Political Pressures and Monetary Mystique*

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Abstract

Central bank independence and transparency have become best practice in monetary policy. This paper cautions that transparency about economic information may not be beneficial in the absence of central bank independence. The reason is that it reduces monetary uncertainty, which could make the government less inhibited to interfere with monetary policy. In fact, a central bank could use monetary mystique to obtain greater insulation from political pressures, even if the government faces no direct cost of overriding. As a result, economic secrecy could be beneficial and provide the central bank greater political independence.

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1 Introduction

Central bank independence and transparency have become best practice in monetary policy. But only 20 years ago when the independence of central banks was not as well established, central banks tended to be notorious for their secrecy. This paper shows that opacity may be desirable when a central bank could be subject to political interference. The reason is that greater monetary uncertainty makes the government more reluctant to intervene in monetary policy. In particular, opacity about the economic shocks to which the central bank responds makes it more difficult to assess the central bank’s intentions from its monetary policy actions. This gives the central bank greater leeway to set monetary policy without government interference. As a result, a central bank could use monetary mystique to insulate itself from political pressures.

This paper helps to explain how central banks managed to gain independence through secrecy before the advent of the new paradigm of central bank independence-cum-transparency. For instance, the ‘monetary veil’ introduced by Chairman Paul Volcker in October 1979 helped to keep US Congress at bay while the Federal Reserve pursued its painful disinflation in the early 1980s. Furthermore, this paper cautions that in the absence of central bank independence, economic transparency may be detrimental as it could lead to greater political interference. Although central bank independence is prevalent in advanced economies, it is much less common in developing countries. In fact, in the survey of 94 central banks by Fry, Julius, Mahadeva, Roger and Sterne (2000, Table 4.4), 93% of central banks in industrialized countries report they enjoy independence without significant qualifications, whereas this holds for only 57% of central banks in developing countries. For those central banks that lack independence, economic secrecy could be an effective way to stave off unwanted political meddling with monetary policy.

This argument is formally developed using a stylized monetary policy game in the spirit of Kydland and Prescott (1977) and Barro and Gordon (1983). The government has a motive to stimulate output beyond the natural rate, whereas monetary policy is set by a conservative central banker (Rogoff 1985). The government can override the central bank’s policy decision at a fixed cost (Lohmann 1992). Uncertainty about the central bank’s true intentions and the economic situation complicate the government’s decision whether or not to interfere. There is rational updating of beliefs (Cukierman and Meltzer 1986, Backus and Driffill 1985, Barro 1986) and the modeling of transparency builds on Faust and Svensson (2001) and Geraats (2005).

Transparency of monetary policy could be defined as the extent to which monetary authorities disclose information that is relevant for the policymaking process; so, perfect transparency amounts to symmetric information. There is a growing literature on central bank transparency that covers many aspects (see the survey by Geraats 2002). In general, an important benefit of transparency is that it reduces uncertainty. But, greater transparency could
also be detrimental. For instance, ambiguity about the central bank’s output preferences makes it easier for the central bank to reach its objectives (Cukierman and Meltzer 1986, Geraats 2007). In addition, opacity about central bank preferences could moderate wage demands by unions (Sorensen 1991) or give rise to beneficial reputation effects (e.g. Faust and Svensson 2001, Geraats 2005), thereby reducing inflation. The publication of voting records could be welfare-reducing in a monetary union when central bankers alter their policy to get reappointed by national politicians (Gersbach and Hahn 2005). The disclosure of information could also cause financial markets to increase their reliance on public signals to coordinate their actions, which could lead to greater volatility if public information is sufficiently noisy (Morris and Shin 2002). Furthermore, transparency about economic information could hamper stabilization policy when the public incorporates supply shocks into inflation expectations and thereby negatively affects the contemporaneous inflation-output trade-off (Cukierman 2001, Gersbach 2003, Jensen 2002). The present paper is the first to focus on the effects of transparency about economic shocks in an institutional framework in which the central bank is subject to political interference. It provides another argument against economic transparency, namely that it could make central banks prone to greater political pressures through government overriding and thereby increase average inflation.

The model extends the seminal analysis by Lohmann (1992) in three important ways. First, it allows for uncertainty about the central bank’s preferences, which are inherently unobservable. Second, it incorporates the realistic assumption that inflation cannot be set directly but can only be influenced indirectly through a monetary policy instrument, such as the money supply. The monetary policy action provides a signal of the central bank’s preferences but it also reflects economic disturbances, such as money market shocks. Third, it is assumed that the government may not have the same information about economic shocks as the central bank.

The main finding of the paper is that opacity about economic shocks gives the central bank greater freedom from political interference. In fact, economic opacity could give the central bank some independence even if the government faces no direct cost of overriding, which is in contrast to Lohmann (1992). Greater economic opacity increases the central bank’s ‘region of independence’, but more preference uncertainty actually reduces it. Thus, in the presence of political pressures, preference uncertainty is detrimental, but mystique about monetary disturbances is beneficial.

The remainder of the paper is organized as follows. The basic model is presented in section 2 and the solution derived in section 3. The comparative statics are shown in section 4 and the main results are summarized in Propositions 1, 2 and 3. Section 5 considers a
few extensions to the basic model that feature more realistic objective functions and a richer economic structure, and it shows that the conclusions are robust. In addition, this section provides some empirical support for the theoretical prediction of this paper that central banks with lower independence are more likely to have low transparency to ward off political interference. Section 6 summarizes the results and concludes that economic opacity could be desirable when the central bank lacks institutional independence. This helps to explain the past practice of independence-through-secrecy. Furthermore, it suggests that countries that wish to adopt the new paradigm of central bank independence-cum-transparency should first grant the central bank political independence before insisting on economic transparency.

2 Model

The structure of the economy is described by the simple money market equation

\[ \pi = m + v \]  

and the Lucas aggregate supply equation

\[ y = \bar{y} + \theta (\pi - \pi^e) \]  

where \( \pi \) is inflation, \( \pi^e \) private sector expectations of inflation, \( m \) money supply growth, \( y \) real aggregate output, \( \bar{y} \) the natural rate of output, and \( \theta \) the extent to which surprise inflation stimulates output (\( \theta > 0 \)). There is a velocity shock \( v \) that is stochastic: \( v \sim N(0, \sigma_v^2) \), with \( \sigma_v^2 > 0 \). More sophisticated specifications of the economic structure, including a New Keynesian Phillips curve, are discussed in section 5 and yield the same qualitative results.

The government has the objective function

\[ W_G = -\frac{1}{2} (\pi - \bar{\tau})^2 + \beta (y - \bar{y}) \]  

where \( \bar{\tau} \) is the government’s inflation target and \( \beta \) the relative weight on output stimulation (\( \beta > 0 \)). The government delegates monetary policy to a central bank, without granting it complete (instrument) independence. The central bank is conservative in the sense that it puts greater weight on inflation stabilization than the government (Rogoff 1985). For simplicity, assume that the central bank only cares about inflation stabilization (\( \beta = 0 \)) and that its objective function is

\[ W_{CB} = -\frac{1}{2} (\pi - \tau)^2 \]  

More plausible objective functions for the government and the central bank that feature output stabilization are discussed in section 5. Although the algebraic expressions become more cumbersome, the conclusions remain the same.
The central bank has an inflation target \( \tau \) that is unknown to the government and satisfies \( \tau \sim N(\bar{\tau}, \sigma^2_\tau) \) with \( \sigma^2_\tau > 0 \), and \( \tau \) and \( v \) independent. The distribution of \( \tau \) could be interpreted as the stochastic process of the inflation target or the government’s prior, where \( \sigma^2_\tau \) is a measure of preference uncertainty.\(^2\) There could be several reasons for the preference uncertainty faced by the government. First, preferences of central bankers cannot be directly observed and are therefore subject to uncertainty. Also, central bank preferences could actually vary because of new appointments to the central bank’s monetary policy committee. In addition, the central bank may have goal independence. Even if there is an explicit inflation target, such a target is often for the medium run and tends to take the form of a range, leaving significant uncertainty about the central bank’s immediate intentions. The assumption that \( E[\tau] = \bar{\tau} \) implies that on average, the inflation target of the central bank and the government coincide.

The central bank does not enjoy complete instrument independence and the government can decide to override the central bank’s policy decision \( m \), either explicitly (e.g. through an act of parliament) or implicitly through political pressure. Following Lohmann (1992), assume that the government suffers a direct cost of overriding \( C > 0 \). This could involve loss of reputation in the form of higher inflation expectations in the future, or electoral losses due to reduced voter confidence. The possibility of government interference is obviously relevant for central banks that lack formal independence, as is still common in developing countries. But even in advanced countries, central banks appear to be prone to political pressures. For instance, Chappell, McGregor and Vermilyea (2005, chapter 9) provide some anecdotal evidence for the Federal Reserve. And the Bank of Japan was widely perceived to succumb to political pressures when it decided not to increase its policy rate on January 18, 2007.\(^3\)

The government’s decision to override the central bank is complicated by two information asymmetries. First, as already mentioned, the government is uncertain about the central bank’s inflation target \( \tau \). Second, the velocity shocks \( v \) are observed by the central bank, but not by the government.\(^4\) Instead, the government only observes a stochastic signal \( s \) such that

\[
v = s + \eta
\]

where \( \eta \sim N(0, (1 - \kappa) \sigma^2_v) \) with \( 0 \leq \kappa \leq 1 \), and \( s, \eta \) and \( \tau \) are independently distributed. The variable \( \eta \) could be interpreted as the government’s forecast error of the velocity shock. In the special case of \( \kappa = 1 \) there is no asymmetric information about the velocity shock so that \( v = s \), whereas for \( \kappa = 0 \) the signal provides no clues about the velocity shock and

\(^2\)The limiting case of no preference uncertainty \( \sigma^2_\tau \to 0 \) amounts to perfect preference transparency as \( \tau \to \bar{\tau} \). When the inflation target \( \tau \) is constant and \( \tau \sim N(\bar{\tau}, \sigma^2_\tau) \) is the government’s prior distribution, a reduction in \( \sigma^2_\tau \) corresponds to an increase in preference transparency.

\(^3\)See for example, “BoJ decision casts doubt on its autonomy”, Financial Times, January 19, 2007.

\(^4\)One could allow for imperfect central bank forecasts, but the conclusions would be the same.
$s = 0$. The parameter $\kappa$ is a measure of economic transparency, where $\kappa = 1$ amounts to perfect transparency.

The timing in the model is as follows. First, the central bank’s inflation target $\tau$ is realized, but only known to the central bank, and the public forms its inflation expectations $\pi^e$. Then, the government receives a (noisy) signal $s$ of the velocity shock $v$. The central bank observes both the signal $s$ and the noise $\eta$ so that it knows the actual velocity shock $v$, which it uses to set the money supply $m_{CB}$. The government observes this policy action and subsequently decides whether to override the central bank and implement policy action $m_G$ under transparency or $m_O$ under opacity about the economic shock $v$. After that, inflation $\pi$ and output $y$ are realized.

The remaining assumption concerns the formation of expectations. The central bank, government and private sector all have rational expectations. The central bank has perfect information, whereas the government and private sector face imperfect information. To be precise, the information set available to the private sector when it forms its inflation expectations $\pi^e$ equals $\Omega \equiv \{\beta, \theta, \bar{y}, \bar{\tau}, \kappa, \sigma^2_{\tau}, \sigma^2_v\}$; the government’s information set when it makes the override decision is $\{m_{CB}, s, \Omega\}$. The solution of the model is described in the next section.

3 Solution

In the absence of political pressures, the conservative central bank would maximize (4) with respect to $m$ subject to (1) and (2), and given $\pi^e$, and it would implement

$$\tilde{m} = \tau - v$$

(6)

to achieve the economic outcome

$$\pi = \tau$$
$$y = \bar{y} + \theta (\tau - \pi^e)$$

However, the government has the objective function (3) and would prefer

$$m_G = \bar{\tau} + \beta \theta - v$$

(7)

to obtain a higher expected level of output (given inflation expectations) but at the cost of higher inflation:

$$\pi = \bar{\tau} + \beta \theta$$
$$y = \bar{y} + \theta (\bar{\tau} + \beta \theta - \pi^e)$$
The government’s desire to stimulate output beyond the natural rate \((\beta > 0)\) leads to the celebrated inflationary bias of discretionary monetary policy \((\pi > \bar{\pi})\) first advanced by Kydland and Prescott (1977).

The discrepancy between (6) and (7) suggests that the government would like to override the central bank if \(\tau\) is sufficiently different from \(\bar{\tau} + \beta \theta\). However, its decision is complicated by the presence of asymmetric information about \(\tau\) and \(v\).

It is instructive to first consider the case of complete economic transparency \((\kappa = 1)\). Then, the velocity shock \(v\) is known to the government, so it can use the central bank’s policy decision \(m_{CB}\) to infer information about its inflation target \(\tau\). The government abstains from overriding \(m_{CB}\) and implementing its preferred policy \(m_G\) if

\[
W_G(m_G) - C \leq W_G(m_{CB})
\]

Using the fact that in the absence of government interference \(m_{CB} = \bar{m}\), and substituting (1), (2), (7) and (6) into (3), it is straightforward to show that this inequality reduces to

\[
\frac{1}{2} (\tau - \bar{\tau} - \beta \theta)^2 \leq C \tag{8}
\]

So, the government decides not to override the central bank if \(\bar{\tau} + \beta \theta - \sqrt{2C} \leq \tau \leq \bar{\tau} + \beta \theta + \sqrt{2C}\). This region of independence is increasing in the cost of overriding \(C\). If the central bank’s desired inflation outcome \(\pi = \tau\) deviates too much from the level preferred by the government \(\pi = \bar{\tau} + \beta \theta\), (8) would no longer hold and the government would interfere with monetary policy. Since the central bank is worse off if the government overrides its policy decision, it adjusts its policy to prevent this. In particular, it optimally implements the monetary policy action that makes the government indifferent between interference and independence. So, for \(\tau < \bar{\tau} + \beta \theta - \sqrt{2C}\) the central bank sets \(m_{CB} = \bar{\tau} + \beta \theta - \sqrt{2C} - v\), and for \(\tau > \bar{\tau} + \beta \theta + \sqrt{2C}\) it sets \(m_{CB} = \bar{\tau} + \beta \theta + \sqrt{2C} - v\). As a result, the government never overrides, but the possibility of political interference does affect the monetary policy outcome. In particular, it leads to higher average inflation: \(E[\pi] > \bar{\pi}\). Intuitively, without political pressures average inflation would be \(\bar{\pi}\), but the threat of overriding brings average inflation closer to the government’s preferred level of \(\bar{\tau} + \beta \theta\). These results are all similar to Lohmann (1992).

When there is incomplete economic transparency \((0 \leq \kappa < 1)\), the government can no longer infer the central bank’s inflation target \(\tau\) from its policy action \(m_{CB}\). But there is an additional complication: The government is unable to implement its preferred policy \(m_G = \bar{\tau} + \beta \theta - v\) because it does not observe the velocity shock \(v\). So, it tries to extract information about \(v\) from the central bank’s actions \(m_{CB}\).

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5This assumes that the government does not override when it is indifferent; otherwise, there is no equilibrium.

6If there were uncertainty about the government’s preferences, overriding could occur.
The government’s preferred policy action under opacity maximizes $E[W_G (m_O) | m_{CB}]$ subject to (1) and (2), and given $\pi^e$. All expectations operators $E[.]$ are implicitly conditional on the public information set $\{s, \Omega\}$. The first order condition implies

$$m_O = \bar{\tau} + \beta \theta - E[v|m_{CB}]$$

(9)

This is the same as the government’s preferred policy under economic transparency, $m_G$ in (7), except that $v$ has been replaced by $E[v|m_{CB}]$.

The government abstains from overriding $m_{CB}$ and implementing its policy $m_O$ if

$$E[W_G (m_O) | m_{CB}] - C \leq E[W_G (m_{CB}) | m_{CB}]$$

(10)

It is shown in Appendix A.1 that this no-override condition reduces to

$$\frac{1}{2} (\bar{\tau} + \beta \theta - E[v|m_{CB}] - m_{CB})^2 \leq C$$

(11)

So, the central bank enjoys independence if

$$\bar{\tau} + \beta \theta - E[v|m_{CB}] - \sqrt{2C} \leq m_{CB} \leq \bar{\tau} + \beta \theta - E[v|m_{CB}] + \sqrt{2C}$$

(12)

This defines a region of independence for $m_{CB} \in [m, \bar{m}]$, where the thresholds $m$ and $\bar{m}$ only depend on publicly available information. The government overrides the central bank only if $m_{CB} < m$ or $m_{CB} > \bar{m}$. But the central bank adjusts its policy to prevent the government from intervening and implementing $m_O$ in (9). Since $m < m_O < \bar{m}$, it follows from (1) and (4) that the central bank optimally sets

$$m_{CB} = \begin{cases} 
  m & \text{if } \tau - v \leq m \\
  \tau - v & \text{if } m < \tau - v < \bar{m} \\
  \bar{m} & \text{if } \tau - v \geq \bar{m}
\end{cases}$$

(13)

To compute the thresholds $m$ and $\bar{m}$, and the government’s preferred policy action $m_O$, it is necessary to obtain an expression for the conditional expectation $E[v|m_{CB}]$, which involves a signal-extraction problem. For $m < m_{CB} < \bar{m}$, (13) implies that $E[v|m_{CB}] = E[v|\bar{m}]$, using (6). Note that (5) and (6) imply that $v$ and $\bar{m}$ are jointly normal because of their common dependence on $\eta$, so

$$E[v|\bar{m}] = s - \frac{(1 - \kappa) \sigma_v^2}{\sigma_v^2 + (1 - \kappa) \sigma_{\bar{m}}^2} (\bar{m} + s - \bar{\tau})$$

$$= \lambda s - (1 - \lambda) (\bar{m} - \bar{\tau})$$

(14)

Alternatively, when (11) is satisfied, the central bank sets $m_{CB}$ equal to (6), which yields

$$\frac{1}{2} (\bar{\tau} + \beta \theta - E[v|m_{CB}])^2 \leq C$$

similar to (8), so $\bar{\tau} + \beta \theta - \sqrt{2C} \leq E[\tau|m_{CB}] \leq \bar{\tau} + \beta \theta + \sqrt{2C}$.

Use the fact that when $x$ and $z$ have a jointly normal distribution then $E[x|z] = E[x] + \frac{\text{Cov}(x,z)}{\text{Var}(z)} (z - E[z])$. Note that all moment operators are implicitly conditional on $s$. 

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where \( \lambda \equiv \frac{\sigma^2}{\sigma^2 + (1 - \kappa) \sigma^2_v} \), so that \( 0 < \lambda \leq 1 \). The magnitude of \( \lambda \) is increasing in the degree of economic transparency \( (\partial \lambda / \partial \kappa > 0) \), reflecting the fact that the signal \( s \) becomes more reliable. In the limiting case of perfect transparency \( (\kappa = 1, \text{ so } s = v) \), \( \lambda = 1 \) and \( E[v|m] = v \). In the case of economic opacity \( (\kappa < 1) \), both the signal \( s \) and the policy decision \( \tilde{m} \) are used to infer information about the velocity shock \( v \). A higher level of \( \tilde{m} \) is partly attributed to a lower velocity shock and therefore reduces the expectation \( E[v|m] \).

For \( m_{CB} = m \), the signal-extraction problem is a bit more complicated since (13) implies \( E[v|m_{CB}] = E[v|\tilde{m} \leq m] \). It follows from (14), (6) and (5) that\(^9\)

\[
E[v|\tilde{m} \leq m] = \lambda s + (1 - \lambda) \bar{\tau} - (1 - \lambda) E[\tilde{m}|\tilde{m} \leq m] = s + (1 - \lambda) \sqrt{\sigma^2_\tau + (1 - \kappa) \sigma^2_v} \frac{\phi(z)}{\Phi(z)}
\]

(15)

where \( \phi(z) \) and \( \Phi(z) \) denote the probability density function and the cumulative distribution function of the standard normal distribution, respectively, and \( z \equiv \frac{m - (\bar{s} - s)}{\sqrt{\sigma^2_\tau + (1 - \kappa) \sigma^2_v}} \) is the normalized lower threshold. The low level of \( \tilde{m} \leq m \) is partly attributed to high velocity shocks so that \( E[v|\tilde{m} \leq m] \geq s \).

Similarly, for \( m_{CB} = \bar{m} \) it holds that \( E[v|m_{CB}] = E[v|\tilde{m} \geq \bar{m}] \), where\(^10\)

\[
E[v|\tilde{m} \geq \bar{m}] = \lambda s + (1 - \lambda) \bar{\tau} - (1 - \lambda) E[\tilde{m}|\tilde{m} \geq \bar{m}] = s - (1 - \lambda) \sqrt{\sigma^2_\tau + (1 - \kappa) \sigma^2_v} \frac{\phi(z)}{1 - \Phi(z)}
\]

(16)

where \( z \equiv \frac{\bar{m} - (\bar{s} - s)}{\sqrt{\sigma^2_\tau + (1 - \kappa) \sigma^2_v}} \) is the normalized upper threshold. The high level of \( \tilde{m} \geq \bar{m} \) is partly attributed to low velocity shocks so that \( E[v|\tilde{m} \geq \bar{m}] \leq s \).

The conditional expectations (14), (15) and (16) show how the government extracts information about the velocity shock \( v \) from the central bank’s policy decision. For perfect economic transparency \( (\kappa = \lambda = 1) \), the expressions reduce to \( E[v|m_{CB}] = s = v \), so the no-override condition (11) reduces to (8).

Using (12), (13), (15) and (16), and substituting \( \lambda \) yields the following conditions for the thresholds \( m \) and \( \tilde{m} \):

\[
m = \bar{\tau} + \beta \theta - E[v|\tilde{m} \leq m] - \sqrt{2C} = \bar{\tau} + \beta \theta - s - \frac{(1 - \kappa) \sigma^2_v}{\sqrt{\sigma^2_\tau + (1 - \kappa) \sigma^2_v}} \frac{\phi(z)}{\Phi(z)} - \sqrt{2C}
\]

(17)

\[
\bar{m} = \bar{\tau} + \beta \theta - E[v|\tilde{m} \geq \bar{m}] + \sqrt{2C} = \bar{\tau} + \beta \theta - s + \frac{(1 - \kappa) \sigma^2_v}{\sqrt{\sigma^2_\tau + (1 - \kappa) \sigma^2_v}} \frac{\phi(z)}{1 - \Phi(z)} + \sqrt{2C}
\]

(18)

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\(^9\)Use the fact that for a normally distributed variable \( x \sim N(\mu, \sigma^2) \), \( E[x|x < \bar{x}] = \mu - \sigma \phi\left( \frac{\bar{x} - \mu}{\sigma} \right) / \Phi\left( \frac{\bar{x} - \mu}{\sigma} \right) \).

\(^{10}\)Now use the fact that for a normally distributed variable \( x \sim N(\mu, \sigma^2) \), \( E[x|x \geq \bar{x}] = \mu + \sigma \phi\left( \frac{\bar{x} - \mu}{\sigma} \right) / [1 - \Phi\left( \frac{\bar{x} - \mu}{\sigma} \right)] \).
The thresholds satisfy $m < \tilde{\tau} + \beta \theta - s < \tilde{m}$. Note that (17) and (18) only provide an implicit expression for $\bar{m}$ and $\bar{m}$ that depends on $\bar{z}$ and $\bar{z}$, respectively. There is no closed-form solution for $\bar{m}$ and $\bar{m}$, except for the special case in which there is perfect economic transparency ($\kappa = 1$, so $s = v$). Then, (17) and (18) reduce to $m = \tau + \beta \theta - v - \sqrt{2C}$ and $\bar{m} = \tau + \beta \theta - v + \sqrt{2C}$, as before. For other values of $\kappa$, $m$ and $\bar{m}$ need to be computed numerically.

To summarize the equilibrium outcome of the model, the central bank’s policy $m_{CB}$ is given by (13), (17) and (18), and there is no overriding by the government. To complete the formal description of the (perfect Bayesian Nash) equilibrium, it is necessary to specify out-of-equilibrium beliefs for the government that sustain the equilibrium outcome. Assume that the government believes (quite reasonably) that off the equilibrium path, the central bank sets some level $m_{CB} < m$ if $\bar{m} < m$ and $m_{CB} > \bar{m}$ if $\bar{m} > \bar{m}$. More precisely, the government believes that off the equilibrium path (i.e. for $m_{CB} \in \mathbb{R} \setminus [\bar{m}, \bar{m}]$), the central bank sets $m_{CB} = m - \delta_L$ if $\bar{m} < \bar{m} < m$ and $m_{CB} = \bar{m} + \delta_H$ if $\bar{m} = \bar{m} < m$, where $\delta_L$ and $\delta_H$ satisfy $\delta_L > 0$ and $\delta_H > 0$ but are not known to the government. Then the government’s preferred policy, which is still given by (9), equals $m_O = \tau + \beta \theta - E[v|m] = m + \sqrt{2C}$ if $m_{CB} < m$, and $m_O = \tau + \beta \theta - E[v|m] = m - \sqrt{2C}$ if $m_{CB} > m$, using (17) and (18). The government always prefers to override off the equilibrium path, because the region of independence $[\bar{m}, \bar{m}]$ is defined by the no-override condition (10) and is independent of out-of-equilibrium beliefs. Furthermore, for the specified out-of-equilibrium beliefs of the government, the central bank always prefers its equilibrium policy (13). In particular, the central bank prefers $m_{CB} = m$ to $m_O = m + \sqrt{2C}$ if $\tau - v < m$; $m_{CB} = \bar{m}$ to $m_O = \bar{m} - \sqrt{2C}$ if $\tau - v > \bar{m}$. As a result, neither the central bank nor the government has an incentive to deviate from the equilibrium outcome. This completes the description of the equilibrium solution.

4 Comparative Statics

The thresholds $m$ and $\bar{m}$ given by (17) and (18) depend on the parameter values. Figure 1 illustrates how $m$ and $\bar{m}$ depend on the degree of economic transparency $\kappa \in [0, 1]$ for $\bar{\tau} = s = 0$, $\beta = \theta = 1$, $\sigma_\tau^2 = \sigma_v^2 = 1$ and $C = 1/2$. These parameter values imply that with perfect economic transparency ($\kappa = 1$), the government’s desired policy is $m_G = 1$ and the region of independence is $[0, 2]$. When there is economic opacity ($0 < \kappa < 1$), Figure 1 shows that the boundaries of the region of independence $\bar{m}$ and $\bar{m}$ are not symmetric around $m_G$. Intuitively, the government has expansionary preferences ($\beta > 0$), so it is willing to give the central bank more leeway to expand the money supply.\footnote{Formally, when $\beta > 0$ the government anticipates a larger surprise shock $|\eta|$ at $\bar{m}$ than at $m$: $|E[v|\bar{m} \geq \bar{m}] - s| > |E[v|m \leq \bar{m}] - s|$. So, the government tolerates greater deviations on the upside than...} Furthermore, Figure 1 shows...
that \( \bar{m} \) is decreasing and \( m \) is increasing in the degree of economic transparency \( \kappa \), thereby shrinking the region of independence \([m, \bar{m}]\). In fact, this result holds more generally:

**Proposition 1** The region of independence \([m, \bar{m}]\) is decreasing in the degree of economic transparency \( \kappa \).

The proof is in Appendix A.2. It shows analytically that \( \frac{d\bar{m}}{d\kappa} < 0 \) and \( \frac{dm}{d\kappa} > 0 \), so that \( \frac{d(\bar{m} - m)}{d\kappa} < 0 \). Intuitively, when there is economic opacity, the government does not observe the velocity shock \( v \), so it is not sure whether it is appropriate to intervene and what level of the money supply to set. Greater economic opacity makes the government more cautious and less likely to interfere with monetary policy. As a result, less economic transparency \( \kappa \) increases the region of independence. Figure 1 shows that reducing transparency (from \( \kappa = 1 \) to \( \kappa = 0 \)) could more than double the size of the region of independence (from 2 to over 5).

Economic opacity also increases the probability that the central bank enjoys independence. Formally, the probability of independence (i.e. no government interference) equals \( P_I = \Phi(\bar{z}) - \Phi(z) \), so \( \frac{dP_I}{d\kappa} = \left( \phi(\bar{z}) \frac{d\bar{m}}{d\kappa} - \phi(z) \frac{dm}{d\kappa} \right) / \sqrt{\sigma_v^2 + (1 - \kappa)\sigma_v^2} < 0 \). The higher probability of independence reduces average inflation because there is less need to adjust on the downside. But for \( \beta = 0 \), \( |E[v|\bar{m} \geq \bar{m}] - s| = |E[v|\bar{m} \leq \bar{m}] - s| \) and the region of independence is symmetric around \( \bar{\tau} - s \).
monetary policy towards the inflation level $\bar{\tau} + \beta\theta$, which exceeds the central bank’s average $\bar{\tau}$. So, greater economic transparency increases the probability of political pressures and leads to higher average inflation.\(^\text{12}\)

The effect of a higher variance of velocity shocks $\sigma^2_v$ is the same as a reduction in economic transparency $\kappa$. The reason is that $m$ and $\bar{m}$ in (17) and (18) only depend on $\kappa$ and $\sigma^2_v$ through $(1 - \kappa)\sigma^2_v$, so that an increase in $\sigma^2_v$ has qualitatively the same effect as a drop in $\kappa$. However, greater uncertainty about the central bank’s inflation target $\sigma^2_\tau$ gives rise to different effects.

**Proposition 2** Under economic transparency ($\kappa = 1$), the region of independence $[m, \bar{m}]$ is not affected by preference uncertainty $\sigma^2_\tau$. Under economic opacity ($0 \leq \kappa < 1$), the region of independence $[m, \bar{m}]$ is decreasing in the amount of preference uncertainty $\sigma^2_\tau$ for $\beta\theta \leq \sqrt{2C}$.

The proof is in Appendix A.2. Intuitively, when there is complete economic transparency ($\kappa = 1$) the government can perfectly infer from the central bank’s policy decision $m_{CB}$ whether or not it is appropriate to intervene. In addition, it also knows exactly what policy to implement. As a result, the amount of preference uncertainty $\sigma^2_\tau$ is immaterial. But when there is some economic opacity ($0 \leq \kappa < 1$), greater preference uncertainty $\sigma^2_\tau$ makes the policy action $m_{CB}$ a more useful indicator of the central bank’s intentions, so the government becomes more responsive to it and allows for less variation in $m_{CB}$ before intervening. The proof shows that $\beta\theta \leq \sqrt{2C}$ is a sufficient condition for the negative relation between preference uncertainty and the region of independence. For $\beta\theta > \sqrt{2C}$, numerical simulations indicate that $\bar{m} - m$ still tends to be decreasing in $\sigma^2_\tau$, although $m$ can be non-monotonic for small $\sigma^2_\tau$.

In the limiting case of perfect preference transparency ($\sigma^2_\tau \to 0$), no finite boundaries $m$ and $\bar{m}$ exist.\(^\text{13}\) With perfect preference transparency ($\sigma^2_\tau \to 0$), the central bank’s inflation target converges to the government’s target $\bar{\tau}$ and the central bank enjoys complete independence for $\beta\theta \leq \sqrt{2C}$. Intuitively, the central bank’s policy already gives an inflation rate of $\bar{\tau}$, so if the government’s inflation bias $\beta\theta$ is sufficiently small, the benefit of overriding is less than the cost $C$. However, for $\beta\theta > \sqrt{2C}$ the government’s expansionary preferences outweigh the overriding cost, so the government always interferes and the central bank has no independence under perfect preference transparency.

More generally, lower overriding costs reduce the independence of the central bank:

\(^{12}\)Interestingly, economic secrecy is not only desired by the central bank but it is also preferred by the government at the beginning of the game, because it gives rise to lower inflation without affecting average output due to rational private sector inflation expectations.

\(^{13}\)To see this, note that $\frac{\phi(\tilde{z})}{1 - \Phi(\tilde{z})}$ has an asymptote of $\tilde{z}$ as $\tilde{m} \to \infty$, so for $\sigma^2_\tau \to 0$ the right-hand side of (18) goes to $\beta\theta + \bar{m} + \sqrt{2C}$. This means that (18) yields no fixed point for $\tilde{m}$. Similarly, the right-hand side of (17) goes to $\beta\theta + \bar{m} - \sqrt{2C}$ as $\sigma^2_\tau \to 0$ so that there is also no fixed point for $\bar{m}$.
Proposition 3  The region of independence \([m, \bar{m}]\) is increasing in the overriding cost \(C\).

The proof is in Appendix A.2. It shows analytically that \(\frac{d\bar{m}}{dC} > 0\) and \(\frac{dm}{dk} < 0\), so that \(\frac{d(\bar{m} - m)}{dk} > 0\). This result is very intuitive. When the government faces a higher overriding cost it becomes more reluctant to interfere with monetary policy. So, the region of independence increases and the probability of independence rises as well. Formally,

\[
\frac{dP_I}{dC} = \left( \frac{\phi(\bar{z}) \frac{d\bar{m}}{dC} - \phi(z) \frac{dm}{dC}}{\sqrt{\sigma^2 + (1 - \kappa)\sigma^2_v}} \right) + 2\sqrt{2C}.
\]

This reveals that in the presence of economic opacity (0 ≤ \(\kappa < 1\)), the size of the region of independence remains strictly positive even if the direct overriding cost \(C\) is zero. The reason is that the government cannot observe the velocity shock, so it faces uncertainty about the appropriate monetary policy stance. This makes the government reluctant to override the central bank, whose policy decision is based on better economic information. Thus, economic opacity could serve as a substitute for direct overriding costs. In particular, a central bank that suffers from a government with low overriding costs \(C\) could envelop itself in economic secrecy to effectively make political interference more costly.

5 Discussion

The model considered so far is based on several simplistic assumptions regarding the economic structure and the objective functions of the central bank and the government. It is now shown that the results in Propositions 1, 2 and 3 hold more generally. First, an extension of the model is considered with standard objective functions that exhibit a concern about the stabilization of both inflation and output. Second, a richer economic structure is discussed.

Suppose that the government not only aims to stimulate output beyond the natural rate but also cares about output stabilization, so that

\[
W_G = -\frac{1}{2} \alpha (\pi - \bar{\tau})^2 - \frac{1}{2} (y - k\bar{y})^2
\]

where \(\alpha\) denotes the concern for inflation stabilization (\(\alpha > 0\)) and \(k\bar{y}\) is the government’s output target (\(k > 1\)). Such a quadratic objective function is consistent with microfoundations and the assumption that the output target exceeds the natural rate (\(k > 1\)) could be based on a plausible market imperfection such as monopolistic competition in the goods market. In addition, suppose that the central bank is no longer an ‘inflation nutter’ that puts no weight on output stabilization. Instead, the central bank cares as much about output stabilization as
the government but it is ‘responsible’ in the sense that it does not attempt to stimulate output beyond the natural rate (Blinder 1997):

\[ W_{CB} = -\frac{1}{2} \alpha (\pi - \tau)^2 - \frac{1}{2} (y - \bar{y})^2 \]  

(20)

Appendix A.3 derives the results for this model extension. It shows that the algebraic expressions become messier but Propositions 1 and 3 continue to hold. Proposition 2 also holds when the sufficient condition \( \beta \theta \leq \sqrt{2C} \) is replaced by \( \frac{\theta}{\sqrt{\alpha + \theta^2}} (k - 1) \bar{y} \leq \sqrt{2C} \), which again means that the overriding cost \( C \) dominates the government’s expansionary preferences \( (k > 1) \).

Now consider a less simplistic economic structure. The money market equation (1) could be replaced by the quantity equation

\[ \pi = m + v - y \]

It is straightforward to check that this only makes the expressions for the money supply \( m \) and the corresponding thresholds more complicated because of an additional intercept term, without affecting any of the qualitative economic results.

A more realistic economic structure would feature aggregate supply shocks \( \varepsilon \), replacing (2) by

\[ y = \bar{y} + \theta (\pi - \pi^e) + \varepsilon \]  

(21)

The introduction of supply shocks \( \varepsilon \) has no effect on the conclusions of the model when there is symmetric information about the supply shocks. When the central bank has private information about the supply shocks \( \varepsilon \), the results in the basic model of section 2 are not affected since \( \varepsilon \) does not affect the money supply \( m \). But in the extended model with the quadratic objectives (19) and (20), opacity about the supply shocks \( \varepsilon \) does influence the outcomes, although in a similar way to opacity about the velocity shocks \( v \). In particular, when the degree of transparency \( \kappa \) is the same for the economic shocks \( \varepsilon \) and \( v \), the results can simply be obtained by replacing \( v \) by \( v \varepsilon = v + \frac{\theta}{\alpha + \theta^2} \varepsilon \) in the algebraic expressions. So, Propositions 1, 2 and 3 continue to hold.

In addition, instead of the neo-monetarist framework in this paper, there could be an interest rate transmission mechanism. Then the monetary policy instrument is the interest rate and (1) would be replaced by an aggregate demand relation with demand shock \( d \), while (21) could be inverted to get the expectations-augmented Phillips curve

\[ \pi = \pi^e + \frac{1}{\theta} (y - \bar{y}) - \frac{1}{\theta} \varepsilon \]

In that case, aggregate demand shocks \( d \) and aggregate supply shocks \( \varepsilon \) matter for economic transparency, but otherwise the conclusions are similar.
Furthermore, the economy could be described by a New Keynesian transmission mechanism with the forward-looking Phillips curve

\[ \pi_t = E_t [\pi_{t+1}^e] + \frac{1}{\theta} (y_t - \bar{y}) - \frac{1}{\theta} \varepsilon_t \]

where the supply shock \( \varepsilon_t \) and inflation target \( \tau_t \) are i.i.d.. Then, the outcomes are exactly the same as in the static model, except that \( \pi^e \) is now replaced by \( E_t [\pi_{t+1}^e] \). Since Propositions 1, 2 and 3 hold for any \( \pi^e \), the results are not affected. So, the conclusions of the paper are robust to this extension with a New Keynesian Phillips curve.

The effect of preference uncertainty on government overriding has also been analyzed by Eijffinger and Hoeberichts (2002), who assume (19), (20), (21) and economic transparency. In contrast to Proposition 2, they find that greater preference uncertainty about the central bank’s preference parameter for inflation stabilization \( \alpha \) increases the expected region of independence. However, it is known that their specification of relative preference uncertainty effectively makes the central bank less conservative, whereas greater uncertainty about the parameter for output stabilization would make the central bank more conservative and reverse their results.\(^{14}\) For an ‘unbiased’ specification that does not distort conservativeness, the effect of greater relative preference uncertainty in the Eijffinger and Hoeberichts (2002) model would disappear, similar to the result in Proposition 2 that preference uncertainty does not affect the region of independence in the case of economic transparency.

The present paper is the first to establish that economic transparency reduces the region of independence for the central bank. Furthermore, it derives the novel result that economic opacity gives the central bank some freedom from political pressures even if there is no direct overriding cost \( (C = 0) \).

Thus, this paper provides a theoretical argument for the observation that central banks could adopt secrecy to obtain greater independence.\(^{15}\) An interesting example is the way in which the Federal Reserve under Chairman Paul Volcker managed to implement a painful disinflation policy during the early 1980s. The introduction of monetary targeting in October 1979 made it more difficult for Congress to assess whether high interest rates were due to restrictive monetary policy or market forces. The change in monetary operating procedures effectively made the monetary policy instrument a less reliable signal of the policy stance due to imperfect information about money market disturbances. So, Congress felt more reluctant to challenge the monetary policy actions of the Federal Reserve. As a result, the ‘monetary veil’ provided cover to pursue the disinflation without political interference.

The present paper suggests that central banks with lower independence benefit more from secrecy to fend off government intervention, so they are less likely to be transparent. Thus,

\(^{14}\) This was first pointed out by Beetsma and Jensen (2003). Geraats (2004) provides further details on the pitfalls of modeling relative preference uncertainty.

\(^{15}\) For instance, Goodfriend (1986, p. 82) argues that “secrecy makes it more difficult for particular political groups to pressure the Federal Reserve regarding current policy actions”.

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it predicts a positive relation between central bank independence and transparency. To investigate this empirically, the comprehensive survey of central banks by Fry, Julius, Mahadeva, Roger and Sterne (2000) is used. Fry et al. (2000, Table 4.6) construct an index for ‘policy explanations’ based on twelve items covering explanations of policy decisions, forecasts and forward looking analysis, and policy assessments and research. This measure is used as a proxy for economic transparency.\(^{16}\) In addition, Fry et al. (2000, Table 4.4) provide an index for central bank independence that captures statutory objectives of price stability, goal and instrument independence, limits on monetary financing of budget deficits, and the length of central bankers’ term of office. It also comprises a separate measure for instrument independence. Data is available for 92 countries.

Table 1: Relation between central bank transparency and independence.

<table>
<thead>
<tr>
<th>Correlation with transparency</th>
<th>[p-value]</th>
<th>Full sample</th>
<th>Excl. fixed FX</th>
<th>Fixed FX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independence</td>
<td>0.430</td>
<td>0.450</td>
<td>0.261</td>
<td>0.157</td>
</tr>
<tr>
<td>Instrument independence</td>
<td>0.339</td>
<td>0.392</td>
<td>0.186</td>
<td>0.316</td>
</tr>
<tr>
<td>Sample size</td>
<td>92</td>
<td>61</td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 shows that there is a statistically significant positive correlation between transparency and central bank independence (with p-values in brackets). Using the more specific measure of instrument independence gives the same finding. This is consistent with the theoretical prediction of this paper that central banks with lower independence are likely to display lower transparency.

However, there is an alternative, public policy argument that also generates a positive relation between central bank independence and transparency. Institutional independence requires public accountability to safeguard democratic legitimacy, and accountability requires transparency. Fortunately, it is possible to distinguish between this public policy motive and the economic explanation advanced in this paper. The former should always apply regardless of the monetary policy framework, whereas the latter relies on the presence of discretionary monetary policy. In particular, the economic argument does not apply to countries that commit to a fixed exchange rate.

Table 1 shows that there is indeed a marked difference between countries with and without a fixed exchange rate regime. The correlation between transparency and (instrument) independence is positive and highly significant for countries without a fixed exchange rate, but it is much weaker for countries that have abandoned discretion over monetary policy by the adoption of a fixed exchange rate regime.\(^{17}\) These findings provide some tentative empiri-

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\(^{16}\)Three out of twelve items do not pertain to economic transparency and have a weight of 15.5%. Reconstructing the index to get a more accurate measure of economic transparency yields similar conclusions.

\(^{17}\)Rank correlations of transparency with independence and instrument independence give similar results: 0.504 [\(<0.001\)] and 0.373 [\(<0.001\)] for the full sample; 0.483 [\(<0.001\)] and 0.381 [0.003] excluding fixed exchange rates; and 0.360 [0.047] and 0.323 [0.073] for countries with a fixed exchange rate regime.
ical support for the economic argument formalized in this paper that the positive relationship between central bank independence and transparency is caused by the greater secrecy that central banks under stronger political pressures adopt to limit government interference.

Finally, it should be emphasized that the present paper analyzes the effects of transparency for a given institutional framework. The override mechanism captures the lack of complete instrument independence that used to be prevalent and still applies to many developing countries. The seminal contributions by Walsh (1995) and Svensson (1997) suggest better institutional frameworks through contracting and inflation targeting. An interesting topic is to analyze how the effects of disclosure policy depend on the institutional settings, but this is left for future research.

6 Conclusion

The new paradigm in monetary policy of central bank independence and transparency has rapidly gained ground. This paper cautions that transparency may not be beneficial without central bank independence. In particular, uncertainty about the economic information to which the central bank responds makes politicians more cautious about intervening in monetary policy because it is harder to interpret the central bank’s actions. As a result, economic secrecy effectively gives the central bank greater political independence.

This paper has formalized this argument using a monetary policy game in which a conservative or responsible central bank without complete independence sets monetary policy. The government, which aims to stimulate output beyond the natural rate, can override the monetary policy decision, but this involves a direct override cost. The government’s decision to override the central bank is complicated by the presence of uncertainty about the central bank’s intentions and imperfect information about the economic situation.

It is shown that the region of independence enjoyed by the central bank is declining in the degree of economic transparency and in the amount of preference uncertainty. Intuitively, economic transparency reduces the government’s uncertainty about whether to override and how to set the policy instrument, so it makes the government less inhibited to interfere with monetary policy. Greater preference uncertainty makes the central bank’s policy action a more useful signal of its intentions, so the government becomes more sensitive to it and leaves the central bank less leeway before overriding.

The region of independence is increasing in the overriding cost for the government. More interestingly, this paper obtains the new result that even in the absence of a direct overriding cost, the size of the region of independence is strictly positive when there is economic opacity. Intuitively, if the government feels uninhibited to interfere with monetary policy, the central bank could effectively make overriding costly by depriving the government of important economic information. Thus, the central bank could insulate itself from political pressures by
enveloping itself in economic secrecy.

The model generates the theoretical prediction that central banks with lower independence are more likely to display less transparency. Empirically, there is indeed a strong positive correlation between central bank independence and transparency. But this could also be for public policy reasons as central bank independence requires accountability and therefore transparency. Interestingly, the positive relation between independence and transparency is much weaker for countries that maintain a fixed exchange rate regime. This supports the economic explanation advanced in this paper, which relies on discretionary monetary policy.

The main conclusion of the paper is that economic opacity could be beneficial if the central bank lacks complete instrument independence because it makes it more difficult for the government to interfere with monetary policy. This helps to explain the past practice of independence-through-secrecy. The paper also has policy implications for countries that wish to adopt the new paradigm of central bank independence-cum-transparency. It is important to ensure that the central bank has political independence before insisting on economic transparency, since monetary mystique is an effective way to prevent political pressures.
A Appendix

Appendix A.1 derives the no-override condition (11) for the basic model of section 2 with objective functions (3) and (4). Propositions 1, 2 and 3 presented in section 4 are proved in appendix A.2. The derivation of the results for the extended model in section 5 with objective functions (19) and (20) is in appendix A.3.

A.1 No-override condition

The condition for no government interference is given by (10):

\[ E[W_G(m_O) | m_{CB}] - C \leq E[W_G(m_{CB}) | m_{CB}] \]

This is equivalent to \( E[D | m_{CB}] \leq C \), where \( D \equiv W_G(m_O) - W_G(m_{CB}) \). Substitute (2) and (1) into (3) to get

\[ W_G = -\frac{1}{2} (m + v - \bar{\tau})^2 + \beta \theta (m + v - \pi^e) \]

So,

\[ D = -\frac{1}{2} (m_O^2 - m_{CB}^2) + (m_O - m_{CB}) (\bar{\tau} + \beta \theta) - (m_O - m_{CB}) v \]

Substituting (9) and rearranging,

\[ D = \frac{1}{2} (\bar{\tau} + \beta \theta - m_{CB})^2 - \frac{1}{2} (E[v|m_{CB}])^2 - (\bar{\tau} + \beta \theta - E[v|m_{CB}] - m_{CB}) v \]

Taking expectations and simplifying gives

\[ E[D | m_{CB}] = \frac{1}{2} (\bar{\tau} + \beta \theta - m_{CB})^2 + \frac{1}{2} (E[v|m_{CB}])^2 - (\bar{\tau} + \beta \theta - m_{CB}) E[v|m_{CB}] \]

\[ = \frac{1}{2} (\bar{\tau} + \beta \theta - E[v|m_{CB}] - m_{CB})^2 \]

Hence, (10) if and only if (11).

A.2 Proof of Propositions 1, 2 and 3

To facilitate the derivation of results for the extended model with objective functions (19) and (20), this section proves Propositions 1, 2 and 3 for a general model in which the no-override condition is

\[ \frac{1}{2} b (B - E[v|m_{CB}] - m_{CB})^2 \leq C \]

(22)

So, the thresholds of the region of independence are determined by

\[ \bar{m} = B - E[v|m \geq \bar{m}] + \sqrt{2C/b} \]

(23)

\[ m = B - E[v|m \leq \bar{m}] - \sqrt{2C/b} \]

(24)
The central bank’s money supply without political pressures is assumed to satisfy \( \tilde{m} \sim N (A - s, a^2 \sigma_z^2 + (1 - \kappa) \sigma_v^2) \). The corresponding expected velocity shock equals

\[
E[v | \tilde{m} \geq \bar{m}] = S - \frac{(1 - \kappa) \sigma_v^2}{\sqrt{a^2 \sigma_z^2 + (1 - \kappa) \sigma_v^2}} \phi (\bar{z}) 
\]

(25)

\[
E[v | \tilde{m} \leq \bar{m}] = S + \frac{(1 - \kappa) \sigma_v^2}{\sqrt{a^2 \sigma_z^2 + (1 - \kappa) \sigma_v^2}} \phi (\bar{z})
\]

(26)

Substituting (25) into (23) and (26) into (24) it follows that the thresholds satisfy

\[
m < B - s < \bar{m}
\]

(27)

The normalized thresholds are

\[
\bar{z} \equiv \frac{\bar{m} - (A - s)}{\sqrt{a^2 \sigma_z^2 + (1 - \kappa) \sigma_v^2}} \quad \text{and} \quad \bar{z} \equiv \frac{m - (A - s)}{\sqrt{a^2 \sigma_z^2 + (1 - \kappa) \sigma_v^2}}
\]

(28)

The coefficients are assumed to satisfy \( B > A, b > 0 \) and \( a > 0 \). For the basic model of section 2, \( B = \bar{\tau} + \beta \theta, A = \bar{\tau} \) and \( b = a = 1 \).

The proofs of Propositions 1, 2 and 3 make use of the following two results:

**Lemma 1** The function \( \frac{\phi(z)}{1 - \Phi(z)} \) is convex and satisfies \( 0 < \frac{d}{dz} \frac{\phi(z)}{1 - \Phi(z)} < 1 \) for \( z \in \mathbb{R} \).

**Proof.** See Sampford (1953). □

Note that \( \frac{\phi(z)}{1 - \Phi(z)} \) is increasing, with a horizontal asymptote of 0 as \( z \to -\infty \) and an asymptote of \( z \) as \( z \to \infty \). So, it follows from convexity that \( \frac{\phi(z)}{1 - \Phi(z)} > z \) for \( z \in \mathbb{R} \).

**Lemma 2** The function \( \frac{\phi(z)}{\Phi(z)} \) is convex and satisfies \(-1 < \frac{d}{dz} \frac{\phi(z)}{\Phi(z)} < 0 \) for \( z \in \mathbb{R} \).

**Proof.** Using the fact that \( \phi(z) = \phi(-z) \) and \( \Phi(z) = 1 - \Phi(-z) \), \( \frac{\phi(z)}{\Phi(z)} = \frac{\phi(-z)}{1 - \Phi(-z)} \). So, the result is a corollary of Lemma 1. □

Note that \( \frac{\phi(z)}{\Phi(z)} \) is decreasing, with an asymptote of \(-z \) as \( z \to -\infty \) and a horizontal asymptote of 0 as \( z \to \infty \). So, it follows from convexity that \( \frac{\phi(z)}{\Phi(z)} > -z \) for \( z \in \mathbb{R} \).

The proof of Proposition 3 is presented first since it is the simplest and uses the same approach as the proofs of Propositions 1 and 2.

**Proof of Proposition 3:**

Differentiate (23) with respect to \( C \) using (25) and (28) to get

\[
\frac{d \tilde{m}}{dC} = \frac{(1 - \kappa) \sigma_v^2}{a^2 \sigma_z^2 + (1 - \kappa) \sigma_v^2} \frac{d \phi(\bar{z})}{d \bar{z}} + \frac{d \tilde{m}}{dC} \frac{d \phi(\bar{z})}{d \bar{z}} + \frac{1}{\sqrt{2bC}}
\]
Rearranging gives
\[
\frac{d \tilde{m}}{dC} = \frac{1}{1 - \frac{(1-\kappa)\sigma_v^2}{\sigma^2\sigma_r^2 + (1-\kappa)\sigma_v^2} \frac{d}{d\tilde{z}} \phi(\tilde{z})} \frac{1}{\sqrt{2bC}} > 0
\]  
using Lemma 1.

Similarly, differentiate (24) with respect to \(C\) using (26) and (28) to get
\[
\frac{d m}{dC} = -\frac{(1 - \kappa)\sigma_v^2}{a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2} \frac{d}{d\tilde{z}} \phi(\tilde{z}) \frac{1}{\sqrt{2bC}}
\]  
Rearranging gives
\[
\frac{d m}{dC} = -\frac{1}{1 + \frac{(1-\kappa)\sigma_v^2}{a^2\sigma_r^2 + (1-\kappa)\sigma_v^2} \frac{d}{d\tilde{z}} \phi(\tilde{z})} \frac{1}{\sqrt{2bC}} < 0
\]  
using Lemma 2.

As a result, \(\frac{(\tilde{m} - m)}{dC} > 0\) so that the region of independence \([\tilde{m}, \bar{m}]\) is increasing in the override cost \(C\).

**Proof of Proposition 1:**

The proof proceeds in two parts. First it is shown that \(\frac{d \tilde{m}}{d\kappa} < 0\), and then that \(\frac{d m}{d\kappa} > 0\).

(I) Differentiate (23) with respect to \(\kappa\) using (25) and (28) to get
\[
\frac{d \tilde{m}}{d\kappa} = -\frac{\sigma_v^2}{\sigma_r^2 + (1 - \kappa)\sigma_v^2} [a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2]^{3/2} \frac{\phi(\tilde{z})}{1 - \Phi(\tilde{z})} + \frac{(1 - \kappa)\sigma_v^2}{a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2} \left( \frac{d}{d\tilde{z}} \phi(\tilde{z}) \right) \left( \frac{d \tilde{m}}{d\kappa} + \frac{1}{2} \frac{\sigma_v^2}{\sigma_r^2 + (1 - \kappa)\sigma_v^2} \frac{d}{d\tilde{z}} (\bar{m} - (A - s)) \right)
\]  
This already gives \(\frac{d \tilde{m}}{d\kappa} \bigg|_{\kappa=1} = -\frac{\sigma_v^2}{\sigma_r^2} \frac{\phi(\tilde{z})}{1 - \Phi(\tilde{z})} < 0\). Rearranging yields
\[
\left[ 1 - \frac{(1 - \kappa)\sigma_v^2}{a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2} \frac{d}{d\tilde{z}} \phi(\tilde{z}) \right] \frac{d \tilde{m}}{d\kappa} = -\frac{\sigma_v^2}{\sigma_r^2 + (1 - \kappa)\sigma_v^2} \frac{\phi(\tilde{z})}{(a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2)^{3/2}} + \frac{1}{2} \frac{(1 - \kappa)\sigma_v^2}{a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2} \left( \frac{d}{d\tilde{z}} \phi(\tilde{z}) \right) \left[ \frac{d \tilde{m}}{d\kappa} + \frac{1}{2} \frac{\sigma_v^2}{\sigma_r^2 + (1 - \kappa)\sigma_v^2} \frac{d}{d\tilde{z}} (\bar{m} - (A - s)) \right]
\]  
Note that the left-hand-side factor in square brackets is strictly positive by Lemma 1 so that \(\text{sgn} \left( \frac{d \tilde{m}}{d\kappa} \right) = \text{sgn} \tilde{R}\). The first term of \(\tilde{R}\) is strictly negative, whereas the second term is positive since \(\frac{d}{d\tilde{z}} \phi(\tilde{z}) > 0\) and \(\tilde{m} > A - s\), using (27) and \(B > A\). To establish \(\text{sgn} \tilde{R}\), use the fact that \(\frac{\phi(\tilde{z})}{1 - \Phi(\tilde{z})} > \tilde{z}\) and \(\frac{d}{d\tilde{z}} \phi(\tilde{z}) < 1\) to get
\[
\tilde{R} < -\frac{\sigma_v^2 a^2 \sigma_r^2 + \frac{1}{2} (1 - \kappa)\sigma_v^4}{[a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2]^{3/2}} \tilde{z} + \frac{1}{2} \frac{(1 - \kappa)\sigma_v^4}{[a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2]^2} [\bar{m} - (A - s)]
\]  
Substituting for \(\tilde{z}\) using (28) and simplifying gives
\[
\tilde{R} < -\frac{\sigma_v^2 a^2 \sigma_r^2}{[a^2\sigma_r^2 + (1 - \kappa)\sigma_v^2]^2} [\bar{m} - (A - s)] < 0
\]
Hence, \( \frac{\bar{m}}{\sigma^2} < 0 \).

(II) Differentiate (24) with respect to \( \kappa \) using (26) and (28) to get

\[
\frac{d \bar{m}}{d \kappa} = -\sigma^2 \left[ \sigma^2 + 2 (1 - \kappa) \sigma^2 + \frac{1}{2} \frac{\sigma^2}{a^2 \sigma^2 + (1 - \kappa) \sigma^2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \right]
\]

This already gives \( \frac{d \bar{m}}{d \kappa} \big|_{\kappa=1} = \frac{\sigma^2}{\sigma^2} = 0 \). Rearranging yields

\[
\left[ 1 + \frac{1}{a^2 \sigma^2 + (1 - \kappa) \sigma^2} \frac{d \phi(\bar{z})}{d \bar{z} \Phi(\bar{z})} \right] \frac{d \bar{m}}{d \kappa} = \frac{\sigma^2}{a^2 \sigma^2 + (1 - \kappa) \sigma^2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \left[ 1 - \frac{1}{2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \right] \left[ m - (A - s) \right] = \frac{\phi(\bar{z})}{\Phi(\bar{z})}
\]

Note that the left-hand-side factor in square brackets is strictly positive by Lemma 2 so that \( \text{sgn} \left( \frac{d \bar{m}}{d \kappa} \right) = \text{sgn} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \). The first term of \( R \) is strictly positive, whereas the second term is ambiguous. However, if \( \bar{m} \geq A - s \) then the second term of \( R \) is non-negative as \( \frac{d \phi(\bar{z})}{d \bar{z} \Phi(\bar{z})} < 0 \), so that \( R > 0 \). To establish \( \text{sgn} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \) when \( \bar{m} < A - s \), use the fact that \( \frac{\phi(\bar{z})}{\Phi(\bar{z})} > -\bar{z} \) to get

\[
R > -\sigma^2 \left[ \sigma^2 + 2 (1 - \kappa) \sigma^2 + \frac{1}{2} \frac{\sigma^2}{a^2 \sigma^2 + (1 - \kappa) \sigma^2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \right] \left[ 1 - \frac{1}{2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \right] \left[ m - (A - s) \right] > 0 \quad \text{for} \quad \bar{m} < A - s
\]

As a result, it always holds that \( R > 0 \). Therefore, \( \frac{d \bar{m}}{d \kappa} > 0 \).

Finally, combining the results under (I) and (II) yields \( \frac{d(\bar{m} - m)}{d \kappa} < 0 \), so that the region of independence is decreasing in the degree of economic transparency. 

**Proof of Proposition 2:**

The proof again proceeds in two parts. First it is shown that \( \frac{d \bar{m}}{d \sigma^2} = 0 \) for \( \kappa = 1 \) and \( \frac{d \bar{m}}{d \sigma^2} < 0 \) for \( 0 \leq \kappa < 1 \), and then that \( \frac{d \bar{m}}{d \sigma^2} = 0 \) for \( \kappa = 1 \) and \( \frac{d \bar{m}}{d \sigma^2} > 0 \) for \( B - A \leq \sqrt{2C/b} \) and \( 0 \leq \kappa < 1 \).

(I) Differentiate (23) with respect to \( \sigma^2 \) using (25) and (28) to get

\[
\frac{d \bar{m}}{d \sigma^2} = \frac{1}{2} \frac{\sigma^2}{a^2 \sigma^2 + (1 - \kappa) \sigma^2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \left[ 1 - \frac{1}{2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} \right] \left[ m - (A - s) \right]
\]
This gives \( \frac{d m}{d \sigma^2} \bigg|_{\kappa=1} = 0 \). Rearranging yields
\[
\left[ 1 - \frac{(1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \frac{d}{d \Phi(\bar{z})} \frac{d \phi(\bar{z})}{d \bar{z}} \right] \frac{d \bar{m}}{d \sigma^2} = -1 \frac{a^2 (1 - \kappa) \sigma_v^2}{2 \left[ a^2 \sigma^2 + (1 - \kappa) \sigma_v^2 \right]^{3/2}} \frac{\phi(\bar{z})}{1 - 1 - \phi(\bar{z})}[\bar{m} - (A - s)]
\]
Note that the left-hand-side factor in square brackets is strictly positive by Lemma 1. In addition, the right-hand side is (strictly) negative (for \( \kappa \neq 1 \)) since \( \frac{d}{d \bar{z}} \frac{\phi(\bar{z})}{1 - \phi(\bar{z})} > 0 \) and \( \bar{m} > A - s \), using (27) and \( B > A \). As a result, \( \frac{d \bar{m}}{d \sigma^2} < 0 \) for \( 0 \leq \kappa < 1 \).

(II) Differentiate (24) with respect to \( \sigma^2_v \) using (26) and (28) to get
\[
\frac{d \bar{m}}{d \sigma^2_v} = \frac{1}{2} \frac{a^2 (1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} - \frac{(1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \left[ \frac{d}{d \Phi(\bar{z})} \frac{d \phi(\bar{z})}{d \bar{z}} \right] \left\{ \frac{d \bar{m}}{d \sigma^2} - \frac{a^2}{2} \frac{m - (A - s)}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \right\}
\]
This gives \( \frac{d \bar{m}}{d \sigma^2_v} \bigg|_{\kappa=1} = 0 \). Rearranging yields
\[
\left[ 1 + \frac{(1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \frac{d}{d \Phi(\bar{z})} \frac{d \phi(\bar{z})}{d \bar{z}} \right] \frac{d \bar{m}}{d \sigma^2_v} = \frac{1}{2} \frac{a^2 (1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \frac{\phi(\bar{z})}{\Phi(\bar{z})} + \frac{1}{2} \frac{a^2 (1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \left[ \frac{d}{d \Phi(\bar{z})} \frac{d \phi(\bar{z})}{d \bar{z}} \right] \left[ m - (A - s) \right] = R_r
\]
Note that the left-hand-side factor in square brackets is strictly positive by Lemma 2. So, \( \text{sgn} \left( \frac{d \bar{m}}{d \sigma^2_v} \right) = \text{sgn} R_r \). Substituting (24) and (26) and rearranging gives
\[
R_r = \left[ \frac{1}{2} \left[ 1 - \frac{(1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \frac{d}{d \Phi(\bar{z})} \frac{d \phi(\bar{z})}{d \bar{z}} \right] \frac{a^2 (1 - \kappa) \sigma_v^2}{\left[ a^2 \sigma^2 + (1 - \kappa) \sigma_v^2 \right]^{3/2}} \frac{\phi(\bar{z})}{\Phi(\bar{z})} + \frac{1}{2} \frac{a^2 (1 - \kappa) \sigma_v^2}{a^2 \sigma^2 + (1 - \kappa) \sigma_v^2} \left[ \frac{d}{d \Phi(\bar{z})} \frac{d \phi(\bar{z})}{d \bar{z}} \right] \left( B - A - \sqrt{2C/b} \right) \right]
\]
Using Lemma 2, the first term is (strictly) positive (for \( \kappa \neq 1 \)) and the second term is non-negative if \( B - A \leq \sqrt{2C/b} \). So, \( B - A \leq \sqrt{2C/b} \) is a sufficient condition for \( R_r > 0 \) when \( \kappa \neq 1 \). Therefore, \( \frac{d \bar{m}}{d \sigma^2_v} > 0 \) for \( B - A \leq \sqrt{2C/b} \) and \( \kappa \neq 1 \). However, for \( B - A > \sqrt{2C/b} \) numerical computations reveal that \( R_r < 0 \) is possible for small \( \sigma^2_v \) (e.g. for \( B - A = 1, C = 0, a = b = 1, \kappa = 0.5, \sigma_v^2 = 1 \) and \( \sigma^2_v = 0.25 \)) so that \( \bar{m} \) can be non-monotonic in \( \sigma^2_v \).

Finally, combining the results under (I) and (II) yields that \( \frac{d(\bar{m} - m)}{d \sigma^2_v} = 0 \) for \( \kappa = 1 \), so the amount of preference uncertainty is immaterial for the region of independence with perfect economic transparency. For \( 0 \leq \kappa < 1 \), \( \frac{d(\bar{m} - m)}{d \sigma^2_v} < 0 \) for \( B - A \leq \sqrt{2C/b} \), so the region of independence is decreasing in the amount of preference uncertainty when the overriding cost is not too small. For \( B - A > \sqrt{2C/b} \), numerical simulations suggest that despite possible non-monotonicity of \( \bar{m} \), the region of independence \( \bar{m} - m \) continues to be decreasing in \( \sigma^2_v \). Note that for the basic model in section 2 the sufficient condition reduces to \( \beta \theta \leq \sqrt{2C} \). ■
A.3 Derivations for extended model

This appendix derives the results for the extended model of section 5 with objective functions (19) and (20). The condition for no government interference is still equal to (10), which is equivalent to $E[D|m_{CB}] \leq 0$, where $D \equiv W_G(m_O) - W_G(m_{CB})$. Substitute (2) and (1) into (19) to get

$$W_G = -\frac{1}{2} \alpha (m + v - \bar{\tau})^2 - \frac{1}{2} \theta (m + v - \pi^e) - (k - 1) \bar{y}^2$$ \hspace{1cm} (31)

So,

$$D = -\frac{1}{2} (\alpha + \theta^2) (m_O^2 - m_{CB}^2) + (m_O - m_{CB}) [(\alpha + \theta^2) \theta (k - 1) \bar{y} - \bar{E}[v|m_{CB}])$$

Taking expectations and simplifying gives

$$D = \frac{1}{2} (\alpha + \theta^2) \left[ \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e + \theta \frac{1}{\alpha + \theta^2} (k - 1) \bar{y} - \bar{E}[v|m_{CB}]) \right]^2 - \frac{1}{2} (\alpha + \theta^2) \left( \bar{E}[v|m_{CB}] \right)^2$$

The policy action desired by the government follows from maximization of $E[W_G|m_{CB}]$ using (31), subject to (2) and (1) and given $\pi^e$:

$$m_O = \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e + \theta \frac{1}{\alpha + \theta^2} (k - 1) \bar{y} - \bar{E}[v|m_{CB}]$$ \hspace{1cm} (33)

Substituting (33) into (32) and rearranging,

$$D = \frac{1}{2} (\alpha + \theta^2) \left[ \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e + \theta \frac{1}{\alpha + \theta^2} (k - 1) \bar{y} - \bar{E}[v|m_{CB}]) \right]^2 - \frac{1}{2} (\alpha + \theta^2) \left( \bar{E}[v|m_{CB}] \right)^2$$

Taking expectations and simplifying gives

$$E[D|m_{CB}] = \frac{1}{2} (\alpha + \theta^2) \left[ \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e + \theta \frac{1}{\alpha + \theta^2} (k - 1) \bar{y} - \bar{E}[v|m_{CB}]) - m_{CB} \right]^2$$

Hence, the no-override condition equals (22) with $B = \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e + \frac{\theta}{\alpha + \theta^2} (k - 1) \bar{y}$ and $b = \alpha + \theta^2$.

The central bank now maximizes (20) subject to (2) and (1) and given $\pi^e$, so in the absence of political pressure it would implement

$$\hat{m} = \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e - v$$

This means that the expressions for $E[v|m_{CB}]$ are affected. Using joint normality of $\hat{m}$ and $v$,

$$E[v|\hat{m}] = s - \frac{(1 - \kappa) \sigma^2_v}{\sigma^2_{\bar{\tau}} + (1 - \kappa) \sigma^2_v} \left( \hat{m} + s - \frac{\alpha}{\alpha + \theta^2} \bar{\tau} - \frac{\theta^2}{\alpha + \theta^2} \pi^e \right)$$

$$= \lambda_2 s - (1 - \lambda_2) \left( \hat{m} - \frac{\alpha}{\alpha + \theta^2} \bar{\tau} - \frac{\theta^2}{\alpha + \theta^2} \pi^e \right)$$ \hspace{1cm} (34)

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where $\lambda_2 \equiv \frac{\alpha^2}{(\alpha + \theta^2) \pi^2 \tau^2 (1 - \kappa) \pi^2}$. Similarly,

$$
E[v|v \geq \bar{m}] = \lambda_2 s + (1 - \lambda_2) \left( \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e \right) - (1 - \lambda_2) E[v|v \geq \bar{m}]
$$

$$
= s - (1 - \lambda_2) \sqrt{\frac{\alpha^2}{(\alpha + \theta^2)^2 \pi^2 \tau^2 (1 - \kappa) \pi^2} \phi(\bar{z})}
$$

$$
E[v|v \leq m] = \lambda_2 s + (1 - \lambda_2) \left( \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e \right) - (1 - \lambda_2) E[v|v \leq m]
$$

$$
= s + (1 - \lambda_2) \sqrt{\frac{\alpha^2}{(\alpha + \theta^2)^2 \pi^2 \tau^2 (1 - \kappa) \pi^2} \phi(\bar{z})}
$$

where the normalized thresholds now equal

$$
\bar{z} \equiv \frac{\bar{m} - \left( \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e - s \right)}{\sqrt{\frac{\alpha^2}{(\alpha + \theta^2)^2 \pi^2 \tau^2 (1 - \kappa) \pi^2}}} \quad \text{and} \quad \bar{z} \equiv \frac{m - \left( \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e - s \right)}{\sqrt{\frac{\alpha^2}{(\alpha + \theta^2)^2 \pi^2 \tau^2 (1 - \kappa) \pi^2}}}
$$

Hence, for $a = \frac{\alpha}{\alpha + \theta^2}$ and $A = \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e$ the expected velocity shock satisfies (25) and (26) and the normalized thresholds equal (28).

As a result, the extended model with objective functions (19) and (20) corresponds to the general model of appendix A.2 for $B = \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e + \frac{\theta}{\alpha + \theta^2} (k - 1) \bar{y}$, $A = \frac{\alpha}{\alpha + \theta^2} \bar{\tau} + \frac{\theta^2}{\alpha + \theta^2} \pi^e$, $b = \alpha + \theta^2$ and $a = \frac{\alpha}{\alpha + \theta^2}$, and it satisfies the conditions $B > A$, $b > 0$ and $a > 0$. Therefore, Propositions 1 and 3 continue to hold for the model extension. Proposition 2 also holds when the sufficient condition $\beta \theta \leq \sqrt{2C}$ is replaced by $B - A \leq \sqrt{2C/b}$, which reduces to $\frac{\theta}{\sqrt{\alpha + \theta^2}} (k - 1) \bar{y} \leq \sqrt{2C}$.
References


