Comparative Performance of Multiple Reaction Function Equations Estimated by Canonical Methods

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This paper considers reaction function or control theory models and the appropriate estimation methods. Initially, we discuss a variety of issues that affect our theory and estimates. These include questions of uncertainty, appropriate assumptions, the mix of theory and empirical work, goals of analysis and criticism of past models, as well as responses to these questions. While we believe that regression analysis is one appropriate empirical method for these problems, in the second part of this paper we briefly introduce the canonical correlation approach to estimation. Finally, in the third section, we have specific estimates for reaction functions using canonical correlation methods for four European countries.

Reaction Function Estimation—General Discussion

In this section, we consider benefits from the use of reaction functions along with some criticism of the technique. Reaction functions actually are an implication of control theory methods. Through evaluation of specific assumptions and critiques of these methods, we can better understand them along with possible alternative techniques for their implementation.

In a prior paper [Resek, 1981] we laid out a mathematical view of reaction functions. Basically the process relies on specific assumptions about the world. First, we have a *structure* of the economy. This structure is combined with a *utility function* held by the policy authority. Combining the information from these sources, the authority adjusts policy tools to maximize his expected utility. Let us consider ways in which that process may be confused or may lead to unreliable estimates.

Varying Parameters and Assumptions about Uncertainty

First, consider the policymaker's utility. Structural econometric models are based largely on linear models—or models at least linear in parameters. These have been found to approximate reality reasonably well. Similarly, it is generally assumed that policymaker's utility is quadratic. This assump-

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tion is one that ultimately must stand the test of time. There may be variables that cross a threshold and lead to a change in utility; but even this may be well approximated by a quadratic function. Quadratic utility does lead to reasonable behavior in general and seems appropriate for many applications.

Utility functions also suffer from division of authority within countries. Our models assume that a single authority determines policy in a unified rational way. However, countries may have separate monetary and fiscal authorities. Furthermore, the policy problem may be sufficiently complex so separate groups of individuals operate on different policy instruments. This difficulty may be handled by carefully examining the timing and the sequence of decisions. The divided authority then is modeled as a sequential policy.

Moreover, the policymaker's utility changes with change in governments. Although sequential governments may have the same utility, the utility structure must be modeled to allow for these structural changes. This question is central to the role of political influences on economic policymaking and estimation must fully consider these policy changes by appropriate parameterization.

Second, consider the economic structure. Several problems arise here. The structure itself is likely to be stochastic. Most structural econometric models have imbedded stochastic elements but in contrast, many simple control models assume the structure is known. Clearly, any reasonable model must bring in stochastic elements, and control theory shows that these elements combined with quadratic utility serve to dampen or reduce optimal policy changes from period to period. The reaction models estimated must allow for these changes.

Additionally, the policymaker may believe a structure is true when it is in fact wrong; and the policymaker's view of true structure may change as governments change. In considering this issue, note that the government acts based on its perceived structure, but the ultimate effect of the actions is based on the true structure. This distinction between true and perceived structure may be handled by two elements. First, the difference impacts the utility function. That is, structural belief in the effectiveness of monetary policy is similar to adding utility to the use of that policy and disutility to other policies. Second, such diversion of true and perceived policy increases errors of structural equations. Since these two elements are already present in the model, no additional adjustments may be necessary.

All of these questions are related to the utility function and changes in utility. Hence models that avoid strong assumptions about known utility structure or nonvarying parameters may have an advantage in application over other models.

Theory versus Empiricism: The Art of Econometrics

The *art* of econometrics selects the theory that is known and parameterizes true unknown areas. It carefully uses the known or reliable structure and avoids unsustainable assumptions.

The dangers of this art come from two sides. First, overspecification of theory introduces those theoretic errors into estimation—since properties of estimation require that the underlying theory be correct. Or overprecise theories may make unsustainable assumptions which then enter into final estimates.

On the other side, too little theory leads to over-parameterization which in turn asks a great deal of the data—and the data are often from limited time series. The data may not be rich enough to estimate the full structure. Hence the final structure must specify as much as possible of what is known so that the data can sustain the overall estimation process. Clearly, we must adequately parameterize the economic structure without asking too much of the data.

The problem of data versus theory is particularly prevalent in reaction theory estimation. Consideration of every political change leads to a shift at every period and provides no data for any real estimation.

Forecasts versus Structural Estimation

Consider standard econometric model building and estimation. In this process we specify stochastic equations of the underlying economic structure. This method requires assumptions of known structural form and generally also a linear structure. The structure is assumed to be unchanging over time—or at least the changes in structure are themselves parameterized in constructing the model. To the degree the structure changes rapidly or structural changes are not embedded in the model, the model will make erroneous estimates and forecasts.

Based on the assumed structure and empirical evidence, estimates are made of the structural parameters. At least two specific goals are possible in this process. First, we may wish to learn the details of the economy. That is, we seek values of specific parameters or the direct and indirect implications of exogenous or endogenous changes. Second, we may wish to forecast the future through direct use of the model. Econometricians understand the differences of these two goals and models need to be evaluated differently based on the goals. Similarly, reaction models designed to estimate the structure should not be judged on their forecasting ability.

In general, questions of changing structure make the forecast process quite difficult, and most serious forecasting groups make broad use of judgment in adjusting forecasts in ways not fully evolving from the original model. Indeed many forecasters who prefer not to rely on econometric simulation for forecasts use structural estimates indirectly.

While the intermixing of econometrics, mathematics, and good judgment are broadly accepted in structural forecasting, we need to insure the same acceptance of the use of judgment with econometrics in reaction functions or control theory estimates.

The Lucas Critique

The so-called Lucas "critique" of Econometric Policy Evaluation also affects our view of policy models [Lucas, 1981]. Good use of reaction functions alters behavior and so alters observed future values. Forecasts themselves are not likely to be correct for they will not include forecasts of altered policy decisions. Therefore, reaction functions are most useful to estimate structural relations of the control model and to be combined with judgment in better understanding the future. At the present time they do not seem to be good techniques for forecasting policy behavior. It is also noteworthy that many alternative tools exist for policy use and it seems impossible to forecast which tool a future government will employ.

Lucas correctly notes that to evaluate policy we must have a *structure* that does not change with the exogenous variable, x. He suggests, of course, that as time passes and tax laws are altered, the means by which investors react in investment functions will vary. And as they learn what transitory income they will have in the future, the knowledge will affect their consumption. So Lucas really has two complaints: the structure of policy itself varies, and consumers' or investors' expectations change as they become aware of these policies, thus altering eventual actions.

However at the end of his work, Lucas clearly agrees that if you go beyond fixed parameters, you can employ changing temporal parameters. Moreover, "agents' responses become predictable . . . when they can become confident that agents and observers share a common view . . ." (p. 125). That is, the past may be studied but the future is hard to predict given different views that agents and observers hold.

We may ask questions such as these: On what historical insights can we base policy decisions? In the light of written records as to goals and methods of policy, does the empirical evidence support the stated policies? And how did this behavior change when the policy was changed? On the other hand, forecasts of the future do not seem to fit well in this analysis.

Furthermore, we look for consistencies across several policy periods with the idea of determining what similarities remain as policy and expectations change but also what differences have occurred in those times. Clearly, the Lucas critique of policy is critical for understanding analysis, but we interpret his work as supporting the present type of analysis.

The major method of reaction functions in this conference, [e.g., Hodgman, 1983]—keeps the issues enumerated above clearly in mind. It considers policy changes, and makes complete tests of alternative parameterization. This paper handles these questions with canonical correlation.

Instruments versus Targets

One simple item discussed in some papers on optimal control theory is the actual number of targets and instruments that exist in a given optimal control or reaction theory problem. The view taken in some control theory articles is the following. We know that policy targets are not achieved exactly each period. Unemployment remains, inflation is not reduced to a manageable level, and so forth. Some theoretical device must be found to insure that in the model targets are *not* met. For that reason it is often assumed that the policymaker seeks many targets. Moreover, he has a smaller number of instruments than targets to deal with. This relation between targets and instruments leads to optimal control that does not satisfy the targets exactly in each period. We emphasize that the assumption is made not because the users of the theory believe it, but because the need to get results that parallel reality requires it. If the number of targets equals the number of instruments or if there are fewer targets, then optimal control can always lead to targets being exactly satisfied every period. Hence, in the context of these models, the assumption that targets exceed instruments is the only way to reach seemingly realistic results.

One of the devices these theorists use is to include the instruments themselves as targets. For example, we may say that no instrument used as a target should be moved more than a small amount and therefore its goal is the past level. Since every instrument is a target, the total number of targets must exceed the total number of instruments.

In this paper we do not make this target-instrument assumption. Rather, uncertainty dominates our analysis or at least affects it considerably. That is, there is great uncertainty as to model structure, as to effect of instruments, and perhaps even as to what the targets are. This uncertainty was discussed above. No policymaker will ever achieve all his targets simply because he does not know the effects of his actions. This is a quite different perception of the world from that where he knows what will happen but does not have sufficient number of instruments to achieve the target.

We carry this view another step. Because of his uncertainty he may try to achieve only a small number of targets, for example, only inflation and unemployment. All other targets are subsumed into two components which really are these two elements. Yet in this situation he can control exchange rates, money supply, a certain number of interest rates, fiscal policy, different types of tax expenditures policies, and so forth. Therefore, he has a very substantial number of policy instruments which he may bring to bear on the problem while he is seeking to achieve only two or three policy targets. In this view of the world the number of instruments *vastly* exceeds the number of targets and therefore the standard control theory approach would yield an indeterminate solution. The theory simply does not tell us how the different instruments will be manipulated.

To this point we have discussed a number of problems of implementation of reaction models. We turn now to canonical correlation estimates as one solution to this question.

Reaction Functions — Theory and Estimation

Simple Reaction Model

Specifically we consider the specification of reaction functions in a system where there exist m endogenous variables, y; k exogenous variables, x; and p instruments, z.

Our simple reaction model arises from a known reduced form

(1) $$y = T_1x + R_1z + u_1$$

and a quadratic loss function

(2) $$L = (y - y_0)'P(y - y_0).$$

This discussion above led us to question the correct specification or constancy of P. Estimation must be implemented solely within single governments that have unchanging utility. Alternatively the model is to be estimated in a dynamic fashion with the structure altered for changes in P at changes in government. For this reason this effect requires a very rich database—or perhaps is difficult or impossible.

Instead consider optimizing the *effect* of the instrument R_1z. That is, the impact of the z_i on equation (1) is the first element of R_1z. Now the optimal values for R_1z arise from differentiating expected loss with respect to R_1z. Setting the result equal to zero, adding an error and multiplying by P^{-1} yields.

(3) $$R_1z^* = y_0 - T_1x + u_1$$

Consider equation (3) where there are more instruments than targets. For example let p = 2 and m = 1. Also let k = 1. Then we have

(4) $$r_{11}z_1^* + r_{12}z_2^* = y_{01} - tx + u_1$$

In this equation u_1 represents the error between optimal policy and the true policy level chosen by the policymaker and observed. This single equation (or m equations in general) must determine p instruments and p is greater than m. Clearly, the structural model is underspecified.

Our belief is that the policymaker chooses one of the z_i by some prior decision rule. This rule may be arbitrary, may be based on completely separate issues such as a decision to do something different from the past, may arise from a reaction model employing uncertainty, or may be based on policymaker judgment or bias. We represent this decision as

(5) $$r_{21}z^* + r_{22}z^* = u_2 \qquad \text{or}$$

(6) $$R_2z^* = u_2$$

where, in general, R_2 is (p-m) by p. This is a linear combination chosen with an error u_2. We believe the variance of u_2 to be very large representing the arbitrary nature of the choice and the large range of either value that can lead to acceptable policy.

Combining (3) and (6) we have an augmented equation

$$(7) \qquad Rz^* = y_{00} - Tx + u$$

The p equations now include the m true policy equations and the p-m supplementary equations representing the arbitrary choice to make the system determinate. For a determinate system, R is nonsingular. Hence

$$(8) \qquad z^* = R^{-1}(y_{00} - Tx) + v$$

where $v = R^{-1}u$

This equation is amenable to econometric reaction function estimation. However, consider the nature of the error vector v. We assume

$$Eu = 0$$

$$Euu' = S$$

where elements of S relating to policy equations (the first m) are "small" u. The remaining (p-m) elements are from arbitrary equations and are large.

For consideration of estimation we need the mean and variance of v. Clearly

$$E\,v = 0$$

$$Evv' = S_v = R^{-1}SR'^{-1}$$

To see the effects, consider an example.

$$\text{Let } R_{11} = R_{12} = R_{22} = 1;\; R_{21} = 0$$

$$S_{u11} = a \text{ (small)} \qquad S_{u12} = 0$$

$$S_{u22} = b \text{ (big)}$$

Then it is easily shown that

$$S_{v11} = a + b \qquad S_{v22} = b$$

That is, *both* equations will have large variance even if the structural equation has a small variance.

Estimation of the system (8) will bring forth very large equation errors. Hence we anticipate that reaction function estimation may have some difficulty.

Canonical Correlation

Instead, return to (7). Rather than solving for z^* as in (8), we multiply by a nonsingular matrix Q.

$$(9) \qquad QRz^* = Qy_{00} - QTx + Qu$$

One estimation method is reduced form as in (8). Instead we may use canonical correlation to estimate (9). It operates by choosing a linear combination of the rows of (7) that maximizes the correlation between the two sides of the equation. Then a second component is chosen that is orthogonal to the first and maximizes correlation remaining. This canonical correlation chooses a specific A which implies a Q that makes u as small as possible in the transformed model. In turn this corresponds well to the theoretical structure we have outlined. In our example, canonical correlation would estimate the true structural equation. The second component would find the arbitrary equation (6).

Reaction function estimation must deal with questions raised to this point in the paper. Varying parameters, uncertainty, and limited data all create difficulties discussed in the first part of this paper. Canonical correlation provides the promise of interesting estimates that avoid some of these problems.

Canonical Correlation Estimates

Presentation of Results

The discussion of results of canonical correlation brings to focus the essential differences between canonical correlation analysis and regression analysis. We are accustomed to making a series of specific tests in this regression context. First we test individual coefficients of explanatory variables to see if they are different from zero. This test has no counterpart in canonical correlation. Since our estimate is of the maximum correlation, it may not affect the actual structural equation but instead a linear combination of two or more such equations. Clearly a test on an individual coefficient is heavily dependent on the implied linear transformation and is not sensible statistically.

Instead, we rely on tests as to whether the equation is significant or not significant and examination of the relationships to see if they are sensible. Moreover, in any given canonical analysis, the number of significant equations relative to the number of variables is extremely interesting. If all variables generate significant equations, then the variables are truly operating independently of one another. However, if there are fewer equations than variables, then the interrelation among both dependent and independent variable sets is of interest.

The reported coefficients in canonical correlation differ from those in regression. In regression analysis, coefficients have a direct interpretation as the unit change in y relative to the unit change in z. In canonical correlations, the variables are normalized. First, all variables—both y and z—are adjusted to a constant variance equal to one. Second, since we have a linear combination of y variables as well as z, there remains a scaling factor. In this analysis, the coefficients are scaled so that the linear combinations of y and z each have a variance of one. Thus a coefficient .5 means the variable accounts for half the variance of the result. Since the y and z are correlated, the coefficients will not sum to one or any other particular value.

Empirical Analysis

To this point we have discussed the rationale behind the use of canonical analysis. Specific models developed should explain the relation between multiple instruments and multiple policy targets. We consider first alternative approaches to instruments and then discuss the targets and the issues that are raised concerning choice of targets. Consider the policy targets. The principal goals are unemployment and inflation. Other similar variables are the utilization rate, growth of GNP or growth of industrial production. A final goal is a reasonable balance of payment stability. This last goal can be represented by the balance of payments itself or by an underlying target such as the interest rate on Eurodollars. These target variables will be employed with each alternate set of instruments that we consider. We have avoided the inclination to fine tune our results by altering those target variables. Instead, we allow all to enter the equations and present the results that occur.

Next turn to possible sets of instrument variables. First, consider a broad view of policy. In this perception, total government policy includes fiscal policy, monetary policy, and exchange rate policy. Therefore, our instruments should include one from each of these areas. For example the federal deficit, an interest rate, and an exchange rate or exchange control variable would encompass one set of three that may broadly describe overall government policy. For each country we shall attempt to develop such a set of variables.

Secondly, one may turn to monetary policy. There is not just one monetary instrument but instead a whole family of possible interest rates or variables of monetary policy or credit restriction. This set of variables will be employed to measure differential monetary effects. Obviously, the method of implementation will vary substantially from country to country.

One problem in this estimation process is that time lags for these policies may differ in a major way. Hence future analysis may benefit from consideration of alternative lag structures. Moreover, some country specific institutions may affect our results.

A third question in setting instruments concerns the timing of policy. Since the monetary authority perceives the policy need and then acts, the exact lag structure from target to instrument may vary. An interesting benefit of canonical correlation is that the same variable may be entered with different lags allowing for possible determinates of the level of policy, timing of policy, and even differential causes for different timing. The purpose of the analyses is to determine which month in the quarter is most important in policy and to determine whether it is the level of the variable or its change that is important. The former is represented by all months having the same sign while the latter brings a sign reversal over the three months. This too will be a subject of scrutiny. Note that since we are not fine tuning any results we may have some potential questions of direction of causation arise in all our efforts. Now we turn to the results for each of the countries. Initially we look at Germany.

Analytic Results—Germany

We present results in Table 1 for several canonical analyses of Germany. An example of interpretation is at the bottom of the table. These results are of interest and generally make good sense. For explanatory or independent variables in all our analyses we use a general group of policy target type variables.

First, we look at three broad types of policy—monetary, fiscal, and foreign exchange. In German CCI, we find the first correlation has an R^2 of .77. The critical y variable is the interest rate, RLQ, which has a .95 coefficient. Its greatest relation is, as expected, with the inflation rate, DCPI, and unemployment rate, UNR. The second correlation has an R^2 of .63 and y represents fiscal policy, the government deficit. Its highest relation is negative with new orders, showing y and z components must be orthogonal to the first canonical correlation, causing the possible reordering of priorities. Finally, the third canonical correlation has an R^2 of only .32 but it is significantly different from zero. This represents the forward exchange premium; it has a high relation with new orders, unemployment, and also with the balance of payments. Interestingly, the current account balance plays a very minor role through the entire German analysis.

It is particularly noteworthy that canonical analysis clearly divides the roles of monetary, fiscal, and exchange policy all of which are significant. There is little multiple instrument selection in this analysis. The last canonical correlation is not close to zero even though the explanatory portions must be orthogonal to the prior variable sets. Moreover, this analysis shows policy targeted for specific results. Monetary policy is targeted at inflation and unemployment; fiscal policy at industrial activity; and exchange policy at the balance of payments.

Next turn to German CCII, the analysis of monetary variables in Germany. In this, we employ five indicators of monetary policy to determine the effects of fine tuning with different monetary tools. This mixture of variables includes the money supply plus four different interest rates. Initially note the values of R^2 and the significance of the correlations. The first three R^2 are .96, .87, and.63 but they are followed by .11 and .05. This indicates that the monetary variables display three orthogonal components—but not more than three.

The first component clearly represents money supply—with a coefficient of 1.00. This has the highest relation to new orders. It is interesting that the inflation rate is not important for this component. The second component represents the loan rate. For it the largest explanatory variable

Table 1
Germany Canonical Correlation

	CCI 1	CCI 2	CCI 3	CCII 1	CCII 2	CCII 3	CCIII 1	CCIII 2	CCIV 1
Set 1									
FWD	.04	.20	.98						
DEFICIT	.15	1.02	−.11						
RLQ	.95	−.40	−.01						
M				1.00	−.16	−.14			
RDISC				.36	−.17	−1.57			
RCALL				−.27	−.64	2.47			
PABY				.20	.64	1.22			
RLQ				.06	1.15	−2.26			
RL1							.25.	3.20	
RL2							−1.11	.70	
RL3							1.85	−3.69	
RD1									.13
RD2									.13
RD3									.77
Set 2									
RED	.49	−.23	.30	.29	.38	−.56	.47	.28	.45
DCPI	.72	−.02	.53	.06	.98	.67	.61	.43	.66
DGNP	.17	.30	.37	−.16	.18	−.03	.12	−.40	.02
BOP	−.02	.06	−.70	.07	−.02	−.21	−.07	.26	−.07
UNR	−.49	−.39	.60	.40	−.31	.58	−.53	.21	−.47
NUORD	−.21	−.58	−1.05	.58	−.50	−.45	.04	−.69	.07
CTACC	.13	−.16	−.22	−.16	.20	.01	.09	.41	.11
R^2	.77	.63	.32	.96	.87	.63	.79	.36	.71
Probability Value									
Correlation									
1		.000			.000		.000		.000
2		.000			.000		.017		.495
3		.003			.000		.624		.690
4					.414				
5					.441				

Observations: 1967-1 through 1980-1 (n = 53).

Variable Names

FWD Forward Exchange Rate/Current Exchange Rate. Dm/$
DEFICIT Government Deficit (−) or Surplus
RLQ Three-Month Loan Rate
M Money Supply
RDISC Discount Rate
RCALL Call Money Rate
PABY Public Authority Bond Yield
RL1/2/3 Three-Month Loan Rate for Specific Month in Quarter
RD1/2/3 Day-to-Day Money Market Rate for Specific Month in Quarter
RED Eurodollar Rate
DCPI One-Year Rate of Change—CPI
DGNP One-Year Rate of Change—GNP
BOP Balance of Payments
UNR Unemployment Rate
NUORD Indes of New Manufacturing Orders
CTACC Current Account

EXAMPLE: Interpretation—CCI, Correlation 1.

$$.04\ FWD + .15\ DEFICIT + .95\ RLQ = R^2\ [.49\ RED + .72\ DCPI + \ldots + .13\ CTACC] + u$$

The multiple correlation between the two variable sets is .77.

EXAMPLE: Probability value for CCIII.
1. H_0 All three correlations are zero.
2. H_0 The last two correlations are zero.
3. H_0 The last correlation is zero.

The first hypothesis is rejected at 0% level. The second hypothesis is rejected at 1.7% level. The third hypothesis is accepted as it would take a 62.4% level for rejection.

is the inflation rate. Yet the role of unemployment is negative probably due to the prior canonical correlation. This relation parallels the results of the first component in CCI where RLQ is related to inflation.

By the time we come to the third component, the interpretation is more difficult due to enforced orthogonality. I feel it most clearly measures the spread between the call rate and the loan rate and is related to unemployment, Eurodollar rate, and inflation. The results for these monetary variables are similar to those above in German CCI in that there are three components, but the similarity ends there. Those results had dependent variables that followed the pattern of the data with a single variable dominating each component. In contrast, the components here are clear combinations of several variables. Second, we note there were five monetary variables and potentially five components but only three were significant. Thus canonical correlation clearly reduced the dimensionality of this question.

Third, consider German CCIII, which analyzes the German loan rate. This considers the three values for three months of the quarter. The critical factor here is that only the first canonical correlation is significant at the 1 percent level—the R^2 of the second variable is .36. We see the coefficients representing the three months in the quarter for the loan rates alternate in sign and become smaller absolutely as time passes. If we ignore the first month in the quarter, with a very small coefficient, the pattern is the same as .7 times the most recent value plus 1.1 times the monthly change in the loan rate. Clearly both the loan rate and change in loan rate play a critical role in this first component. The second component, although of marginal significance, represents the two-month change in the rate.

It is interesting to contrast the loan rate of German III with the day-to-day rate of German CCIV. Here the y coefficients in succession do not alternate for the three months but have the same sign. They are complementary but the most recent month is most important. The month-to-month difference is represented by the second component but this is not significant at all. As expected, the primary explanatory variables are the inflation rate and unemployment rate.

Analytic Results—Italy

The types of analysis run for Italy closely parallel those employed for Germany. As we consider these results, we contrast them with those found above.

Italy CCI considers the relation of fiscal, monetary, and foreign exchange policy. The first correlation is .73 and represents the monetary base. This is similar to the result for Germany. Moreover, the inflation rate has the largest effect here. The second component has an R^2 of only .38. It represents a combination of all three policy variables. The new elements affecting y are government deficit and the forward premium on the lira. These factors are related negatively to poor global balance of payments and

Table 2
Italy Canonical Correlation

	CCI		CCII			CCIII
	1	2	1	2	3	1
Set 1						
DEFICIT	.12	1.20				
PRXR	−.08	−1.07				
MB3	1.16	1.74				
RD			.15	2.76	−.16	
RGB			.89	−2.98	1.83	
BMPOL			.04	.08	.75	
RTB			.13	−.04	.27	
MB3			−.07	.31	−2.07	
MB1						1.14
MB2						.62
MB3						−.77
Set 2						
DCPI	.92	−.20	.91	.20	−.02	.91
DGNP	.06	−.69	−.05	.83	−.33	.07
RED	.07	.04	.15	−.01	.23	.08
UNQ	.36	.09	.24	−.08	−.62	.38
UTR	.17	.81	.18	−.06	−.14	.16
CAB	.24	−.04	.16	−.63	−.54	.29
GB	−.01	−.77	.06	.17	.04	.00
R^2	.73	.38	.83	.68	.33	.73
Probability Value						
Correlation						
1	.000		.000			.000
2	.002		.000			.230
3	.262		.026			.472
4			.581			
5			.860			

Observations: 1966-1 through 1980-3 (n = 59).
Variable Names

DEFICIT	Government Deficit (−)
PRXR	Forward Premium on Lire
RD	Discount Rate
RGB	Government Bond Yield
BMPOL	Controlled Changes in Monetary Base
RTB	Treasury Bill Rate
MB1/2/3	Monetary Base—Specific Month in Quarter
DCPI	One-Year Change—CPI
DGDP	One-Year Change—GDP-Deflated
RED	Rate on Eurodollar
UNQ	Unemployment Rate
UTR	Industrial Capacity Utilization Rate
CAB	Current Account Balance
GB	Global Balance of Payments

high utilization rate. In contrast with Germany CCI, the third component here is not significant.

In Italy CCII we relate several monetary variables to the policy targets. The first correlation, with an R^2 of .83, represents the rate on government bonds. This variable has a very high relation to the inflation rate. The next variable signifies the differences between the discount rate, RD, and the rate on government bonds. The high discount rate is associated with growth of real GNP and with a negative balance on the current account. This relation has an R^2 of .68.

The third correlation is of marginal significance and has an R^2 of .33. The crucial variable is the monetary base and the most interesting element is the failure of this variable to be significant in the earlier correlations. The highest relation shows an increase of the monetary base with high unemployment.

Since we entered five monetary variables, five correlations are possible but the last two are of no statistical significance.

Finally, consider Italy CCIII. This examines the importance of the timing of the total monetary base. Only the first component is significant and the three coefficients sum to about 1.00. Yet the result is like a negative first difference in that the most recent month is minus and the earlier months positive. It indicates that inflation is associated with a higher level of money supply but also that the level of money supply tends to be decreasing. No other variable plays a really major role here.

Analytic Results—United Kingdom

For the United Kingdom, our data base is directed at monetary variables so we have a smaller set of canonical results. First, we related foreign exchange and monetary policy to targets. This relation omitted a fiscal policy variable and the omission should be corrected in the future. With only two policy tools, there are a maximum of two correlations. The first represents mainly the interbank rate and shows it is highly related to the Eurodollar rate. The interesting element of this relation is the small role of inflation and of unemployment. The second correlation represents the foreign exchange variable, the forward premium in the dollar relative to the pound. This variable has the highest relation to inflation, but the correlation is small with an R^2 of .20. It is significant at the 5 percent level but not at 1 percent.

In the United Kingdom CCII we relate a set of monetary variables to the targets. We computed estimates employing three interest rates and three measures of the monetary base. However, colinearity among monetary variables led to results emphasizing differences of monetary measurement. In the estimate presented we include three interest rates and the monetary base. The first correlation chooses the monetary base as the significant monetary variable. Its greatest relation is with unemployment. The second monetary variable selects the interbank rate, or more precisely combines the interbank rate, RIB, and the Bank of England minimum lending rate, RB. The former is positive and the latter is negative and smaller absolutely. The implication is that the critical factor is the degree that RIB exceeds RB. This difference is related to the Eurodollar rate and negatively to unemployment. Recall again that the explanatory factors are

Table 3
United Kingdom Canonical Correlation

	CCI		CCII		
	1	2	1	2	3
Set 1					
FPR	−.59	.98			
RIB	1.13	.05			
RB			−.36	−2.81	8.29
RIB			.18	3.44	−4.99
RCM			.20	.31	−2.15
MB			1.00	−.46	−1.26
Set 2					
DCPI	.05	1.06	.04	.32	.69
UNQ	.10	−.38	.72	−.77	−.89
PSBRQ	.11	.34	.18	−.14	.82
REDQ	.87	−.46	.27	.99	−.40
DGDP	−.04	.17	.02	−.00	−.04
R^2	.89	.20	.93	.52	.27
Probability Value					
Correlation					
1	.000			.000	
2	.011			.000	
3				.005	
4				.862	

Observations: 1965-1 through 1980-4 (n = 64).
Variable Names

FPR	Forward Premium Dollar
RIB	Interbank Rate (3 months)
RB	Minimum Lending Rate
RCM	Call Money Rate
MB	Monetary Base
DCPI	One-Year Change—CPI
UNQ	Number Unemployed
PSBRQ	Public Sector Borrowing Requirement
REDQ	Eurodollar Rate
DGDP	One-Year Change Gross Domestic Product Deflated

required to be orthogonal to the prior correlation set.

Finally, a third component is significant although the R^2 is only .27. It emphasizes the bank rate and its relation to the public sector borrowing requirement, PSBRQ, and inflation.

Analytic Results—France

For France, we consider France CCI, which analyzes the role of the three types of policy. The first component is dominated by monetary policy in the form of interbank lending rate, IBR. Its greatest relation is to the Eurodollar rate. Additionally, it is affected by inflation rate. The second component represents the forward premium on the franc. This is related with an R^2 of .57 and is most affected by inflation. The third component represents fiscal policy—the deficit—but is not statistically significant. The

Table 4
France Canonical Correlation

	CCI 1	CCI 2	CCII 1	CCII 2
Set 1				
FPR	.58	.91		
DEFICIT	−.03	−.21		
IBR	−1.12	.12		
MB			.97	.44
IBR			.03	1.12
M1BRQ			.03	−2.17
Set 2				
LABCONF	.04	.11	.01	.02
DIPI	.03	.31	.07	−.18
UN	.05	−.48	.90	.88
BOP	−.05	−.37	.02	.15
DCPI	−.37	1.38	.11	−1.02
RED	−.79	−.77	.09	−.42
R^2	.78	.57	.98	.71
Probability Value				
Correlation				
1	.000		.000	
2	.000		.000	
3	.374		.546	

Observations: 1966–1 through 1979–4 (n = 64).
Variable Names

FPR	Forward Exchange Rate/Spot Rate (FR/$)
DEFICIT	Government Deficit (−)
MB	Money Supply — M3
IBR	Overnight Interbank Rate
M1BRQ	One-Month Rate
LABCONF	Labor Conflicts — Days Lost
DIPI	One-Year Change — Industrial Production Index
UN	Unemployed (Number)
BOP	Balance of Payments
DCPI	One-Year Change — CPI
RED	Eurodollar Rate

most important result then is the failure of fiscal policy in the form of the deficit to play any role.

In France CCII, we consider three monetary variables, the money supply and two interest rates. Despite the key relation found above of the interbank rate with the Eurodollar rate, these variables do not seem to belong in any part of this analysis. The most important monetary variable is the money supply. And it is highly related to the unemployment rate. The second component takes the monthly interbank rate directly and as an increment over the quarterly rate. It is highly related to inflation. These two components exhaust the significant relations found for France.

Cross Country Comparisons

It is interesting to compare the various types of results across the four countries. Contrasting the policy types leads to very different results for the different countries. In Germany, these policies are quite separated by the canonical correlations, but in contrast, the United Kingdom had only two policies and both provided significant correlations, although the second value was low. In both Italy and France, fiscal policy and exchange rate policy were intermixed and provided a single correlate of moderate magnitude. In each country, monetary policy dominated the first correlate.

The second major type of analysis related several monetary variables. In the United Kingdom, France, and Germany, money supply was of greater importance than interest rates, but in Italy the loan rate dominated the results. In France, United Kingdom, and Italy the difference between interest rates and not the rates themselves dominated the second correlate. In contrast, Germany showed more importance for single variables and therefore clearly differentiated policy variables than in the other countries.

These results may be affected by errors in timing, or may be estimating structural equations rather than policy. Nevertheless, these estimates of country relations and contrasts between countries provide an interesting and useful application of canonical correlation methods.

References

Hodgman, Donald R. and Robert W. Resek. "Determinants of Monetary Policy in France, the Federal Republic of Germany, Italy and the United Kingdom: A Comparative Analysis." This volume.

Lucas, Robert E. "Econometric Policy Evaluation: A Critique" in *Studies in Business Cycle Theory*, pp. 104–130, Cambridge; M.I.T. Press, 1981.

Resek, Robert W. "Theoretical and Empirical Aspects of Reaction Functions," Monetary Policy Conference, UIUC, November 1981.

Discussion

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This is a difficult paper to discuss. The difficulty does not concern so much the development of the technical part of the paper; rather, it concerns a proper understanding of the author's objectives and of his interpretation of the actual results. In my comments I will try to follow the structure of Professor Resek's paper, discussing first a number of technical points and then his estimates of correlations between sets of "policy instruments" and sets of "policy targets" for a number of European countries.

Reaction Functions

In the first section of his paper Professor Resek makes a number of penetrating and interesting comments on the traditional approach to the estimation of policy rules, the "reaction" functions of policymakers. One can only agree with his discussion of the many problems connected with the view of reaction functions "as an implication of control theory." Among these problems, that of parameter variability (not only of structural models, to which the well-known criticism by Professor Lucas is related) appears to be the most important. Indeed, it is certainly true that "policy-maker's utility changes with change in governments." Even if it might be possible to find something like an "encompassing" policy rule with variable coefficients related to changes in the preferences of policymakers, there are certainly great problems in estimating such a rule from the available observations.

Professor Resek also emphasizes that the assumption that targets exceed instruments is made in optimal control theory "not because it is believed by the users of the theory, but because it is required to get reasonable results," that is, to avoid exactly achieving targets every period. Instead, the author appears to suggest that in reality there are many instruments at the disposal of the policymaker and only a few targets he is really interested in achieving. The policymaker ends up using all these instruments, without, however, being able to reach all his targets, mainly because the world in which he operates is greatly affected by uncertainty. This is an interesting proposition. It is not, however, totally new, and one could recall the works of Brainard (1967), Henderson and Turnovsky (1972) and Johansen (1973) as the pioneering attempts at a proper understanding of this issue. My impression is, however, that in reality not many *independent* instruments are under the *direct control* of the policymaker and that there are also limits

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to the ranges within which they can be used (which is incidentally an important reason for their introduction in the loss functions and the consequent joint treatment as instruments and targets at the same time).

Instead of relying on optimal control theory, and in order to avoid the assumption of an unchanging utility, Professor Resek chooses to start directly from a known reduced form:

$$y = T_1x + R_1z + u_1 \tag{1}$$

where y is an $m \times 1$ vector of endogenous variables (the objectives of the policymaker), z is a $p \times 1$ vector of instruments, x is a $k \times 1$ vector of noncontrollable exogenous variables (for instance, time, weather, world demand, raw material prices for nonproducing countries, etc.). T_1 and R_1 are proper conformable matrices of coefficients and u_1 is a vector of residuals. If the policymaker wants to achieve a given set of target values, y_0, one might consider the problem of finding a proper set of instruments, z^*, so that

$$R_1z^* = y_0 - T_1x + u_1 \tag{2}$$

where, this time, "u_1 represents the error between optimal policy and the true policy level chosen by the policymaker and observed." Clearly, only when $p = m$ could (2) be solved directly for z^* and the "reaction function" be directly estimated, within a framework close to that of Tinbergen of one instrument for each target. Professor Resek considers instead, at this stage, the situation $p > m$, so that (2) is overdetermined. He suggests, however, that policymakers choose $p - m$ instruments by means of *ad hoc* rules of the form

$$R_2z^* = u_2 \tag{3}$$

where R_2 is a $(p - m) \times p$ matrix. But this is in my opinion a very strange way of making policy and it is by no means clear how (3) could be a consequence of uncertainty or similar factors. Anyway, it is clear that the combination of (2) and (3) is capable of producing a determinate system such as

$$Rz^* = y_{00} - Tx + u \tag{4}$$

where, obviously, $R = (R_1', R_2')'$, $y_{00} = (y_0', 0')'$, $T = (T_1', 0')'$, and $u = (u_1', u_2')'$.

(4) could be solved for z^*, premultiplying both sides by R^{-1}, and one could estimate the derived reaction function by standard regression techniques. Professor Resek, however, shows that since the variance of u_2 is likely to be extremely large (and this seems to be quite obvious given the oddness of (3)!), the variance of the errors of the reaction functions will also be very large so that it will be quite difficult to obtain a good fit from estimates of such functions. But such is nature! Therefore, instead of solving (4) for z^*, the proposal is to apply canonical correlation to estimate an "innocuous"

linear transformation of (4), that is:

(5) $$QRz^* = Q(y_{00} - Tx) + Qu$$

where Q is a nonsingular matrix. It is obvious that (5) gives exactly the same solution as (4). Considering a sample of n observations, so that Z^*, Y_{00} and U would be matrices of dimensions $n \times p$, $n \times m$ and $n \times p$, respectively, and assuming for simplicity that there were no other exogenous variables, x, (5) could be rewritten as

(6) $$Z^*R'\,Q' = Y_{00}Q' + UQ'$$

or

(7) $$Z^* = Y_{00}P + V$$

where $P = R'^{-1}$ and $V = UR'^{-1}$, so that the ordinary least squares estimate of P would be

(8) $$\hat{P} = (Y'_{00}\,Y_{00})^{-1}\,Y'_{00}\,Z^*.$$

Professor Resek's proposal amounts, instead, to computing the canonical variables of (5), (or (6)). The method consists in standardizing the original variables z^* and y_{00}, obtaining, say, $\bar{z}^*$ and $\bar{y}_{00}$ and finding the estimates of the column vectors a_i and b_i in the two canonical variables.

(9) $$t_i = a_i'\bar{z}^* \quad , \quad s_i = b_i'\bar{y}_{00}$$

which maximize the correlation between t_i and s_i. A property of this technique is that one can compute pairs of canonical variables up to the order $j = \min(p,m)$ and that they will always be orthogonal with previous canonical variables (see Dhrymes (1971) for further details). To each pair will correspond a canonical correlation coefficient, r_i, which will also be, as shown by Vinod (1968) (but see also Chetty (1969), Dhrymes and Mitchell (1969) and Vinod (1969)), the estimate of the regression coefficient in

(10) $$t_i = \rho_i\,s_i + e_i.$$

But, then, an immediate relation between this technique and the previous ordinary least squares regression estimates can be made, provided that *all* pairs of canonical variables are computed. Consider, for simplicity, the case of $p = m$ (but the analysis could be easily extended, even if for $p > m$ one should work with generalized inverses). Following Vinod (1968), one can then rewrite (10) as

(11) $$t = \rho s + e \quad , \quad t = A\bar{z}^*, \quad s = B\bar{y}_{00}$$

where $t = (t_1, \ldots, t_p)'$, $\rho = \text{diag}(\rho_1, \ldots, \rho_p)$, $s = (s_1, \ldots, s_p)'$, $e = (e_1, \ldots, e_p)'$, and A and B are matrices with i-th rows equal to a_i' and b_i', respectively, for $i = 1, \ldots, p$. But then

(12) $$\bar{z}^* = A^{-1}\rho B\bar{y}_{00} + A^{-1}e$$

or

$$\bar{Z}^* = \bar{Y}_{00}\,\bar{P} + \bar{V} \tag{13}$$

where $\bar{P} = B'\,\rho A'^{-1}$ and $\bar{V} = EA'^{-1}$. But it is immediately clear that (13) is a linear system of equations involving exactly the same variables as (7), except that they are now *standardized*. Furthermore, if one used the canonical correlation estimates r_i and, say, $\hat{a}_i$, b_i to estimate $\bar{P}$, obtaining $\hat{\bar{P}}$, as Chetty (1969) and Dhrymes and Mitchell (1969) have shown, it will also hold that

$$\hat{\bar{P}} = (\bar{Y}'_{00}\,\bar{Y}_{00})^{-1}\,\bar{Y}'_{00}\,\bar{Z}^* \tag{14}$$

which is identical to (8) except for the standardization. But, then, the problem associated with estimates such as (8), namely that there will be large equation errors so that "we anticipate that reaction function estimates may have some difficulty," does not seem to me to be resolved by means of a different estimation technique such as canonical correlation, as suggested by the strong relationship between (14) and (8). Indeed, this problem is really a fact of life, associated with the original specification of the *ad hoc* policy rules (3), and, for that matter, with possibly large errors in the set of equations (2).

It is true, however, that not all pairs of canonical variables need to be calculated. Indeed, Professor Resek relies on a test of significance of the estimated canonical correlation coefficients, discarding those components associated with correlations not significantly different from zero. In this case the strict relationship between ordinary least squares and canonical correlation estimates does not hold anymore, since (13) can no longer be derived from the pairs of canonical variables. I suspect, however, that this does not alleviate significantly the problem due to the large errors in the original specification of the policy rules. Furthermore, in the case considered by Professor Resek in the technical part of his paper, with more instruments than targets, one should expect a priori that for $p - m$ pairs of canonical variables the correlation coefficients should be equal to zero, due to the specification of (3). I think that a proper interpretation of Professor Resek's formalization is not that there are more instruments than targets, which could be justified on the grounds of structural uncertainty of a kind different from that considered in that formalization, but rather that the instruments are not independent among themselves. However, one can consider different instruments which are linear combinations of the original ones, and whose number is equal to that of the targets, given the framework considered by Professor Resek (system (2), with $p > m$), which is very similar to that first examined by Tinbergen (1952).

Indeed, before passing to consider the empirical results, it might be useful to recall that Professor Resek's structural model does not contain "uncertain" structural coefficients. Furthermore, these coefficients are constant and independent of changes in policy, so that Lucas' (1976) critique necessarily holds also in the present case and a claim that his work is "supporting the present type of analysis" cannot be accepted. Even if one agrees with Professor Resek's observation that "Varying parameters, uncertainty, and limited data all create difficulties," it is difficult to support the further observation that "canonical correlation provides the promise of interesting estimates that avoid some of these problems."

The Canonical Correlation Estimates

As one can see from the set of equations (5), one would expect to find, in applying canonical correlation analysis, on the left hand side the values of policy instruments and on the right hand side the values of policy targets *and* of *important* exogenous variables. To start with, instead, the latter disappear in the empirical analysis and, what I think is even more important, rather than considering the correlations between linear combinations of instruments and targets, Professor Resek considers the correlations between linear combinations of a number of instruments and of the *actual* values of a number of endogenous variables, presumably the objects of policymaking.

Furthermore, contrary to the proposition put forward in the theoretical development, the actual calculations are always conducted, for the four major Western European countries, with a number of instrumental variables that is always smaller than the number of variables assumed to be the target of policy. It is also to be noticed that the canonical correlation is always conducted among contemporaneous variables, without allowing for *dynamic* effects which, however, should be quite naturally included through the presence of predetermined variables among the x variables in a system such as (5) (system (9) in Professor Resek's paper). I think that, even if Professor Resek's objective is only that of *describing* the policy decisions actually taken, and not that of examining possible optimal policies, one should be extremely careful in considering his empirical estimates as estimates of the actual policy process. I would prefer to interpret these results as exploratory data analysis by means of which a number of significant relationships can be identified. I am not clear, however, whether the pairs of canonical variables estimated by Professor Resek are really a description of actual policy rules or somewhat incomplete estimates of structural equations or even of pseudo-reduced forms relating endogenous variables to other endogenous variables.

Consider, indeed, the actual figures of Table 1. One can see that there is a strict positive relationship in Germany between the loan rate and the rate of inflation (a similar result also occurs for Italy, see Table 2). What does this mean? It might simply describe the correlation which one could expect out of any variant of a Fisher effect. It is very difficult instead to interpret it as a direct estimate of a policymaker's decision rule. One might think, only to exemplify, that it is some measure of money supply which is directly controlled to counteract increases in inflation, thus producing an increase in nominal interest rates and therefore a *contemporaneous* correlation between the rate of inflation and the interest rate. But then the policy instrument would be money and not the interest rate. This would have the obvious consequence that a measure of money supply and not an interest rate should be considered as "the" monetary instrument in what Professor Resek calls the "broad view of policy" (that is set CCI of Table 1). Furthermore one should then look for a negative correlation between such an instrument (or its rate of growth) and actual (possibly lagged) values of the rate of inflation. Also, one should not have *both* money and interest rates in the set of monetary instruments CCII. Indeed if money is the instrument, one should consider interest rates as endogenous variables freely moving to equilibrate the financial markets. As a further point, I think that if the monetary instrument is the money base, it might be very difficult to interpret in a sensible way the single coefficient estimates in a set of instruments which includes the government deficit (as in CCI of Table 2 for Italy). Finally, I don't think that there is much hope of discriminating among the *months* in which a given policy instrument is most important, within a quarter, on the basis of its correlation with a set of values of endogenous variables observed in the same quarter. One would probably benefit, instead, from allowing the lagged values of the instruments to be present among the set of the right hand side variables of (5), and trying to specify the target values (rather than considering the actual ones) with respect to which the correlation of a set of instruments should be calculated.

To close these critical comments, I wish to point out that I find Professor Resek's attempt to use a different technique from standard regression analysis to investigate such a difficult field as the setting of policy a courageous one. Even if I don't share his faith in the power of canonical correlation analysis, I think that a contribution will be made to the investigation of the mixture of policy instruments when improvements in the treatment of the dynamics of policy rules and in the specification of target values are obtained.

References

Brainard, W.C. "Uncertainty and the Theory of Policy," *American Economic Review,* May 1967.

Chetty, V.K. "Econometrics of Joint Production: A Comment," *Econometrica,* October 1969.

Dhrymes, P.J. *Econometrics*. New York: Harper and Row, 1970.

——— and B.M. Mitchell. "Estimation of Joint Production Functions," *Econometrica,* October 1969.

Henderson, D.W. and S.J. Turnovsky. "Optimal Macroeconomic Policy Adjustment under Conditions of Risk," *Journal of Economic Theory,* 4, 1972.

Johansen, L. "Targets and Instruments under Uncertainty," in H.C. Bos, H. Linnemann and P. de Wolff (eds.), *Economic Structure and Development — Essays in Honour of Jan Tinbergen*. Amsterdam: North Holland, 1973.

Lucas, R.E. "Econometric Policy Evaluation: A Critique," in K. Brunner and A.M. Meltzer (eds.), *The Phillips Curve and Labor Markets*. Carnegie-Rochester Series on Public Policy, vol.1, 1976.

Tinbergen, J. *On the Theory of Economic Policy*. Amsterdam: North Holland, 1952.

Vinod, H.D. "Econometrics of Joint Production," *Econometrica,* April 1968.

——— "Econometrics of Joint Production — A Reply," *Econometrica,* October 1969.