THE COMMISSION SERVICES' CYCLICAL ADJUSTMENT METHOD

Comparison of the Hodrick-Prescott filter with other trend estimation methods

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1. Introduction

For the analysis of the budgetary policy situation in the euro-area and the European Union (EU), the European Commission services calculate cyclically adjusted budget balance figures for each of the Member States.

The cyclical adjustment method used by the Commission services (European Commission, 1995) to calculate these cyclically adjusted balances involves two main steps (Blanchard, 1990):

- first, trend output is estimated. Estimates of the cyclical fluctuations are obtained by subtracting these trend output estimates from actual output.
- secondly, the effects of these output gaps on government budget receipts and expenditure are calculated via the use of revenue and expenditure elasticities. These cyclical effects are then deducted from the actual government budget balance to obtain the cyclically adjusted budget balance.

The cyclical adjustment method currently used by the Commission services applies the Hodrick-Prescott filter (Hodrick and Prescott, 1980; Prescott, 1986) to obtain estimates for the trend output and output gaps.

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The Hodrick-Prescott trend estimation method is currently widely used as a simple technique for the detrending of economic time series. In the Hodrick-Prescott method, a smooth average trend with gradually changing growth rates is calculated over the cycle via the application of weighted moving averages to the actual output series.

With this method, the estimated trend is not an economic policy objective and the deviations of actual output from trend output are symmetric over the full period. The choice of an average trend as a benchmark for the Commission services' cyclical adjustment method was preferred because it offered a pragmatic approach, whereby the reference path has no normative significance but corresponds to an attainable average level of output.

Indeed, since the cyclically adjusted budget balance measures the size of the budget under *normal* economic conditions, trend GDP corresponds to the mean value of GDP over the cycle and must therefore be distinguished from potential GDP. In contrast to potential GDP, which measures the maximum attainable output if all factors of production are fully utilized, trend GDP measures the level of output under average utilization of production factors.

The properties of the output gaps calculated on the basis of potential output are clearly different from those obtained on the basis of average trend output. As the potential level of output tends to lie above the average output level, deviations of actual output from potential output are systematically negative and asymmetric, whereas deviations of actual output from average trend output are symmetric over the series. Consequently, the output gaps which deviate from trend GDP have mean value of zero, while the output gaps calculated as deviations from potential output exhibit larger negative values.

When potential output is used as a reference benchmark for the cyclical correction of budget balances, the cyclical component risks to be overestimated in a downturn and to be underestimated in an upturn. When the average level of output is used instead as a benchmark, cyclical influences cancel out over the series and are therefore note interpreted differently during a slowdown as compared to an expansionary phase. Such an interpretation is in line with the approach adopted in the Stability and Growth Pact. Under the Pact, EU Member States have to aim for a medium-term budgetary position of "*close to balance or in surplus*". The Pact assumes that the actual budget outcome can be either above or below this medium-term position due to the influence of normal cyclical fluctuations in economic activity. However, the Pact also assumes that these fluctuations of the budget around the medium-term position should cancel out over the cycle, so that the chosen position can be maintained on average over the cycle.

The Hodrick-Prescott method is a simple and transparent method which provides a uniform framework for the calculation by the Commission services of trend output and output gaps for each of the EU Member States. Thus, the comparability of treatment among the Member States is guaranteed.

As the trend and output gap estimates are calculated mechanically in the Hodrick-Prescott method, they do not require judgmental fine-tuning and can therefore easily be replicated. Thus, the Commission services preferred a mechanical technique to be applied without judgmental intervention over more complex methods.

Nevertheless, the Hodrick-Prescott method is subject to various underlying limitations. It has been mainly criticized in the literature for the arbitrariness in the choice of the smoothing parameter λ and for its end point bias problem. The Commission services therefore closely examined the choice of the smoothing parameter λ and introduced a correction for the end point bias problem via the inclusion of mechanical projections.

In addition, the Commission services decided to check the plausibility of the trend and output gap estimates obtained via the Hodrick-Prescott filter against those obtained via other methods, such as the band-pass filter, the linear time trend technique or the production function approach.

It should be noted in this respect that any trend estimation procedure is confronted with the problem that the trend of a macroeconomic time series is not observable. This makes it difficult to directly assess the quality of a specific estimate of the trend component. To overcome this lack of observability, the decomposition of a time series into a trend and a cyclical component is achieved by making certain identifying assumptions on the functional form and stochastic properties of the trend component and on the correlation between trend and cycle. Besides certain technical differences relating to the estimation procedures, alternative methods differ mainly concerning these identifying assumptions.

The first section of this note presents the Hodrick-Prescott filter and characterizes its main features. The selection of the smoothing parameter λ and its impact on the extraction of the cyclical component are examined more closely. Attention is also given to the so-called end point bias problem. In the second section, the Hodrick-Prescott filter is compared other trend estimation methods, such as the band pass filter, linear time trend methods and the production function approach. In the third section, the output gap estimates resulting from these different methods are compared to other business cycle indicators, such as the degree of capacity utilization and the producer sentiment indicator. The main results that were obtained in this note are summarized in the conclusions.

2. The Hodrick-Prescott filter

2.1 Main characteristics

To obtain estimates for trend output, the Commission services' cyclical adjustment method applies the Hodrick-Prescott filter to the actual output series (see box for the technical specification of the filter).

The Hodrick-Prescott filter is obtained by minimizing the regular fluctuations in actual output around trend output subject to a constraint on the variation of the growth rate of trend output.

The technical specification of the Hodrick-Prescott filter imposes a trade-off between the smoothness and fit to actual output of trend output: the smoother trend output, the poorer its fit to actual output and vice versa. This trade-off is determined by the choice of the value for the Lagrange multiplier λ. The value of the Lagrange multiplier λ also determines the smoothness of the trend estimates. A low value of λ produces trend output estimates that closely follow actual output and are therefore very volatile, while a high value of λ produces very smooth trend estimates that follow actual output less closely. For values of λ which tend to infinity, the growth rates of trend output would remain constant and would therefore correspond to the results obtained with the split time-trend estimation method.

Box: Technical specification of the Hodrick-Prescott filter

$$Min \sum_{t=1}^{T} \left(\ln Y_t - \ln Y_t^*\right)^2$$

subject to

$$\sum_{t=2}^{T-1} \left[\left(\ln Y_{t+1}^* - \ln Y_t^*\right) - \left(\ln Y_t^* - \ln Y_{t-1}^*\right)\right]^2 \leq e$$

where: Y_t = actual GDP at constant market prices

Y_t^* = trend GDP at constant market prices

e = small number arbitrarily chosen

The problem can be re-written as follows:

$$Min \sum_{t=1}^{T} \left(\ln Y_t - \ln Y_t^*\right)^2 + \lambda \sum_{t=2}^{T-1} \left[\left(\ln Y_{t+1}^* - \ln Y_t^*\right) - \left(\ln Y_t^* - \ln Y_{t-1}^*\right)\right]^2$$

where: λ = Lagrange multiplier

It has been shown by King and Rebelo (1992), that applying the HP filter, results effectively in a moving average filter. Each estimate of trend output is calculated by the application of a weighted average, which extends over several years, to the actual output data. The weighting coefficients of the moving averages are fixed in such a way that higher weights are assigned to the years closest to the reference year, i.e. the year for which trend output is calculated. The filter weights are symmetric, i.e. observations in a similar position on each side of the central observation are given equal weights.

It has been shown by Baxter and King (1995) that moving average filters whose weights sum to zero have a trend reduction property, i. e. they make time series stationary which have linear and quadratic deterministic trends and they also render series with stochastic trends up to the order 2 stationary[1]. Symmetry is also a desirable property since it implies that no phase shift - i.e. a in time of the turning points of the series - is introduced.

2.2 Choice of the smoothing parameter λ

Any degree of smoothness can be achieved with the Hodrick-Prescott filter by setting specific values of the smoothing parameter λ. In order to gain better insight into the question which cyclical components of GDP are actually extracted by this filter for specific values of the smoothing parameter, it is useful to use frequency domain methods[2].

This framework can be used to analyze which cyclical frequencies are suppressed by the Hodrick-Prescott filter. Which cyclical components are dampened and which components are kept is expressed by the gain function $C^F(\theta)$, which attaches a weight to each component with frequency, θ or correspondingly cycle length $2\pi/\theta$.

The Hodrick-Prescott filter eliminates cycles with infinite duration or trends while it passes cycles with higher frequencies. The smoothing parameter λ determines to what extent low frequency cycles are eliminated from the data. When using the Hodrick-Prescott filter in the cyclical adjustment method, the Commission services set λ equal to 100, which corresponds to standard practice.

Figure 1 plots C^F for two alternative values of λ. It can be seen that for $\lambda = 100$ the Hodrick-Prescott filter hardly dampens cyclical components up to a period of 15 to 16 years, while it practically eliminates all cycles with a period larger than 20 years. In contrast, for λ=10 only cycles up to 8 years would be retained fully in the cyclical

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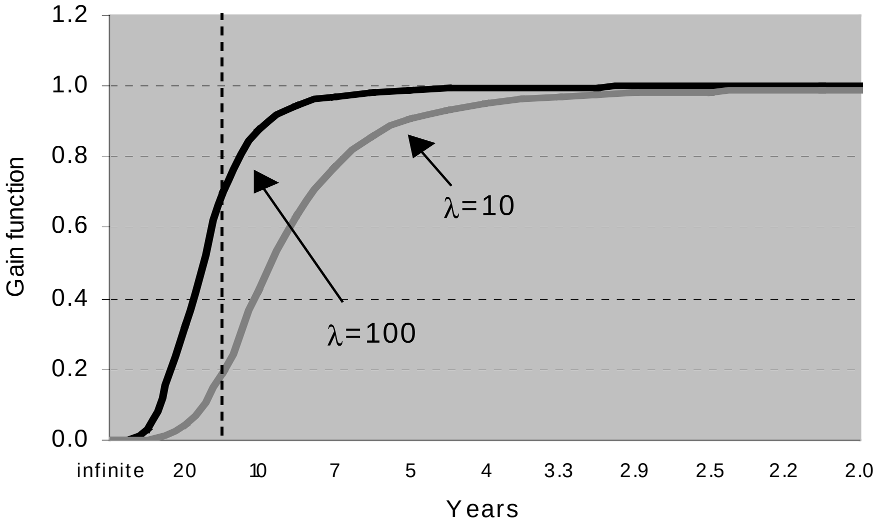
1 The level of a process with stochastic trend of order one contains a random walk component. The growth rate of a process with stochastic trend of order two contains a random walk component. Most macroeconomic time series can be regarded as integrated of order one; this holds in particular for real GDP. The price level is an example of a process which is integrated of order 2, since the inflation rate often exhibits random walk properties.

2 A brief introduction of the relevant concepts is given in the annex.

component. King and Baxter (1995), for example, have suggested setting the smoothing parameter λ equal to 10. In light of what is generally regarded as typical business cycle length, the Commission services' choice seems a rather generous definition of the cycle.

Fig. 1

Gain function Hodrick-Prescott filter



2.3 End point bias problem

With actual data and finite samples a problem arises with symmetric filters, because the theoretically infinite moving average must be truncated at a finite lag or the filter must be constructed in such a way that the filter weights become asymmetric at the end points.

Baxter and King (1995) show that, close to the end points - especially the last 3 to 4 observations -, the Hodrick-Prescott filter not only eliminates the low frequency cycles it is supposed to eliminate, i. e. cycles with a cycle length of more than 16 years, but also has a tendency to dampen the influence of cycles with higher frequencies. This will affect cyclical components with a period larger than 4 years. Only cycles with shorter periods will be passed fully. This implies that the Hodrick-Prescott filter produces a series for the output gap that underestimates the length of the cycle close to the end point, if no corrective measures are taken.

Since this phenomenon especially occurs for the last 3 or 4 observations, one possibility to correct for this bias is to extend the data set by adding GDP forecasts over a range of 3 to 5 years. This is standard practice and also followed by the Commission services.

Mechanical projections - obtained with ARIMA-models[3] - are added by the Commission services to extend the actual output series on which the Hodrick-Prescott filter is then applied. The approach chosen for generating these projections is in keeping with that followed by the Commission services for the cyclical adjustment method. The projections are calculated mechanically and can be easily reproduced. The aim is to add observations which are characteristic for the actual output series, i.e. which show average growth rates, to counterbalance observations which are uncharacteristic for the rest of the series.

Of course, the addition of forecasts to prolong the series is only a second-best solution for the end point bias problem: this problem only disappears once all observations influencing the calculation of the trend estimate have become available as historical data. By adding forecasts to the data series, a forecast error problem is added and it has to be checked how severe this problem may be. Since the Hodrick-Prescott filter uses leads for calculating the trend component, the forecasted values of GDP will obviously influence the trend and consequently the output gap estimate to some extent.

Figure 2 illustrates this end point problem of the Hodrick-Prescott filter by comparing the Commission's estimate of the output gap for Italy with an output gap constructed on the basis of the standard

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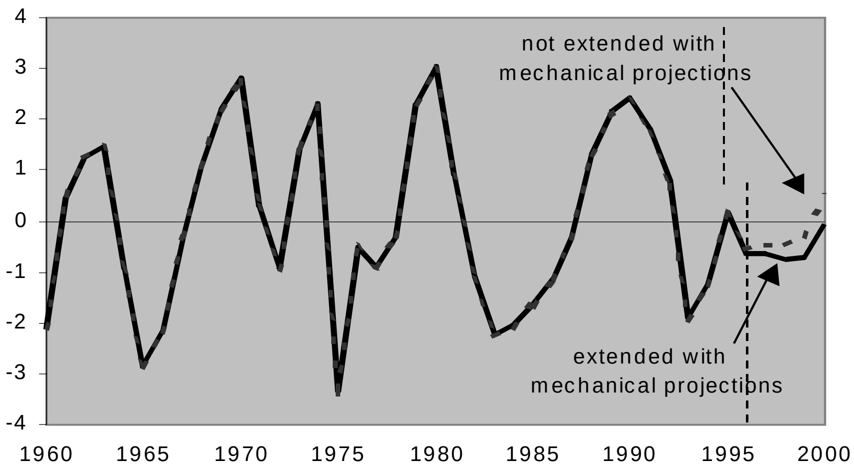
3 To calculate the output gaps used in the cyclical adjustment method, the Commission services add two years of forecasts made by the country desks and published officially as the Commission forecasts. This series is then extended with 4 years of mechanical projections obtained via "Auto-Regressive Integrated Moving Average" (ARIMA, Box Jenkins) specifications of the time series. These models are characterised in the literature as powerful forecasting procedures for the short term for a large number of time series.

Hodrick-Prescott method without extending the output series with forecasts[4].

Fig. 2

Correction for the end point bias problem - Italy



Two observations can be made on the basis of this graph:

- not extending the series clearly dampens the period of the last cycle;
- noticeable deviations between the two output gaps occur about 3 to 4 years before the final observation, which confirms the findings of Baxter and King.

In order to analyse how sensitive the output gap estimate reacts to changes in the forecast, the output gaps calculated by the Commission services have been compared with two alternatives. Under the first

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4 The effect is clearly visible in the case of Italy where, due to the low real GDP growth rates recorded over the past years, the trend growth rate is pulled downward if no correction for the end point bias is applied. The inclusion of projections showing higher growth rates for future years pulls up the trend estimates and widens the negative output gap. Similar results are found for the other countries.

alternative, it is assumed that GDP growth exceeds the mechanical projection by 0.5 percentage points each year over the period 2001 to 2004 while under the second alternative it is assumed that the ARIMA-forecast is too optimistic and over-predicts growth by 0.5 points in each year after 2000. The results indicate that a change in the forecast of 0.5 percentage points translates into a change in the output gap of close to 0.1 percentage points of trend GDP in 1998 and 0.2 percentage points in 1999. This sensitivity is strikingly similar across all countries and does not seem to be related strongly to the cyclical position.

3. Comparison with other trend estimation methods

3.1 Band pass filter

The Hodrick-Prescott filter has some desirable properties in that it retains fully those cycles above a certain frequency and then quickly reduces the weight to zero for cycles below that frequency. However, it is not an *ideal filter* with a sharp cut-off point at a specified cycle length.

In recent years suggestions have been made to construct such ideal or band pass filters (see, for example, Baxter and King, 1995). These filters attach a weight equal to one to the selected cyclical components. They eliminate all cycles with a period larger than the maximum selected business cycle period by attaching a weight equal to zero to them. This is in fact a theoretical possibility, which is more difficult to implement in practice.

It is interesting to observe that the band pass filter suggested by Baxter and King (1995) shares some common properties with the HP filter. First, it is also symmetric in order to avoid a phase shift of the filtered series; second, the filter weights are fixed in such a way that stochastic and deterministic trends are fully eliminated and third, the resulting moving average filter is of infinite order.

However, it is impossible to achieve such an ideal filter in practice with a finite set of weights. The smaller the number of weights used, the less sharp will be the cut-off property of the filter. For practical applications to finite time series, it is therefore also necessary to find a good approximation to this ideal filter. Moreover, there also exists an end point problem. After doing extensive sensitivity analysis, Baxter and King (1995) recommend adding six observations in order to eliminate the end point bias.

How does the Hodrick-Prescott filter compare to this band pass filter? In general, the output gaps constructed with the Hodrick-Prescott filter resemble closely those based on the band pass filter if the same data extension is used. A test was set up to examine the similarity of output gaps obtained with the Hodrick-Prescott filter against those obtained with a band pass filter. For this test, the filter design for the band pass filter was set up in such a way that it eliminated cycles larger than 16 years, in correspondence with the smoothing assumption currently used by the Commission services for the Hodrick-Prescott filter. Under these conditions, there is virtually no difference between the two output gaps.

Table 1

Correlation between output gaps from Hodrick-Prescott and band pass filter

Member State	Correlation coefficient
Belgium	0.98
Denmark	0.98
Germany	0.99
Greece	0.98
Spain	0.98
France	0.98
Ireland	0.99
Italy	0.99
Luxembourg	0.99
Netherlands	0.98
Austria	0.98
Portugal	0.99
Finland	0.99
Sweden	0.98
United Kingdom	0.99
All EU countries	**0.98**

Source: Commission services.

As can be seen in table 1, the correlation between them is very close to one for all EU countries. These results suggest, as found previously by Baxter and King (1995) for US GDP, that the Hodrick-Prescott filter is practically indistinguishable from a band pass filter in practical applications with finite data. Also the theoretical properties as derived from the gain function of the Hodrick-Prescott filter seem to carry over to the band pass filter.

3.2 Linear time trend

Both the Hodrick-Prescott filter and the band pass filter are designed to eliminate both stochastic and deterministic trends from the data and only leave strictly stationary components.

To regard time series as being composed of a linear deterministic trend and a cyclical component - with the implication that trend and cycle are completely uncorrelated with each other - has been the standard view until the early 1980s. In the 1980s, several time series studies (see, for example Nelson and Plosser, 1982) arrived at the conclusion that many macroeconomic variables - including real GDP - contain a unit root component or a stochastic trend. This is now the standard view and it implies that the trend component in GDP is now regarded as subject to irregular stochastic shocks, which have a permanent effect on the level of GDP. Though it is difficult to distinguish empirically with very high precision between deterministic and stochastic trends, it is nevertheless the case that standard unit root tests generally support the view that trends are subject to random shocks. In particular, the hypothesis that GDP contains a unit root cannot be rejected at the usual significance level.

If deterministic trends are used in order to calculate output gaps, there is a risk that trend components are not completely eliminated. It is difficult to reject the unit root hypothesis for real GDP in EU countries. The critical value (at the 5% level) of the Augmented Dickey Fuller-test is -3.0 for a sample of 25 observations. ADF-tests on real GDP have been carried out for all EU Member States. The tests showed that none of the reported t-statistics comes close to the critical value. A short sample period was chosen in order to avoid running a test over the years 1973-1974, which is often regarded as a year in which a significant trend break has occurred in many industrialised countries. Running unit root tests over a period with a trend break would therefore bias the results in favour of the unit root hypothesis.

How does the Hodrick-Prescott trend differ from a linear deterministic trend? The trend growth rate generated with the Hodrick-Prescott filter is correlated with the actual growth rate, expressing the view that trend and cycle are not completely independent phenomena. The correlation coefficient is around 0.5 for most EU countries, while for Spain, France and Ireland it is considerably higher. The economic assumption underlying a deterministic trend is of course that trend and cycle are completely independent phenomena. Since the Hodrick-Prescott filter also eliminates a stochastic trend component, it tends to generate smaller output gaps compared to output gaps calculated from a deterministic trend.

In the split time trend method, linear trends are fitted over periods spanning a full cycle. The latter are determined by taking the period between cyclical peaks or troughs. These have to be determined *a priori*. When a turning point in the cycle is located at the end of the series, the split time trend method performs worse than the Hodrick-Prescott method. When the start of a new cycle occurs at the end of the observed series, it will be difficult to locate. From the observations available, it will not yet be clear that a new cycle has started. As a result, the estimate trend growth rate will have to be revised considerably as soon as information showing the progression of the cycle becomes available. The trend growth rate estimated at first may differ considerably from the estimate given when more information becomes available and may therefore provide a misleading interpretation of the situation.

As the Hodrick-Prescott filter immediately processes all information available, the local turning point at the end of the series will directly affect the trend estimates for the surrounding interval. As a result, these estimates will have to be revised only slightly when more information becomes available. In addition, the Hodrick-Prescott filter does not require a prior analysis to locate the turning points is necessary.

One major reason why it is so difficult to reject the unit root hypothesis for GDP data may be the presence of a trend break of an otherwise deterministic trend. For example, Perron (1989) has demonstrated that by allowing for a single trend break or a trend shift in 1974, GDP for many industrialized countries can actually be modelled as stationary around a deterministic trend. However, this approach has been criticized as relying on an *a priori* known date for the trend break.

Recently Perron (1997) has presented an estimation method that searches for the trend break that minimizes the t-statistic used for testing for the presence of a unit root. Applying this procedure obviously gives a higher chance of rejecting the stochastic trend hypothesis.

Tests on the presence of a unit root were again carried out on real GDP data, while allowing for a trend break. The tests show that, even taking into account a trend break, the hypothesis of a unit root over the period 1974 to 1998 cannot be rejected. With this method, significant trend breaks can be detected for most EU countries. When trend breaks are allowed for, the size of the output gap using a deterministic trend is generally not larger than the Hodrick-Prescott output gap.

3.3 Production function approach

An alternative way[5] of extracting a cyclical component from the data is to use a production function approach and to make assumptions on the functional form of the production technology, returns to scale, the trend growth of technical progress as well as on the 'average' utilization

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5 The performance of the univariate and multivariate Beveridge-Nelson and the Blanchard-Quah methods was test by M. Forni and L. Reichlin, in a study for the Commission services (Forni and Reichlin, 1998). These methods do not seem to generate convincing results in practical applications. The Beveridge-Nelson method produces very volatile cyclical components as well as negative correlations between its trend and GDP growth, especially in situations where the GDP growth rate is positively autocorrelated.
The Blanchard-Quah decomposition assumes that trend and cycle correspond to independent causes. Output gap estimates depend strongly on the second variable which is used in the VAR analysis and there is no criterion on which the choice of this second variable can be based.
The Beveridge-Nelson and Blanchard-Quah estimates do not show a uniformly positive correlation with other business cycle indicators. The property of these methods that they can be negatively correlated with the trend shows up as a negative correlation with other cyclical adjustment methods. This property goes against any intuitive economic reasoning and makes it difficult to use them in practice.

rate of production factors, including an estimate of the trend employment and unemployment rates.

In this section trend estimates for GDP are presented which are based on the production function used in the Commission's QUEST model (Röger and In 't Veld, 1997). In QUEST, technology is described by a Cobb Douglas specification with employment and capital as inputs:

$$Y = L^{\alpha} K^{1-\alpha} TFP$$

Total factor productivity follows an exogenous deterministic trend, possibly with a trend break, and it also depends on current and past investment activities, which is captured by the mean age of the capital stock and an irregular (possibly auto-correlated) component which captures technology shocks of various kinds[6].

$$TFP = e^{\pi t} \frac{Age(K)}{K} \mathrm{Z}$$

Using this framework, trend output is calculated on the basis of a mean employment rate for period t, a value for total factor productivity, which would be attained for Z=1 and the actual value of the capital stock. Though the production function approach is an attractive method for calculating trend output, the problem of trend elimination from GDP or production output is in a sense shifted to calculating trend values of production inputs (Ongena and Röger, 1997).

Generally the NAIRU concept could be used to define an equilibrium employment rate. However, there does not seem to be a stable NAIRU for EU countries (Röger and In't Veld, 1996). Though current labor market theories (bargaining models, search models efficiency wage models, relative wage rigidity) make predictions on what determines shifts in the NAIRU, the problem remains that it is generally difficult to find stationary deviations from the estimated NAIRU. One

6 Among other things, this term captures changes in hours of work over the business cycle, changes in relative prices of intermediate production inputs and scale effects.

way out of this problem is to resort to technical detrending methods in order to arrive at a normal employment level. Here two alternative methods are used. In a first approach, trend employment is calculated by simply applying the Hodrick-Prescott filter to the employment series itself:

$$LT = HP(L)$$

Since it can be argued that using the Hodrick-Prescott filter for employment biases the comparison between the production function approach and the Commission services' trend estimation method too much in favor of finding similar output gaps, an alternative method is also used which avoids the use of the Hodrick-Prescott filter. By the second method, trend employment is calculated as follows:

$$LT = L*(1+(lur-lurt))$$

Where lurt is the trend unemployment rate and the trend is calculated by using Perron's segmented trend method.

Also, the calculation of trend total factor productivity is not straightforward since growth theory provides many alternative specifications for trend total factor productivity. The following three alternatives can be distinguished:

Vintage Models (used in QUEST);

R&D Models (quality ladder models, variety of products models);

Human Capital Models.

Under these three alternatives, trend growth of total factor productivity would be determined either by the age of the capital stock, the stock of R&D capital or the stock of human capital[7].

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[7] If one adopts a less theoretical approach and calculates trend total factor productivity via the Hodrick-Prescott filter and uses a Cobb Douglas production function together with an Hodrick-Prescott employment trend, then the resulting output trend component would be nearly indistinguishable from detrending GDP directly (Ongena and Röger, 1997)

	Age of capital (A = K + (1-d)K(-1))
TFPT =	RD capital stock (RDK = RD + (1-d)RDK(-1)
	Human capital stock (HK = EDU + (1-d)HK(-1)

The production function approach thus gives the following filter (in logs)

$$y_t^{pf} = \alpha l_t^T = (1-\alpha)k + tfp_t^T$$

The trend value of GDP depends to a large extent on current and past investments as well as current and past observations of unemployment. This has the practical advantage that an end point problem does not exist[8].

Since the trend depends for a large part on stock variables, the production function trend is likely to be smooth, but unlike a linear deterministic trend it will be influenced by booms and recessions. These two properties implied by the production function approach are similar to the properties that are generated by the Hodrick-Prescott trend. This implies that Hodrick-Prescott and production function output gaps in general move closely together.

Indeed, as can be seen in table 2, the correlation coefficient between Hodrick-Prescott output gaps and those generated by the production function method with Hodrick-Prescott trend unemployment is around 0.9 on average for all EU Member States. It drops to 0.7 for the production function method with a trend break in the unemployment rate.

8 The end point problem remains to the extent that trend employment is calculated via the Hodrick-Prescott filter or another symmetric filter.

Table 2

Correlation between output gaps from Hodrick-Prescott and QUEST production function

Member State	Correlation coefficient between Hodrick-Prescott filter and production function	Correlation coefficient between Hodrick-Prescott filter and production function with trend break
Belgium	0.69	0.83
Denmark	0.87	0.53
Germany	0.68	0.29
Greece	0.86	0.18
Spain	0.93	0.15
France	0.93	0.17
Ireland	0.82	0.54
Italy	0.92	0.36
Netherlands	0.88	0.25
Austria	0.80	0.39
Portugal	0.96	0.45
Finland	0.94	0.55
Sweden	0.93	0.26
United Kingdom	0.98	0.40
All EU countries	0.88	0.73

Source: Commission services.

The production function approach also implies a moving trend growth rate. Trend growth is, however, more volatile than the trend growth rate implied by the Hodrick-Prescott filter of total GDP. This can be explained by the fact that the Hodrick-Prescott filter implicitly smoothes both the unemployment rate and the level of employment, while in the production function approach only the unemployment rate is smoothed. That means that shocks to the participation rate or the population of working age are treated as affecting trend output directly while Hodrick-Prescott filtering of output smoothes these shocks over time.

The trend growth rates generated by the production function are also positively correlated with GDP growth rates and thus with the cycle. As in the Hodrick-Prescott filter, trend and cycle are not viewed as completely independent phenomena. The link between cyclical fluctuations and the trend runs via investment: reduced investment during recessions reduces the capital stock, thus lowering attainable output.

The production function approach also requires that certain *a priori* assumptions are made - for example, on the type of technical progress -. Moreover, the method imposes heavy data requirements.

4. Performance against business cycle indicators

An indication of the usefulness of the output gap measures discussed so far is to compare them to other business cycle indicators, such as the rate of capacity utilisation and the producer sentiment indicator. In general a positive correlation between output gap measures and these indicators is to be expected. Such a comparison also provides information on whether the cycle length is suitable.

It should be noted, however, that the capacity utilisation measure may not be comprehensive enough to characterise total economic activity since it focuses entirely on the manufacturing sector. The sentiment indicator is broader, since it includes industry confidence (1/3), consumer confidence (1/3), share prices (1/6) and construction sector confidence (1/6).

Table 3 presents correlations between the different measures for the output gap and capacity utilisation and the sentiment indicator. It is generally the case that the Hodrick-Prescott output gap is more strongly correlated with both the capacity utilisation rate and the sentiment indicator compared to output gap measures based on a deterministic trend method. Moreover, the turning points identified with the Hodrick-Prescott filter coincide with those shown in the business cycle indicators (Canova, 1999). The highest correlation is, however, found for the production function output gaps. However, in general, it is generally not significantly different from the results obtained with the Hodrick-Prescott filter[9].

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9 Summary statistical indicators on each of the trend estimation methods that have been examined in this paper are given in Annex II. The results show that the Hodrick-Prescott and band pass filters and the linear time trend methods (both without and with correction for trend breaks) produce symmetric

(continues)

Table 3: Correlation of output gap measures with capacity utilisation and sentiment indicator

Member State	Capacity utilisation					Sentiment indicator				
	Hodrick Prescott	Linear trend	Trend break	PF	PF with break	Hodrick Prescott	Linear trend	Trend break	PF	PF with break
Belgium	0.34	0.30	0.35	0.38	0.40	0.43	0.45	0.50	0.64	0.68
Denmark	0.27	0.36	0.39	0.34	0.48	0.16	0.25	0.21	0.10	0.41
Germany	0.47	0.33	0.40	0.70	0.69	0.26	0.12	0.34	0.71	0.70
Greece	0.32	0.01	0.04	0.39	0.44	0.33	0.26	0.22	0.16	0.19
Spain	0.30	0.24	0.43	0.42	0.24	0.28	0.38	0.04	0.34	0.35
France	0.74	0.69	0.56	0.86	0.86	0.51	0.45	0.33	0.72	0.76
Ireland	-0.37	0.06	-0.21	0.05	0.31	-0.05	0.39	-0.03	0.23	0.39
Italy	0.67	0.34	0.61	0.72	0.67	0.50	0.49	0.56	0.50	0.45
Netherlands	0.27	0.09	0.25	0.28	0.40	0.24	0.25	0.23	0.35	0.43
Portugal	0.11	0.06	0.06	0.11	0.14	0.34	0.32	0.35	0.36	0.34
United Kingdom	0.71	0.63	0.60	0.78	0.76	0.62	0.59	0.56	0.67	0.73

Note: The following methods for the estimation of output gaps are presented: (1) Hodrick-Prescott, (2) deterministic trend, (3) deterministic trend with break, i.e. the Perron method, (4) QUEST production function with Hodrick-Prescott trend employment, (5) QUEST production function with trend break in the unemployment rate.
The calculations could not be carried out for the three Member States that joined the EU in 1995 - Austria, Finland and Sweden - because sufficiently long time series for the capacity utilisation and sentiment indicators are not yet available for these countries.

Source: Commission services.

results. The standard deviation of the gaps produced by the linear time trend method is larger than for the other methods. Both linear time trend methods also produces gaps which are less evenly distributed than is the case for the other methods. Graphs per Member States are provided in Annex III.

5. Conclusions

The Hodrick-Prescott filter is a simple and transparent method and does not require any judgmental fine-tuning. It allows filtering out deterministic and stochastic trends up to second order. It does not induce shifts in the turning points of the series and the cyclical component that is retained by this filter is symmetric over the cycle. When the value for the smoothing parameter λ is set at 100 - as is done in the Commission services' cyclical adjustment method -, cycles with a length of up to 16 years are retained. If only cycles with a length of up to 8 years are kept - which would correspond more to the commonly accepted definition of business cycles - the output gaps become smaller. The end point bias problem can be addressed by extending the series with projections and the sensitivity of the forecast error appears to be limited. The Hodrick-Prescott filter has similar properties as the band pass filter and is as easy to compute.

The linear trend method is not widely used anymore since the hypothesis that GDP has a stochastic trend component is difficult to reject. The output gaps obtained via this method are large because trend and cycle are independent. If trend breaks are allowed for - which is strongly suggested by the data -, then the output gaps generated by the deterministic trend method are not uniformly larger anymore.

Instead of making statistical assumptions on the time series properties of trends and their correlation with the cycle, the production function approach makes assumptions based on economic theory.

However, the economic assumptions underlying the production function approach appear to be consistent with those retained in the Hodrick-Prescott filter. Indeed, both approaches produce smooth trend growth rates and show a positive correlation between trend and cycle.

The performance of the Hodrick-Prescott output gaps against other business cycle indicators - such as capacity utilisation or the sentiment indicator - is better compared to that of those generated by the two variants of the deterministic trend method. The performance of the production function output gaps is even slightly better than that of the Hodrick-Prescott filter. All in all, most features of the Hodrick-Prescott filter appear acceptable and, as was shown in this note, it often performs better than most other methods. The validity of the use of the Hodrick-Prescott filter as a simple method for trend estimation was thus confirmed by the tests carried out in this note. The production function approach could present an alternative for the Hodrick-Prescott filter. In practice, the Commission services already use trend and output gap estimates obtained via the QUEST production function to check the plausibility of the results obtained via the Hodrick-Prescott filter.

ANNEX I

Frequency Representation of HP Filter

Economic time series are composed of cyclical functions of time such as:

$$y_t = a\cos(\theta t)$$

Since y goes through a complete cycle as $x = \theta t$ goes from 0 to 2π, the parameter θ (or the frequency) implicitly defines the time it takes for y to complete a full cycle, or in other words the *period* of the cycle. The period (p) is implicitly defined as $p = 2\pi / \theta$. For example, the frequency of a cycle that repeats itself every 8 years would be given by $2\pi / 8$ with annual data. Also notice, with annual data one can only analyze cycles with a period of at least 2 years. That implies the interval of possible frequencies is given by $[0, \pi]$.

This framework can be used to analyze which frequencies are suppressed of certain types of filters and in particular the Hodrick-Prescott filter. King and Rebelo (1992) have shown that the cyclical component of the Hodrick-Prescott filter can be written as follows

Let the cyclical component of GDP after filtering with Hodrick-Prescott be given by:

$$y_t^c = [1 - HP(L)]y_t$$

Define $C(L) = (1 - HP(L))$, then the Fourier transform is given by

$$C^F(\theta) = \frac{2\lambda\sqrt{(1-\cos(\theta))}}{1 + 2\lambda\sqrt{(1-\cos(\theta))}}$$

Since $\cos(\theta) = 1$ for $\theta = 0$ and $\cos(\theta) = -1$ for $\theta = \pi$ it is easy to see that the Hodrick-Prescott filter eliminates cycles with zero and low frequencies while it passes cycles with higher frequencies. It can also be seen from this expression that the smoothing parameter λ determines to what extent low frequency cycles are eliminated from the data. For $\lambda = 100$, the Hodrick-Prescott filter leaves a cyclical component with cycles up to a period of 20 years. Given standard definitions of business cycle duration this seems a rather generous definition. King and Baxter (1995), for example, have suggested to set the smoothing parameter to 10, this would only leave cycles up to 8 years in the cyclical component.

Band Pass Filter

The idea underlying the band pass filter is as follows. Time series can equivalently be represented in the time domain as an infinite moving average process or in the frequency domain, namely as an integral over all its random periodic components from 0 to π as follows

$$y_t = \int_{-\pi}^{\pi} f(\theta) d\theta$$

Given this representation of the time series, the cyclical component could be calculated after determining the maximum cycle length that can still be regarded as a business cycle (e. g. cycle with period of 16 years (corresponds to frequency 2π/16) by giving a zero weight to periodic components f() with $\theta \leq 2\pi/16$ and a weight equal to one to all periodic components with $\theta > 2\pi/16$. Let $\alpha(\theta)$ be a frequency response function with

$$\alpha(\theta) = \begin{cases} 1 \; for \; \theta > 2\pi/16 \\ 0 \; for \; \theta \leq 2\pi/16 \end{cases}$$

then, in frequency domain the cyclical component would be represented as:

$$y_t^c = \int_{-\pi}^{\pi} \alpha(\theta) f(\theta) d\theta \, .$$

There exists a time domain representation of this frequency response function, i. e. there exists a filter $(1 - BP(L))$ with filter weights BP_h which can be found via the inverse Fourier transform of $\alpha(\theta)$ as

$$BP_h = \int_{-\pi}^{\pi} \alpha(\theta) e^{i\theta h} d\theta \, .$$

In other words, the band pass filter leads to the following cyclical component

$$y_t^c = (1 - BP(L)) y_t$$

ANNEX II

Statistical indicators on output gaps produced via different methods (all EU countries, 1974-98)

	Average	Standard deviation	Fisher coefficient	Pearson coefficient
Hodrick-Prescott filter	-0.1	2.3	0.2	3.9
Band pass filter	0.0	2.5	0.1	4.0
Linear time trend	0.1	3.4	0.7	7.6
Linear time trend with trend breaks	0.0	2.2	-0.2	5.6
Production function with Hodrick-Prescott trend for unemployment	-0.6	2.2	0.1	3.2
Production function with trend break for unemployment	-0.6	2.3	0.0	2.9

ANNEX III

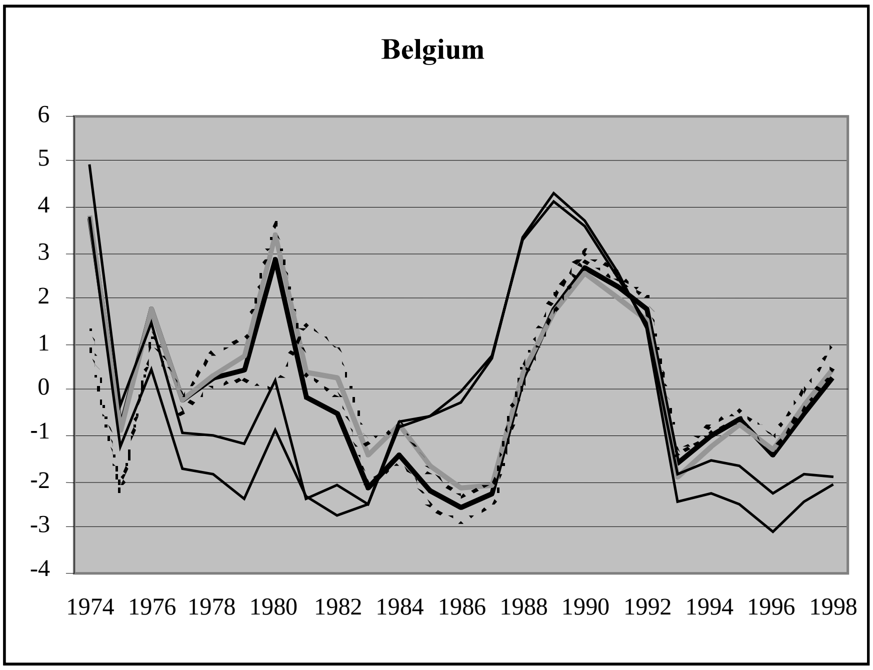
Output gaps produced by various trend estimation methods

Graphs legend:

Dotted lines: linear trends (with and without trend breaks)

Thin lines: production function (with and without trend breaks)

Thick lines: Hodrick-Prescott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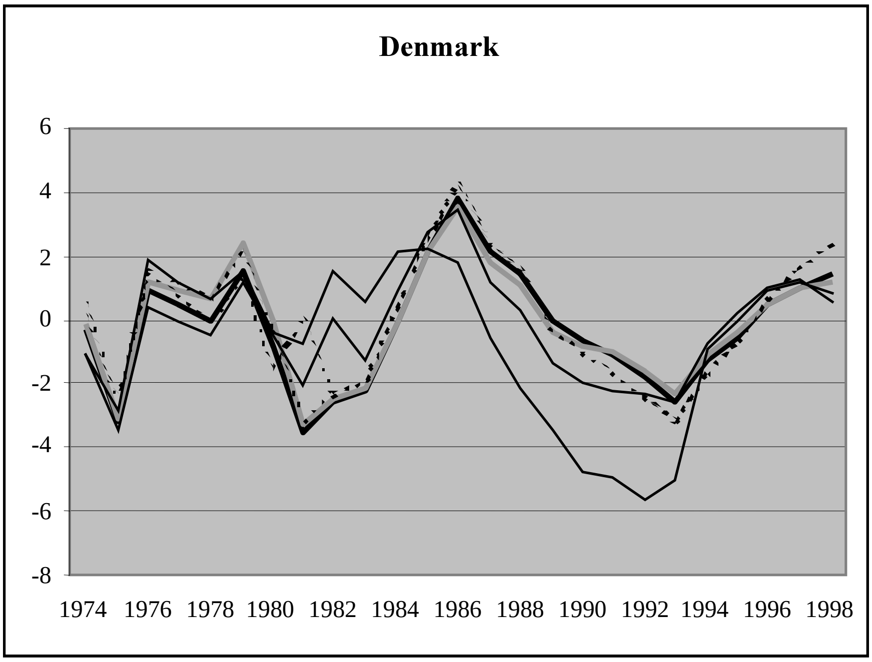
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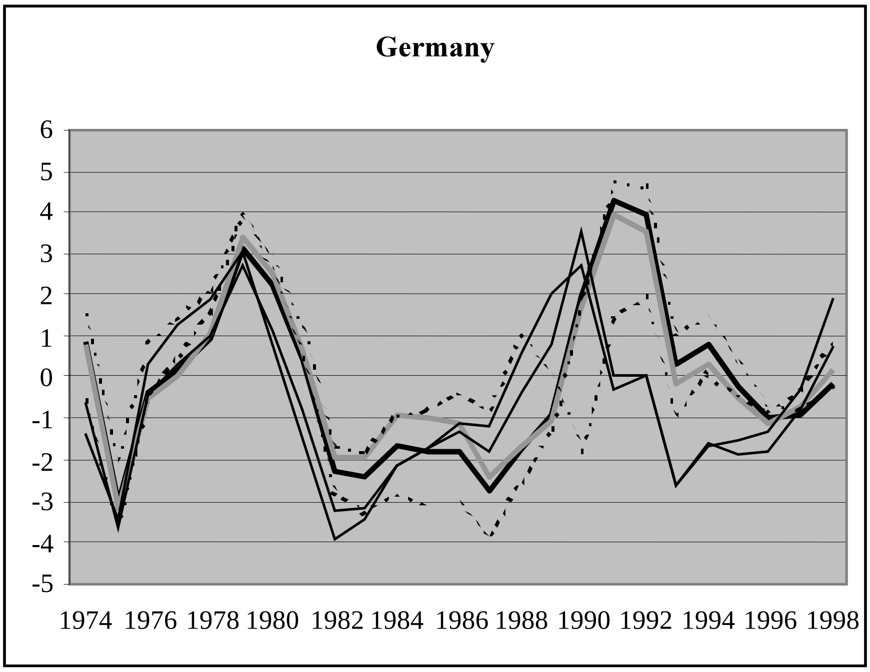
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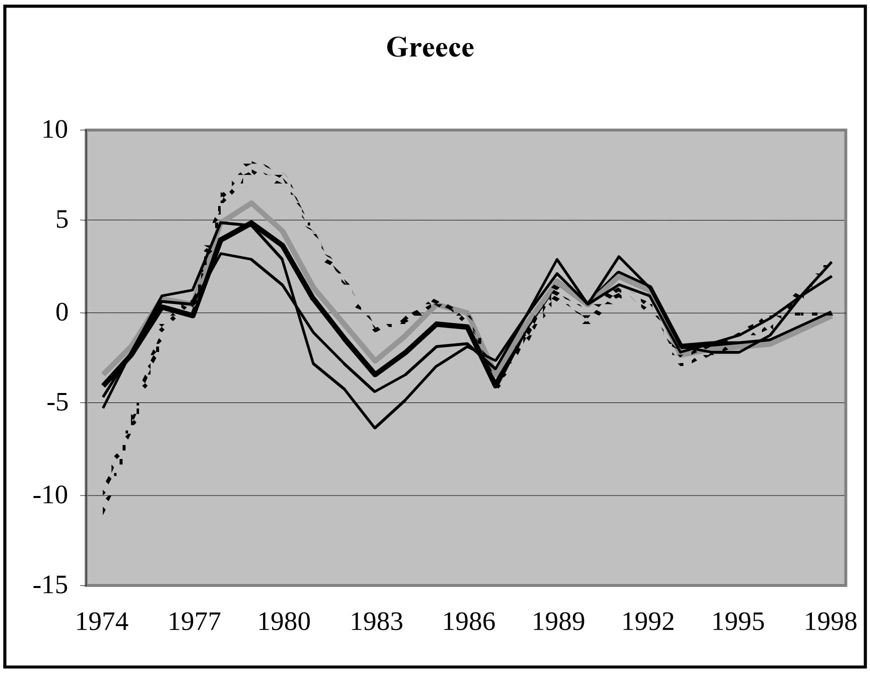
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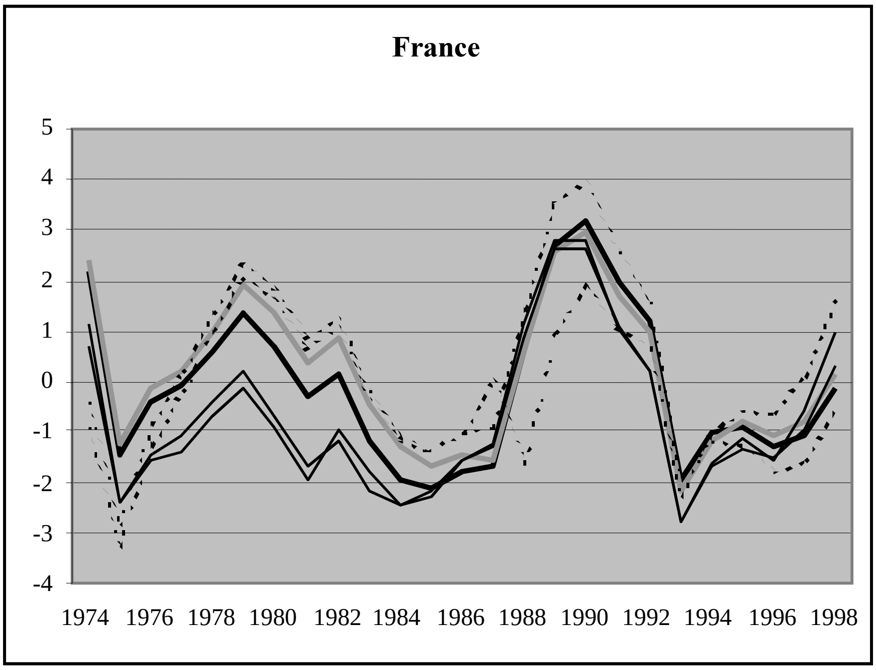
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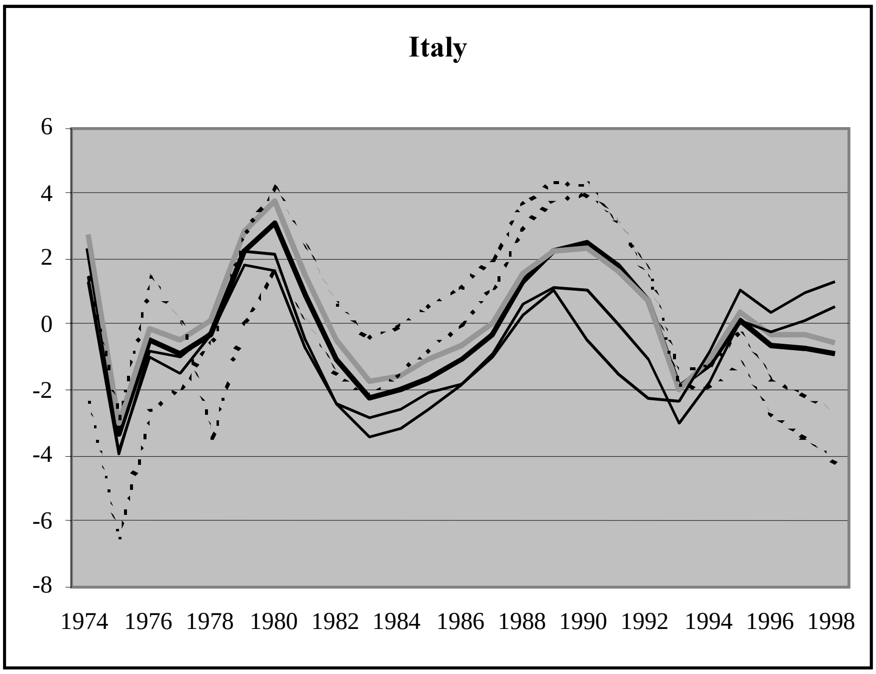
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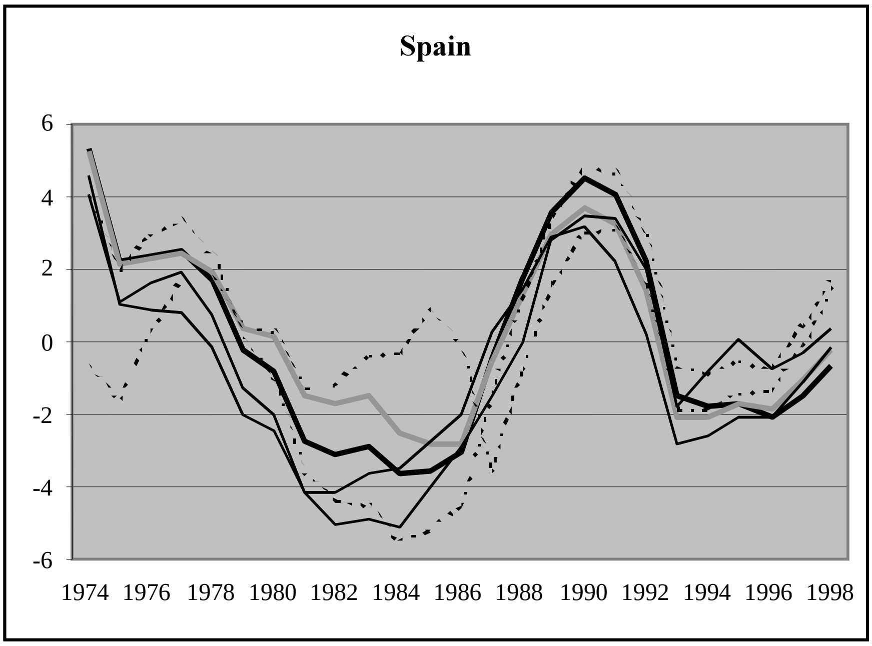
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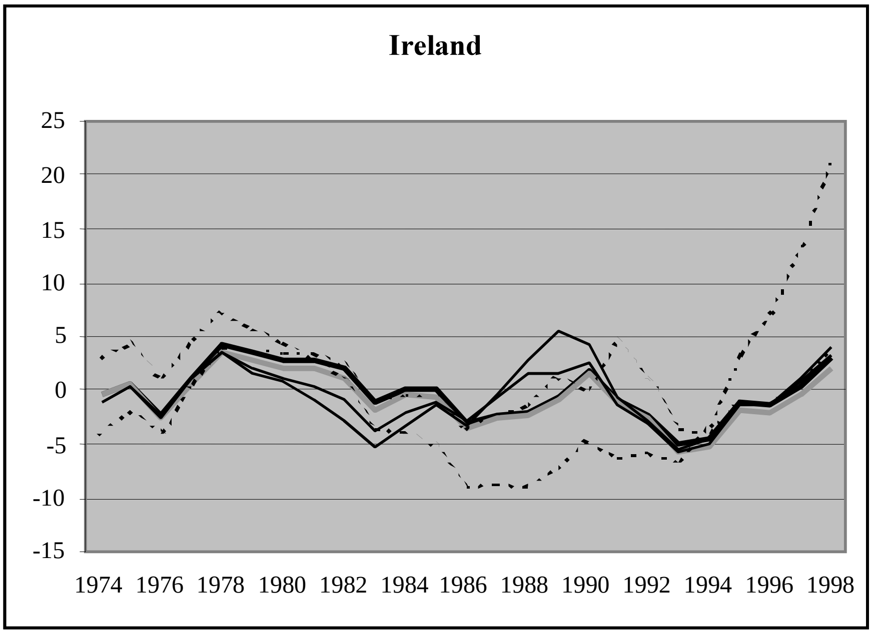
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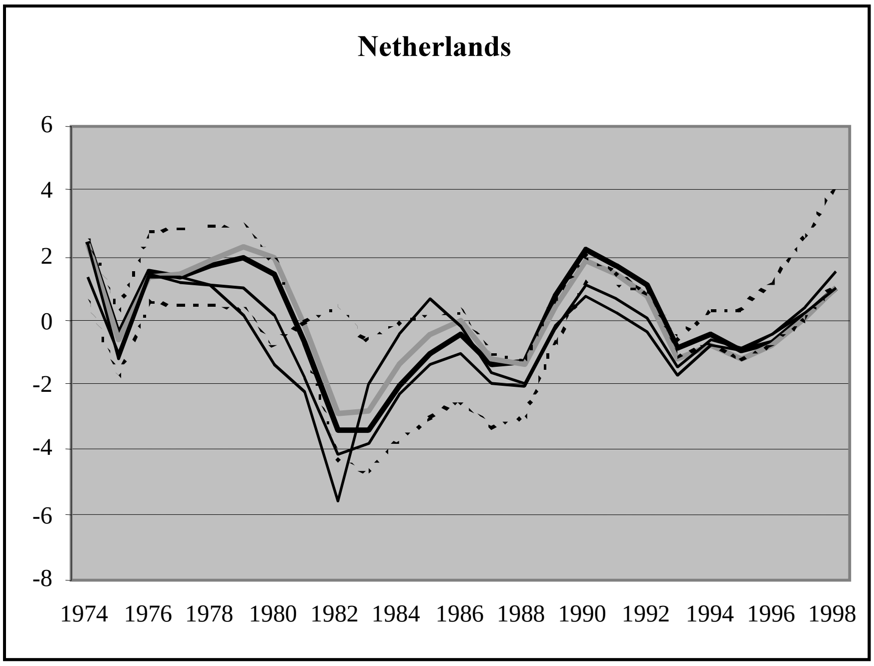
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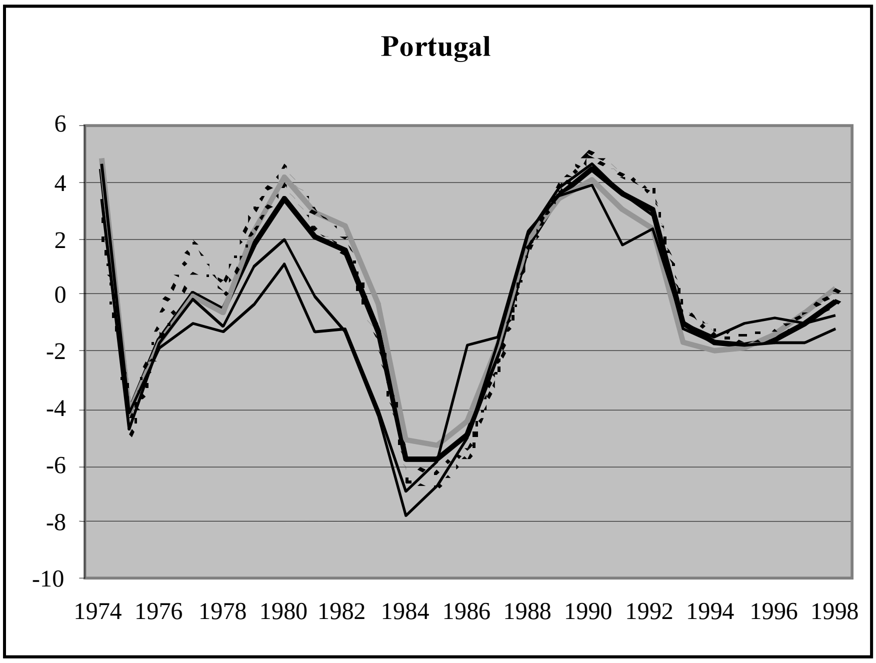
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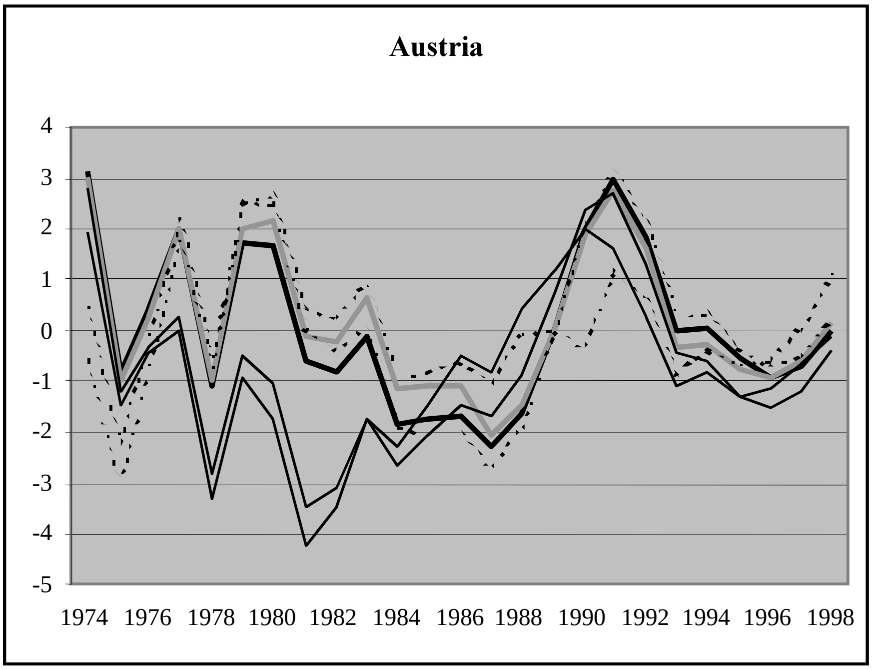
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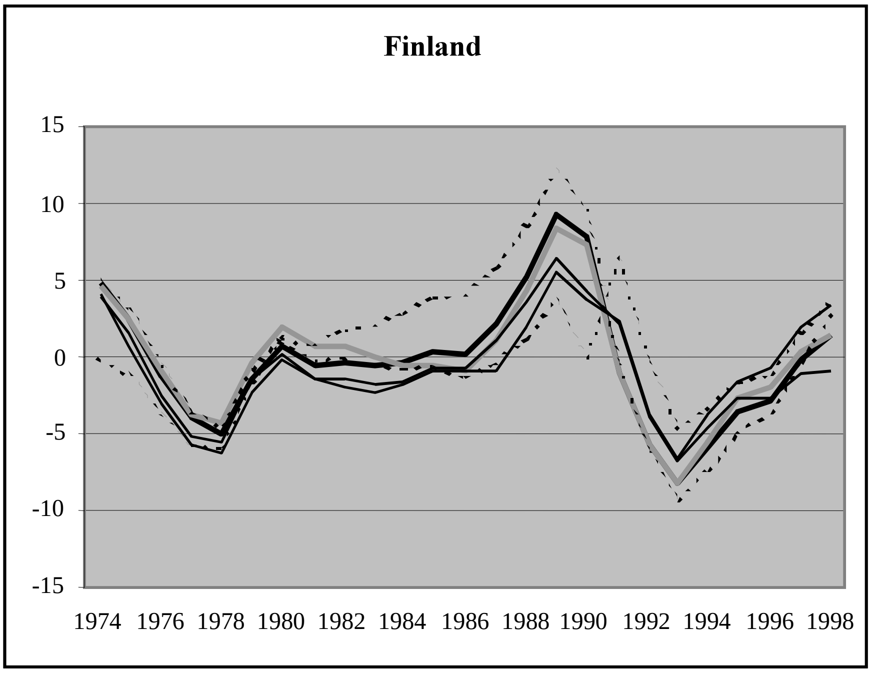
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