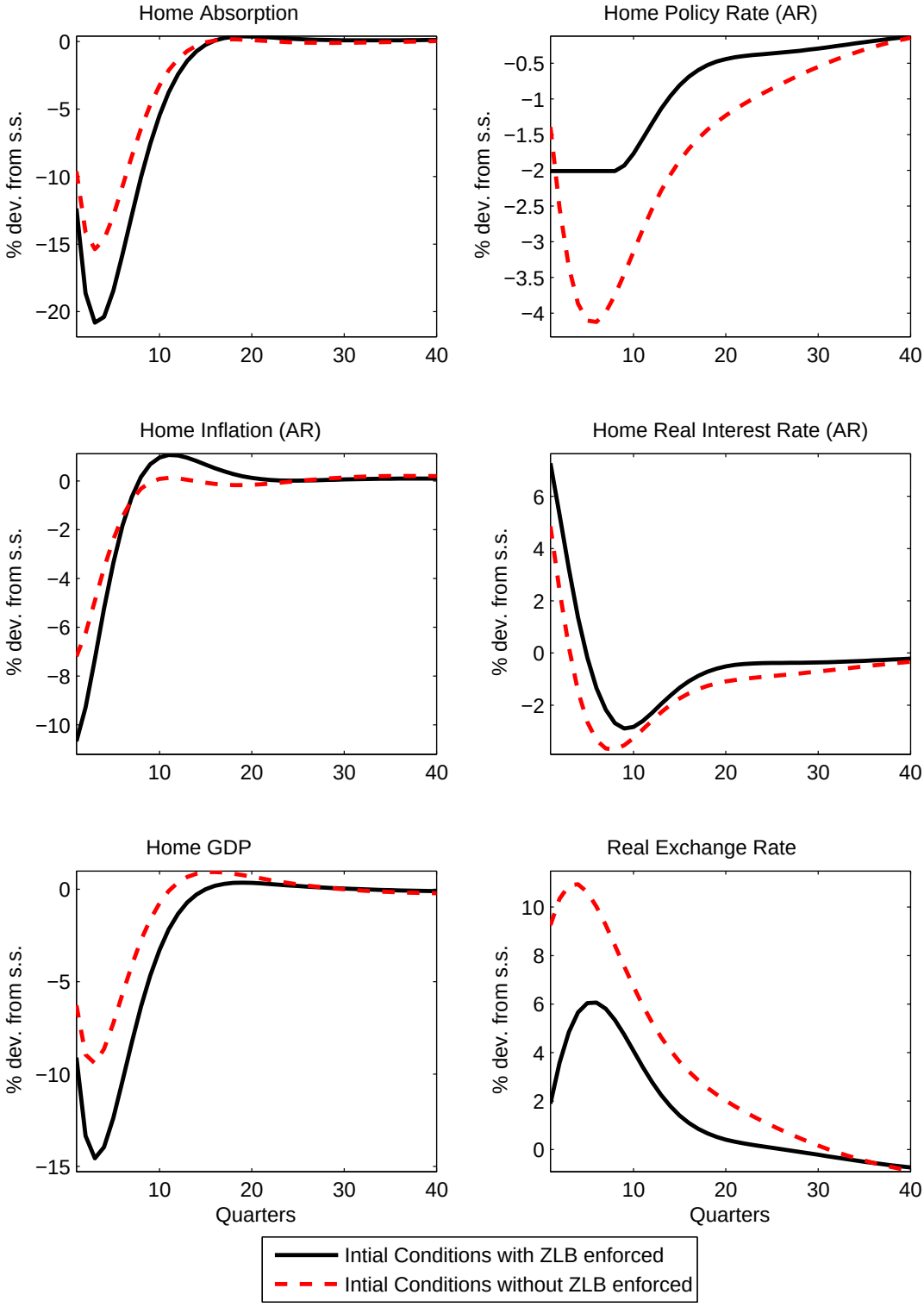

Figure 2: Effects of Foreign Consumption Shock against Backdrop of Domestic Recession

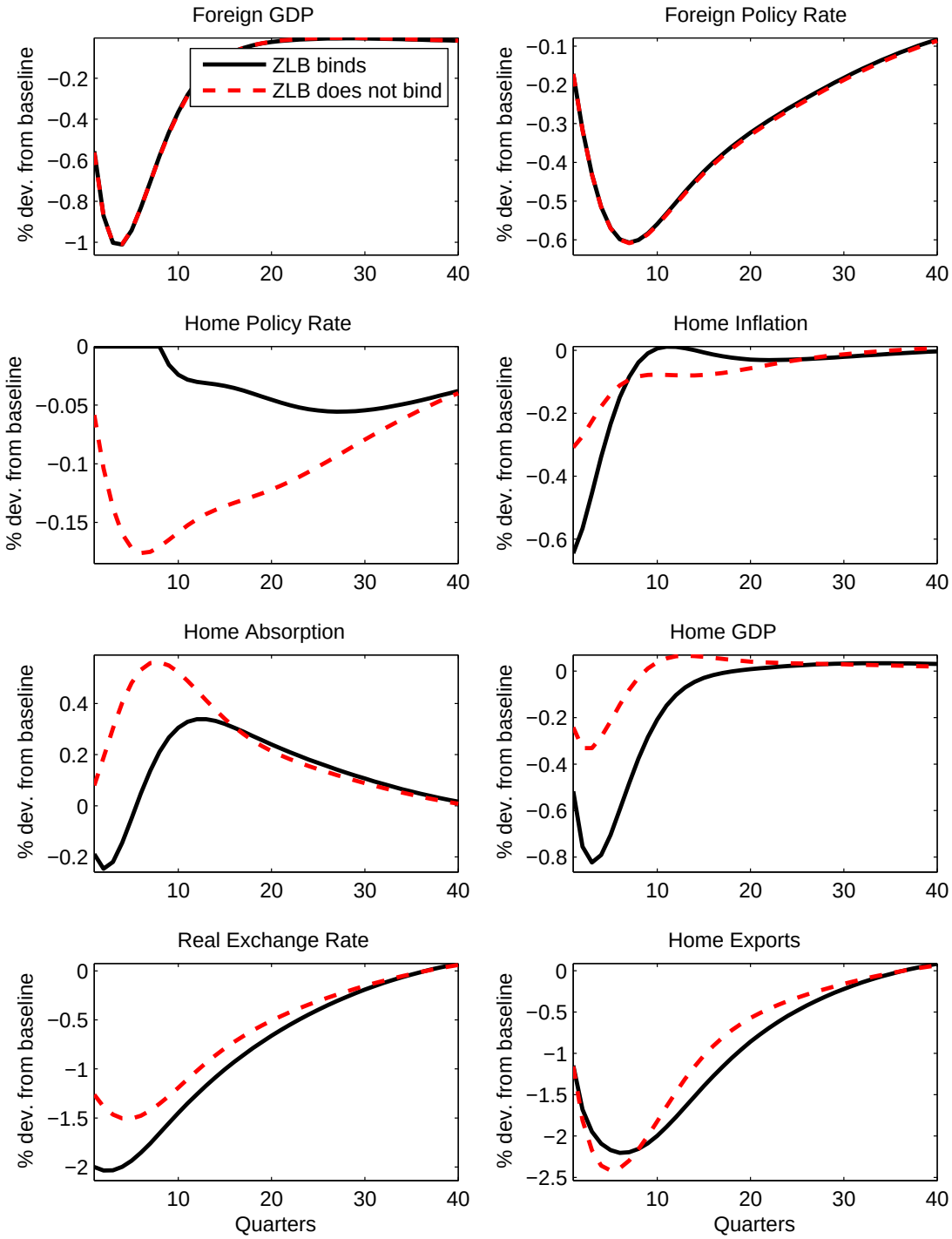

Figure 3: Marginal Foreign Consumption Shock under Different Initial Baseline Paths

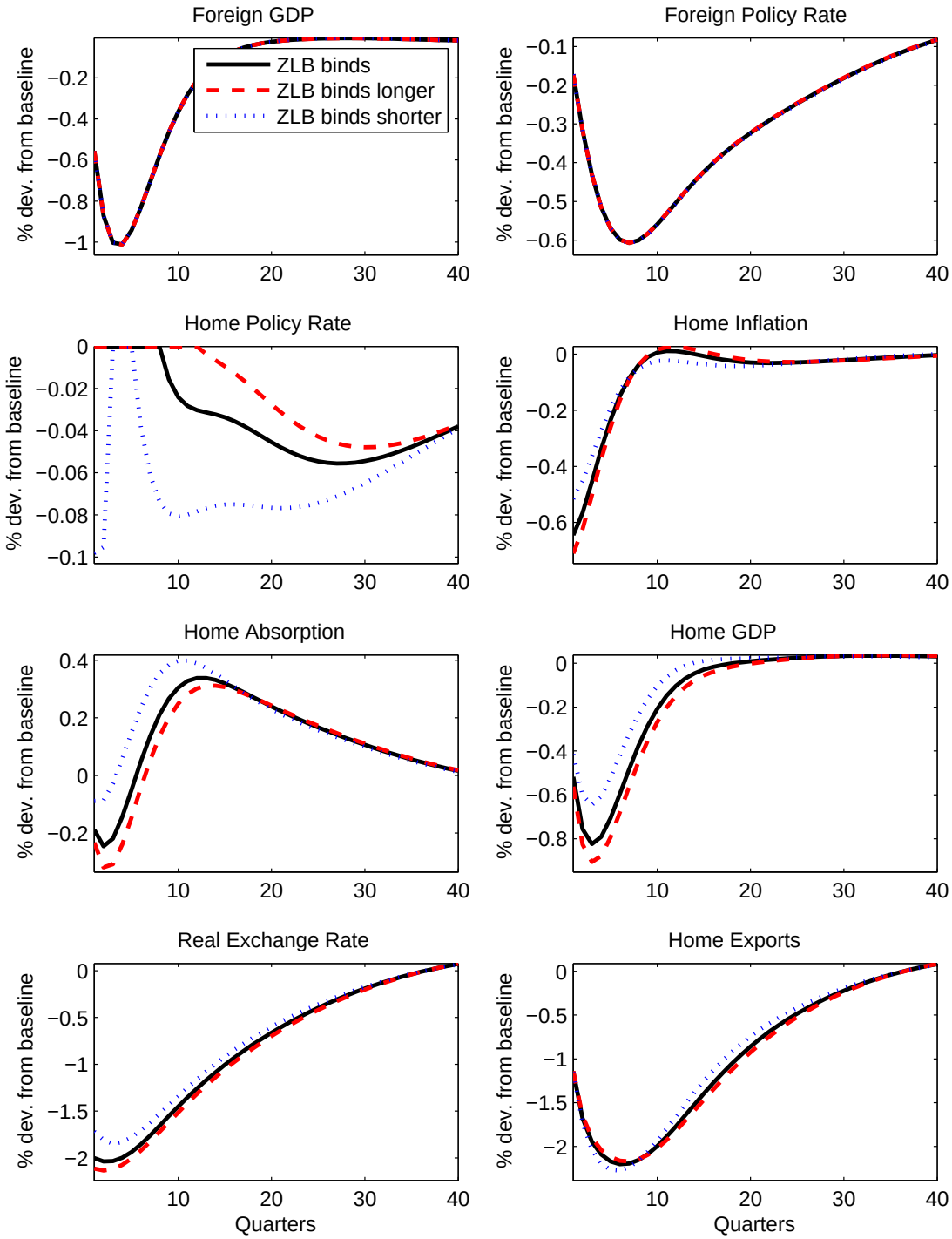

Figure 4: Positive vs Negative shocks

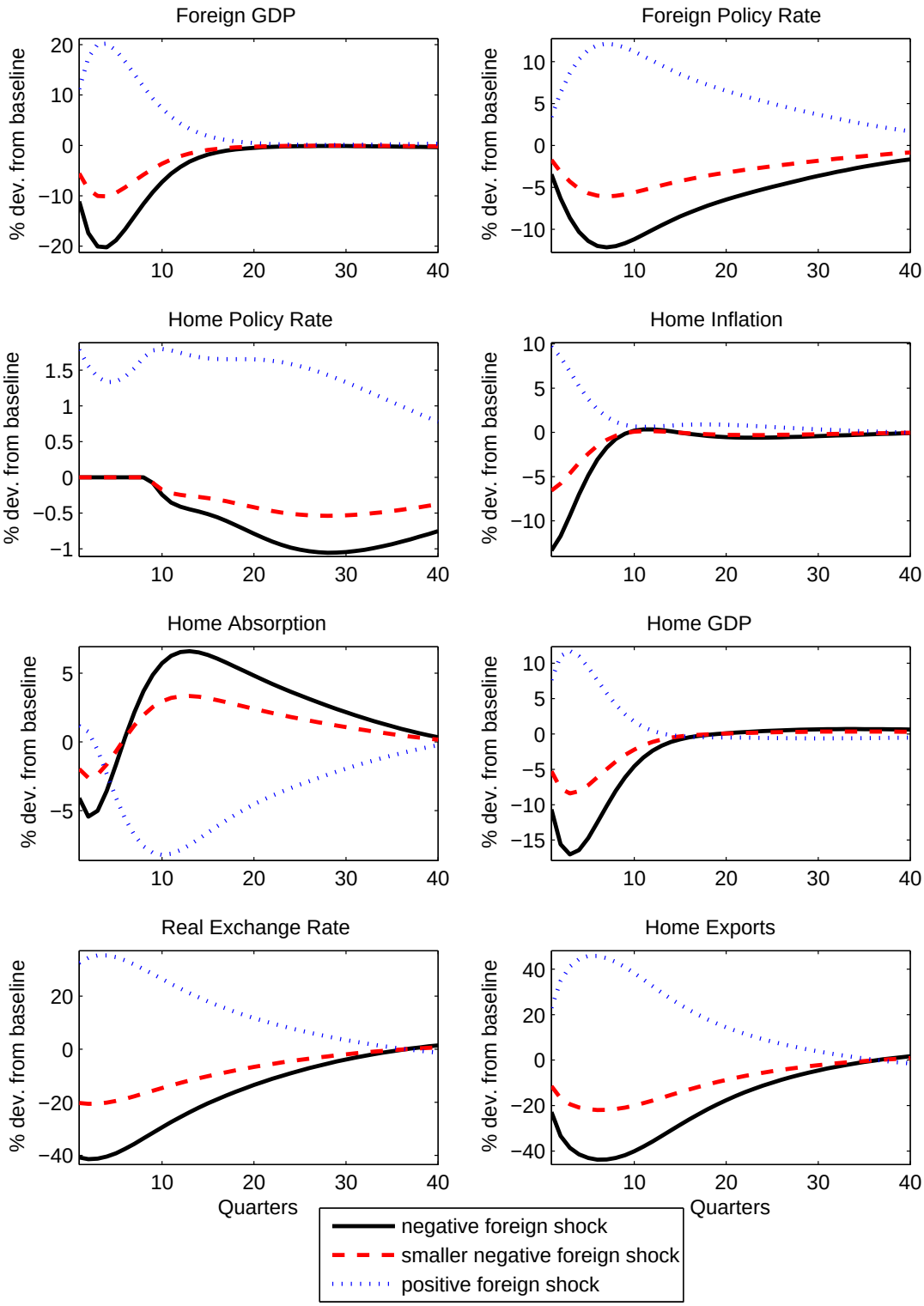

Figure 5: Zero Lower Bound Binds at Home and Abroad

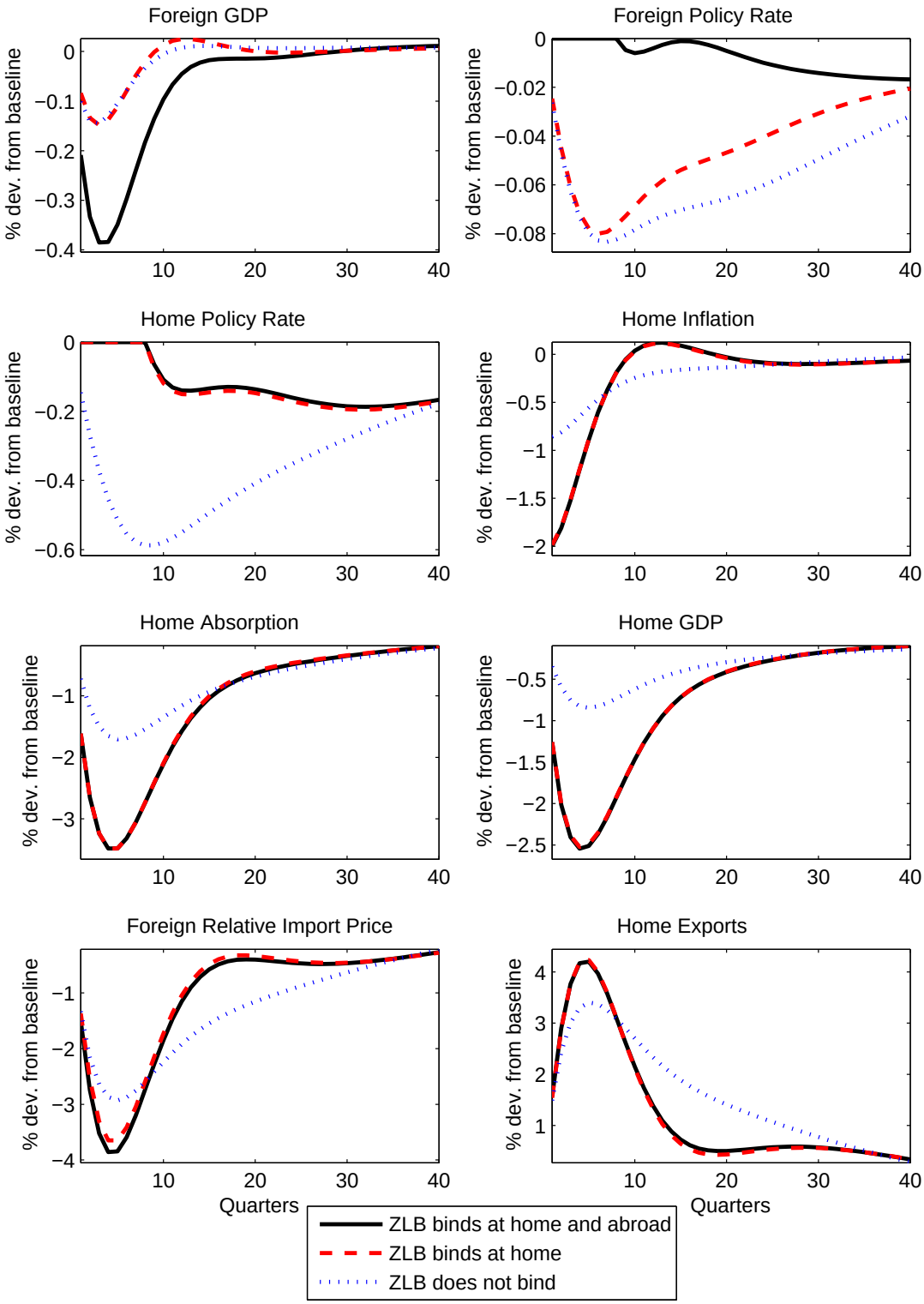

Figure 6: Sensitivity to Degree of Inflation Indexation

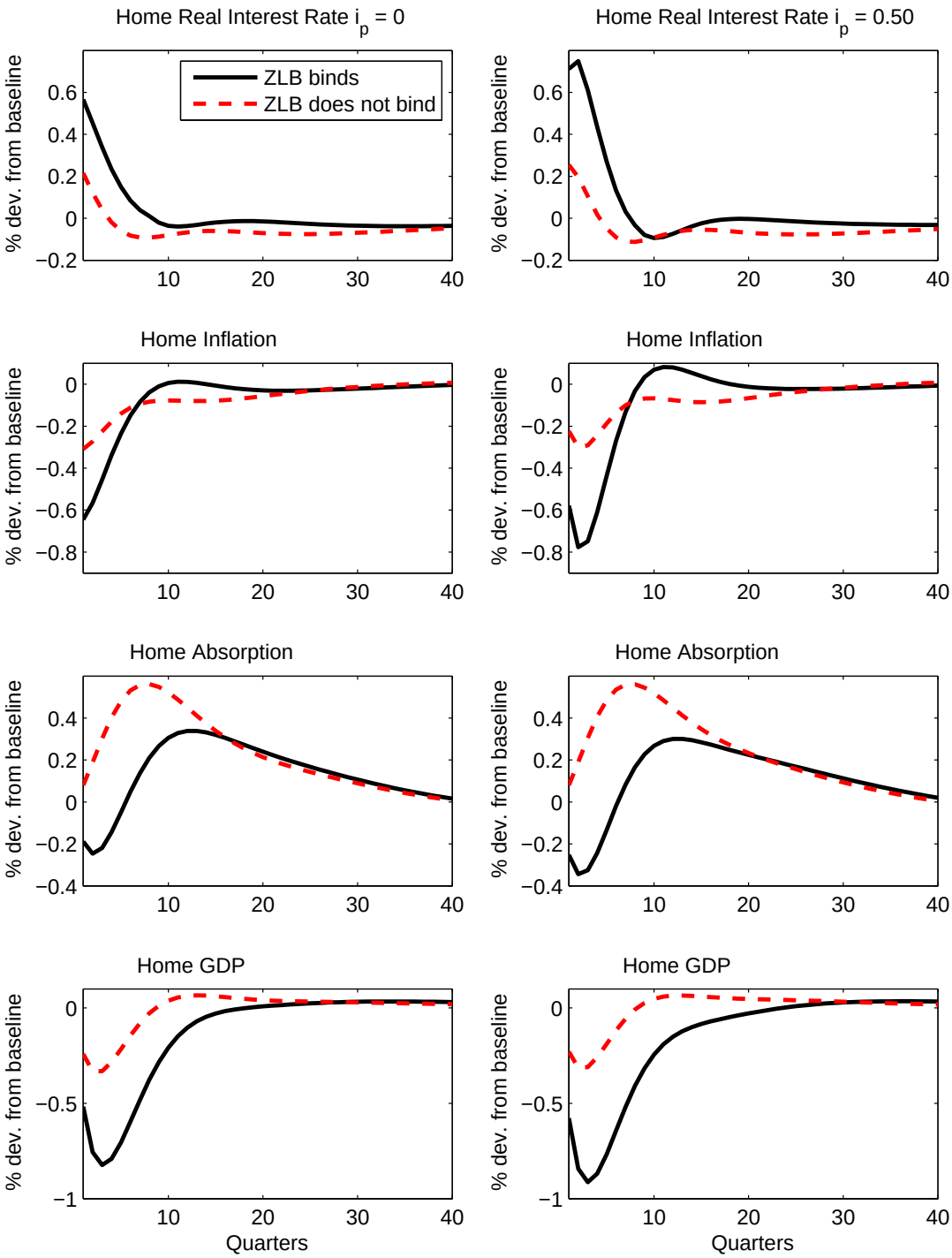

Figure 7: Different Interest Sensitivity

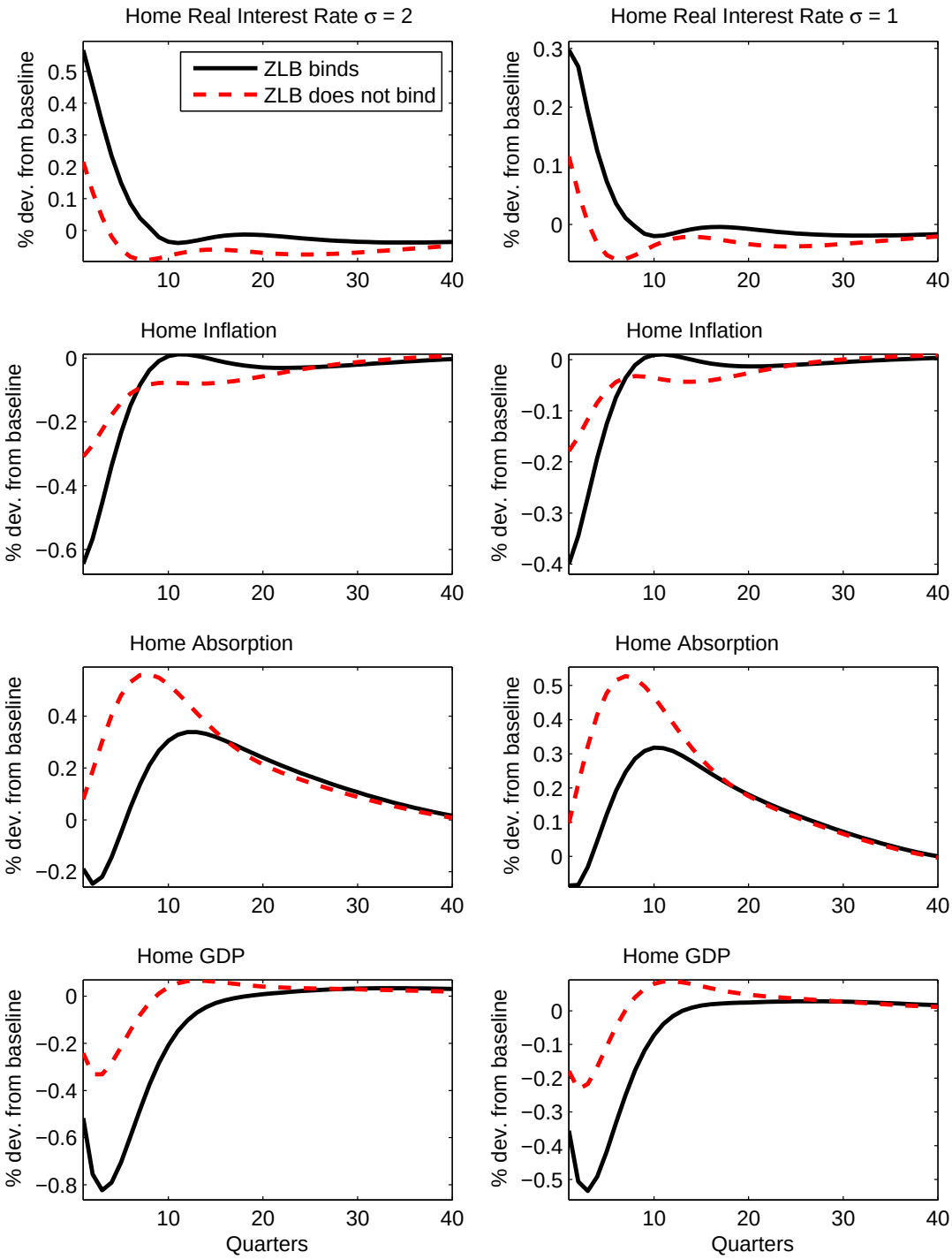

Figure 8: Foreign Government Spending when Home Country is at Zero Lower Bound

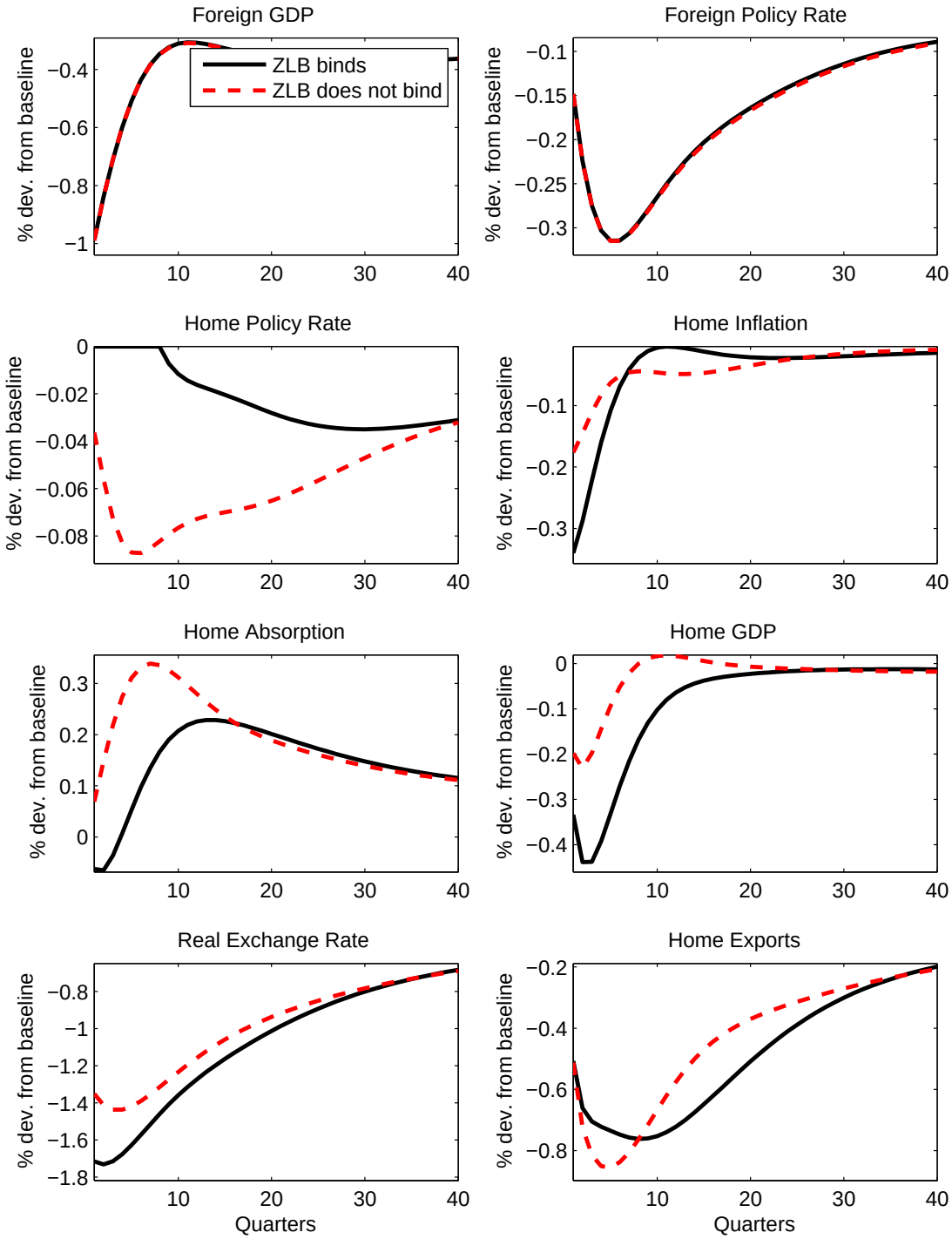

Figure 9: An Increase in the Capital Tax Rate Abroad when Home Country is at Zero Lower Bound