MEASURING STRUCTURAL BUDGET BALANCES IN A FAST-GROWING ECONOMY: THE CASE OF IRELAND

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1. Introduction

The most popular method of measuring structural, or cyclically-adjusted, budget balances is the gap-elasticities approach. This involves the undertaking of two critical estimation procedures. The first procedure involves estimating the economy's potential level of output. The estimate of potential output allows a measure of the output gap, the difference between actual and potential output, in any particular year to be calculated, which, in turn, provides an assessment of the position of the economy in the business cycle in that year. The second estimation procedure involves estimating the elasticity of cyclically-sensitive government revenue and expenditure categories with respect to GDP. These elasticities provide a measure of the cyclical responsiveness of these particular revenue and expenditure categories.

With the output gap and elasticity measures to hand, the calculation of the structural budget balance then involves multiplying each of the cyclically-sensitive government revenue and expenditure categories by the output gap, expressed as a proportion of actual GDP, to the power of the appropriate elasticity estimate. The structural budget balance is then calculated as being equal to the sum of the cyclically-

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adjusted revenue categories and the unadjusted revenue categories less the sum of the cyclically-adjusted expenditure categories and the unadjusted expenditure categories.

This paper assesses the difficulties that arise in measuring structural budget balances in Ireland. An examination of the behaviour of key macroeconomic and government sector variables over the past twenty years illustrates the challenge posed in estimating accurately structural budget balances in Ireland. With current and prospective Irish economic performance somewhat different to that of other EU countries, this review seeks to add some illumination to the debate on how structural budget balance measurement should be best approached, particularly in a small, fast-growing economy such as Ireland.

Given the critical importance of output gap measurement to structural balance estimation, it is important to be aware of why significant differences arise between alternative approaches to potential output. In section 2, some characteristics of Irish economic performance are examined that may help explain why there is such a large discrepancy between measures of the output gap in Ireland. It is equally important to be aware of issues that can impact on the measurement of the cyclical response parameters. In section 3, aspects of the cyclical behaviour of Irish fiscal variables that may impact on the appropriateness of using constant elasticity parameters are highlighted. Section 4 concludes.

2. Output Gap Estimation in Ireland

Output gaps, defined as actual less potential output, are generally estimated for two purposes. One is to provide information on the level of excess capacity in the economy at a particular point in time. This allows an assessment to be made of the economy's cyclical position and the cyclically-adjusted state of the public finances. The second use of output gap estimates is to provide a measure of the impact that capacity utilisation conditions are likely to have on price and wage outcomes. In particular, upward pressure on price levels can emerge when the growth rate of actual output exceeds the growth rate of potential output.

The difficulty with assessing output gaps, however, is that since potential output is not directly observable, neither is the output gap.

Estimating output gaps, regardless of the intended use, is therefore fraught with uncertainty. This paper focuses solely on the use of output gaps in measuring structural fiscal balances. It is worth noting, in any case, that, even with rigorous estimation, the output gap has been a particularly poor indicator of inflation in Ireland (Kenny (1996))[1].

Potential Output Growth in Ireland: Overview

Some recent comparisons of output gap calculations across EU countries suggest that Ireland may be something of an outlier in terms of a comparison of Commission and OECD estimates of output gaps. Briotti (1998) reports that Ireland has the lowest correlation among EU countries between the OECD's production-function based potential output estimates and the European Commission's trend-smoothing based potential output estimates over the period 1982 to 1999.

The relatively low correlation for Ireland could be attributable to the manner in which the respective production-function and trend-smoothing approaches can produce different output gap estimates in an economy characterised by relatively volatile real economic performance. In particular, the estimation of potential output in Ireland is likely to be complicated both by a highly-elastic labour supply and by highly-mobile capital flows relative to most other EU countries.

The high elasticity of labour supply arises from the traditional significance that both inward and outward migration flows have on the labour market in the Irish economy. Over the last decade, for example, the domestic supply of labour has been significantly enhanced by favourable demographic factors, higher female labour force participation rates and large inward migration. The effective labour supply has also been enhanced by improvements in the quality and educational levels of

[1] There seems to have been significant structural change in the inflationary process itself in Ireland over the last two decades. In particular, successful social partnership arrangements in Ireland since the late 1980s have helped moderate wage increases in return for tax rate reductions and increased social welfare expenditure. Also, the increased openness of the Irish economy raised the importance of the exchange rate and foreign inflationary pressures in price determination. In this context, it is perhaps unsurprising that output gap measures have not proved to be good indicators of inflation in Ireland.

the labour force. However, while Ireland has experienced increased labour supply in the 1990s, this should be compared to substantial outward migration of labour throughout the 1980s, which in itself reversed the inward trend of returning migrants in the 1970s. This variability in the supply of labour has been a traditional feature of the Irish economy.

In addition, the outward-oriented industrial policies pursued over the last three decades have encouraged significant levels of foreign direct investment and the expansion of the modern sectors of the economy. While these policies have been extremely successful in recent years, there has also been considerable churning in the number and types of firms investing in Ireland reflecting the mobility of capital. The conjunction of highly-elastic labour supply with highly-mobile capital flows can make output growth in Ireland highly variable.

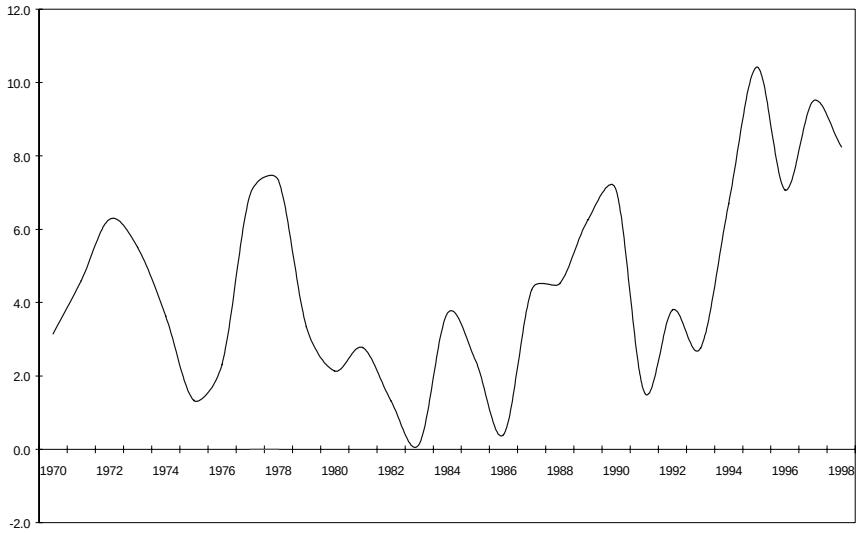
Economic Growth and the Behaviour of Factor Inputs in Ireland

Over the period 1971-1996, average real GDP growth in Ireland was 4.1 per cent compared to 2.25 per cent for the EU as a whole (Kenny, 1996). While this differential is consistent with the convergence hypothesis of Ireland catching up with its more developed EU partners, Irish growth rates also displayed significant variation throughout the period (as evidenced in Figure 1). This might not be unexpected given the scope for variability in the supply of the factors of production outlined above.

From Figure 1, a number of distinct growth phases in the Irish economy since the early 1970s can be identified. The economic performance of the early to mid 1970s in Ireland reflected the international experience of growth slow down following the first OPEC oil shock. High growth resumed during the late 1970s but was followed by protracted recessions and lower growth in the early- to mid-1980s. This continued until 1987 when an upturn in growth occurred at the same time as a successful fiscal consolidation programme was initiated. A cyclical downturn in the early 1990s coincided with a disimprovement in the international economy. Since then the Irish economy has entered a very high growth phase with GDP growth averaging 9 per cent over the period 1993-1998.

Fig. 1

Real GDP Growth in Ireland, 1971-1998



Source: CSO (1998)

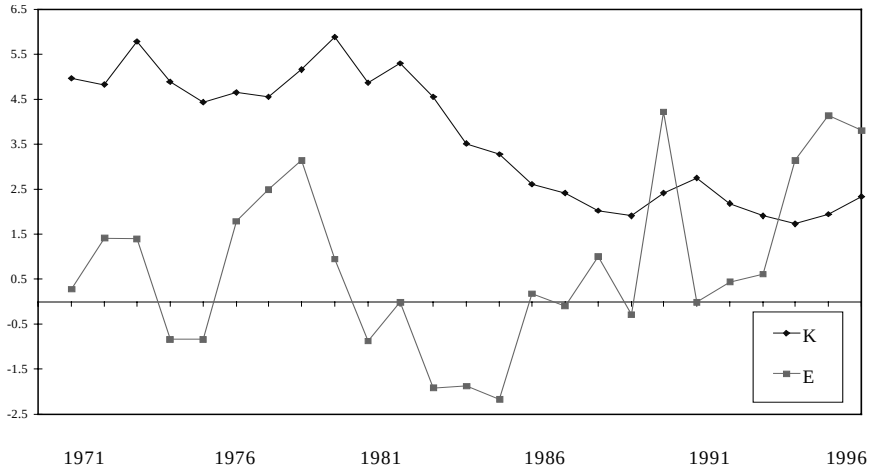
Kenny (1996) used a growth accounting analysis to identify the contribution of the different factors of production to Ireland's growth experience over the 1971-1996 period. He found that the breakdown of the contribution to the 4.1 per cent average growth rate was 13 per cent from labour, 28 per cent from capital and 59 per cent from total factor productivity. These contributions were based on average shares of 0.68 and 0.32 for labour and capital, respectively, over the period.[2].

The variability of labour, capital and total factor productivity over time gives an insight into why actual output growth varied considerably over the 1971-1996 period. Figure 2 shows how employment, as a measure of labour's contribution to growth, tended to fall and rise over the period. Strong employment growth in the late 1970s was followed by

[2] These average shares of labour and capital mask the dramatic change since 1987 in factor incomes, where labour's share of factor incomes dropped from 75 per cent to nearly 65 per cent by 1996 resulting from moderate wage growth under the national partnership arrangements (Lane, 1998a).

Fig. 2

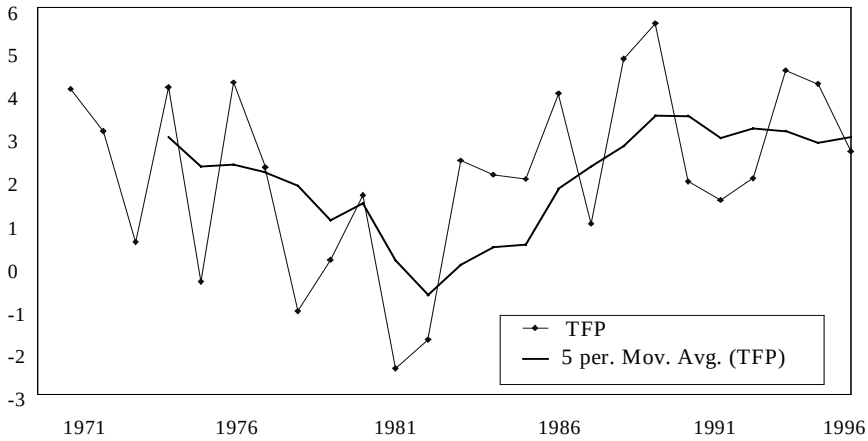
Annual Growth Rate of Employment (E) and the Capital Stock (K) in Ireland: 1971-1996



Source: Kenny (1996)

Fig. 3

Annual Growth in Total Factor Productivity (TFP) in Ireland: 1971-1996



Source: Kenny (1996)

labour shedding in the 1980s. Strong employment growth resumed again in the late 1980s and is also in evidence in more recent years.

The contribution of capital to growth is also captured in Figure 2. Capital stock growth rates were positive over the period but there was a trend decline in the rate of capital accumulation through the 1980s reflecting negative real rates of growth in investment. Another feature of the 1971-1996 period is that capital and employment tend to be complementary in production, as evidenced by the pick-up in the rate of capital accumulation when employment increases were at their highest.

Like employment and capital growth, total factor productivity (TFP) growth in Ireland was quite volatile in the 1971-1996 period (see Figure 3). The overall efficiency in production is captured by TFP, which is a measure of technological progress. Figure 3 shows that TFP displayed a high degree of cyclicality but its growth in the 1990s was quite strong. The years when TFP growth was negative tend to correspond to recessionary periods in the Irish economy.

Methods of Potential Output Determination

The above analysis of actual output and factors of production patterns over the period 1971 to the present suggests two practical difficulties in attempting to estimate accurately potential output in Ireland:

i. there is the general difficulty in how best to approach potential estimation when there is considerable variability in both output and the supply of factors of production;

ii. there is also the contemporaneous difficulty of how to estimate potential output levels when growth rates are at a historical high.

Kenny (1995,1996) utilised both trend smoothing and production function approaches to tackle the difficulties in potential output estimation for Ireland. The trend smoothing approaches used by Kenny were a simple linear time trend, a split time trend, a Hodrick-Prescott filter (under the alternative assumptions of setting the smoothing parameter λ equal to 100 and 500), and peak-to-peak extrapolation estimates. Alongside these five trend smoothing measures, two production function-based measures were also estimated. These both

used a Cobb-Douglas production functions, one with polynomial trends and the other with moving average trends.

The trend smoothing approaches share a common characteristic that past events have a direct bearing on future levels of potential growth. However, conversely, there is also the counter-intuitive implication that future levels of potential growth can impact on past estimates. This is particularly the case for the simple linear time trend which uses a constant growth rate over the entire estimation period, such that events in the 1970s and the 1990s can influence the estimate of potential growth for the 1980s. To overcome this problem somewhat the split time trend approach and the peak to peak extrapolation methods used by Kenny disallow the influence of past and future events beyond the segmented end-points. However, both these methods require a large degree of subjective judgement in selecting the different growth segments or major peaks when resources are assumed to be fully utilised. The Hodrick-Prescott (HP) filter allows for more variability in the potential output measure. It tries to minimise the deviation of output from its trend subject to smoothness constraint. The higher the smoothness parameter λ the more smooth is the series, the lower is λ the more ragged is the series as it follows actual output closely. If $\lambda = 0$ then potential output would be the same as actual output, thus not allowing for an output gap.

The structural models utilised a production function approach adapted from You (1979) and Brox (1984). In contrast with You and Brox, who assumed constant natural rates of unemployment for the US and Canada respectively, Kenny specifically allowed for a large increase in the natural rate of unemployment (on account of the strong growth in Irish unemployment in the 1980s which is likely to have increased the natural rate in those years). This would mean that an intense utilisation of capital alone could drive output above potential, a feature of the model that Kenny found particularly attractive given the possibility of rigidities in the Irish labour market. The modification used allowed output to exceed potential even when the unemployment rate was above its long-term natural rate.

Kenny found that there is a consistency across all seven methods in the *sign* of the output gaps over the period. The scope, however, for there being considerable differences in the *size* of potential output estimates is evident in his research. This uncertainty with regard to the

size of the output gap arises both when comparing production function estimates with trend smoothing estimates and when comparing trend smoothing-based estimates with one another. This feature of Kenny's research is illustrated in Table 1 which shows his estimates of potential output for three distinct growth periods calculated using two trend smoothing techniques and a production function model.

Table 1

Implied Average Percentage Growth of Potential Output

	Split Trends	*Hodrick-Prescott*	*Production Function*
1971-1979	4.28	3.72	3.68
1979-1987	2.49	3.11	2.88
1987-1996	5.00	4.48	4.30

Source: Kenny (1996)

Although these three estimation methods generate output gap measures of common sign, there is considerable difference in the size of the output gaps. In particular, the split-trends estimate tend to be more variable than the other two estimates. The production function and the Hodrick-Prescott filter estimates tend to be more closely aligned to each other, although the production function approach consistently provides the lower estimates of the two.

Issues to be Addressed in Potential Output Measurement in Ireland

(i) Differences between Various Potential Output Measures

Overall, in comparing trend smoothing-based and production function-based potential output measures, Kenny's results mirror the earlier comparison of the OECD and European Commission measures for Ireland in that there is a considerable range in the size of the potential output estimates. This is not an encouraging finding for the task of attempting to measure structural budget balances in Ireland.

The production function approach, based on estimates of factor inputs, has a more solid theoretical basis than trend smoothing the trend smoothing approach. However, the estimation of the full employment level of factor inputs has significant problems, particularly in separating the causes of output fluctuations into demand and supply shocks. Supply shocks permanently change the level, not the growth rate, of potential output while demand shocks only impact actual not potential output. Potential output in the production function approach tends to lie above the average output such that the output gap tends to be negative and asymmetric. In contrast the trend smoothing approach has symmetric gaps over the business cycle. However, the trend smoothing approach is both poor at detecting structural breaks and has an endpoint problem arising from the application of moving averages. The endpoint problem arises when the more recent data points do not reflect similar points in the cycle so leading to potential output that is distorted upwards or downwards depending on whether the economy is at a particularly strong or weak growth point.

(ii) High Output Growth in the late 1990s

The particularly strong growth in the Irish economy in the late 1990s is likely to have made estimating output gaps more difficult, particularly with respect to end-point sample bias. Using a Hodrick-Prescott filter (with λ set equal to 100) points to a rise in trend growth rate to 6.75 - 7.75 per cent in 1997-1998, with a 6.75 per cent growth rate projected over the medium term. Production function-based approaches suggest similar estimates of 7.25 per cent in 1997-98 and 7 per cent over the medium term. However, the extent of the end-point bias is clearly evident in the Irish case when 1997 and 1998 are removed from the sample with the potential growth rate for 1996 falling from 7 per cent to 5.5 per cent, and the medium term projection falling to 5 per cent. The removal of the 1997 and 1998 observations from the estimation sample, therefore, leads to between 2-4 percentage points lower medium-term potential estimate. Some international agencies estimate that potential output growth rate in Ireland at present may be as high as 9 per cent. The high actual growth rates in Ireland at present, therefore, seem to leave scope for significant disparities in potential output growth estimates and underline the need to consider carefully how to model the late 1990s end-sample data.

3. Cyclical Response Parameters

The categories of revenue and expenditure that are usually considered to fluctuate automatically in response to changes in economic activity are, on the revenue side, direct taxes on households, direct taxes on enterprises, indirect taxes, actual social contributions and, on the expenditure side, current transfers to households. Given the output gap, the cyclical sensitivity of the budget balance depends on the elasticity of these revenue and spending categories with respect to movements in output. These parameters are the links between the output gap calculation and the cyclically-adjusted balance figure.

Government Revenue and Expenditure Categories, 1977-1998

When expressed as a percentage of GDP, total government revenue and expenditure in Ireland, display a bell-like shape over the period 1977-1997, see Figure 4). Both aggregates rose sharply and steadily from 1977 through until the early 1980s. They then remained at that high level through until the mid-to-late 1980s. However, from that time on, a secular downward movement occurred in both revenues and expenditures.

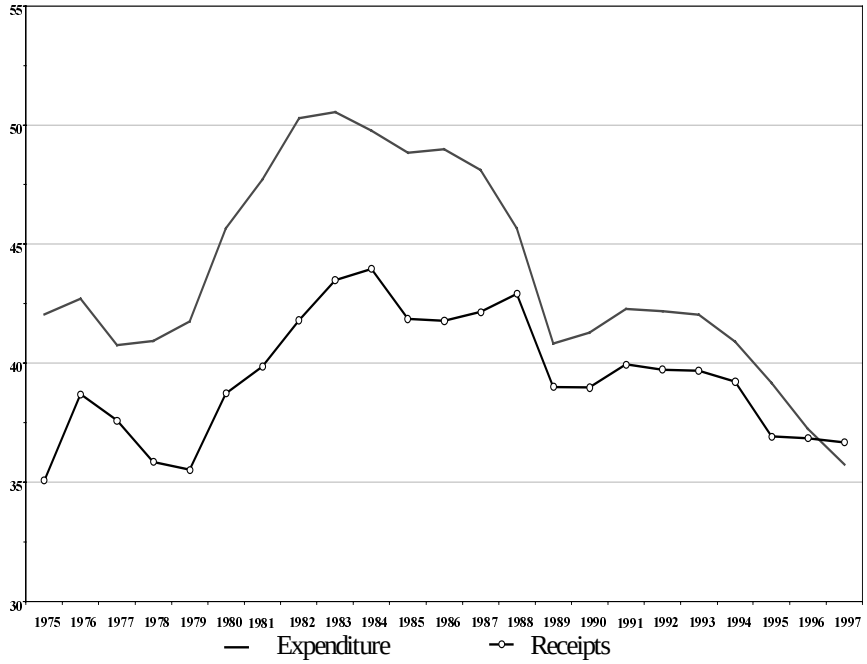
The rate of decline has been particularly pronounced for total expenditure. In 1987, faced with fiscal developments that were felt to be unsustainable, a broad political consensus facilitated the implementation of significant cut-backs in public expenditure. These cutbacks coincided with the beginning of a period of improvement in macroeconomic performance, an improvement that has, in general, continued through to the present.

It has been posited that there is a causal connection between these two developments, in particular that Ireland experienced an "expansionary fiscal contraction" in the late 1980s. Essentially, this hypothesis implies that the fiscal contraction undertaken from 1987 onwards signalled to the private sector a reduction in its future tax burden. In turn, this stimulated an immediate upturn in private sector activity more than sufficient to offset the impact of the fiscal contraction on GDP growth.

Recently, there has been some revision of whether it is appropriate to say that an expansionary fiscal contraction took place. Bradley and Whelan (1997) suggest that the pick-up in growth was principally attributable to favourable external factors at the time (unexpectedly strong world growth, large tax cuts in the UK, and a fall in international interest rates). Whatever the causal factors, the improvement in the Irish public finances has been dramatic since 1987.

Fig. 4

Total Receipts and Expenditure as percent of GDP in Ireland



Source: CSO (1998)

The Cyclicality of Fiscal Policy in Ireland

Bradley and Whelan argue that short-term budgetary policy has been pro-cyclical in Ireland in recent times. A pro-cyclical fiscal policy would be contrary to the neoclassical theory of fiscal policy which predicts countercyclicality in the fiscal deficit. According to this theory, tax-smoothing is the optimal fiscal policy - that is choosing a constant tax

rate which avoids the intertemporal distortion that would arise if tax rates were to change over the economic cycle. If a constant tax rate is chosen then tax revenues as a proportion of GDP will be acyclical.

On the expenditure side, the optimal policy is to maintain a smooth absolute level of government spending over the cycle (in which case the ratio of government spending to GDP moves countercyclically) or to change absolute government spending in a countercyclical manner in an effort to smooth incomes over time (again, in this case the government spending to GDP ratio will move countercyclically). An optimal fiscal policy, therefore, implies the ratio of government revenue to GDP is acyclical and the ratio of government expenditure to GDP is countercyclical. The ratio of the government deficit to GDP should, consequently, move countercyclically[3].

Gavin and Perotti (1996) found that the fiscal deficit in a panel of OECD countries is significantly countercyclical. In contrast, Lane (1998b) notes that the fiscal deficit in Ireland appears to be acyclical over the period 1990-1996 - an outcome that would suggest that fiscal policy may be procyclical in Ireland. He provides a systematic, econometric analysis of the behaviour of government spending, government revenues and the government deficit over the years 1980 to 1994. Controlling for the discrete permanent step adjustment in government expenditure initiated in 1987 by including a dummy variable in his equations, Lane's salient results are as follows:

- Expenditure: Total government expenditure as a proportion of GDP is acyclical; consequently, the level of government expenditure is procyclical.
- Revenue: Total government revenue as a proportion of GDP is acyclical or possibly even countercyclical. The ratio being acyclical would imply that the level of government revenue is procyclical; the ratio being countercyclical would imply that the level of government revenue is either acyclical, procyclical or countercyclical.

[3] Appendix 1 outlines the relationship between the cyclical behaviour of fiscal variables expressed as a proportion of GDP and expressed in levels, and also outlines the implications of such behaviour for the elasticity of those variables with respect to GDP.

- Deficit: His econometric results support the initial hypothesis that the unadjusted budget balance, as a proportion of GDP, is acyclical. For the structural budget balance, using the OECD measurement of that variable, he finds that this variable, again as a proportion of GDP, is acyclical. There is, however, also some support for the structural balance ratio being procyclical. As Lane points out, this latter result confirms what is implied for the structural deficit by the unadjusted deficit being acyclical.

These results, therefore, point to absolute government expenditure in Ireland being procyclical. If we simplify total tax revenue, TR, as being equal to an average tax rate, *t*, multiplied by GDP, then the econometric results for the government revenue ratios (indicating that that ratio is acyclical or possibly countercyclical) imply that the tax rate will either be acyclical or move countercyclically. Tax rates moving countercyclically would constitute procyclical fiscal policy.

In summary, fiscal policy in line with the neoclassical theory would lead to the ratios of government expenditure, government revenue, and the unadjusted government deficit to GDP being, respectively, countercyclical, acyclical, and countercyclical. Lane's results suggest that these ratios in Ireland are, respectively, acyclical, acyclical or countercyclical, and acyclical.

Implications for Cyclical Sensitivity Parameters

The implications of procyclical fiscal policy for the measurement of structural fiscal balances is that revenue and expenditure elasticities are unlikely to be time-invariant.[4] Looking at the implications for the

[4] There are similar, if converse, implications for the measurement of revenue and expenditure elasticities when the government pursues an active countercyclical fiscal policy, i.e. where tax rates are increased and social welfare payment rates are reduced when economic activity picks up and where tax rates are reduced and social welfare payment rates are increased when activity turns down. For simplicity, we confine the discussion to the procyclical case. The general point being made is that if the government is changing tax and social welfare payment rates over the cycle then there will arise the possibility of significant time-variance in revenue and expenditure elasticity values.

revenue elasticities first, the neoclassical theory would predict that tax rates would be constant over the cycle in which case the revenue elasticity, (ΔTR/TR)/(ΔY/Y), where TR refers to revenue and Y to GDP, would be acyclical. In contrast, were tax rates to move countercyclically this would mean that revenue elasticities would be cyclically-variant.

Although total government expenditure may be procyclical, total social security payments may, as found by Lane for Ireland, move countercyclically. This is as one would expect if social security acts as an automatic stabiliser. However, this finding does not imply that social security rates are acyclical. To illustrate this, let SS denote total social security expenditure and U the level of unemployment. For simplicity, assume there is a single security rate per person unemployed, denoted s, and that SS is equal to s.U. It will then be the case that

$$\frac{d\,SS}{d\,Y} = \frac{ds.U}{d\,Y} = s.\frac{\partial\,U}{\partial\,Y} + U.\frac{\partial\,s}{\partial\,Y}$$

It is possible for $d\,SS/d\,Y$ to be negative, as found by Lane, but for $\partial\,s/\partial\,Y$ to be positive. The total term $d\,SS/d\,Y$ would still be negative so long as the positive term U.$\partial\,s/\partial\,Y$ is less in absolute terms than the negative term s. $\partial\,U/\partial\,Y$.[5] If social security rates move procyclically then the marginal cost in unemployment benefits with respect to the unemployment rate may also be cyclically-variant[6].

The implication of procyclical fiscal policy, and systematic discretionary fiscal policy more generally, for elasticity measurement is

[5] The other factor impinging on the social security expenditure elasticity is the Okun coefficient. In the Irish case, the earlier discussion on output gaps in Ireland is relevant here. The problems pertaining to measuring both potential output and the natural level of unemployment render finding accurate estimates of the Okun coefficient difficult in Ireland.

[6] See Appendix 2 for a fuller outlining of the implications of variability in tax rates and social security rates on elasticity measures.

that such policy will result in revenue and expenditure elasticities not being constant over time. There may be a need, therefore, to assess and, possibly, quantify the significance and consequences of this time variance in elasticity measures. Other, more familiar sources of time variance in elasticity measures also arise. For example, fiscal drag will tend, ceteris paribus, to increase over time the elasticity of revenue categories with respect to output growth. Such factors need also to be taken into consideration when addressing elasticity estimation.

4. Conclusions

This paper highlights issues that, in general, complicate the use of the gap-elasticity approach to measuring structural budget balances in Ireland. The high rates of actual output growth in Ireland in the late 1990s make it difficult to assess potential output growth so hindering the accurate estimation of recent and prospective output gaps. Identifying potential output is further exasperated in an economy, like Ireland, that has experienced significant changes in its productive factors over the past two decades. Another factor impinging on the gap-elasticities approach is that procyclical fiscal policy may well mean that it is inappropriate to use constant, cross-sectionally-generated elasticity estimates when calculating the structural budget balance. In conclusion, it seems vital that the idiosyncratic features of an economy, as exemplified in this case study of the Irish economy, be accounted for when seeking to achieve good estimates of structural budget balances.

APPENDIX 1

Cyclicality and Elasticity in Shares and Levels

Depending on the revenue or expenditure variable's elasticity with respect to output, it may seem to move differently within the economic cycle when it is expressed in levels or in shares. The table below outlines the different cases when the elasticity value ranges from being negative to being positive and greater than 1 in magnitude. For example, consider the case where the elasticity is greater than 0 but less than 1. In this case when output (Y) increases the share of the fiscal variable in Y declines, i.e. the share moves countercyclically, whereas it simultaneously increases in levels so that the level moves with the cycle, or procyclically. Cyclicality in this context is not attributing causation but merely correlation between variables.

Output	Shares	Share Cyclicality	Levels	Levels Cyclicality	Elasticity
Y $\uparrow$	$\frac{X}{Y}$ $\uparrow$	**Pro**	**X** $\uparrow$	**Pro**	$\eta_{xy} > 1$
Y $\uparrow$	$\frac{X}{Y}$ —	**Acyc**	**X** $\uparrow$	**Pro**	$\eta_{xy} = 1$
Y $\uparrow$	$\frac{X}{Y}$ $\downarrow$	**Counter**	**X** $\uparrow$	**Pro**	$0 < \eta_{xy} < 1$
Y $\uparrow$	$\frac{X}{Y}$ $\downarrow$	**Counter**	**X** —	**Acyc**	$\eta_{xy} = 0$
Y $\uparrow$	$\frac{X}{Y}$ $\downarrow$	**Counter**	**X** $\downarrow$	**Counter**	$\eta_{xy} < 0$

Let X = TR or E, where TR is Government Tax Revenue,
E is Government Expenditure,
Y is GDP

$\eta_{xy} = \frac{dx}{dy} . \frac{y}{x}$ is the absolute value of the GDP elasticity of X.

APPENDIX 2

Impact of Variable Rates of Tax and Social Security

Let TR = t.Y and SS = s.U

where TR = Tax Revenue,
SS = Social Security Expenditure,
t = Tax Rate,
s = Rate Social Security Payments,
y = GDP,
u = Unemployment Level

(i) CONSTANT RATES

Tax case: $\frac{dTR}{dY} = \frac{d(t.Y)}{dY} = t$ which is a constant, implying that revenue is acyclical in shares and procyclical in levels.

Social Security case: $\frac{dSS}{dY} = \frac{d(s.U(Y))}{dY} = \frac{d(s.U(Y))}{dU}.\frac{dU}{dY} = s.\frac{dU}{dY}$

Because $\frac{dU}{dY} < 0$ this implies that social security expenditure is countercyclical in both shares and levels.

(ii) VARIABLE RATES

Tax case: $TR = t(Y).Y$

$$\frac{dTR}{dY} = t(Y).\frac{\partial Y}{\partial Y} + Y\frac{\partial t}{\partial Y}$$

$$\frac{dTR}{dY} = t + Y\frac{\partial t}{\partial Y} = t\left[1 + \frac{Y}{t}.\frac{\partial t}{\partial Y}\right] = t\left[1 + \frac{1}{\varepsilon_t}\right]$$

where $\varepsilon_t = \frac{t}{Y}.\frac{\partial Y}{\partial t}$

If $\varepsilon_t > 0$ when $\frac{\partial Y}{\partial t} > 0$ this implies procyclicality of revenues in levels and shares

If $\varepsilon_t = 0$ when $\frac{\partial Y}{\partial t} = 0$ this implies procyclicality of revenues in levels and acyclicality in shares

If $\varepsilon_t < 0$ when $\frac{\partial Y}{\partial t} < 0$ the cyclicality depends on the value of ε.

when $|\varepsilon| > 1$ this implies procyclicality in levels and countercyclicality in shares

$|\varepsilon| = 1$ this implies acyclicality in levels and countercyclicality in shares

$|\varepsilon| < 1$ this implies procyclicality in levels and countercylicality in shares

Social Security case: $SS = s(Y).U(Y)$

$$\frac{dSS}{dY} = \frac{d(s(Y).U(Y))}{dY} = s.\frac{\partial U}{\partial Y} + U\frac{\partial s}{\partial Y}$$

where $\frac{\partial U}{\partial Y} < 0$ is the inverse Okun coefficient.

If $\frac{\partial s}{\partial Y} \leq 0$ then the social security expenditure is countercyclical in levels and shares.

If $\frac{\partial s}{\partial Y} > 0$ then the cyclicality of the social security expenditure depends on the relative size of the $\frac{\partial s}{\partial Y}$ and $\frac{\partial U}{\partial Y}$ terms.

If $s.\frac{\partial U}{\partial Y} > U\frac{\partial s}{\partial Y}$ would imply countercyclical in expenditure in levels and shares.

$s.\frac{\partial U}{\partial Y} = U\frac{\partial s}{\partial Y}$ would imply acyclical expenditure in levels and countercyclical in shares.

$s.\frac{\partial U}{\partial Y} < U\frac{\partial s}{\partial Y}$ would imply procyclical expenditure in levels, while the impact for the share will depend on the value of the social services expenditure elasticity with respect to output.

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