Credit Constraints, Firms' Precautionary Investment, and the Business Cycle*

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Abstract

This paper studies the macroeconomic implications of firms' precautionary investment behavior in response to expected future credit constraints. Firms increase their demand for liquid, safe and short-run investments in anticipation of future borrowing constraints, and this is shown to be at the source of a powerful amplification channel of macroeconomic shocks. This mechanism increases our understanding of the "financial accelerator" by focusing not only on the investment *capacity* of firms, but also on the effects of the expectation of future financial frictions on the *willingness* to invest and on the preference for the *type* of investment. I also show in a calibrated model that this mechanism is quantitatively significant, and that it accounts for the observed business cycle patterns of the aggregate and firm-level composition of investment, which are at odds with the existing models studying the macroeconomic implications of financial frictions, in which the expectation of future credit constraints does not influence firms' current actions directly.

Keywords: Investment, Financial Frictions, Business Cycles, Idiosyncratic Production Risk

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1 Introduction

There is a large body of research on the role of financial frictions in amplifying business cycles and monetary policy shocks. Most of this work is focused on studying how firms' investment *capacity* is affected by tighter borrowing constraints in recessions or following a tightening of monetary policy, either directly through a balance sheet channel (Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999)) or indirectly through a decreased supply of intermediated finance (Holmstrom and Tirole (1997), Bolton and Freixas (2003), Van den Heuvel (2007)). In either case, all of these theories describe how firms are *constrained in the amount* they can invest following a shock.

There has been little focus in the literature however on an amplification and propagation mechanism that studies how changes in the likelihood of being credit constrained in the future may be affecting firms' *willingness* to invest and firms' *preference* for the *type* of investment they carry out. In short, firms that anticipate being credit constrained in the future may wish to retain more liquid balance sheets by investing less or investing differently.

Empirical evidence suggests indeed that firms' precautionary behavior in anticipation of future expected financial constraints is a key determinant of their financial and investment decisions. Recent surveys by Graham and Harvey (2001) and Bancel and Mittoo (2002) find that CFOs consider financial flexibility (having enough internal funds to avoid having to fore-go positive Net Present Value projects in the future) to be the primary determinant of their policy decisions. Almeida, Campello and Weisbach (2004) report that the expectation of future financing problems significantly affects firms' investment policies, and Caggesse and Cunat (2007) find that it significantly affects hiring decisions.

The relevance of this approach is further enhanced by two observations. On the one hand, despite the fact that a small fraction of agents are observed to be financially constrained at any given point in time, a much larger fraction may anticipate the possibility of being constrained in the future. The importance of this distinction between the effect of the *anticipation* of constraints and the *contemporaneous* effect of constraints has already been pointed out in studies of the buffer stock behavior of consumers. On the other hand, the subset of firms that suffer most from financial constraints and hence for which these considerations are relevant (small and privately-owned firms) is a very sizeable portion of economic activity, and in the US accounts for about one half of private-sector GDP and employment. [1]

As is suggested in the empirical evidence mentioned above, firms may insulate them-

[1] Data from the U.S. Small Business Administration Report 2003 show that non-farm businesses with less than 500 employees account for about half of private-sector GDP, employ more than half of private-sector labour, and over 1992-2002 generated between 60-80% of net new jobs annually.

selves from potential future credit rationing by adjusting their capital structure, their cash holdings, the characteristics of employment contracts or their investment strategies. This paper focuses on the effects on investment decisions.[2] A number of questions arise. Can a mechanism capturing this precautionary element in firms' behavior have significant effects on aggregate investment and output dynamics? Can it account for the behavior of the composition of real investment across the business cycle, which current models studying the macroeconomic implications of agency costs cannot account for? Are frictions preventing optimal risk and liquidity management by firms a powerful amplification mechanism of macroeconomic shocks?

These questions are dealt with by analyzing a dynamic stochastic general equilibrium model of a production economy subject to aggregate and idiosyncratic uncertainty. In the model, entrepreneurial firms in the investment good-producing sector have access to a highly profitable technology that is subject to liquidity risk. They also have access to safe but low-return alternative investment opportunity. Their wealth is limited, and they enter into state contingent contracts with financial intermediaries, which resemble a combination of standard loans and credit lines.[3] Entrepreneurs are subject to limited commitment and collateral constraints, and this will limit the extent to which financial intermediaries can spread the idiosyncratic risk faced by entrepreneurs.

In this paper I *first* describe theoretically the mechanism for the proposed *precautionary channel* of amplification of macroeconomic shocks. Entrepreneurs need to collateralize their borrowing using their fixed capital. If a negative aggregate productivity shock hits the economy, fire sales of capital will cause valuations to drop, and this decreases the pledgeability of entrepreneurial returns. Given the persistence of aggregate shocks, firms anticipate being less able to rely on asset liquidations or spot borrowing to deal with any possible future idiosyncratic liquidity shocks, and shift the composition of their investment towards less volatile and more liquid, but less profitable, activities. This amplifies the effect of the initial shock.

Secondly, I show in a calibrated model that this mechanism is quantitatively significant. Furthermore, I find that the amplification mechanism has two features which match observed business cycle regularities. On the one hand, it is highly asymmetric, delivering

[2]With regards to financial policies, I take the approach that firms have a limited ability to use their capital structure to gain financial flexibility due to financial constraints. For example, a large fraction of firms do not have the flexibility to switch between debt and equity, or the ability to issue commercial paper. In any case, in the model firms borrow using state contingent contracts subject to collateral constraints, and decide optimally the extent to which they want to hedge using that contract or by adjusting their investment decisions.

[3]Small and medium sized enterprises rely overwhelmingly on financial intermediaries rather than financial markets for their financing and risk management activities (Cantillo and Wright (2000), Faulkender (2003), Petersen and Rajan (1994)) and do so mostly using loan commitment facilities (Kashyap et al. (2002) document that 70% of bank lending to U.S. small firms is done on a loan commitment basis).

short and sharp recessions, and prolonged moderate boom periods. The extent to which the amplification mechanism is symmetric or not depends on how positive and negative technology shocks affect the trade-off between current and future marginal rates of return on different types of investment differently, and on the extent to which credit constraints are more likely in recessions. On the other, this channel requires relatively smaller negative technology shocks to generate recessions than it does positive shocks to generate booms. In the extreme, shocks to the volatility of the stochastic productivity process can generate downturns without any change to fundamental technology parameters.

The *third* main result is that this model is able to account for the business cycle patterns of aggregate and firm-level composition of investment. This is in line with evidence presented in a number of recent empirical papers. Aghion, et al. (2007) find using a firm-level data-set that while the share of R&D investment over total investment is countercyclical for firms that do not face credit constraints, it becomes pro-cyclical for credit constrained firms. Furthermore, this is only observed in downturns, when the share of R&D for these firms falls drastically. Almeida, Campello and Weisbach (2004) find on the other hand that financially constrained firms' cash flow sensitivity of cash increases significantly in recessions, while it is unchanged for unconstrained firms. Aghion, et al. (2005) give evidence using data on the aggregate composition of investment of a panel of countries that the share of structural (long-term) investment over total investment decreases following shocks that can be expected to make firms more likely to be credit constrained in the near future, and also document that this effect is stronger for less financially developed economies. They find, importantly, that the effect of financial development on the strength of the financial accelerator does not act through a mechanism that alters the amount of investment, but rather the composition, something which is at odds with the main prediction in existing macro models of credit frictions, in which the effects of the expectations of future potential financial constraints are ignored.

These observations are at odds with the existing models of macroeconomic implications of agency costs in which expectations of future constraints do not affect firms' current actions. In my model, however, a worsening of expected credit conditions causes the composition of investment to shift to safer but lower return technologies (contrary to the Schumpeterian idea of "cleansing" recessions). Also, composition shifts to activities with a higher degree of asset tangibility, and towards activities that use more liquid collateral and collateral whose value is less pro-cyclical. Absent alternative safer investment technologies, firms increase their investment in liquid, marketable securities and cash.

The *fourth* result concerns the role of credit frictions in financial intermediaries, whose ability to satisfy firms' liquidity demand may itself be subject to similar countercyclical constraints as non-financial firms, creating the potential for feedback effects between firms' investment decisions and intermediaries' balance sheet conditions. Following a negative aggregate shock, firms increase their demand for ex-ante protection by financial intermediaries through credit lines. Intermediaries, however, are also subject to limited commitment and collateral constraints, and need to back their loan commitments using the loans extended to entrepreneurs as collateral. Intermediaries' ability to provide these loan commitments may decrease both due to lower valuation of existing loans, and lower demand for loans. This introduces a premium on liquidity services by banks, and forces firms to rely even more on operational hedging by adjusting the riskiness of their production technologies, reinforcing the initial effect. This further depresses the valuation of capital, and in turn the valuation of the loan portfolio of banks, further limiting their liquidity commitment capacity. A feedback effect from entrepreneurial investment composition choices to asset prices, loan portfolio valuations and financial intermediaries' liquidity provision capacity arises.

Relationship with the Literature

This paper is closely related to the strand of literature studying the macroeconomic implications of endogenous borrowing constraints for firms, such as Bernanke and Gertler (1989), Holmstrom and Tirole (1997), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Bolton and Freixas (2003), Krishnamurthy (2003), Rampini (2004) and Van den Heuvel (2007). The majority of the papers in this literature does not study issues of risk-sharing and insurance, and instead focus mainly on how credit frictions affect the ability of firms to invest.[4] Krishnamurthy (2003) and Rampini (2004) are an exception however. Krishnamurthy (2003) studies how introducing state-contingent claims eliminates the Kiyotaki and Moore (1997) mechanism, and shows that an aggregate constraint on the capacity of the economy to provide such insurance against aggregate shocks reinstates the mechanism, only that the constraint is on the side of the suppliers of finance. I extend that analysis along three key dimensions. Firstly, Krishnamurthy (2003) does not study the ex-ante effects of limited insurance capacity on the optimal investment choice of firms, which is the key element of the new mechanism I introduce in this paper. Secondly, I extend the model to a fully dynamic setup. Finally, I integrate the analysis in a fully general equilibrium model to be able to assess quantitatively the importance of this channel. A paper closely related in spirit is Rampini (2004), in which a model is introduced that delivers pro-cyclical entrepreneurial activity and amplification of technology shocks. The main difference with my paper is that his mechanism relies on entrepreneurs' risk aversion as the only motive for risk management, while in my setup demand for insurance is production-related.

There is another strand of literature that studies the macroeconomic impact of unin-

[4] Stochastic models in this literature abstract from issues of risk management by making certain modelling choices that make risk irrelevant for entrepreneurs, such as assuming risk neutrality, linear production technologies, or permanently binding credit constraints.

surable idiosyncratic labor-income risk (Huggett (1993), Aiyagari (1994), Krusell and Smith (1998)) or uninsurable investment risk (Acemoglu and Zilibotti (1997), Angeletos and Calvet (2005, 2006), Covas (2006)) in the neoclassical growth model, to analyze issues related to capital accumulation, equilibrium real interest rates and output growth rates. They do not study however if and how market incompleteness varies across the cycle, and how this endogeneity of the risk-sharing opportunities affects cyclical fluctuations.

Regarding the corporate finance literature, a number of theoretical papers have identified the different sources of firms' insurance demand. One such motive is that if firms face costs of raising external finance, or indeed the prospect of being credit rationed, they may find it optimal to hedge against low cash-flow realizations to avoid having to fore-go positive NPV projects, a motive studied formally in Froot, Scharfstein and Stein (1993). Another important source is the risk-aversion of entrepreneurs who, for incentive reasons, have most of their personal wealth invested in the venture they manage, and who also hold a controlling stake in that venture (Stulz (1984)).[5]

The remainder of the paper is organized as follows. Section 2 studies in detail the problem faced by entrepreneurial agents and financial intermediaries in a partial equilibrium set-up. Section 3 embeds this analysis in a fully general equilibrium dynamic stochastic model. The steady state of the model, and the calibration, are discussed in section 4. Section 5 presents the main results of the model. The main model is extended to study the role of financial intermediaries' constraints in section 6. Section 7 presents empirical evidence. Finally, section 8 concludes.

2 Partial Equilibrium Analysis of Entrepreneurs and Financial Intermediaries

In this section I focus on the partial equilibrium analysis of entrepreneurs and financial intermediaries, and in the following section I embed this partial equilibrium setup in a general equilibrium framework. For clarity, I begin here by making a brief description of the whole economy in which the entrepreneurial and intermediary sectors will be embedded. An explanatory chart to aid in understanding the interrelationships in the model economy is in figure (1).

Consider an infinite horizon, discrete-time economy, populated by *four types of agents*: households (measure $1 - \eta$), entrepreneurs (measure η), firms (measure 1) and banks

[5] Other motives have also been pointed out in the literature, such as hedging as a way to avoid non-linear costs of financial distress (Greenwald and Stiglitz (1993), Smith and Stulz (1985)), to resolve conflicts of interest between bond-holders and equity-holders, or between managers and providers of finance, and hedging to avoid tax non-linearities (Smith and Stulz (1985)).

(measure 1), where within each type there is a continuum of agents. There are *three types of goods*: consumption goods, investment goods, and entrepreneurial capital ("capital" from now on). Entrepreneurs produce the investment good using capital, and are subject to agency problems when seeking external finance. They are financed using their own net worth and external funds from households through financial intermediaries. Firms produce the consumption good using labour (from households and entrepreneurs) and the investment good, and are not subject to any agency problems. The model uses consumption goods as the numeraire.

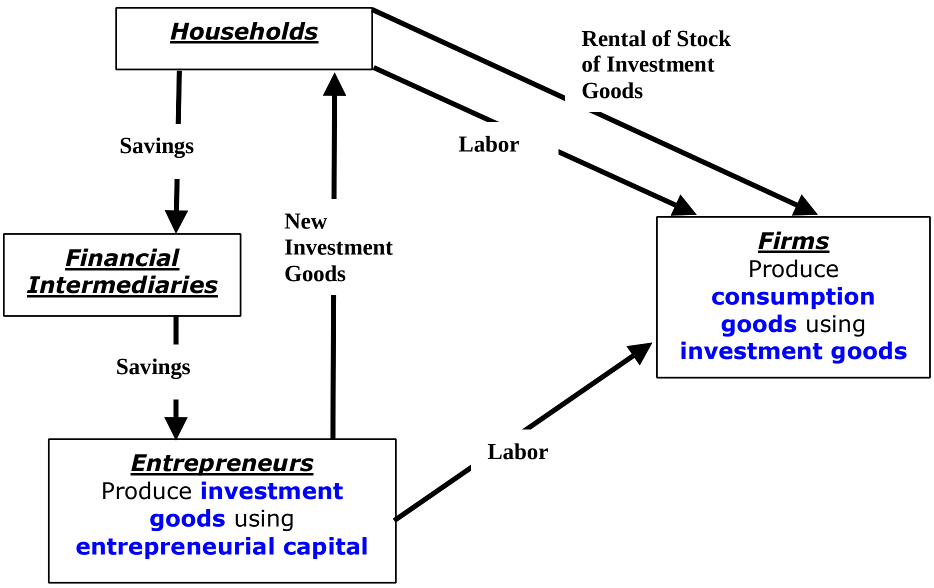

Figure 1: The Economy - Agents and their Economic Relationships

Now I turn to analyze the entrepreneurs' and intermediaries' problem in detail.

2.1 Entrepreneurs

Entrepreneurs are risk-neutral and live for two full periods (they are born in period t, and die at the *beginning* of period $t+2$), and face an investment opportunity at the beginning of their young period and at the beginning of their old period.[6]

[6] Risk neutrality is introduced to highlight that demand for insurance is production-related rather than derived from a particular assumption on entrepreneurial preferences. Also, recent empirical evidence related to the "private equity premium puzzle" suggests that entrepreneurs may be relatively less risk averse. In particular, Gentry and Hubbard (2000) find that there is no significant difference between entrepreneurs' and non-entrepreneurs' financial portfolios (not counting entrepreneurs' private equity), when one might expect entrepreneurs' portfolios to be more conservative to compensate for the riskiness of their private equity.

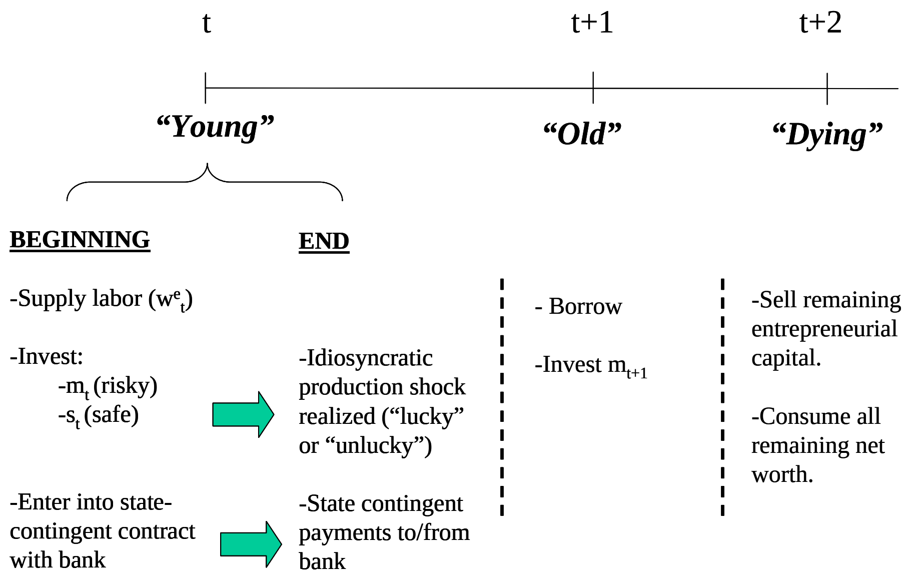

Figure 2: Sequence of Events in the Life-Time of an Entrepreneur

Entrepreneurs are born with a unit endowment of labour, which they supply inelastically at the wage rate w_t^e when young. w_t^e is thus all their net worth when born. Their objective is to maximize consumption at the end of their lifetimes. The timeline for their actions is captured in figure (2).

2.1.1 Technology

Entrepreneurs produce the investment good using a technology that uses entrepreneurial capital, m_t, as an input. Investment by *young* entrepreneurs is risky: it may yield very profitable early results, or it may generate additional liquidity needs. Specifically, investing an amount m_t at the beginning of period t produces $y_t^e = \omega m_t$ units of the investment good (which will be sold on to firms at price q_t) at the end of period t with probability 1/2, where $\omega > 1$ and is large. However, with probability 1/2, production is not successful and instead requires an additional injection of resources of xm_t consumption goods, where $x < 0$. Lack of payment of that amount means the entrepreneur cannot operate his technology in the second period.

Investment of m_t by an *old* entrepreneur (i.e. an entrepreneur at the beginning of his second period of life) yields $y_t^e = Af(m_t)$ with certainty at the end of period t, where $Af(m_t)$ is such that $A > 1$, $f\prime(\cdot) > 0$ and $f''(\cdot) < 0$.

2.1.2 Finance and Insurance: within-period state-contingent financial contract with financial intermediaries

Young entrepreneurs can enter into state-contingent financial contracts with intermediaries.[7] Financial intermediaries offer *within-period contracts* that specify an amount to be paid to/from the entrepreneur at the beginning of the period (before the liquidity shock), and an amount at the end of the period, contingent on the idiosyncratic production shock realization. Financial intermediaries hold liquidity during the period, not from one period to another, and hence only offer within-period contracts. For this reason, *state-contingency* refers to idiosyncratic risk only, as there is no aggregate risk within a period. Financial intermediaries will be studied in more detail in the next subsection.

I will denote by i_t (denoting "insurance") the units of the consumption good that the financial intermediary commits to pay the young entrepreneur should he suffer a liquidity shock in his first period production. We will denote by b_t ("borrowing") the units of the consumption good the entrepreneur commits to repay the intermediary should his production result in success.

The budget constraint for a young entrepreneur at the beginning of period t is hence:

$$p_t m_t + \phi_t i_t - \frac{b_t}{2} = w_t^e \tag{1}$$

where p_t is the price of a unit of entrepreneurial capital, $\phi_t \geq 1/2$ is the price of insurance, and $1/2$ is the price of debt b_t. It is important to note the different pricing dynamics of debt and insurance. The cost of borrowing is zero, given that repayment is intra-period and intermediaries have no alternative use for those funds, so the price is the actuarially fair value of $1/2$. The price of insurance ϕ_t may be larger than its actuarially fair price of $1/2$ if there are any constraints in the supply of insurance. This will become clearer when we analyze financial intermediaries.

Following the realization of the first period production, a lucky young entrepreneur is left with $z_t^l = q_t \omega m_t - b_t$ units of the consumption good, and $(1-\delta)m_t$ units of (depreciated) entrepreneurial capital. An *unlucky* young entrepreneur reaches the end of period t with $z_t^u = x m_t + i_t$ units of the consumption good (where z_t^u can be positive or negative) and $(1-\delta)m_t$ units of (depreciated) entrepreneurial capital.

At the beginning of the second period, a lucky entrepreneur has a net worth equal to

$$n_{t+1}^l = z_t^l + p_{t+1}(1-\delta)m_t = q_t \omega m_t - b_t + p_{t+1}(1-\delta)m_t. \tag{2}$$

[7] Borrowing by old entrepreneurs is not state-contingent given that production in the second period is not subject to idiosyncratic uncertainty: investing m_{t+1} delivers $y_{t+1}^e = Af(m_{t+1})$ with certainty.

And an unlucky entrepreneur has a net worth equal to

$$n^u_{t+1} = z^u_t + p_{t+1}(1-\delta)m_t = xm_t + i_t + p_{t+1}(1-\delta)m_t. \tag{3}$$

Entrepreneurs in their second period (old entrepreneurs), lucky and unlucky alike, invest in the entrepreneurial technology, an amount m^i_{t+1} (for $i = l, u$) given by:

$$p_{t+1}m^i_{t+1} = n^i_t + b_{t+1}. \tag{4}$$

Borrowing by old entrepreneurs, b_{t+1} in (4), is not state-contingent given that production in the second period is not subject to idiosyncratic uncertainty: investing m_{t+1} delivers $y^e_{t+1} = Af(m_{t+1})$ with certainty.

2.1.3 Financial Constraints

As was mentioned above, borrowing by entrepreneurs is subject to limited commitment and collateral constraints. In particular, entrepreneurs can only borrow up to a fraction θ of the expected discounted value next period of their entrepreneurial capital, or

$$b_t \leq \theta E_t\left(\frac{p_{t+1}}{1+r_{t+1}}\right)(1-\delta)m_t. \tag{5}$$

The reason that the constraint values capital at next period's price is that intermediaries, should they wish to liquidate entrepreneurial capital following an event of default, would need to wait until the next period to be able to sell it. In addition, this carries an opportunity cost of $(1+r_{t+1})$, where r_{t+1} is the equilibrium rate of return in this economy and will be defined in the next section.

2.1.4 Optimal Solution

I now solve the entrepreneur's individual problem. I focus on an equilibrium solution in which entrepreneurial returns and the strictness of borrowing constraints are such that borrowing constraints are *always* binding for entrepreneurs. The necessary conditions for this to be satisfied for young entrepreneurs are:

Assumption 1 *Entrepreneurial returns are high enough and θ is tight enough, such that young entrepreneurs' borrowing constraint against their high-return idiosyncratic state is always binding. A sufficient condition for this assumption to hold is that*

$$\theta < \frac{\omega + |x|}{2(1-\delta)}. \tag{6}$$

Entrepreneurs maximize expected consumption at the beginning of their third period (they die immediately after liquidating their remaining capital in their third period of life). The choice variables are m_t (investment in the entrepreneurial activity) and i_t (insurance). The optimality condition for this choice is:

$$E_t\left\{R^L_{m,t+1}\left[\frac{q_t\omega+(1-\delta)p_{t+1}-\theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}{p_t-\frac{\theta}{2}(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}\right]\right\}+$$

$$E_t\left\{R^U_{m,t+1}\left[\frac{x+p_{t+1}(1-\delta)}{p_t-\frac{\theta}{2}(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}\right]\right\} \tag{7}$$

$$= \quad E_t\left\{R^U_{m,t+1}\left(\frac{1}{\phi_t}\right)\right\},$$

where $R_{m,t+1}$ is the return to a unit of the consumption good invested in the risky technology in the second period, and is given by:

$$R^i_{m,t+1}=\frac{q_{t+1}f'(m^i_{t+1})+(1-\delta)p_{t+2}-\theta(1-\delta)E_{t+1}\left(\frac{p_{t+2}}{1+r_{t+2}}\right)}{p_{t+1}-\theta(1-\delta)E_{t+1}\left(\frac{p_{t+2}}{1+r_{t+2}}\right)} \quad \text{where } i=\{L,U\}. \tag{8}$$

The expectations in (7) are taken over next period's realization of the aggregate productivity shock (which is continuous). The idiosyncratic state can be either lucky or unlucky.

The interpretation of (7) is very intuitive. The left hand side of the expression captures the effect of investing one additional unit in the risky entrepreneurial activity in the first period. The first term captures what happens if the investment turns out to be successful at the end of this first period. It yields in the margin an amount $q_t\omega$, and the state-contingent debt contract becomes payable, which equals an amount $\theta(1-\delta)E_t\left(\frac{p_{t+1}}{1+r_{t+1}}\right)$ per unit of capital invested. Finally the entrepreneur still holds the capital, which per unit is valued at $(1-\delta)E_t(p_{t+1})$. These funds are reinvested in the second period at a marginal rate of $E_t(R_{m,t+1}|i=lucky)$, after being leveraged at the rate of:

$$\frac{1}{p_{t+1}-\theta(1-\delta)E_{t+1}\left(\frac{p_{t+2}}{1+r_{t+2}}\right)}. \tag{9}$$

However, if the investment turns out to be unsuccessful (the second term), then it yields a negative amount x per unit of capital, but again means that the entrepreneur keeps an amount of depreciated capital for the following period.

The right hand side captures the returns to not investing one additional unit of capital, but instead purchasing insurance with the amount saved. The amount saved is the downpayment required to purchase one unit of entrepreneurial capital in period t, or $p_t - 0.5\theta(1-\delta)E_t\left(\frac{p_{t+1}}{1+r_{t+1}}\right)$, and the cost of an insurance security is ϕ_t. This means that should the entrepreneurial activity be unsuccessful, the entrepreneur can claim $\frac{1}{\phi_t}\left[p_t - 0.5\theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}\right]$ units of the consumption good as insurance payment, which it will leverage up the following period according to the multiplier (9) and invest at the marginal rate of $E_t(R_{m,t+1}|i = unlucky)$.

2.1.5 Entrepreneurs' Optimal Reaction to Changes in Current and Expected Credit Conditions

One of the central objects of study in this paper relates to entrepreneurs' optimal reaction to variations in *expected* financial constraints. For clarity of the analysis I am going to assume temporarily that there is no aggregate uncertainty, so the agent knows the exact future path of all prices. In that context, when borrowing capacity next period decreases due to a decrease in p_{t+2} the share of resources invested in the risky technology in period t will change in general. In particular, the share will decrease if:

$$\frac{dR^L_{m,t+1}}{dp_{t+2}}R^L_{m,t} + \frac{dR^U_{m,t+1}}{dp_{t+2}}R^U_{m,t} > 2\frac{dR^U_{m,t+1}}{dp_{t+2}}\frac{1}{\phi_t} \tag{10}$$

where:

$$R^L_{m,t} = \frac{q_t\omega + (1-\delta)p_{t+1} - \theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}{p_t - \frac{\theta}{2}(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}, \tag{11}$$

$$R^U_{m,t} = \frac{x + p_{t+1}(1-\delta)}{p_t - \frac{\theta}{2}(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}, \tag{12}$$

and $R^L_{m,t+1}$ and $R^U_{m,t+1}$ are as in (8) above.

The parameter space that will tend to make condition 10 be satisfied corresponds to a high curvature of $f(m)$, and a small volatility of period 1 returns. Intuitively, a high curvature of the date 2 production function, $f(m)$ increases the incentives to hedge, or the wedge between the marginal product of capital in the unlucky and lucky states. On the other hand, a large difference in the beginning-of-period wealth when entering period 2 means that a negative change in the leverage ratio will affect the lucky state's investment more, given that the multiple is applied over a larger amount of net worth. The result would be unambiguous if credit constraints only bound in the unlucky state.

It is important to note that the share of risky investment would be constant if no motive for insurance was present. This result is at the heart of the mechanism analyzed, and captures in the simplest possible way the idea that the anticipation of future financial constraints has an effect in the current investment behavior of firms. Other papers studying the potential of agency costs to amplify macroeconomic shocks have either assumed that technologies are linear, or that firms face no choice other than to invest fully in their entrepreneurial technology, in such a way that by construction future financial constraints, even if anticipated, do not alter current behavior. In particular, this would be the case if $f(m)$ was linear in this model.

2.2 Financial Intermediaries

Financial intermediaries in this model channel savings received from households and lend to entrepreneurs. At the beginning of every period, all of the households' savings are deposited in financial intermediaries, which commit to purchase investment goods from entrepreneurs and return it to households by the end of the period. Financial intermediaries use that liquidity to provide loans and insurance to entrepreneurs.

2.2.1 Contract between a household and a bank

A household provides q_t units of the consumption good at the beginning of period t to the Bank in return for one unit of the investment good at the end of the period. Banks' commitment to deliver on such a promise is assumed to be complete, and this assumption can be rationalized under depositor protection schemes combined with large penalties for defaulting institutions.

2.2.2 Contract between an entrepreneur and a bank

Banks themselves are also subject to collateral constraints:

$$i_t \leq b_t. \tag{13}$$

Expression (13) captures banks' need to collateralize all their obligations (their obligations are the insurance payments to the unlucky entrepreneurs). The only assets they can use to collateralize are the loans they extend to entrepreneurs. I show later that the price of insurance when there is no aggregate shortage in the supply side will be equal to the actuarially fair price ($\phi = 0.5$). If there is an aggregate insurance capacity shortage, then $\phi > 0.5$.

Table 1: Sequence of Events within One Period

1	θ_t, the aggregate productivity shock, is realized.
2	Firms hire labor from households and entrepreneurs and rent capital from households. These inputs are used to produce the consumption good, $Y_t = \theta_t F(K_t, H_t, H_t^e)$
3	Households make their consumption and savings choice. All savings are deposited in financial intermediaries, which commit to purchase capital from entrepreneurs and return it to households by the end of the period.
4	The Financial Intermediaries use the resources obtained from households to provide loans and insurance to entrepreneurs.
5	Entrepreneurs borrow resources from the Intermediaries. Young entrepreneurs decide how much to invest in the risky capital-creation technology and how much in insurance. Old entrepreneurs place all of these resources (along with their entire net worth) into their capital-creation technology. Dying entrepreneurs (in their third period) liquidate their entrepreneurial capital holdings, and consume all of their remaining net worth.
6	The idiosyncratic technology shock of each young entrepreneur is realized. The successful entrepreneurs repay their loans to the Intermediaries, and the unsuccessful ones claim their insurance payments.
7	Intermediaries purchase all of the investment goods from entrepreneurs, and hands them to households. Banks end the period with no liquidity.

3 General Equilibrium

In this section, I embed the entrepreneurial and financial intermediation sectors in a general equilibrium framework. I will start by explaining the choices faced by households and firms, and then discuss how the entrepreneurial sector and the financial intermediaries are introduced into the general equilibrium framework. In order to understand the sequence of events in this economy, Table (1) summarizes what happens within each period.

In what follows, all variables in upper case indicate aggregate quantities.

3.1 Households

There is a continuum of risk-averse households, who maximize expected lifetime utility of consumption, c_t, and leisure, $(1 - L_t)$, taking as given wages w_t, the price of investment goods q_t, and the equilibrium rate of return on the investment goods r_{t+1}:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - L_t). \tag{14}$$

At the beginning of every period households choose their labour supply, and their optimal labor-leisure choice is given by:

$$\frac{u_L(t)}{u_c(t)} = w_t \tag{15}$$

They then choose their optimal consumption. All savings are deposited in financial intermediaries, which commit to purchase investment goods from entrepreneurs and return them to households by the end of the period. These investment goods are then rented to firms, which use it for production the following period and pay in return an interest rate of $1 + r_{t+1}$ (which is stochastic and depends on the realization of θ_{t+1}). The optimal savings and consumption choice is given by:

$$u_c(t) = \beta E_t \left[u_c(t+1) \frac{q_{t+1}(1-\delta) + (1 + r_{t+1})}{q_t} \right] \tag{16}$$

where $u_c(t)$ is the marginal utility of consumption in period t.

3.2 Firms

Firms produce the consumption good using a constant returns to scale production function:

$$Y_t = \theta_t F(K_t, H_t, H_t^e) \tag{17}$$

where K_t is the stock of investment goods, H_t is aggregate labour supplied by households, and $H_t^e = H^e$ is labour supplied by entrepreneurial agents (which is constant).

Perfect competition in the factor markets implies the following factor prices:

$$r_t = \theta_t F_1(t) - 1 \tag{18}$$

$$w_t = \theta_t F_2(t) \tag{19}$$

$$w_t^e = \theta_t F_3(t) \tag{20}$$

3.3 Market Clearing Conditions

There are seven markets that need to clear in this economy: the markets for entrepreneurial capital, investment goods, insurance, consumption goods, entrepreneurial credit, entrepreneurial labour, and household labour.

3.3.1 Entrepreneurial Capital Market

Entrepreneurial capital can be created one-for-one using consumption goods with an instantaneous technology. This imposes an upper bound on the price of entrepreneurial capital in terms of consumption goods at one. However, certain entrepreneurs are sellers in the secondary capital market, in particular those dying and exiting the economy, and those in financial distress that need to liquidate some assets, and this opens the possibility of the capital price p_t being driven below one.

There are two dimensions of heterogeneity in the entrepreneurial sector: age and, for the old entrepreneurs, having been hit by the idiosyncratic liquidity shock in their first period or not. That means that at any given point in time there are five types of entrepreneurs: the young (indexed by superscript Y), the lucky old (indexed by superscript L), the unlucky old (indexed by superscript U), and the dying (who may have been lucky or unlucky in their young age, and indexed respectively by superscripts DL and DU).

The dying entrepreneurs are always sellers, as they are in their third and last period of life and all they do is liquidate their remaining undepreciated entrepreneurial capital and consume the proceeds of that sale along with the proceeds of the sale of their output to intermediaries in the previous period. Young entrepreneurs, on the other hand, are always buyers of capital, and their demand is given by (7).

The old entrepreneurs (beginning their second period of life) are either sellers or buyers depending on their idiosyncratic state, and on the aggregate state of the economy. Lucky entrepreneurs are always buyers of capital, while unlucky ones may be buyers or sellers. What determines if the unlucky ones need to liquidate capital in order to satisfy their liquidity shock xm_{t-1} are two things: the amount of insurance they bought in the previous period, i_t, and their borrowing capacity this period, $\theta(1-\delta)E_t(p_{t+1}/1+r_{t+1})$. Low aggregate insurance capacity coupled with low ex-post borrowing capacity may result in unlucky entrepreneurs having to fire sell entrepreneurial capital.

Whenever expression:

$$\sum_i \pi_i M_{it}(p_t) \geq \sum_i \pi_i (1-\delta) M_{it-1}, \text{ for } i = Y, L, U, DL, DU \tag{21}$$

holds with equality, p_t may be driven below 1. Otherwise $p_t = 1$. In expression (21), π_i indicates the measure of type i entrepreneur in the entrepreneurial sector (for example $\pi_Y = 1/3$, and $\pi_L = \pi_U = \pi_{OL} = \pi_{OU} = 1/6$).[8]

3.3.2 Insurance Market

Aggregate insurance demand by young entrepreneurs is given by:

$$I_t = \pi_Y \frac{1}{\phi_t}(w_t^e + B_t^Y - p_t M_t^Y). \tag{22}$$

[8] Some examples of evidence of significant price drops in episodes of fire sales can be found in Pulvino (1998) (in a study of commercial aircraft transactions) and in Coval and Stafford (2007) (in an analysis of mutual fund asset sales that demonstrates that these effects may be present even in highly liquid markets).

Insurance supply is provided by the intermediaries. The insurance contract is intraperiod; it commences after the financial intermediaries have received the deposits from households, and it ends before intermediaries use those deposits to purchase investment goods and give them to households. Financial intermediaries hence have at their disposal all of those funds to use to provide both entrepreneurial credit (which is also intraperiod), and entrepreneurial insurance. Hence intermediaries have ample resources to provide both, and supply is not limited by the liquidity available to banks.

Financial intermediaries, however, are also subject to limited commitment and collateral constraints. The only asset that can be used as collateral in this economy is capital. Banks only hold claims to any capital *indirectly* through the loans they extend to entrepreneurs, as these loans are themselves subject to collateral constraints. Hence banks' collateral capacity is given by the collateral value of the loans they have extended.

The aggregate collateral constraint of intermediaries is then:

$$I_t^{FI} \leq B_t^{FI} , \tag{23}$$

where I_t^{FI} is the aggregate amount of insurance that banks commit to provide at the end of period t, and B_t^{FI} are the total loans that are due for repayment at the end of period t.

When the collateral constraint for insurance supply (23) is not binding, then insurance is priced at the actuarially fair price of $\phi_t = 0.5$ (given that the probability of incurring in the insurable event is 0.5). However, when this constraint binds, insurance will only be sold at a premium and $\phi_t > 0.5$.

3.3.3 Goods, Labour, Investment Goods and Credit Markets

The labour supplied by households is equal to $H_t = (1-\eta)L_t$, while entrepreneurs supply labour inelastically and in the aggregate provide $H_t^e = \eta$.

The aggregate resource constraint (goods market equilibrium) in terms of expenditures is given by:

$$\begin{aligned} Y_t \;=\; & (1-\eta)C_t + \eta C_t^e + \eta \sum_{i=Y,L,U} \pi^i \left[M_t^i - (1-\delta) M_{t-1}^i \right] + \\ & \sum_{i=L,U,DL,DU} \pi^i \left[Z_t^i - (1-\delta) Z_{t-1}^i \right] + \eta \pi^Y 0.5 x M_{t-1}^Y, \end{aligned} \tag{24}$$

The first two terms in (24) capture aggregate consumption in this economy, by both households (C_t) and entrepreneurs (C_t^e). The third term captures additional investment in the aggregate stock of entrepreneurial capital, while the fourth term deals with the variation in aggregate savings of entrepreneurs (Z_t^i are the amount of consumption goods entrepreneurs of group i carry over from period t to period $t+1$).[9] The last term of the expression reflects the aggregate reinvestment costs of entrepreneurs who suffered the negative idiosyncratic shock the previous period.

The market for investment goods used by consumption goods producing firms clears at the price of q_t, according to the expression:

$$K_{t+1} = (1-\delta)K_t + \frac{Y_t^e}{q_t}.$$

Entrepreneurial credit clears at the interest rate of zero, given that financial intermediaries have no alternative use for the funds during the duration of the contract (which is intra-period). The supply of credit by intermediaries is captured by B^{FI}, and the total amount of loans demanded by entrepreneurs is $\sum_i \pi_i B_t^i$, where π_i is the measure of each of the five types of entrepreneurs and B_t^i are the aggregate amounts of credit demanded by each type. Equilibrium in the credit market requires that:

$$B^{FI} = \sum_i \pi_i B_t^i$$

3.4 Recursive Equilibrium Conditions

The recursive competitive equilibrium is defined by decision rules for K_{t+1}, C_t, H_t, M_{it}^Y, M_{it}^L, M_{it}^U, Z_{it}^L, Z_{it}^U, Z_{it}^{OL}, Z_{it}^{OU}, I_t, C_t^E, B_{it}^Y, B_{it}^L, B_{it}^U, q_t, p_t, and ϕ_t, as a function of K_t, θ_t, $\{M_{i,t-1}\}$ and $\{Z_{it-1}\}$. The appendix provides a detailed explanation of these recursive equilibrium conditions, and of the computational procedure used to solve this model.

4 Calibration and Analysis of Steady State

The model is parameterized at the non-stochastic steady state using values to replicate long-run empirical regularities in U.S. post-World War II macro data. In addition the calibration is designed so the results are comparable with the existing quantitative studies on agency costs and business cycle fluctuations, such as Carlstrom and Fuerst (1997).

The final good production technology is assumed to be Cobb-Douglas of the form

$$Y_t = \theta_t K_t^{\alpha^K} H_t^{\alpha} H_t^{\alpha^e}$$

with a capital share (α^K) of 0.36, a household labour share (α) of 0.63, and an entrepreneurial labour share (α^e) of 0.01. The share of entrepreneurial labour is positive to ensure

[9] Households can only transfer resources from one period to the next by purchasing capital (even if they could use a safe storage technology with no return they would not use it as it would be rate-of-return dominated by investment in k_{t+1}). Entrepreneurs on the other hand can only transfer any resources they have at the end of the period through a safe (zero-return) storage technology.

that young entrepreneurs have positive net worth with probability one. It is chosen to be small so that the model dynamics closely resemble the standard RBC dynamics when the financial frictions in the model are removed. The capital depreciation rate is set to $\delta = 0.02$.

The technology shock, θ_t, follows the process

$$\log \theta_{t+1} = \rho \log \theta_t + \sigma_\varepsilon \varepsilon_{t+1}$$

where $\sigma = .01$ and $\rho = 0.95$, and $\varepsilon_{t+1} \sim N(0, 1)$.

The utility function for households is of the form

$$U = \frac{c^{1-\gamma} - 1}{1-\gamma} + v(1 - L)$$

with v chosen so that the steady-state level of hours is equal to 0.3. The intertemporal preference rate is set at $\beta = 0.99$, and the risk aversion parameter γ is set at 1, but higher values (up to 4) are also tested for robustness.

With regards to the calibration of the entrepreneurial sector parameters, we start by calibrating the pledgeability of entrepreneurial capital (captured by θ) to match empirically documented Loan-to-Value (LTV) ratios for commercial mortgage lending to small and medium-sized enterprises. Titman, Tompaidis, and Tsyplakov (2005) find that the LTV ratios (measured as the loan amount divided by the appraised value of the property) have values between 60% and 80% for over 75% of the loans the study, and an average of 65%.[10] In numerical simulations, the choice of this parameter is shown to be quite important for my results. For that reason I use a conservative choice in my baseline calibration of 70%. The two remaining parameters relate to the entrepreneurial risky technology (the multiplicative productivity factor, and the parameter regulating its curvature and hence the intensity of the demand for risk and liquidity management), and they are calibrated to match two empirical regularities: (1) the risk premium, and (2) the share of loans that are issued on a commitment basis. Regarding the latter, I use the value document by Kashyap et al. (2002), who find that 70% of bank lending by U.S. small firms is through credit lines. Regarding the former, I follow Carlstrom and Fuerst (1997) and use the average spread between the 3-month commercial paper rate and the prime rate (which for the period from April 1971 to June 1996 equals 187 basis points).

The analysis of the steady state of this model yields some interesting results. The steady state is obtained by eliminating the volatility of the aggregate productivity parameter, but preserving the idiosyncratic uncertainty element. The steady state chosen is one in which the aggregate collateral constraint does not bind, and in which the entre-

[10] They use data on 26,000 individual commercial mortgages originated in the U.S. between 1992 and 2002.

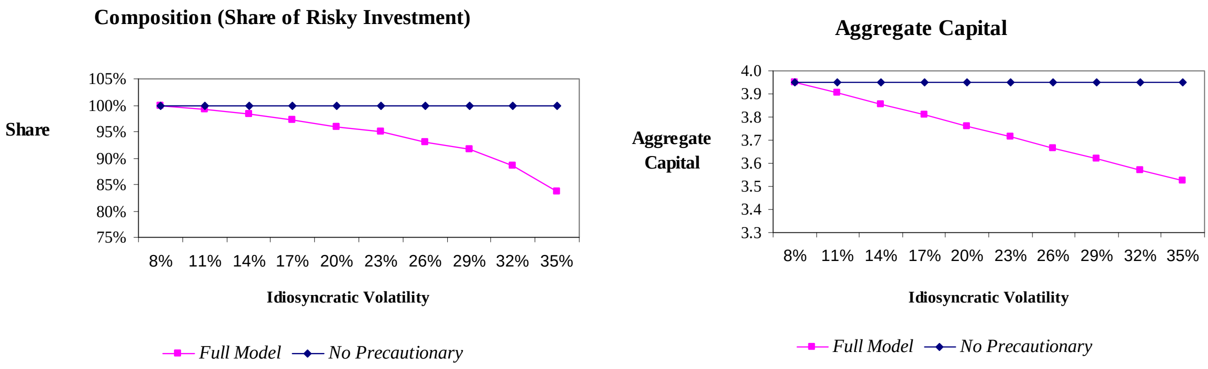

Figure 3: Composition of entrepreneurial investment and aggregate capital in the steady state, as a function of changes in idiosyncratic volatility.

preneurial capital market is such that the price always remains at its fundamental value of $p_t = 1$.

I then conduct an analysis by which I perform a mean-preserving increase in the idiosyncratic volatility parameter. The results are that the steady state composition of entrepreneurial investment shifts to a safer profile with higher volatility, and that the aggregate stock of investment goods in the economy is substantially lower, as is clear from figure (3). This is in contrast to standard models of credit frictions in which the anticipation of *future* financing problems do not affect *current* investment decisions. Those models predict that the share of risky productive investment is not sensitive to idiosyncratic volatility, and I reproduce that result in this model by removing the source of precautionary behavior in firms, as is shown as well in figure (3) in the series labeled "*No Precautionary*".[11]

5 The Precautionary Channel of Amplification

I analyze the dynamics of this model by studying the behavior of different aggregates in response to changes in aggregate productivity, or total factor productivity (TFP). I compare the response of the relevant aggregate variables in three models: a completely standard real business cycle (RBC) framework, a model with borrowing constraints but no precautionary channel, and the full model introduced in the previous section. The purpose of this section is to clarify how the amplification mechanism described in the previous

[11] The transformation of the model into one in which there is no precautionary behavior is straightforward and is done by altering the functional form of the second period entrepreneurial production opportunity to one with constant returns to scale in the only factor, entrepreneurial capital (as opposed to the benchmark setup with decreasing returns to scale in that same factor). This implies that the entrepreneur is no longer concerned with smoothing his net worth at the beginning of the second period and hence has no demand for insurance.

sections works and in particular to highlight what the contribution of this mechanism is with respect to the standard financial accelerator.

I focus the attention on some aggregate quantities and prices that relate to entrepreneurial investment; in particular I will study the dynamic behavior of the composition of entrepreneurs' investment between safe and risky investment, the price q_t of the investment goods produced by entrepreneurs and bought by firms, aggregate investment by consumption good-producing firms, and finally aggregate output. The results are in figure (4).

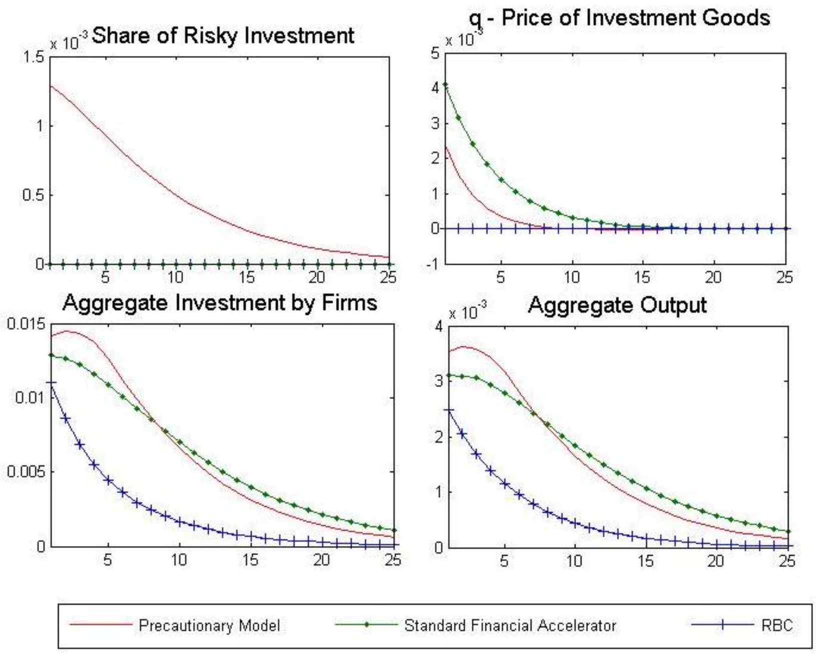

Figure 4: Impulse Resonse to a Positive One Standard Deviation Shock to Productivity (Periods = quarters).

First, the dynamics of the standard RBC model are well known. I obtain these dynamics by eliminating the capital-producing sector (entrepreneurs) and assuming an infinitely elastic supply of capital at the price of unity. The response of investment and output mimics closely the evolution of the underlying technology process. In essence, there is little propagation in this version. The price of capital q_t does not react to changes in technology because of the assumption of infinite elasticity, and there are no compositional effects of investment changes.

Secondly, the dynamics of the standard financial accelerator can be obtained in the current framework through several ways. One is by *not* giving firms an investment choice and assuming that they are permanently credit constrained: firms will simply invest as much as possible in the risky activity every period. Another is by linearizing the second period production function: this way, firms have no incentive to smooth second-period investment, and simply maximize first period investment in the risky activity and do not invest in the safe asset. I adopt the latter approach, without loss of generality. In either case, there is no compositional effect and no precautionary behavior: even if firms anticipate rationally that the severity of credit constraints may increase the following period, this does not affect their current behavior. This is the essence of the contribution of the precautionary mechanism introduced in this paper. The idea is not that firms behave in an irrational way by not reacting to the anticipation of future credit constraints, but that either the are *unable* to react (because they have no discretion as to how to invest or save, or because they always operate in a corner solution because they are assumed to be permanently credit constrained, etc...) or that they are *unwilling* to react (they have no motive to smooth end-of-period wealth, which implies they are risk neutral and that returns to investment the following period are linear in beginning-of-period net worth).

The cyclical dynamics in the standard financial accelerator are well known as well. Financial imperfections may amplify and add persistence to the effects of technology shocks, as is the case in figure (4), but do not affect the composition of investment. Two comments are in order. First, it is worthwhile noting that there is some controversy in the literature as to the extent to which financial imperfections dampen or amplify cycles, and different papers analyze scenarios in which one or the other result obtains. The focus of the results in this paper are on how taking into account firms' precautionary behavior changes the way we should understand the way financial frictions affect aggregate investment dynamics, and in that respect contributes to that discussion. Secondly, there is less controversy in the literature surrounding the persistence effects of financial frictions, at least in terms of the qualitative effects. In the results in this paper the persistence effects are dampened with respect to frameworks in which entrepreneurs are modelled as infinitely lived and hence their net worth (the aggregate level and its distribution) becomes an important state variable that adds substantial persistence. In my model, entrepreneurs live for two periods, and hence the effect of net worth dynamics is significantly smaller. In any event, in the context of my framework it affects both the standard financial accelerator version and the precautionary channel version in the same way, and thus does not affect the comparison of both, which is the object of study.

Finally, if we observe the changes that occur in the aggregate dynamics as a result of considering firms' precautionary behavior, we can notice that they are significant. The main idea of the precautionary channel is that if *future expected borrowing* conditions worsen, then entrepreneurs will adjust the riskiness of their investment portfolio by reducing their exposure to the risky technology. When a negative shock hits, firms understand that the shock will be persistent and that it means that the probability of being financially constrained next period increases. They react by decreasing their share of risky investment. This works both ways, so when a positive shock hits the economy

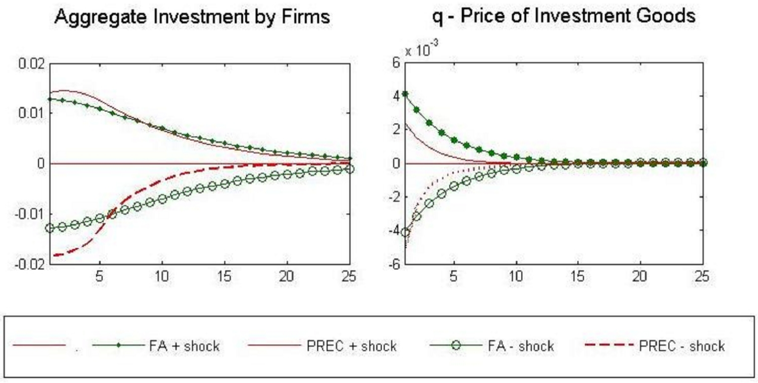

Figure 5: Asymmetry of Effects: Impulse Response Functions to a Positive and a Negative One Standard Deviation Shock to Productivity (Periods = quarters). (FA = Financial Accelerator Model; PREC = Precautionary Model)

and future expected borrowing conditions improve, entrepreneurs increase the riskiness of their investment portfolio. The precautionary model implies a larger contemporaneous response to shocks (more amplification), and smaller persistence. The intuition for this result is that firms anticipate future financial restrictions and react immediately. In the standard financial accelerator framework, in papers such as Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (2000), firms invest as much as they can at every point in time. This adds persistence to their effects; a negative shock may imply that entrepreneurs' net worth decreases slowly to reach its minimum several periods later, with entrepreneurial investment following that pattern. Taking into account a precautionary behavior and the flexibility to adjust the investment portfolio means forward-looking firms may choose to react in advance to that to minimize future credit rationing.

Another important result is the asymmetry of effects, which can be seen in figure (5). Hansen and Prescott (2002) and Sichel (1993) find evidence that positive shocks produce smaller positive output effects than negative shocks produce negative output effects. The existing theory tries to explain this on the basis of capacity constraint models (Hansen and Prescott (2002), Danziger (2003)) and sticky price models (Devereux and Siu (2003)). In my framework, the key element to these results is in the nonlinear dynamics of the price of entrepreneurial capital p_t. Given that consumption goods can be turned into entrepreneurial capital one-for-one, this means $p_t \leq 1$. However, following a negative shock such that there is not enough demand at $p_t = 1$ for the existing stock of entrepreneurial capital (remember dying entrepreneurs liquidate all of their holdings), then $p_t < 1$. This asymmetry in the pricing behavior generates asymmetry in both the

	Data	RBC	Standard Accelerator	Prec ($\rho = 0.5$)	Prec ($\rho = 0.95$)	Extension
$\sigma_Y \ / \ \sigma_{(Tech\ Shock)}$	2.01	1.43	1.86	2.73	2.13	2.94
$\sigma_I \ / \ \sigma_{(Tech\ Shock)}$	11.14	4.84	6.05	7.46	6.48	8.13
σ_q (%)	-	0	2.3	4.6	3.3	5.0

Table 2: Summary of Numerical Results - Comparison of Outcomes (Real U.S. data for 1985-2005 from Alvarez and Jermann (2006))

model with and the model without the precautionary element, but more so in the latter, the reason being that an asset price feedback effect kicks in in the precautionary model: firms do not internalize the future pricing effects of their current actions.

6 Extension - Binding Insurance Capacity

The previous section has assumed that the constraint on banks' ability to supply i_t is not binding; the price of insurance is at the actuarially fair price throughout, or $\phi_t = 0.5$. compromised. However, the ability of financial intermediaries and capital markets to satisfy firms' liquidity demand may itself be subject to similar countercyclical constraints as non-financial firms, creating the potential for feedback effects between firms' investment decisions and intermediaries' balance sheet conditions. As was stressed in the introduction, a main source of risk and liquidity management for firms are financial intermediaries, both using ex-ante protection through credit lines, and ex-post protection by borrowing on the spot market. Several empirical studies have found that loan supply to small firms is curtailed in downturns and following monetary policy shocks (Gertler and Gilchrist (1993)).[12]

In order to analyze this possibility, we introduce an extension to the model. The definition of composition in the model presented in the previous sections distinguishes only between investment in the only risky technology and investment in the insurance security. This set-up is introduced as it is more adequate to describe the mechanism in a starker way. However, in order to study the effects of banks' collateral constraints, it is necessary to extend the model to consider an additional investment opportunity; not doing so would mean that given banks' limited ability to supply the insurance security,

[12] Other empirical studies examining the extent to which there is a bank lending channel of shock transmission are Iacoviello and Minetti (2007) and Den Haan, Sumner, and Yamashiro (2007).

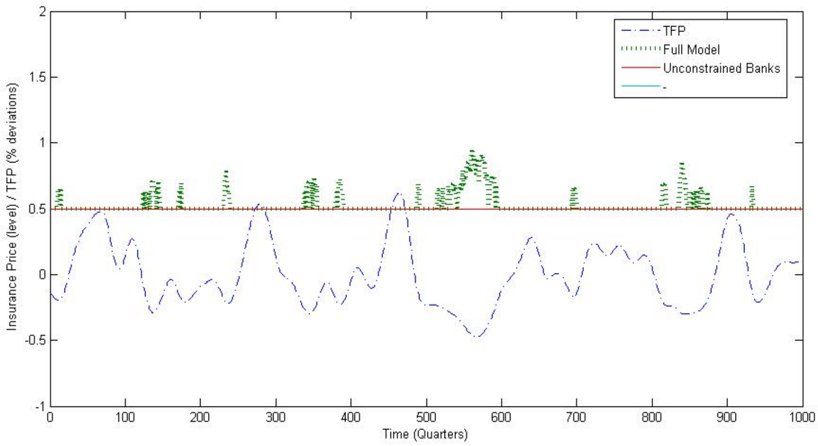

Figure 6: Insurance Price

we would be arbitrarily limiting the investment opportunities of firms and eliminating the option of more alternative low return investment opportunities, or, in the limit, storage. The details of the extension to this model are in the appendix.

The feedback effect through entrepreneurial capital valuations and financial intermediaries' commitment capacity works as follows. Following a negative aggregate shock, firms increase their demand for ex-ante protection by financial intermediaries through credit lines. Intermediaries, however, are also subject to limited commitment and collateral constraints, and need to back their loan commitments using the loans extended to entrepreneurs as collateral. Intermediaries' ability to provide these loan commitments may decrease both due to lower valuation of existing loans, and lower demand for loans. This introduces a premium on liquidity services by banks, and forces firms to rely even more on operational hedging by adjusting the riskiness of their production technologies, reinforcing the initial effect (the insurance price dynamics can be seen in figure (6)). This further depresses the valuation of capital, and in turn the valuation of the loan portfolio of banks, further limiting their liquidity commitment capacity. A feedback effect from entrepreneurial investment composition choices to asset prices, loan portfolio valuations and financial intermediaries' liquidity provision capacity arises.[13]

7 Empirical Evidence

In this section I present evidence that provides support to the predictions of the model analyzed in the previous sections. The predictions refer broadly to ex-ante reactions

[13] A similar relationship between banks' financial state and entrepreneurs' technology choice arises in Minetti (2007), although in that paper the mechanism is based on how banks' balance sheet condition may affect entrepreneurs' incentives to provide effort, rather than on issues of risk and liquidity demand.

by entrepreneurial firms when the expectations about future risk-sharing conditions vary. These reactions may manifest themselves in particular decisions with respect to the choice of production technology along the dimensions of riskiness, length or collateralizability of the capital used, the choice of the share of cash and liquid securities as a share of total assets, and the choice of the level of investment.

Special care has to be taken to distinguish the effects of the specific channel identified in this paper, with the effects of the traditional credit channel. In particular, some of the empirical studies carried out to test the standard credit channel could be picking up the effects of the insurance channel identified in this paper. If firms' investment sensitivity to monetary policy shocks or productivity shocks is higher for small firms with a high degree of agency problems, this could be due to either a lack of ability to borrow to invest (a *corner solution*), or a lack of willingness to carry out such investments as an optimal decision that weighs in the prospect of being credit constrained in the future and not being able to undertake profitable investment opportunities that may arise (an *interior solution*). If banks' loan supply is sensitive to monetary policy shocks or productivity shocks, and small firms with high agency problems are especially bank-dependent, then their investment reaction may be due to an inability to borrow today, or to the expectation that the current credit crunch will persist in time and may result in an inability to borrow in the future to withstand liquidity shocks or undertake investment opportunities. The empirical tests carried out in this section take this observational equivalence into account.

The broad prediction tested is that if risk-sharing conditions worsen in the present, or are expected to worsen in the future, then the asset composition strategies of high agency cost firms should reflect this in a particular way. We need to operationalize both elements of the prediction, the exogenous explicative component, and the endogenous reaction. We do so in a number of ways below, and we divide the analysis into two subsections, one analysing a firm-level panel data set of European firms, and another using aggregate U.S. investment data.

7.1 U.S. Aggregate Investment Data, the Business Cycle and Credit Conditions

In order to distinguish between different types of investment along the riskiness dimension, one strategy is to study the behavior of Research & Development investment as a fraction of total investment. Another strategy is to study the behavior of long-term, structural investment, again as a share of total investment. The U.S. is particularly convenient to study these aspects of investment as there is abundant data on industrial R&D activity, provided by the National Science Foundation.

7.1.1 R&D Investment Behavior Across the Business Cycle

A component of investment which is likely to be very sensitive to liquidity insurance supply conditions is Research & Development spending. Some authors in the literature have pointed out the potential effect of business cycle fluctuations on research and development investment. Geroski and Walters (1995), Fatas (2000) and Barlevy (2004) all find evidence of a positive relationship between output and R&D.[14] Other studies have looked further into the topic by analysing the composition of R&D spending, and how that varies across the cycle. Rafferty (2003a and 2003b) documents that basic research increases in downturns, while development is procyclical. He also analyses in that work if cash flow constraints have a role in the variations of total R&D spending, and finds that they do, which suggests that availability of means to insure against negative liquidity shocks to those R&D projects should encourage investment in them. Interestingly, Hall (1992) finds that most R&D is financed by internal funds, which makes this type of investment especially reliant on being able to implement an optimal risk management strategy that does not leave a firm willing to engage in R&D development at some future stage totally dependent on external funds for that venture.[15]

I show in figure (7) some evidence for the cyclical pattern of R&D spending using data from the National Science Foundation for the United States from 1953 to 2005. I plot the share of R&D investment as a share of total fixed capital formation and compare the evolution of this ratio against NBER dated recessions in the United States. Again, this chart shows evidence of sharp contractions in the share of R&D spending at the onset of recessions and fast recoveries following the beginning of the upward section of the cycle.

I have conducted some further analysis studying variations in the share of R&D investment exploiting certain differences at the sectoral level. The main premise is that certain types of firms should show a higher sensitivity in their ratios of R&D investment as a fraction of total investment than others. In particular, the model suggests that smaller firms (a proxy for higher agency costs), firms in more volatile sectors, and firms in sectors with a higher external finance dependence, should show a higher sensitivity.

Some tentative evidence, without resorting to formal econometric analysis, for all these three is shown below. One of the analyses looks at sectoral variation in investment across the cycle, where sectors are classified according to their volatility using a number of different criteria.[16] My criterion to classify industries as per their volatility uses a

[14] Geroski and Walters (1995) measure R&D spending by the number of patent applications, while Fatas (2000) and Barlevy (2004) look at R&D expenditures as reported by companies in the United States.

[15] More evidence in this line is provided by Himmelberg and Petersen (1994), who document that R&D spending at the firm level is very sensitive to cash flow.

[16] This measure is in line with that used by Huizinga (1992), and I compare my classification with the one in that paper for robustness.

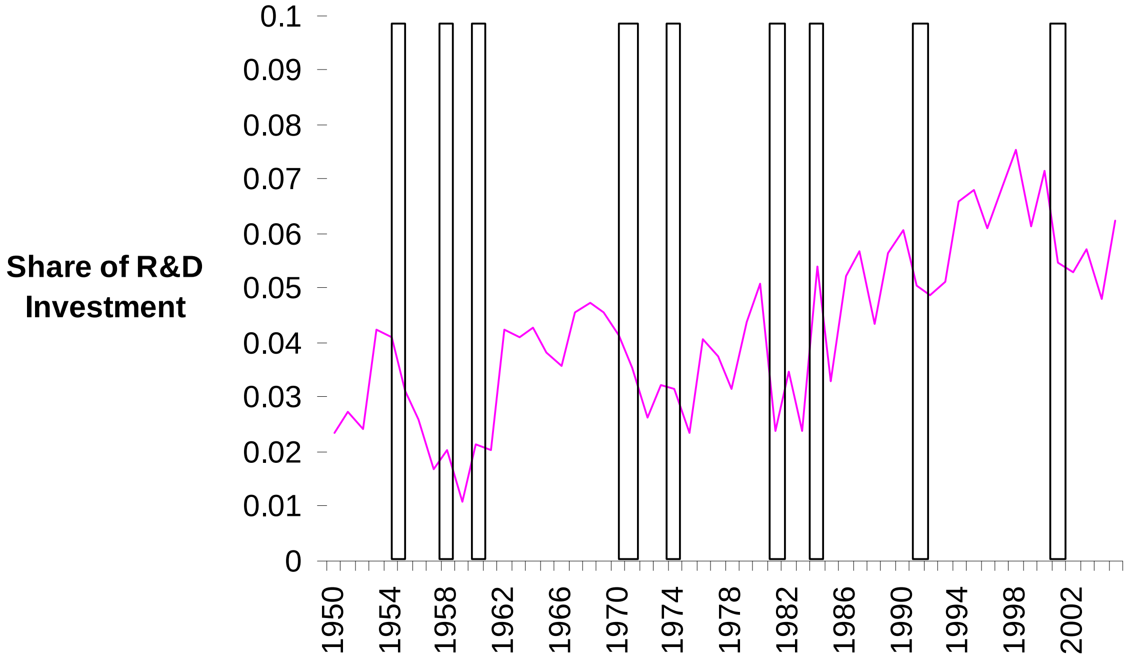

Figure 7: **R&D Investment as a Share of Total Fixed Capital Formation** (Data for investment for the U.S. from National Science Foundation)

combination of measures such as the standard deviation of real wages, of input prices, of output prices, and the average horizon of investment projects within sectors. The data is divided into very low volatility sectors and very high volatility sectors (ignoring moderate sectors), and shown in figure (8) below. The data suggests that R&D spending is more sensitive in highly volatile sectors, as a share of total investment, in line with my predictions.

Another interesting measure is that of external dependence, where the precise definitions and classification are taken from Rajan and Zingales (1998). Again the data is divided into very low dependence sectors and very high dependence sectors (ignoring moderate sectors), and shown in figure (9) below. The data suggests that R&D spending sensitivity is not significantly different in both groups of firms. This lack of evidence may be due to either a lack of the effect posited, or indeed a failure in the specific index used, and I am currently investigating this more deeply.

Finally, I use average firm size within each sector to again divide the data into very low average size sectors and very high average size sectors (ignoring moderate sectors), and the results are shown in figure (10) below. The data suggests that R&D spending is more pro-cyclical in sectors with smaller sized firms, in line with my predictions.

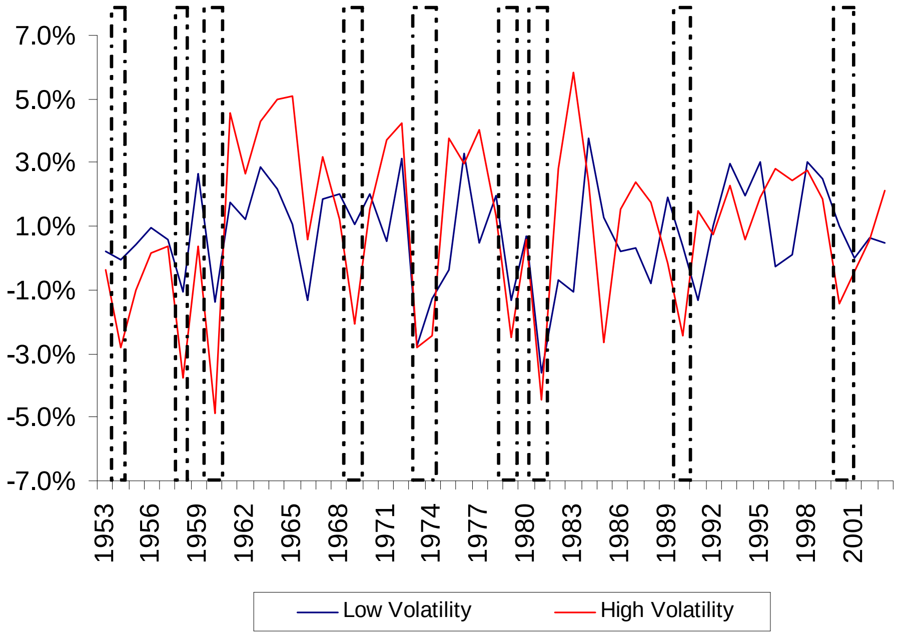

Figure 8: Annual % Variation in R&D Private Spending as a Share of Total Investment - Industries Classified by Volatility (using criterion that weighs input and output price volatility, uncertainty in outcome of investment projects, average duration of projects, etc...)

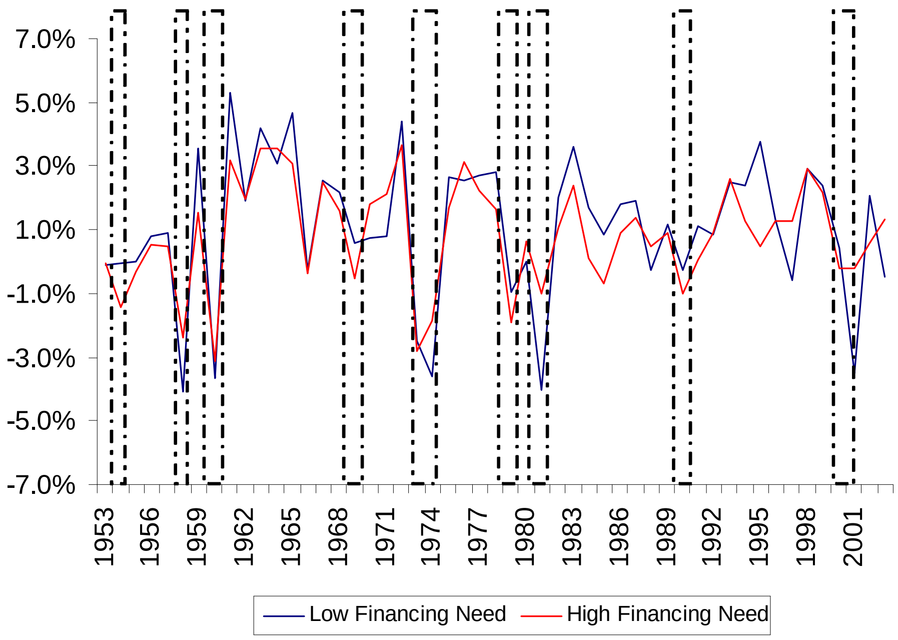

Figure 9: Annual % Variation in R&D Private Spending as a Share of Total Investment - Industries Classified as per the Rajan and Zingales (1998) index of External Dependence

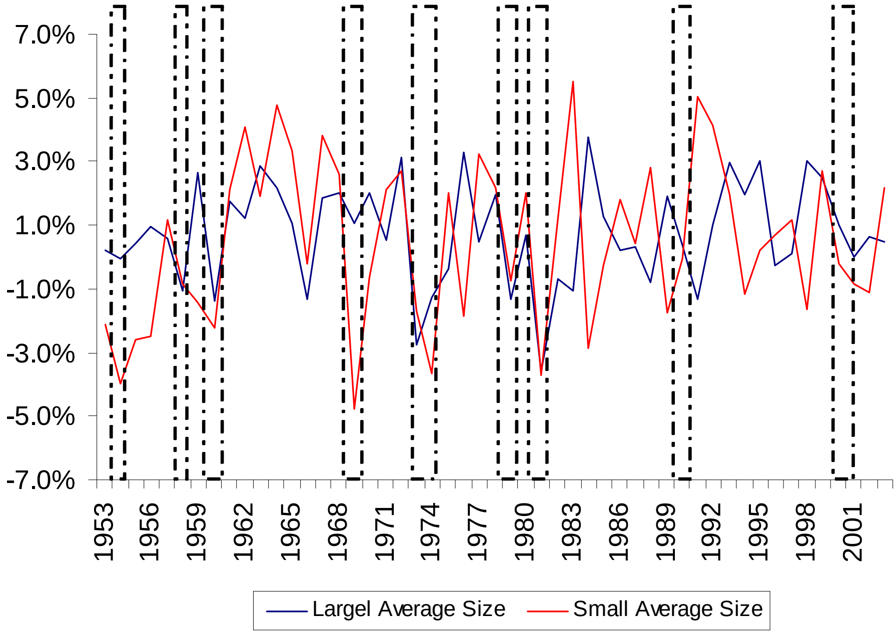

Figure 10: % Variations in ratio of R&D expenditures as a share of total investment

7.1.2 Long-Term Structural Investment and Credit Standards

One broad classification of investment with relevance for the topic of uncertainty is along the dimension of duration of the project. Longer projects, which carry a higher risk of facing intermediate episodes of reinvestment requirements, and a higher risk about returns inherent in that the conditions about demand and other aspects so far into the future will be more uncertain, will not be undertaken in case the risk-sharing opportunities are low. I construct a measure of the share of long-term investment as a proportion of total investment, using data from the OECD, and study how it varies across the cycle. The raw numbers for the United States are plotted in figure (11), which captures the evolution of this ratio over the past 50 years. Also plotted are the NBER dated recessions that have taken place during this period of time. The chart shows a clear cyclical pattern that is common to most of the recession episodes that occurred: the share of long-term investment falls significantly during downturns, and recovers with some lag as the boom begins.

With regards to the first element in the insurance channel, the worsening of expected insurance conditions, available U.S. data provides the opportunity to measure varying Bank credit conditions through the Senior Loan Officer Opinion Survey on Bank Lending Practices. This is a survey of approximately sixty large domestic banks and twenty-four U.S. branches and agencies of foreign banks conducted by the Federal Reserve. It is conducted quarterly, and questions cover changes in the standards and terms of the banks' lending and the state of business and household demand for loans.

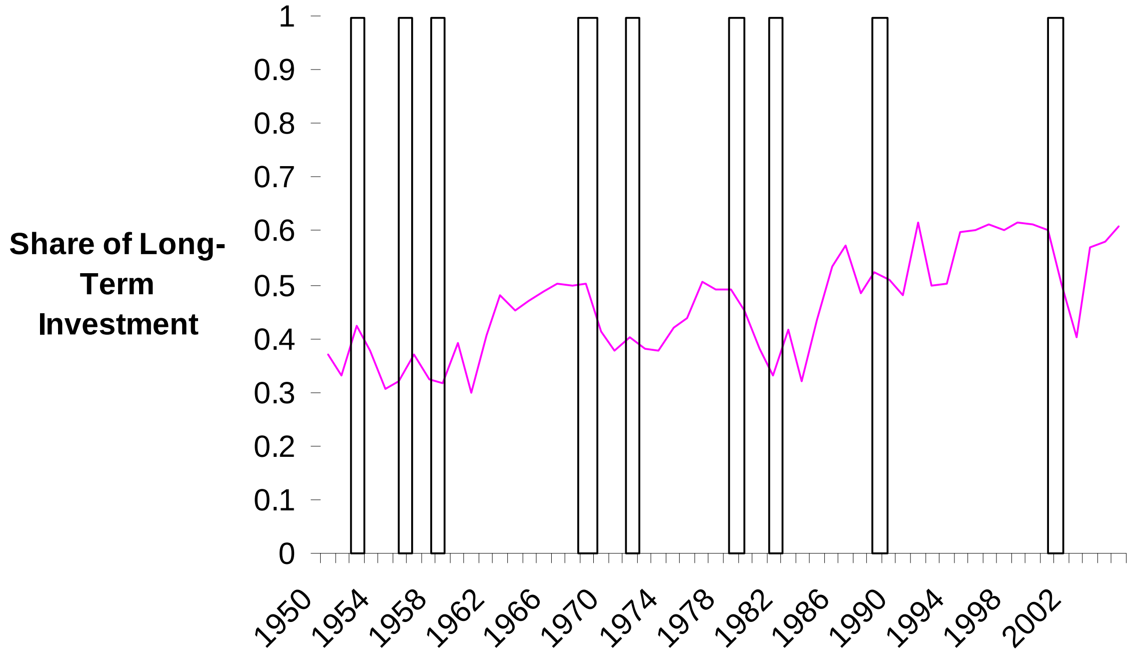

Figure 11: **Long-Term Investment as a Share of Total Investment** (Data for investment for the U.S. from OECD)

The premise is that if credit conditions worsen (standards for credit lines increase,...) or are expected to worsen (bank liquidity expected to fall, collateralizable asset values expected to fall,...), high agency cost firms, and firms in industries with (a) riskier profiles and (b) higher financing needs, should be hit worst, and hence should see a higher reaction of their long-term structural investment ratio (as a fraction of total investment). As preliminary evidence, I show below, in chart (12), the reaction of the share of long-term structural investment as a share of total investment for firms of all sizes for the U.S. The evidence is not in line with the predictions of this paper, as the graph shows that riskier, long-term investment responds positively to credit conditions. Lack of availability of data disaggregation by firm-size may explain this puzzling result, and I am currently studying this issue further.

Credit Standards are measured as the percent of Loan Officers reporting that they have tightened their credit standards during the past 3 months (Minus percent which have eased), and the composition of investment is calculated according to three different measures:

- Share 1 = (Structures + Residential Investment) / Gross private domestic investment
- Share 2 = (Structures + Residential Investment) / Fixed investment

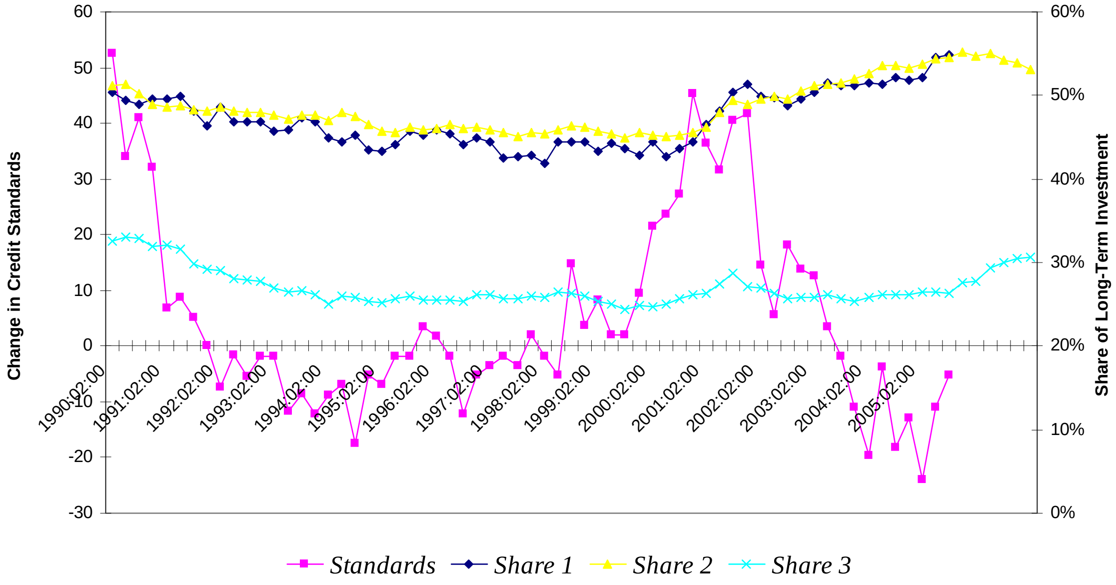

Figure 12: Credit Standards and the Share of Long-Term Structural Investment as a Fraction of Total Investment (US Data)

- Share 3 = Structures / Nonresidential Fixed investment

where gross private domestic investment = Fixed Investment (Structures + Equipment and software + Residential) + Change in private inventories for small firms.

8 Discussion and Conclusion

There exists a large body of research on the role of financial frictions in amplifying macroeconomic shocks. Most work has been focused on how firms' *investment capacity* is affected in recessions by tighter borrowing constraints or by a decreased supply of intermediated finance, and has studied how firms are constrained in the amount they can invest ex-post. There has been little focus however on a propagation mechanism that studies how cyclical changes in the risk-sharing capacity of the financial system may be affecting *firms' willingness to bear risk* and acting to propagate the cycle by affecting the risk profile of their investment portfolio (the *composition* as well as the amount).

This paper is motivated by two sets of observations. On the one hand, there is evidence that constrained firms shift the composition of their investment towards safer and more liquid technologies in recessions, while this is not the case for unconstrained firms. On the other hand there is evidence that credit constrained firms display a precautionary behavior induced by future expected financing constraints that significantly affects their real and financial policies.

Based on these observations, I incorporate these precautionary effects into a dynamic stochastic general equilibrium framework to study their macroeconomic implications. Additionally, I model the aggregate availability of risk management opportunities for firms. I use this framework to address two important questions. Can a mechanism capturing this precautionary element have significant consequences for aggregate investment and output dynamics? Can this mechanism account for the observed variation in the composition of aggregate and firm-level investment across the business cycle?

This paper identifies a novel amplification mechanism of macroeconomic shocks based on time-varying risk-sharing opportunities that affect firms' preference for the risk profile of their portfolio of investment projects. This amplification mechanism is shown to be quantitatively large and asymmetric. Furthermore, it is shown to be potentially enhanced by financial intermediaries' own credit constraints, creating a powerful feedback mechanism between entrepreneurial investment choices, asset prices, and banks' balance sheet conditions and insurance capacity.

On the other hand, this framework is able to account for the empirically documented cyclical variation in the composition of real investment, a feature which the existing models studying the macroeconomic implications of financial constraints cannot account for. In particular, it is shown how following worsening expected financing conditions, firms shift to safer but lower return investments, or, absent alternative investment opportunities, to liquid securities and cash, and how these effects are stronger for high agency cost firms and for firms in highly volatile industries.

9 Appendix

9.1 Entrepreneur's Individual Problem

Assumption 1 *Entrepreneurial returns are high enough and θ is tight enough, such that young entrepreneurs' borrowing constraint against their high-return idiosyncratic state is always binding. A sufficient condition for this assumption to hold is that*

$$\theta < \frac{\omega + |x|}{2(1-\delta)}. \tag{25}$$

Given that young entrepreneurs face a fully state-contingent set of securities, they will want to borrow to the limit against their lucky idiosyncratic state if the marginal return to a unit of resources in period $t+1$ in the unlucky state is higher than the return to investing in their idiosyncratic lucky state in period $t+1$, taking into account the cost of implementing the transfer of resources from the lucky state in $t+1$ to the unlucky state in $t+1$. That involves borrowing from the lucky state in period $t+1$ to the current date (period t), at no cost, and investing in the insurance security i_t, at cost ϕ_t.

In other words, from an ex-ante perspective they want to equalize their future net worth in both possible states in $t+1$, which implies they need to transfer an amount of resources equal to $(\omega + |x|)\phi_t m_t$ from their lucky state to their unlucky state. They will not be able to implement this perfect smoothing of net worth if their borrowing capacity is not large enough to transfer those funds, or:

$$\theta E_t \left(\frac{p_{t+1}}{1 + r_{t+1}} \right) (1-\delta) < (\omega + |x|)\phi_t. \tag{26}$$

Given that $p_t \leq 1$, $\phi_t > 0.5$, and assuming $r_{t+1} > 0$, this implies that a sufficient condition for borrowing constraints to be binding is that $2\theta(1-\delta) < \omega + |x|$, borrowing constraints for young entrepreneurs are always binding. It is not necessarily the case that $r_{t+1} > 0$ however, and in the numerical simulations I check that (26) is always satisfied.

9.2 General Equilibrium - Recursive Equilibrium Conditions

The recursive competitive equilibrium is defined by decision rules for K_{t+1}, C_t, H_t, M_{it}^Y, M_{it}^L, M_{it}^U, Z_{it}^L, Z_{it}^U, Z_{it}^{OL}, Z_{it}^{OU}, I_t, C_t^E, B_{it}^Y, B_{it}^L, B_{it}^U, q_t, p_t, and ϕ_t, as a function of K_t, θ_t, $\{M_{i,t-1}\}$ and $\{Z_{it-1}\}$. A reminder of what each variable is is contained in table (3).

The recursive equilibrium conditions are given in (27)-(45) below. First, there is a savings supply decision by households, and a labor supply decision, given respectively by:

$$u_c(t) = \beta E_t \left\{ u_c(t+1) \frac{[q_{t+1}(1-\delta) + \theta_t F_1(t)]}{q_t} \right\}, \tag{27}$$

Table 3: Explanation of Variables

K_{t+1}	Aggregate capital used by consumption-good producing firms.
C_t	Aggregate consumption of households
H_t	Aggregate labor supplied by households
M_{it}^Y	Aggregate investment in entrepreneurial capital by young entrepreneurs
M_{it}^L	Aggregate investment in entrepreneurial capital by old (second period) entrepreneurs that were lucky in their first period
M_{it}^U	Aggregate investment in entrepreneurial capital by old (second period) entrepreneurs that were unlucky in their first period
Z_{it}^L	Aggregate net worth of young entrepreneurs at the end of period t, having not had a liquidity shock (lucky)
Z_{it}^U	Aggregate net worth of young entrepreneurs at the end of period t, having had a liquidity shock (unlucky)
Z_{it}^{OL}	Aggregate net worth of old (second period) entrepreneurs that where lucky in their first period, at the end of period t
Z_{it}^{OU}	Aggregate net worth of old (second period) entrepreneurs that where unlucky in their first period, at the end of period t
I_t	Aggregate intra-period insurance purchases by young entrepreneurs.
C_t^E	Aggregate consumption of entrepreneurs (the consumption of the dying, third period entrepreneurs)
B_{it}^Y	Aggregate intra-period borrowing in period t by young entrepreneurs
B_{it}^L	Aggregate intra-period borrowing in period t by old (second period) entrepreneurs that where lucky in their first period
B_{it}^U	Aggregate intra-period borrowing in period t by old (second period) entrepreneurs that where unlucky in their first period
q_t	Price of capital used by consumption good producing firms
p_t	Price of entrepreneurial capital
ϕ_t	Price of insurance
π^i	Weight of each type $i = \{Y, L, U, OL, OU\}$ of entrepreneur in the total population of entrepreneurs.

and:

$$\frac{u_L(t)}{u_c(t)} = \theta_t F_2(t). \tag{28}$$

The investment good market clearing obtains when the following equation is satisfied:

$$K_{t+1} = (1-\delta)K_t + \eta \sum_i \pi_i A_t f(M_t^i) \tag{29}$$

The aggregate resource constraint requires that

$$\begin{aligned} Y_t \;=\; & (1-\eta)C_t + \eta C_t^e + \eta \sum_{i=Y,L,U} \pi^i \left[M_t^i - (1-\delta) M_{t-1}^i \right] + \\ & \sum_{i=L,U,DL,DU} \pi^i \left[Z_t^i - (1-\delta) Z_{t-1}^i \right] + \eta \pi^Y 0.5 x M_{t-1}^Y. \end{aligned} \tag{30}$$

The aggregate productivity factor θ follows the stochastic process:

$$\log\theta_{t+1} = \rho\log\theta_t + \sigma_\varepsilon\varepsilon_{t+1} \tag{31}$$

The entrepreneurial fixed factor market clearing requires that

$$p_t = \begin{cases} 1 \quad \text{if } \sum\limits_i \pi_i M_{it}(p_t) > \sum\limits_i \pi_i(1-\delta)M_{it-1} \\ \text{given by } \sum\limits_i M_{it}(p_t) = \sum\limits_i (1-\delta)M_{it-1} \text{ otherwise} \end{cases} \tag{32}$$

be satisfied, while the young entrepreneurs demand for entrepreneurial capital is given by:

$$E_t\left\{R^L_{m,t+1}\left[\frac{q_t\omega + (1-\delta)p_{t+1} - \theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}{p_{t+1} - 0.5\theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}}\right]\right\} +$$

$$E_t\left\{R^U_{m,t+1}\left[\frac{x + p_{t+1}(1-\delta)}{p_{t+1} - 0.5\theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}}\right]\right\} \tag{33}$$

$$= \quad E_t\left\{R^U_{m,t+1}\left[\frac{\frac{1}{\phi_t}\left[p_t - 0.5\theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}\right]}{p_{t+1} - 0.\theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}}\right]\right\},$$

where $R_{m,t+1}$ is the return to a unit of the consumption good invested in the risky technology by old entrepreneurs in period $t+1$, and is given by:

$$\begin{aligned} R^L_{m,t+1} &= \left[q_{t+1}f'(M^L_{t+1}) + (1-\delta)p_{t+2} - \theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}\right] \\ R^U_{m,t+1} &= \left[q_{t+1}f'(M^U_{t+1}) + (1-\delta)p_{t+2} - \theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}\right]. \end{aligned}$$

Old entrepreneurs' investment for those who were lucky in the previous period is given by:

$$M^L_t = \frac{Z^L_{t-1} + p_t M^Y_{t-1}(1-\delta)}{p_t - \theta E_t(\frac{p_{t+1}}{1+r_{t+1}})}, \tag{34}$$

and for those who were unlucky by:

$$M^U_t = \frac{Z^U_{t-1} + p_t M^Y_{t-1}(1-\delta)}{p_t - \theta E_t(\frac{p_{t+1}}{1+r_{t+1}})}. \tag{35}$$

Liquidity of young entrepreneurs who have been lucky (unlucky) at the end of period t is given respectively by

$$Z_{it}^{L} = q_t \omega M_t^Y - B_t^Y \tag{36}$$

and:

$$Z_{it}^{U} = x M_t^Y + I_t. \tag{37}$$

Liquidity of old entrepreneurs who had been lucky (unlucky) when young at the end of period t is given respectively by

$$Z_{it}^{OL} = q_t A f(M_t^L) - B_t^L \tag{38}$$

and:

$$Z_{it}^{OU} = q_t A f(M_t^U) - B_t^U. \tag{39}$$

Insurance demand by young entrepreneurs satisfies:

$$I_t = \frac{1}{\phi_t}(w_t^e + B_t^Y - p_t M_t^Y),$$

and the market clearing condition for insurance is:

$$\phi_t = \begin{cases} 0.5 \quad \text{if} \;\; B_t^{FI} > I_t^{FI} \\ < 0.5, \text{ and given by } I_t = B_t^{FI} \;\; \text{otherwise} \end{cases} \tag{40}$$

Aggregate bank lending to all entrepreneurs is:

$$B^{FI} = \sum_i \pi_i B_t^i$$

Aggregate insurance supply is:

$$I_t^{FI} = I_t$$

Borrowing by young, old lucky, and old unlucky entrepreneurs is respectively:

$$B_t^Y = 0.5\theta(1-\delta)E_t\left(\frac{p_{t+1}}{1+r_{t+1}}\right)M_t^Y, \tag{41}$$

$$B_t^L = \theta(1-\delta)E_t\left(\frac{p_{t+1}}{1+r_{t+1}}\right)M_t^L, \tag{42}$$

and:

$$B_t^U = \theta(1-\delta)E_t\left(\frac{p_{t+1}}{1+r_{t+1}}\right)M_t^U. \tag{43}$$

The market clearing condition for entrepreneurial credit is:

$$\sum_i \pi_i B_{it} = B_t^{FI} \tag{44}$$

And finally, entrepreneurial consumption (that of the dying entrepreneurs) is:

$$C_t^e = \eta\{\pi^{OL}[Z_{t-1}^L + p_t(1-\delta)M_{t-1}^L] + \pi^{OU}[Z_{t-1}^U + p_t(1-\delta)M_{t-1}^U]\} \tag{45}$$

9.3 Computational Appendix

The equilibrium of this model is solved using the Parameterized Expectations Algorithm (PEA), a method commonly used to solve nonlinear stochastic dynamic models (see Marcet (1988), den Haan and Marcet (1990), and Christiano and Fisher (2000)). It is a non-finite state-space algorithm that approximates the conditional expectation of one or more equilibrium conditions by using a parametric function of the state variables. I have chosen this solution method as it allows me to deal with *(a)* the relatively large number of endogenous state variables (applying discrete state-space methods might be problematic because of the 'curse of dimensionality') and *(b)* the occasionally binding inequality constraints.

To solve this model I need to approximate three expectational equations. First, I approximate the households' euler equation (27), from which I obtain current period consumption C_t. Second, I also need to approximate the optimality condition of entrepreneurs (33), from which I obtain current period investment in the risky technology by the young entrepreneurs, M_t^Y. Finally, I approximate the discounted value of one unit of entrepreneurial capital next period, or $E_t(p_{t+1}/(1+r_{t+1}))$, which I can do by using any of the equations that incorporate that term, such as (41), (42) or (43).

For the choice of approximating function for (27) I can use homotopy and introduce a function based on the closed form solution that exists for the one-sector stochastic growth model with logarithmic utility and full depreciation. For the second and third equations I have tried with polynomial functions of different orders, discarding terms for which the explanatory power is small.

For every period in the simulated time series I check whether the markets for insurance and entrepreneurial capital clear at the prices of, respectively, $\phi_t = 0.5$ and $p_t = 1$. If the do not, then prices are adjusted until both markets clear.

The rest of the endogenous variables, K_{t+1}, H_t, M_{it}^L, M_{it}^U, Z_{it}^L, Z_{it}^U, Z_{it}^{OL}, Z_{it}^{OU}, I_t, C_t^E, B_{it}^Y, B_{it}^L, B_{it}^U, and q_t are calculated each period, where the length of simulation for each iteration is $T = 5,000$. The parameters of the approximating functions are recalculated after each iteration until convergence.

9.4 Extension - Recursive Equilibrium Conditions

The extension I focus on is one in which entrepreneurs have the option to invest in the final consumption good producing firms in the *first* period (not in the second period).

They are assumed to do so by purchasing investment goods and renting them to firms to obtain the equilibrium rate of return on that investment, r_t.

Their budget constraint at the beginning of period t is now:

$$p_t m_t + \phi_t i_t + s_t - 0.5 b_t = w_t^e, \tag{46}$$

where s_t is the amount invested in investment goods, delivering an expected return $E_t(1 + r_{t+1})$ the following period. At the beginning of the second period, a lucky entrepreneur has a net worth equal to:

$$n_t^l = q_t g(m_t) - b_t + p_{t+1}(1 - \delta)m_t + s_t(1 + r_{t+1}), \tag{47}$$

and an unlucky entrepreneur has a net worth equal to:

$$n_t^u = x m_t + i_t + p_{t+1}(1 - \delta)m_t + s_t(1 + r_{t+1}). \tag{48}$$

This extension requires a minor adjustment of the risky technology specification. For an interior solution to obtain we need to introduce decreasing returns to scale in the risky technology in the first period as well: now instead of investing m_t and obtaining $q_t \omega m_t$ with probability 1/2, entrepreneurs obtain $q_t f(m_t)$ with probability 1/2, where $g(m_t)$ is such that $g\prime(\cdot) > 0$ and $g''(\cdot) < 0$. This adjustment is without loss of generality.

Entrepreneurs in their second period (old entrepreneurs), lucky and unlucky alike, invest in the entrepreneurial technology, an amount m_{t+1}^i (for $i = l, u$) given by:

$$p_{t+1} m_{t+1}^i = n_t^i + b_{t+1}. \tag{49}$$

The optimal choice of entrepreneurs is given by expression (50) below.

$$E_t \left\{ R_{m,t+1}^L \left[\frac{q_t g'(m_t) + (1 - \delta)p_{t+1} - \theta(1 - \delta)\frac{p_{t+1}}{1+r_{t+1}}}{p_t - 0.5\theta(1 - \delta)\frac{p_{t+1}}{1+r_{t+1}}} \right] \right\} + \tag{50}$$

$$E_t \left\{ R_{m,t+1}^U \left[\frac{x + p_{t+1}(1 - \delta)}{p_t - 0.5\theta(1 - \delta)\frac{p_{t+1}}{1+r_{t+1}}} \right] \right\}$$

$$\begin{aligned} &= E_t \left\{ R_{m,t+1}^U \left(\frac{1}{\phi_t} \right) \right\} \\ &= E_t \left\{ R_{m,t+1}^U (1 + r_{t+1}) \right\} + E_t \left\{ R_{m,t+1}^L (1 + r_{t+1}) \right\} \end{aligned} \tag{51}$$

where $R_{m,t+1}$ is the return to a unit of the consumption good invested in the risky technology in the second period, and is given by:

$$R^i_{m,t+1} = \frac{q_{t+1}f'(m_{t+1}) + (1-\delta)p_{t+2} - \theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}}{p_{t+1} - \theta(1-\delta)E_{t+1}\left(\frac{p_{t+2}}{1+r_{t+2}}\right)} \text{ where } i = \{L, U\}. \quad (52)$$

Expression (50) is the condition for optimal investment: the marginal return to investing a unit of the consumption good in the risky entrepreneurial activity, in the alternative technology, and in the insurance security has to be equalized.

Following a change in the expectations about credit conditions in the following period, entrepreneurs react by adjusting their levels of risky entrepreneurial investment, safe alternative investment and insurance security. In particular, a decrease in the expected ex-post borrowing capacity of entrepreneurs in period $t+1$, captured by a decrease in the term

$$E_t\left[\theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}\right]$$

in a situation where ex-post resources have not been smoothed across lucky and unlucky entrepreneurs, results in a decrease in the resources invested in the risky technology in period t as a share of total investment (including investment in the alternative technology and in the insurance security). Absent frictions in the supply of insurance (i.e. when ϕ_t remains at the actuarially fair price) , then the share of investment in i_t increases, and the share invested in the alternative investment s_t can increase or decrease, or

$$\frac{dm_t}{dE_t(p_{t+2})} < 0, \frac{di_t}{dE_t(p_{t+2})} > 0, \frac{ds_t}{dE_t(p_{t+2})} \gtrless 0.$$

However if frictions in the supply of insurance are severe enough, and ϕ_t increases sufficiently as a result of worsening expected future financial frictions, then

$$\frac{dm_t}{dE_t(p_{t+2})} < 0, \frac{di_t}{dE_t(p_{t+2})} \gtrless 0, \frac{ds_t}{dE_t(p_{t+2})} > 0.$$

The extension requires changes to the following equilibrium conditions. First, the entrepreneurial choice between the risky investment, insurance, and the safe alternative, is determined by (53) and (55) respectively.

$$E_t\left\{R^L_{m,t+1}\left[\frac{q_t g'(M^Y_t)+(1-\delta)p_{t+1}-\theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}{p_t-0.5\theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}\right]\right\}+ \tag{53}$$

$$E_t\left\{R^U_{m,t+1}\left[\frac{x+p_{t+1}(1-\delta)}{p_t-0.5\theta(1-\delta)\frac{p_{t+1}}{1+r_{t+1}}}\right]\right\}$$

$$= \quad E_t\left\{R^U_{m,t+1}\left(\frac{1}{\phi_t}\right)\right\}$$

where $R_{m,t+1}$ is the return to a unit of the consumption good invested in the risky technology in the second period, and is given by:

$$R^i_{m,t+1}=\frac{q_{t+1}f'(m_{t+1})+(1-\delta)p_{t+2}-\theta(1-\delta)\frac{p_{t+2}}{1+r_{t+2}}}{p_{t+1}-\theta(1-\delta)E_{t+1}\left(\frac{p_{t+2}}{1+r_{t+2}}\right)} \text{ where } i=\{L,U\}. \tag{54}$$

$$E_t\left\{R^U_{m,t+1}\left(\frac{1}{\phi_t}\right)\right\} = \tag{55}$$

$$E_t\left\{R^U_{m,t+1}\left(1+r_{t+1}\right)\right\}+E_t\left\{R^L_{m,t+1}\left(1+r_{t+1}\right)\right\}$$

Old entrepreneurs' investment is now given by (56), for those who were lucky in the previous period, and (57), for those who were unlucky.

$$M^L_t=\frac{Z^L_{t-1}+p_tM^Y_{t-1}(1-\delta)+S^Y_t(1+r_t)}{p_t-\theta E_t(\frac{p_{t+1}}{1+r_{t+1}})} \tag{56}$$

$$M^U_t=\frac{Z^U_{t-1}+p_tM^Y_{t-1}(1-\delta)+S^Y_t(1+r_t)}{p_t-\theta E_t(\frac{p_{t+1}}{1+r_{t+1}})} \tag{57}$$

And finally, insurance demand by young entrepreneurs is given by:

$$I_t=\frac{1}{\phi_t}(w^e_t+B^Y_t-p_tM^Y_t-S^Y_t) \tag{58}$$

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