Loose Commitment*

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Abstract

Due to time-inconsistency or political turnover, policymakers' promises are not always fulfilled. We analyze policy problems combining commitment and discretion. We consider three settings where the planner occasionally defaults on past promises. In the first setting, a default may occur in any period with a given probability. In the second, a planner does not default during a finite tenure but disregards the promises of previous planners. In the third, we make the likelihood of default a function of endogenous variables. We formulate these problems recursively, and provide techniques that can be applied to a general class of models. Our method can be used to analyze the plausibility and the importance of commitment. In a fiscal policy application, we find that average allocations become substantially closer to discretion. In addition, we find that most of the welfare gains from commitment are only achieved when credibility is already high. This result provides theoretical support for the low investment in fiscal policy credibility.

JEL *classification*: C61, C63, E61, E62

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1 Introduction

1.1 Motivation and Contribution

In a general class of problems, households' behavior depends on expectations of future variables. Characterizing optimal policy in such circumstances is intricate. A planner influences households' expectations through its actions, and in turn households' expectations influence the planner's actions. Following the seminal papers by Kydland and Prescott (1977) and Barro and Gordon (1983a), the literature has taken two different approaches to tackle this problem - commitment and discretion.

In the commitment approach it is assumed that the planner will never default on its past promises. Under discretion, a planner can never make and fulfil a promise. These two settings are clearly extreme, it seems more reasonable to assume that institutions and planners sometimes fulfill their promises and sometimes do not. In addition, the commitment and discretion solutions can differ significantly, leaving the researcher without a clear answer.

This paper proposes several frameworks combining commitment and discretion. We first consider a setting where current promises will be fulfilled with a given probability, another setting where promises are only kept during a finite tenure, and lastly we make the likelihood of default a function of endogenous variables. There may be several interpretations for the *loose commitment* settings just described. A political economy interpretation is that governments fulfil their own promises but it is possible that another government is elected and today's promises will not be kept. Another interpretation is that a government commits to future plans, but if particular events arise, such as wars or political instability, defaulting becomes inevitable. As it is common in the discretion literature, we consider that a default on past promises occurs whenever a reoptimization takes place. For the purpose of this paper it is equivalent whether such reoptimization is made by the same planner or by a newly appointed one.

The contribution of this paper is in part methodological, we considerably generalize and extend the work of Roberds (1987) and Schaumburg and Tambalotti (2005). We prove that the solution of these problems is recursive, and provide a methodology that can be applied to a large class of microfounded models. It is not possible to tell a priori whether allocations and welfare under *loose commitment* will be closer to the full commitment or the full discretion cases. Since such results are model dependent, we believe it is important to apply our methodology to any model.

We also provide an application to a fiscal policy problem. In our application, we find that average allocations are substantially closer to discretion. We also conclude that most of the welfare gains from commitment are only achieved when credibility is already high. If one believes that fiscal policy credibility is unlikely to be high, our result provides theoretical support for the low interest in building more credibility. Nevertheless, one could also argue the opposite, i.e. that fiscal authority can fully commit. In such interpretation, our results suggest that fiscal commitment is crucial since small temptations to act discretionally are very costly.

1.2 Methodology

In a very specific model, Roberds (1987) considers that promises may not always be kept. The author's methodology is not generalizable to other models. Schaumburg and Tambalotti (2005) propose a setup equal to one of the three settings described here, and apply it to a monetary model without state variables. Nevertheless, the authors follow a restrictive linear quadratic approach that was criticized by Klein et al. (2004). Moreover, there is an additional drawback of applying the linear quadratic approach in these types of problems. As shown by Debortoli and Nunes (2006), a correct linear-quadratic approximation can in general be derived if one imposes a timeless perspective assumption. The *loose commitment* framework requires a departure from the timeless perspective. As a consequence, using the linear-quadratic approach with *loose commitment* is inappropriate not only because solutions may be inaccurate, but also because the specification of the original model is violated.

The main tools to analyze time-inconsistent and time-consistent policy are recent. The key reference for solving time-inconsistent models is Marcet and Marimon (1998). Klein and Rios-Rull (2003) show how to solve for the time-consistent policy with linear quadratic techniques. Klein et al. (2004) recognize that the techniques proposed in Klein and Rios-Rull (2003) do not deliver controlled accuracy and propose a technique based on generalized Euler equations and a steady state local analysis. Judd (2004) proposes global approximation methods instead of steady state local analysis.

We prove the recursivity of the solution using the tools of Marcet and Marimon (1998). In the solution procedure, we use a global method and generalized Euler equations taking the recent contributions of Judd (2004) and Klein et al. (2004). We show how to solve for linear and non-linear models without and with state variables relying only on one fixed point. As a side product, our methodology can be used as a homotopy method to obtain the time-consistent solution.

1.3 Literature Review

Reputational equilibria as in Backus and Driffill (1985) is a recurrent topic in this literature. Unlike the reputational equilibria literature we are not aiming at building setups where a planner of a certain type resembles another type. We aim at characterizing the solution of planners that can make credible promises, but may be out of charge when it is time to fulfill them. Our results hold in a more plausible and standard infinite horizon framework and we are not limited to models without state variables, as is often the case in reputation models.

Another recurrent topic in this literature is the trigger strategies as in Barro and Gordon (1983b). This paper is not aimed at building equilibria where private agents try to enforce a given equilibrium. To enforce a given equilibrium atomistic private agents need to develop and coordinate on highly sophisticated expectations mechanisms. Even if such strategies are possible, they are very hard to implement and may not be enforced every period. Hence, the planner may not always be forced to fulfil its promises, as in the *loose commitment* setting.

Flood and Isard (1989) consider a central bank commitment to a rule with escape clauses. The rule does not incorporate some important shocks affecting the economy. When such shocks hit the economy it may be better to abandon the rule. One can interpret that our probability of default is their probability of anomalous shocks. Another interpretation is that we consider more rational policymakers, who do not leave important shocks aside the commitment rule. In such interpretation, the rule is always better and the planner only defaults if the commitment technology becomes inoperative. An important difference is that our setting can have state variables being dynamic, while theirs is static.

Persson et al. (2006), elaborating on an earlier proposal of Lucas and Stokey (1983), suggest a mechanism that makes the commitment solution to be time-consistent. Each government should leave its successor with a carefully chosen maturity of nominal and indexed debt for each contingent state of nature and at all maturities. Even though such strategies do eliminate the time-consistency problem, this structure of debt is not observed in reality. The view of this paper is that at certain points in time the commitment solution may be enforced, but in some contingencies discretion is unavoidable.

The paper is organized as follows: section 2 introduces the probabilistic model, section 3 describes the T-periods model, section 4 applies the previous setups to an optimal taxation model, section 5 considers an extension with endogenous probabilities and section 6 concludes.

2 The probabilistic model

We will consider a general model where a planner is not sure whether its promises will be kept or not. As we had anticipated, this uncertainty can be due to several factors. For simplicity, we assume that these events are exogenous and that in any period the economy will experience default or commitment with given exogenous probabilities. In Section 5, we will relax this assumption. Since it is indifferent whether it is the same or a new planner that defaults and reoptimizes, we use the terms "reelection", "new planner" and "default" interchangeably.

To make matters simple, we abstract from any shock other than the random variable s_t describing default (D) or commitment (ND) in period t. It is a straightforward generalization to include other sources of uncertainty, but the notation would be harder to follow. More formally, suppose the occurrence of Default or No Default is driven by a Markov stochastic process $\{s_t\}_{t=1}^{\infty}$ with possible realizations $\bar{s}_t \in \Phi \equiv \{D, ND\}$, and let Ω^t be the set of possible histories up to time t:

$$\Omega^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j \in \Phi, \forall j = 1, ..., t\} \tag{1}$$

We only consider the histories $\omega^t = \{D, \bar{s}_1, \bar{s}_2, ..., \bar{s}_t\}$ that start with default. This is because in the initial period there are no promises to be fulfilled or equivalently the current government has just been settled. Before turning to the planner we describe the problem of individual agents.

2.1 Individual agents and constraints

The economy is populated by individual agents such as rational utility maximizing households and profit maximizing firms. As it is standard to assume, economic agents maximize their objectives taking as given the actions of the government. We describe a very general setting where the first order conditions (FOCs) of households and firms fit the following functional form:

$$b_1(c_t(\omega^t), k_t(\omega^t)) + \beta E_t b_2(c_{t+1}(\omega^{t+1}), k_{t+1}(\omega^{t+1})) = 0 \tag{2}$$

where b_1 and b_2 are vectors of functions, β is the discount factor, E_t denotes rational (mathematical) expectations using available information. The vectors k and c denote the set of states and controls from the perspective of the government.

Given our institutional setting, consumers will believe the promises of the current planner, but will consider that if a different planner comes into play, then different policies will be implemented and past promises will not be kept. As it is common in the time-consistency literature, economic agents will take future controls that can not be committed upon as functions of the state, i.e. $c_{t+1}(\{\omega^t, D\}) = \Psi\{k_{t+1}(\{\omega^t, D\})\}$ where we use the short notation $\{\omega^t, D\}$ to denote $\{\omega^t, \bar{s}_{t+1} = D\}$. $\Psi(.)$ denotes the policy function that rational agents anticipate to be implemented in future periods.[1] The constraint therefore becomes:

$$\begin{aligned} b_1(c_t(\omega^t), k_t(\omega^t)) + \beta Prob(\{\omega^t, ND\}|\omega^t) b_2(c_{t+1}(\{\omega^t, ND\}), k_{t+1}(\{\omega^t, ND\})) \\ + \beta Prob(\{\omega^t, D\}|\omega^t) b_2(\Psi\{k_{t+1}(\{\omega^t, D\})\}, k_{t+1}(\{\omega^t, D\})) = 0 \end{aligned} \tag{3}$$

where we use the short notation $Prob(\{\omega^t, ND\}|\omega^t)$ to denote $Prob(\{s_j\}_{j=0}^{t+1} = \{\omega^t, ND\}|\{s_j\}_{j=0}^{t} = \omega^t)$. The planner will then take as given the FOCs of economic agents. In addition, the planner will have other constraints such as feasibility and its own budget constraint, which either fit the functional form of Eq. (3) or the following functional form:

$$k_{t+1}(\omega^{t+1}) = \ell(c_t(\omega^t), k_t(\omega^t)) \tag{4}$$

Eq. (4) describes the evolution of the states, being ℓ a vector of functions and where it is understood that $k_{t+1}(\{\omega^t, ND\}) = k_{t+1}(\{\omega^t, D\})$, $\forall \omega^t$.[2]

2.2 The planner

When default occurs, a new planner is appointed and it will be taking decisions from that point onwards. Therefore, it is convenient to separate all histories ω^t with respect to the first time that default occurs. This is because we want to know which histories correspond to which planner. We now define the subset of Ω^t of histories where only commitment as occurred up to time t as:

$$\Omega^t_{ND} \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j = ND, \forall j = 1, ..., t\} \tag{5}$$

and the subsets of histories where the first default occurs in period i,

$$\begin{aligned} &\Omega^t_{D,i} \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : (\bar{s}_i = D) \wedge (\bar{s}_j = ND), \forall j = 1, ..., i-1\}, \text{ if } i \leq t \\ &\Omega^t_{D,i} = \oslash, \text{ if } i > t \end{aligned} \tag{6}$$

[1] For further discussions on this issue see Klein et al. (2004).

[2] We consider this formulation for notational convenience. In the presence of additional sources of uncertainty one should consider the more general form $k_{t+1}(\omega^{t+1}) = \ell(c_t(\omega^t), k_t(\omega^t), \omega^{t+1})$.

By construction note that $\{\Omega^t_{ND}, \Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ is a partition of the set Ω^t. Moreover, it can be seen that the sets Ω^t_{ND} and $\Omega^i_{D,i}$ are singletons.[3] Therefore, in order to avoid confusion between histories and sets of histories, we will refer to these singleton sets as ω^t_{ND} and $\omega^i_{D,i}$ respectively.

In figure 1 we show a more intuitive representation of the particular partition of histories specified above, where we use the name of the unique history ending in a given node to denote the node itself. White nodes indicate when a new planner is settled (default has occurred), while black nodes indicate the cases where the first planner is still in power (no default has occurred). We can see that in any period t there is only one history ω^t_{ND} such that commitment has always occurred in the past, or in other words the planner settled in period 0 is still in charge. Moreover, there is also only one history $\omega^i_{D,i} = \left\{\omega^{i-1}_{ND}, D\right\}$, meaning that the first default occurred in period i. In our institutional setting, a new planner is then settled from the node $\omega^i_{D,i}$ onward and it will make its choices over all the possible histories passing through the node $\omega^i_{D,i}$, that is the sets $\Omega^t_{D,i}, \forall t \geq i$.

We will now write the problem of the current planner where to simplify notation, and without loss of generality, we abstract from the presence of constraints in the maximization problem:

$$\begin{aligned} W(k_0) = & \max_{\substack{\{c_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t \in \Omega^t}} [\sum_{t=0}^{\infty} \sum_{\omega^t_{ND}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \\ & + \max_{\substack{\{c_t(\omega^t)\}_{t=1}^{\infty} \\ \omega^t \in \Omega^t_{D,1}}} \left\{ \sum_{t=1}^{\infty} \sum_{\omega^t \in \Omega^t_{D,1}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \right\} \\ & + \max_{\substack{\{c_t(\omega^t)\}_{t=2}^{\infty} \\ \omega^t \in \Omega^t_{D,2}}} \left\{ \sum_{t=2}^{\infty} \sum_{\omega^t \in \Omega^t_{D,2}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \right\} \\ & + ...] \end{aligned} \tag{7}$$

where we are using the short notation $Prob(\omega^t) = Prob(\{s_j\}^t_{j=0} = \omega^t)$. Eq. (7) makes it explicit that inside the maximization problem of the current government there are other planners maximizing welfare during their tenures. Given that $\{\Omega^t_{ND}, \Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ is a partition of the set Ω^t, all the histories are contemplated

[3] Ω^t_{ND} only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_t = ND\}$ and similarly the set $\Omega^i_{D,i}$ only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_{i-1} = ND, \bar{s}_i = D\}$.

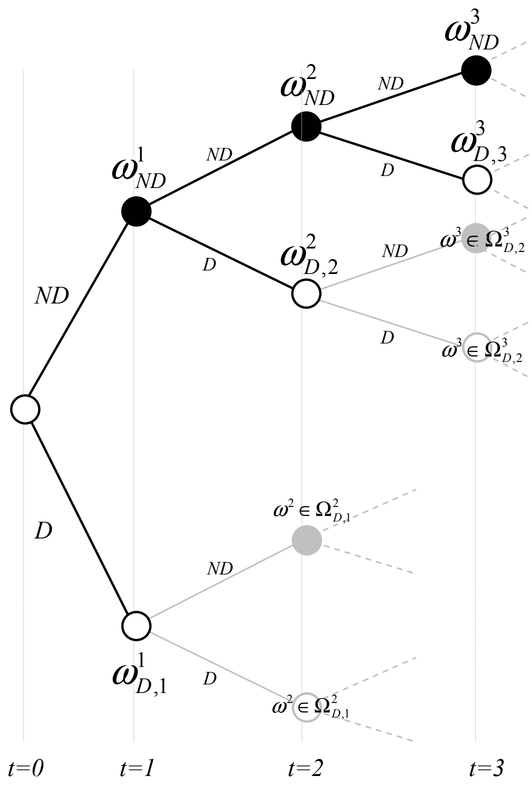

Figure 1: Diagram of the possible histories

in our formulation. Since $\forall\ t > i$, $\Omega^t_{D,i} = \{\omega^i_{D,i}, \{\bar{s}_j\}^t_{j=i}\}$, we can rewrite the probabilities for $\omega^t \in \Omega^t_{D,i}$ in the following way:

$$Prob(\omega^t) = Prob(\omega^i_{D,i} \wedge \omega^t) = Prob(\omega^t|\omega^i_{D,i})Prob(\omega^i_{D,i}), \forall\ \omega^t \in \Omega^t_{D,i},\ t \geq i. \quad (8)$$

Substituting for these expressions into Eq. (7) and collecting the common term in the summation, we obtain:

$$W(k_0) = \max_{\substack{\{c_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t \in \Omega^t}} \left\{ \sum_{t=0}^{\infty} \sum_{\omega^t_{ND}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \right. \tag{9}$$

$$\left. + \sum_{i=1}^{\infty} \beta^i Prob(\omega^i_{D,i}) \left[\max_{\substack{\{c_t(\omega^t)\}_{t=i}^{\infty} \\ \omega^t \in \Omega^t_{D,i}}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega^t_{D,i}} \beta^{t-i} \{Prob(\omega^t | \omega^i_{D,i}) u(c_t(\omega^t), k_t(\omega^t))\} \right] \right\}$$

Since we are assuming that any future planner is also maximizing we can define the value functions:

$$\xi_i(k_i(\omega^i_{D,i})) \equiv \max_{\substack{\{c_t(\omega^t)\}_{t=i}^{\infty} \\ \omega^t \in \Omega^t_{D,i}}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega^t_{D,i}} \beta^{t-i} \{Prob(\omega^t | \omega^i_{D,i}) u(c_t(\omega^t), k_t(\omega^t))\} \tag{10}$$

where it was made explicit that each planner assigns probability one to its initial node. The value functions $\xi_i(k_i)$ summarize the happenings after the node $\omega^i_{D,i}$. Since $\Omega^t_{D,i} \cap \Omega^t_{D,j} = \oslash$ for $i \neq j$, the choices of future planners are independent between themselves. This formulation is very general since one can assume several institutional settings that the future planners will face. For example, one can assume that some future planners have full commitment while others do not. For simplicity we will assume that all future planners face the same institutional settings which at this stage we do not specify, thus we assume that $\xi(k_i) = \xi_i(k_i)\ \forall i$.[4] Since all the histories $\{\Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ are already being maximized by other planners, it is equivalent to consider that the initial planner maximizes over the single history $\{\omega^t : \omega^t \in \Omega^t_{ND}\} \equiv \omega^t_{ND}$ instead of $\omega^t \in \Omega^t$. We can therefore rewrite the problem at period $t = 0$ as:

$$W(k_0) = \max_{\{c_t(\omega^t_{ND})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left\{ \beta^t \{Prob(\omega^t_{ND}) u(c_t(\omega^t_{ND}), k_t(\omega^t_{ND}))\} \right.$$
$$\left. + \sum_{i=1}^{\infty} \beta^i Prob(\omega^i_{D,i}) \xi(k_i(\omega^i_{D,i})) \right\} \tag{11}$$

We will now assume that the random variable s_t is i.i.d. to further simplify the problem. It is straightforward to generalize our formulation to Markov processes.

[4]In a companion paper we relax this assumption, focusing on political disagreement issues.

Also to simplify notation denote $Prob(\{\omega^t, ND\}|\omega^t) = \pi$ and $Prob(\{\omega^t, D\}|\omega^t) = 1-\pi$, which implies that:

$$Prob(\omega^t_{ND}){=}\pi^t \tag{12}$$

$$Prob(\omega^t_{D,t}){=}\pi^{t-1}(1-\pi). \tag{13}$$

With this formulation at hand we are ready to show that our problem can be written as a saddle point functional equation (SPFE), and that the optimal policy functions of the planner are time-invariant and depend on a finite set of states.

2.2.1 The recursive formulation

Collecting results from the previous section, the problem of the current planner is:

$$\max_{\{c_t(\omega^t_{ND})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}(\beta\pi)^t\{u(c_t(\omega^t_{ND}), k_t(\omega^t_{ND})) + \beta(1-\pi)\xi(k_{t+1}(\omega^{t+1}_{D,t+1}))\} \tag{14}$$

$$\begin{aligned} s.t: \quad & k_{t+1}(\omega^{t+1}_{ND}) = k_{t+1}(\omega^{t+1}_{D,t+1}) = \ell(c_t(\omega^t_{ND}), k_t(\omega^t_{ND})) \\ & b_1(c_t(\omega^t_{ND}), k_t(\omega^t_{ND})) + \beta(1-\pi)b_2(\Psi\{k_{t+1}(\{\omega^t_{ND}, D\})\}, k_{t+1}(\{\omega^t_{ND}, D\})) \\ & \qquad + \beta\pi b_2(c_{t+1}(\omega^{t+1}_{ND}), k_{t+1}(\omega^{t+1}_{ND})) = 0 \end{aligned}$$

Due to the fact that we do have future controls in the constraints through the term $\beta\pi b_2(c_{t+1}(\omega^{t+1}_{ND}), k_{t+1}(\omega^{t+1}_{ND}))$, the usual Bellman equation is not satisfied.[5] Building on the results of Marcet and Marimon (1998), we show that problems of this type can be rewritten as a SPFE that generalizes the usual Bellman equation. This result is summarized in proposition 1.

Proposition 1 *Problem (14) can be written as saddle point functional equation as:*

$$\begin{aligned} W(k,\gamma) = & \min_{\lambda\geq 0}\max_{c}\{h^m(c,k,\lambda,\gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k',\gamma')\} \\ & s.t: k' = \ell(c,k) \\ & \gamma' = \lambda, \ \gamma_0 = 0 \end{aligned} \tag{15}$$

where

$$h^m(c,k,\lambda,\gamma) = u(c,k) + \lambda g_1(c,k) + \gamma g_2(c,k) \tag{16}$$

$$g_1(c,k) = b_1(c,k) + \beta(1-\pi)b_2(\Psi\{l(c,k)\}, l(c,k)) \tag{17}$$

$$g_2(c,k) = b_2(c,k) \tag{18}$$

[5] For details see Stokey et al. (1989).

Proposition 1 makes it clear that the current planner maximizes utility of the representative agent subject to the constraints $k' = \ell(c,k)$ and $g_1(c,k)+\beta\pi g_2(c',k') = 0$, where the latter is incorporated in h^m. If there is no commitment, the continuation of the problem is $\xi(k')$. If the current promises will be fulfilled, then the continuation of the problem is $W(k',\gamma')$, and promises are summarized in the co-state variable γ'. The optimal policy functions of such problem are time invariant and depend on a finite number of states, as proposition 2 describes.[6]

Proposition 2 *The solution of problem (14) is a time invariant function with state variables (k_t,γ_t), that is to say:*

$$\begin{aligned} \psi(k,\gamma) \in & \arg\min_{\lambda\geq 0}\max_{c}\{h^m(c,k,\lambda,\gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k',\gamma')\} \\ & s.t: k' = \ell(c,k) \\ & \gamma' = \lambda,\ \gamma_0 = 0 \end{aligned} \tag{19}$$

2.3 Equilibrium

In the institutional setting built in Eq. (14), we only assume that all planners from period 1 onward will face the same problems. From now on, we also assume that all future planners face the same institutional setting as we specify in period 0. In other words, we specify their problems in the same way as the problem of the planner in period 0.[7] Thus we can use the following definition of equilibrium.

Definition 1 *A Markov Perfect Equilibrium where each planner faces the same institutional setting must satisfy the following conditions.*

1. Given $\Psi(k)$ and $\xi(k)$, the sequence $\{c_t\}$ solves problem (14);
2. The value function $W(k,\gamma)$ is such that $\xi(k) = W(k,0) \equiv W(k)$;
3. The policy functions $\psi(k,\gamma)$ solving problem (14) are such that $\Psi(k) = \psi(k,0)$.

The second part of the definition imposes directly that the problem of the initial and future planners must be equal. The third part of the definition imposes a consistency requirement in the constraints. More precisely, we require the policy function $\Psi(k)$ that agents expect to be implemented under default to be consistent

[6]As it is common in the time-consistent literature we do not prove that the optimal policy function is unique. Nevertheless, we found no evidence of multiple solutions.

[7]In a companion paper, we relax this assumption and focus on political disagreement issues.

with the optimal policy function. We refer to the notion of Markov Perfect Equilibrium because the function Ψ only depends on the natural state variables k. Also, in this equilibrium neither the planner nor individual agents desire to change behavior. Individual agents are maximizing and their beliefs are correct. The planner, taking as given Ψ and $\xi = W$, is also maximizing.

2.4 Solution strategy

There are different ways to solve our problem. One approach would be to prove that iterating on the SPFE is a contraction. By doing so, we could solve our problem in a very similar way to the usual value function iteration. We will follow a different approach, we will solve our problem using FOCs to the lagrangian. Our generic problem is:

$$W(k_0) = \underset{\{c_t, k_{t+1}\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} (\beta\pi)^t \left[u\left(c_t, k_t\right) + \beta\left(1-\pi\right)\xi\left(k_{t+1}\right)\right] \tag{20}$$

$$\begin{aligned} s.t. \quad & k_{t+1} = \ell\left(c_t, k_t\right) \\ & g_1\left(c_t, k_t\right) + \beta\pi g_2\left(c_{t+1}, k_{t+1}\right) = 0 \\ & \qquad\qquad\qquad\qquad \forall t = 0, ..., \infty \end{aligned}$$

where g_1 and g_2 are defined by Eqs. (17, 18) respectively.

Details on the FOCs can be found in the appendix. It is important to mention that the term $\xi_{k,t+1}$ appears in the FOCs. As we had anticipated, the current planner will try to influence future planners. The value function $\xi\left(k_{t+1}\right)$ summarizes the welfare that agents will achieve with a planner appointed at $t+1$. From the perspective of the planner appointed at $t+1$, the state variables k_{t+1} can not be changed. Nevertheless, from the perspective of the current planner, who is in charge at period t, k_{t+1} can be manipulated.[8] The FOC with respect to k_{t+1}, thus considers both the possibility that the current planner stays in power and the possibility that a new planner is appointed. In the case that a new planner is appointed the current planner can only affect future decisions through the states k_{t+1}, which in turn influence the value function $\xi\left(k_{t+1}\right)$.

The FOCs expressed in Eqs. (39-42) allows us to solve for the optimal policy. If the problems of the current and future planners differs, we could proceed in the following way. We could first obtain the value functions $\xi\left(k_{t+1}\right)$ and the optimal policy

[8]Note that, when default occurs, the lagrange multiplier is set to zero and cannot be used to influence incoming planners.

functions $\Psi(k_{t+1})$ corresponding to future planners. Given $\xi(k_{t+1})$ and $\Psi(k_{t+1})$, we could then solve for the policy functions of the current planner.

As described in Definition 1, we are particularly interested in the formulation where future planners face the same problem as the current planner, i.e. where $\xi(k_t) = W(k_t)$ and hence $\xi_{k,t+1} = W_{k,t+1}$. In this case, one possible solution strategy relies on the solution of two fixed points. In a first step, we could guess the functions $W_{k,t+1}$. We could then solve for the optimal policy $c_t = \psi(k_t, \lambda_{t-1})$. This second step would involve solving a fixed point problem, because according to our equilibrium definition $\Psi(k) = \psi(k, 0), \forall k$. Once obtained the policy function, we could update our guess of W and $W_{k,t+1}$ and repeat the procedure until convergence.

We will show a solution method that only relies on solving one fixed point. To obtain the derivative $W_{k,t+1}$ we can use envelope results, which are summarized in result 1.

Result 1 *Using envelope results it follows that:*

$$\frac{\partial W(k_t)}{\partial k_t} = \frac{\partial u[c_t(k_t), k_t]}{\partial k_t} + v_t \ell_{k,t} + \lambda_t g_{1,k,t} \tag{21}$$

where all variables are evaluated using the optimal policy of a planner appointed in period t, given the state k_t.

Result 1 uses the fact that the planners are maximizing a function, which allows the use of envelope principles.[9] It is important to note that in Eq. (21) all the variables are evaluated with the optimal policy that the government elected at t implements. For instance, one has to bear in mind that the policy function at time t of a planner appointed at t does not depend on the lagrange multiplier.

By Definition 1, the policy functions that the current and future planners implement are equal. If we use the envelope result to substitute $\xi_{k,t+1} = W_{k,t+1}$, the FOCs only depend on the functions $\psi(k_t, \lambda_{t-1})$ and $\Psi(k)$, where $\Psi(k) = \psi(k, 0), \forall k$. We can use a collocation method to solve for the optimal policy functions. This solution method is simpler, because it relies on one fixed point instead of two. As a side product, our methodology can be used as a homotopy to obtain the time-consistent solution. Starting from the time-inconsistent solution, one can gradually reduce the probability of commitment to zero in order to obtain the time-consistent solution.

We want to stress that in our framework global solution methods proposed in Judd (1992) and Judd (2004) are much more appropriate. The linear quadratic approximation proposed in Benigno and Woodford (2004) or Benigno and Woodford

[9] A proof of this envelope result is available upon request.

(2006) is only valid in a timeless perspective. The timeless perspective assumes that initial commitments are equal to the steady-state commitment, not being useful to analyze transition dynamics. There are several reasons that make the linear quadratic approach inappropriate in our framework. Firstly, we consider that commitments may be broken and consequently we need to focus on transition dynamics. Secondly, our model does not have a steady state point around which one can take an approximation. Thirdly, under discretion the allocations can be very far from the commitment steady-state. Our method is more suitable and it is also simpler. Even for an exactly linear quadratic model Schaumburg and Tambalotti (2005) need to solve three fixed points to get their solution using a less reliable method.

Beside these numerical considerations, there is an important drawback of applying the linear-quadratic approach to study problems with *loose commitment* settings. Indeed, as shown by Debortoli and Nunes (2006), a correct linear-quadratic approximation of a general model can only be derived by imposing the timeless perspective approach. However, allowing for the occurrence of a default explicitly violates the timeless perspective assumption. Therefore, applying the linear-quadratic approach to study problems characterized by *loose commitment* contradicts the microfoundations of the original model.

3 T-periods model

We will now consider another institutional setting, where a planner knows that it will be in charge during T periods. After that a new planner is appointed. As in previous sections, we will assume that the future planner faces the same institutional settings as the initial planner. Using the same notation as in section 2, we can write the problem as:

$$
\begin{aligned}
W(k_0) = & \max_{\{c_t\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} (\beta)^t \{u(c_t, k_t)\} + \beta^T W(k_T) \qquad (22)\\
s.t: & \; k_{t+1} = \ell(c_t, k_t), \qquad t = 0, 1, ..., T-1. \\
& b_1(c_t, k_t) + \beta b_2(c_{t+1}, k_{t+1}) = 0, \qquad t = 0, 1, ..., T-2. \\
& b_1(c_t, k_t) + \beta b_2(\Psi_0\{k_{t+1}\}, k_{t+1}) = 0, \; t = T-1.
\end{aligned}
$$

The objective function includes the instantaneous utility of all the periods during the tenure and the value function of the future planner. The constraints that the planner faces reflect the institutional setting just described. Up to the last period, the current planner can credibly commit. In the last period of the tenure, private agents know that in the next period another planner will be appointed and no credible promises can be made. Therefore, private agents expect that in period T a new planner implements the policy function Ψ_0.

By appealing to standard dynamic programming techniques, it is clear that the policy functions of the planner appointed at $t = 0$ are equal to the policy functions of the planner appointed at $t = T$. The proof of such result is simple and only requires to consider the tenure of each planner as one big period and use infinite horizon dynamic programming results.[10]

Proposition 3 *Denote* $\Psi_{j,i}$ *as the optimal policy function of a planner appointed at* $t = j$ *in the time period* $t = j + i$*. That is to say,*
$\{\Psi_{j*T,0}(k_{j*T}), \Psi_{j*T,1}(k_{j*T}), ..., \Psi_{j*T,T-1}(k_{j*T})\} = \psi(k)$ *where*

$$\psi(k) \in \arg\max_{c} \textit{Problem (22)}$$

Then $\Psi_{j*T,i}(k) = \Psi_{(j+1)*T,i}(k)$, $\forall k, j, 0 \leq i \leq T - 1$.

The previous proposition states that the solution of problem (22) is a tenure invariant function with state variables (k). It is important to stress that we are not claiming that the policy functions are time-invariant. Indeed, the policy function that a planner implements in one period is different from the policy function that the same planner implements in another period. Another important remark in Proposition 3 is that we are only considering the state variables when the tenure begins. A planner appointed in $t = 0$ will implement policy functions for all the periods $t = 0, 1, ...T - 1$ that only depend on the initial state k_0. If the model would have some sources of shocks, such as productivity, the state-space would be huge. Since we just kept shocks away for notational convenience, we want to use techniques that can easily incorporate exogenous shocks. To do so we will solve for policy functions that depend on the past lagrange multiplier and the past state variable.[11] These policy functions are still time variant but tenure invariant. As before, we can apply envelope results, which allow us to simplify our problem to a single fixed point. The FOCs of this problem are easily obtained and for brevity we will not state them here.

[10]Another proof follows from applying finite horizon dynamic programming results and using the definition that the problems of different planners are equal.

[11]We can do so by using the results of Marcet and Marimon (1998) in a finite horizon economy.

4 An optimal taxation problem

In the previous sections we have formulated optimal policy problems in a general form. We will now refer to a specific optimal taxation problem mentioned in Marcet and Marimon (1998) and Klein et al. (2004). We chose this model because it is a benchmark in the literature, where both the commitment and discretion solutions have been analyzed. A representative household derives utility both from private $\{c_t\}$ and public consumption $\{g_t\}$. The representative agent rents capital $\{k_t\}$ to a firm and inelastically supplies one unit of labor. Capital and labor markets are competitive, but financial markets are not available to the government. Thus, the government collects taxes $\{\tau_t\}$ and provides the public good under a balanced budget constraint. The household problem is:

$$\max_{\{k_{t+1},c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{23}$$
$$s.t : c_t + k_{t+1} = k_t + (1-\tau_t)[w_t + (r_t - \delta)k_t]$$

where r, w, β and δ refer to the interest rate, wage, the discount factor and the depreciation rate respectively. There is uncertainty in this economy because it is not know in advance whether the planner will default or not. Wages and interest rates are determined in perfectly competitive markets:

$$r_t = f_k(k_t) \tag{24}$$
$$w_t = f(k_t) - f_k(k_t)k_t \tag{25}$$

where $y_t = f(k_t)$ is production. The FOCs of the households are:

$$u_c(C(k_t, k_{t+1}, g_t), g_t) = \beta E_t u_c(C(k_{t+1}, k_{t+2}, g_{t+1}), g_{t+1})\{1+[1-T((k_{t+1}, g_{t+1})][f_k(k_{t+1})-\delta]\} \tag{26}$$

where we have already substituted the interest rate, the resource constraint and the balanced budget condition, which are described by:

$$c_t \equiv C(k_t, k_{t+1}, g_t) = f(k_t) + (1-\delta)k_t - k_{t+1} - g_t \tag{27}$$
$$\tau_t \equiv T(k_t, g_t) = g_t/(f(k_t) - \delta k_t) \tag{28}$$

As in Klein et al. (2004), we also consider the possibility that the government only taxes capital income and therefore $\tau_t \equiv T(k_t, g_t) = g_t/((r_t - \delta)k_t)$. In these models the government would like to manipulate expectations. If the government commits to low taxes tomorrow, then private agents will accumulate more capital and the government can have low taxes today. In principle, lower taxes will mean less public consumption. Note that the government may promise high taxes tomorrow, because this gives an incentive to tax more today and provide more public consumption. For further discussions on this issue see Klein et al. (2004).[12]

In order to proceed to the numerical solution, we specify a per-period utility function:

$$u(c_t, g_t) = \log(c_t) + \gamma_g \log(g_t) \tag{29}$$

and a standard production function:

$$y_t = k_t^\alpha \tag{30}$$

We use a standard calibration for an annual model of the US economy. Table 1 summarizes the values used for the parameters.

Table 1: Parameter values

β	δ	α	γ_g
0.96	0.08	0.36	0.50

4.1 Probabilistic Model

We now consider the probabilistic model introduced previously. In case of default, households believe that k_{t+2} and g_{t+1} will be given by the functions $h(k_{t+1})$ and $\Psi(k_{t+1})$ respectively. Households anticipate the changes in power and therefore Eq. (26) is written as:

$$\begin{aligned} &u_c(C(k_t, k_{t+1}, g_t), g_t) = \\ &= \beta\pi u_c(C(k_{t+1}, k_{t+2}, g_{t+1}), g_{t+1})\{1 + [1 - T((k_{t+1}, g_{t+1})][f_k(k_{t+1}) - \delta]\} \\ &+ \beta(1-\pi)u_c(C(k_{t+1}, h(k_{t+1}), \Psi(k_{t+1})), \Psi(k_{t+1}))\{1 + [1 - T((k_{t+1}, \Psi(k_{t+1}))][f_k(k_{t+1}) - \delta]\} \end{aligned} \tag{31}$$

If one is tempted to match political cycles with commitment cycles, then the value of 0.75 is realistic, since it corresponds to a planner being in office during 4 years on average. A calibration based on the political history of the US implies a

[12] The result that the government commits to higher taxes may seem strange to the reader. However, note that Chamley (1986) setup is very different from ours. For simplicity we are not considering different tax rates on capital and labor and we have balanced budget every period.

Table 2: Capital Income Tax - Average Values

	0,00	**0,25**	**0,50**	**0,75**	**1,00**
k	1,602	1,663	1,755	1,918	2,366
g	0,232	0,230	0,227	0,222	0,203
y	1,185	1,201	1,224	1,264	1,364
c	0,825	0,838	0,857	0,889	0,971
τ	0,776	0,768	0,756	0,735	0,673
λ	0,000	-0,205	-0,541	-1,227	-3,644

value of 0.8, while the political history of Italy would imply a calibration around 0. We will first examine the model where only capital income is taxed. Table 2 shows average allocations in the economy. In this case, discretion implies higher taxes, which induce lower capital accumulation and consumption but more provision of the public good. The first feature that one should stress is that average allocations in the economy seem to be closer to the discretion solution rather than to the commitment one. It may be expected that decreasing the probability of commitment by 25% would make the allocation to move by 25% of the difference between commitment and discretion. Nevertheless, a decrease in the probability of commitment from 1 to 0.75 leads to a bigger change in the average allocations of the economy towards the discretion steady state. For example, in the capital income tax model the absolute drop in capital is already 59% of the difference between full commitment and discretion.

Figure 2 plots the average path during the first 25 quarters, if one starts at the steady state value of capital under default and no promises have been made. The picture confirms the results of table 2, since the path for $\pi = 0.75$ is relatively closer to the discretion path. Also notice that in the commitment solution, as credibility starts to build, taxes and public consumption start to be lowered gradually. In figure 3 we plot the paths followed for a given history. We consider the history where by chance a new planner is reappointed every four years. There are noticeable differences in the accumulation path of capital. It is not just the realization of default or commitment that makes allocations to change. Even considering the same history, the policies that the government implements lead to different allocations. The figure also shows that, when a new planner is reappointed, taxes and public consumption jump to high levels. Only after this initial increase do these variables start to be decreased gradually.

We now turn attention to the case where total income is taxed. Average allo-

Table 3: Total Income Tax - Average Values

	0,00	**0,25**	**0,50**	**0,75**	**1,00**
k	4,391	4,387	4,379	4,363	4,259
g	0,408	0,409	0,411	0,416	0,447
y	1,703	1,703	1,702	1,700	1,685
c	0,944	0,943	0,940	0,934	0,897
τ	0,302	0,303	0,304	0,308	0,333
λ	0,000	-0,048	-0,135	-0,334	-1,453

cations are shown in table 3. Again the result is that average allocations get away from the commitment steady state quite quickly. Figure 4 plots average paths and figure 5 plots the paths for a specific realization, where a new government is reappointed every four years. The conclusions are the same as before. The fact that a new planner is reappointed creates a change in policy. Nevertheless, for the same history different policies induce non-negligible differences in capital accumulation and output.

We finally focus on implications for welfare. Table 4 shows the welfare gains for different probabilities of commitment. We have normalized the welfare gain of moving from complete discretion to full commitment to 1. The first line refers to the situation where only capital income is taxed. When the probability of default increases from 0 to 0.25, we see that only 15% of the benefits of commitment are achieved. Even when the probability of commitment is 0.75, the gains of commitment are 60%. The pattern of this table is that at low levels of credibility the welfare gains from increasing commitment are low. Most of the gains from commitment can only be achieved when credibility is already high. This pattern is even more pronounced in the model where all income is taxed. Even if the probability of keeping past promises is 0.75, only 39% of the gains are achieved. This is an important result in our paper. For low levels of credibility, increasing credibility leads to small relative gains in welfare.

Schaumburg and Tambalotti (2005) found the opposite result in a linear-quadratic application without state variables to the Barro-Gordon model. Our results together with theirs suggest why economists and policy makers have devoted considerably more attention to increase monetary policy credibility, but have almost ignored fiscal policy credibility. The institutional changes aimed at building central bank credibility in the 80's were justified by the potential welfare gains when credibility is low. Nowadays, central banks have more credibility but they have not fully com-

Table 4: Welfare Gain

	0	**0,25**	**0,5**	**0,75**	**1**
Capital Income Tax	0,000	0,150	0,344	0,608	1,000
Total Income Tax	0,000	0,066	0,175	0,392	1,000

mitted to any future action or rule. The reason seems to be that the benefits of increasing credibility even further are small.

Our results suggest that, in this regard, fiscal policy is very different from monetary policy. The relative welfare gains from commitment are convex, not concave. If one believes that fiscal policy credibility is unlikely to be high, increasing credibility of the fiscal authority from low to moderate levels will just lead to small relative gains in welfare. One could alternatively interpret our welfare results in an opposite way; one could claim that fiscal credibility must be fully achieved, since small temptations to act discretionally lead to high relative welfare losses. Nevertheless, it may be difficult to establish a highly credible fiscal authority, because such institution would interfere with democratic and political choices. Direct welfare comparisons indicate that monetary policy credibility may be more important. In the capital and total income model presented in this paper, the commitment solution improves welfare by 3,5% and 0,4% respectively.[13] In Barro-Gordon type of models commitment improves welfare by much larger amounts.[14] These considerations may provide a justification for the low interest among academics and policy makers in increasing fiscal policy credibility.

4.2 T-Periods Model

In this section we will apply the T-periods setting to the fiscal policy model described previously. For brevity considerations we skip the FOCs and we proceed directly with the analysis. This model displays political cycles. Figure 6 and figure

[13] To keep comparability with Schaumburg and Tambalotti (2005) welfare was computed as life-time utility. The relative gains (values reported in the tables) are unchanged if we use the compensating variation in (private and public) consumption. The improvement from discretion to commitment does change to 2,19% and 0,13% in the capital and total income model respectively.

[14] In Barro-Gordon models the welfare loss penalizes quadratically deviations of inflation from zero and deviations of the output gap from a target level. The inflation and output gap under commitment are nearly zero. Under discretion the inflation is quite high and the output gap is still zero. Since standard calibrations give a much higher weight to inflation deviations in the loss function, the gains from commitment are substantial.

Table 5: T-period model - Capital Income Tax - Average Allocations

	DEF	2	4	8	COM
k	1,602	1,700	1,837	1,995	2,366
g	0,232	0,229	0,225	0,219	0,203
y	1,185	1,211	1,245	1,282	1,364
c	0,825	0,846	0,873	0,904	0,971
τ	0,776	0,764	0,746	0,725	0,673
λ	0,000	-0,315	-0,825	-1,538	-3,644

Table 6: T-period model - Total Income Tax - Average Allocations

	DEF	2	4	8	COM
k	4,391	4,385	4,375	4,357	4,259
g	0,408	0,410	0,413	0,418	0,447
y	1,703	1,703	1,701	1,699	1,685
c	0,944	0,942	0,938	0,932	0,897
τ	0,302	0,303	0,306	0,310	0,333
λ	0,000	-0,071	-0,199	-0,406	-1,453

7 plot the paths for planners facing different tenure lengths. During each tenure, allocations move towards commitment values. When past promises are broken there is a sudden movement towards the discretion value. Longer tenures allow allocation to be closer to the full commitment solution.

We compare average allocations when the planner knows with certainty that will be in charge during 1,2,4,8 and infinitely many periods. Obviously, the extreme values considered correspond to the default and commitment cases. Given the political history of the US, tenures of 8 periods can be considered an upper bound, while tenures of 4 years have been the norm. Tables 5 and 6 present the average allocations for the capital income and total income taxation model respectively. In the capital income model, for the benchmark tenure of 4 periods, capital moves towards discretion by 69% of the total difference between discretion and commitment. Regarding the total income model, the same analysis shows that capital moves by 88% of the difference between commitment and discretion. Hence, results show that average values are still close to the discretion case both for the capital and the total income taxation model. Table 7 shows the results for welfare. In the more realistic total income taxation model, when the planner knows that will stay in power during 4 years, only 27% of the welfare gain is achieved.

Table 7: Welfare Gain T-period model

	DEF	**2**	**4**	**8**	**COM**
Capital Income Tax	0,000	0,256	0,528	0,745	1,000
Total Income Tax	0,000	0,102	0,266	0,492	1,000

5 Extension - endogenous probabilities.

We are finally going to consider an extension where the probability of defaulting depends on the states of the economy. Since capital is the only natural state variable in the economy and all allocations depend on capital, we will consider that the probability of defaulting today depends on the current capital stock. We think it is plausible to assume that when capital is higher there is a higher probability of reelection. We will consider the following probability function:

$$F(k_t) = 1 - \frac{1}{(\frac{k_t}{\tilde{k}})^\rho + 1} \tag{32}$$

where $\tilde{k}$ and ρ are parameters to be defined. Note that $\tilde{k}$ is a normalization such that $F(\tilde{k}) = 0.5$ and that the higher is ρ, the easier it is for the planner to influence its reelection probability. In the case of $\rho = 0$ the probability is always constant. The planner and households will consider that the probability of commitment in the next period is $F(k_{t+1})$ instead of π. For instance, the objective function of the planner is:

$$\sum_{t=0}^{\infty} \beta^t \frac{\Pi_{j=0}^{t}(F(k_t))}{F(k_0)} \{u(c_t, k_t) + \beta(1 - F(k_{t+1}))W(k_{t+1})\} \tag{33}$$

All the proofs considered previously also apply in this setting using minor modifications.[15] We can use a homotopy from the model in section 2 to this model by changing ρ from 0 to the desired value. This model raises an extra difficulty, because both the derivative and the level of the value function appear in the FOCs, hence one also needs to approximate the value function. We choose $\rho = 5$ and $\tilde{k}$ to be equal to the average capital allocation when $\pi = 0.5$. Our normalization of $\tilde{k}$ allows us to directly compare the results with the probabilistic model when $\pi = 0.5$.

[15] It is useful to redefine the objective of the planner using the definition $\theta_{t+1} = \theta_t F(k_{t+1})$, with $\theta_0 = 1$. Note that the special term on $F(k_0)$ in the objective function does not induce any time-inconsistency problem because k_0 is predetermined.

Results are presented in table 8 both for the capital income and the total income model. We see that capital is now higher. Since the probability of commitment is increasing in capital the planner has a further motive to accumulate capital. In the capital income model this effect is quite visible, while in the total income model this effect is more subtle.[16] Similarly, if we turn our attention to welfare, we notice that in the capital income model with endogenous probability the gain is 54.3% of the the total gain from commitment. This value is much higher than the welfare gain of 34.5% obtained in the benchmark case of $\pi = 0.5$ (as reported in table 4). In the total income model, welfare with endogenous probability is almost identical to the benchmark case of $\pi = 0.5$. There are two reasons for this discrepancy. First, in the capital income model higher probability of commitment leads to higher capital, which in turn increases the probability of commitment. This self reinforcing mechanism is not present in the total income model. The second reason is that the pure discretion and pure commitment solutions in the first model are very different, while this is not so in the second one. Overall, our results suggest that governments accumulate more capital to be reelected, and this is a good policy since it reduces political turnover increasing the commitment probability.

Table 8: Endogenous Probability - Average Values

	Capital Income Tax		**Total Income Tax**	
	$\pi = 0.5$	**End. Prob.**	$\pi = 0.5$	**End. Prob.**
k	1.755	1.896	4.379	4.381
g	0.227	0.222	0.411	0.411
y	1.224	1.259	1.702	1.702
c	0.857	0.885	0.940	0.941
τ	0.756	0.738	0.304	0.304
λ	-0.541	-0.864	-0.135	-0.134

6 Conclusions

The time-consistent and time-inconsistent solutions can differ dramatically in some models. It is not clear which assumption on the planners commitment technology is more plausible. It seems more realistic to consider that planners only face some *loose commitment* technology. We have considered different formulations of

[16]If we increase ρ in the total income taxation model then capital starts to be visibly higher.

loose commitment and applied these tools to optimal fiscal policy. We considered a setup where the fiscal authority is reappointed every period with a given probability, another setup where the fiscal authority stays in power for T periods and finally a setup where the probability of reelection is endogenous. Combining these polar cases are straightforward extensions. Even though our settings may be naturally interpreted in the spirit of political turnover, one can also consider that the same planner may default on its own plans and reoptimize.

From the methodological point of view, our contribution is to show a solution technique for problems of limited commitment with the following main features. First, it can be applied to a wide class of non-linear models, with or without state-variables keeping the model's micro-foundations structure intact. Second, building on the results of Marcet and Marimon (1998), we proved that the solution to our problem is recursive. Third, we implement an algorithm which is relatively inexpensive, because it only requires the solution of one fixed-point, and makes use of global approximation techniques which are pointed out in the literature as more reliable. Finally, as a by-product, our procedure can be used as a homotopy method to find the time-consistent solution.

We show that in our optimal taxation model under *loose commitment* average allocations seem to be closer to the time-consistent solution. We find out that at low levels of credibility, further credibility does not lead to large welfare gains. To achieve most of the commitment gains, credibility has to be high. These results are in sharp contrast with those obtained in the literature regarding monetary policy. We believe that our results give support for the low interest in building independent or credible fiscal authorities.

There are many interesting applications of our frameworks. We are continuing and extending this line of research. In a companion paper we analyze the interactions between fiscal and monetary policy. In another companion paper we consider the case where different planners have different objectives. For the sake of tractability, but not of plausibility, the literature on political economy has not yet considered that parties with different tastes may still commit taking into account the possibility of reelection.

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A Proofs

Proof. of Proposition 1

Drop history dependence and define:

$$\begin{aligned}
r(c_t, k_t) &\equiv u(c_t, k_t) + \beta(1-\pi)\xi(l(c_t, k_t)) \\
g_1(c_t, k_t) &\equiv b_1(c_t, k_t) + \beta(1-\pi)b_2(\Psi\{l(c_t, k_t)\}, l(c_t, k_t)) \\
g_2(c_{t+1}, k_{t+1}) &\equiv b_2(c_{t+1}, k_{t+1})
\end{aligned}$$

Our problem is thus:

$$\begin{aligned}
&\max_{\substack{\{c_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t = \omega^t_{ND}}} \sum_{t=0}^{\infty} (\beta\pi)^t \{r(c_t, k_t)\} \qquad (34) \\
s.t: \quad & k_{t+1} = \ell(c_t, k_t) \\
& g_1(c_t, k_t) + \beta\pi g_2(c_{t+1}, k_{t+1}) = 0
\end{aligned}$$

which fits the definition of Program 1 in Marcet and Marimon (1998). To see this more clearly note that our discount factor is $\beta\pi$ and we have no uncertainty. Since ω^t_{ND} is a singleton, we have previously transformed our stochastic problem into a non-stochastic problem. Therefore, we can write the problem as a saddle point functional equation in the sense that there exists a unique function satisfying

$$\begin{aligned}
W(k, \gamma) = &\min_{\lambda \geq 0} \max_{c} \{h(c, k, \gamma, \lambda) + \beta\pi W(k', \gamma')\} \qquad (35) \\
s.t: \quad & k' = \ell(c, k) \\
& \gamma' = \lambda, \;\; \gamma_0 = 0
\end{aligned}$$

where

$$h(c, k, \lambda, \gamma) = r(c, k) + \lambda g_1(c, k) + \gamma g_2(c, k) \qquad (36)$$

or in a more intuitive formulation define:

$$h^m(c, k, \lambda, \gamma) = u(c, k) + \lambda g_1(c, k) + \gamma g_2(c, k) \qquad (37)$$

and the saddle point functional equation is:

$$\begin{aligned}
W(k, \gamma) = &\min_{\lambda \geq 0} \max_{c} \{h^m(c, k, \lambda, \gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k', \gamma')\} \qquad (38) \\
s.t: \quad & k' = \ell(c, k) \\
& \gamma' = \lambda, \;\; \gamma_0 = 0
\end{aligned}$$

■

Proof. of Proposition 2: Using Proposition 1, this proof follows trivially from the results of Marcet and Marimon (1998). ■

B First Order Conditions - Probabilistic Model

To solve the problem first set up the Lagrangian, using ν_t and λ_t as Lagrange multipliers for the two constraints. Thus, we need to find the FOCs of the following problem:

$$\underset{\{\nu_t,\lambda_t\}_{t=0}^{\infty}}{Min}\ \underset{\{c_t,k_{t+1}\}_{t=0}^{\infty}}{Max}\ \mathscr{L} = \sum_{t=0}^{\infty} (\beta\pi)^t\, u\left(c_t,k_t\right) + \beta\left(1-\pi\right)\xi\left(k_{t+1}\right)$$
$$+\,\nu_t\left(\ell\left(c_t,k_t\right) - k_{t+1}\right) + \lambda_t\left(g_1\left(c_t,k_t\right) + \beta\pi g_2\left(c_{t+1},k_{t+1}\right)\right)$$

The FOCs are[17]:

$$\frac{\partial\mathscr{L}}{\partial c_t} : u_{c,t} + v_t\ell_{c,t} + \lambda_t g_{1,c,t} + \lambda_{t-1}g_{2,c,t} = 0 \tag{39}$$
$$\frac{\partial\mathscr{L}}{\partial k_{t+1}} : \beta\left(1-\pi\right)\xi_{k,t+1} - v_t + \beta\pi\left(\lambda_t g_{2,k,t+1} + u_{k,t+1} + \lambda_{t+1}g_{1,k,t+1} + v_{t+1}\ell_{k,t+1}\right) = 0 \tag{40}$$
$$\frac{\partial\mathscr{L}}{\partial \nu_t} : k_{t+1} = \ell\left(c_t,k_t\right) \tag{41}$$
$$\frac{\partial\mathscr{L}}{\partial \lambda_t} : g_1\left(c_t,k_t\right) + \beta\pi g_2\left(c_{t+1},k_{t+1}\right) = 0 \tag{42}$$
$$\forall t = 0,...,\infty \qquad \lambda_{-1} = 0$$

where, using Eqs. (17,18) it follows that:

$$g_{1,c,t} = b_{1,c,t} + \beta\left(1-\pi\right)\left[\ell_{c,t}\left(b_{2,c,t+1}\Psi_{k.t+1} + b_{2,k,t+1}\right)\right]$$
$$g_{2,c,t} = b_{2,c,t}$$
$$g_{1,k,t} = b_{1,k,t} + \beta\left(1-\pi\right)\left[\ell_{k,t}\left(b_{2,c,t+1}\Psi_{k.t+1} + b_{2,k,t+1}\right)\right]$$
$$g_{2,k,t} = b_{2,k,t}$$

[17] For notational simplicity we treat k and c as scalars instead of vectors. The symbol $f_{x,t}$ indicates the partial derivative of the function $f(x_t)$ with respect to x_t. We suppressed the arguments of the functions for readability purposes.

Figure 2: Capital Income - Average Allocations

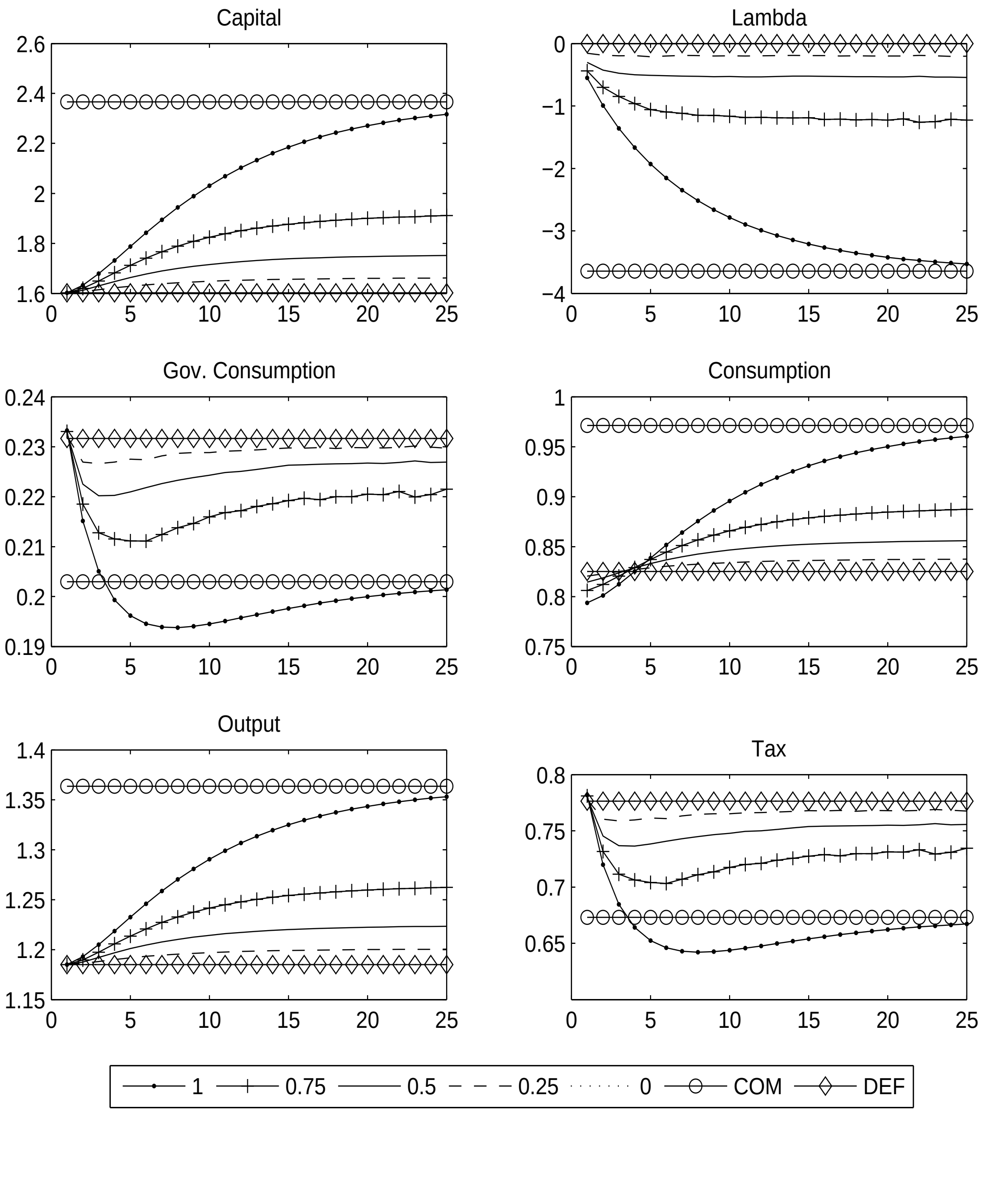

Figure 3: Capital Income - Default every 4 periods

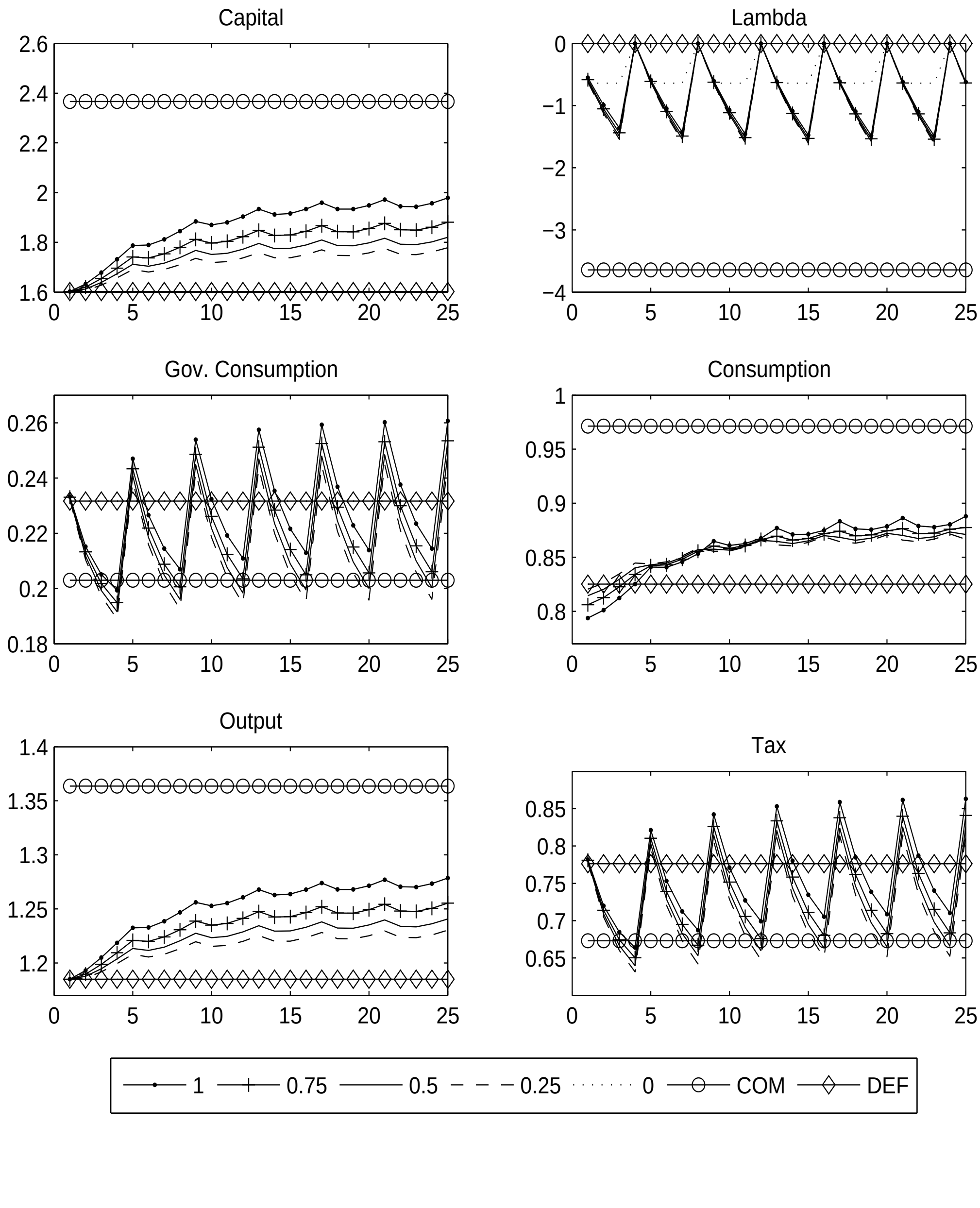

Figure 4: Total Income - Average Allocations

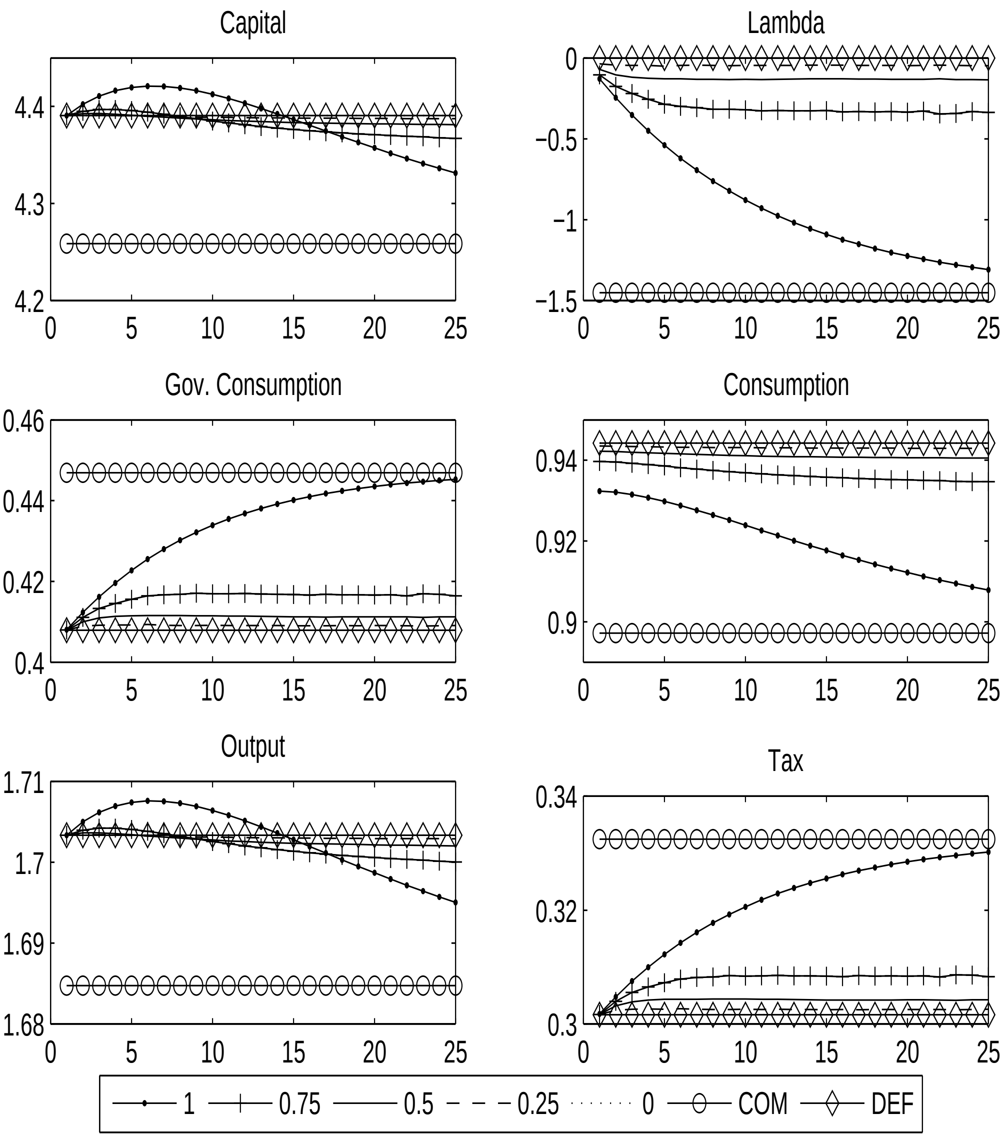

Figure 5: Total Income - Default every 4 periods

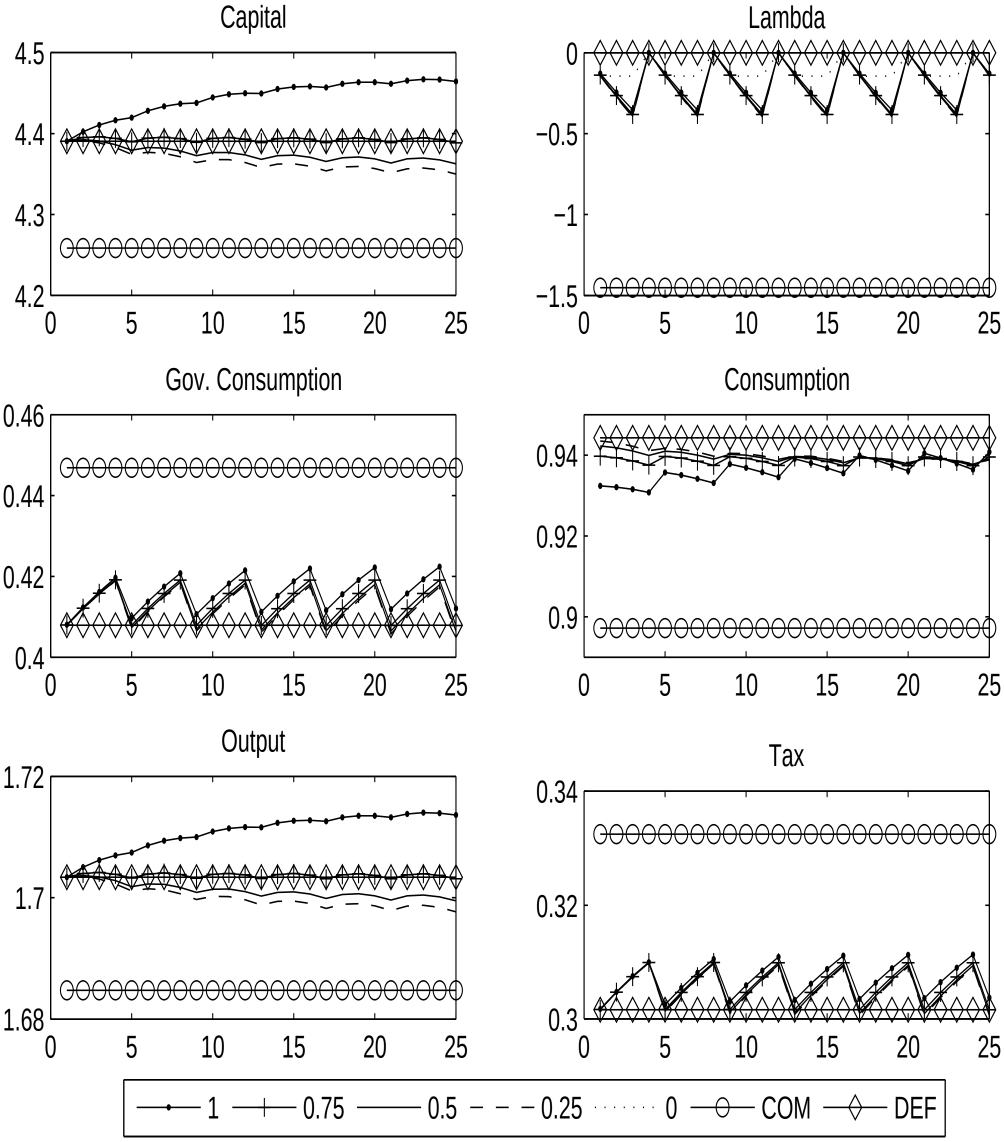

Figure 6: Capital Income - T-periods model

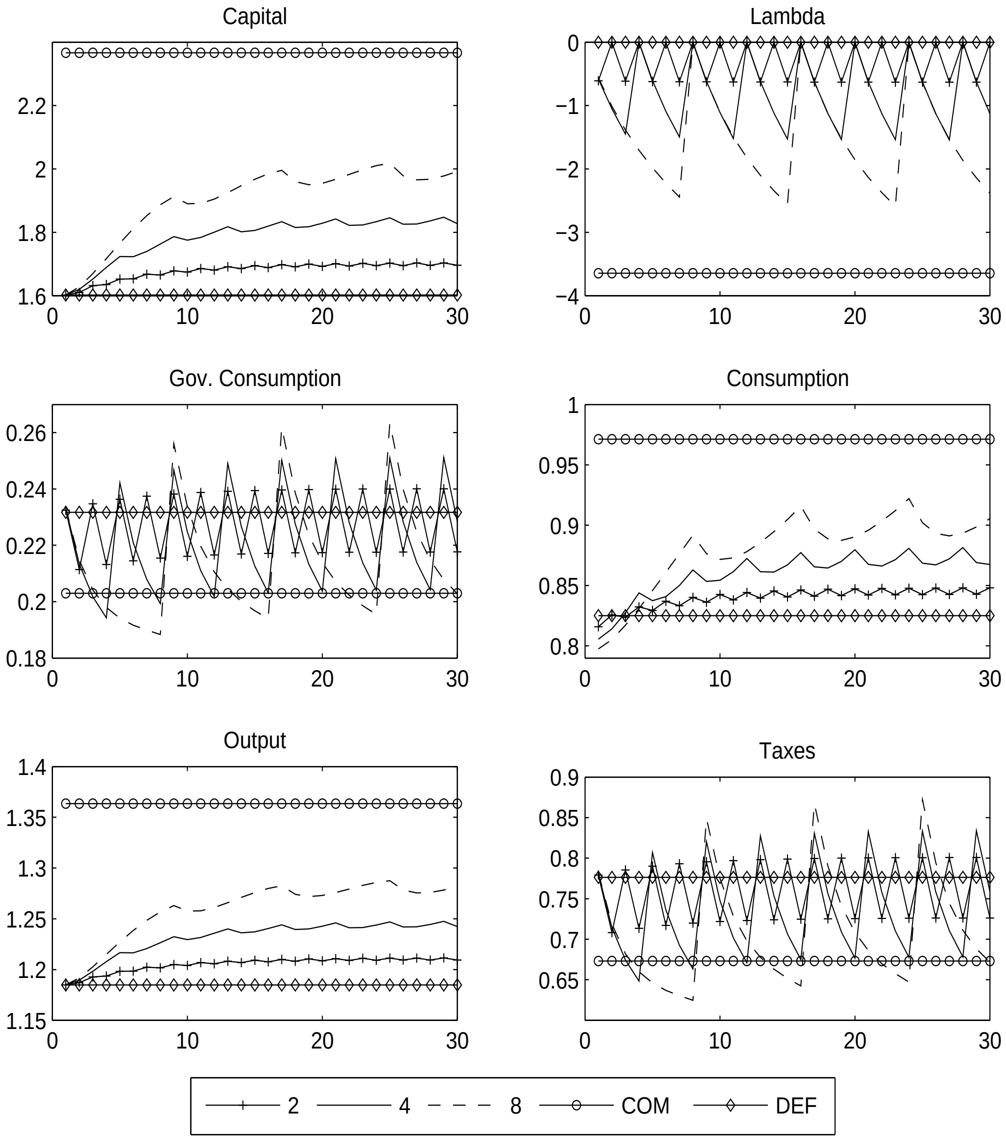

Figure 7: Total Income - T-periods model