Fertility, Education, and Mortality: A Unified Theory of the Economic and Demographic Transition

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Abstract

This paper provides a unified theory of the economic and demographic transition. Optimal decisions about fertility, education of children as well as own education are affected by different dimensions of mortality and technological progress which change endogenously during the process of development. The model generates an endogenous transition from a regime characterized by limited human capital formation, little longevity, high child mortality, large fertility and a sluggish income and productivity growth to a modern growth regime in which lower net fertility is associated to acquisition of human capital and improved living standards. Simulations illustrate the dynamics of the model which are found to be in line with empirical observations and stylized facts.

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1 Introduction

Transitions from stagnant economic environments to developed economies with permanent growth and improving living conditions are characterized by substantial changes in many dimensions of human life. In the Western world, for example, aggregate and per capita income displayed a virtual explosion from the second half of the 18th century onwards, after almost stagnant development during the entire history. The changes that this transition brought about, however, were not confined to the economic domain only. Rather, this transition to modern societies also involved changes in other important dimensions of the human development and indicators of general living conditions and health reflected in fertility behavior, population density, educational attainment, child mortality and adult longevity. In fact the economic transition from stagnation to growth was associated to a demographic transition from a regime characterized by high child mortality, little longevity, large fertility, and a positive correlation between living conditions and fertility, to a regime with large improvements in terms of adult longevity, reduced child mortality and widespread education, as well as lower total and net fertility.[1] The recent episodes of escape from stagnation are characterized by similar changes with the improvements in economic and living conditions being associated with demographic transitions.

This paper provides a unified theory of endogenous economic and demographic transitions, which result from the interactions between the economic domain, in particular human capital formation and technological improvements, and the demographic environment, reflected by adult longevity and child mortality, as well as fertility. The framework allows for a systematic investigation of the role of health for education, fertility and the evolution of the economy. The model builds on, and unifies, the theories proposed by Galor and Weil (2000), who consider a trade-off between quantity and quality of children, and Cervellati and Sunde (2005), who study how life expectancy affects individual education decisions. We extend and generalize these frameworks, in which biased technological change eventually triggers a phase transition, in several directions. In this paper, we investigate the dynamic evolution of fertility and human capital formation, and the simultaneous transitions in different domains of human life.

We consider an overlapping generations setting in which heterogenous adults, who have successfully survived childhood, maximize their utility, which is affected both by their own consumption and the well-being of their surviving offspring. Adults decide about the number of children, the amount of time invested in providing their children with basic education, as well as about their own education in terms of type of human capital they want to acquire, and how much time to spend on its acquisition. The crucial state variables for these decisions are child mortality, i.e. the probability that the new-born children survive until adulthood, adult longevity, i.e. the life horizon that an adult has available, as well as the technological environment that determines the returns to human capital. Dynamically, the human capital acquired by a generation of adults affects the technology as well as living conditions in terms of adult longevity and child mortality faced by future generations. Changes in these state variables, in turn, affect the profitability of investing in the different types of human capital for those children who survive until adulthood, and therefore their decisions.

The paper presents an analytical characterization of the transition from stagnation to growth, which is based on the interplay between human capital formation, technological progress, and health. Education decisions are crucially affected by adult longevity which determines the horizon over which investment in skills can amortize. Technological progress changes the relative productivity of education and consequently the incentives both for providing children with basic education as well as for the acquisition of human capital as adult. Faster progress induces parents to invest more in the quality of their children. Furthermore, technological progress alters the

[1] Compare e.g. the data provided by Maddison (1991). We provide and discuss the stylized facts for Sweden, England and Wales in section 3.

relative wages for different types of human capital, leading to a changing population structure in terms of acquired skills. A change in the the relative costs for education change, for example in terms of the effectiveness of education time that is determined by the available schooling technology or the health status of individuals has similar implications. Therefore, changes in the environment crucially affect education and fertility behavior, which are intimately related.

We identify several distinct effects of mortality on education and fertility decisions. Increases in adult longevity induce an *income* effect that leads adults to invest more time in children as well as in their own education. Falling child mortality causes a *substitution* effect, allowing adults to reduce the total number of children to which they give birth. A changing technological environment gives rise to a larger return to basic education, and therefore a trade-off between the investment in *quantity and quality* of offspring. Finally, a *differential fertility* effect arises because high skilled individuals, for whom the opportunity cost of raising children is higher, give birth to fewer children. The overall effect on fertility and human capital acquisition depends on the interactions between these different effects, and evolves during the course of development.

From a dynamic perspective, the human capital and income of a generation affects the technological environment and the mortality of the following generations, creating a bi-directional feedback mechanism. The feedback between human capital, fertility and adult longevity eventually creates sufficiently high returns to investments in human capital for a sufficiently large fraction of the population and thus triggers a transition. The model generates a development path with the endogenous emergence of a phase transition from an environment with high mortality, high fertility, and little investment in education and skills to an environment with low mortality, large life expectancy, low fertility and widespread education. During the transition, the environment changes rapidly within just a few generations.

The pattern of this development path is consistent with available evidence. Human capital is the driving force behind the phase transition, which is initiated by changes in optimal human capital investments. This implies that changes in income are a consequence, not a cause, of the transition.[2] These changes in optimal in human capital investments in turn affect optimal fertility decisions. As larger fractions of the population optimally decide to acquire skills and spend more time in higher education, they reduce their fertility. This drop in fertility is reinforced by a reduction child mortality due to the substitution effect. As a result, eventually overall net fertility in the population declines. This reduction in net fertility, which is in line with empirical evidence, has been difficult to rationalize with reductions in child mortality alone in the previous literature.[3]

While the main features of the economic and demographic transitions observed in reality are similar across countries, the precise time structure of these transitions can be very different. For example, Galor (2005) and Doepke (2005) note that in some countries like England, fertility declined before child mortality fell. In contrast, adult longevity began to increase well before the economic transition, preceding reductions in fertility by as much as a century.[4] These authors note that these dynamics are not compatible with existing theories that link fertility behavior to child mortality. In our model, the precise timing of the different dimensions of the phase transition crucially depends on how the decisions of a generation affect the environment of their offspring, and thus their offspring's optimal decisions. By simulating the model for illustrative purposes we show that the behavior of key variables like adult longevity, infant mortality, gross and net fertility, and literacy and income per capita are in line with empirical observations and

[2]See Clark (2005) for a discussion of this point.

[3]Kalemli-Ozcan (2002, 2003) has investigated the role of uncertainty about child survival in inducing forward looking parents to 'hoard' children due to a precautionary motive. In her model, a substitution effect implies that gross fertility declines if child survival chances increase. However, Doepke (2005) investigates different versions of the quantity-quality model, including a version with sequential fertility, and concludes that child mortality cannot explain the fertility drop alone.

[4]See also Lorentzen *et al.* (2005).

historical evidence in different countries.[5]

An earlier strand of the literature analyzes models with multiple steady state equilibria and explains the transition from a stagnant regime to an environment of sustained growth by scale effects, see Goodfriend and McDermott (1995), exogenous technological change, see Hansen and Prescott (2002), or shocks, that move the dynamic system from one steady state to another, see Blackburn and Cipriani (2002) and Boucekkine, de la Croix, and Licandro (2003).

By considering an endogenous phase transition in a unified framework, our paper contributes to the recent literature of unified growth theories.[6] As in Galor and Weil (2000), which initiated the literature on unified theories, and Galor and Moav (2002), who investigate the role of natural selection forces, we consider a framework in which parents face a quantity-quality trade-off. The change of this trade-off in the course of development brings about a decline in fertility that is associated to a more intensive caring for children: a shift from quantity to quality.[7] In our model, however, individuals also face another trade-off, namely between resources devoted to child bearing and the parents' education investment in the acquisition of their own human capital. The consideration of this additional trade-off is crucial for the results of the model.

The role of adults longevity, child mortality and fertility for the economic and demographic transition has been investigated by Soares (2005), who studies the effects of exogenous changes in longevity and mortality. In contrast, the focus in this paper is on the endogenous interactions between human capital, fertility, and mortality. This focus on endogenous life expectancy is shared by Galor and Moav (2005) who investigate the role of changing environmental conditions, like population density, for longevity. Differently from our approach, which is based on Cervellati and Sunde (2005) and where life expectancy depends on human capital, Galor and Moav (2005) consider a model of natural selection in which the change of longevity depends on the survival of different human types in response to changes in the extrinsic mortality environment.

The paper proceeds as follows. Section 2 introduces the basic framework, discusses the individual decision problem concerning education choice and fertility behavior, and derives the optimal decisions. Section 3 then derives and characterizes the general equilibrium allocation within a given generation. Section 4 embeds the model in a dynamic framework, derives and characterizes the dynamic equilibrium and derives the long-term development patterns. The section ends with illustrative simulations of the model and a brief discussion of the simulations in light of empirical and historical evidence. Section 5 concludes. For convenience, all proofs are relegated to the Appendix (STILL TO BE ADDED).

2 A Model of Human Capital and Fertility Choices

2.1 The Framework

Individual Endowments and Timing. Time is continuous, $\tau \in \mathbb{R}^+$. The economy is populated by an infinite sequence of overlapping generations of individuals, which are denoted with subscript t, where $t \in \mathbb{N}^+$. A generation of individuals t, born at some moment in time τ_t enjoys a childhood of length $k_t = k$ after which individuals turn adult. Reproduction is asexual and takes place once individuals become adults. Consequently, every generation is born $k_t = k$

[5]The framework can generate different development patterns and therefore be used to investigate the reasons for different transition patterns. This point is illustrated using the cases of England and Sweden.

[6]Empirical findings related to assumptions and implications of our model are discussed in detail below.

[7]Other contributions studying the role of fertility for long-term development and the demographic transition, include Kogel and Prskawetz (2001) (2001), Hazan and Berdugo (2002), Lucas (2002), Kalemli-Ozcan (2002), Lagerlof (2003) and Doepke (2004), among others. See also the extensive survey by Galor (2005) for a detailed discussion of the theoretical literature and empirical evidence regarding these issues.

periods after the birth of the respective previous generation.[8] Not all children of generation t survive childhood because of infant and child mortality. The fraction of children surviving to adulthood is denoted by $\pi_t \in (0, 1)$. The timing of the model is illustrated in Figure 1.

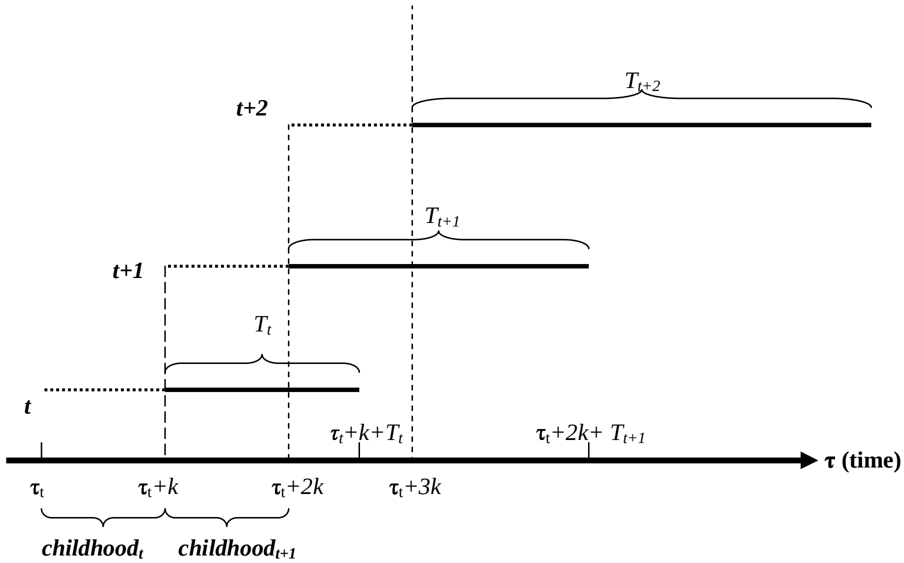


Figure 1: Timing of Events

Each generation t is formed by a continuum of individuals denoted by i. At birth, every individual is endowed with ability $a \in [0, 1]$. The distribution of ability within a given generation of new-born individuals is uniform over the unit interval. Since child mortality affects every child the same way, the ability distribution of adults is also uniform.[9] We assume that individuals make no decisions during childhood. But as soon as they become adults (i.e. at age k) individuals make decisions concerning their own education, fertility and the time invested in raising their offspring. As we want to concentrate on the endogeneity of fertility choice and hence family size, we abstract from issues of non-divisibility. The number of children is therefore a continuous choice variable, $n \in \mathbb{R}_0^+$. In order to highlight the mechanism we also restrict attention to a deterministic framework without sequential child birth.[10] A generation of adults consequently consists of a continuum of agents with population size N_t, which is determined by size and fertility of the previous generation, as well as the survival probability of children. Adults of generation t face a (deterministic) remaining life expectancy T_t. The determinants of both child survival probability π_t and adult longevity T_t are investigated below.

[8] Instead of assuming a fixed frequency of births, the length of the time spell between the births of two successive generations, hence the timing of fertility, could be modelled as a function of the life expectancy of the previous generation. This would modify the results concerning population size, but would leave the main results concerning the economic and demographic transition unchanged. See Blackburn and Cipriani (2002) for a paper on long-term development that deals with changes in the timing of fertility. See also for a model of the interactions between labor market participation and fertility timing.

[9] The assumption of a uniform distribution is for simplicity since the central results can be generated with any distribution of ability a among the surviving adults including the degenerate distribution in which all individuals are equally able. The *ex ante* distribution of innate ability or intelligence does not change over the course of generations.

[10] As investigated by Doepke (2005), accounting for the fact that in reality the number of children is discrete can affect the optimal choice if the parents have a precautionary demand for children. In the current framework the assumption of n being a continuous variable is only made for simplicity. Uncertainty giving rise to precautionary motives in fertility behavior is realistic, but strictly complementary to our analysis of fertility. Sequential fertility decisions would complicate the analysis without adding any additional insights.

Preferences and Production Function. Adults care about their own consumption as well as about the potential aggregate income of their (surviving) offspring, along the lines of Galor and Weil (2000). This formulation reflects the trade-off between the resources devoted to own consumption and to raising children. We denote by c_t^i the total lifetime consumption of an agent i at generation t, and by n_t^i its total number of offspring. Individual preferences are represented by a lifetime utility function which is strictly monotonically increasing, strictly quasi concave and by satisfies the standard boundary conditions that insure interior solutions. In particular, lifetime utility is given by

$$u(c_t^i, y_{t+1}^i \pi_t n_t^i) = \left(c_t^i\right)^{(1-\gamma)} \left(y_{t+1}^i \pi_t n_t^i\right)^{\gamma} \text{ with } \gamma \in (0,1) \tag{1}$$

where y_{t+1}^i is the (potential) lifetime income of an offspring of individual i. The second component generates a link between generations that can be interpreted as a warm glow type of altruistic preferences. We abstract from life cycle considerations, the choice of the optimal consumption and savings path over the life cycle, discounting etc.[11]

Income y_{t+1}^i results from supplying human capital on a competitive labor market as studied below. A unique consumption good is produced with an aggregate production technology that uses all human capital available in the economy at any moment in time, i.e. embodied in all generations alive at that date, as the only factors of production. We consider two types of human capital. The first type is interpreted as high-quality human capital characterized by a higher content of abstract knowledge. We refer to this as *skilled* or theoretical human capital and denote it by s. The second type is labelled *unskilled*, or practical human capital, denoted by u, and contains less intellectual quality, but more manual and practical skills that are important in performing tasks related to existing technologies.[12] Apart from their different role in the production process, the main difference between the two types of human capital concerns the intensity in which they require time and ability in the education process. A detailed discussion of these issues is provided in the next section.

The unique consumption good is produced with an aggregate production function with constant returns to scale. We adopt a simple formulation in which generation specific vintage technologies are identified by the total factor productivity A_t. A given generation t can only operate the respective vintage t.[13] In particular, generation t produces Y_t units of the consumption good using its stock of human capital, H_t^u and H_t^s, in the CES production function

$$Y_t = A_t \left[x_t \left(H_t^u\right)^{\eta} + (1 - x_t)\left(H_t^s\right)^{\eta}\right]^{\frac{1}{\eta}} \tag{2}$$

with $\eta \in (0,1)$ and the relative production share $x_t \in (0,1)\ \forall t$.[14] This allows us to concentrate on the production process during a given generation when determining the labor market equilibrium. Technological progress, studied below, takes place in the form of vintage of technology

[11] This formulation also implies that individuals can perfectly smooth consumption as well as the utility from children over their life. At the same time, however, individuals cannot perfectly substitute utility from their own consumption by utility derived from their offspring.

[12] Hassler and Rodriguez-Mora (2000) use a related perception of abstract versus applied knowledge.

[13] Human capital is inherently heterogenous across generations, because individuals acquire it in an environment characterized by the availability of different vintages of technologies. Human capital acquired by agents of a generation allows them to use technologies up to the latest available vintage. This implies that a generation's stock of human capital of either type is not a perfect substitute of that acquired by older or younger generations, and is sold at its own price.

[14] The focus of the paper is not on the macroeconomic role of demand for different consumption goods. The role of different income elasticities for different goods for structural change from agriculture to industry has been studied by Laitner (2000)

with larger productivity. The rate of technological progress is given by,

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}} \tag{3}$$

Below we will also consider the case of skill biased technological change by restricting attention to the case in which $x(A_t)$ where $x'(A_t) < 0$.

Human Capital. In order produce income y_t^i and consume, individuals have to acquire human capital, which they can then supply on the labor market. Every generation has to build up the stock of human capital from zero, since the peculiar characteristic of human capital is that it is embodied in people.

We model human capital production as the outcome of an education process that involves both the decisions of the individual as well as those of his parents. In particular, investments in education by the individual and by his parents are complementary inputs in the production of human capital. Parents can affect the level of human capital, and therefore income, of their children. But this is possible only to a certain extent. The type and amount of human capital acquired by each individual ultimately depends on his own choices, his ability and the effort he devotes to his own education. All that parents can do is facilitate the adoption of any type of human capital for their children by spending time with them during childhood to give them some preparatory education.[15] In order to isolate the development effects related to the various dimensions of health and human capital formation, any links between generations through savings or bequests are excluded.[16]

Human capital acquisition involves a time intensive education process. We denote by $e^{i,j}$ the time devoted by an individual i to his own education to acquire human capital of either type, unskilled or skilled, $j = u, s$. The different types of human capital differ in terms of the returns they generate. On the other hand, the different types of human capital are inherently different with respect to the time intensity of their acquisition, and the effectiveness of ability. The acquisition of human capital characterized by more abstract knowledge requires a longer time investment devoted to the acquisition of the building blocks of the elementary concepts without being productive in the narrow sense. Once the basic concepts are mastered, the remaining time spent on education is more productive. This is captured by a fix cost $\underline{e}^j$ measured in time units, which an agent needs to pay when acquiring h^j units of human capital type $j = \{u, s\}$.

The effectiveness of the time an individual spends in formal education depends on both individual innate ability and the preparatory education he received by his parents. Ability magnifies time investments human capital j by a factor $m^j\left(a^i\right)$ with $\partial m^j\left(a^i\right)/\partial a^i \geq 0$. Similarly, the larger the time devoted by parents to raise their offsprings, the more effective will the children be in acquiring any type of knowledge. Denote by $r_{t-1} \in [0, 1]$ as the fraction of lifetime of parents of generation $t-1$ spent in raising each of their children. Then the effect of parental preparatory education is reflected in the higher productivity of every unit of time spent by children in own education $e^{i,j}$ given a higher time investment of the parents,

$$f\left(r_{t-1}, g_t\right) \tag{4}$$

with $f_r\left(.\right) > 0$, $f_g\left(\cdot\right) < 0$, $f_{rr}\left(.\right) < 0$, $f_{gg}\left(.\right) > 0$ and $f_{rg}\left(.\right) > 0$ for any $\left(r_{t-1}, g_t\right) \geq 0$.[17] These assumptions follows Galor and Weil (2000) and imply that the larger the time spent raising

[15] This novel way of modeling human capital formation as the outcome of optimal choices of parents and children has important theoretical implications for the relationship between fertility, human capital and socio-economic conditions (health and technological development).

[16] We also abstract from real resources as input for the human capital formation process, as well as issues related to capital market development and public provision of education.

[17] The assumption $f_{rg} > 0$ represents a necessary but not sufficient condition for the emergence of a quantity-quality trade-off in fertility choices.

children r_t the larger the impact on resulting human capital. On the other hand, the negative effect of the rate of technological progress g_t reflects an obsolescence or erosion effect. Faster technical change implies a lower effectiveness of education, although a larger r_t tends to reduce this negative effect of a rapidly changing technological environment.

These characteristics are formalized in the human capital production function

$$h^j\left(a^i, r_{t-1}, e_t\right) = \alpha^j f\left(r_{t-1}, g_t\right)\left(e_t - \underline{e}^j\right) m^j(a^i); \quad \forall \quad e \geq \underline{e}^j \quad , \; j = u, s \tag{5}$$

and $h^j = 0 \; \forall e < \underline{e}^j$ with $\alpha^j > 0$. Individual ability, the time spent by an individual in formal education, and the time spent by his parents during the upbringing are complementary factors in the production of human capital. This implies, in particular, that the larger the time spent by parents in child-raising the larger is the amount of human capital acquired by the child for any degree of formal education.

The education process inherently differs among different types of human capital with respect to the time intensity of the education process and the effectiveness of ability. In particular, the larger the content of abstract knowledge incorporated in human capital the larger is the time required to master the building blocks and basic concepts of this type of human capital. In other words, the minimum education time required to make investment in human capital of type j productive is increasing in the degree of skills, which implies $\underline{e}^s > \underline{e}^u \geq 0$. After the building block are mastered, that is after a period of time $\underline{e}^s$ or $\underline{e}^u$ has been spent in education, every unit time of education is productive in terms of acquiring human capital of either type. These assumptions jointly imply that the time spent in the unskilled education process is more rapidly effective in producing human capital albeit with a lower overall productivity. Finally, we assume that ability is relatively more important (and effective) when acquiring advanced skills. For analytical convenience we assume that the production of skilled human capital is linearly increasing in individual ability $m^s(a) = a$ while the acquisition of unskilled human is independent from a so that $m^u(a) = 1$.

This formulation of the education process implies that, an individual i that has received an education r_{t-1} from his parents and acquires human capital of type j by investing an amount of $e_t^{i,j}$ in education can earn a total lifetime income of

$$y_t^{i,j}\left(a^i\right) = y_t^j\left(a^i, r_{t-1}^i, e_t^{i,j}\right) = w_t^j h^j\left(a^i, r_{t-1}^i, e_t^{i,j}\right)\left(T_t - e_t^{i,j}\right) . \tag{6}$$

2.2 Education and Fertility Decisions

The individual optimization problem. We now turn to the choice problem of adult members of a given generation t. Investments in own human capital as well as rasing children imply costs in terms of time that is not available for market work. While spending time in education $e_t^{i,j}$ an individual cannot work and therefore earns no income. Similarly, with respect to fertility parents face time equivalent costs of r_t to raise children that survive until adulthood. In particular, raising children involves costs in terms of foregone working time equal to $r_t \pi_t n_t T_t$.[18] As in Barro and Becker (1989) and Galor and Weil (2000) this feature implies the existence of a trade-off in fertility choices between the quantity and the quality of offsprings: parents need to choose both number of children and the time devoted to raising them. Additionally, since individuals have to choose their own type of human capital as well as the optimal time in formal education, the problem also implies a trade-off between acquisition of *own* human capital and fertility and education of offspring. This implies that own education, quantity and quality of the children influence one another so that these optimal choices must be treated jointly.

[18] One could also assume that the birth of each child entails a separate cost equivalent to a share b of lifetime so that the total cost of births is given by bn_tT_t. The consideration of this cost would leave all qualitative results unchanged.

Formally, the problem of an individual with ability a born in generation t can be characterized as follows. The individual has to choose the type of human capital $j \in \{u, s\}$ he wants to acquire and the optimal education time spent on its accumulation, $e_t^{i,j}$.[19] The individual also chooses the number of offsprings n_t^i and the time spent with each of them r_t^i.[20]

Denote by $w_t^j > 0$ the wage rate paid at any moment in time to every unit of human capital h_t^j of type $j = u, s$ acquired by generation t. Since each individual is of negligible size and price taker on the market, in making optimal choices individuals take life expectancy T_t, child survival probability π_t and the wage rates w_t^j and w_{t+1}^j as given. Optimal choices are made under the lifetime budget constraint of an individual of generation t acquiring human capital j. The solution of the individual maximization problem (7) denoted by $\{j^*, e_t^{i,j*}, n_t^{i*}, r_t^{i*}\}$ can be characterized by

$$\{j^*, e_t^{i,j*}, n_t^{i,j*}, r_t^{i*}\} = \underset{\{n_t, r_t,, e_t^j, j=u,s\}}{\arg\max} \quad u_t\left(c_t^i, \pi_t n_t^{i,j} y_{t+1}^j\left(a^i, r_t^i, e_{t+1}^{i,j}\right)\right) \quad \text{subject to} \quad (7)$$

$$c_t^i \quad \leq \quad (T_t\left(1 - r_t^i \pi_t n_t^{i,j}\right) - e_t^j) w_t^j h_t^j\left(a^i, r_{t-1}^i, e_t^{i,j}\right) \text{ subject to (5) and (6) for } \; j = u, s\,.$$

In order to derive the optimal choices of an individual we proceed as follows. We first characterize the optimal education, fertility and child raising choices that maximize individual utility conditional on choosing to acquire a particular type of human capital $j = u, s$. We then identify the optimal education decision in terms of the type of human capital by comparing the utility that each agent derives from acquiring each type of human capital. The optimal individual choice h_t^{j*} is given by that type of human capital that offers the highest lifetime utility given the optimal choices of education time and fertility.

Education, fertility and child raising for j-type human capital. The optimization problem is strictly globally concave so that first order conditions uniquely identify the optimal choices made by any individual, conditional on the acquisition of a particular type of human capital. The optimal choices of education time and number of children for an individual of ability a acquiring human capital type l are given by the solution to the following optimization problem,

$$\{e_t^{i,j*}, n_t^*, r_t^{i*}\} = \arg\max\left[\left(T_t\left(1 - r_t^i \pi_t n_t^{i,j}\right) - e_t^{i,j}\right) w_t^j h_t^j\left(a, r_{t-1}, e_t^{i,j}\right)\right]^{(1-\gamma)}\left[y_{t+1}^i \pi_t n_t^{i,j}\right]^{\gamma} \quad (8)$$

Solving the optimization problem one obtains optimal education time and optimal fertility of agents acquiring human capital of type j.[21] The first order conditions read

$$e_t^{i,j} = \frac{T_t\left(1 - r_t^i \pi_t n_t^{i,j}\right) + \underline{e}^j}{2} \quad (9)$$

and

$$n_t^{i,j} = \frac{\gamma\left(T - e_t^{i,j}\right)}{T r_t \pi} \quad (10)$$

[19]Allowing individuals to acquire various amounts of both types of human capital would not change the formal arguments, but would imply a somewhat different interpretation of human capital.

[20]Note that the formulation (5) implies that the time parents spend on the education of a child r improves the ability of the child in acquiring any type of human capital without creating a bias. In equilibrium it will be optimal to spend the same share r_t with each offspring as shown next. This feature of the model also implies that the optimal choice of the type of education chosen by the children does not depend on the time that their parents spent raising them. This neutrality of parental education represents a natural benchmark and greatly simplifies analytical tractability.

[21]See appendix for derivations and Proofs.

The inspection of these first order conditions illustrates that, *ceteris paribus*, having more children decreases the time invested in own education and *vice versa*. A higher fix cost $\underline{e}^j$ involved with the acquisition of high skill human capital requires a larger time investment in education, however. Furthermore the quantity-quality trade-off implies that the optimal number of children is decreasing with the time invested in each of them.

Concerning the optimal time spent in raising children, the first order condition for the interior solution is given by,

$$(1-\gamma) T_t \pi_t n_t^{i,j} = \gamma \left[T_t \left(1 - r_t^i \pi_t n_t^{i,j} \right) - e_t^{i,j} \right] \frac{\partial f\left(r_t^i, g_t\right)}{\partial r_t^i} \frac{1}{f\left(r_t^i, g_t\right)}$$

Making use of the first order condition with respect to fertility, this can be rewritten as the following intuitive condition,

$$\varepsilon_{f,r} \equiv \frac{\partial f\left(r_t^i, g_t\right)}{\partial r_t^i} \frac{r_t^i}{f\left(r_t^i, g_t\right)} = 1 \,. \tag{11}$$

Notice that the optimal investment in raising children r_t^{i*} implicitly defined by (11) depends on g_t but not on own education choices, fertility or ability. Given the optimal $r_t^{i*} = r_t^*$ implied by (11) and solving the system of equations implied by (9) and (10), we obtain the individual optimal choices of e and n for each type of human capital j.

Proposition 1. *For any* $\left\{ w_t^j, T_t, \pi_t \right\}$, *individual optimal education, fertility and time devoted to children of an agent deciding to acquire human capital of type* $j = \{u, s\}$, $\left\{ e_t^{j*}, n_t^{j*}, r_t^* \right\}$ *given by,*

$$n_t^{i,j*} = n_t^{j*} = \frac{\gamma}{2-\gamma} \frac{T_t - \underline{e}^j}{T_t r_t^* \pi_t} \quad \text{and} \tag{12}$$

$$e_t^{i,j*} = e_t^{j*} = \frac{T_t (1-\gamma) + \underline{e}^j}{(2-\gamma)} \tag{13}$$

where r_t^* *solves* (11)*, for all* i*.*

By implicit differentiation of (11) and given the assumptions about $f(\cdot)$, we have

$$\frac{\partial r_t^*}{\partial g_t} > 0 \,. \tag{14}$$

This implies that the key determinant of the "quality of children" is the rate of technological change g_t. From (12) we also get a negative relationship between quantity and quality of children. This result is in line with the treatment by Galor and Weil (2000), where the trade-off between quantity and quality of children crucially changes with the rate of technical change. Everything else equal, stagnant economies are characterized by large fertility and little investment in children quality while larger improvements in technical progress induce a reduction in the number of children and an increase in the time devoted to raise each of them.

Concerning the individuals' own education choice conditions (12) and (13) imply that the acquisition of skilled and unskilled human capital induces individual to spend a different amounts of time in formal education and have a different amount of children. In particular, individuals that decide to acquire high quality human capital spend more time in their own education and choose to have a lower number of offsprings, $n_t^{s*} < n_t^{u*}$ while $e_t^{s*} > e_t^{u*}$. This "differential fertility" effect is in line with empirical observations.[22]

[22] This differential fertility behavior is empirically well documented, see, e.g., Castro-Martín (1995), Rindfuss *et al.* (1996), and Mare (1997), as well as Caldwell (1999) for a historical discussion of the role of education for fertility. See also de la Croix and Doepke (2003) for a theoretical investigation of the role of differential fertility in explaining the timing of the demographic transition and further empirical references.

Individual Choice of Human Capital. In order to fully characterize the optimal individual choice we now turn to the individual problem of choosing the type of education, s or u. This choice depends, among other things, on the level of wages which are determined in general equilibrium on the labor markets and which individuals take as given.

Using e_t^{s*} and e_t^{u*} from conditions (5) and (13) one obtains the respective levels of human capital,

$$h_t^{j*}\left(a, r_{t-1}^i, e_t^{j*}\right) = \alpha f\left(r_{t-1}^i, g_t\right) \frac{(1-\gamma)\left(T_t - \underline{e}^j\right)}{(2-\gamma)} m^j\left(a^i\right) \qquad \text{for } j = u, s : . \tag{15}$$

Note that agents with higher ability have a comparative advantage in acquiring h^s, so that the amount of h_t^s monotonically increases in a, and consequently lifetime utility for those investing in s increases monotonically in the ability parameter. Hence, conditions (12) and (13) imply that for any individual of ability a, there is a unique e_t^{j*} and level of fertility n_t^{i*} which maximize his lifetime utility conditional on acquiring a given type of human capital. Concerning the individual decision problem (8) this signifies that the indirect utility enjoyed by acquiring $s-type$ human capital, $u_t^{s*}(a)$ is strictly monotonically increasing in ability. Consequently, for every vector of wage rates, there exists a unique ability threshold $\widetilde{a}_t$ for which the indirect utilities of acquiring either types of human capital are equal. Denoting by $\alpha \equiv \alpha^u/\alpha^s < 1$ the relative productivity of a unit time of education time in the formation of the two types of human we have,

Lemma 1. *For any* $\{w_t^s, w_t^u, T_t, \pi_t\}$ *there exists a unique* $\widetilde{a}_t \in (0,1)$ *given by,*

$$\widetilde{a}_t = \alpha \left(\frac{T - \underline{e}^u}{T - \underline{e}^s}\right)^{\frac{2-\gamma}{1-\gamma}} \frac{w_t^u}{w_t^s} \tag{16}$$

such that all agents with $a \leq \widetilde{a}_t$ *optimally choose to acquire u-type human while all agents such that* $a > \widetilde{a}$ *acquire s-type human capital as in* (15).

This ability threshold that characterizes individuals acquiring h^s instead of h^u is increasing with the relative wage w_t^u/w_t^s, which measures the the relative returns of the different human capital investments. At the same time, the ability threshold is decreasing with the relative fix cost necessary to master the basic knowledge about the particular type of human capital, $\underline{e}^u/\underline{e}^s$. Furthermore the threshold is decreasing with life expectancy, since a longer life increases the period over which the education investment is amortized, which facilitates the acquisition of high quality human capital even for individuals with lower ability.

Consequently, for any given distribution of abilities $d(a)$, the threshold $\widetilde{a}_t$ determines the fractions of the population of adults of a given generation that acquire high skilled human capital denoted by λ_t. This implies that, for any given distribution of ability, the share of population acquiring human capital is a monotonic function of the threshold $\widetilde{a}_t^*$. When adopting a uniform distribution for simplicity, we have

$$\lambda_t = \lambda(\widetilde{a}_t^*) := \int_{\widetilde{a}_t^*}^1 d(a)da = (1 - \widetilde{a}_t^*) \quad \text{and} \quad (1-\lambda_t) = (1 - \lambda(\widetilde{a}_t^*)) := \int_0^{\widetilde{a}_t} d(a)da = \widetilde{a}_t^* \ . \tag{17}$$

2.3 Health, Fertility and Human Capital

In the previous section we investigated the relationship between fertility behavior and human capital formation, which is the result of the effort of parents and children. Before investigating the dynamic evolution of the model we briefly discuss the role of adult longevity and child mortality for individual decisions.

Several effects determine human capital formation and fertility behavior. The first effect is the well-known *quantity-quality* trade-off between the number of children and the time spent in raising them. From (11) and (14) it is clear that this trade-off depends crucially on the technological environment, which changes the effectiveness of the time spent by parents in raising their children. However, this trade-off is independent both from adult longevity and child mortality. This neutral effect of longevity and child mortality on the quantity-quality trade-off has been pointed out previously by Moav (2005) and has been analyzed in more detail by Hazan and Zoabi (2005).

Optimal fertility and human capital decisions are also affected by own education choices. From (13), a longer lifetime duration T_t implies the acquisition of more human capital of either type by inducing individuals to spend more time in formal education. Similarly, (12) shows that, conditional on acquiring a certain type of human capital, longer life expectancy leads to an increase in gross fertility. This *income effect* implied by a longer life therefore tends to increase both human capital acquisition and fertility by relaxing the lifetime constraint. The reason is that a longer life expectancy increases the time horizon over which the time devoted to education can be amortized during the working life as well as the time available to raise children.

While child mortality does not affect own education choices it crucially affects (gross) fertility, which, from (12), is strictly monotonically decreasing in π for all individuals. This *substitution effect* due to the change in the relative price of consumption and children, implies that lower child mortality is a key determinant of fertility. This effect has been discussed in the literature, see e.g. Kalemli-Ozcan (2003) and Doepke (2005). There it has also been shown that the existence of uncertainty and a precautionary demand motive for children would tend to reinforce this effect.

The income and substitution effects work in the same direction for all individuals regardless of the type of human capital they acquire. From condition (16), $\widetilde{a}_t$ is decreasing in adults longevity T and from (17) this implies a larger $\lambda_t(\widetilde{a}_t)$. Hence, a longer life expectancy makes the acquisition of human capital h^s profitable for a larger share of population and induces more agents to be skilled. But since $n_t^{u*} > n_t^{s*}$ this implies a *differential fertility* effect: by inducing individuals to acquire skilled human capital and to adjust their fertility choices accordingly, life expectancy tends to reduce (gross) fertility in the population.

As a result, health in the form of adult longevity and child mortality affects gross and net fertility in the population both directly, by inducing a change in the optimal choice of education time and individual fertility, and indirectly by affecting individuals' education choices concerning the different types of human capital. The overall effect on the population wide fertility rate depends on the relative strength of the different effects at work. This discussion of the demographic and technological determinants of gross fertility is summarized in

Proposition 2. *For any $\{A_t, T_t, \pi_t, g_t\}$ the average gross fertility rate is given by,*

$$\begin{aligned} n_t^* &= n_t(T_t, \pi_t, \lambda_t, g_t) = (1-\lambda_t)n_t^{u*} + \lambda_t n_t^{s*} \\ &= \frac{\gamma}{2-\gamma}\left[(1-\lambda_t(T_t))\frac{T_t - \underline{e}^u}{T_t r_t^* \pi_t} + \lambda_t(T_t)\frac{T_t - \underline{e}^s}{T_t r_t^* \pi_t}\right] \\ &= \frac{\gamma}{2-\gamma}\frac{T_t - ((1-\lambda_t(T_t))\underline{e}^u + \lambda_t(T_t)\underline{e}^s)}{T_t r_t^*(g_t)\pi_t}, \end{aligned} \tag{18}$$

with

$$\frac{\partial n_t^*}{\partial \pi_t} < 0, \quad \frac{\partial n_t^*}{\partial T_t} \gtrless 0 \text{ and } \frac{\partial n_t^*}{\partial g_t} < 0\,. \tag{19}$$

In short, the previous Proposition states that an economy with a lower child mortality and a quickly changing technological environment is characterized by lower gross fertility due to the *substitution* effect and the change in the *quantity-quality* trade-off. Adults longevity has in principle an ambiguous effect on fertility due to the interaction between the *income* and the *differential fertility* effect. In Section 3 we show that, taken together, the different effects can account for the demographic transition from an environment with large fertility, low education and slow growth to an environment with low fertility, widespread education and rapid and sustained growth. The actual timing and the features of this transition, however, will depend on the interactions between the different driving forces. This implies the possibility for idiosyncratic differences in the development pattern of different economies.

Finally notice that the net fertility rate is affected not only by gross fertility, but also by how many children actually reach the reproductive age. Hence, net fertility is given by $\pi_t n_t^*(T_t, \pi_t, \lambda_t)$. Consequently, the size of the adult population of generation $t+1$ is therefore given by $N_{t+1} = N_t(1 + n_t\pi_t)$.[23] This discussion concludes the analysis of the economy for a given generation t in a partial equilibrium setting with exogenous wages. The following section investigates the determination of λ_t, T_t and π_t in the dynamic general equilibrium setting.

3 Dynamic Evolution of the Economy

We now turn to the investigation of the mutual interactions between the process of economic development, the acquisition of human capital and the various dimensions of health.[24] We then study the dynamics of the economy over the course of generations, before investigating the capability of the model to replicate the process of development including the endogenous economic and demographic transition in Section 4.

Each generation t of individuals takes adult longevity T_t, the survival probability of children π_t and the level of technological advancement, as expressed by A_t and x_t as given. Before being able to analyze the dynamics of the economy, we therefore need to introduce the dynamics of these state variables across generations.

3.1 Equilibrium Investments in Human Capital

In the previous section optimal individual choices were determined conditional on market wages. We now characterize the equilibrium investment in human capital that is compatible with endogenously determined wages. The aggregate levels of the two types of human capital supplied by generation t are denoted by

$$H_t^u = \int_0^{\widetilde{a}_t} h_t^u(a)d(a)da \qquad \text{and} \qquad H_t^s = \int_{\widetilde{a}_t}^1 h_t^s(a)d(a)da\,.$$

Wage rates are determined on competitive labor markets, and wages equal the respective marginal productivity.[25] Wages are therefore given by

$$w_t^s = \frac{\partial Y_t}{\partial H_t^s} \quad \text{and} \quad w_t^u = \frac{\partial Y_t}{\partial H_t^u}\,. \tag{20}$$

[23]To allow for more realism, below we consider the possibility that not all surviving adults are able to reproduce themselves, where this reproduction probability depends positively on the general health status of the society: $N_{t+1} = N_t(1 + n_t\pi_t\zeta)$ with $\zeta \in (0, 1]$ and where ζ depends positively on the general health status. In the following discussion, we simply assume that $\zeta = 1$.

[24]For illustration purposes, we postpone the discussion of the dynamic evolution of a variable reflecting the overall health status of society, which may affect the effectiveness of education for the acquisition of human capital α^j. While, as seen below, adding this dimension of health provides additional insights, the main results to be discussed next remain unchanged.

[25]Empirical evidence supports the view that different sectors competed for labor, and wage payments reflected productivity even at early stages of industrial development, see e.g. Magnac and Postel-Vinay (1997).

The corresponding ratio of instantaneous wage rates is then given by:[26]

$$\frac{w_t^u}{w_t^s} = \frac{x_t}{1-x_t}\left(\frac{H_t^s}{H_t^u}\right)^{1-\eta} \tag{21}$$

We now solve for the equilibrium allocation of the economy for a given generation of adult individuals. The intra-generational allocation of time and ability resources on the two types of human capital, and consequently also fertility behavior, is fully characterized by the unique threshold ability that splits the population. The solution of the individual decision problem in Section 2.2 has shown that, for each ratio of wages w_t^u/w_t^s, there exists a unique ability threshold $\widetilde{a}_t$ that splits the population such that individuals with higher ability acquire human capital of type h^s, while individuals with lower ability acquire h^u. Moreover, condition (21) implies that the wage ratio is determined by the relevant aggregate levels of human capital available in the economy for a given generation that arise from the equilibrium allocation of adults to the two types of human capital. Since for a given T_t the wage ratio is decreasing in the unique ability threshold, there exists a unique intragenerational equilibrium. Substituting for the optimal human capital supplies and the resulting wages into condition (16), the intragenerational equilibrium can implicitly be characterized by

$$\left(\frac{\left(1-\widetilde{a}_t^{*2}\right)^{1-\eta}}{\widetilde{a}_t^{*2-\eta}}\right)^{1-\gamma}\left[\left(\frac{x}{1-x}\right)^{1-\gamma}\left(\frac{1}{2}\right)^{(1-\eta)(1-\gamma)}\alpha^{\eta(1-\gamma)}\right]\left(\frac{T-\underline{e}^u}{T-\underline{e}^s}\right)^{1+\eta(1-\gamma)} = 1 \tag{22}$$

After some manipulations, one can get,

$$\left(\mathcal{G}(\widetilde{a}_t^*)\cdot\mathcal{F}_t - 1\right)T_t = \underline{e}^s\left(\mathcal{G}\left(\widetilde{a}_t^*\right)\mathcal{F}_t\right)^{1/(1+\eta(1-\gamma))} - \underline{e}^u \tag{23}$$

where $\mathcal{G}(\widetilde{a}_t^*) = \left(\frac{\left(1-\widetilde{a}_t^{*2}\right)^{1-\eta}}{\widetilde{a}_t^{*2-\eta}}\right)^{1-\gamma}$ and $\mathcal{F}_t = \left[\left(\frac{x_t}{1-x_t}\right)\left(\frac{1}{2}\right)^{(1-\eta)}\alpha^{\eta}\right]^{(1-\gamma)}$. Equation (23) implicitly identifies the unique equilibrium share of the population that optimally decides to acquire skilled human capital.

Proposition 3. [Human Capital Investment in Equilibrium] *For any given* $\{T_t, \pi_t, A_t, x_t\}$ *generation* t *there exists a unique*

$$\lambda_t^* \equiv 1 - \widetilde{a}_t^*$$

that solves (23) *and represents the equilibrium share of population optimally acquiring skilled human capital. Accordingly in equilibrium we have a unique vector,* $\left\{H_t^{j*}, w_t^{j*}, r_t^*, e_t^{j*}, n_t^{j*}, h^{j*}(a)\right\}$ *for each* $a \in [0,1]$ *and* $i = u, s$*, such that the individual choices of education investments and fertility,* (12) *and* (13)*, the implied optimal individual levels of human capital given by* (15)*, the corresponding population structure defined by the threshold* $\widetilde{a}_t$ *in condition* (16)*, and the resulting aggregate levels of human capital and wages in* (21) *are mutually consistent.*

The discussion so far implies that the problem of determining the equilibrium vector is well defined. We can now investigate the equilibrium relationship between life expectancy and human capital that is implied by (23),

$$T_t = \frac{\underline{e}^s\left(\mathcal{G}\left(\widetilde{a}_t^*\right)\mathcal{F}_t\right)^{-1/(1+\eta(1-\gamma))} - \underline{e}^u}{\left(\mathcal{G}(\widetilde{a}_t^*)\mathcal{F}_t\right)^{-1/(1+\eta(1-\gamma))} - 1}\,.$$

Given $\widetilde{a}_t^*$, all variables characterizing the equilibrium human capital formation of each generation are uniquely identified, because the implicit function relating the cut-off $\widetilde{a}_t^*$ to life expectancy T_t

[26] Decreasing marginal productivity of human capital of any type and Inada conditions insure interior equilibria, but these assumptions are not necessary to obtain the key results.

is monotonically decreasing in T_t. Hence, in equilibrium the relation between life expectancy T and the fraction of population acquiring h^u, $\widetilde{a}_t^*$ implies that the higher the life expectancy, the more people will invest in the time-consuming human capital acquisition of h^s in equilibrium. In turn, this implies that the fraction of individuals acquiring skilled human capital is monotonically increasing in T_t.

Proposition 4. [Human Capital and Life Expectancy] *For any generation t with $T_t \in [\underline{e}^u, \infty)$, the equilibrium fraction of individuals acquiring human capital, $\lambda_t(T)$, is implicitly defined by* (23) *and is an increasing function of expected lifetime duration T_t, with zero slope for $T \longrightarrow 0$ and $T \longrightarrow \infty$.*

Proof. In the Appendix. □

This finding is in line with evidence that suggests that life expectancy of adults is the key determinant of human capital acquisition and consequently income differences across countries, see Soares (2005) and Shastry and Weil (2003). Lorentzen, McMillan, and Wacziarg (2005) provide evidence that life expectancy is crucially associated to economic development through human capital acquisition.[27] Microevidence, such as that provided by Behrman and Rosenzweig (2004) using data on monoyzygotic twins, also shows a causal effect of health during childhood, measured by birth weight, on schooling attainment.

Apart from the monotonic relationship between adult longevity and human capital acquisition, the theory predicts that the effect of longevity on human capital is not necessarily smooth. Rather, the effect of longevity on the ability threshold is stronger and more pronounced for intermediate values of T and $\widetilde{a}_t^*$. For low levels of life expectancy, the share of population investing in h^s is small due to the fix cost involved with acquiring h^s, which prevents a large part of the population from receiving sufficient lifetime earnings to be worth the effort. The larger the fix cost, the more pronounced is the concavity of the equilibrium locus. In this situation, substantial increases in adult life expectancy are needed to give incentives to a significant fraction of individuals to acquire skilled rather than unskilled human capital. On the other hand, when the ability threshold is very low, and a substantial share of the population is engaged in h^s, very large increases in T are necessary to make even more individuals acquire h^s instead of h^u. Rather than insufficient life time to amortize investment in human capital, the reason for this are the decreasing returns to human capital of either type, which drives down the relative wage w^s/w^u as consequence of the high supply of h^u. This wage effect dampens the attractiveness of investing in h^s for the individuals with low ability, even though life expectancy is very high, rendering the equilibrium locus convex in this range.

3.2 Endogenous Mortality

The theoretical analysis so far illustrates the quite distinct roles of different dimensions of life expectancy for individual decisions. In particular, while adult longevity is a key determinant for human capital acquisition, the probability that children survive until adulthood primarily affects fertility behavior. In this section we endogenize the mortality environment faced by individuals.

Ample empirical evidence suggests that better knowledge about diseases and better technological conditions as well as public policies help to avoid or cure them, thereby reducing mortality (see Mokyr, 1993, Schultz, 1993 and 1999, Easterlin, 1999). Empirical findings also suggest that income, wealth and particularly the overall level of education affect mortality and health, see

[27] In contrast, Acemoglu and Johnson (2006) use predicted mortality as instrument for actual mortality to elicit the causal effect of life expectancy on growth. They find no evidence that a large exogenous increase in life expectancy during the 20th Century has led to a significant increase in per capita economic growth. In their view this confirms the effectiveness of combating poor health in LDCs, but also casts doubt on the conjecture that bad health conditions are the root of poverty of most LDCs.

Mirovsky and Ross (1998) and Smith (1999), and that a better educated society also invents and adopts more and better drugs (Lichtenberg, 1998, 2002, 2003). Recent evidence provided by van den Berg *et al.* (2006) shows on basis of individual data and using non-parametric and duration models that the general economic conditions faced by individuals during early childhood have a causal effect on these individuals' longevity, even as adults.[28]

In light of this evidence, we model the probability that members of generation t survive until adulthood, π_t, and adult life expectancy T_t faced by members of generation t as an externality that depends positively on the skilled human capital embodied in the parent generation $t-1$, reflecting the level of knowledge on which society can build their decisions, and on the income of the parents during the childhood y_{t-1}.[29]

We formalize this positive externality by linking a generation t's life expectancy to the fraction of population the previous generation $t-1$ that acquired human capital of type h:

$$T_t = \Upsilon\left(\lambda^*_{t-1}, y_{t-1}\right) \tag{24}$$

with $\partial T_t/\partial \lambda_{t-1} > 0$ with $\Upsilon(0) > 0$ and $\Upsilon(1) < \infty$ and $\partial T_t/\partial y_{t-1} \geq 0$, where $y_{t-1} = Y_{t-1}/N_{t-1}$. This formulation implies that the positive link and the dynamic process does not rely on scale effects or family-specific education. Similarly, the probability to survive childhood, which essentially reflects the inverse of child mortality, is assumed to be a function of the level of development in terms of average per capita income and the supply of skilled human capital, at the time of birth of children

$$\pi_t = \Pi\left(y_{t-1}, \lambda^*_{t-1}\right) \ , \tag{25}$$

where $y_{t-1} = Y_{t-1}/N_{t-1}$, $\partial \pi_t/\partial \lambda^*_{t-1} \geq 0$, $\partial \pi_t/\partial y_{t-1} > 0$, and $\Pi\left(0, \lambda^*_0\right) = \underline{\pi}$. Equivalently for our results, life expectancy and child survival probability could be related to average or total human capital or total income of the previous generation(s).[30] If one accepts a positive effect of the level of human capital on aggregate income, our assumptions are also consistent with evidence indicating that the aggregate income share spent on health care increases with aggregate income levels.[31] Note that improvements in adult longevity and child survival chances involve no scale effects.

3.3 The Phase Diagram

The bi-directional feedbacks between human capital and mortality can be analysed by studying the phase diagram of the discrete non-linear dynamic system linking human capital of generation t to the its longevity and future longetivy to past human capital acquisition.

We analyze the behavior of the economy by looking at the dynamic adjustment of human capital and lifetime duration conditional on the value of relative productivity. Also note that the intra-generational equilibrium does not depend on the mortality of children, π, as is illustrated

[28] They use macroeconomic conditions like the state of the business cycle at birth as instrument for individual economic conditions during childhood and show that these affect mortality later in life.

[29] Of course, in reality individuals can influence their life expectancy by a healthy life style, smoking habits, drug and alcohol consumption, sports and fitness behavior health care expenditures etc. Likewise can parents affect the survival probability of their children by their own behavior. Our model reflects the view that individuals lacked a detailed knowledge about which factors and activities are healthy or detrimental for average life duration during early phases of development. In the historical context, beneficial factors, such as leisure, were often simply not available. An explicit consideration of a positive correlation between life expectancy and the level of education would reinforce the results, but also complicate the analysis substantially, since it would give rise to a distribution of life expectancy across the population that changes during the process of development. While in principle such an extension would be feasible, we abstract from this possibility in this paper.

[30] See Tamura (2002) and Boucekkine, de la Croix, and Licandro (2002), or Blackburn and Cipriani (2002) for papers taking this approach.

[31] See Getzen (2000) and Gerdtham and Jönsson (2000) and the references there for respective evidence.

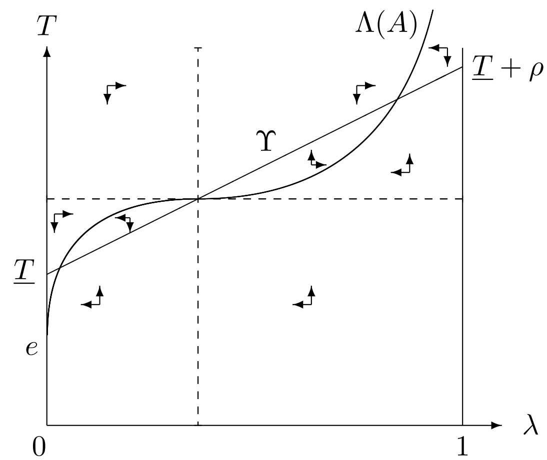


Figure 2: Phase Diagram of the Conditional Dynamic System

by condition (23). Denote the locus $T_t = \Lambda(\lambda_t, A)$ in the space $\{T, a\}$, which results from the intragenerational equilibrium condition (23), by $\Lambda(A,)$, and the locus $T_t = \Upsilon(a_{t-1})$ representing the intergenerational externality on lifetime duration by Υ. We can therefore illustrate the dynamics of the economy by first considering the reduced conditional system:

$$\begin{cases} \lambda_t &= \Lambda(T_t, A_t) \\ T_t &= \Upsilon(\lambda_{t-1}, y_{t-1}) \end{cases} \tag{26}$$

which delivers the dynamics of human capital formation and life expectancy for any given level of technology $A > 0$. From the previous discussion we know that the first equation of the conditional system is defined for $T \in [e, \infty)$.

Any steady state of the conditional system is characterized by the intersection of the two loci $\Lambda(A)$ and Υ.

Definition 1. [Steady States] *For any given A_t, a steady state equilibrium of the dynamic system* (26) *is a vector $\{\lambda(A), T(A)\}$ with $\lambda(A) \in [0,1]$ and $T(A) \in [e, \infty)$, such that, for any $A \in (0, \infty)$: $\lambda(A) = \Lambda^{-1}(T(A), A)$ and $T(A) = \Upsilon(\lambda(A))$. The associated equilibrium aggregate levels of unskilled and skilled labor are denoted by $H^u(A)$ and $H^s(A)$.*

Existence of at least one steady state equilibrium is ensured by the assumptions that $T(\lambda_t = 1) > \min\{\underline{e}^u, \underline{e}^s\}$ and $T(\lambda_t = 1) < \infty$. As a consequence of the non-linearity of Λ, the system (26) therefore displays at least one, and at most three steady state equilibria with different properties. Figure 2 illustrates the system (26) in the case when three equilibria exist. The possibility of the existence of several, inherently different equilibria requires a closer investigation to which we turn next.

3.4 Characterization of Steady State Equilibria

The results presented so far immediately allow us to make the following statement,

Proposition 5. [Properties of Steady State Equilibria] *For any $A \in (0, \infty)$, $a \in (\underline{a}(A), 1)$, and $T \in (\underline{e}, \infty)$, the conditional dynamic system* (26) *has steady state equilibria with the following properties: (1) There exists least one steady state equilibrium; if it is unique, it is globally stable; (2) there are at most three steady states denoted by $\{\lambda^z(A), T^z(A)\}$, where $z = M, \sim, G$ with the following properties: (i) $\lambda^M(A) \leq \lambda^\sim(A) \leq \lambda^G(A)$ and $T^M(A) \geq T^\sim(A) \geq T^G(A)$; (ii) $\{\lambda^M(A), T^M(A)\}$ and $\{\lambda^G(A), T^G(A)\}$ are locally stable; (iii) $\{\lambda^\sim(A), T^\sim(A)\}$ is unstable; (iv) in any steady state equilibrium positive amounts of both types of human capital are supplied on the labor market.*

Unstable equilibria $\{\lambda^{\sim}(A), T^{\sim}(A)\}$ are characterized by an intersection of the Υ-locus with the Λ-locus from below for intermediate levels of λ and G. In the following we only consider equilibria of types M and G. The M-type equilibrium describes the situation of an economy in a nearly stagnant environment that exhibits Malthusian features of low longevity, high child mortality, high fertility and a small share of skilled individuals in the population. Adult life expectancy T_A^S is low in such an equilibrium, and only a small share λ_A^S of the population acquires human capital of type h. An equilibrium of type G, on the other hand, more closely resembles modern, growing economies G with large adult longevity T_A^G, low child mortality, low fertility and a large fraction λ_A^G of the current adult population acquiring growth-enhancing human capital. Note that in any steady state equilibrium, the supply of both types of human capital is strictly positive.

Before studying the dynamics of the system and characterizing the economic and demographic transitions we briefly pause to comment on the role of the different state variables for human capital formation in equilibrium. In particular, the discussion above has illustrated that the crucial state variables that affect individual education choices, and hence the demographic as well as economic structure of the economy, are the technological environment and the reward structure for the different types of human capital. The state of technological development, as reflected by the importance of high-skilled labor, x_t, in the production process crucially affects the relative returns for high-skilled human capital. Conversely, the relative time intensity of acquiring human capital, α, crucially affects the costs of becoming high-skilled in terms of forgone earnings during the education process. Hence, it is conceivable that changes in these environmental parameters are complementary, even isomorphic, for changing the structure of society. These ideas can be made more concrete as

Lemma 2. *The equilibrium fraction of individuals acquiring skilled human capital λ_t^* increases, with the relative productivity x of human capital and with the relative productivity of the education processes α :*

$$\frac{\partial \lambda_t^*}{\partial x_t} > 0 \text{ and } \frac{\partial \lambda_t^*}{\partial \alpha} > 0 \tag{27}$$

Notice that both x and α can substitute adult longevity as state variable in inducing human capital acquisition. The more productive skilled human capital s is relatively to unskilled human capital u, the less restrictive is the fix cost requirement associated with its acquisition, because the break-even of the investment in education is attained at a lower age. The previous Lemma also implies that improvements in the returns of skilled human capital, leading to higher relative better wages due to, e.g., skill biased technological progress, or higher returns to education, resulting from a higher efficiency of the time spent in education due to, e.g., better schooling technologies, both spur the acquisition of high skilled human capital. Graphically, both an increase in x_t or in α implies a downwards shift of the locus $\Lambda(A,)$. In the following investigation of the dynamic implications of a changing technological environment, we restrict attention to endogenous skill biased technical change in the form of a relative total factor productivity which is increasing overtime. Notice, however, that improvements in the production technologies of human capital would deliver qualitatively similar dynamics.

The empirical evidence also suggests that adult and child mortality are affected by related, but somewhat distinct, determinants. Cutler *et al.* (2006) provide a survey of recent findings concerning the determinants of mortality. They show that the increase in life expectancy of about 30 years in the past century and the large cross-country variation in mortality is strongly correlated with changes and differences in income per capita. They review the determinants of these patterns over history, over countries and across groups within countries and identify the application of scientific advance and technical progress (which is induced and facilitated by human capital) as ultimate determinant of health and mortality. Taken together, the evidence implies that the level of knowledge and the amount of human capital is relatively more important for adult longevity than the level of development per se (reflected by, for example, the level of per capita income). Hence, adult longevity primarily appears to depend on the ability to cure diseases, that is the level of medical knowledge, the availability of surgery and other medical treatments that allow to repair physical damage and extend the aging process. Soares (2005) reports macroeconomic evidence that suggests that adult longevity is barely affected by improvements in income or nutrition, but is rather related to 'structural' factors that depend on the knowledge available in a society. To simplify the analysis and highlight the relevant mechanisms, we make the assumption that adult longevity of generation t only depends on the level of knowledge embodied in its parent generation and not on average income, rendering (24) to λ_{t-1}, $T_t = \Upsilon\left(\lambda^*_{t-1}\right)$. Because of the definition of λ as a fraction, the lifetime duration is bounded from above and thus cannot be increased beyond a certain level. We take this biological limit to extending life expectancy as a commonly agreed empirical regularity (see also Vaupel, 1998). The minimum lifetime duration without any skilled individuals in the parent generation is given by $\underline{T}$. The functional form of this relation entails no consequences for the main results, any monotonic relationship can be used without changing the main mechanism.[32]

In contrast, concerning the probability to survive childhood, empirical findings that suggest that higher incomes, public health expenditures, but also access to electricity or vaccines, increases the probability of children to survive to adulthood, see e.g. Wang (2003) for a recent survey. The latter is consistent with empirical evidence on the effect of maternal education on child health reported by Desai and Alva (1998) on the basis of data from Demographic and Health Surveys for 22 countries. Despite a strong positive correlation, they find little evidence for a causal effect of higher maternal education on child health, but rather an indirect effect where education mainly reflects socioeconomic status and area of residence. In particular, access to clean, piped water and toilets has a more immediate causal effect on health than education than maternal education. In other words, child mortality is primarily a function of the level of development at the time of birth of children, the possibility to avoid diseases, the availability of adequate and sufficient nourishment and an environment that prevents or facilitates infectious diseases. In light of this evidence we make the simplifying assumption that the child survival rate π_t only depends on the level of economic development at the time of the birth, reflected by the average per capita income y_{t-1}, and not on the level of knowledge, such that (25) becomes $\pi_t = \Pi\left(y_{t-1}\right)$.[33] While a larger total income Y_{t-1} in the population improves the probability of children reaching adulthood, this formulation also implies a Malthusian element since a larger population size N_{t-1} decreases living conditions and therefore child survival rates.[34]

The differential roles of human capital and income per capita for child mortality and adult longevity have important dynamic implications as will be discussed below. While the extreme assumptions are made for simplicity and mainly affect the time structure of the phase transition, all main results are unchanged when considering the more general specifications.

[32]In the illustrative simulation below we we adopt a simple linear formulation $T_t = \underline{T} + \rho\lambda^*_{t-1}$ that implies a lower and an upper bound for adult longevity, where $\rho > 0$ is a parameter reflecting the strength of the positive externality in terms of the potential amount of time life can be extended by medical knowledge.

[33]In the illustrative simulations of the model presented below, we assume that

$$\pi_t = 1 - \frac{1 - \underline{\pi}}{1 + (q y_{t-1})^{\mu}}$$

with $y_{t-1} = Y_{t-1}/N_{t-1}$, $q > 0$ and $\underline{\pi} \in (0,1)$ being the baseline survival in a non-developed society, in order to ensure that π_t is bounded between zero and one.

[34]Considerable evidence documents the negative effect of population density and urbanization on child mortality, especially during the first stages of the demographic transition, see e.g. Galor (2005). The relevance of population density and potential alternative, more explicit effects of density for health and mortality are discussed in the extensions below.

4 The Economic and Demographic Transitions

The analysis of the full dynamic system must account for the evolution of all variables of interest. We next assume that high-skilled human capital s helps in adopting new ideas and technologies, and thus creates higher productivity gains than low-skilled human capital u.

4.1 Technological Progress

Apart from changes in the health environment, the second dynamic element is the endogenous evolution of the production technology. Technological improvements occur with the birth of a new generation of individuals. Two features of the specification of technological progress are crucial for our results. First, human capital is the engine of growth through an externality working towards higher productivity along the lines of Lucas (1988) and Romer (1990). New technological vintages are characterized by larger TFP A. Secondly, human capital induces a non-neutral technological process, as studied e.g. by Nelson and Phelps (1966), Acemoglu (1998), and Galor and Moav (2000), among others. In particular, technological progress is biased towards high-skill intensive production and depends on the stock of human capital already available in the economy. Empirical evidence, provided e.g. by Doms, Dunne, and Troske (1997) supports this feature. These assumptions imply that the more individuals of a generation acquire skilled human capital the more attractive is the accumulation of skilled human capital for future generations. Using a simple vintage representation, advances in technology embodied in the latest vintage evolve according to:

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}} = F\left(\widetilde{a}_t^*, A_{t-1}\right) = \delta H_{t-1}^s(\lambda_{t-1})A_{t-1} \, ; , \tag{28}$$

where $\delta > 0$.[35] In order to incorporate the feature of a skill biased technical change we assume that the relative productivity of low-skilled human capital in production, x decreases with the level of technological advancement,

$$x_t = X\left(A_t\right) \text{ with } \frac{\partial X\left(A_t\right)}{\partial A_t} < 0 \tag{29}$$

Note that there are no scale effects involved in the specification of technological progress. The crucial relation is between the level of development and the fraction of the previous generation of adults investing in skilled human capital.[36]

4.2 Global Dynamics

The process of development of the economy, and in particular the economic and demographic transition, can be characterized as an interplay of individually rational behavior and macroeconomic externalities. The global dynamics of the economy are fully described by the trajectories

[35] While highlighting the role of human capital for technological progress, the specific functional form of this relationship has little impact. Any specification implying a positive correlation between technological progress $(A_t - A_{t-1})/A_{t-1}$ and H_{t-1} would yield qualitatively identical results. In the simulations below, we adopt Jones' (2001) specification, which is a generalization of the original contribution of Romer (1990) allowing for decreasing returns,

$$A_t = \left(\delta H_{t-1}^{\psi} A_{t-1}^{\phi} + 1\right) A_{t-1} \, ; ,$$

where $\delta > 0$, $\psi > 0$, and $\phi > 0$. As will become clearer below, assuming exogenous technical change would be equivalent for the main results of the model. The only consequence of assuming exogenous advances would be a missing reinforcing feedback effect as the economy develops.

[36] In the simulations below we adopt the simple formulation $x_t = A_0/A_t$.

of the fractions of the population acquiring either type of human capital, characterized by $\lambda := (1 - \widetilde{a}_t^*)$, adult longevity, T_t, the probability of children surviving to adulthood, π_t, the level of technological development, A_t, and the respective production shares of human capital, x_t. The dynamic path is fully described by the infinite sequence $\{\lambda_t, T_t, \pi_t, A_t, x_t\}_{t\in[0,\infty)}$, resulting from the evolution of the nonlinear first-order dynamic system consisting of equations (23), (24), (25), conditional on the evolution of the technological environment (28) and (29):

$$\begin{cases} \lambda_t &= \Lambda(T_t, A_t) \\ T_t &= \Upsilon(\lambda_{t-1}) \\ \pi_t &= \Pi(\lambda_{t-1}, T_{t-1}, A_{t-1}) \\ A_t &= F(\lambda_{t-1}, A_{t-1}) \\ x_t &= X(A_t) \end{cases} \tag{30}$$

We now turn to the analysis of the dynamic equilibrium of the economy. Consider the following,

Lemma 3. *The technology index A_t increases monotonically over generations with* $\lim_{t\longrightarrow\infty} A_t = +\infty$.

The strict monotonicity of A_t over generations depends on the assumption that A is monotonically increasing as generations pass. However, this assumption is not necessary for the main argument. What is crucial is that productivity will eventually be increasing once a sufficiently large fraction of the population acquires h. As A_t increases, the fraction of the population investing in h^s also increases. The levels of life expectancy necessary to make an agent of ability a indifferent between acquiring either types of human capital tend to decrease and the locus $HH\,(A)$ shifts down for any a (excluding the extremes). These results imply that technological development is monotonous, and that this monotonous development leads to continuous change in the allocation of ability and time towards human capital acquisition. The following Proposition combines these findings with the result on the structure of steady state equilibria in the previous section.

Proposition 6. [Dynamic Emergence of Steady State Equilibria] *Consider an economy with $\min\{\underline{e}^u, \underline{e}^s\}$ sufficiently large, so that $\forall A \leq A^0$, the system (26) is characterized by a unique steady state equilibrium of type M. There exist $A^0 < A^1 < A^2 < \infty$ such that the system (26) is characterized by: (i) a unique type-M equilibrium $\forall A_t \leq A^1$; (ii) two steady states $\{\lambda^M(A^1), T^M(A^1)\}$ and $\{\lambda^\sim(A^1), T^\sim(A^1)\}$ at $A_t = A^1$; (iii) three steady states: $\{\lambda^M(A_t), T^M(A_t)\}$ $\{\lambda^\sim(A_t), T^\sim(A_t)\}$ and $\{\lambda^G(A_t), T^G(A_t)\}$ $\forall A_t \in (A^1, A^2)$; (iv) two steady states $\{\lambda^\sim(A_t), T^\sim(A_t)\}$ and $\{\lambda^G(A_t), T^G(A_t)\}$ at $A_t = A^2$; (v) a unique G-type equilibrium $\forall A_t > A^2$.*

With these results, we can characterize the development path of the economy by describing the unique dynamic equilibrium. Consider a non-developed economy in which adult life expectancy is low.[37] Since A is low, investing in h is relatively costly for a large part of the population as the importance of the fix cost for education, $\underline{e}$, is large. This means that the concave part of the $\Lambda(A)$-locus is large and the conditional system is characterized by a unique dynamic equilibrium of type $\{\lambda_A^S, T_A^S\}$, exhibiting low life expectancy and a small fraction of individuals deciding to acquire theoretical human capital. This situation is depicted in Figure 3(a). During this early stage of development, the feedback effects on adult longevity and productivity are very minor.

[37] As will become clear below, starting from this point is without loss of generality. However, even though the model is also capable of demonstrating the situation of developed economies, the main contribution lies in the characterization of the transition from low to high levels of development.

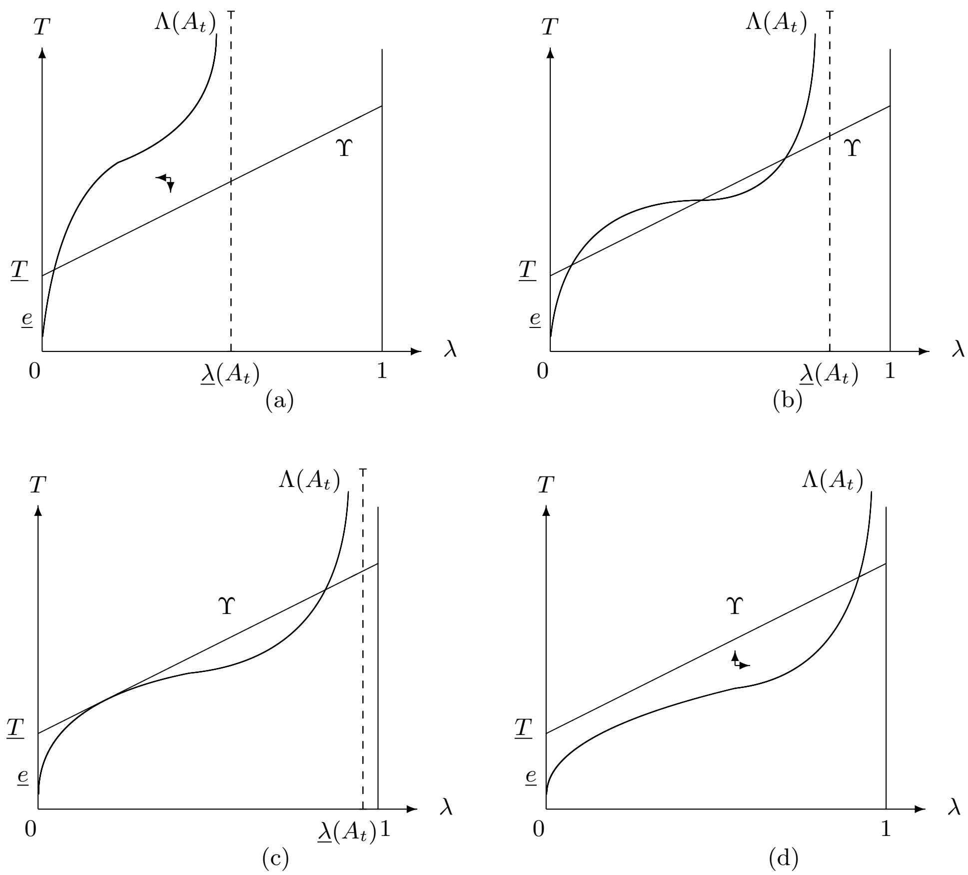


Figure 3: The Process of Development

As generations pass, productivity growth makes investing in h more profitable for everybody, and adult life expectancy increases slowly. Graphically, the locus $\Lambda(A)$ shifts downwards clockwise as time passes, and the importance of the concave part decreases. After a sufficiently long period of this early stage of sluggish development, $\Lambda(A)$ exhibits a tangency point, and eventually three intersections with Υ. From this generation onwards, in addition to E^S, also steady states of type E^u and E^G emerge. Since the S-type equilibria are locally stable, however, the economy remains trapped in the area of attraction of the S-type equilibria, as depicted Figure 3(b).

As generations pass, the dynamic equilibrium induced by initially low adult life expectancy moves along Υ. The consecutive downward shifts of $\Lambda(A)$, however, eventually lead to a situation in which the initial dynamic equilibrium lies in the tangency of the two curves, as shown in Figure 3(c). In the neighborhood of this tangency, the intra-generational equilibrium locus $\Lambda(A)$ lies below the linear Υ-locus and the equilibrium is not anymore stable. Already the following generation faces an adult longevity that is high enough to induce a substantially larger fraction to acquire human capital h than in the previous generation. At this point a unique E^G steady state emerges, as is shown in Figure 3(d). This triggers a period of rapid development, during which the fraction of population acquiring skilled human capital increases fast within few consecutive generations, and adult life expectancy increases rapidly as consequence of the intergenerational externality. This economic transition towards a path of permanent growth lasts for a few consecutive generations. After this transition, life expectancy converges asymptotically to its (biologically determined) upper bound $\rho + \underline{T}$. Even though the fraction of the population acquiring human capital h^s keeps growing, there is always some fraction of the population acquiring applied knowledge h^u, but due to endogenous growth mechanisms, economic development remains fast even though changes in adult longevity and human capital structure in the economy abate.

The following proposition summarizes these global dynamics. The evolution of the system is given by the sequence of ability thresholds, life expectancies and productivity levels $\{\lambda_t, T_t, A_t\}_{t\in[0,\infty)}$, starting in a situation of an undeveloped economy:

Proposition 7. [THE PHASE TRANSITION] *There exists a unique generation t^2 such that for all $t \leq t^2$ the economy is characterized by a sequence of $S-$type equilibria, while for all $t > t^2$ the economy is characterized by a sequence of $G-$type equilibria. This generation t^2 is the first generation for which the level of technology exceeds A^2 as given by Proposition 6.*

It is important to note that the actual trajectory of the system depends on the initial conditions and cannot be precisely identified in general. Proposition 7 in fact states that the system moves generation by generation in the area of attraction of the locally stable steady state E^S until this steady state disappears, and the system converges to a series of globally stable steady states E^G.

In historical terms, the model therefore exemplifies the different stages of development. For example, Europe could be thought of as being trapped in a sequence of E^S equilibria during ancient times and the middle ages. These equilibria are characterized by a very low technological level and very poor living conditions. At the beginning ot time, that is for $A_0 = \lim_{t\to 0} A_t = 0$ we have both adult longevity and child survival probability are at a minimum: $T_0 \simeq \underline{T}$ and $\pi_0 = \underline{\pi}$. From Proposition 4 under these conditions the fraction of individual optimally acquiring high quality human capital is close to zero: $\lambda_0 \simeq 0$. Total net fertility under these conditions is given by,

$$n_0^u \underline{\pi} = \frac{\gamma}{2-\gamma} \frac{T_0 - \underline{e}^u}{T_0} \frac{1}{r_0^*} \,. \tag{31}$$

The phase transition stated in Proposition 7 implies that for $A_\infty = \lim_{t\to\infty} A_t = \infty$ and that eventually the whole population optimally decides to acquire h^s human capital: $\lambda_\infty \simeq 1$ since $T_\infty \simeq \underline{T} + \rho$ and $\pi_\infty \simeq 1$. This, in turns, implies that net fertility is eventually given by,

$$n_\infty^s = \frac{\gamma}{2-\gamma} \frac{T_\infty - \underline{e}^s}{T_\infty} \frac{1}{r_\infty^*} \,. \tag{32}$$

The economy experiences an economic transition and a demographic transition passing from an environment characterized by poor living conditions, in terms of high adult and child mortality, little human capital acquisition and stagnant environment, to an economy characterized by all population being educated, long life expectancy and little child mortality.

In terms of change in net fertility, and comparing (31) and (32), the model predicts that,

Lemma 4. *Following the economic and demographic transitions net fertility drops if, and only if,*

$$\frac{T_\infty r_\infty^*}{T_\infty - \underline{e}^s} > \frac{T_0 r_0^*}{T_0 - \underline{e}^u} \,. \tag{33}$$

As expected the direct effect of an increase in adult longevity leads to an increase in the number of children due to the income effect. In line with the previously cited literature we also find that a reduction in child mortality, due to the substitution effect, cannot explain alone the observed drop in net fertility rates. However, the decline in net fertility can be observed in the model even in the absence of precautionary demand for children if the reduction in fertility associated with the switch from unskilled to skilled human capital is large enough. The previous Lemma states that the differential fertility effect can account for the observed drop in net fertility. Equation (33) implies that net fertility declines whenever the total time spent raising each child, relative to the time spent working, increases after the demographic transition. Two effect contribute to make this conditions hold. The education cost of acquiring human capital must be large enough as compared to the increase in adult longevity.[38] The acquisition of skilled human capital involves a larger period of basic education $\underline{e}^s > \underline{e}^u$ which tend to reduce the number of years spent working. This effect is reinforced by the switch from quantity to quality of children, implying $r^*_\infty > r^*_0$ which is associated to the modern growth regime.

Historical data suggest that these conditions are very likely to hold in practice. For instance, child mortality in England and Wales fell substantially from around 20 percent in the period 1550-1600 to less than 0.5 percent at the end of the 20th century. Adult longevity measured by life expectancy at the age of 30 experienced an increase from around 60 years to around 75 years.[39] Considering that the acquisition of higher education is currently associated to a time investment, $\underline{e}^h$, from 10 to 15 years this can help rationalize the reduction in fertility.[40] Hence, the model shows that the dramatic and fast changes in human capital acquisition that accompanied the demographic transition can rationalize and explain the substantial decline in gross fertility and the fact that net fertility eventually decreases. The current framework therefore offers an explanation that can generate the observed drop in gross and net fertility, which is based on differential fertility choices depending on the education choice. For illustrative porpuses we assume in the following that condition (33) holds.[41] Notice that this implies, *a fortiori,* that gross fertility eventually drops.

These observations are the result stated in Proposition 7 are jointly recorded in the following,

Proposition 8. [Economic and Demographic Transitions] *The economy is characterized by the following phases in the process of development:*

(i) A (potentially very long) phase of stagnant development with little longevity, $T_0 \simeq \underline{T}$*, large child mortality* $\pi_0 = \underline{\pi}$*, very few individuals acquiring human capital* h^s, $\lambda_0 \simeq 0$ *and large gross and net fertility rates as in* (31)*;*

(ii) A rapid transition involving rapid increase in T_t*,* π_t*,* λ_t *income per capita* y_t *and technological level* A_t*;*

(iii) A phase of permanent growth in technology and income with long life expectancy $T_\infty \simeq \underline{T} + \rho$*, negligible child mortality* $\pi_\infty \simeq 1$ *all population acquiring* h^s *human capital* $\lambda_\infty \simeq 1$ *and low gross and net fertility rates as in* (32)*.*

Development begins with an extended phase characterized by low living standards and low adult longevity, high child mortality and large fertility. The economic and demographic environment barely changes with technological progress being almost undetectable. Any increases in longevity and income directly funnel into higher levels of fertility. Eventually, when the economic development picks up momentum, more people become skilled, productivity growth accelerates and living standards improve. In this second phase of development, dubbed the 'Post-Malthusian Regime' by Galor and Weil (2000), adult longevity already increases, child mortality is still high, and fertility increases as incomes grow. Eventually, as child mortality falls and long adult longevity induces large fractions of the population to become skilled, also

[38] Notice that interpreting h^u as unskilled labor with $\underline{e}^u = 0$ and h^s as human capital involving $\underline{e}^s > 0$ this condition is satisfied for any increase in longevity.

[39] Data are from Wrigley and Schofield (1981) and UK national statistics.

[40] In the following section we discuss the relevant data with particular attention to the historical experience of England.

[41] The only difference in the following discussion would concern the different limit values in net fertility while all other features are unchanged.

fertility starts declining, and the economy enters a modern growth path on which income grows while fertility declines to low levels.

The model predicts that the economic and the demographic transition are closely related phenomena. In the model the economic take-off crucially depends on the improvement in living conditions which are a key determinant of individual human capital investments. Longevity and child mortality are, in turn, crucially affected by the level of economic development and the availability of human capital. This close interrelationship between economic and demographic development implies that the exit from the low (economic and human) development is triggered a joint improvement in all the dimensions.

While the features of the economy before and after the transition can be characterized analytically the actual timing in the change of the different variables *during* the transition to sustained growth are governed by several opposing mechanisms. As child mortality decreases, i.e. the survival probabilities of children increase, adults of either type of education optimally decide to have fewer offspring. This *substitution* effect arises since adults care about the total number of surviving children. In the current framework, there is no uncertainty that would require the hoarding of children and, as discussed above, if only this effect is at work no net fertility drop can be observed in the population following a reduction in child mortality. However, improvements in health conditions also have other effects. Consider again the individually optimal fertility decisions that are implied by the choice to either types of human capital h^j stated in condition (15). All individuals increase their fertility as their adult life expectancy increases, regardless of which type of human capital they decide to acquire. This increase arises due to the *income* effect. The joint consideration of these two effects implies that, conditional on education choices, net fertility increases after improvements in living conditions. the third effect arises due to the shift toward the acquisition of h^s human capital. Conditional on facing the same adult longevity T_t and child survival probability π_t, adults who decide to become high-skilled optimally choose to have fewer children than adults who acquire human capital of type u. This *differential fertility* effect, which is linked to the structural change of the economy, gains importance along the development path, since, as has been shown before, eventually the population structure shifts from acquisition of u-type human capital to the acquisition of $s-$type human capital as the economic transition takes off. As a result nearly the entire population eventually acquires skilled human capital which is associated to longer education and lower fertility. The change in the different dimensions of health is not necessarily simultaneous, however. In particular if, as assumed above, child mortality is crucially related to income per capita while adult longevity is linked to knowledge and human capital, then improvements in the different dimension of mortality may not be simultaneous. In the following, also by use of a simple illustrative simulation, we show that this consideration can help reconciling the debated British experience with theoretical predictions of unified growth theories.

As discussed above several authors have pointed out that the process of decline in mortality started long before the drop in fertility casting doubt on the importance of longevity for fertility changes. The predictions of the framework presented above imply that different dimensions of longevity may have very different effects on fertility. Furthermore, which of the effects is expected to prevail depends on their relative strength as well as the timing of the delays involved in the intergenerational transmission mechanisms. Dynamically, the fertility reducing effects are likely to be minor at the onset of the economic transition and to be dominated by the income effect. Therefore, and in line with the common wisdom concerning the observation of the most demographic transitions, the model predicts a possible initial increase in gross and net fertility during the early stages. This is reflected by phase (ii) of demographic development in Proposition 8, and consistent with the second phase of development, dubbed the 'Post-Malthusian Regime'. Note, however, that this transitional phase can be very short when convergence to the E^G-steady state is rapid. If the change in human capital structure is sufficiently swift over the course of a few generations, implying a strong differential fertility effect, or likewise if the substitution effect is strong since child survival rates improve strongly with little delay to the economic transition, the Post-Malthusian phase (ii) might be short as society quickly enter phase (iii) when the two fertility-reducing effects of increasing child survival rates and larger importance of differential fertility from investments in skilled human capital s gain momentum. Thus, under condition (33) gross and net fertility eventually start declining despite adult longevity and per capita income still growing. The economy therefore enters a 'Modern Growth Regime', once more following the classification of Galor and Weil (2000).[42]

Apart from these global dynamics, the model of education and fertility decisions generates different dynamics in the optimal decisions in the context of changes in adult and child mortality. Condition (13) also implies that the effect of life expectancy on the education time is stronger for those acquiring skilled human capital,

$$\partial e_t^{s*}/\partial T_t > \partial e_t^{u*}/\partial T_t \geq 0\,. \tag{34}$$

This is also true for fertility $\partial n_t^{s*}/\partial T_t > \partial n_t^{u*}/\partial T_t > 0$. This implies that individuals acquiring higher skills are more responsive to changes in longevity (since they choose longer education periods). On the other hand, the optimal individual fertility decision (12) implies that the drop in fertility following a reduction in child mortality is stronger for unskilled individuals,

$$\partial n_t^{u*}/\partial \pi_t < \partial n_t^{s*}/\partial \pi_t < 0\,. \tag{35}$$

This completes the discussion of the theoretical results.

4.3 Discussion

This section presents a brief discussion of the results in light of historical evidence, and then present an illustrative simulation of our model to demonstrate its capability to replicate different patterns of development.

The Economic and Demographic Transition: Stylized Facts Most experiences of economic and demographic transitions in the Western world exhibit similar patterns. When the economic transition took off century, mortality fell significantly and average life expectancy at birth as well as at later ages, which had virtually been unchanged for millennia, increased sharply within just a few generations. The improved environment and living conditions associated with better health conditions, lower mortality and overall life expectancy, as well as changing economic demands triggered a more widespread acquisition of human capital. Along the process of development, fertility increased from high levels, before reproduction rates fell substantially. Nevertheless, population size increased, mostly due to increased life expectancy. The timing of events was not identical in all countries, and in fact shows some interesting differences, which we discuss by referring to the examples of England and Sweden. We choose these countries because of reasons of data availability, but also since England represents the first country in which the phase transition occurred, while Sweden shows similar patterns as many other European countries that experienced somewhat later transitions.

The onset of the economic and demographic transition took place in England during the second half of the 18th Century, while Sweden developed more than half a century later. Figure

[42]Consider the dynamic pattern when condition (33) fails to hold. Even under this scenario, the observed fertility rates experience a marked slow-down due to the transition to a low-fertility high-education equilibrium allocation of large parts of the population, reflected by a E^G steady state.

4 illustrates the Swedish case.[43] In Sweden, the early phases of income growth are preceded by increases in life expectancy and increases in primary school enrolment. Life expectancy at birth and later ages, for example age 30, seems to have improved from about the same time at the beginning of the 19th Century onwards. Consequently, fertility, both in gross and net terms, increased, but eventually, with larger shares of the population entering secondary and tertiary education, fertility rates dropped below the pre-transition levels. Nevertheless, the population went on to grow as consequence of more cohorts being alive at the same time.

The timing of events in England, the first country to experience an economic and demographic transition, are a bit more complicated. The development of the different dimensions of life in England is illustrated in Figure 5.[44] The onset of the transition preceded that of Sweden by several decades. Basic education and human capital indicators like average literacy rates, and years spent in formal schooling and year of apprenticeships improved substantially after the take off.[45] Most interestingly, however, improvements in life expectancy at birth lags adult longevity in terms of life expectancy at age 30. This is an indication that infant mortality began to fall much later than mortality at later ages.

The timing of the events in the different dimensions of the economic and demographic transition is in line with the predictions of Proposition 8. In particular, improvements in adult longevity seem to have affected fertility as well as economic growth in a causal sense as has recently been documented by Lorentzen, McMillan, and Wacziarg (2005). Similarly, Soares (2005) reports that gains in adult longevity preceded the decline in net fertility. This historical evidence suggests that the acquisition of human capital by part of parents and the increase in the lifetime devoted to education represents an first order determinant of fertility behavior.[46] Simultaneously, or slightly delayed compared to the improvements in health and life expectancy, fertility behavior changed substantially. Fertility rates dropped from high levels in less developed countries to substantially in the context of economic development and improvements in living conditions. This demographic transition typically lags health improvements somewhat, as during the second half of the 19th century, see Galor and Weil (2000), and as in developing countries today. Due to health improvements and despite fertility reductions, the size of population started to increase substantially in European countries. The increase in population size even after the decrease of fertility suggests that the reduction in reproduction is more than compensated by an increase in lifetime duration. Apart from changing overtime, both education and fertility choices differ substantially across the population, however. A large evidence documents that education choices depend on individual living conditions, in particular economic environment, health and life expectancy. Individual fertility decisions in turn depend on individual income and education.[47]

The differences in the timing of the transition in England and Sweden has raised some de-

[43]The data for Sweden have been collected from the following sources. Data for GDP per capita is provided by the internet portal for historical Swedish statistics, `www.historia.se`. Life expectancy and fertility data are taken from Wrigley and Schofield (1981), Keyfitz and Flieger (1968). Population data are taken from the Swedish Central Statistical Office, `www.scb.se` and the internet portal for historical Swedish statistics, `www.historia.se`. Data on schooling enrolment have been constructed by de la Croix, Lindh, and Malmberg (2006). Missing values are obtained by linear intrapolation.

[44]Data for the U.K. or England and Wales, respectively, are from the following sources. GDP data is provided by Floud and McCloskey (1994). Education data and information on apprenticeships is taken from Cipolla (1969) and Floud and McCloskey (1994). Life expectancy and fertility data are taken from Wrigley and Schofield (1981), Keyfitz and Flieger (1968) and the websites of the Office of National Statistics (`http://www.statistics.gov.uk`) and the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat (World Population Prospects: The 2004 Revision and World Urbanization Prospects: The 2003 Revision, `http://esa.un.org/unpp`). Missing values are obtained by linear intrapolation.

[45]Extensive evidence on this is reported by Cipolla (1969) and Floud and McCloskey (1994).

[46]This is true, *a fortiory*, once considering the increased education of work participation by women.

[47]de la Croix and Doepke (2003) provide theoretical and empirical arguments for which differential fertility affects the occurrence and the timing of the transition.

Table 1: Parameter Values Used for Simulation

α	0.95	β	0.05	ρ	25.0	$\underline{\pi}$	0.75
γ	0.5	η	0.4	$\underline{e}$	15.0	$\widetilde{a}(0)$	0.995
ψ	0.42	ϕ	1.25	$\underline{T}$	50.0	$A(0)$	0.75
μ	2	ν	6	$\underline{r}$	1.5	b	1

bate. In particular, the decline in fertility well before the onset of the substantial drop in child mortality, but substantially after the increase in adult longevity in England has been recognized as problematic for theories linking mortality and fertility, see e.g. Doepke (2005) and Galor (2005).[48] While these puzzling findings can be rationalized by the model presented before by considering the different dimensions of adults and child mortality and the possibility that improvements in these different domains where not always contemporaneous. Historically, one can argue that health, adult longevity and child mortality were affected differently by the process of development. Consider again Proposition 8. The duration and extent of the post-malthusian phase crucially depends on the relative strength and timing of the different effects at work. How could one generate a demographic transition with adult longevity improving well before child mortality declines?[49] A delay in the decline of child mortality implies that after the start of the transition the income effect is relatively stronger than the substitution effect. Under these conditions it is likely that gross fertility initially increases, and only declines with a lag once the differential fertility effect associated to the sharp changes in acquisition of human capital becomes sufficiently strong. This can be the case even if the substitution effect is not too strong since child mortality is still large. The answer to the question why these lags occurred in England, but not in Sweden is likely to have to do with the fact that England was the first country to experience a transition. Followers, like Sweden, could already resort to medical knowledge, or technologies that reduce child mortality and that proved effective in England.

Illustrative Simulation of the Development Path: INCOMPLETE VERSION We simulate the model for 600 generations, with new generations born at a frequency of one year.[50] The parametric specifications used for the intergenerational externalities are given in the text for T in equation (24), for π in footnote 33, for A in footnote 35, and for x in footnote 36, respectively. Parameter values and initial conditions used in the simulation are contained in Table 1. The time costs of surviving children is parameterized as function $r(\pi_t) = \underline{r} \cdot \pi_t^{\nu}$

Marginal productivity of time spent in education, given a specific level of ability, is assumed to be the same in the production of both types of human capital. A maximal life expectancy of 75 years cannot be exceeded, while the minimum adult longevity is assumed to be 60 years. Initial child survival probability is 0.8. All these numbers are roughly in line with historical data from England, where life expectancy at age 30 was around 60 in the 16th century, and mortality of infants between age 0 and 9 was around 220/1000. The fix cost of acquiring theoretical human capital h, $\underline{e}$, is 15 years. Initially, the share of production accruing to L, $x = 0.95$. Clearly, the model is capable of producing a deliberately long stagnancy period before

[48] See also the evidence provided by Boucekkine, de la Croix, and Licandro (2003).

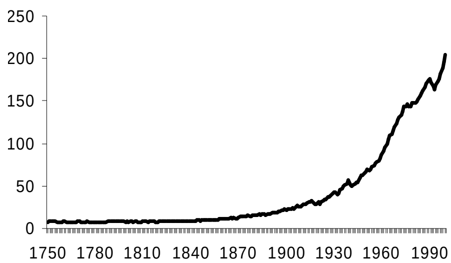
[49] In fact there are several reasons why this could have been the case in the first experiences of demographic transition including the increase in population density and urbanization and the fact that living conditions (and per capita income) improved several decades after the increase in acquisition human capital and medical knowledge.

[50] Interpreting every year as the arrival of a new generation, this reflects a horizon from year 1500 to 2100, which includes the period of economic and demographic transition.

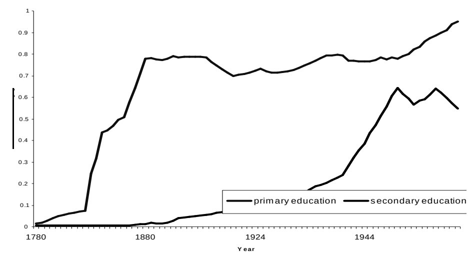
the transition. The main patterns of development generated by the model are depicted in Figure 6. The patterns resemble closely those in the historical data presented before and illustrate the discussion of the main results in section 4. Figure 7 makes clear that the model is capable of generating complex fertility dynamics. In particular, during the demographic transition, fertility intermediately increases due to income effects. Eventually, fertility declines in the later stages of the demographic transition. However, net fertility peaks well after gross fertility has started to decline. This can be explained by the differential timing of the improvements in the different dimensions of health, i.e. adult longevity and child mortality.

5 Conclusion

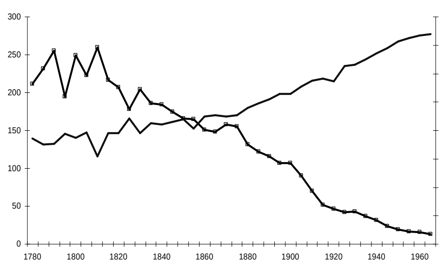
— TO BE ADDED —



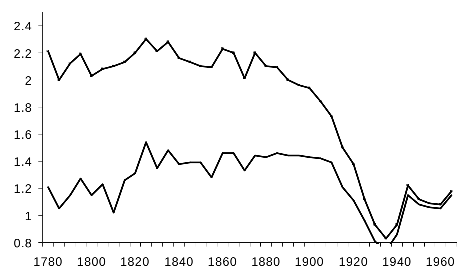
(a) GDP per capita



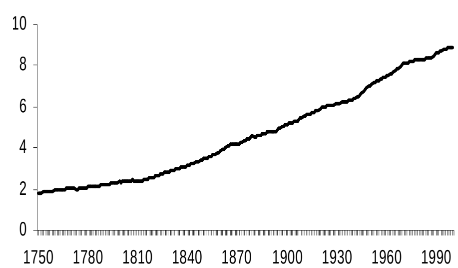
(b) School Enrolment



(c) Life Expectancy at Birth (left axis, lower graph) and at Age 30 (right axis, upper graph)

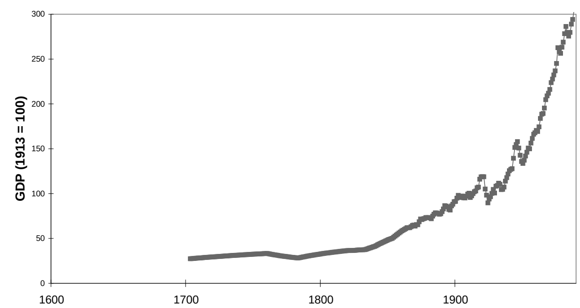


(d) Gross and Net Reproduction Rates

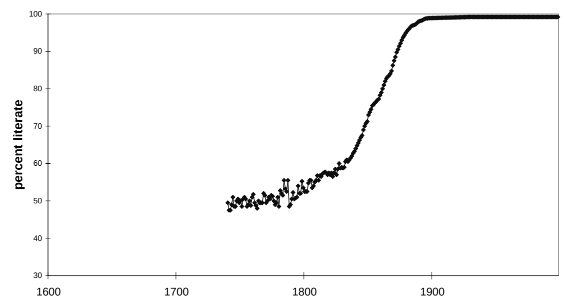


(e) Population Size

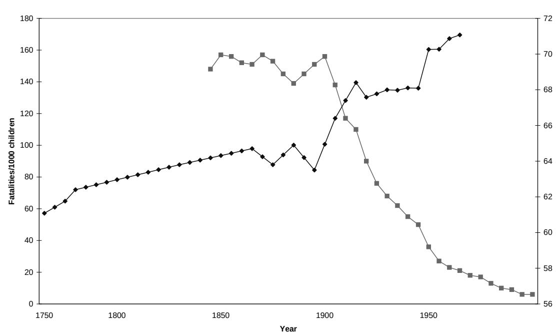
Figure 4: The Stylized Facts of Long-Run Development for Sweden



(a) GDP per capita (U.K.)

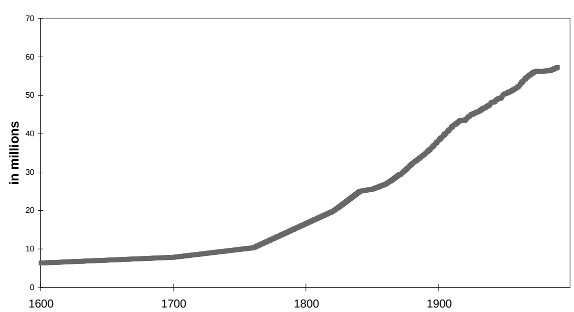


(b) Literacy Levels (England and Wales)



(c) Infant Mortality (left axis, short graph) and Life Expectancy at Age 30 (right axis, long graph) (England and Wales)

(d) Gross and Net Reproduction Rates (England and Wales)



(e) Population Size (U.K.)

Figure 5: The Stylized Facts of Long-Run Development for the U.K.

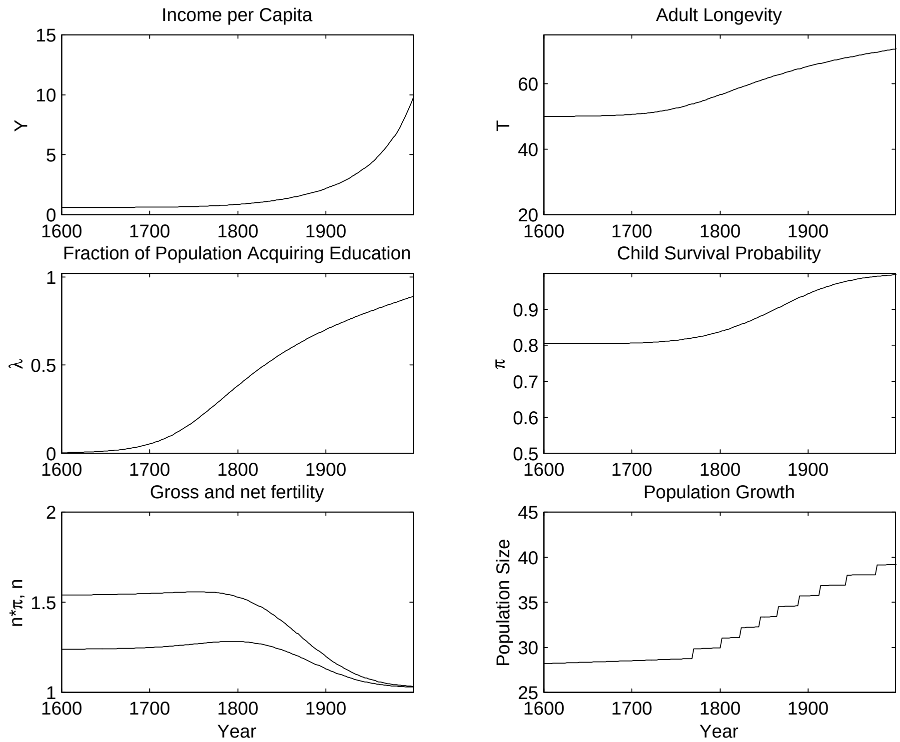


Figure 6: A Simulation of the Development Process

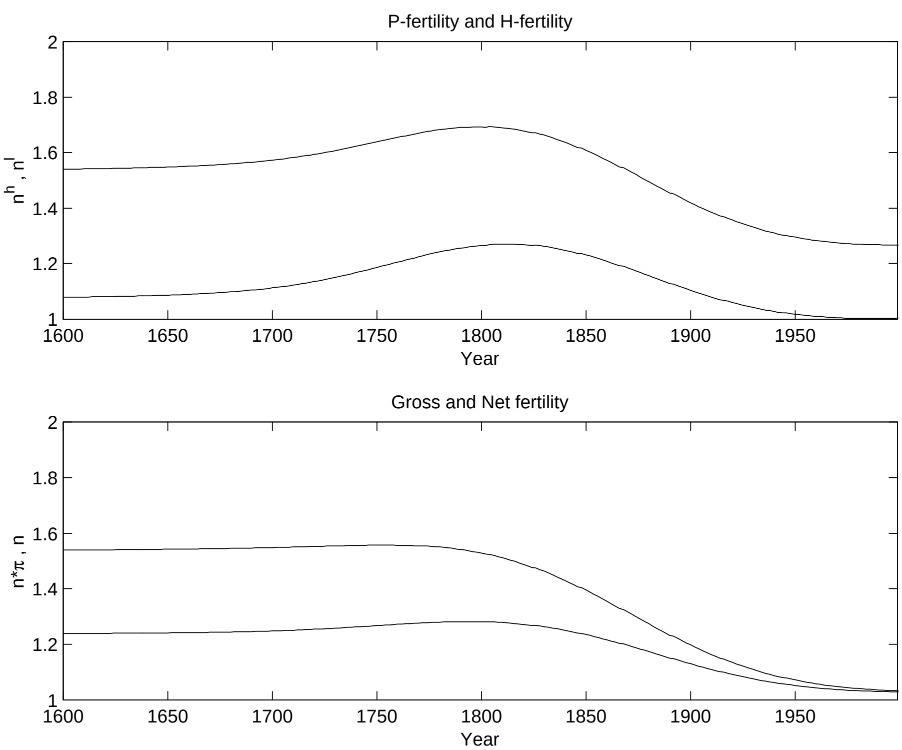


Figure 7: The Timing of the Demographic Transition

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A Relegated Proofs of Propositions

— TO BE ADDED —